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Luís Aguiar-Conraria

Pedro C. Magalhães

Maria Joana Soares

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Cycles in Politics: Wavelet Analysis of Political Time-Series[*]

Luís Aguiar-Conraria[†]   Pedro C. Magalhães[‡]   Maria Joana Soares[§]

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Abstract

Spectral analysis and ARMA models have been the most common weapons of choice for the detection of cycles in political time-series. Controversies about cycles, however, tend to revolve about an issue that both techniques are badly equipped to address: the possibility of irregular cycles without fixed periodicity throughout the entire time-series. This has led to two main consequences. On the one hand, proponents of cyclical theories have often dismissed established statistical techniques. On the other hand, proponents of established techniques have dismissed the possibility of cycles without fixed periodicity. Wavelets allow the detection of transient and coexisting cycles and structural breaks in periodicity. In this paper, we present the tools of wavelet analysis and apply them to the study to two lingering puzzles in the political science literature: the existence to cycles in election returns in the United States and in the severity of major power wars.

[†]NIPE and Economics Department, University of Minho, E-mail address: lfaguiar@eeg.uminho.pt

[‡]Social Sciences Institute, University of Lisbon, E-mail address: pedro.magalhaes@ics.ul.pt

[§]NIPE and Department of Mathematics and Applications, University of Minho, E-mail address: jsoares@math.uminho.pt

1 Introduction

Cyclical theories regarding social phenomena are abundant in the social sciences, mostly in economics but also in other disciplines, including political science.[1] However, among empirically minded social researchers, the ability to amass evidence concerning the existence of cyclicality in time series data has remained constrained by the assumptions shared by the two conventional weapons of choice for its detection, i.e., spectral analysis and autoregressive moving-average (ARMA) models. In this paper, we propose an alternative approach to the detection of cycles in time series data - wavelet analysis - which addresses the main limitation shared by spectral analysis and ARMA models: their inability to detect transient and irregular cycles and structural breaks in the periodicity of those cycles.

As van der Eijk and Weber elegantly put it, the study of cycles has been, for political scientists and other social researchers, a sort of "Lorelei," one of the Rhine nymphs whose singing led boatmen to the rocks where they inevitably perished: "researchers are lured to it because of its theoretical promise, only to become entangled in (if not wrecked by) messy problems of empirical inference" (van der Eijk and Weber 1987, 271). In political science, two longstanding debates in two different subfields exemplify the difficulties posed by cyclical theories and by the methods employed so far to test them. In American politics, the existence of cycles in election returns has played a central role in debates about electoral realignment, but the periodicity of those cycles and even their very existence remains a contentious issue among students of the American parties and elections (see Mayhew 2002 for a critique and Rosenof 2003 for a detailed historical review of the realignment literature). Similarly, in international relations, the question of whether there are long cycles of war in the international system has intrigued scholars for a long time, but still remains one of the fundamental sources of conflicting findings in the literature (Mintz 2005).

As we will show in greater detail later on in the paper, much of the controversy around these issues results from the possibility of transient or irregular cycles without fixed periodicity. In most cases, we have good substantive and theoretical reasons to expect irregularity and structural breaks in the periodicity of a particular time-series. In that case, both spectral analysis and

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[1] For a synthesis of cyclical theories in the social sciences, see, for example Mohler (2001).

ARMA models will be inappropriate from the start. Cycles that are predominant in a particular time period will either not be detected or they will be overdetected, and attributed to the whole period under analysis. "As a result of these features, some advanced time-series techniques such as spectral analysis or transfer function modeling cannot be usefully applied to most political data" (Weakliem 2010: 637). The rejection of these methods, however, has often led to the adoption of alternative approaches that fall short of meeting rigorous standards of statistical inference (Goldstein 1988). In other cases, the assumptions of particular techniques have been turned into fundamental substantive conclusions about the nature of social and political phenomena, leading to the argument that only "fixed period models are meaningful models of cyclic phenomena" (Beck 1991). The result, however, is that either cyclical theories end up inevitably being rejected or they are forced to acquire a somewhat mechanic and simplistic character, oblivious to the plausible reasons why the parameters of the process may suffer gradual (or even sudden) changes.

In the extant literature, techniques such as duration analysis (Connybeare 1992) or Markov-switching regressions (Sayrs 1993; Jones, Kim, and Startz 2010) have been presented as potential solutions to these problems. We provide another viable alternative to examine cycles in time-series data, which has the advantage of staying close to the frequency domain approach of spectral analysis while addressing the main limitations it shares with ARMA models: wavelet analysis.

As a coherent mathematical body, wavelet theory was born in the mid-1980s. It is now used in physics, geophysics, astronomy, epidemiology, signal processing, oceanography, and, more recently, after the pioneering works of Ramsey and Lampart (1998) and Gençay, Selçuk and Witcher (2001), in economics.[2] Its main contribution comes from performing spectral analysis as a function of time, thus revealing how different periodic components may change over time. As a result, wavelet analysis allows for the detection of changes in the periodicity of cycles and, at the same time, meets rigorous standards of statistical inference. We show that the potential of wavelet analysis also extends to the study of political time-series, by demonstrating its applicability to the study of two puzzles in the political science literature: the existence of cycles in US election returns and in the severity of wars.

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[2]For detailed reviews of wavelet applications to economic data, the reader is referred to Crowley (2007) and Aguiar-Conraria and Soares (2011).

The paper proceeds as follows. The next section uses empirical and constructed numerical examples to illustrate the use of ARMA models and spectral analysis in the detection of cycles and their shared inability to deal with irregular and transient cycles. We will show that this inability does not extend to wavelet analysis. Following that section, we present the three basic tools in wavelet analysis: the wavelet power spectrum, cross-wavelets, and phase-differences. Equiped with these tools, we will develop two applications to real world political time-series data. The first is the analysis of presidential and congressional election returns in the United States from 1854 to 2008. The second application of wavelet analysis concerns the severity of major power wars from 1495 to 1975, using well-established data in the literature (Levy 1983; Goldstein 1988). In the last section, we sketch a research agenda in political science where the use of wavelet analysis may shed light on important empirical and theoretical discussions. In the appendix, we describe how to computationaly implement the wavelet tools.

2 Why Do We Need Wavelet Analysis?

In order to understand why we need wavelet analysis, we can start with a basic and very familiar example: the presidential vote in the United States, and how it can be modeled as a second order autoregressive process. Norpoth (1995) argued that the two-term limit rule, respected by all presidents but one and enshrined since 1951 in the 22nd amendment, has allowed majority parties to reap the advantages of incumbency after a first term is completed while saving minority parties from being forced to challenge an incumbent president in the subsequent election. This results in a regular electoral cycle. Norpoth estimated the following model for the Republican presidential share of the vote from 1860 to 1992:[3]

$$RVote_t = 0.52RVote_{t-1} - 0.55RVote_{t-2} \qquad (1)$$

This type of analysis is what we call a time-domain analysis. ARMA processes, as long as

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[3]We omit the intercept and the error term as they unnecessarily complicate the discussion. For similar approaches, see, for example, Lin and Guillén (1998) and, for the British Conservatives vote lead, Lebo and Norpoth (2007).

either the autoregressive or the moving average component is of order 2 or higher, can capture the cycling behavior of stationary time-series. For this particular case, we can have a look, in figure 1.a, at the impulse-response function produced by equation (1): IRF1. As we can see, after a shock (the impulse), the line wiggles up and down as it approaches zero. There is a peak in period 1, period 6, and then, again, in period 11. This suggests that there it a second-order autoregressive process in the presidential two-party vote, through which, since 1860, peaks in electoral support for a particular party tend to be reached in five period cycles. Taking into consideration that presidential elections occur every four years, what Norpoth has found is that there is a 20-year cycle in election returns. In other words, the half-cycles of ascendancy of a particular party are estimated to last for about ten years, suggesting that, after two terms, majorities become much less likely to be able to hold on to power.

Equivalently, we could have reached the same conclusion gathering the information in a different way. Equation (1) is a second order difference equation, which has the following solution:[4]

$$RVote_t = 0.74^t \left(A_1 \cos\left(1.2t\right) + A_2 \sin\left(1.2t\right) \right), \tag{2}$$

where A_1 and A_2 are two arbitrary constants to be determined from two initial conditions. Noting that the co(sine) is a periodic function with period 2π and that $2\pi/1.2 \approx 5.2$, one can immediately conclude that this process has a cycle of period close to 5.2, which in turn implies a cycle of periodicity slightly below 21 years ($5.2 \times 4 = 20.8$). We can also use equation (2) to draw an Impulse response function (see IRF2 in figure 1.a).

Instead of performing this analysis on the time domain, we can do it on the frequency domain. Frequency domain analysis – also called spectral or Fourier analysis – allows us to decompose the overall variance of a time-series, by decomposing the observed pattern over time into a spectrum of cycles of different lengths. If one finds that a cycle of a given length is particularly important to explain the total variance, then one has found a predominant cycle. This is the approach to the cyclicality of American election returns adopted, for example, by Merrill, Grofman, and Brunell (2008), who, using spectral analysis of the Democratic share of the presidential vote from

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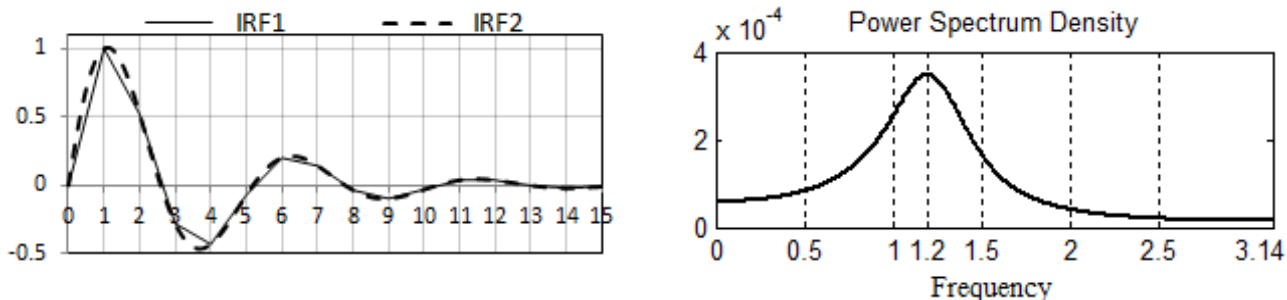
[4]See chapter 1 of Hamilton (1994).

Figure 1: (a) Impulse-Response of the Republican Presidential vote in the US. IRF1 is based on equation (1) and IRF2 is based on equation (2). (b) Power Spectrum Density for the AR(2) exemplified by equation (1).

1854 to 2006, found a 26-year cycle, see figure 6.b.[5]

Under some conditions, time domain and frequency domain analysis are equivalent. As Beck reminds us, "every stationary series meeting some minimal conditions has a spectral representation as well as a ARMA representation" (Beck 1991: 461). As Hamilton shows (1994, p.155, equation 6.4.14), ARMA estimates can be straightforwardly converted into the power spectral density of a time-series. In figure 1.b, we use Hamilton's formula to plot the power spectrum associated with a second-order auto-regression model – AR(2) – model, with coefficients given by equation (1). The peak of the power spectrum is at 1.2. Just like before, this leads us to conclude that there is a cycle of period 5.2 $(\approx 2\pi/1.2)$, which, converting periods into years, would lead us to conclude that the cycle has a periodicity of about 21 years $(5.2 \times 4 = 20.8)$. In sum, when one is estimating an ARMA model, one is also estimating the power spectral density.

There is, however, a potentially serious problem with all this. Both approaches used so far, and the Spectral Representation theorem that asserts their fundamental equivalence, rely on the assumption that the time-series is stationary. Stationarity means not only that a series has a constant mean and constant and finite variance, but also that the autocovariances are not functions of time (see Hamilton 1994, in particular pages 44-45). What if, however, the time-series fails to meet these criteria? One scenario for that to happen is of particular interest to us in this study: the existence of transient and irregular cycles. The approaches we have discussed so far are inappropriate to analyze data exhibiting those features. They require stationarity, which

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[5]See also Lin and Guillén (1998).

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in turn demands a time invariant generating process where autocovariances are not functions of time and, thus, where cyclicality needs to be assumed as time invariant.[6] Furthermore, under the Fourier transform, the time information of a time-series is completely lost. Because of this loss of information, it becomes an almost impossible task to distinguish transient cycles and to identify any structural changes that may cause shifts from one predominant cycle to another.

To illustrate this point, consider the following numerical example. We generate 100 yearly observations according to the following data generating process:

$$y_t = \cos\left(\frac{2\pi}{p_1}t\right) + \cos\left(\frac{2\pi}{10}t\right), \qquad \text{with} \quad \begin{cases} p_1 = 3 & \text{if } t < 40 \text{ or if } t > 60 \\ p_1 = 5 & \text{if } 40 \leq t \leq 60 \end{cases} \qquad (3)$$

Formula (3) tells us that the time-series y_t is the sum of two periodic components. The second periodic component is a cosine function that generates a 10 year cycle while the first generates some transient dynamics. In the beginning, it represents a 3 year cycle that, temporarily, changes to a 5 year cycle between the fourth and the fifth decades.

Figure 2.a displays the time-series generated by equation (3). If we estimate the power density spectrum, the information on the transient dynamics is completely lost, as we can see in figure 2.b. The power spectral density estimate is able to capture both the 3-year (note that $2\pi/2.045 \approx 3.07$) and the 10-year cycles ($2\pi/0.685 \approx 9.17$) but it completely fails to capture the 5-year cycle that temporarily coexisted with the 10-year cycle.

Consider a different example. Again, for $t = 1, 2, \cdots, 100$, assume that:

$$x_t = \cos\left(\frac{2\pi}{p}t\right), \qquad \text{with} \quad \begin{cases} p = 4 & \text{if } t < 50 \\ p = 6 & \text{if } t \geq 50 \end{cases} \qquad (4)$$

Formula (4) tells us that the time-series x_t is a periodic series, with a 4 year cycle which is replaced by a 6 year cycle in the second half of the sample. In figure 2.d and 2.e, we have the

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[6]We should remark that not having a unit root is not a sufficient condition for stationarity. It is true, as Williams (1992) showed, that several political time-series do not possess a unit root. This, however, does not imply stationarity. As long as there are irregular cycles, the autocovariances will change with time and, therefore, the data are not stationary. Moreover, this type of nonstationarity does not disappear by taking first differences.

resulting time-series and the power spectrum density estimate. Note that, while spectral analysis is able to capture both cycles, it is completely silent about their timing. The power spectrum density tells us that 4 $(2\pi/1.59 \approx 3.95)$ and 6-year cycles $(2\pi/1.06 \approx 5.93)$ are important to explain the total variance of the time-series, but it does dot tell us that one is relevant only in the first half of the sample while the second one only appears in the second half.
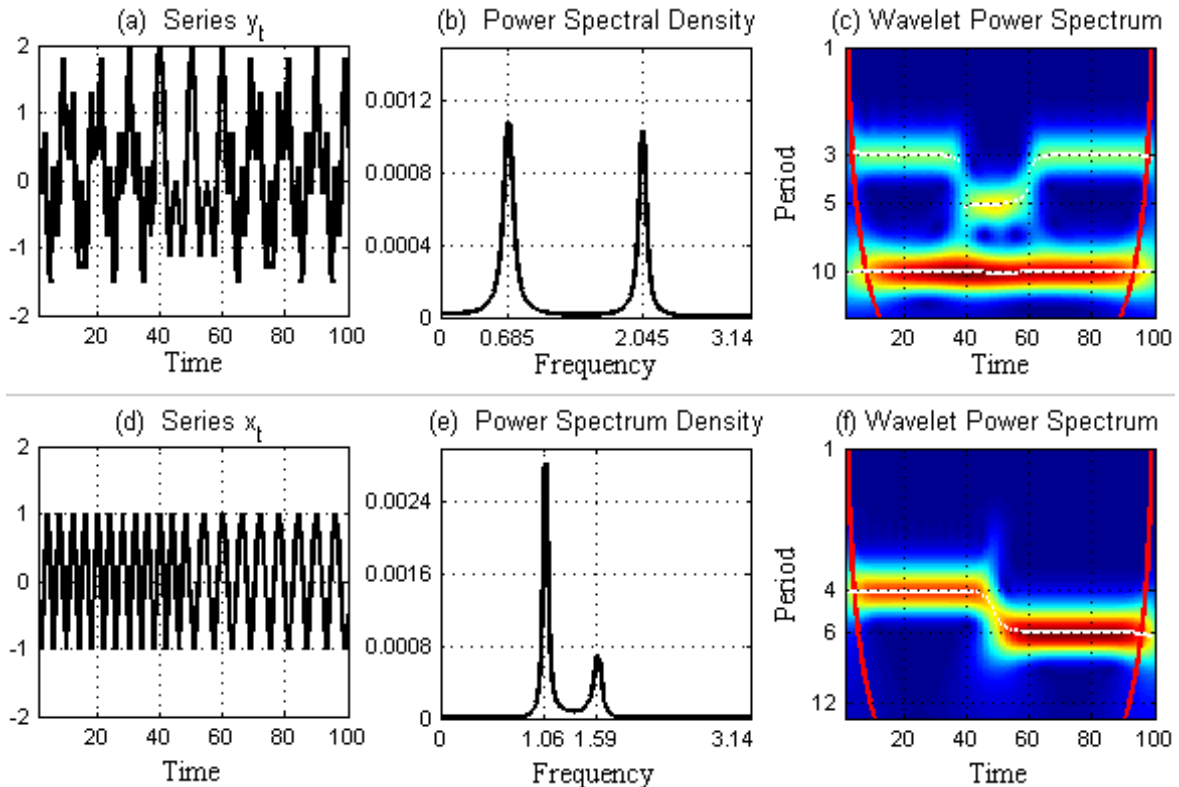

Figure 2: (a) y_t, equation (3); (b) standard power spectrum density estimate for y_t; (c) wavelet power spectrum estimate of y_t. (d) x_t, equation (4); (e) standard power spectrum density estimate for x_t; (f) wavelet power spectrum estimate of x_t.

The two main disadvantages of spectral analysis are evident from figure 2. First, it leads us to lose the time information of a time-series. Second, it makes it impossible to determine whether a particular cycle is transient or characterizes the entire series, as well as to identify the structural changes from one predominant cyclicality to another.

This is precisely where wavelet analysis comes to the rescue. When one estimates the wavelet power spectrum, one is estimating the power spectrum over time. Therefore, changes that happen accross time will be recorded in the wavelet power spectrum. Unlike what occurred with spectral

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analysis, we can now determine the parts of the series were these cycles were predominant and those where they were not.

In the next section, we will provide the mathematical details. For now, let us just reexamine the examples provided by equations (3) and (4), based on their wavelet power spectral estimates, displayed in figure 2.c and 2.f. In the horizontal axis, we have the time dimension. The vertical axis gives us the periods of the cycles. The power is given by the color. The color code for power ranges from blue (low power) to red (high power). Regions with warm colors represent areas of high power. The white lines show the maxima of the undulations of the wavelet power spectrum, therefore giving us a more precise estimate of the cycle period. The region outside the thick red lines is called the cone of influence (COI). As with other types of transforms, the continuous wavelet transform (CWT) applied to a finite length time-series inevitably suffers from border distortions; this is due to the fact that the values of the transform at the beginning and the end of the time-series are always incorrectly computed, in the sense that they involve missing values of the series, which are then artificially prescribed. Because of that, we pad the series with zeros. In the COI, the results have to be interpreted carefully. In particular, if one pads the series with zeros, as we do, the wavelet power in beginning and the end of the time-series will tend to be underestimated.

Figure 2.c, which analyses the data generated by equation (3), shows us some of the information that we had already obtained early on with the power density spectrum, i.e. a white line on period 10 across the entire time-series, meaning that there is a permanent 10-year cycle. However, we can now spot that a 3-year cycle does not characterize the entire series, but only from year 1 to year 40 and, again, after year 60. Furthermore, we are also able to spot a yellow/orange region between years 40 and 60, with the white stripes identifying a cycle of period five.

In figure 2.f, corresponding to the data generated by equation (4), we also obtain all the information that spectral analysis had provided us in figure 2.d, i.e., the existence of a 4 and a 6-year cycles. However, we are no longer blind to changes that occured accross time. With the help of the wavelet power spectrum, we can clearly see that the 4-year cycle predominated in the first half of the sample, being replaced, after year 50, by a 6-year cycle.

In sum, figure 2 illustrates the big advantage of wavelet analysis: we gain information about the time dimension, something that is entirely lost with spectral analysis.

3 Wavelets: Frequency Analysis Across Time

In this section, we introduce several wavelet tools: (1) the *wavelet power spectrum*, which describes the evolution of the variance of a time-series at the different frequencies; (2) the cross-wavelet power of two time-series, which describes the local covariance between the time-series, and the *wavelet coherency*, which can be interpreted as a localized correlation coefficient in the time-frequency space; and (3) the phase, which can be viewed as the position in the cycle of the time-series as a function of frequency, and the *phase-difference*, which gives us information on the synchronization between oscillations of the two time-series. The technical details are explained in the online appendix. In the main text, we will simply discuss some concepts, illustrate them, and present some of the necessary formulas.

3.1 The Wavelet

To overcome the problems pointed out before, wavelet analysis has been proposed. The Fourier basis functions are sines and cosines,[7] which have constant oscillations. Instead, a wavelet function drops towards zero. In our applications, it is enough to require that the wavelet function has zero mean and to assume a decaying property. This means that the function wiggles up and down the $t-$axis while it approaches zero; i.e. it behaves like a small wave, hence the term "wavelet", that looses its strength as it moves away from the centre. It is this property that allows, contrary to the Fourier transform, for an effective localization in both time and frequency.

The choice of the wavelet is important and will depend on the particular application one has in mind. If information about cycles is required, complex wavelets are necessary. We need complex numbers to gather information about the phase, which, in turn, tells us the position in the cycle of the time-series as a function of frequency. The Morlet wavelet, which we use in this

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[7]The main idea of spectral analysis is that a stationary time series is a weighted sum of periodic functions: $\cos(2\pi\xi t)$ and $\sin(2\pi\xi t)$, where ξ denotes a particular frequency.
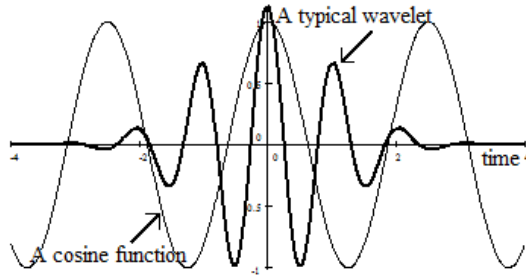
Figure 3: The typical wavelet function versus a cosine function. While the cosine function alway ranges between -1 to 1, the wavelet function approaches zero when it moves away from the center.

paper, satisfies these requirements. On top of that, one can also argue (see Aguiar-Conraria and Soares, 2011) that the Morlet wavelet has optimal joint time-frequency concentration, meaning that it is the wavelet which provides the best possible compromise in these two dimensions.[8]

3.1.1 The Continuous Wavelet Transform

We start with a wavelet function, ψ. Given a time-series $x(t)$, its continuous wavelet transform (CWT) with respect to the wavelet ψ is a function of two variables, $W_x(\tau, s)$:

$$W_x(\tau, s) = \int_{-\infty}^{\infty} x(t) \left[\frac{1}{\sqrt{|s|}} \psi^* \left(\frac{t - \tau}{s} \right) \right] dt, \tag{5}$$

where the asterisk denotes complex conjugation. The wavelet transform maps the time-series into a function of two variables, τ and s. The dilation factor, s, controls the width of the wavelet: it stretches it into a long wavelet function to measure the low frequency movements (long-run cycles); and it compresses into a short wavelet function to measure the high frequency movements (short-run cycles). With the Morlet wavelet there is an one to one relation between wavelet scales and frequencies ($\xi \approx \frac{1}{s}$), so we use the terms scale and frequency interchangeably. The position of the wavelet in time is given by τ. The wavelet transform, by mapping the original series in τ and s, gives us information simultaneously on time and frequency. The formulas of the wavelet and the Fourier transforms are very similar. The main difference is that in the wavelet transform we

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[8]In our toolbox, the Morlet wavelet is the default option. We also allow for the use the Generalized Morse Wavelets (GMW), which have been shown to have some desirable characteristics. Our advice is to use the Morlet wavelet. In specific situations, if one is interested in having a better time resolution (or frequency resolution), GMWs may be used with profit.

have a wavelet function instead of the cosine plus sine that we have under the Fourier transform.[9]

In analogy with the terminology used in the Fourier case, the wavelet power spectrum (sometimes called scalogram or wavelet periodogram) is defined as

$$(WPS)_x(\tau, s) = |W_x(\tau, s)|^2.$$ (6)

This gives us a measure of the variance distribution of the time-series in the time-frequency (time-scale) domain. In other words, it describes the relative importance of frequencies for each time step. To draw the wavelet power spectra in figure 2, we relied on this formula.

3.2 Cross Wavelets: Wavelet Coherency and the Phase-Difference

If one is interested in detecting and quantifying relationships between different times-series, Fourier cross-spectrum analysis will be inadequate, unless one is willing to assume there is a time invariant generating process for both series. Fortunately, wavelet analysis has also developed tools for that purpose. The concepts of cross-wavelet power, cross-wavelet coherency, and wavelet phase-difference are natural generalizations of the basic wavelet analysis tools that enable us to deal with the time-frequency dependencies between two time-series.[10]

A simple example may help bring home why it is important to measure the association in the time-frequency domain rather than simply in the frequency or the time domains. Consider the Federal Reserve Board's policy reaction to news about inflation. If inflation raises, the Fed, typically, increases interest rates to cool down the economy and fight inflation. If the Fed is well succeeded, that means that while in the short-run cycle, an increase in inflation leads to an increase in the interest rates (positive correlation), in the longer run an increase in the interest rates will lead to a decrease in inflation (negative correlation). This means that the link between interest rates and inflation will be different at different frequencies. Also note that these relations are time-varying, e.g. they depend on who is the Chairman of the Federal Reserve Board (e.g. see Taylor 1999). This simple example shows us that both the causality and the sign of the

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[9]The Fourier transform of x_t is given by $X(f) = \int_{-\infty}^{\infty} x_t e^{-i2\pi\xi t} dt = \int_{-\infty}^{\infty} x_t [\cos(2\pi\xi t) - i\sin(2\pi\xi t)] dt$.

[10]Aguiar-Conraria and Soares (2011) have already proposed a generalization of these tools that will allow us to move beyond bivariate analysis.

correlation can differ at different frequencies and can change with time. Not taking this into account may lead to mix a of all these effects, which will be hard to disentangle.

The wavelet coherency gives us a measure of the correlation between two variables in time and in frequency, while the phase-difference tells us which variable is leading and which one is lagging. The cross-wavelet transform of two time-series, $x(t)$ and $y(t)$, is defined as $W_{xy}(\tau, s) = W_x(\tau, s) W_y^*(\tau, s)$, where W_x and W_y are the wavelet transforms of x and y, respectively. The cross-wavelet power is $|W_{xy}(\tau, s)|$. The cross-wavelet power of two time-series depicts the local covariance between two time-series at each time and frequency. In analogy with the concept of coherency used in Fourier analysis, given two time-series $x(t)$ and $y(t)$ one can define their wavelet coherency R_{xy}, which depicts the local correlation between two time-series at each time and frequency, by:

$$R_{xy}(\tau, s) = \frac{|W_{xy}(\tau, s)|}{\sqrt{|W_{xx}(\tau, s)| \, |W_{yy}(\tau, s)|}}. \tag{7}$$

Formula (7) is similar not only to the Fourier coherency formula, but also to the formula for the correlation between two variables. This is not by accident. While the correlation measures the association between two time-series in the time domain and the Fourier coherency measures the association in the frequency domain, the wavelet coherency measures the association in the time-frequency domain. With a complex-valued wavelet, we can compute the phase of the wavelet transform of each series, which gives us information about the position in the cycle of a time-series. With the cross-wavelet transform one can obtain the information about the phase difference between two time-series, and thus obtain information about the possible delays of the oscillations of the two series as a function of time and scale (frequency). The phases and phase differences are angular measures, because of that they only take values between $-\pi$ and π: $\phi_{xy} \in [-\pi, \pi]$.[11] A phase-difference of zero indicates that the time-series move together at the specified frequency; if $\phi_{xy} \in (0, \pi/2)$, then the series move in phase, but the time-series x leads y; if $\phi_{xy} \in (-\pi/2, 0)$, then it is y that is leading; a phase-difference of π $(-\pi)$ indicates an anti-phase relation; if $\phi_{xy} \in (\pi/2, \pi)$, then y is leading; time-series x is leading if $\phi_{xy} \in (-\pi, -\pi/2)$. These

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[11]The wavelet phase of a single variable, e.g. $x(t)$, is given by $\phi_x(s, \tau) = \tan^{-1}\left(\frac{\Im(W_x(s, \tau))}{\Re(W_x(s, \tau))}\right)$. The phase difference between $x(t)$ and $y(t)$ is simply $\phi_x(s, \tau) - \phi_y(s, \tau)$, which can be computed from the cross-wavelet transform, by using the formula $\phi_{x,y}(s, \tau) = \tan^{-1}\left(\frac{\Im(W_{xy}(s, \tau))}{\Re(W_{xy}(s, \tau))}\right)$.
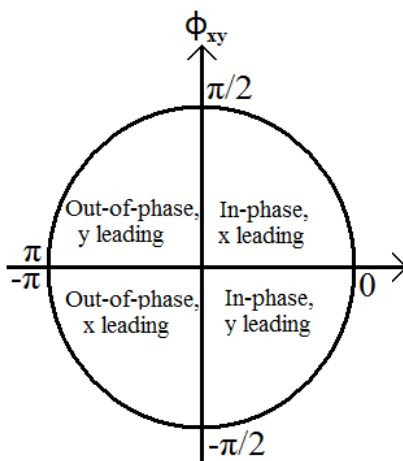
relations are represented in figure 4.

Figure 4: Phase-difference relations

Previously, we have argued that any ARMA model has a spectral representation. We have also shown that wavelet analysis addresses some of the major limitations of Spectral Analysis. With comparable arguments, the same case can be made to any Vector Autoregression (VAR), which is simply a multivariate extension of the AR models.

After the seminal paper of Freeman, Williams and Lin (1989), who introduced vector autoregressions in the Political Science literature, vector autoregressions became a standard tool and are now pervasive in modeling political time-series.[12] Typically, the authors do not report the coefficients estimates. This happens for good reasons. For example, Goldstein 1991a, runs a VAR with 4 variables (production, prices, real wages and war severity). If Goldstein were to report the coefficients estimates that would imply reporting 240 coefficients.[13] The cycles are usually detected by the undulations of the impulse-response functions. However, in principle, one could find the general solution to the estimated VAR, just as we found the solution for equation (1).[14]

To see how wavelet analysis is suitable to identify changes in the dynamic relations of two

[12]Another early application was provided by McGinnis and Williams (1989), who estimate a VAR model to capture the dynamics of military expenses and diplomatic hostility between the United States and the Soviet Union.

[13]In addition, because VARs imply the inclusion of the same variable several times (with different lags), multicollinearity is almost certainly a big problem. That's why individual coefficient estimates have almost no meaning.

[14]Using the data on war severity and inflation with 15 lags, like Goldstein (see next), our results show that the estimated VAR can be decomposed as a sum of several components which include a sum of thirteen pairs of sines and cosines representing thirteen cycles of different periodicities, ranging from 4 to 330 years.

time-series, consider two time-series that share two common cycles, with some leads and lags:

$$x_t = \sin\left(\tfrac{2\pi}{5}t\right) + \sin\left(\tfrac{2\pi}{12}t\right), \qquad t = 0, 1, 2, \ldots, 100.$$

$$y_t = \begin{cases} \sin\left(\tfrac{2\pi}{5}\left(t + \tfrac{7.5}{12}\right)\right) - \sin\left(\tfrac{2\pi}{12}\left(t - \tfrac{18}{12}\right)\right), & \text{for } t \leq 50 \\[2mm] \sin\left(\tfrac{2\pi}{5}\left(t - \tfrac{7.5}{12}\right)\right) - \sin\left(\tfrac{2\pi}{12}\left(t + \tfrac{18}{12}\right)\right), & \text{for } t > 50. \end{cases} \qquad (8)$$

Looking at the system of equations (8), it is clear that x_t and y_t share 5-year and 12-year cycles. However, the way their cycles relate to each other evolves with time and is different across frequencies. Consider the shorter period cycle, the 5-year cycle. The cycles are positively correlated (both sines have a plus signal). However, while for the first half of the sample the y_t cycle precedes the x_t cycle by seven and a half months, in the second half of the sample, the y_t cycle lags the x_t cycle. There is also a 12-year common cycle. In this case they are negatively correlated. In the first half of the sample, at the 12 year frequency, y_t lags x_t, in the second half it is y_t leading by one year and a half.

These features are captured in figure 5. Figure 5.b returns the wavelet coherency. In the bottom, we have the phase and phase-difference computed for two different frequency bands. On the left (Fig 5.c), we compute the phases for the 4∼6 year frequency band. On the right, we consider the 11∼13 year frequency band. The green line represents the y_t phase and the blue represents x_t's phase. The red represents the phase difference. That both series have common and highly correlated 5-year and 12-year cycles is revealed by the regions of strong coherency around those frequencies. That the 5-year cycles are in phase (positively correlated) is revealed by the phase difference (red line in the upper right graph), which is consistently situated between $-\pi/2$ and $\pi/2$ (this indicates that the series are in-phase, as we can see in figure 4). Finally, we can see that the 5-year y_t cycle was leading for the first half of the time and lagging in the second half by noting that in the first half of the sample the phase-difference is between zero and $\pi/2$, while in the second half it is between $-\pi/2$ and zero. Looking at the 12-year cycle, we observe that the series are out of phase (negatively correlated) with x_t leading in the first half and y_t leading in the second half of the sample. All this information about structural breaks in the data would have been lost if Fourier cross spectral analysis had been used.
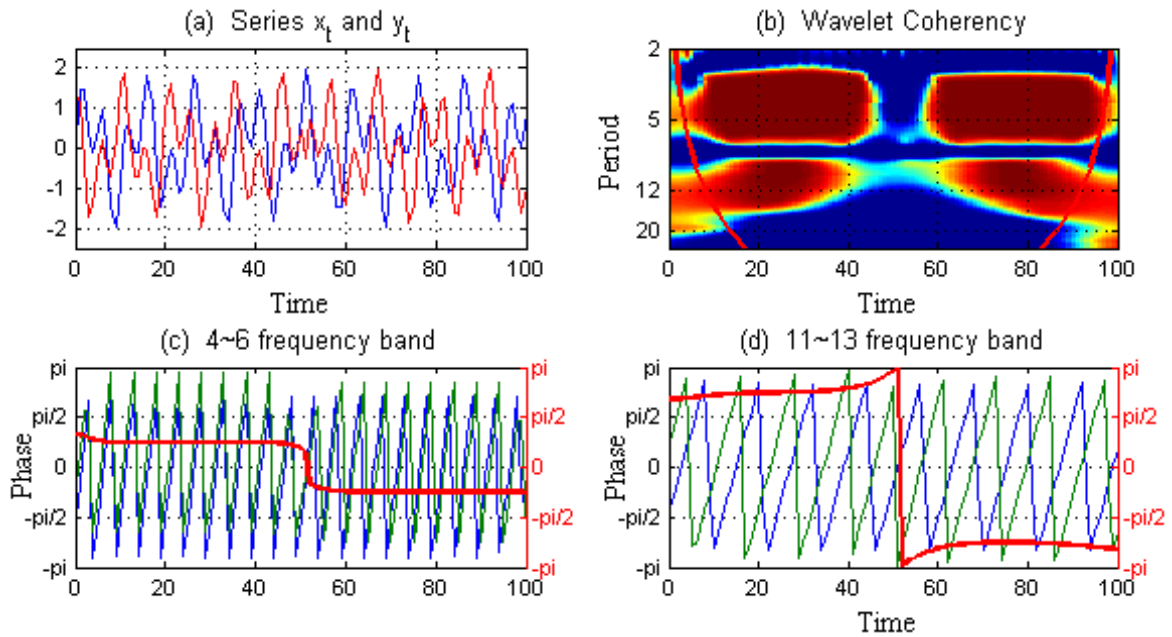
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Figure 5: (a) x_t and y_t. (b) Wavelet coherency - The cone of influence is shown with a red line. Coherency ranges from blue (low coherency) to red (high coherency). (c) and (d): Phase and phase-difference. The green line represents the y_t phase, the blue represents x_t's phase and the red represents the phase difference.

4   Two Applications: Waves in Elections and in Wars

One major objection to our line of argument so far would be to argue that the kind of irregular and transient cycles exhibited by our numerical examples are rare in real world political times series. If so, we would be adding unnecessary complication to existing approaches. In this section, we address this objection by examining two phenomena where the existence of cycles has been exhaustively studied but where major controversies remain about the periodicity and even the very existence of cycles. We have already alluded to those two phenomena in our explanation of wavelet analysis. The first is the alleged existence of regular cycles in presidential and congressional election returns in the United States. The second is the alleged – and disputed - existence of irregular cycles in the severity of major power wars.

Both issues play an important role in two political science subdisciplines. The first, in American politics, particularly as it relates to discussions about the theory and empirics of electoral realignment and negative feedback cycles in policy preferences (Merrill, Grofman, and Brunell

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2008). The second in international relations, where the existence of cycles in wars has been classified as one of the major sources of conflicting findings in the IR literature (Mintz, 2005). As we will show, wavelet analysis reveals that the assumption of regular cycles in these two phenomena is untenable, and provides a description of the features of these two series that contributes to elucidate previous controversies. Furthermore, the patterns that emerge from wavelet analysis are more consistent with other established and related theoretical and empirical findings than those that emerged from spectral analysis. Finally, they raise new relevant substantive explananda that deserve the attention of future research.

4.1   Cycles in American National Electoral Politics

The notion that there has been a "regular political oscillation" (Lowell 1898, 73) in election returns has been around for a long time in the study of American politics,[15] and has survived the introduction of modern social science methods in electoral studies. Stokes and Iversen (1962), for example, confirmed that a "law of the pendulum" was in place in presidential elections, and suggested a number of reasons why that might be the case, including the inevitable costs of ruling, a "coalition of minorities" effect, the value placed by voters on alternation itself, or the existence of shifting public moods of conservatism and liberalism. However, they explicitly stopped short of posing that such reasons forced us to assume a "regular or periodic nature" in electoral cycles (Stokes and Iversen 1962, 162-163). It is true that, more recently, strong hypotheses and evidence in favor of a fixed periodicity have begun to emerge. Nevertheless, there seems to be agreement neither on the plausibility of those hypotheses nor on how to interpret the available evidence concerning what kind of fixed periodicity we should expect.

One line of argument points to the likelihood of regular and relatively short cycles. As we have seen, Norpoth (1995) estimated equation (1), which implied a 20 year cycle. More recently, Merrill, Grofman, and Brunell (2008) extended this reasoning in several ways. First, using spectral analysis, they revealed that these short cycles (slightly longer than Norpoth's, about 26-years long) characterize not only presidential elections but also those for the House and the

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[15]For a detailed discussion of different cyclical theories of elections, see Rosenof (2003).

Senate. Second, they developed and tested a model where the advantages of incumbency also play an important role in extending the length of cycles, but are countervailed by policy motivations moving parties away from the median voter and by voters' negative feedback reaction to liberal (or conservative) policies.

An alternative line of argument, however, has suggested different reasons for the existence of regular cycles of party ascendancy and decline, as well as different lengths for those cycles. Burnham, for example, argued that "critical realignments" – abrupt changes in the voter coalitions behind American parties – occurred "with a remarkable uniform periodicity" (Burnham 1970, 8), "approximately once every thirty years" (Burnham 1967, 288), i.e., in full cycles of 60 years. Others, adopting more nuanced approaches that allowed for realigning "periods" or "eras", have nevertheless continued to propose the possibility of long regular cycles in American politics. Beck (1979), for example, suggested a socialization theory of party realignment, through which generational replacement produces predictable changes in the overall composition of the electorate. Newer generations will be less connected to the existing party system and have weaker partisan loyalties, rendering them more available to be mobilized into new alignments. This has resulted in "the cyclical creation, stabilization, and decay of mass electoral coalitions over the last one hundred and fifty years of American politics", through which "realignments have appeared about every thirty years" (Beck 1979, 131). In other words, the generational replacement approach suggests that the ascendancy of a particular party in the system is likely to extend for a 30-year half-cycle, pointing to the existence of full cycles of 60 years.[16]

Critics of cyclical views of American elections, however, have retained considerable ammunition on their side. Some have dispelled the notion altogether: if the very notion of the existence of abrupt periodic changes – critical realigning elections – is debunked, the notion of their periodicity is obviously prejudicated (Lichtman 1976 and Mayhew 2002). However, even if one resists linking periodicity to the questionable notion of "critical elections", there are reasons to believe that, at the very least, a fixed periodicity is unlikely to have characterized the entire

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[16]This does not exhaust completely the entire discussion about cycles in the American politics literature. For example, others have suggested the existence of 60 to 70-year half-cycles, with only three major realignments: Jefferson in 1800, Lincoln in 1860 and FDR in 1930 (Reichley 2000). Evidence supporting the statistical significance of such predominant cyclicality, however, is virtually impossible to obtain on the basis of the actual historical length of the American party system.

American electoral history and to be similarly prevalent in presidential and congressional elections. Take, for example, the Civil War and Reconstruction era, the beginning of which serves as the starting point of the series analyzed by Norpoth (1995) and Merrill, Grofman, and Brunell (2008). Whatever 'cycles' of ascendancy and decline we may find in this period, it is highly unlikely that they can be easily explained by extant explanations of cyclicality. After all, this period includes the exclusion of Southern states from federal elections until the end of the war, U.S Army rule over the South until the 1870s, and successive enfranchisement and disenfranchisement of different classes of citizens, not to mention rampant political violence. The shift that seems to have occurred in the 1870s – from Republican dominance to a highly sectionalized but nationally balanced and competitive system until the end of the century – cannot be dissociated from these macro-level factors and events, which bring considerable complications to any assumption of "normal politics" under which, say, institutional or generational factors might exert their influence (Beck 1979; Rusk 2001; Robbins and Norpoth 2010).

Similarly, the last few decades of the series, particularly those since the 1950s and 60s, offer reasons to be somewhat skeptical about the continuation of whatever cyclical patterns that may have prevailed until then. In some cases, there are historically recent trends that are likely to have served as inhibitors of electoral alternation in comparison to earlier years. That is clearly the case with the dramatic rise in the "incumbency advantage" in House elections, which can be dated to the beginning of the 1950s (Gelman and King 1990). As Campbell (2006) and Norpoth and Rusk (2007) have argued, this is the most plausible explanation for why the House election results failed to register the kind of shifts that realignment theorists expected to be happening in the 1960s, leading them to talk about a "delayed" or "staggered" realignment that ended up taking place only in the 1990s.

Other recent trends, however, have combined to generate more uncertain expectations about the cyclicality of elections. First, there are reasons to believe that electoral cycles may have accelerated in the recent decades. This is particularly the case if we consider the strong evidence in favor of an increased polarization of American parties and their electorates (McCarty, Poole, and Rosenthal 2006). As Stimson notes (1999: 129), such increased polarization seems to have

resulted in quicker and stronger public reactions to policies, accelerating changes in the public's ideological mood and, thus, creating pressures towards shorter cycles of electoral dominance by one party or another. Second, other developments in the last decades may have served to simply turn election outcomes more dependent on contingent factors and, thus, less predictable and attached to any regular cyclicality. These developments include, for example, a heightened instability in partisanship among the generations that have been politically socialized since the 1960s (Lewis-Beck, Norpoth, and Jacoby 2008), the steady increase in the share of "movers" in terms of party identification from the 1950s to the 1990s (Clarke and McCutcheon 2009), the transformative role of television in the way election campaigns are conducted and in the direction of a "horse race" coverage (Patterson 1994), and the related increased relevance – particularly since the 1980s – of valence issues and "electability" considerations in the way voters evaluate parties and candidates (Clarke, Kornberg and Scotto 2009; Jacoby 2010). Therefore, it is not entirely surprising that the search for a clear electoral edge of one party over the other in recent decades has often proved fruitless. Campbell (2006), for example, finds that both the pre-1896 and the post-1968 periods are characterized by an evenly balanced and highly competitive system, while studies of House elections also find extended periods without the dominance of any party (Robbins and Norpoth 2010, 324) and more irregular cycles than what traditional realignment theories suggest (Norpoth and Rusk 2007, 402).

As Stokes and Iversen (1962: 160) put it, "all such explanations share the premise that there is something to be explained." When we look at presidential and congressional election returns, are we likely to find regular electoral cycles, irregular ones, or no cycles at all? Wavelet analysis is particularly well equipped to answer this question in a way that avoids a potentially fruitless "all or nothing" debate. While not excluding the possibility of regular cycles, it allows for the detection of transient patterns, irregular cycles, and structural breaks in the periodicity of the series. And with the help of cross-wavelet tools, we can clearly identify differences and similarities in the cyclicality of different types of elections – Presidential and Congressional –, while preserving both the time-information in the series and rigorous standards of statistical inference.

Our data is essentially the same used by Merrill, Grofman, and Brunell (2008): the share of the Democratic vote in presidential elections since 1856 and the Democratic share of the House seats since 1854, including, in our cases, the 2008 results. First, we compute the wavelet power spectrum, for the presidential and House results. Second, we explore the varying relationship between these two series, through cross wavelets and also phase-differences, giving us a measure of the delays in the oscillations between them.

Figure 6, on top, displays the presidential share of the vote series. In figure 6.b, we can see how the Fourier power spectrum reveals a predominant cycle with 26-year periodicity, i.e., a 13-year half-cycle of ascendancy of a particular party. These results are essentially the same obtained by Merrill, Grofman, and Brunell (2008). However, let us shift now our attention to figure 6.a, where the wavelet power spectrum is estimated. Several new crucial pieces of information emerge.

The wavelet power spectrum shows that the statistically significant evidence for that main cycle is limited to the period between 1900 and 1970, revealing that the assumption of time invariance lying at the heart of spectral analysis is likely to be wrong in this particular case. Before the 1890s, we have no evidence for any prevalent cycles. Conversely, between the late 1950s and 1980, the analysis reveals that the results of a particular party seem to have improved (or worsened) in an even shorter cycle, slightly less than two consecutive terms. And after 1980, evidence for ciclicality must be seen as weak. True, due to the edge effects captured by the cone of influence, wavelet power may be underestimated in the beginning and the end of the series, before 1875 and after 1985. However, the results nevertheless point to the fact that cyclicality is temporally localized. Therefore, in the case of presidential elections, the "system of 1896" that followed the economic panic and the ensuing regional redistribution of the vote does not seem to be, at least in this respect, a mere fiction engendered by realignment theorists. Instead, it marks the beginning of cyclical patterns in election returns for which we do not have evidence in earlier periods. Similarly, by the 1950s and 60s, these cyclical patterns start being disturbed up to the point where we cease to find evidence for them since the early 1980s. Finally, we acknowledge that detecting the long full cycles of 60 years proposed by the generational replacement approach on the basis of only 150 years of data was going to a difficult proposition in the first place. In
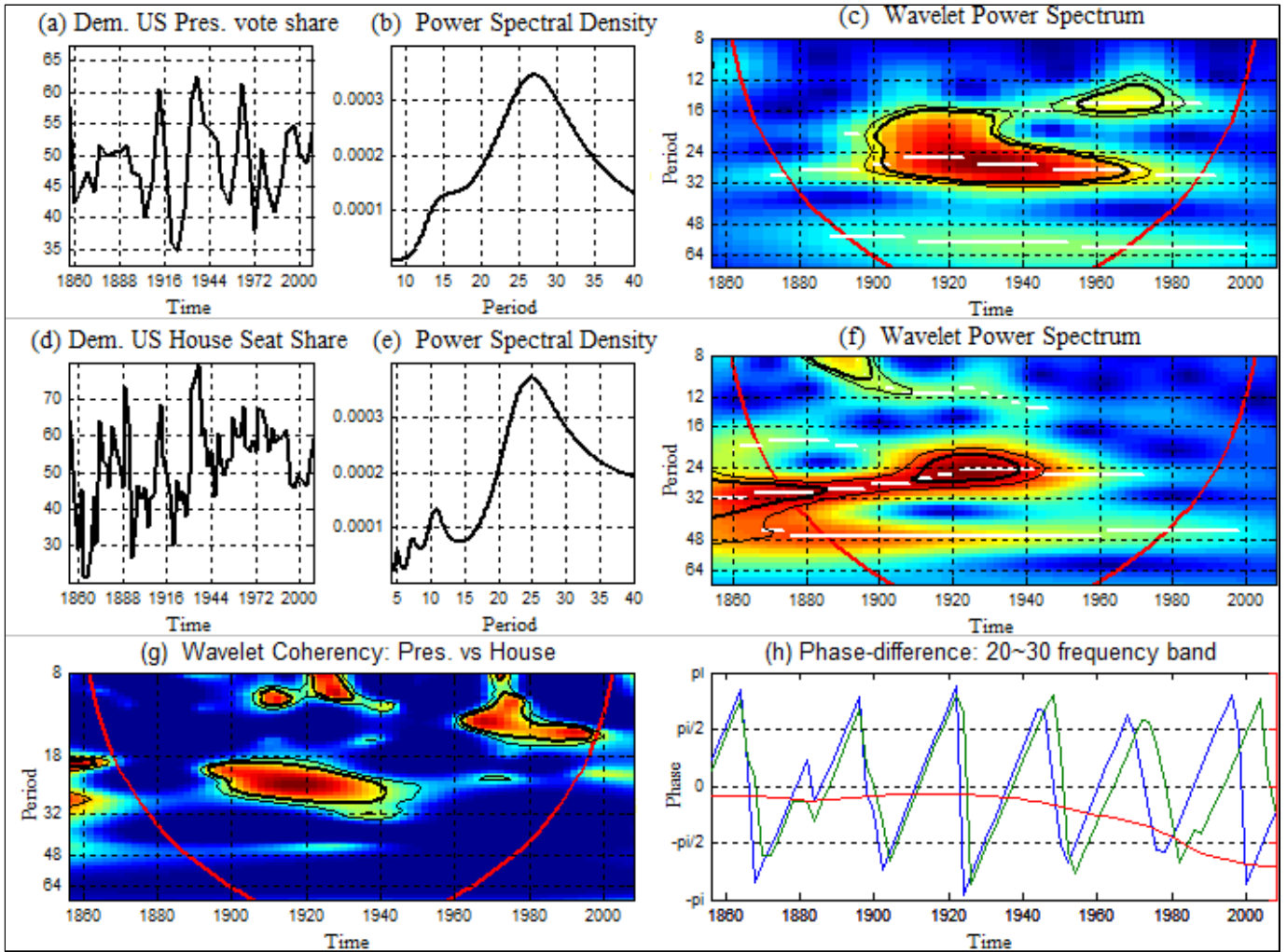
Figure 6: U.S. president democratic vote share — (a) time-series, (b) Fourier power spectrum, (c) wavelet power spectrum. Democrat House seat share — (d) time-series, (e) Fourier power spectrum, (f) wavelet power spectrum. Presidential Election vs House of Representatives — (g) Wavelet Coherency, (h) Phase and Phase-difference. In (c) and (f) the white lines show the maxima of the undulations of the wavelet power spectrum. In (a), (f), and (g) the cone of influence, which indicates the region affected by edge effects, is shown with a thick red line; the thick/thin black contour designates the 5%/10% significance level; the color code ranges from blue (low power/coherency) to red (high power/coherency). In (h), the blue line represents the House of Representatives phase, the green line represents the Presidential election's phase, and the red line gives us the phase-difference between the two series.

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any case, as figure 6.a shows, evidence that such long cycles coexist with the short cycles must been seen as clearly insufficient: it is not significant even at the 25% level.

In the middle of figure 6 we perform the same analyses, but this time for the House of Representatives.[17] Again, in figure 6.e, we observe the results of spectral analysis, i.e., the 25-year cycles, almost the same as the ones detected by Merrill, Grofman, and Brunell (2008). However, again, the wavelet power spectrum in figure 6.d allows us to see that such a cycle is not predominant throughout the entire series: for the House of Representatives, the 25/26-year cycle is present since the beginning of the series, but evidence for it disappears after the 1940s.

We also estimate the wavelet coherency and the phase-difference between both series.[18] As we can see in figure 6.g, there are two main regions of high coherency between the two series: between the 1890s and 1940 at the 20∼30 year frequency band; and between the 1960s and 2000 at the 10∼16 frequency band. However, only one of them corresponds to regions where we had detected relevant cycles in the wavelet power spectrums for both series, i.e., the 20∼30 frequency band between 1890 and 1940. In other words, that region in figure 6.g corresponds roughly to the intersection of the high power regions of figures 6.a and 6.d. The phase information for that period and frequencies (figure 6.h) tells us that the series were in phase, i.e. their cycles were highly coordinated.[19] Thus, in what concerns the 20∼30 cycle that spectral analysis told us characterized both series – Democratic vote share for the Presidency and Democratic seat share for the House – it is now clear that synchronization occurs only for a 50-year period between the late 19th century and the mid-20th century. Different elections have had different dynamics, and the biggest divergence started around the 1940s and 50s. Unsurprisingly, the timing of this divergence matches the increase of the incumbency advantage in congressional elections.

What we just saw is that the possibility of irregular cycles in election returns is not a fanciful and unnecessary complication that exists only in constructed numerical examples. It is, in fact, a crucial feature of the data that emerges only when we move from the frequency domain of

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[17]We use biannual data from 1854 to 2008.

[18]Because presidential elections only occur each 4 years, to be able to match the biannual data for the House election, we linearly interpolated the data on presidential elections. If, instead, we had chosen to delete the in-between observations for the House elections, similar conclusions would have been reached.

[19]After 1940, however, the phase-difference (red line) moves away from zero and the House series (blue line) leads the presidential series (green line). One should not pay too much attention to this phase lead because low coherency suggests that the two series are quite independent from each other.

spectral analysis to the time-frequency domain of wavelet analysis. Admittedly, the picture that emerges is more complex and nuanced than that which would emerge from a frequency domain approach. However, what is also striking is that, with just three figures, wavelet analysis allows us to parsimoniously address and bring new light to long-standing controversies about the cyclicality of election returns, as well as produce a picture that is not incongruent with what we know about trends in the last decades of American politics.

The results uncover the possibility that the clash between proponents of the existence of cycles and those who see them as implausible may be unnecessary. As our analysis shows, with the use of wavelets, it is possible to find cycles in election returns and, at the same time, periods without any pronounced cyclical ascendancy of any particular party. This is, of course, quite different from assuming either that the 26-year (or any other) cycle predominates for the entire series or that election returns have no cyclical features whatsoever. Furthermore, it creates a set of new explananda about the cyclicality of election results for which we already have a set of competing and intriguing hypotheses linked to recent trends in American politics (increased polarization, instability in party identification, or the role of valence judgments in the evaluation of candidates, just to give a few examples). These findings also serve as a warning against assuming that cyclicality remains today a good assumption for forecasting models of presidential elections. There is no evidence that the 13-year half-cycle has been relevant since the 1960s, or that this cycle remains either a relevant interpretative tool for recent events – such as the Democratic victory of 2008 – or a relevant predictive tool for future ones – such as any "wave of Democratic ascendancy" for the next decade or so.[20]

Finally, wavelet analysis also allowed us to uncover that the dynamics in presidential and House elections are less similar than conventional methods would lead us to believe. While the 26-year cycle for presidential elections started in the late 1890s and lasted until the late 1960s, a cycle of the same periodicity characterizes the House election series only until the 1940s,

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[20]In a June 2008 piece in the Huffington Post, Thomas B. Edsall thoughtfully commented on several pieces of political science research, including Merril, Grofman, and Brunell (2008), as indicating that "Barack Obama is riding the leading edge of a Democratic wave, benefiting from a potential – although by no means certain – cyclical shift in the partisanship of American voters which could last at least until through 2016" ("Obama Rides The Wave", http://www.huffingtonpost.com/2008/06/24/obama-rides-the-wave_n_108848.html). See also Lee Drutman, "The Wheel of Political Fortune Keeps Spinning", Miller-McCune, July 28th, 2008. Available at: http://www.miller-mccune.com/politics/the-wheel-of-political-fortune-keeps-spinning-4383/

something we would have missed altogether had we relied solely in classical spectral analysis. Synchronism in the presidential and House series is a localized phenomenon, lasting until the 1940s, and with the latter displaying an increasing lag since then.

Again, such finding is more consistent with established facts about House elections, particularly the surge in incumbency advantage, which seems to have put an end to the regular ebb and flow that House elections had exhibited until then and disturbing the relationship between its short-term cyclical component and that of presidential elections. If we want to study cycles in national elections, it is clearly not indifferent to know which specific elections we are talking about, and a single model might be inadequate to explain cyclical behavior in presidential and congressional elections.

4.2 Cycles in the Severity of Wars

Our second application of wavelet analysis concerns cycles in wars. The question of whether wars occur cyclically has intrigued observers at least since the 16th century (Clark 1958), but the answer has remained extremely elusive for modern social scientists. On the one hand, variations between both data sources and dimensions of the phenomenon of war – outbreak, duration, magnitude, or severity, for example – seemed to yield rather different answers (Singer and Small 1972; Houweling and Siccama 1988; Mansfield 1988). On the other hand, theories that might explain why cyclical rhythms were likely to be found in wars were never in abundant supply.

However, the focus on Great Power wars brought some theoretical and empirical clarity to the discussion about the cyclicality of warfare.[21] Goldstein's theory of long cycles, in particular, offers both a cyclical theory of major wars and a cyclical explanation for their intensity (1985, 1988, 1991a and 1991b).

Goldstein built on Kondratieff's observation – (Kondratieff, 1935, pp. 112-113) – that the most extensive wars tend to occur in the rising phases of economic long-cycles (cycles that range approximately from forty to sixty years in length), and argued for a basic relationship between major wars and the economy in the last five centuries. His theory features a "lagged negative

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[21]For reviews of cyclical theories of war, see Rosecrance (1987), Levy (1991), and Kohout (2003).

feedback loop" of economic growth and war: upswings in production, investment and innovation lead both to competition for scarce resources and to increases in the "war chests" of great powers, making war both desirable and possible. War severity, in turn, leads to rises in prices and declines in real wages, but as wars come to an end, economic recovery reinitiates, leading us back to where we started. Using intuitive methods, together with the autocovariance function analysis and VAR modelling, Goldstein identified a 50 to 60-year cycle in Levy's (1983) war severity series and a relationship between war fatalities and long-wave prices' upswings.

Several theoretical, empirical and methodological questions have been raised, however, about these general propositions. On the one hand, it seems implausible that the same cyclicality should prevail throughout the entire historical period under examination. Structural changes are likely to have fundamentally altered the cycles of war and their relationship with other phenomena, such as trends in economic growth or technological innovation. For example, the Industrial Revolution has allowed states to improve their power in the system without resorting to war, increased the overall destructiveness of warfare, and permitted the creation of permanent military establishments, thus changing the overall incentives for warfare and breaking the connection between rises in production and war capabilities (Levy 1991). On the other hand, the $50 \sim 60$ year cycle for which Goldstein found empirical support seems, in fact, non-existent on the basis of classical techniques. As Beck (1991) shows, using spectral analysis, although there is some evidence for coherence between inflation and war severity – especially in the post-1815 period – the war series lacks a strong periodic component itself. And if rigorous testing does not allow rejecting the null hypothesis of no predominance of a long period cycle, "we would normally conclude that a cycle does not exist" (Beck 1991, 471).

Goldstein's response, however, is that cycles in war (and other social and political phenomena) do not have to be characterized by the fixed periodicity typical of physical phenomena, but rather by "repeating sequences" and a "inner dynamic that gives rise to repetition" (Goldstein 1988: 176). In a single time-series, only periodicity allows us to conclude that cycles exist. "But when ups and downs correlate throughout a worldwide political economic-system, it is safe to conclude that there is a deeper cyclical dynamic at work, not just a scatter of random ups and

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downs" (Goldstein 1988, 177). Thus, Goldstein proposes shedding techniques that presume fixed periodicity (such as spectral analysis – Goldstein 1991b, 478) and, instead, detects cycles that are spaced irregularly in time: while the 50~60 year cycle does characterize the period between the beginning of the 17th and the 19th century, he argues that other periodicities characterize the periods immediately before and after (1988, 239-242). And, crucially to establish cyclicality in his own definition, Goldstein poses that war leads prices by one to five years (1988, 249-254).

Wavelet analysis allows us to examine these different claims. Our data on war severity is taken from Goldstein (1988). It measures fatalities in great power wars and runs from 1495 to 1975 (see figure 7.a). For comparison with Goldstein, Beck (1991) analyzes the full sample, however he warns that caution must be used when interpreting his results. This is so because of the structural change that occurred around 1815. As he points out, the time-series is not stationary because the mean and variance of the time-series before 1815 are, respectively, 1.45 and 0.72, while after 1815 they become 0.46 and 1.28. Because of this non-stationarity, rigorous Fourier spectral analysis would require the sample to be divided in two. As we have pointed out earlier, this is one major advantage of wavelet analysis: stationarity is not required. Therefore, in our setup, there is no reason to break the sample in two.

Figure 7.b shows the wavelet power spectrum of the war severity series. The first and most striking result is that, in accordance with Goldstein's proposal, wavelet analysis does uncover a statistically significant cycle of period around 60 years, as we can see by the white stripe and by the dark red area for that frequency. Furthermore, such cycle is shown to be localized in time, rather than prevalent throughout the entire series: the thin black contour identifies the 10% statistically significant region for the 60-year cycle, which runs from early 1700s to mid-1800s, but is replaced in the 20th century by a shorter and also significant cycle, with a period around 30 years. Another transient cycle, although not as strong, is also identified between 1750 and 1800, at the 20-year frequency band. In short, unlike what had been suggested so far on the basis of spectral analysis or ARMA techniques, we do have enough evidence to clearly reject the null hypothesis that there are no predominant cycles in the war severity data. Furthermore, wavelet analysis provides us with a previously unexplored set of explananda concerning the cyclicality

of war: the absence of cyclicality until the mid 17th century, the prevalence of a 60-year cycle until the late 19th century, and the structural change that brings about a shorter periodicity afterwards.
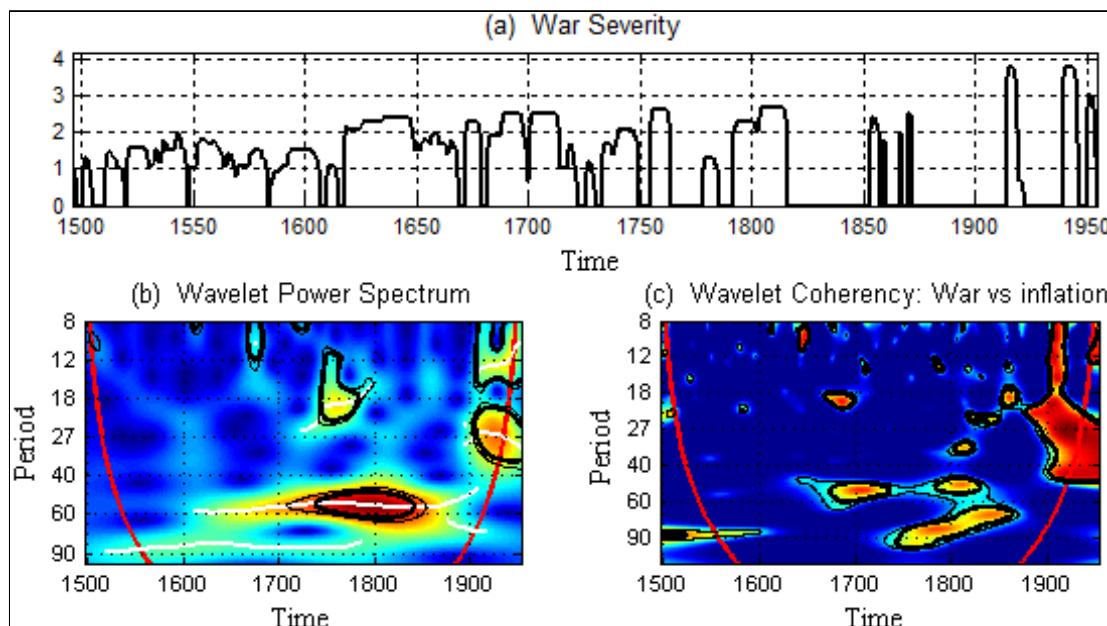

Figure 7: (a) War Severity. (b) Wavelet power spectrum. (c) Wavelet coherency between war severity and inflation.

Is it plausible that inflation provides the explanations for these patterns? This issue can also be examined with wavelet analysis. Figure 7.c gives us the wavelet coherency between war severity and British inflation, again using the Goldstein 1988 data also employed in Beck 1991. Here, the results are mostly consistent with theirs. Beck found the series to be significantly coherent after 1815, a pattern that we also observe in the $18 \sim 45$ frequency band (but only after 1875). Before that period, we observe high coherency at frequencies around 75 years between the mid-18th and the mid-19th centuries, as well as small regions of high coherence in the $45 \sim 55$ frequency band around 1700 and 1800. These, however, are not regions of high power in the war series, as we see in figure 7.b. Therefore, the wavelet coherency results do not lend particular credence to the notion that cycles in war severity consistently precede cycles in inflation, and much less that they are preceded by upswings in economic growth. This is hardly surprising. There are several economic theories that attempt to explain inflation. Some relate prices to

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the monetary base, others consider interest rates, government expenditures, unemployment rate, output gap, agents' expectations, etc. One should not expect to produce substantial claims about the influence of war on inflation without controlling for several of these possible causes.

In sum, what have we learned from the analysis of war severity? Do we need to presume fixed periodicity in cycles of war in order to perform rigorous statistical testing of their existence? Or should we reject "orthodox statistical inferences" (Goldstein 1991b: 480) when testing theories proposing time-varying cyclicality? What we have seen is that wavelet analysis allows us to answer the two questions negatively. It did not assume fixed periodicity or stationarity in war severity cycles, but it did not rule them out either. Because it did not assume them, it allowed us to detect transient and evolving cycles. With wavelet coherency, we were able investigate Goldstein's proposed hypothesis – the relationship between inflation and war – even in the presence of irregular cycles. And all this could be done with straightforward and rigorous standards of statistical testing.

5   Directions for Future Research

Spectral analysis and, especially, ARMA models, have served political scientists quite well in summarizing the dynamics of time-series in countless studies and applications. However, although very useful, they cannot capture all the features of the data one may be interested in. Recent approaches, such as the development of dynamic conditional correlation (DCC) models under the ARCH/GARCH framework (Engle 2002), have allowed researchers to explore the notion that correlations between series are time-varying, a development that has already found highly relevant applications in the study of political time-series (Lebo and Box-Steffensmeier 2008). Wavelet analysis constitutes, we submit, another important breakthrough in the same general direction, i.e., addressing time-varying features of time-series data. Like spectral analysis, it is particularly adapted to the detection of cycles. But unlike spectral analysis, it can be applied whenever one is interested in estimating the spectrum as a function of time, revealing how the different periodic components of the series change throughout the series. Furthermore, unlike DCC (and other time-varying approaches like Kalman filtering), wavelet analysis explicitly addresses relationships

across frequencies.

In this paper, we used three tools in wavelet analysis that, to our knowledge, have not yet been used by political scientists: the wavelet power spectrum, the cross-wavelet coherency and the phase-difference. While the wavelet power spectrum quantifies the main periodic component of a given time-series and its time evolution, the cross-wavelet transform and the cross-wavelet coherency are used to quantify the degree of linear relation between two non-stationary time-series in the time-frequency domain. Phase analysis is a nonlinear technique that makes possible to study synchronization and delays between two time-series across different frequencies. We applied these tools to the study of cycles in American elections and war severity. In both cases, we uncover patterns that spectral analysis or ARMA models have kept hidden in the existing literature.

Our focus on cycles in American election returns and in Great Power wars provides just two of many possible illustrations of the potential of wavelet analysis. But what is that potential, and what might a future research agenda based on the use of this technique look like? First of all, it should be noted that the uses of wavelet analysis go well beyond the mere detection of cycles. For example, wavelets can been used to construct band-pass filters (Yogo 2008), to estimate the memory of fractional integrated processes (Hsu 2006) or to improve forecasting models (Yousefi, Weinreich and Reinarz 2005). However, even if focusing exclusively on the study of cycles in politics, there are probably three major lines of research that could be followed. The first and simplest extension of the work we developed here would concern the replication of existing empirical studies which aimed at detecting cycles with the use of ARMA models or spectral analysis, but where confidence in findings and further theoretical refinement might be obtained if the employed techniques would allow – as wavelet analysis does – for the possibility of transient and irregular cycles. This includes studies of cycles in military expenditures (William and McGinnis 1992), British electoral returns (Lebo and Norpoth 2007; Merrill, Grofman, and Brunell 2011), party incumbency in Europe (Jerôme and Jerôme-Speziari 2010), terrorism (Enders and Sandler 2006), and political discourse (Namenwirth and Weber 1987), to give a few examples.

A second avenue of research could entail the application of wavelet analysis to cases where the

assumptions of regular periodicity have, so far, made researchers hesitant of employing statistical inference to detect cycles. Although collective action and political violence have long been argued to exhibit cyclical patterns, broader generalizations have been made difficult precisely because the assumption of structural stability in these patterns is so obviously untenable (Tarrow 1988: 433). Very similar words could be said about research on civil war and ethnic strife (Garrison 2008) or regime change away or towards autocracy (Hale 2006), for example. Some areas seem particularly ripe for further advances in this regard. One of the recent trends in the study of protest is, precisely, the increasing availability of time-series data and the use of sophisticated quantitative research methods to uncover underlying patterns, which seem to present both cyclicality and irregularity (Oliver and Myers 2002). In fact, it is not surprising that the study of these themes, together with terrorism, generate cyclical hypotheses: explicitly or implicitly, their conceptualization has often resorted to predator-prey, diffusion, or contagion models to describe interactions between authorities and challengers, all of which tend to generate cyclical patterns (Tsebelis and Sprague 1989; Olzak 1994; Francisco 1996; Das 2008).

A final possible route of research concerns the examination of the extent to which cycles in different series might be related. For example, scholars have hypothesized and investigated the possibility of contagion of diffusion effects across different political systems of phenomena such as ideological self-placement and policy moods (Kim and Fording 2001), regime change (Brinks and Coppedge 2006), or protest activities (Reising 1999). As we have seen in the paper, cross-wavelet and phase-difference analysis would allow an assessment of the extent to which different cyclical patterns in these different series are effectively synchronized as well as the detection of lagging and leading series, and thus the detection of the original sources of an hypothesized diffusion process. The same tools can be employed, for example, in the study of the dynamic nationalization of politics (Morgenstern, Swindle, and Castagnola 2009), by showing the extent to which changes in electoral returns across time are synchronized across territorial subunits – i.e., electoral districts, regions, or states – and the extent to which such synchronism has increased or decreased. Perhaps most important of all, it is by now clear that wavelet analysis provides a very useful tool to detect and examine economic cycles. If that is the case, one obvious field of future inquiry becomes

the relation between economic time-series such as those in unemployment, inflation, growth, spending, and stock-market returns, for example, and political time-series with manifest cyclical features, such as vote shares (Lewis-Beck and Stegmaier 2000), policy mood (Stevenson 2001), preferences for spending (Wlezien 1995) or election timing (Gartner and Wellershof 1995). And as wavelet analysis has also been used to examine the extent to which business cycles across different countries have become increasingly synchronized (Aguiar-Conraria and Soares forthcoming), it provides an obvious tool with which to further investigate whether such synchronization is a source of "partisan waves" of government control across highly integrated economies (Kayser 2009).[22]

These are just a few illustrations of how this technique may contribute to the discipline. As Kennedy notes (2008: 317), wavelet analysis addresses the limitations of frequency domain approaches "so successfully that it has become very widely used in a broad range of disciplines, and is becoming more common in economics". We believe this can also be the case in political science. Political scientists have gained increasing access to historical politically relevant time-series data going sometimes several centuries back, as well as to a wealth of high frequency time-series data – coming either from official statistics or surveys – about contemporary politics in many different societies, and there are good reasons to believe that such wealth and the diversity of its sources is likely to increase quite dramatically in the near future (King 2009). The alluring search for patterns in those sorts of data, however, has been made particularly difficult by the assumptions of the kind of techniques employed so far, which have prevented attention to contextual variations over time, short-term dynamics, and structural breaks, thus leading either to negative findings or to positive findings of questionable face validity. Wavelets will contribute to change this.

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[22]This type of analysis is impossible to perform with ARMA models (because these are univariate models). It is also rather difficult to do with the typical VARs, because information on the cycle periods is not given directly. If one had the possibility to perform multivariate wavelet analisys, estimating partial coherences (akin to partial correlations), wavelets would allow us to go from a-theoretical discussions about the existence of cycles to debates about causal relationships. Fortunately, Aguiar-Conraria and Soares (2011) have already proposed ways to perform multivariate wavelet analysis.

6 Appendix — Analytic Wavelets Toolbox

Due to its increasing popularity and applicability into a wide range of fields, the amount of wavelet-related software has been growing at an enormous rate. Some commercial scientific computing software, such as *Matlab*, now integrate wavelet analysis packages.[23] The reader can find and freely download our toolbox, which runs in MatLab, in http://sites.google.com/site/aguiarconraria/joa wavelets. This toolbox was written with social science applications in mind. To our knowledge, ours is the first toolbox that performs multivariate wavelet analysis, allowing for the possibility of computing multiple and partial wavelet coherencies as well as partial phase-differences. Our toolbox is divided into two folders:

1. **Functions** – containing all the Matlab functions. This has two sub-folders:

   - **Auxiliary** – containing some auxiliary functions to, e.g. generate surrogate series or compute Fourier spectra; it also contains a function to compute measures associated with generalized Morse wavelets.

   - **WaveletTransforms** – containing functions to compute the (analytic) wavelet transform, cross-wavelet transform, wavelet coherency, wavelet phase-difference and time-lag, multiple coherency, partial coherency and partial phase-difference.

2. **Examples** – containing Matlab scripts to generate the pictures associated with each example and application of this paper (and other papers as well). Therefore, it is easy for a research to replicate our results and, then, adjust our codes to his/her own research.

When implementing the transforms, some choices have, naturally, to be made. The most important choice is the wavelet choice. To our knowledge, every economics application of the CWT has made use of the Morlet wavelet. This is our default. Our toolbox also allows for the use of the Generalized Morse Wavelets, which encompass the most popular analytical wavelets (such as the Paul wavelet). Our advice is to use the Morlet. The GMWs can be used for robustness

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[23]E.g. Math Work's *Wavelet Toolbox* for Matlab is one such package. The choice of wavelets is large. The ability to compute the wavelet coherence and cross spectrum was only very added to the toolbox (Wavelet Toolbox 4.6, released in September 2010). However this toolbox still does not include significance testing, which is a major shortcoming.

checks. The second most important choice is about significance tests. In our toolbox, the tests of significance are always based on Monte Carlo simulations. The simulations use two different types of methods to construct surrogate series: (1) fitting an ARMA(p,q) model and building new samples by bootstrap or (2) fitting an ARMA(p,q) model and construct new samples by drawing errors from a Gaussian distribution. The 'Econometrics toolbox' is necessary to perform these tests.[24] Our experience tells us that either ARMA(1,0) or ARMA(1,1) fit the data well enough.[25]

The other options are less important. For example, in order to convert frequencies into periods, one has to declare the periodicity of the data. When computing the wavelet coherency, smoothing is necessary, because, otherwise, coherency would be identically one at all scales and times.[26] Smoothing is done by convolution with window functions in time and in frequency. By default, we use the Bartlett window. The other options — Hamming, Hanning, Blackman, etc — require the use of the Signal Processing toolbox. Our experience tells us that the final pictures are quite insensible to this choice.

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[24]The user that does not have the Econometrics toolbox can perform significance tests by choosing an ARMA$(p,0)$ model with bootstrap. In this case, the model is estimated by OLS and the code is self-contained.

[25]One has to avoid overfitting, otherwise the null may be almost impossible to reject.

[26]The same happens with the Fourier coherency.

References

[1] Aguiar-Conraria, L., Soares, M. J. (forthcoming). "Business cycle synchronization and the Euro: a wavelet analysis", *Journal of Macroeconomics*, doi: 10.1016/j.jmacro.2011.02.005

[2] Aguiar-Conraria, L. and Soares, M. J. (2011), "The Continuous Wavelet Transform: A Primer", mimeo.

[3] Beck, N. (1991), "The Illusion of Cycles in International Relations", *International Studies Quarterly*, 35(4), 455-476.

[4] Beck, P. A. (1979), "The Electoral Cycles and Patterns of American Politics", *British Journal of Political Science* 9(2), 129-156.

[5] Brinks, Daniel, and Michael Coppedge. (2006), "Diffusion Is No Illusion", *Comparative Political Studies* 39(4): 463 -489.

[6] Burnham, W. D. (1967), "Party Systems and the Political Process",In *The American Party Systems: Stages of Political Development*, New York: Oxford University Press, 277–307.

[7] Burnham, W. D. (1970), *Critical Elections and the Mainsprings of American Politics*. New York: Norton.

[8] Campbell, J. E. (2006), "Party Systems and Realignments in the United States, 1868-2004"*Social Science History,* 30(3), 359–386.

[9] Clark, G. (1958), *War and Society in the Seventeenth Century.* Cambridge: Cambridge University Press.

[10] Clarke, H., Kornberg, A. and Scotto, T. (2009), *Making Political Choices: Canada and the United States*, University of Toronto Press.

[11] Clarke, H. D., and McCutcheon, A. L. (2009), "The Dynamics of Party Identification Reconsidered." *Public Opinion Quarterly* 73(4), 704-728.

[12] Conybeare, J. A. (1992), "Weak Cycles, Length and Magnitude of War: Duration Dependence in International Conflict", *Conflict Management and Peace Science*, 12(1), 99–116

[13] Crowley, P. (2007), "A Guide to Wavelets for Economists", *Journal of Economic Surveys*, 21 (2), 207–267.

[14] Das, S. P. (2008), "Some mechanisms of terror cycles", *Journal of Economic Behavior & Organization* 67(3-4): 644–656.

[15] van der Eijk, C. and Weber, R. P. (1987), "Notes on the Empirical Analysis of Cyclical Processes", *European Journal of Political Research,* 15 (2), 271-280.

[16] Enders, W., and Sandler, T. (2006), *The Political Economy of Terrorism*, Cambridge University Press.

[17] Engle, R. (2002), "Dynamic Conditional Correlation – A Simple Class of Multivariate GARCH", *Journal of Business and Economics Statistics,* 20, 339-50.

[18] Francisco, R. A. (1996). "Coercion and protest: An empirical test in two democratic states", *American Journal of Political Science* 40(4): 1179–1204.

[19] Freeman, J., Williams, J. and Lin, T.-M. (1989), "Vector Autoregression and the Study of Politics, American Journal of Political Science, 33(4), 842-877.

[20] Gartner, M., and K. W Wellershoff. (1995), "Is there an election cycle in American stock returns?", *International Review of Economics & Finance* 4(4): 387–410.

[21] Garrison, S. (2008), "The Road to civil war: An interactive theory of internal political violence", *Defence and Peace Economics* 19(2): 127–151.

[22] Gelman, A. and King, G. (1990), "Estimating Incumbency Advantage Without Bias", *American Journal of Political Science* 34 (4), 1142-1164.

[23] Gençay, R., Selçuk, F. and Witcher, B. (2001), *An Introduction to Wavelets and Other Filtering Methods in Finance and Economics*, Academic Press, San Diego, CA.

[24] Goldstein, J. (1985), "Kondratieff Waves as War Cycles", *International Studies Quarterly* 29(4), 411-444.

[25] Goldstein, J. (1988), *Long Cycles: Prosperity and War in the Modern Age*, Yale University Press.

[26] Goldstein, J. (1991a), "A War-Economy Theory of the Long Wave", N. Thygesen and K. Velupillai (eds.), *Business Cycles: Theories, Evidence, and Analysis.* Basingstoke: MacMillan, 303-325.

[27] Goldstein, J. (1991b), "The Possibility of Cycles in International Relations", *International Studies Quarterly* 35(4), 477-480.

[28] Goodhart, C., and Bhansali, R. (1970), "Political Economy", *Political Studies,* 18(1), 43-106.

[29] Hale, H. E. (2006), "Regime cycles: Democracy, autocracy, and revolution in post-Soviet Eurasia", *World Politics* 58(1): 133–165.

[30] Hamilton, J. (1994), *Time Series Analysis*, Princeton University Press, New Jersey.

[31] Houweling, H. and Siccama, J. G. (1988), *Studies of War.* Dordrecht: Martinus Nijhoff.

[32] Hsu, N.J., (2006), "Log-Memory Wavelet Models", *Statistica Sinica* 16, 1255-1271

[33] Jacoby, W. G. (2010), "The American Voter", in J. E. Leighley (ed.), *The Oxford Handbook of American Elections and Political Behavior.* Oxford: Oxford University Press, 262-277.

[34] Jerôme, Bruno, and Véronique Jerôme-Speziari. (2010), "Forecasting partisan dynamics in Europe", *International Journal of Forecasting* 26(1): 98-115.

[35] Jones, B., Kim, C. J., and Startz, R. (2010), "Does Congress Realign or Smoothly Adjust? A Discrete Switching Model of Congressional Partisan Regimes", *Statistical Methodology,* 7 (3), 254-276.

[36] Kayser, M. A. (2009), "Partisan waves: International business cycles and electoral choice", *American Journal of Political Science* 53(4): 950–970

[37] Kennedy, P. (2008), *A Guide to Econometrics*, Wiley-Blackwell, 6th edition.

[38] Kim, H., and R. C Fording. (1998), "Voter ideology in Western democracies, 1946–1989", *European Journal of Political Research* 33(1): 73–97.

[39] King, G. (2009), "The Changing Evidence Base of Social Science Research", in G. King, K. L. Schlozman, and N. H. Nie (eds.), *The Future of Political Science: 100 perspectives.* New York: Routledge, 91-93.

[40] Kohout, F. (2003), "Cyclical, Hegemonic, and Pluralistic Theories of International Relations: Some Comparative Reflection on War Causation", *International Political Science Review* 24(1), 51-66.

[41] Kondratieff, N. D. (1935), "The Long Waves in Economic Life", *Review of Economic Statistics*, 17, 105-115.

[42] Lebo, M. J. and Box-Steffensmeier, J. (2008), "Dynamic Conditional Correlations in Political Science", *American Journal of Political Science*, 52(3), 688-704.

[43] Lebo, M. J. and Norpoth, H. (2007), "The PM and the Pendulum: Dynamic Forecasting of British Elections", *British Journal of Political Science* 37(1), 71-87.

[44] Levy, J. S. (1983), *War in the Modern Great Power System, 1495-1975.* Lexington, KY: University Press of Kentucky.

[45] Levy, J. S. (1991), "Long Cycles, Hegemonic Transitions, and the Long Peace", in C. W. Kegley, Jr. (ed.), *The Long Postwar Peace.* New York: Harper Collins, 147-176.

[46] Lewis-Beck. M., Norpoth, H. and Jacoby, W. G. (2008), *The American Voter Revisited.* Ann Arbor: University of Michigan Press.

[47] Lewis-Beck, M. S, and M. Stegmaier. (2000), "Economic determinants of electoral outcomes", *Annual Review of Political Science* 3(1): 183–219.

[48] Lichtman, A. J. (1976), "Critical Election Theory and the Reality of American Presidential Politics, 1916-40", *The American Historical Review* 81(2), 317-351.

[49] Lin, T.-M., and Guillén, M. (1998), "The Rising Hazards of Party Incumbency: A Discrete Renewal Analysis", *Political Analysis,* 7(1), 31-57.

[50] Lowell, A. L. (1898), "Oscillations in Politics", *The Annals of the American Academy of Political and Social Science* 12, 69-97.

[51] Mansfield, E. D. (1988), "The Distribution of Wars Over Time", *World Politics* 41(1), 21-51.

[52] Mayhew, D. R. (2002), *Electoral Realignments: A Critique of an American Genre.* New Haven: Yale University Press.

[53] McCarty, N., Poole, K. T., and Rosenthal, H. (2006), *Polarized America: The Dance of Ideology and Unequal Riches.* Cambridge, Mass: MIT Press.

[54] McGinnis, M. D. and Williams, J. T. (1989), "Change and Stability in Superpower Rivalry", *The American Political Science Review,* 83(4), pp. 1101–1123

[55] Merrill, S., B. Grofman, and Brunell, T. L. (2008), "Cycles in American National Electoral Politics, 1854–2006: Statistical Evidence and an Explanatory Model", *American Political Science Review,* 102(1), 1–17.

[56] Merrill, Samuel, Bernard Grofman, and Thomas L. Brunell. (2011), "Do British Party Politics Exhibit Cycles?", *British Journal of Political Science* 41(01): 33-55.

[57] Mintz, A. (2005), "The Method-of-Analysis Problem in International Relations", in A. Mintz and B. Russett (eds.), *New Directions for International Relations. Confronting the Method-of-Analysis Problem.* Lanham, MD: Lexington Books, 3-20.

[58] Mohler, P. P. (2001), "Cycles: Social", in N. J. Smelser and P. B. Baltes (eds.), *International Encyclopedia of the Social and Behavioral Sciences.* Amsterdam: Elsevier, 3204-3207.

[59] Morgenstern, S., S. M Swindle, and A. Castagnola. (2009), "Party Nationalization and Institutions", *The Journal of Politics* 71(04): 1322–1341.

[60] Namenwirth, J. Zvi, and Robert Philip Weber. (1987), *Dynamics of culture.* Allen & Unwin.

[61] Norpoth, H. (1995), "Is Clinton Doomed? An Early Forecast for 1996", *PS: Political Science and Politics,* 28(2), 201-207.

[62] Norpoth, H. and Rusk, J. G. (2007), "Electoral Myth and Reality: Realignments in American Politics", Electoral Studies 26 (2), 392-403.

[63] Oliver, P. E, and D. J Myers. (2002), "The coevolution of social movements", *Mobilization: An International Quarterly* 8(1): 1–24.

[64] Olzak, Susan. (1994), *The dynamics of ethnic competition and conflict.* Stanford University Press.

[65] Patterson, T. E. (1994). *Out of Order.* New York: Vintage.

[66] Ramsey, J. and Lampart, C. (1998), "Decomposition of Economic Relationships by Time Scale Using Wavelets: Money and Income", *Macroeconomic Dynamics,* 2, 49–71.

[67] Reichley, A. J. (2000), *The Life of the Parties: A History of American Political Parties.* Lanham, Md.: Rowman & Littlefield.

[68] Reising, Uwe K. H. (1999), "United in Opposition?" *Journal of Conflict Resolution* 43(3): 317 -342.

[69] Robbins, S. M. and Norpoth, H. (2010), "Balance or Dominance? Party Competition in Congressional Politics", *Political Research Quarterly,* 63(2), 316-327.

[70] Rosecrance, R. (1987), "Long Cycle Theory and International Relations", *International Organization* 41(2), 283-301.

[71] Rosenof, T. (2003), *Realignment: The Theory that Changed the Way we Think About American Politics.* Lanham, MD: Rowman & Littlefield.

[72] Rusk, J. G. (2001), *A Statistical History of the American Electorate.* Washington, DC: CQ Press.

[73] Sayrs, L. (1993), "The Long Cycle in International Relations: A Markov Specification", *International Studies Quarterly*, 37(2), 215–237.

[74] Singer, J. D. and Small, M. (1972), *The Wages of War, 1816-1965: A Statistical Handbook.* New York: John Wiley and Sons.

[75] Stevenson, R. T. (2001), "The Economy and Policy Mood: A Fundamental Dynamic of Democratic Politics?", *American Journal of Political Science* 45(3): 620–633.

[76] Stimson, J. A. (1999), *Public Opinion in America: Moods, Cycles, and Swings.* Boulder: Westview Press.

[77] Stokes, D. E. and Iversen, G. R. (1962), "On the Existence of Forces Restoring Party Competition", *Public Opinion Quarterly* 26 (2), 159-171.

[78] Tarrow, S. (1988), "National politics and collective action: Recent theory and research in Western Europe and the United States." *Annual Review of Sociology* 14: 421–440.

[79] Taylor, J. B. (1999), "An Historical Analysis of Monetary Policy Rules," in *Monetary Policy Rules*, John B. Taylor, ed., University of Chicago Press, pp. 319–41.

[80] Tsebelis, G., and J. Sprague. (1989). "Coercion and revolution: Variations on a predator-prey model", *Mathematical and Computer Modelling* 12(4-5): 547–559.

[81] Weakliem, D. L. (2010), "Time Series Analysis of Political Change" *Handbook of Politics*: 637–651.

[82] Williams, J. (1992), "What Goes Around Comes Around: Unit Root Tests and Cointegration", *Political Analysis*, 4(1), 229–235.

[83] Williams, J. and McGinnis, M., (1992), "The Dimensions of Superpower Rivalry: A dynamic factor analysis", *Journal of Conflict Resolution* 36(1), 86–118.

[84] Wlezien, C. (1995), "The public as thermostat: dynamics of preferences for spending." *American Journal of Political Science* 39(4): 981–1000.

[85] Yogo, M. (2008). Measuring business cycles: A wavelet analysis of economic time series. *Economics Letters* 100: 208–212.

[86] Yousefi, Shahriar, Ilona Weinreich, Dominik Reinarz (2005), "Wavelet-based prediction of oil prices", *Chaos, Solitons & Fractals* 25(2), 265–275.

WEB APPENDIX

Supplementary Information File associated with "Cycles in Politics:
Wavelet Analysis of Political Time-Series"

A The Wavelet

In what follows, $L^2(\mathbb{R})$ denotes the set of square integrable functions, i.e. the set of functions defined on the real line such that $\|x\| := \{\int_{-\infty}^{\infty} |x(t)|^2 \, dt\}^{1/2} < \infty$, with the usual inner product, $\langle x, y \rangle := \int_{-\infty}^{\infty} x(t) y^*(t) dt$. The asterisk superscript denotes complex conjugation. Given a function $x(t) \in L^2(\mathbb{R})$,

$$X(\xi) := \int_{-\infty}^{\infty} x(t) e^{-i2\pi\xi t} dt = \int_{-\infty}^{\infty} x(t) \left[\cos(2\pi\xi t) - i\sin(2\pi\xi t)\right] dt. \tag{A.1}$$

will denote its Fourier transform. We recall the well-known Parseval relation, valid for all $x(t), y(t) \in L^2(\mathbb{R})$, $\langle x(t), y(t) \rangle = \langle X(\xi), Y(\xi) \rangle$, from which the Plancherel identity immediately follows: $\|x(t)\| = \|X(\xi)\|$. The minimum requirements imposed on a function $\psi(t)$ to qualify for being a *mother (admissible or analyzing) wavelet* are that $\psi \in L^2(\mathbb{R})$ and also fulfills a technical condition, usually referred to as the *admissibility condition*, which reads as follows:

$$0 < C_\psi := \int_{-\infty}^{\infty} \frac{|\Psi(\xi)|}{|\xi|} d\xi < \infty, \tag{A.2}$$

where $\Psi(\xi)$ is the Fourier transform of $\psi(t)$,(see Daubechies 1992, 24).

The wavelet ψ is usually normalized to have unit energy: $\|\psi\|^2 = \int_{-\infty}^{\infty} |\psi(t)|^2 \, dt = 1$. The square integrability of ψ is a very mild decay condition; the wavelets used in practice have much faster decay; typical behavior will be exponential decay or even compact support. For functions with sufficient decay, it turns out that the admissibility condition (A.2) is equivalent to requiring $\Psi(0) = \int_{-\infty}^{\infty} \psi(t) \, dt = 0$. This means that the function ψ has to wiggle up and down the $t-$axis,

i.e. it must behave like a wave; this, together with the decaying property, justifies the choice of the term wavelet (originally, in French, *ondelette*) to designate ψ.

A.1 The Continuous Wavelet Transform

Starting with a mother wavelet ψ, a family $\psi_{s,\tau}$ of "wavelet daughters"can be obtained by simply scaling ψ by s and translating it by τ

$$\psi_{s,\tau}(t) := \frac{1}{\sqrt{|s|}} \psi\left(\frac{t-\tau}{s}\right), \quad s,\tau \in \mathbb{R}, s \neq 0. \tag{A.3}$$

The parameter s is a scaling or dilation factor that controls the length of the wavelet (the factor $1/\sqrt{|s|}$ being introduced to guarantee preservation of the unit energy, $\left\|\psi_{s,\tau}\right\| = 1$) and τ is a location parameter that indicates where the wavelet is centered. Scaling a wavelet simply means stretching it (if $|s| > 1$), or compressing it (if $|s| < 1$).[27]

Given a function $x(t) \in L^2(\mathbb{R})$ (a time-series), its continuous wavelet transform (CWT) with respect to the wavelet ψ is a function $W_x(s,\tau)$ obtained by projecting $x(t)$, in the L^2 sense, onto the over-complete family $\{\psi_{s,\tau}\}$:

$$W_x(s,\tau) = \langle x, \psi_{s,\tau} \rangle = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{|s|}} \psi^*\left(\frac{t-\tau}{s}\right) dt. \tag{A.4}$$

When the wavelet $\psi(t)$ is chosen as a complex-valued function, the wavelet transform $W_x(\tau,s)$ is also complex-valued. In this case, the transform can be separated into its real part, $\mathcal{R}(W_x)$, and imaginary part, $\mathcal{I}(W_x)$, or in its amplitude, $|W_x(\tau,s)|$, and phase, $\phi_x(\tau,s)$: $W_x(\tau,s) = |W_x(\tau,s)| e^{i\phi_x(\tau,s)}$. The phase-angle $\phi_x(\tau,s)$ of the complex number $W_x(\tau,s)$ can be obtained from the formula:

$$\phi_x(\tau,s) = \tan^{-1}\left(\frac{\Im\left(W_x(s,\tau)\right)}{\Re\left(W_x(s,\tau)\right)}\right), \tag{A.5}$$

using the information on the signs of $\Re(W_x)$ and $\Im(W_x)$ to determine to which quadrant the angle belongs to.

For real-valued wavelet functions, the imaginary part is constantly zero and the phase is,

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[27]Note that for negative s, the function is also reflected.

therefore, undefined. Hence, in order to separate the phase and amplitude information of a time-series, it is important to make use of complex wavelets. As Lilly and Olhede (2009) explain, analytic wavelets, i.e. wavelets $\psi(t)$ satisfying $\Psi(\xi) = 0$, for $\xi < 0$, are ideal for the analysis of oscillatory signals, since the continuous analytic wavelet transform provides an estimate of the instantaneous amplitude and instantaneous phase of the signal in the vicinity of each time/scale location (τ, s).[28] The importance of the admissibility condition (A.2) comes from the fact that it guarantees that it is possible to recover $x(t)$ from its wavelet transform. When ψ is analytic and $x(t)$ is real, a reconstruction formula is given by

$$x(t) = \frac{2}{C_\psi} \int_0^\infty \left[\int_{-\infty}^\infty \mathcal{R} \left(W_x(s, \tau) \, \psi_{s,\tau}(t) \right) d\tau \right] \frac{ds}{s^2}. \tag{A.6}$$

Therefore, we can easily go from $x(t)$ to its wavelet transform, and from the wavelet transform back to $x(t)$. Note that one can limit the integration over a range of scales, performing a band-pass filtering of the original series. See Daubechies (1992, 27-28) or Kaiser (1994, 70-73) for more details about analytic wavelets.

A.2 Localization Properties

Let the wavelet ψ be normalized so that $\|\psi\| = 1$. Also, assume that ψ and its Fourier transform Ψ have sufficient decay to guarantee that the quantities defined below are all finite. We define the center μ_t of ψ by

$$\mu_t = \int_{-\infty}^\infty t \, |\psi(t)|^2 \, dt. \tag{A.7}$$

In other words, the center of the wavelet is simply the mean of the probability distribution obtained from $|\psi(t)|^2$. As a measure of concentration of ψ around its center one usually takes the standard deviation σ_t:

$$\sigma_t = \left\{ \int_{-\infty}^\infty (t - \mu_t)^2 \, |\psi(t)|^2 \, dt \right\}^{\frac{1}{2}}. \tag{A.8}$$

In a total similar manner, one can also define the center μ_ξ and standard deviation σ_ξ of the Fourier transform $\Psi(\xi)$ of ψ.

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[28]Note that an analytic function is necessarily complex.

The interval $[\mu_t - \sigma_t, \mu_t + \sigma_t]$ is the set where ψ attains its "most significant" values, whilst $[\mu_\xi - \sigma_\xi, \mu_\xi + \sigma_\xi]$ plays the same role for $\Psi(\xi)$. The rectangle $[\mu_t - \sigma_t, \mu_t + \sigma_t] \times [\mu_\xi - \sigma_\xi, \mu_\xi + \sigma_\xi]$ in the (t, ξ) −plane is called the Heisenberg box or window in the time-frequency plane. We then say that ψ is localized around the point (μ_t, μ_ξ) of the time-frequency plane with uncertainty given by $\sigma_t \sigma_\xi$.

The uncertainty principle, first established by Heisenberg in the context of quantum mechanics, gives a lower bound on the product of the standard deviations of position and momentum for a system, implying that it is impossible to have a particle that has an arbitrarily well-defined position and momentum simultaneously. In our context, the Heisenberg uncertainty principle tells us that there is always a trade-off between localization in time and localization in frequency; in particular, we cannot ask for a function to be, simultaneously, band and time limited. To be more precise, the Heisenberg uncertainty principle establishes that the uncertainty is bounded from below by the quantity $1/4\pi$:

$$\sigma_t \sigma_\xi \geq \frac{1}{4\pi}. \tag{A.9}$$

If the mother wavelet ψ is centered at μ_t, has standard deviation σ_t and its wavelet transform $\Psi(\xi)$ is centered at μ_ξ with a standard deviation σ_ξ, then one can easily show that the daughter wavelet $\psi_{\tau,s}$ will be centered at $\tau + s\mu_t$ with standard deviation $s\sigma_t$, whilst its Fourier transform $\Psi_{s,\tau}$ will have center $\frac{\mu_\xi}{s}$ and standard deviation $\frac{\sigma_\xi}{s}$.

From the Parseval relation, we know that $W_x(s, \tau) = \langle x(t), \psi_{s,\tau}(t) \rangle = \langle X(\xi), \Psi_{s,\tau}(\xi) \rangle$. Therefore, the continuous wavelet transform $W_x(s, \tau)$ gives us local information within a time-frequency window $[\tau + s\mu_t - s\sigma_t, \tau + s\mu_t + s\sigma_t] \times [\frac{\mu_\xi}{s} - \frac{\sigma_\xi}{s}, \frac{\mu_\xi}{s} + \frac{\sigma_\xi}{s}]$. In particular, if ψ is chosen so that $\mu_t = 0$ and $\mu_\xi = 1$, then the window associated with $\psi_{\tau,s}$ becomes

$$[\tau - s\sigma_t, \tau + s\sigma_t] \times \left[\frac{1}{s} - \frac{\sigma_\xi}{s}, \frac{1}{s} + \frac{\sigma_\xi}{s} \right] \tag{A.10}$$

In this case, the wavelet transform $W_x(s, \tau)$ will give us information on $x(t)$ for t near the instant $t = \tau$, with precision $s\sigma_t$, and information about $X(\xi)$ for frequency values near the frequency $\xi = \frac{1}{s}$, with precision $\frac{\sigma_\xi}{s}$. Therefore, small/large values of s correspond to information about $x(t)$

in a fine/broad scale and, even with a constant area of the windows, $A = 4\sigma_t\sigma_\xi$, their dimensions change according to the scale; the windows stretch for large values of s (broad scales s – low frequencies $\xi = 1/s$) and compress for small values of s (fine scale s – high frequencies $\xi = 1/s$).

Figure A1 illustrates this major advantage afforded by the wavelet transform, when compared to the Short Time Fourier Transform: its ability to perform natural local analysis of a time-series in the sense that the length of wavelets varies endogenously. It stretches into a long wavelet function to measure the low frequency movements; and it compresses into a short wavelet function to measure the high frequency movements.
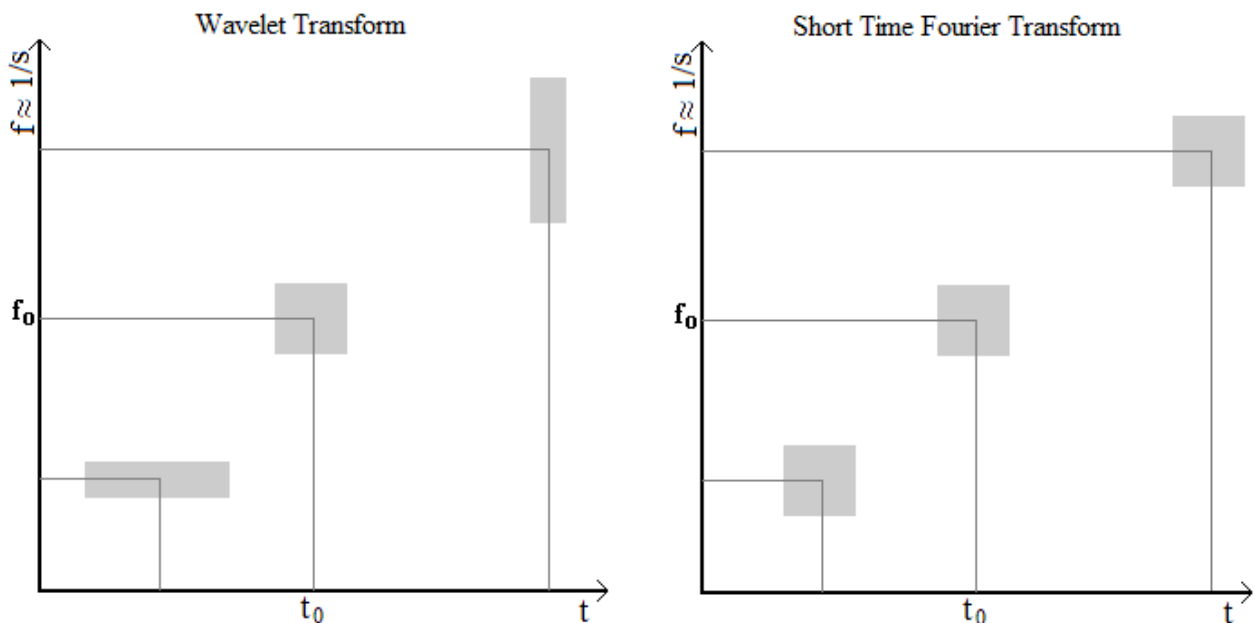

Figure A1: Time-frequency resolution

A.3 The Morlet Wavelet: Optimal Joint Time-Frequency Concentration

There are several types of wavelet functions available with different characteristics, such as Morlet, Paull, Cauchy, Mexican hat, Haar, Daubechies, etc. Since the wavelet coefficients $W_x(s,\tau)$ contain combined information on both the function $x(t)$ and the analyzing wavelet $\psi(t)$, the choice of the wavelet is an important aspect to be taken into account, which will depend on the

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particular application one has in mind. To study cycles, it is important to select a wavelet whose corresponding transform will contain information on both amplitude and phase, and hence, a progressive complex-valued wavelet is a natural choice (the advantage of using a progressive wavelet has already been referred).

We will use the Morlet wavelet, proposed by Goupillaud, Grossman and Morlet (1984):

$$\psi_{\eta}(t) = \pi^{-\frac{1}{4}}\left(e^{i\eta t} - e^{-\frac{\eta^2}{2}}\right)e^{-\frac{t^2}{2}}. \tag{A.11}$$

The term $e^{-\frac{\eta^2}{2}}$ is introduced to guarantee the fulfillment of the admissibility condition; however, for $\eta \geq 5$ this term becomes negligible. The simplified version

$$\psi_{\eta}(t) = \pi^{-\frac{1}{4}}e^{i\eta t}e^{-\frac{t^2}{2}} \tag{A.12}$$

of (A.11) is normally used (and still referred to as a Morlet wavelet).
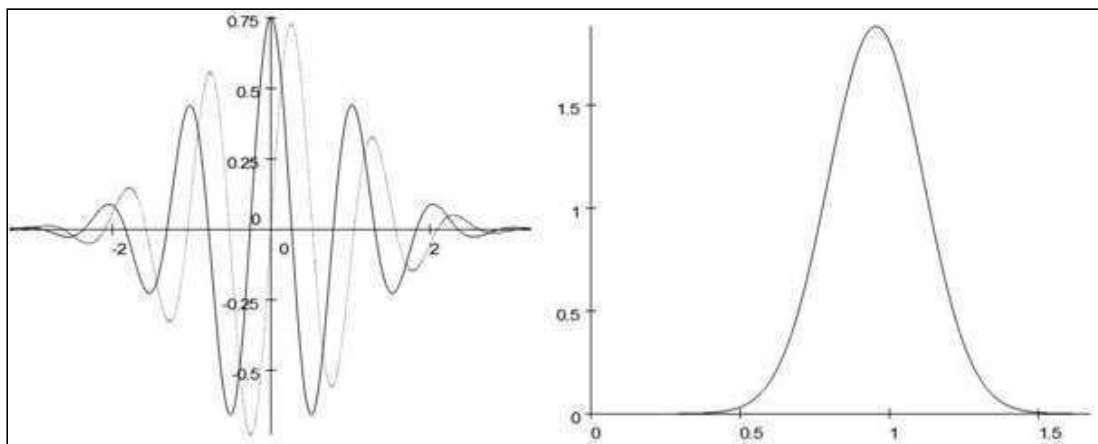

Figure A2: On the left: the Morlet wavelet $\psi_6(t)$ — real part (solid line) and imaginary part (dashed line). On the right: its Fourier transform.

This wavelet has interesting characteristics. For $\eta > 5$, for all practical purposes, the wavelet can be considered as analytic; see Foufoula-Georgiou and Kumar (1994).[29] The wavelet (A.12) is centered at the point $\left(0, \frac{\eta}{2\pi}\right)$ of the time-frequency plane; hence, for the particular choice $\eta = 6$, one has that the frequency center is $\mu_{\xi} = \frac{6}{2\pi}$ and the relationship between the scale and

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[29]We used $\eta = 6$.in all our computations.

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frequency is simply $\xi = \frac{\mu_\xi}{s} \approx \frac{1}{s}$. Thanks to the clear inverse relation between scale and Fourier frequency there is a one-to-one relation between scale and frequency and we will use both terms interchangeably.[30]

It is simple to verify that the time standard deviation is $\sigma_t = 1/\sqrt{2}$ and the frequency standard deviation is $\sigma_\xi = 1/\left(2\pi\sqrt{2}\right)$. Therefore, the uncertainty of the corresponding Heisenberg box attains the minimum possible value $\sigma_t \sigma_\xi = \frac{1}{4\pi}$. In this sense, the Morlet wavelet has optimal joint time-frequency concentration.

A.4 Transform of Finite Discrete Data

If one is dealing with a discrete time-series $x = \{x_n,\ n = 0, \ldots, T-1\}$ of T observations with a uniform time step δt, which we can take as the unity $(\delta t = 1)$, the integral in (A.4) has to be discretized and is, therefore, replaced by a summation over the T time steps; also, it is convenient, for computational efficiency, to compute the transform for T values of the parameter τ, $\tau = m\delta t;\ m = 0, \ldots, T-1$. In practice, naturally, the wavelet transform is computed only for a selected set of scale values $s \in \{s_k, k = 0, \ldots, F-1\}$ (corresponding to a certain choice of frequencies f_k). Hence, our computed wavelet spectrum of the discrete-time series x will simply be a $F \times T$ matrix W_x whose (k, m) element is given by

$$W_x(k, m) = \frac{1}{\sqrt{s_k}} \sum_{n=0}^{T-1} x_n \psi^* \left((n - m) \frac{1}{s_k} \right) \quad k = 0, \ldots, F-1, \quad m = 0, \ldots, T-1. \qquad \text{(A.13)}$$

Although it is possible to calculate the wavelet transform using the above formula for each value of k and m, one can also identify the computation for all the values of m simultaneously as a simple convolution of two sequences; in this case, one can follow the standard procedure and calculate this convolution as a simple product in the Fourier domain, using the Fast Fourier Transform algorithm to go forth and back from time to spectral domain; this is the technique prescribed by Torrence and Compo (1998).

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[30]As Meyers, Kelly and O'Brien (1993) say, "for a general wavelet, the relation between scale and the more common Fourier wavelength is not necessarily straightforward; for example, some wavelets are highly irregular without any dominant periodic components. In those cases it is probably a meaningless exercise to find a relation between the two disparate measures of distance. However, in the case of the Morlet wavelet, which is a periodic wavelet enveloped by a Gaussian, it seems more reasonable."

A.5 Cone of Influence

As with other types of transforms, the CWT applied to a finite length time-series inevitably suffers from border distortions; this is due to the fact that the values of the transform at the beginning and the end of the time-series are always incorrectly computed, in the sense that they involve missing values of the series which are then artificially prescribed. When using the formula (A.13), a periodization of the data is assumed. However, before implementing formula (A.13), we pad the series with zeros, to avoid wrapping. Because of this zero padding, regions afected by edge effects will under estimate the wavelet power. The region in which the transform suffers from these edge effects is called the cone of influence (COI) and, therefore, its results have to be interpreted carefully.

A.6 Wavelet Power Spectrum

In view of the energy preservation formula, and in analogy with the terminology used in the Fourier case, we simply define the (local) wavelet power spectrum as

$$(WPS)_x(s, \tau) = |W_x(s, \tau)|^2, \tag{A.14}$$

which gives us a measure of the local variance.

The seminal paper by Torrence and Compo (1998) is one of the first to give guidance for conducting significance tests for the wavelet power. By using a large number of Monte-Carlo simulations, they derived empirical distributions for the wavelet power corresponding to an AR(0) or a stationary AR(1) process with a certain background Fourier power spectrum (P_ξ), under the null, the corresponding distribution for the local wavelet power spectrum,

$$D\left(\frac{|W_n^x(s)|^2}{\sigma_x^2} < p\right) = \frac{1}{2}P_\xi\chi_v^2,$$

at each time n and scale s. The value of P_ξ is the mean spectrum at the Fourier frequency ξ that corresponds to the wavelet scale s — in our case $s \approx \frac{1}{\xi}$, — and v is equal to 1 or 2, for real or complex wavelets respectively. For more general processes, like an ARMA process, one has to

rely on Bootstrap techniques or Monte Carlo Simulations.

Sometimes the wavelet power spectrum is averaged over time for comparison with classical spectral methods. When the average is taken over all times, we obtain the so-called global wavelet power spectrum:

$$GWPS_x(s) = \int |W_x(\tau, s)|^2 \, d\tau. \tag{A.15}$$

A.7 Cross-Wavelets

A.7.1 Cross-Wavelet Power and Phase-Difference

The cross-wavelet transform of two time-series, $x(t)$ and $y(t)$, first introduced by Hudgins, Friehe and Mayer (1993), is simply defined as

$$W_{xy}(s, \tau) = W_x(s, \tau) \, W_y^*(s, \tau), \tag{A.16}$$

where W_x and W_y are the wavelet transforms of x and y, respectively. The cross-wavelet power is given by $|W_{xy}|$. While we can interpret the wavelet power spectrum as depicting the local variance of a time-series, the cross-wavelet power of two time-series depicts the local covariance between these time-series at each scale and frequency. Therefore, cross-wavelet power gives us a quantified indication of the similarity of power between two time-series.

Torrence and Compo (1998) derived the cross-wavelet distribution assuming that the two time-series have Fourier Spectra P_ξ^x and P_ξ^y. Under the null, the cross-wavelet distribution is given by

$$D\left(\frac{|W_x W_y^*|}{\sigma_x \sigma_y} < p\right) = \frac{Z_v(p)}{v} \sqrt{P_\xi^x \, P_\xi^y},$$

where $Z_v(p)$ is the confidence level associated with the probability p for a pdf defined by the square root of the product of two χ^2 distributions. For more general data generating processes one has to rely on Monte Carlo simulations.

The phase difference, $\phi_{x,y}(s, \tau)$, can be computed from the cross-wavelet transform, by using the formula

$$\phi_{x,y}(s, \tau) = \tan^{-1}\left(\frac{\Im\left(W_{xy}(s, \tau)\right)}{\Re\left(W_{xy}(s, \tau)\right)}\right). \tag{A.17}$$

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It is possible to show that $\phi_{xy} = \phi_x - \phi_y$,[31] justifying its name. A phase difference of zero indicates that the time series move together at the specified time-frequency; if $\phi_{xy} \in (0, \frac{\pi}{2})$, then the series move in phase, but the time-series x leads y; if $\phi_{xy} \in (-\frac{\pi}{2}, 0)$, then it is y that is leading; a Phase-Difference of π (or $-\pi$) indicates an anti-phase relation; if $\phi_{xy} \in (\frac{\pi}{2}, \pi)$, then y is leading; time-series x is leading if $\phi_{xy} \in (-\pi, -\frac{\pi}{2})$.

With the Phase-Difference, one can also calculate the Instantaneous Time-Lag between the two time-series x and y:

$$(\Delta T)_{xy}(\tau, s) = \frac{\phi_{xy}(\tau, s)}{2\pi\xi(\tau)}, \tag{A.18}$$

where $\xi(\tau)$ is the frequency that corresponds to the scale s.

A.7.2 Wavelet Coherency

As in the Fourier spectral approaches, wavelet coherency can be defined as the ratio of the cross-spectrum to the product of the spectra of both series, and can be thought of as the local correlation, both in time and frequency, between two time-series. The wavelet coherency between two time-series, $x(t)$ and $y(t)$, is defined as follows:

$$R_{xy}(s, \tau) = \frac{|S(W_{xy}(s, \tau))|}{|S(W_{xx}(s, \tau))|^{\frac{1}{2}} |S(W_{yy}(s, \tau))|^{\frac{1}{2}}}, \tag{A.19}$$

where S denotes a smoothing operator in both time and scale. Smoothing is necessary. Without that step, coherency is identically one at all scales and times. Smoothing is achieved by a convolution in time and scale. The time convolution is done with a Gaussian and the scale convolution is performed by a rectangular window (see Cazelles et al. 2007 for details). As in the case of the traditional (Fourier) coherency, or the (absolute value of the) correlation coefficient, Wavelet Coherency satisfies the inequality $0 \leq R_{xy}(\tau, s) \leq 1$.

Theoretical distributions for wavelet coherency have recently been derived, by Ge (2008), but only when two stationary Gaussian white noises processes are assumed and we use the Morlet wavelet; for more general processes, one again has to rely on Monte Carlo simulation methods.

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[31]To be more precise, the above relation holds after we convert $\phi_x - \phi_y$ into an angle in the interval $[-\pi, \pi]$.

References

[1] Cazelles, B., Chavez, M., de Magny, G. C., Guégan, J-F and Hales, S. (2007), "Time-Dependent Spectral Analysis of Epidemiological Time-Series with Wavelets", *Journal of the Royal Society Interface*, 4, 625–36.

[2] Daubechies, I. (1992) *Ten Lectures on Wavelets*, CBMS-NSF Regional Conference Series in Applied Mathematics, vol. 61 SIAM, Philadelphia.

[3] Foufoula-Georgiou, E. and Kumar, P. (1994), *Wavelets in Geophysics*, volume 4 of *Wavelet Analysis and Its Applications*. Academic Press, Boston.

[4] Ge, Z. (2008), "Significance Tests for the Wavelet Cross Spectrum Power and Wavelet Linear Coherence", *Annales Geophysicae*, 26, 3819-3829.

[5] Goupillaud, P., Grossman, A. and Morlet, J. (1984), "Cycle-Octave and Related Transforms in Seismic Signal Analysis", *Geoexploration*, 23, 85–102.

[6] Hudgins, L., Friehe, C. and Mayer, M. (1993), "Wavelet Transforms and Atmospheric Turbulence", *Physical Review Letters*, 71:20, 3279-82.

[7] Kaiser, G. (1994), *A Friendly Guide to Wavelets*, Birkhäuser, Basel.

[8] Lilly, J. M., and Olhede, S. C. (2009). "Higher-order Properties of Analytic Wavelets", *IEEE Transactions on Signal Processing*, 57 (1), 146-160.

[9] Meyers, S. D., Kelly, B. G. and O'Brien, J. (1993) "An Introduction to Wavelet Analysis in Oceanography and Meteorology: with Application to the Dispersion of Yanai Waves", *Monthly Weather Review*, 121, 2858-2866.

[10] Torrence, C. and Compo, G. P. (1998), "A Practical Guide to Wavelet Analysis", *Bulletin of the American Meteorological Society*, 79, 605–618.

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