

Assessment of the structural properties of timber members in situ – a probabilistic approach

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Abstract The assessment of the structural performance of existing timber structures is dependent, among other factors, on the capacity to evaluate the physical and mechanical properties of structural timber elements in situ. This paper discusses the possibilities/advantages of using a probabilistic approach to obtain a more reliable prediction of the reference properties of these timber members in situ. The presented approach combines information from common non-destructive techniques (NDT), such as visual assessment and ultrasounds, and those from semi-destructive tests (SDT), as meso tension specimens and wood cores. An application of this approach to maritime pine (*Pinus pinaster* Ait.) and chestnut (*Castanea sativa* Mill.) timber pieces of structural dimension is presented.

Keywords bending strength, modulus of elasticity, non-destructive techniques, structures, visual assessment

1. INTRODUCTION

The assessment of the structural performance of existing timber structures is dependent, among other factors, on the capacity to evaluate in situ the physical and mechanical properties of structural timber elements. This task is significantly more difficult for timber members in comparison with other materials, in part due to the high variability of timber properties, both within and between members (heterogeneous material). Nevertheless, in some circumstances (e.g. alteration of use, detection of deterioration) the structural performance of existing structures has to be addressed.

Generally the way of dealing with the uncertainty involved while assessing the mechanical properties is based in the same principles used for selecting and applying timber in new constructions. In Europe the design of new timber structures is based on the concept of limit states (ultimate and serviceability limit states) and follows the rules described in the European standard EN 1995-1-1, also known as Eurocode 5. In this case, the properties of a particular batch of timber are controlled by selecting a particular visual strength grade or a strength class, and the mechanical properties of this grade or class are used for deriving the design values, to be used in the design of the construction.

When assessing existing structures, a frequently adopted procedure is to use a visual strength grading standard (specific to the employed wood species) to assign the material properties to timber members

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in service. This method allows to group sawn timber into more homogeneous batches (visual grades) and assign, for all members, the same value for the mechanical property(ies), usually corresponding to the 5th percentile of its probability distribution (parametric or non-parametric). This approach will be discussed in more detail in section 2.

Eq. 1 gives a general representation of the way Eurocode 5 defines the design value of material property (strength or stiffness). This design value is function of the characteristic value of a strength property, the mean value of modulus of elasticity or the mean value the shear modulus. The design value takes also into account the effect of moisture content and duration of load in wood strength properties (k_{mod} , k_{def}). Finally, a safety factor for the material property (γ_M) is applied, to account for model uncertainties, dimensional variations, the possibility of an unfavourable deviation from the characteristic value; and the random part of k_{mod} and k_{def} .

$$X_d = \frac{k_1}{\gamma_M} \cdot X_k \cdot k_h \quad (1)$$

where X_d = design value of a material property, k_1 = modification factor taking into account the effect of the duration of load and moisture content, $k_1 = k_{\text{mod}}$ for strength or $k_1 = 1/(1+k_{\text{def}})$ for stiffness properties, γ_M = material safety factor, X_k = characteristic or mean value of timber strength or stiffness property, respectively, k_h = size factor.

Other member and system modification factors are also considered in Eurocode 5, but they will not be referred to in the present paper.

In the case of timber member in situ, other factors affecting material properties should also be taken into consideration. A conservation factor linked to the degradation by fungi or insects with implication in the reduction of the material properties (k_{con}) and an aging factor that attends to physical and mechanical deterioration of the timber materials due to time in service (k_a). Thus Eq. 1 becomes Eq. 2.

$$X_d = \frac{k_1}{\gamma_M} \cdot X_k \cdot k_h \cdot k_{\text{con}} \cdot k_a \quad (2)$$

where k_{con} = reduction of wood properties due to conservation status, k_a = reduction of wood properties due to time in service (aging factor).

The Eurocode 5 factors in Eq. 2, are generally taken as constants, dependent on the worst situation to which the members are exposed while in service. The other two factors (k_{con} and k_a) are more difficult to define, because the way in which the degradation of the wood material by fungi or insects leads to the deterioration of the mechanical performance of structural elements is left to the judgment of the expert evaluating the structure. As regards aging, a long discussion exists about the need to consider this factor for structural members (gross cross-section). From the experience with white pine timber, Suter (1982) suggested a value of 0.9 for k_a .

All the discussion on adjustment or safety factors is relevant if the major source of uncertainty linked to the true timber's strength or stiffness distribution can be predicted with some accuracy. In most situations, no prior information exists about the mechanical properties of the wooden material in service. The lack of information alongside with the uncertainty associated to timber mechanical properties leads to the adoption of over conservative options.

The present paper aims to discuss the possibilities/advantages of using an alternative probabilistic approach to obtain a more reliable assessment of the mechanical behaviour of timber members in situ. This approach uses current information taken from visual strength grades and crosses that information with the one obtained by using non and semi destructive methods. The possible results obtained from

the application of the different approaches are discuss taking into account their application to maritime pine (*Pinus pinaster* Ait.) and chestnut (*Castanea sativa* Mill.) timber pieces of structural dimension.

2. CURRENT APPROACH

Current in situ assessment of the mechanical properties of timber members is carried out following the procedure outlined in Fig. 1. A similar procedure is presented by the Italian standard UNI 11119. This standard considers the necessity of a factor similar to k_{con} , Eq.2, which should be defined by the expert while conducting the survey. The standard also makes reference to a possible use of non destructive techniques (NDT) but it does not state the available NDT and in what way they could assist in the definition of the mechanical properties of the timber members.

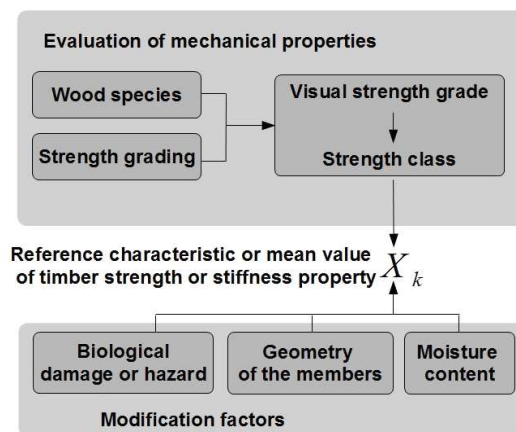


Figure 1 – Current factors used to define the mechanical performance of timber elements in situ.

The allocation of mechanical properties to a timber element in situ is currently made by applying a visual strength grading standard or by the simple use of the strength class system (standard EN 338). This approach relies on the concept of *reference material properties* and *other material properties*. *Reference properties* include density, bending strength and modulus of elasticity. These properties are the ones that are experimentally determined while developing a visual strength grading system according with the European standardization. *Other properties* are obtained from the *reference properties* using a series of expressions described in the standard EN 384. These same two groups of properties are used by the probabilistic model code proposed by the Joint Committee for Structural Safety (JCSS 2006), Fig. 2.

Reference Material Properties	Other Material Properties
R_m = bending strength	$R_{t,0.90}$ = tension strength
E_m = bending MOE	$E_{t,0.90}$ = tension MOE
ρ_{den} = density	$R_{c,0.90}$ = compression strength
	$E_{c,0.90}$ = compression MOE
	R_v = shear strength
	G_v = shear modulus
	$R_{i,0.90}$ = embedding strength

Figure 2 – Reference material properties and other material properties (JCSS 2006).

Since the full application of visual strength grading rules is difficult (if not impossible) for a timber element in situ (Bonamini 1995), the prediction of mechanical properties is often conservative. This procedure requires the identification of the wood species, probable origin and quality, which allows the choice of a proper strength class from the European standard EN 1912.

Following this procedure, uncertainty on the material properties is dealt by using the 5th percentile values and following application of a safety factor (χ_M). This procedure is simple and conservative. It

can comprise: a) allocation of a safe (lower) strength class (SC) according with the employed wood species; b) allocation of the strength class that represents the average quality (visual strength grade) of all the timber elements examined or; c) the allocation to each timber member of a particular strength class that corresponds to the features present in that particular piece of timber (again, through visual strength grading).

The plain use of visual strength grading principles does not take into consideration major differences between visual strength grading of sawn timber and timber elements in situ. The former are carried out without knowing the exact way in which the beam will be used and the location of defects in relation with the stresses distribution along the length and cross-section of the elements. In the case of timber elements in situ, this information is known and can be used to restrain some of the variability normally associated to a visual strength grading, Fig. 3.

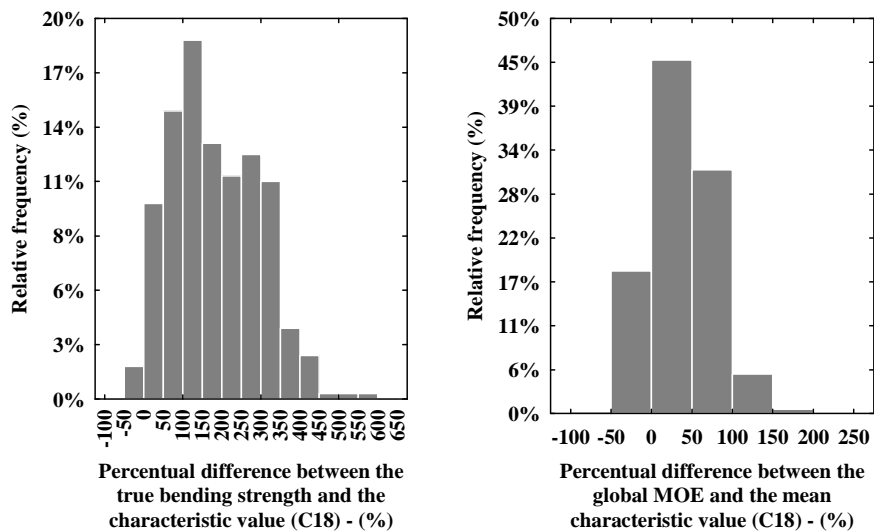


Figure 3 – Percentage difference between true strength and modulus of elasticity in bending and the corresponding characteristic values (*reference properties*) for visual strength grade E (NP 4305) applied to maritime pine.

From Fig. 3 it can be seen that using visual strength grading to predict strength and modulus of elasticity underestimates the true values as much as 600 % or 200 %, respectively. In this respect, Yeomans (1999) mentions some of the problems that the straightforward application of current visual strength grading standards can bring to the conservation of historic buildings.

3. A PROBABILISTIC APPROACH

The probabilistic modelling of uncertainties together with the definition of acceptable failure probabilities defines the framework for the reliability based assessment of structures (JCSS 2006). In this code, probabilistic models for *reference properties* are provided along with the expected uncertainty.

Table 1 – Probabilistic models for *reference properties* for structural timber (JCSS 2006)

	Distribution	COV
Bending strength	Lognormal	0.25
Bending MOE	Lognormal	0.13
Density	Lognormal	0.10

A complete application of this code requires the capability to predict the expected values for the *reference properties*. These properties are generally chosen in accordance with the visual strength grade (based on the concepts mentioned in the previous section) or from tests conducted using similar material (Toratti et al 2007).

Two alternative probabilistic models for assessing the reference properties can however be used. One approach uses information about the original quality of the timber element (before being exposed to load, humidity and temperature history). This *a priori* information can be obtained from the simple consideration of the most likely visual strength grade to be allocated to a structural member. Indirect or direct tests made at the structural elements or at the structure (proof-loading) can lead to *a posteriori* information about the quality of the timber members. This last information will be used to update the *a priori* information (probabilistic Bayesian approach), as discussed in section 3.2).

The other approach does not take into consideration any *a priori* information about the quality of the timber elements, but only the information from direct or indirect measurements made during the appraisal of the structural elements (classical inference), as presented in section 3.1.

3.1. Classical inference

Classical inference characterizes a random variable X solely by using the data obtained from an experiment $X = (x_1, x_2, \dots, x_n)$. This data is drawn from a population with a distribution of probability $f_x \cap p(X|\Theta, \Omega)$ defined by a space of unknown parameters Θ and the space Ω of all possible values of X . No previous assumption or information about Ω is made.

Prediction on wood species, density, moisture content and mechanical properties of timber structural elements can be obtained from intrusive (semi-destructive) as well as non intrusive (non-destructive) techniques. Other factors that can affect the mechanical properties can also be reported (deterioration and aging effects).

In the case of timber structures the tests currently applied and the quality of the information obtained regarding the *reference properties* are shown in Table 2. This table clearly shows a high variability regarding the efficiency (r value) and reliability (interval of r values found) for each method.

Table 2 – Correlation matrix (r) for non and semi-destructive tests for assessing the reference properties of timber structural members in-situ (values taken from various bibliographic sources)

		Pylodin	Drill resistance	Ultrasounds	Testing of small specimens
Reference properties	Density	0.02 - 0.89	0.06 - 0.88	-	-
	Modulus of elasticity	-	-	0.30 - 0.74	0.30 - 0.80
	Bending strength	-	0.86 - 0.93	0.32 - 0.80	-

The integration of the data obtained from these NDT is clearly a matter of discussion. The results shown in Table 2 can be explained by the difference between wood species, the elements' conservation level, the elements' size, and the different procedures followed by different authors. Therefore, the extrapolation of results to other cases is difficult and justifies the efforts currently ongoing in several committees to harmonize the use of NDT in the assessment of structural timber elements in situ.

An example of the combination of several techniques was proposed by Machado and Palma (2010b). The studies already conducted (Machado and Palma 2010a, Machado and Palma 2010b) show clearly that a good prediction of density and modulus of elasticity can be achieved by core drilling, ultrasounds and tension (meso-specimens) tests. Bending strength is however more difficult to predict. This aspect is related with the fact that the ultimate strength is a function of local weak points,

generally not easily detected by visual inspection and dependent on the stress level and distribution on the structural element.

In the present study, semi and non-destructive techniques were used to infer the location parameter (in this case the mean value) of a probability density function (pdf). The choice of the pdf for each reference property was made according to JCSS (2006). The uncertainties (scale parameter) in the predictions can be considered equal to the variability found for the wood's properties within and between timber members. Table 3 shows the coefficients of variability pointed out by two bibliographic references and the data obtained in the present study.

Table 3 – Coefficients of variation (%) for the *reference properties*

	Wood Handbook (clear wood)	JCSS*	Present study (clear wood)			
			Within the element		Between elements	
			Maritime pine	Chestnut	Maritime pine	Chestnut
Density	10	10	6	5	13	8
Reference properties Modulus of elasticity in bending	22	13	10	12	26	21
Bending strength	16	25	9	14	17	22

* Data related with structural timber (pieces presenting defects affecting clear wood's behaviour) – Forest Products Laboratory (2010).

3.2. Bayesian inference

Bayesian inference assumes that *a priori* information about the material exists. Let Y be a random variable representing the space of possible values for a certain property of the material and f_Y the probability density function that represents the distribution of the space of possible values of Y . The pdf f_Y is characterized by a vector of parameters $\theta - g_\theta$. If a new data set is collected, $\hat{Y} = \{y_1, y_2, \dots, y_n\}$, a Bayesian updating of Θ distribution can be obtained, Eq. 3.

$$g_\theta''(\theta|\hat{Y}) = \frac{g_\theta'(\theta) \cdot L(\theta|\hat{Y})}{\int_{-\infty}^{+\infty} g_\theta'(\theta) \cdot L(\theta|\hat{Y}) dq} \quad (3)$$

where $g_\theta(\theta)$ represents the pdf of the uncertain parameter Θ , $L(\theta|\hat{Y})$ is the likelihood function of the results contained in \hat{Y} , " means the *a posteriori* and ' the *a priori* pdf of Θ .

In the majority of timber structures in Portugal, information about the principles used to select the timber elements is not available. Therefore, simple facts that are known to affect the mechanical behaviour of a timber member (origin of the material and growth conditions) are often impossible to determine during an inspection.

Therefore the *a priori* information about the quality of the wooden material can only be provided by the visual strength grading of the elements, which just requires information on the wood species and the elements' features. Some intrusive (semi-destructive) and non intrusive (non-destructive) techniques can be used to obtain additional information to predict the mechanical behaviour of the elements. This information can then be used to update the information provided by the visual grading. The application of these concepts is presented in section 4.

4. CASE STUDY

4.1. Experimental work

4.1.1. Materials

Home-grown maritime pine (*Pinus pinaster* Ait.) and chestnut specimens (*Castanea sativa* Mill.) were selected for this study. These specimens were visually graded according with visual strength grading standards developed for each of these two wood species. For maritime pine timber it was applied the Portuguese standard NP 4305, and for chestnut wood the Italian standard UNI 11035-2. This last standard was chosen since no Portuguese standard exists for the visual grading of home-grown chestnut for structural applications.

From each species, thirty beams with dimensions $40 \times 100 \times 2000 \text{ mm}^3$ were selected. The beams were conditioned and tested at the standard environment defined in the European Standard EN 408 ($20 \pm 2 \text{ }^\circ\text{C}$ air temperature and $65 \pm 5 \%$ relative humidity). The testing of the beams was conducted after they showed a mass variation below 0.1 % between weight measurements taken each two hours.

4.1.2. Methods

The sequence of tests performed in each beam was the following:

- It was visually strength graded according with one of two standards (NP 4305 for maritime pine and UNI 11035-2 for chestnut);
- Five ultrasonic time-of-flight readings were obtained from different clear wood zones. A PUNDITplus was used with 150 kHz transducers and mineral gel as coupling agent. An indirect method was used (transducers were located in same surface at a distance of 40 cm). More details about the ultrasonic method can be found in Machado and Palma (2010b);
- A four-point bending test was conducted according to EN 408. A proof-loading test was applied to obtain the global modulus of elasticity ($E_{m,g}$). The maximum proof-load applied was determined according with Eq. 4;

$$F_p = \frac{0.96 \cdot t \cdot h \cdot k_h \cdot f_{m,k}}{1.8} \quad (4)$$

where F_p corresponds to the applied proof-load, t to the thickness of the beam, h is the depth of the beam, k_h is the size factors given in EN 384; and $f_{m,k}$ is the characteristic value of bending strength associated to each visual strength grade for a 150 mm depth.

- Two tension meso-specimens (Brites et al 2010) were taken for determination of the modulus of elasticity ($E_{0,t,m}$).
- Two wood cores were collected for density prediction (ρ_c).

The mean value of the dynamic modulus of elasticity given by ultrasounds was obtained considering the combination between the density values (ρ_c) obtained from the two core drills and the five ultrasonic velocities (V) determined from the time-of-flight readings, Eq. 5.

$$E_{dyn} = \frac{\sum_{i=1}^5 V_i^2 \cdot \left(\sum_{j=1}^2 \rho_{c,j} \right)}{10} \quad (5)$$

For evaluating the prediction capacity of the wood cores, a regression analysis was conducted. The mean density value given by the two wood cores was compared with the values obtain from standard prismatic specimens taken from the beams. The obtained results show that wood cores can provide, as expected, a good prediction of the density of the whole element, Fig. 4.

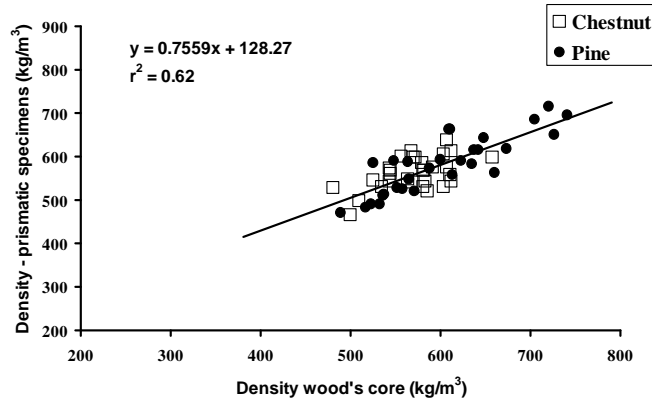


Figure 4 – Correlation between density values obtained from wood cores and current EN 408 method.

The bending modulus of elasticity for each beam was predicted by a cross-validation process. This process consisted in the combination of the information from the non destructive technique (ultrasounds – E_{dyn}) and a semi destructive technique (tension test mesospecimens – $E_{0,t,m}$).

The combination of these two predictions was carried out using the parametric method for combining estimators, initially proposed by Cochran (1937). Considering that there are two estimators of E (\hat{E}_1 and \hat{E}_2) with pdf defined as $N(E_1, \sigma_1^2)$ and $N(E_2, \sigma_2^2)$, respectively, the combined unbiased estimator is given by Eq. 6.

$$E = \omega_1 \cdot E_1 + \omega_2 \cdot E_2 \tag{6}$$

with

$$\omega_1 = \sigma_2^{-2} / (\sigma_1^{-2} + \sigma_2^{-2}) \text{ and } \omega_2 = \sigma_1^{-2} / (\sigma_1^{-2} + \sigma_2^{-2})$$

As stated before, it was considered that the location parameter (mean value) is a random variable and that the scale variable is know (standard deviation). In this case, a coefficient of variation of 13 % (JCSS 2006) was considered (see Table 3) for tension modulus, and of 10 % for the dynamic modulus (value taken from the results obtained in this study).

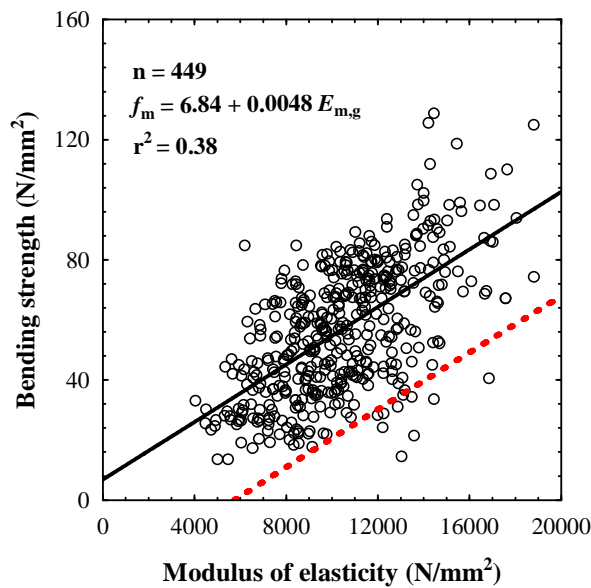


Figure 5 – Regression curve and 95% lower prediction confidence limit curve adjusted to the relation f_m and $E_{m,g}$ for maritime pine beams.

The bending strength was predicted using the same procedure used for machine strength grading. For maritime pine, the lower 95% confidence limit of the regression curve between the global modulus of elasticity and the bending strength was used, Fig 5. For chestnut, the prediction of the bending strength was not performed since there is no information about the previous correlation, and the correspondent regression curve, between the modulus of elasticity and the bending strength for home-grown chestnut timber.

As for the Bayesian inference, the initial data (*a priori* information) was provided by the assignment of a particular strength class (Table 4) according to the visual grade of the timber element.

Table 4 – Allocation of strength classes to the visual strength grades considered for maritime pine and chestnut

Wood species	Maritime pine			Chestnut		
Visual strength grade	NP 4305			UNI 11035-2		
	EE	E	R	S	R	
Strength class	C30	C18	C14	D28	D14	
Reference properties	f_m [N/mm ²]	30	18	14	28	14
	$E_{m,g}$ [N/mm ²]	12000	9000	7000	9000	7000
	ρ_{mean} [kg/m ³]	460	390	350	550	350

R – Rejected

* – Strength classes considering the values of the grade S of UNI 11035-2 and those mentioned in EN 338. For chestnut, the R grade was considered equal (same values) as for C14 for Softwoods and designated as D14.

For both the modulus of elasticity and the bending strength, the same prior and posterior probability distribution function (Table 1) was considered. Thus, the lognormal distributions $f_m \cap LN(Z_f, A_f)$ and $E_{m,g} \cap LN(Z_E, \Lambda_E)$ were assigned to the bending strength (f_m) and to the global modulus of elasticity ($E_{m,g}$) variables, correspondingly.

When the precision is known, but the mean is a random variable, the natural conjugate of the kernel of the likelihood function is the Normal distribution. The precision was considered as 13 %, as indicated by the JSCC (2006).

The parameters associated to the prior distribution (m' , n') and the information collected from the tests made using non and semi destructive techniques (m , n) can be combined as indicated in Eq. 7 (Raiffa and Schlaifer 1961). In this paper, an equal weight for the *a priori* and the *a posteriori* information ($n' = n$) was considered, regardless of the number of observation supporting either one.

$$m'' = \frac{n' \cdot m' + n \cdot m}{n' + n} \quad (7)$$

where: m' is the mean value assumed for the *a priori* distribution, n' is the sample size assumed for the *a priori* distribution, m is the mean estimated from the test results and n is the sample size that supports the test results.

4.2. Results and discussion

The comparison of the predicted values of modulus of elasticity, using both the classical and Bayesian approach, with the experimental values is presented in Fig. 6. A normal distribution function was fitted to the errors obtained with each approach. In the same figure it is also plotted the error distribution considering solely the visual grading system applied.

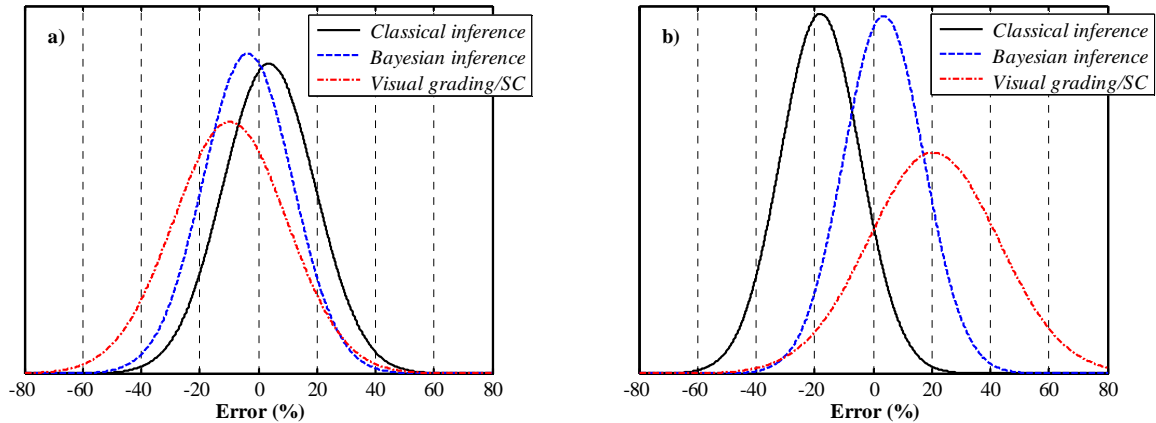


Figure 6 – Error distribution of the different approaches, in relation to the experimentally determined global modulus of elasticity in bending: a) maritime pine; b) chestnut

Fig. 6 shows that Bayesian inference, using information from current approach (visual grading → Strength Class) and from semi and non destructive methods, provided for both wood species the less biased result.

Fig. 6 also shows the differences that can be found between the two species. In the case of maritime pine, an exhaustive mechanical characterization was conducted between 1989 and 1991, leading to the establishment of a visual strength grading standard. These studies can explain that the application of the current approach delivered, as expected, significant conservative values. The two proposed approaches resulted in a shift of the curves to the right and a more unbiased prediction of the modulus of elasticity.

In the case of home-grown chestnut, the mechanical characterization of structural elements was never carried out and, therefore, a specific strength grading standard (considering the geographic origin) does not exist. The application of the Italian visual grading standard led to a significant over estimation of the global modulus of elasticity. Nevertheless, there is again a clear contribution of the Bayesian inference for a more unbiased prediction of the modulus of elasticity.

These results shows the importance that the information provided by semi or non-destructive testing can bring to improve the assessment of timber members in situ, when the geographic origin of the wooden material is not known. It should be stressed that this uncertainty about the origin of timber elements in situ is common for many timber structures.

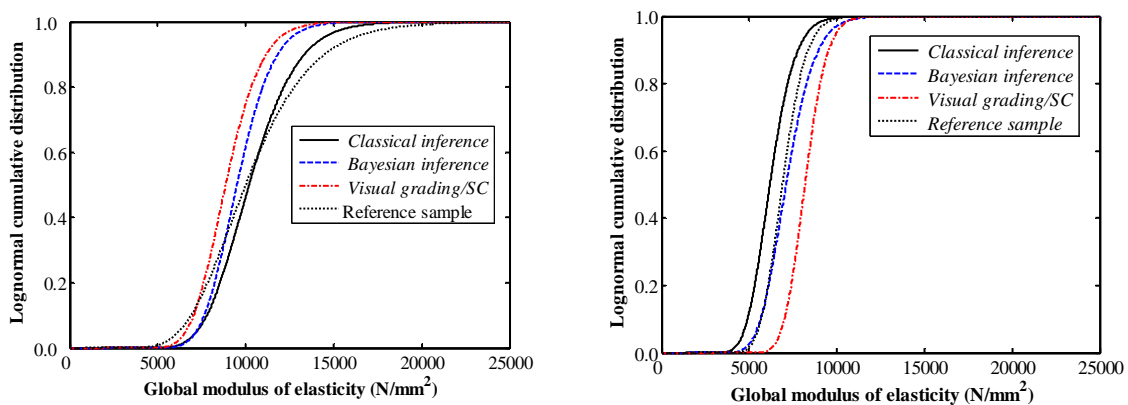


Figure 7 – Global modulus of elasticity cdfs for the different approaches and the cdf of a lognormal distribution fitted to a sample of thirty specimens (reference sample). a) maritime pine, b) chestnut

Fig. 7 shows the lognormal cumulative distribution (cdf) fitted to the results obtained by the different approaches and to the reference sample. These results showed a clear improvement of the prediction of the real cdf of the reference sample by the use of the non and semi-destructive methods. The use of information based on these tests seems to support therefore a more reliable structural analysis of the structural behaviour of timber elements in situ.

As for the bending strength, the results are only shown in the basis of a predicted cumulative distribution function. It was not possible to determine the error, since the specimens were not tested until failure. However, the results (Fig. 8) showed that the two approaches seem to restrain some of the understandable large variability associated with the simple use of visual grades and strength classes.

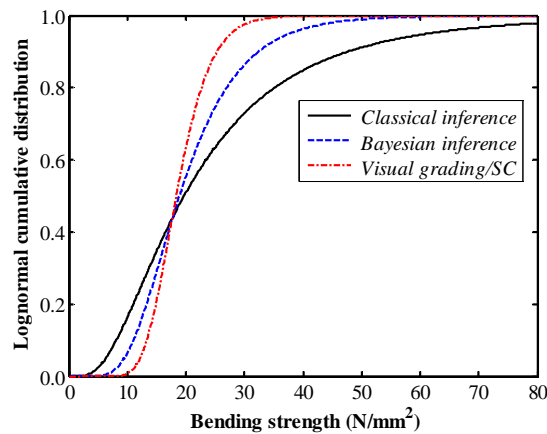


Figure 8 – Lognormal cdf according with the different approaches followed for maritime pine

The information provided by error probability density distributions of the type shown in Fig. 6 can be used in probability modelling of the behaviour of structural timber, by adding an error component (ε) to each prediction, Eq. 8, using a Monte Carlo or other similar method.

$$E_m = \widehat{E}_{m,g} + \varepsilon \quad (7)$$

5. CONCLUSIONS

The application of new approaches to the evaluation of the *reference properties* of timber structural members shows that a clear benefit can be attained by combining information from visual strength grading with information gathered from semi and non-destructive techniques.

In the case of the modulus of elasticity, the application of the two studied approaches resulted in a less biased estimator (mean error closer to zero) than the simple use of visual grades or strength classes.

Regarding bending strength, the proposed approaches need to be validated by tests conducted until failure. However, the results obtained with the application of the two approaches are promising, given the similarity with the results obtained for the modulus of elasticity and the correlation between the bending strength and the bending modulus of elasticity.

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