FLEXURAL BEHAVIOR OF STEEL FIBER REINFORCED CONCRETE:
TESTING AND MODELING

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In this paper the results of tests performed on specimens and structural elements made of steel fiber reinforced concrete are presented. Fiber content ranged from 0 to 60 kg/m^3 of concrete. Using the results of the uniaxial compression tests performed under displacement control condition, a stress-strain relationship for fiber concrete in compression was derived. Three-point bending tests on notched beams were carried out in order to simulate the post-cracking behavior and to evaluate the fracture energy. Based on the constitutive relationships derived from the experiments, a layered model for the analysis of steel fiber reinforced concrete cross sections was developed. The model performance and the benefits of fiber reinforcement on thin slabs reinforced with steel bars were assessed by carrying out tests on slab strips. The main results are presented and discussed.

Key words: structural concrete, fiber reinforcement, fracture energy, constitutive relations, experimental tests, flexural model.

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INTRODUCTION

Steel fiber reinforced concrete (SFRC) is a cement based material reinforced with short steel fibers. When steel fibers are added to a concrete mix, they are randomly distributed and act as crack arrestors. Debonding and pulling out of fibers require more energy, giving a substantial increase in toughness and resistance to cyclic and dynamic loads ("ACI 544" 1982). SFRC has been used for a wide variety of applications, namely, pavements and overlays, industrial floors, hydraulic and marine structures, repairing and rehabilitation works ("ACI 544" 1982, Balaguru and Shah 1992). However, there is still a lack of information on the modeling of SFRC structures. The present work aims to contribute to the on going research effort in the field of these composites. For this purpose, a research program has been carried out combining experimental investigation and numerical modeling.

The results obtained with the uniaxial compression tests with fiber reinforced concrete have revealed a slight increase in the compression strength, stiffness and strain at peak load, and a substantial increase in the post-peak energy absorption capacity (Fanella and Naaman 1985, Otter and Naaman 1986, Ezeldin and Balaguru 1993, Mansur et al. 1997). The magnitude of these changes depends on the matrix properties and on the fiber type and content. Several stress-strain relationships are available for plain concrete (CEB-FIP 1993, Wee et al. 1996), but only few equations are published for SFRC (Fanella and Naaman 1985, Ezeldin and Balaguru 1993). Based on the experimental results obtained, and following the procedures proposed by Mebarkia and Vipulanandan (1992), a compression stress-strain relationship was put forward.

The most significant improvement imparted by adding fibers to a concrete mix is the substantial increase in the energy absorption capacity. To measure this property, some procedures have been proposed. "ASTM C1018" (1991) proposes the evaluation of the toughness indices and "JSCE-SF4" (1984) recommends the determination of the flexural toughness factor. The practical use of these toughness parameters have been questioned, since they are not material properties (Trottier and Banthia 1994). The toughness parameters of "ASTM C1018" and "JSCE-SF4" are not adjusted to define the tensile strain-softening law (Hordijk 1991), which simulates the behavior of the damaged region ahead of a continuous crack, the so-called fracture process zone. To define the softening behavior, Hillerborg et al. (1976) have introduced the concept of fracture energy, G_f. The standard test for the evaluation of this property was established by RILEM (1985). In this work, three-point bending tests on notched beams were performed in order to evaluate G_f. Based on the results obtained, expressions are proposed to predict G_f for the *SFRC* tested.

The majority of the methods for analyzing fiber reinforced concrete presents difficulties in predicting the flexural behavior of bidimensional SFRC structures because the law used to simulate the post-cracking behavior is not derived from the material fracture parameters (Henager and Doherty 1976, Swamy and Al-Ta'an 1981, Jindal 1982, Craig 1987). In the present work a model for designing cross sections of SFRC structural elements was developed based on the compression and tensile constitutive laws derived from experimental tests. The cross section is discretized in layers of plain or fiber reinforced concrete, to which, reinforcing layers of conventional steel bars can be added. The post cracking tensile behavior of the concrete layers not influenced by the reinforcing bars is

simulated by a bilinear tension softening diagram, based on the concrete fracture properties (Bazant and Oh 1983). The post cracking tensile behavior of concrete layers near the conventional reinforcement is modeled by a multilinear (tension stiffening) diagram that takes into account the concrete fracture properties and the reinforcement characteristics (Massicote et al. 1990, Barros 1995a). The model performance was assessed by using available experimental data and the bending tests carried out in thin SFRC slabs reinforced with a wire mesh. The results given by the model were also compared with the results obtained from a finite element analysis.

MATERIALS AND MIXTURES

Table 1 presents the composition of the mixtures used in the experimental program. Five series were manufactured. The weight of the mixtures was about 2400 kg/m^3. The fiber percentage, fiber aspect-ratio (length to diameter ratio) and the water-cement ratio (*w/c*) were the main variables which were changed in order to evaluate their influence on the behavior of the SFRC composites analyzed. The fracture energy and the post-cracking behavior were evaluated from bending tests performed on specimens of these five series. The compression behavior was determined using specimens of series *s3* and *s4*. The mixture specified in Table 1 for series *s2* was used to cast the slab strips.

An ordinary portland cement (*OPC*) was used in all mixes. Hooked-end steel fibers of trademark Dramix *ZP30/.50* and *ZX60/.80* have been utilized (Bekaert Specification 1991). The *ZP30/.50* fibers are 30 mm in length, 0.5 mm diameter and have a strength of about 1250 MPa, while *ZX60/.80* fibers are 60 mm in length, 0.8 mm diameter and have a strength of about 1100 MPa.

The water, cement, aggregates and sand were first mixed for two minutes. The fibers were then slowly added. The mixing time for approximately 0.10 m^3 of *SFRC* ($\approx$ 50% of the electrical portable rotary mixer capacity) was about 3 minutes. A controlled internal vibration was used for the compactation of the cylinder and prismatic specimens, while for the slab strips an external vibration was applied to the forms. Until demoulding, (at approximately 7 days) the specimens and the slab strips were covered with wet cloths. After demoulding the specimens were kept at 65% RH and 20ºC until the date of the test.

COMPRESSION BEHAVIOR

The main objective of the uniaxial compression tests performed with SFRC cylinder specimens was to define a stress-strain law ($\sigma_c - \varepsilon_c$) to simulate the complete compression behavior of the composites analyzed. Cylinder specimens 150 mm in diameter and 300 mm in height were tested under displacement control condition. A MTS closed-loop, servocontrolled compressive testing machine having a capacity of 2700 kN was used. A loading rate ranging from 10 µm/s to 30 µm/s was applied following the recommendations of "JSCE-SF5" (1984).

Previous work (Fanella and Naaman 1985, Ezeldin and Balaguru 1993, Barros 1995b) has shown that the complete stress-strain expressions proposed for plain concrete (CEB-FIP 1993, Wee et al. 1996) can not fit the post-peak response of the fiber concrete. Fanella and Naaman (1985) proposed an expression to predict the complete $\sigma_c - \varepsilon_c$ relationship for fiber-reinforced mortar. Using the experimental data and taking into

account the fiber geometry, volume fraction and fiber shape, eight parameters were evaluated, four of them to characterize the $\sigma_c - \varepsilon_c$ ascending branch and the other four parameters to define the descending branch. Like the expression of Ezeldin and Balaguru (1993), the one proposed in the present work is based on one parameter only. The expression is based on the following stress-strain relationship:

$$\sigma_c = f_{cm} \frac{\dfrac{\varepsilon_c}{\varepsilon_{c1}}}{(1-p-q) + q\left(\dfrac{\varepsilon_c}{\varepsilon_{c1}}\right) + p\left(\dfrac{\varepsilon_c}{\varepsilon_{c1}}\right)^{\frac{1-q}{p}}} \tag{1}$$

with

$$q = 1 - p - \frac{E_{c1}}{E_{ci}}, \quad p + q \in \left]0,1\right[, \quad \frac{1-q}{p} > 0 \tag{2}$$

which was proposed by Vipulanandan and Paul (1990) for polymer and plain concrete, and used by Mebarkia and Vipulanandan (1992) for glass-fiber-reinforced polymer concrete. The strain at peak stress, ε_{c1}, the average compression strength, f_{cm}, and the ratio between the secant modulus of elasticity and the tangent modulus of elasticity, E_{c1}/E_{ci}, ($E_{c1} = f_{cm}/\varepsilon_{c1}$) for each type of fibers are expressed in function of the fiber percentage. The parameter p, which takes a value in the range 0 to 1, is obtained by minimizing the following expression:

$$e^2 = \sum_{i=1}^{n} \left(\frac{\sigma_{ci}^r - \sigma_{ci}^c}{f_{cm}}\right)^2, \tag{3}$$

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where σ_{ci}^r and σ_{ci}^c are the stresses obtained in the experiments and calculated from expression (1), respectively, and n is the number of scan readings in a test.

Each series of tests is composed by, at least, four specimens. The average compression strength values are included in Table 2. Applying the method of least squares to the results of series $s3$ and $s4$ (see Table 1), the following expressions were obtained:

$$\varepsilon_{c1} = \varepsilon_{c1o} + 0.0002 W_f , \tag{4}$$

$$p = 1.0 - 0.919 \exp(-0.394 W_f) \tag{5}$$

for $ZP30/.50$ fibers and

$$\varepsilon_{c1} = \varepsilon_{c10} + 0.00026 W_f , \tag{6}$$

$$p = 1.0 - 0.722 \exp(-0.144 W_f) \tag{7}$$

for $ZX60/.80$ fibers where ε_{c10} is the strain at peak for plain concrete (2.2×10^{-3} according to CEB-FIP Model Code 1990) and W_f is the fiber weight percentage in the mixture. The expressions proposed for ε_{c1} gives the tendency observed in the experiments carried out by several researchers (Fanella and Naaman 1985, Otter and Naaman 1986, Ezeldin and Balaguru 1993). In Fig. 1 the $\sigma_c - \varepsilon_c$ experimental results (series $s4$) are compared with the $\sigma_c - \varepsilon_c$ curve obtained analitically.

In practice, the uniaxial compression strength, f_{cm}, is usually the only material parameter experimentally evaluated. For a given f_{cm}, the corresponding E_{ci} can be obtained from the CEB-FIP Model Code 1990 (1993) recommendations, $E_{ci} = 21500 \left[f_{cm}/10 \right]^{1/3}$ (MPa), since this property is only marginally changed by the fiber reinforcement. For f_{cm} values ranging from 30 to 60 MPa and for concrete reinforced with a percentage of fibers similar to those used in the present work, the values of ε_{c1} and p can be obtained from the expressions proposed. As $E_{c1} = f_{cm}/\varepsilon_{c1}$, the parameter q can be evaluated and the $\sigma_c - \varepsilon_c$ expression defined.

The ability of the proposed law to predict the complete stress-strain curve of concrete reinforced with hooked-end steel fibers, given by other authors, is illustrated in Fig. 2. The experimental results obtained by Ezeldin and Balaguru (1993) and Mansur et al. (1997) are compared with the analytical curve given by the present model.

TENSILE BEHAVIOR

The tensile behavior of SFRC under bending was assessed by performing three-point bending tests on notched beams of 600x150x150 mm³ with 450 mm of span. The tests were performed under displacement control condition. The load was applied through an actuator of 250 kN maximum capacity, with a load cell calibrated for 25 kN. In order to avoid extraneous deformations, the middle point deflection was measured by a LVDT placed on a frame attached to the beam, the so-called "Japanese yoke" ("JSCE-SF4" 1984, Gopalaratnam et al. 1991), see Fig. 3. As the ratio between compressive and tensile strength of the SFRC tested in this work was in the range 5 to10 (Hillerborg 1983) the

procedures recommended by RILEM (1985) were applied to measure the fracture energy. This property was determined (see Fig. 3) from the following expression (Petersson 1982, RILEM 1985):

$$G_f = \frac{W_0 + W_1 + W_2}{b(d-a)} = \frac{W_o}{b(d-a)} + \frac{m(1-\alpha^2)g\delta_u}{b(d-a)} = G_f^F + G_f^{pp} \tag{8}$$

where m is the mass of the beam between the supports (length, l), g is the gravity acceleration, $\alpha = L/l - 1$ (see Fig. 3), δ_u is the final deformation of the beam response and G_f^F and G_f^{pp} are the fracture energy supplied by the actuator and by the beam weight, respectively.

The notch depth/beam depth ratio (*a/d*) of the specimens of each series of tests performed are presented in Table 3. The notches were 4 mm wide.

The plain concrete specimens were tested using a deflection rate of 2 μm/s, while SFRC specimens were tested using two deflection rate regimes: 4 μm/s until 2 mm of deflection, e.g. until a stable softening state, and 15 μm/s up to the end of the test. To avoid readings outside the linear range of the external LVDT, the tests on fibrous specimens were stopped at approximately 20 mm of deflection. However, for this deflection, the fracture energy was not totally dissipated, thus, a linear branch was assumed for data between the last reading and the 25 mm deflection, for which the force is assumed to be zero. This procedure is similar to that applied by Hordijk (1991) in uniaxial tensile tests. In Fig. 4, the load-displacement diagrams obtained in some representative tests are illustrated.

The fracture energy supplied by the actuator and by the weight of the beam until the estimated ultimate deflection are represented by G_f^{Fe} and G_f^{ppe}, respectively. To estimate the variation of the fracture energy with the fiber content, the contribution of G_f^{ppe} was not taken into account, because it depends on the geometric variables, weight of the specimen and ultimate deflection, which were taken as constant values. Therefore, the dependence of the fracture energy on the fiber content was specified through a relationship between the $G_f^{Fe}\big/G_{fo}^{Fe}$ ratio and the fiber percentage in weight, W_f, for the two types of fibers used, where G_{fo}^{Fe} is the fracture energy of the plain concrete specimens due to the applied load. Applying the method of least squares to the results the following expressions were obtained:

$$\frac{G_f^{Fe}}{G_{fo}^{Fe}} = 19.953 + 3.213 W_f \tag{9a}$$

for the specimens reinforced with *ZP30/.50* fibers and

$$\frac{G_f^{Fe}}{G_{fo}^{Fe}} = \exp\left(3.408 - \frac{0.376}{W_f}\right) + 1.0, \tag{9b}$$

for the specimens reinforced with *ZX60/.80* fibers. Fig. 5 illustrates the curves corresponding to both expressions. Due to the scatter of results, expression (9) should be used with caution. The different age of the specimens at testing and the existence of mixtures with different *w/c* ratio in the analysis might have contributed to this fact.

Simplified post-cracked model

To define the tensile post-cracking behavior, the softening constitutive relationship must be derived from the fracture parameters, namely, the tensile strength, f_{ctm}, the width of the fracture-process zone, l_b, the fracture energy, G_f, and the shape of the softening diagram. For the amount of fibers used ($V_f < 1.5\%$), the concrete strength is only marginally changed, so the f_{ctm} can be obtained from CEB-FIP Model Code 1990 (1993) recommendations. The fracture process zone is a microcracked zone with some remaining ligaments for stress transfer (Hillerborg 1980). Some attempts have been made to measure the fracture-process zone in concrete (Bazant and Oh 1983, Hu and Wittmann 1990, Foote et al. 1987). However, difficulties due to the influence of test conditions and specimen geometry have been reported. In the absence of a more refined model, l_b can be taken as the average crack spacing in members under stabilized cracking conditions (CEB-FIP 1993), or approximately three times the maximum aggregate size, as proposed by Bazant and Oh (1983), for plain concrete. The fracture energy is evaluated from expressions (9).

By finite element numerical simulation of the three-point bending tests on notched beams, it was concluded (Barros and Figueiras 1995a) that the softening law can be modeled, with enough accuracy, by the bilinear diagram shown in Fig. 6. The characteristic point of the softening diagram is defined by α and p_1 parameters. The values of these parameters were evaluated from the aforementioned numerical simulation. The parameter p_2 of the softening diagram is given by the following relationship (Barros 1995b):

$$p_2 = \frac{2G_f}{\alpha\, l_b\, f_{ctm}\varepsilon_{cr}} - \frac{p_1 - \alpha}{\alpha} \quad . \tag{10}$$

FLEXURAL MODEL

A numerical model was developed for the analysis of SFRC cross sections under bending and axial forces. The complete moment-curvature relationship can be calculated by the model and used in a material non-linear analysis of SFRC structures. The cross section is discretized in a given number of concrete layers and reinforcing layers, as it is shown in Fig. 7. A linear distribution of strains throughout the cross section is assumed.

Concrete

The uniaxial compression behavior of plain concrete is simulated from the stress-strain law ($\sigma_c - \varepsilon_c$) proposed in the CEB-FIP Model Code 1990 (1993). For SFRC the expression (1) is applied. After cracking, ($\varepsilon_{cti} > \varepsilon_{cr}$) the stress in a concrete layer depends on the state of the layer, which can be under tension softening or under tension stiffening. The tension-softening phenomenon is associated with the fracture-process zone observed in plain (or fibrous) concrete members after tensile strength has been reached. The tension-stiffening phenomenon can be defined as the stiffening of a cracked reinforced concrete tie given by the interaction between concrete and reinforcement. The criterion to decide the state of a given layer was that recommended by CEB-FIP Model Code 1990 (1993). The tension softening diagram is represented in Fig. 6, while the tension stiffening diagram used in the present model is shown in Fig. 8. The definition of the characteristic

points A, B and C of the tension stiffening diagram is discussed in a previous work (Barros 1995b).

Reinforcement

In the present model the reinforcement behavior can be simulated by a multilinear diagram or by a linear-parabola $\sigma_s - \varepsilon_s$ diagram (Barros and Figueiras 1996). The linear-parabola diagram is characterized by the following expressions:

$$\sigma_s = E_s\,\varepsilon_s \quad \text{for } \varepsilon_s \leq \frac{\beta\,f_{su}}{E_s}\,, \tag{11}$$

$$\sigma_s = f_{su} + \gamma\left(\varepsilon_s - \varepsilon_{su}\right)^2 \quad \text{for } \frac{\beta\,f_{su}}{E_s} < \varepsilon_s \leq \varepsilon_{su}\,. \tag{12}$$

The stress-strain relationship for a bare reinforcing bar is different from the average stress-average strain relationship for reinforcing steel embedded in concrete (Stevens et al. 1987, Okamura and Maekawa 1991). To take into account this fact the yield envelope of the bare reinforcing bar is reduced according to the expression proposed by Stevens et al. (1987):

$$\Delta f_{su} = f_{su} - \frac{C_t}{\phi_s}\,f_{ctm}\,. \tag{13}$$

where $C_t = 75$ mm and ϕ_s is the reinforcement diameter in mm.

Numerical procedure

The numerical procedure is based on two loops. An inner loop which determines the depth of the neutral axis by accomplishing the axial force equilibrium equation:

$$F - \left(\sum_{i=1}^{k} F_{ci} + \sum_{j=1}^{n} F_{sj} \right) = tolef \tag{14}$$

where F is the external force, *tolef* is a given tolerance value, $F_{ci} = b\, h_i\, \sigma_{ci}$ is the internal force in the concrete layer number i, $F_{sj} = A_{sj}\, \sigma_{sj}$ is the force in the reinforcement layer j. The meaning of the other variables is represented in Fig. 7.

In the outer loop the strain at top surface, ε_c^{top}, (see Fig. 7) is incremented by a given value and the flexural moment, M, is evaluated from the following expression:

$$M = \sum_{i=1}^{k} F_{ci}\, z_{ci} + \sum_{j=1}^{n} F_{sj} z_{sj} \tag{15}$$

where z_{ci} and z_{sj} are the distances between the corresponding layers and the cross section reference (the geometric axis).

Model appraisal

In order to assess the model performance with the results given by other authors, the moment-curvature relationship of beams tested by Kormeling et al. (1980) was calculated. Table 4 specifies the data used in the numerical simulation. Only the beam reinforced with 70 kg/m^3 of hooked-end fibers ($W_f = 2.92$ percent), the beam reinforced with two bars of 4 mm diameter and the beam reinforced with 70 Kg/m^3 of fibers and two bars of 4 mm diameter were analyzed. The cross section was discretized into forty layers of equal thickness. For the beam reinforced with two steel bars, the post cracking tensile behavior of the first seven bottom layers was simulated by the tension stiffening diagram, while the post cracking tensile behavior of the remainder layers was simulated by the tension softening diagram. Fig. 9 shows the correlation between experimentally measured values and the diagrams given by the model.

SLAB STRIPS

To assess the performance of the model developed and to appraise the benefits of steel fiber reinforcement on the behavior of thin slabs, slab strips reinforced with an ordinary steel wire mesh and with different percentages of steel fibers were tested under bending.

Experiments

The concrete composition used for the slabs was that of series *s2* of Table 1. Fig. 10 shows the schematic representation of the measuring devices and load arrangement applied in a typical test (Barros and Figueiras 1996). The reinforcement of each slab strip consists

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of a mesh with wires of 2.7 mm diameter and a global area of longitudinal reinforcement of 40 mm^2 in the slab width (see Fig. 10). This reinforcement was placed in the slab tensile face with a concrete cover of approximately 3 mm. The yield and the ultimate strength of the steel wires, was 560 MPa and 800 MPa, respectively. The average compression strength of the slab strips specimens reinforced with 0, 30, 45 and 60 kg/m^3 of fibers was 65.8 MPa (217 d), 61.5 MPa (204 d), 59.9 MPa (176 d) and 59.1 MPa (124 d), respectively. The age of the tests is between brackets.

The slab strips reinforced with 30, 45 and 60 kg of fibers per m^3 of concrete exhibited, in the central region, an average crack spacing of 100, 80 and 40 mm respectively, while the slab strip reinforced with wire mesh exhibited one large crack only. The observed decreasing in crack spacing with the increment of fiber percentage (see Fig. 11) is followed by an increase in the failure load, as it can be observed in Fig. 12, wherein the relationship between the load and the displacement at midspan is shown. The average failure load of the slab strips reinforced with 0, 30, 45 and 60 kg/m^3 of fibers was 10.2, 13.4, 16.0 and 21.1 kN, respectively.

Modeling

The numerical model described in this work is now applied to the slab strip tested. The cross section was discretized in ten concrete layers of equal thickness. The wire mesh was positioned at 71 mm from the slab bottom surface. Table 5 includes the data used in the analysis.

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The experimentally obtained moment-curvature relationship ($M - \chi$) for the slab strips (section S_c, see Fig. 10) and the moment-curvature relationship determined by using the present model are compared in Fig. 13. A fairly good agreement can be observed.

CONCLUSIONS

To study the structural behavior of steel fiber reinforced concrete (SFRC) a series of tests was carried out in laboratory. Stress-strain relationships were derived in order to formulate numerical models for analyzing SFRC structures.

Based on the results obtained from the uniaxial compression tests on steel fiber reinforced concrete (SFRC) cylinder specimens, a compression stress-strain law was proposed for the composites analyzed. From the results of the three-point bending notched beam tests, the post-peak tensile behavior of SFRC structures was assessed by using the fracture energy concept.

To evaluate the flexural resistance and the ductility of cross sections of SFRC members under bending, a numerical model was developed. This model applies the constitutive laws and the material fracture parameters determined from the experiments carried out. The model performance was assessed by simulating either the response of SFRC beams tested by other researchers, as well as, the behavior of SFRC slab strips tested in this work. The concrete slab strips reinforced with wire mesh and with different percentage of steel fibers were tested in bending. The increasing of fiber percentage has significantly improved the load carrying capacity and decreased the crack opening and crack spacing. It was observed

that mixes with 60 kg/m^3 of fibers exhibited an ultimate load twice the ultimate load of the slab strips with the reinforcing bars (wire mesh) but without fibers.

The simple but yet accurate model developed gives the complete moment-curvature relationship of a SFRC cross section. Either the moment-curvature or the stress-strain relationships derived in the present work are fitted to be used in finite element models for the nonlinear analysis of SFRC bidimensional structures.

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