

The ideal structure of nilpotent-generated transformation semigroups

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Abstract

In 1987 Sullivan determined the elements of the semigroup $N(X)$ generated by all nilpotent partial transformations of an infinite set X ; later in 1997 he studied subsemigroups of $N(X)$ defined by restricting the index of the nilpotents and the cardinality of the set. Here, we describe the ideals and the Green's relations on such semigroups, like Reynolds and Sullivan did in 1985 for the semigroup generated by all idempotent total transformations of X . We then use this information to describe the congruences on certain Rees factor semigroups and to construct families of congruence-free semigroups with interesting algebraic properties. We also study analogous questions for X finite and for one-to-one partial transformations.