

Use of AI techniques and updating in geomechanical characterisation

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0. General

The determination of geomechanical parameters is one of the main issues where higher uncertainty lies. The natural heterogeneity of the formations together with the punctual character of the most commonly used tests hinders the establishment of accurate geological/geotechnical models. This is especially true in the preliminary stages of the projects where limited access to the rock mass exists. The rock mass characterisation is normally carried out using *in situ* and laboratory tests together with the application of empirically based rock mass classification systems. These tools together with experience provide a relatively sound basis for geotechnical engineers in their projects.

In the recent years, many advances have been reached in every field of rock mass characterisation. New tests, with particular emphasis for dynamic methods, and improvements on the classification systems are available and constitute as important aids for present and future projects. Also in the numerical fields, developments have been reached. In the Artificial Intelligence (AI) scope, for instance, several successful applications have been developed which prove that these tools can be important in decision support.

During construction, a great amount of geotechnical information is produced which, normally, is not properly stored and analysed. Data Mining techniques, which use AI tools, can help in a more correct analysis of this data taking advantage on the embedded knowledge in a database. The gathered data can be used to update the geotechnical model. This is normally carried out based only on the engineering experience which turns the process variable and user dependent. Bayesian techniques can provide a

framework for a general updating procedure in order to reduce uncertainties. In this work, a general approach to Data Mining and Bayesian updating techniques and how they can be used in geotechnical engineering will be performed. This approach will be complemented by simple application examples.

1. Knowledge discovery in databases and Data Mining

1.1 Introduction

Currently, there is a great expansion of information that needs to be stored and processed. It is important to use computational tools to explore this data which often presents high complexity and can hold valuable information such as trends or patterns that can be very useful (Goebel & Gruenwald, 1999).

In the past, two major approaches have been used for this goal: classical statistics and knowledge from experts. However, the number of human experts is limited and they may overlook important details, while classical statistic analysis does not give adequate answer when large amounts of complex data are available. The alternative is to use automated discovery tools to analyze the raw data and extract new and useful knowledge (Hand et al., 2001).

Due to the awareness of the great potential of this subject there has been an increasing interest in the Knowledge Discovery from Databases (KDD) and Data Mining (DM) fields. These terms are often confused. KDD denotes the overall process of transforming raw data into knowledge and DM is just one step of the KDD process, aiming at the extraction of useful patterns from the observed data. The knowledge derived through DM is often referred to as models or patterns and it is very important that this knowledge is both novel and understandable.

The KDD process consists in the following steps (Figure 1):

- Data selection: the application domain is studied and relevant data is collected.

- Pre-processing or data preparation: noise or irrelevant data is removed, multiple data sources may be combined and prior knowledge can be incorporated.
- Transformation: data is transformed in appropriate forms for the DM process.
- DM: intelligent methods are applied in order to extract models or patterns.
- Interpretation: results from the previous step are studied and evaluated.

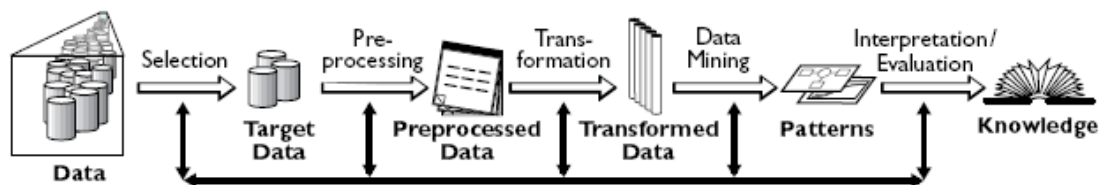


Figure 1 - Phases of the KDD process (Fayyad et al., 1996)

DM is a relatively new area of computer science that lies at the intersection of statistics, machine learning, data management, pattern recognition, artificial intelligence and others. DM is thus emerging as a class of analytical techniques that go beyond statistics and concerns with automatically find, simplify and summarize patterns and relationships within large data sets.

There are several DM techniques, each one with its own purposes and capabilities. Examples of these techniques include Multiple Regression Analysis, Decision Trees and Rule Induction, Neural and Bayesian Networks, Learning Classifier Systems and Instance-Based algorithms (Lee & Siau, 2001; Berthold & Hand, 2003).

1.2 Data Mining

DM consists in the searching and inference of patterns or models in the data which can represent useful knowledge. Depending on what kind of patterns to be found, DM tasks are normally classified into two categories: descriptive and predictive. Descriptive tasks characterize the general properties of the data while predictive perform inference on data in order to make predictions (Han & Kamber, 2000). Descriptive models intend to summarize data in convenient ways to improve the understanding of data while predictive models aim to forecast the unknown value of a variable given known values of other variables (Hand, 2001). In the following items the main DM tasks, models and techniques will be described.

1.2.1 Tasks

Classification is the process of finding a model (or function) which describes different classes in data in order to allow associating a new object to a class according to its characteristics. Normally, the derived model is induced by the classification algorithm based on the analysis of a training set of data. In other words, classification categorizes a certain object into one of several predefined classes. Each object belongs to a certain class among a pre-defined set of classes. The objective of the classification algorithm is to find some relation between attributes and one class in order that the classification process can use that relation to predict the class of a new and unknown object.

Figure 2 presents a hypothetical classification example of a rock mass classification system. It is intended to develop a simpler system only based on the uniaxial compressive strength (UCS in MPa) and the *Rock Quality Designation* (RQD) as classification parameters based on the results of application of another more complex system. The algorithm is applied over a set of examples of the classification system (training data) to find the classification rules. Their accuracy is tested over a different set of examples not used for training (testing data). If the model shows acceptable results it can be used to classify new cases.

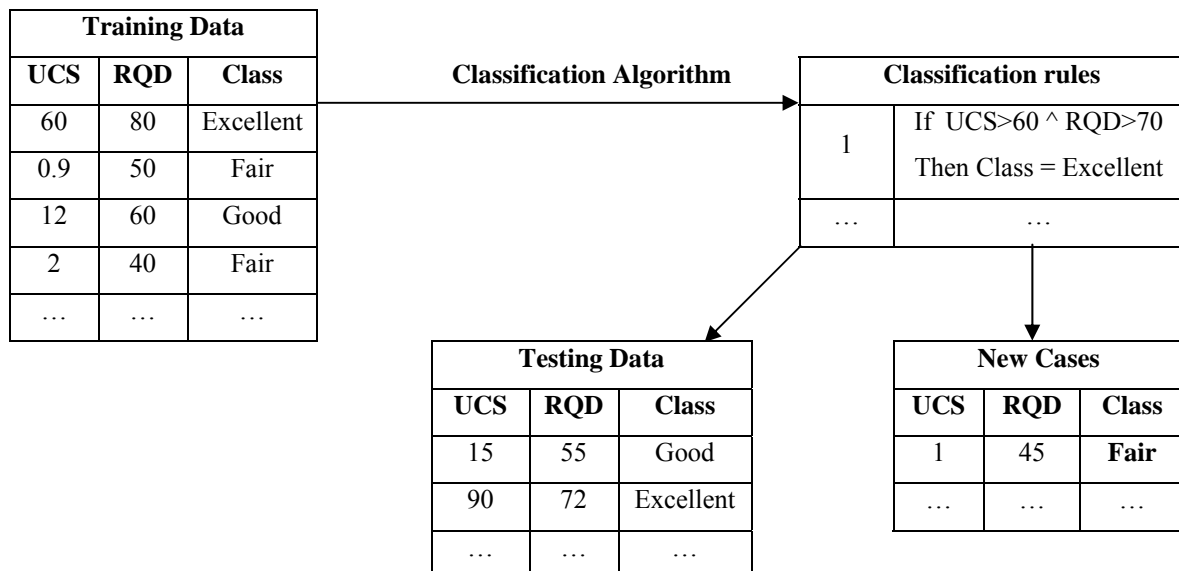


Figure 2 – Classification example with rock mass classification data

Regression is a predictive model, very similar to classification, used for continuous values (in classification the variables are categorical). In fact, the main difference is the nature of the response variable which is, in this case, numerical instead of nominal.

Regression allows obtaining other important information. Using this technique, it can be possible to know the relative importance of each parameter in the prediction of the target variable. This information can be very useful for the comprehension of the physical phenomenon supporting the inferred model. Moreover, regression presents flexibility concerning the input parameters allowing that empirical and/or specialized knowledge is considered in the models. For instance, it is possible to consider an input variable that, based on experience, should be in the model, even though it leads to a small predictive improvement. Inversely, it is possible to exclude variables which one considers should not appear in the model or lead to a substantially reduced model complexity in exchange of some predictive accuracy loss. Finally, it is possible to explore interaction between input variables in the sense that the influence of one input in the target variable depends on the values taken by others.

Association or dependencies deal with finding interesting relationships between items of a given data set. These models describe significant dependencies between variables through the identification of groups of highly associated data. These dependencies can exist at two levels:

- Structural: the model presents locally dependent variables in a graphical way.
- Quantitative: the model specifies the strengths of the dependencies using a numerical scale.

Clustering is the process of grouping *similar* objects into classes. In classification an object is associated in one of several predefined classes while in clustering the classes must be determined by the data. It is a kind of learning by observation other than learning by examples as in classification. Cluster analysis is also referred as unsupervised learning. The clusters are defined by finding groups in data which presents certain similarities. These similarities are evaluated by metrics or probability tools.

1.2.2 Models and techniques

The main issue of the DM task is building a model to represent data. In this step of the KDD process learning occurs by adopting a search algorithm for training. This process occurs over a training set until a given criteria is met. After training the model is built and its quality is normally evaluated over a test set not used for training.

There are several different models but there is no universal one to efficiently solve all the problems (Harrison, 1998). Each one presents specific characteristics (advantages and drawbacks) which make them better suited in a certain case. This section will present the modelling techniques used in this work with exception to the linear and multiple regression which is a widely known topic.

Decision trees and rule induction

A decision tree is a direct and acyclic flow chart that represents a set of rules distinguishing classes or values in a hierarchical form. These rules are extracted from the data, using rule induction techniques, and appear in an “If-Then” structure, similar to the rule presented in Figure 2, expressing a simple and conditional logic. Source data is spitted into subsets, based on the attribute test value and the process is repeated in a recursive manner. Graphically they present a tree structure and are formed by three main components (Figure 3):

- The top node or root that represents all the data.
- Branches which connect nodes. Each internal node represents a test to an attribute while the branches denote the outcome of the test.
- Leafs which are terminal nodes represent classes or values.

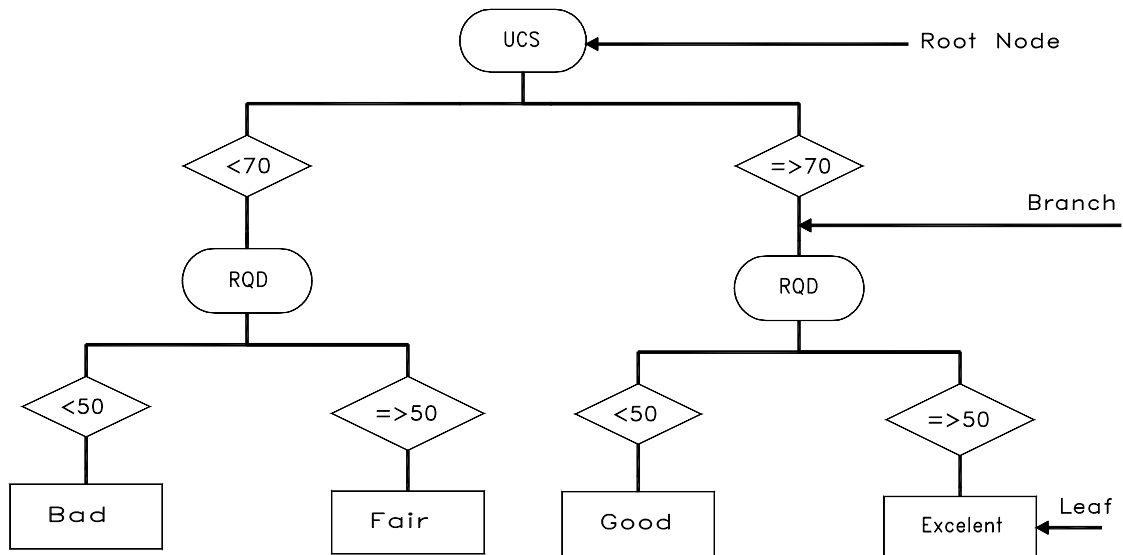


Figure 3 – Example for a decision tree

Considering again the previous example of the hypothetical classification system, in Figure 3 it is presented a possible classification tree for this case where the different components are identified. Each path between the root to a leaf correspond to a decision rule. In this case an example of a decision rule could be:

If $UCS < 70$ and $RQD < 50$ then class = Bad

After a tree is learned it can be used to classify or calculate the value of a new object. There are two types of decision trees namely classification and regression trees (Berry & Linnof, 2000). These two types of trees use the same structure. The only difference is the type of the target variable. Classification trees are used to predict the class to which data belongs while regression trees are used to estimate the value of a continuous variable based on induced mathematical expressions.

The greatest benefits of decision trees approach are that they are easy to understand and interpret. They use a “white box” model i.e. the induced rules are clear and easy to explain as they use a simple conditional logic. Additionally, they can deal with categorical and continuous variables. The main drawback is that they get harder to manage as the complexity of data increases leading to increasing number of branches in the tree.

Artificial Neural networks

Artificial Neural Networks (ANN) were conceived to imitate the biological networks of neurons found in the brain. They are formed by groups of connected artificial neurons in a simplified but very similar structure to the brain neurons. Like the biological structures, ANN can be trained and learn from a set of examples to find solutions to complex problems, recognize patterns and predict future events. The acquired knowledge can then be generalized to solve new problems. This means that they are self-adaptive systems.

ANN are complex parallel computational structures based on connected processing units (neurons) organized in layers. Neurons communicate using signals through input/output connections and each connection has an associated weight. The neuron multiplies each input with the weight of the associated connection. The total input is the sum of all weighted inputs. Finally, an activation function is applied in order to relate the input (stimulation) to the output (response) (Sakellariou and Ferentinou, 2005). This way, the artificial neuron (Figure 4) is composed by three main elements (Cortez, 2002):

- A set of connections which represent synapses.
- The neuron which reduce several inputs to one output.
- An activation function which limits the output amplitude of the neuron and introduce a non-linear component.

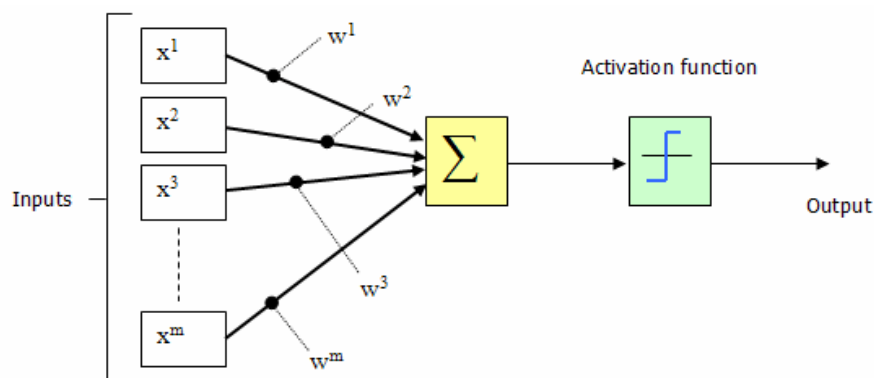


Figure 4 – Scheme of an artificial neuron configuration

where a is a slope parameter. In an ANN, neurons can be connected and organized in many different ways (Santos & Azevedo, 2005). The most used organization is the Multi-layer networks which are composed by different parallel layers. The first is the input and the last the output layer. Intermediate ones are called hidden layers (Figure 5).

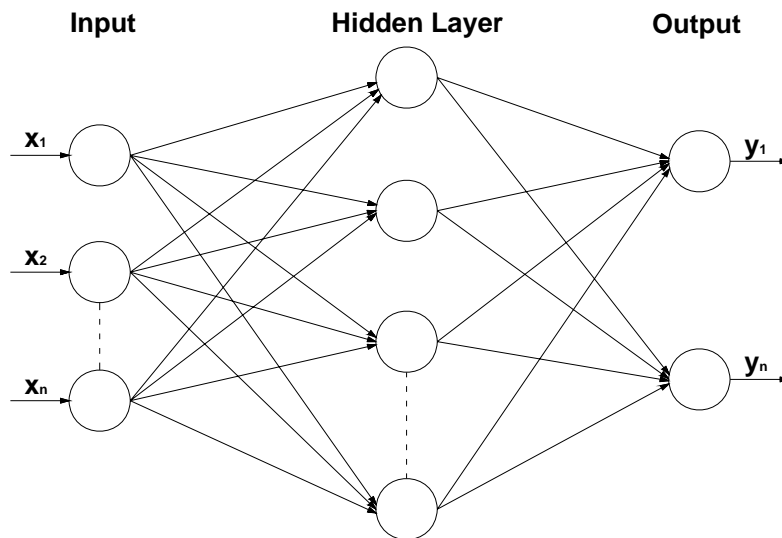


Figure 5 – Scheme of a Multi-Layer network

The connection structure of neurons in a network is normally called architecture or topology. There are several architectures, each one with its own potentialities, but the most used is the multilayer feed-forward (also represented by Figure 5).

The learning process of an ANN is carried out using specific algorithms with very well defined rules. Supervised learning is one of the main method where examples of the inputs together with the correspondent outputs are used in the training process. This allows the network to learn the patterns embedded in the examples. During training, the outputs of the network are compared with the real values resulting in an error measure. This error is used to adjust the weights of the connections in order to minimize it in an iterative process.

There are several different models that have been implemented on ANN. Perceptron networks were the first to be developed. They are one layered feed forward networks with several inputs and outputs. Perceptrons are very simple to use however they are only applicable to problems with low complexity.

Back-propagation networks are the most widely used paradigm in supervised learning. They consist in networks where neurons are distributed on two or more layers. The back-propagation algorithm performs learning in multilayer feed-forward networks. It is based on the selection of an error function whose value is determined by the difference between the outputs of the network and the real values.

Back-propagation networks are powerful learning tools and have been used with success in several applications. They are able to learn from noisily and highly non linear data and can recognize different sets of data within a broader data set.

Model evaluation

After generating the models it is necessary to evaluate their future performance. Normally, this is carried out applying the model to a set of examples not used to induce the model. Holdout and cross-validation are two common techniques for assessing the models accuracy, based on randomly-sampled partitions of the data.

In the holdout method data is randomly partitioned into two independent sets, a training set and a test set. Typically, two thirds of the data are allocated to the training set and the remaining to the test set. Nevertheless, there is no theoretical background to support these values. The training set is used to induce the model whose accuracy is estimated with the test set. The estimate is pessimistic since only a portion of the initial data is used to derive the classifier.

In cross-validation, data is randomly partitioned into k mutually exclusive subsets randomizing for each one the cases within the training and test set. Training and testing is performed k times and the overall error of the model is taken as the average of the errors obtained in each iteration. The values of k can vary between 2 and n (number of cases) however a commonly considered value is 10. It allows using all the available cases in training and test. The accuracy of this technique involves a considerable computational effort (Cortez, 2002).

There are several evaluation techniques that can be applied to the models depending if it is a regression or classification problem. In regression problems the goal is to induce the

model which minimizes an error measurement e between real values and the ones predicted by the model. The most used error measurements are the following:

$$\text{Mean Absolute Deviation: } MAD = \frac{\sum_{i=1}^N |e_i|}{N} \quad 1$$

$$\text{Sum Squared Error: } SSE = \sum_{i=1}^N e_i^2 \quad 2$$

$$\text{Mean Squared Error: } MSE = \frac{SSE}{N} \quad 3$$

$$\text{Root Mean Squared Error: } RMSE = \sqrt{SSE} \quad 4$$

where N is the number of examples. More than one measurement should be used when evaluating the performance of a model since they measure different types of errors. Another way to evaluate the capabilities of the models is to compute the determination coefficient (R^2) which is very common in many statistical applications.

For classification problems one of the most used techniques is the confusion matrix (Kohavi & Provost, 1998). It is used to evaluate the results of a classification indicating the predicted values versus the correct ones. In the lines are disposed the real classes while in the columns the predictions performed by the model. In the main diagonal it is indicated the number of correct guesses while the remaining indicate errors. In Table 1 it is presented the confusion matrix for an example with two classes. In this example the classes are designated as positive and negative.

Table 1 – Confusion matrix for two classes

Class	Predicted C₁	Predicted C₂
Real C₁	True positive TP	False negative FN
Real C₂	False positive FP	True negative TN

With this matrix it is possible to calculate important measures for the model evaluation:

$$\text{Specificity: } spec = \frac{TN}{TN + FP} \times 100\% \quad 5 \quad \text{Sensitivity: } sens = \frac{TP}{TP + FN} \times 100\% \quad 6$$

$$\text{Accuracy: } tacc = \frac{TP + TN}{N} \times 100\% \quad 7 \quad \text{Precision: } prec = \frac{TP}{TP + FN} \times 100\% \quad 8$$

1.3 Application example

1.3.1 Description

Two of the most used classification systems are the RMR (Bieniawski, 1989) and the Q-system (Barton et al, 1974). Most of the times there are some difficulties to apply these classifications systems. Some of the data required to their application may not be available, can lack of reliability or may be difficult/expensive to obtain. Also, the considered parameters may have different importance depending on the type of rock mass being analyzed.

In this work it is intended to develop new alternative models to calculate the RMR index for the particular case of granite rock masses which are very important in the North of Portugal (Miranda et al., 2007 a)). They are intended to use only the most important parameters in the behaviour of granite rock masses with a good predictive accuracy.

This study was carried out using a large database of the empirical systems application in an important underground structure built in a granite rock mass. On this database DM techniques were applied to obtain the new models. Multiple regression techniques and artificial neural networks (ANN) were used. The first are simpler to use and analyze and allow having an insight of which parameters are the most important in the indexes prediction while the latter are more complex and suitable for highly non-linear problems.

1.3.2 The database

Some of the variables histograms presented skewed distributions and others only assumed a few different values. Figure 6 presents the histogram of the RMR variable

one example of a skewed distribution. This fact can influence the quality of the induced models specially those based on neural networks since this kind of algorithm can learn better the behaviour of normally distributed variables. This way, and after some preliminary trial calculations, it was decided to proceed to the transformation of some variables in order to maximize their normality.

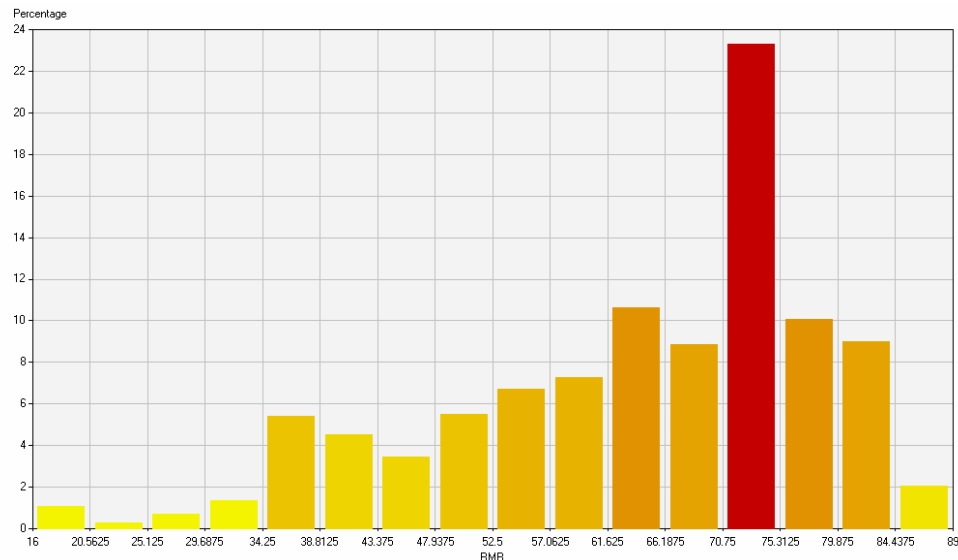


Figure 6 – Histogram of the RMR variable where is possible to observe the skewness of the distribution

The data for these models was assembled from the Venda Nova II powerhouse complex which is an important underground work recently built in the North of Portugal. The interested rock mass is a granite formation so the conclusions drawn in this study are only applicable to formations with similar characteristics.

The collected data was composed by applications of the empirical RMR and Q systems. After some data cleaning work it was then organized and structured in a database composed of 1230 examples and 21 attributes which are described in Table 2. Other attributes were added to the database in order to check their possible influence on the models. Globally, 9 new attributes were added and are presented in Table 3.

The data is based on the results obtained in a granite rock mass. More specifically, the main limitations that should be considered are high uniaxial compressive strength (>100 MPa), RQD values over 65% and slightly wet to dry rock mass. The models developed in this work should only be applied to rock masses with similar characteristics.

Table 2 – Name and description of the attributes in the original database

Name	Description
RQD	<i>Rock Quality Designation</i>
J_w	Factor related with the underground water
J_n	Factor related with the number of discontinuities sets
J_r	Factor related with discontinuities rugosity
J_a	Factor related with the weathering degree of discontinuities
SRF	Factor related with the stress state in the rock mass
Q	Rock mass quality index proposed by Barton et al. (1974)
Q'	Altered form of the Q index ($Q' = RQD/J_n * J_r/J_a$)
RCU	Uniaxial compressive strength
P_1	Weight related with the uniaxial compressive strength of the intact rock
P_2	Weight related with the RQD
P_3	Weight related with discontinuities spacing
P_4	Weight related with discontinuities conditions
P_5	Weight related with the underground water conditions
P_6	Weight related with discontinuities orientation
P_{41}	Discontinuities conditions – persistence
P_{42}	Discontinuities conditions – aperture
P_{43}	Discontinuities conditions – rugosity
P_{44}	Discontinuities conditions – filling
P_{45}	Discontinuities conditions – weathering
RMR	Rock Mass Rating proposed by Bieniawski (1978)

Table 3 – List of attributes added to the original database

Name	Description
RQD/J_n	Ratio which represents the compartmentation of the rock mass
J_r/J_a	Ratio which represents the shear strength of discontinuities
J_w/SRF	Ratio which represents an empirical factor named “active stress”
$\log Q$	Base 10 logarithm of the Q value
$\log Q'$	Base 10 logarithm of the Q' value
GSI	Geological Strength Index proposed by Hoek et al., 2002
N	Altered form of the Q index ($Q' = RQD/J_n * J_r/J_a * J_w$)
RCR	Altered form of the RMR index ($RCR = P_2+P_3+P_4+P_5+P_6$)
RCU	Uniaxial compressive strength

1.3.3 Modelling and evaluation

The SAS Enterprise Miner was used as modelling tool (www.sas.com). It was developed by the SAS Institute to perform DM tasks and combines statistical analysis with graphical interfaces and delivers a wide range of predictive models. The evaluation of the models was carried out using the results provided by this software and complementary calculations on spreadsheets. In the SAS Enterprise Miner, the DM tasks are carried out programming and connecting nodes in a graphical workspace, adjust settings, and run the constructed workflow. In Figure 7 the workflow used for in this work is presented.

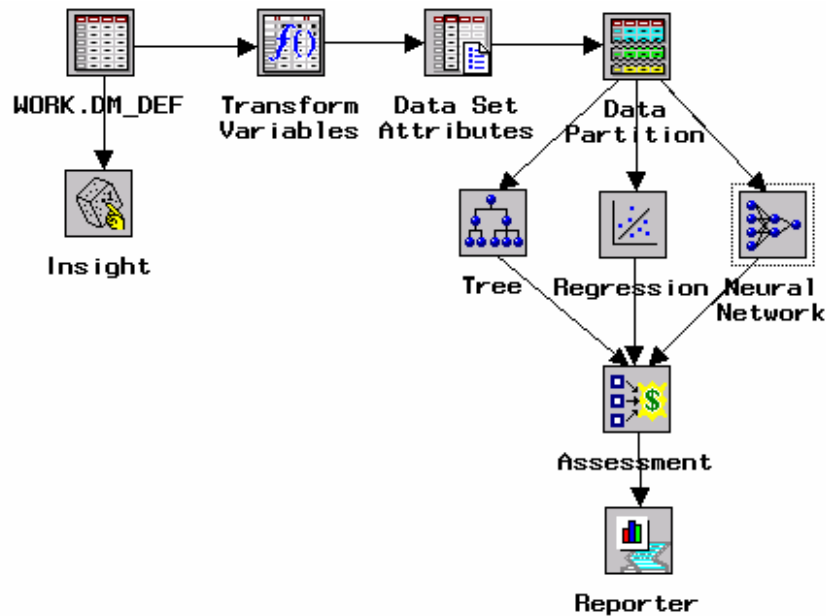


Figure 7 – Workflow used for the DM tasks

The algorithms used for the regression models were multiple regression and ANN. The applied artificial network was a multilayer feed-forward network with one hidden layer of six neurons. Focus was drawn to the multiple regression models because it was intended to obtain the explanatory physical knowledge behind the models (for instance, which were the main attributes in the prediction of a certain variable). Moreover, these models are simpler to use and to implement. The neural network models were used more for comparison purposes. It was not tried to optimize their behaviour for instance changing the numbers of neurons or topology and are an open issue for further research. The used error measures were the MAD and the RMSE.

To validate and assess the models accuracy the holdout method was used. In this method data is randomly partitioned into two independent sets, a training set and a test set. In this case, 2/3 of data was used for training and 1/3 for testing. The training set is used to induce the model and its accuracy is estimated with the test set. For each model 10 runs were carried out randomizing the data within the training and testing sets. The mean and confidence intervals for the error measures were then computed considering the results of the 10 runs and a 95% confidence interval of a T-student distribution. These statistical measures define the range of expected errors for future predictions of the final model which is induced using all the data for training. In addition to the error measures also the determination coefficient (R^2) was used.

1.3.4 Results

The study of all target variables started considering firstly all the attributes. This was done to determine, for the linear regression models and, which were the most important attributes in the prediction of this variable. In Figure 8, a graph of the relative importance of the main attributes for the RMR variable is presented.

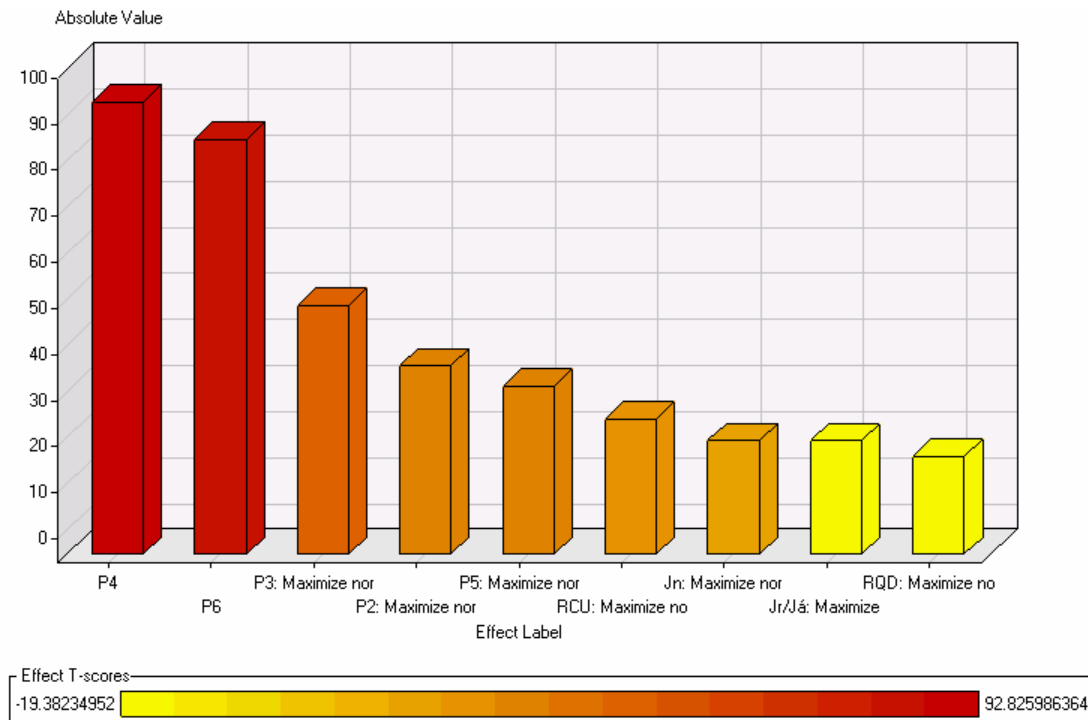


Figure 8 – Relative importance of the attributes for the prediction of the RMR variable

As it was expected, the main parameters which influence the prediction of RMR are the ones directly related to its calculation even though P_1 appears only in an indirect way in the form of the unconfined compressive strength (defined as RCU in the plot). It is important to notice that, among these parameters, the most important are, by far, the ones related with the discontinuities. In particular the parameters related with conditions (P_4) and orientation of discontinuities (P_6) are very good predictors of RMR. Moreover, in the scale of relative importance, the parameters of the Q system also related with discontinuities appear (J_n and J_r/J_a). These facts point out to the conclusion that, in granite formations, the data related with the discontinuities is a very good predictor of the overall quality of the rock.

The next step was to induce models considering only the most important parameters: P_3 , P_4 and P_6 . The obtained regression model was the following:

$$RMR = 35.77 + 0.065 \times P_3^2 + 1.369 \times P_4 + 0.977 \times P_6 \quad 9$$

In Table 4 the results for the regression and ANN models are presented in terms of average errors and determination coefficient and correspondent t-student 95% confidence intervals. These results are concerned only with the testing set since they are the ones related with the behaviour of the models when facing new cases. The results for the models which use all the attributes are presented only for comparison matters.

Table 4 – Results for the models considering all the attributes and the most important ones for the RMR coefficients

All attributes				P_3, P_4 and P_6			
Regression			ANN	Regression			ANN
R^2	MAD	RMSE	RMSE	R^2	MAD	RMSE	RMSE
0.995	0.650	1.094	1.070	0.944	2.565	3.522	2.857
± 0.001	± 0.050	± 0.073	± 0.070	± 0.005	± 0.083	± 0.169	± 0.114

As it was expected, the models which use all attributes are very accurate. The error measures are low and the determination coefficient is close to 1. Using only the three main parameters, the error significantly increases. This is because only half of the parameters used in the original expression are applied. Nevertheless, the error can be considered low for engineering purposes. Analysing the MAD and RMSE values a prediction error around 3 is expected. This means that, for instance, if a rock mass has a “real” RMR value of 65, a value within [62; 68] will be predicted which is acceptable. This expression can be useful for preliminary stages of design or when only information about discontinuities is available or is reliable.

Considering the RMSE, the ANN slightly outperforms the regression models. Only for the ones with less attributes the difference can be considered significant. In this case the RMSE for the ANN is approximately 20% less than the correspondent value of the regression model. In Figure 9 the plot of computed versus predicted RMR values is presented.

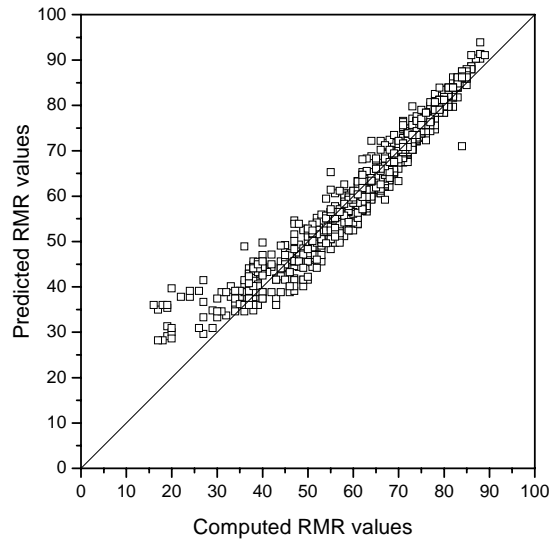


Figure 9 – Computed versus Predicted RMR values for regression model with parameters P_3 , P_4 and P_6

It can be seen that the values lay near a 45 degree slope line which means that the prediction model shows a good accuracy. However, the deviations between real and predicted values increase with decreasing rock mass quality. For RMR values below 30-35 the prediction error increases and the model tends to overestimate the RMR. Above RMR values of 85 this overestimation trend is also observed. Since the model is based in the discontinuities characteristics this fact can be explained by the importance loss of discontinuities for poorer and massive rock masses.

The plot of Figure 9 shows a tail with an almost quadratic trend. In order to minimize this fact a transformation of the RMR variable was performed. Calculations were repeated using RMR^2 and the following regression model was obtained.

$$RMR^2 = 1036.7 + 7.148 \times P_3^2 + 166.3 \times P_4 + 116.7 \times P_6 \quad 10$$

Table 5 resumes the results and Figure 10 presents a plot of real versus predicted values for this model.

Table 5 – Results for the multiple regression model considering parameters P_3 , P_4 and P_6 and using the transformed form of the target variable

Regression		
R^2	MAD	RMSE
0.954 ± 0.004	2.179 ± 0.081	3.172 ± 0.119

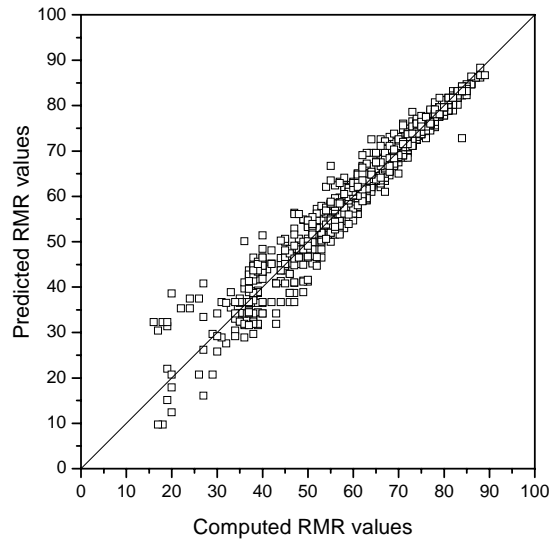


Figure 10 – Real versus Predicted RMR values for regression model with parameters P_3 , P_4 and P_6 and considering the transformation RMR

This transformation led to a slight reduction on the error measurements (approximately 0.4 for each) and confidence intervals and a small increase on R^2 . In Figure 10, a loss of accuracy for lower RMR values can still be observed. However, this happens with higher significance for RMR values below 30 and the overestimation trend is no longer observed as in the previous model. The points are almost equally distributed along the 45 degree slope line which means that the mean prediction error is close to 0. Table 6 summarizes the main issues of the regression models for the two approaches, considering RMR and RMR^2 as the target variables.

Table 6 – Comparison of the main results between the regression models which use RMR and RMR^2 as target variables

Target variable	
RMR	RMR^2
<ul style="list-style-type: none"> - Overestimation trend for $RMR < 35$ and $RMR > 85$. - Good behaviour in a central range of RMR values. - Accuracy lost for poorer rock masses. 	<ul style="list-style-type: none"> - Very good results for $RMR > 50$. - Higher dispersion than previous model in a central range of RMR values ($35 < RMR < 50$). - For the lower range also accuracy lost with no specific trend.

In a merely statistic point of view, the model which uses RMR^2 as the target variable presents a better performance since it has lower error measures and higher R^2 . Also, in this case the error does not follow a specific trend presenting a mean value close to 0. However, for design purposes, the conclusion may not be necessarily the same. In fact, the error measures are very close but when using the model with RMR, one knows that

in a certain range of values, an overestimation trend is expected. When using the other model the expected error is random.

As it was already referred, the Q system related parameters J_n and J_r/J_a are also important to the RMR prediction. These attributes were added to this model and calculations were again performed. However, only negligible increased performance was achieved.

2. Bayesian updating in geomechanics

2.1 Introduction

Determination of geomechanical parameters is an exercise of subjective nature. The inherent uncertainty about their real value hinders the establishment of a deterministic set of values for the parameters. In practice, for each geotechnical zone, a range of values is assigned to the parameters based on the geotechnical survey and, in the case of rock masses, often by application of the empirical classification systems.

In the initial stages, the available information about the rock masses is limited. However, the construction of geotechnical models is a dynamic process and, as the project advances, it can be updated as new data is gathered. Data can have different sources each with its own precision and accuracy. Data uncertainty involves an objective (frequentist) and subjective component: the latter is usually dependent on the geotechnical engineer's experience. Nowadays, a methodology to consistently treat the problem of geomechanical model updating is needed in order to reduce the uncertainty.

The characteristics of the Bayesian methods of data analysis make them well suited for geotechnical purposes where uncertainty is present at several levels and data is compiled in different stages and with different properties. In Figure 11 a general scheme for the deformability modulus (E) calculation and updating during preliminary and construction stages is presented. It consists of a Bayesian framework where E is considered a random variable with a given distribution function. Uncertainty about the

parameter is represented by its standard deviation which can be reduced as more data are obtained through *in situ* tests, mapping of the tunnel face and backanalysis.

In this work, the general Bayesian framework for the geotechnical model updating is presented. It is shown how data from a preliminary geotechnical survey can be updated using *in situ* tests. More specifically, information about E is available by application of the empirical systems and then updated using the results of LFJ tests (LNEC, 1983). Real data from the Venda Nova II powerhouse complex was used for the updating process. Different types of prior information and distributions were considered and results were compared to evaluate the sensitivity of the results to prior assumptions.

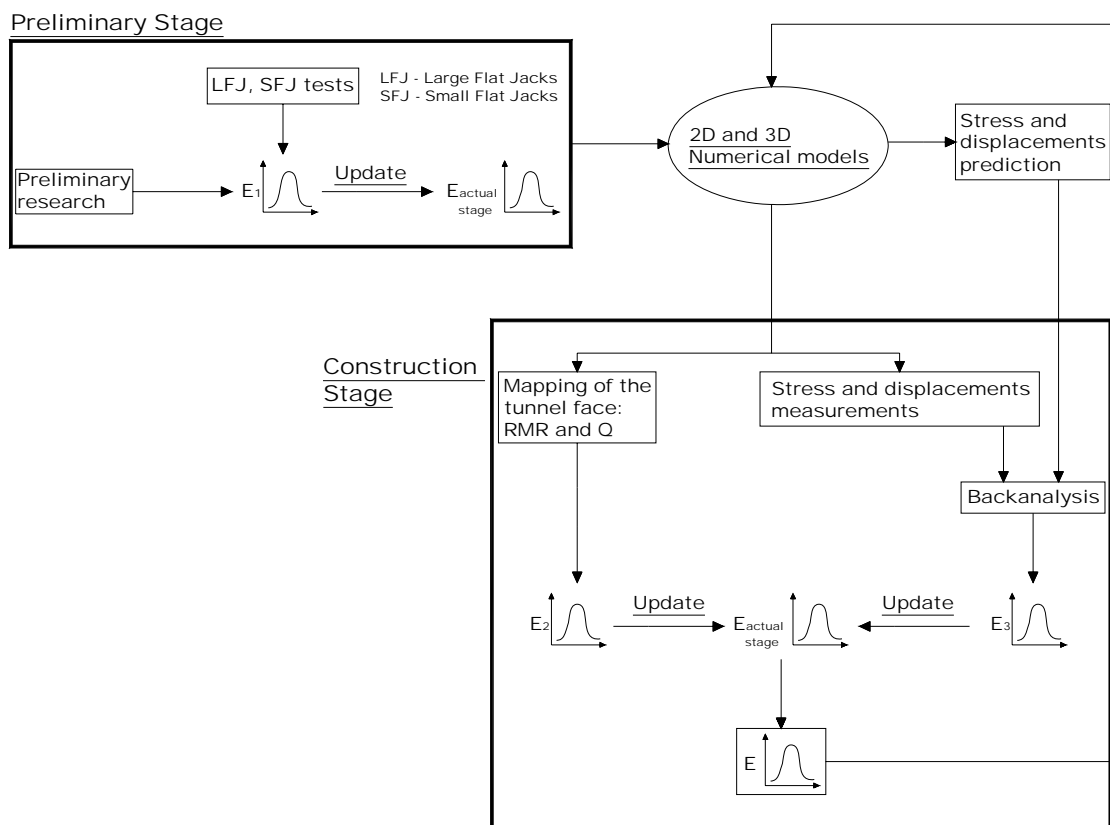


Figure 11 - Scheme of the updating process

2.2 Bayesian data analysis and uncertainty

Uncertainties can be represented in terms of mathematical concepts (Ditlevsen and Madsen, 1996; Einstein, 2006). In many cases it is enough to model the uncertain

quantities using random variables with given distribution functions and parameters estimated on the basis of statistical and/or subjective information (Faber, 2005). The principles and methodologies for data analysis that derive from the subjective point of view are often referred to as Bayesian statistics. Its central principle is the explicit characterisation of all forms of uncertainty in a data analysis problem. The knowledge about an unknown parameter is described by a probability distribution. Probability is used as the fundamental measure of uncertainty.

Bayesian techniques allow one to update random variables when new data are available using a mathematical process in order to reduce uncertainties. This process can be divided into the three following steps (Ditlevsen and Madsen, 1996):

- 1 – Set up a joint probability distribution for all variables.
- 2 – Calculate the conditional posterior distribution of the variables given new data.
- 3 – Evaluate the fit of the model to the data analysing if the conclusions are reasonable and how sensitive, the results are to the modelling assumptions in step 1.

The posterior distribution is a compromise with reduced uncertainty between the prior information and the one contained in the new data (Bernardo & Smith, 2004). As it contains prior and new information the posterior is the updated distribution for the random variable with reduced uncertainty.

Bayesian methods provide tools to incorporate data and external information into the data analysis process. In a Bayesian approach, the data analysis process starts with a given probability distribution. Its parameters may be chosen or estimated based on previous experimental results, experience and professional judgement. This distribution is called prior distribution and represents the uncertainty about the parameter states.

When additional data become available, this is used to update this prior distribution into a posterior distribution using the Bayes theorem. Figure 12 summarizes this overall process.

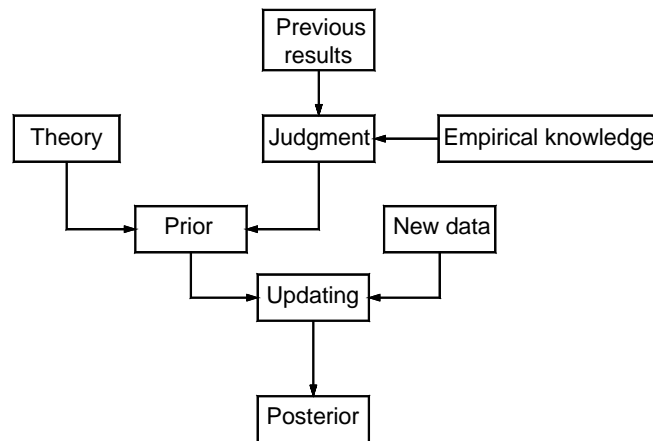


Figure 12 - Scheme of the updating process (adapted from Faber, 2005)

If the prior distribution of a parameter θ , with n possible outcomes $(\theta_1, \dots, \theta_k)$, is continuous and the new information x is available, then the Bayes theorem is given by:

$$p(\theta | x) = \frac{p(\theta)p(x | \theta)}{\int p(\theta)p(x | \theta)d\theta} \quad 11$$

where, $p(\theta)$ is the prior distribution of the possible θ values which summarizes the prior beliefs about the possible values of the parameter, $p(x|\theta)$ is the conditional probability (or likelihood) of the data given θ and $p(\theta|x)$ is the posterior distribution of θ given the observed data x .

The joint probability distribution of the data and the parameter is given by $p(x|\theta)$ which is called the likelihood and is defined by:

$$p(x | \theta) = L(\theta) = \prod_i p(x_i | \theta) \quad 12$$

Bayes theorem consists of multiplying the prior with the likelihood function and then normalizing (term in nominator), to get the posterior probability distribution, which is the conditional distribution of the uncertain quantity given the data. The posterior density summarizes the total information, after considering the new data, and provides a basis for posterior inference regarding θ .

2.3 Bayesian inference

The process of Bayesian inference involves passing from a prior distribution $p(\theta)$ to a posterior distribution $p(\theta|x)$ using the likelihood function of the data. Because the posterior integrate information from the data it will be less variable than the prior. The consideration of normal likelihood, i.e. that data follows a normal distribution, has the computational advantage of allowing the use of conjugate priors or uninformative priors which result in proper posteriors. The central limit theorem helps to justify the use of the normal likelihood and the results are often perfectly acceptable (Dietlevsen & Madsen, 1996). However, the modelling assumptions should always be checked analysing the posterior distribution.

In the Bayesian approach the parameters of interest are assumed to follow certain probability distributions with one or more unknown distribution parameters. These parameters are also considered to have given distributions with known prior hyperparameters. The hyperparameters are then updated given the data and will be used to infer to the parameter distribution. The consideration of variable moments rather than fixed ones intends to incorporate several levels of uncertainty in the model.

In this work a multiparameter model that involves the consideration of both mean and variance as unknowns was used. In the developed Bayesian framework it was considered that both mean (μ) and variance (σ^2) of E were random variables. A normal likelihood was considered together with the conjugate prior. The natural conjugate prior has the following form:

$$p(\mu | \sigma^2) \propto \left(\frac{n_0}{\sigma^2}\right)^{1/2} \exp\left[-\frac{n_0}{2\sigma_0}(\mu - \mu_0)^2\right] \times \left(\frac{1}{\sigma^2}\right)^{\frac{\nu_0}{2}+1} \exp\left[-\frac{S_0}{2\sigma^2}\right] \quad 13$$

where n_0 is the initial number of E values taken from the analytical solutions. This means that the prior is the product of the density of an inverted Gamma distribution with argument σ^2 and ν_0 degrees of freedom and the density of a normal distribution with argument μ , where the variance is proportional to σ^2 . In other words, it is the

density of the so-called normal-gamma distribution. Therefore, the prior for μ conditional on σ^2 is a normal with mean μ_0 and variance σ^2/n_0 :

$$\mu | \sigma^2 \sim N\left(\mu_0, \frac{\sigma^2}{n_0}\right) \quad 14$$

The prior for the precision ($1/\sigma^2$) is a gamma distribution with hyperparameters $\nu_0/2$ and $S_0/2$:

$$\frac{1}{\sigma^2} \sim \text{gamma}\left(\frac{\nu_0}{2}, \frac{S_0}{2}\right) \quad 15$$

The appearance of σ^2 in the conditional distribution of $\mu | \sigma^2$ means that μ and σ^2 are necessarily interdependent. The conditional posterior density of μ , given σ^2 , is proportional to $p(\mu, \sigma^2)$ with σ^2 held constant. After some algebra, it can be shown that

$$\mu | \sigma^2, x \sim N\left(\mu_1, \frac{\sigma^2}{n_1}\right) \quad 16$$

where

$$\mu_1 = \frac{n_0}{n_0 + n} \cdot \mu_0 + \frac{n}{n_0 + n} \cdot \bar{x} \quad 17$$

$$n_1 = n_0 + n \quad 18$$

The parameters of the posterior distribution combine the prior information and the information contained in the data. For example, μ_1 is a weighted average of the prior and of the sample mean, with weights determined by the relative precision of the two pieces of information. The marginal posterior density of $1/\sigma^2$ is gamma:

$$\frac{1}{\sigma^2} | x \sim \text{gamma}\left(\frac{\nu_1}{2}, \frac{S_1}{2}\right) \quad 19$$

where

$$\nu_1 = \nu_0 + n \quad 20$$

$$S_1 = S_0 + (n-1) \cdot s^2 + \frac{n_0 \cdot n}{n_0 + n} \cdot (x - \mu_0)^2 \quad 21$$

The posterior sum of squares, S_1 , combines the prior sum and the sample sum of squares, and the additional uncertainty given by the difference between the sample and the prior mean.

2.4 Posterior simulation

Obtaining the posterior distribution is the fundamental objective of Bayesian analysis. To obtain the complete posterior distributions of the parameters it is normally necessary to use simulation methods. There are several different algorithms to simulate the posterior distributions. One of the most popular is the Markov Chain Monte Carlo (MCMC). Markov chain simulation is a general method based on a sequential draw of sample values with the distribution of the sampled draws depending only on the last value (Brooks, 1998). In probability theory, a Markov chain is a sequence of random variables $\theta_1, \theta_2, \dots, \theta_n$ for which, for any time t , the distribution of θ_t depends only on the most recent value, θ_{t-1} .

The Gibbs sampler is a particular Markov chain algorithm. It is the most popular one and is normally chosen for simulation in conditionally conjugate models, where it is possible to directly sample from each conditional posterior distribution. In this work, the Gibbs sampler was implemented in order to simulate the posterior distributions.

2.5 Application of the Bayesian framework to update the deformability modulus of a rock mass

In this work the general Bayesian framework for updating E was applied in an underground structure (Miranda et al., 2007 b)). The data were collected in the Venda Nova II project and consisted of 40 LFJ tests performed in a gallery. In each test 4 cycles were conducted resulting in a total of 160 values of E . The mean and standard deviation of the tests were 36.9 GPa and 6.1 GPa, respectively.

The mean and variance of E were considered to be unknown variables. The prior was developed considering the information of 76 values of E with mean value of 38.5 GPa, calculated using analytical expressions found in literature based on the empirical classification systems. The parameters of the gamma distribution were computed using this information and the abovementioned formulae.

The normal and lognormal distributions were tested as the probability density functions of the tests and initial data in order to compare the sensitivity of the results to this

assumption. The prior and correspondent updated posterior distributions are presented in Table 7.

Table 7 – Prior and posterior distributions considering the conjugate prior

	Normal distribution	Lognormal distribution
Priors	$\mu \sigma^2 \sim N\left(38.5; \frac{\sigma^2}{76}\right)$	$\mu \sigma^2 \sim N\left(3.489; \frac{\sigma^2}{76}\right)$
	$\frac{1}{\sigma^2} \sim \text{gamma}\left(38.5; \frac{1}{11573.5}\right)$	$\frac{1}{\sigma^2} \sim \text{gamma}\left(38.5; \frac{1}{16.586}\right)$
Posteriors	$\mu \sigma^2 \sim N\left(37.4; \frac{\sigma^2}{236}\right)$	$\mu \sigma^2 \sim N\left(3.560; \frac{\sigma^2}{236}\right)$
	$\frac{1}{\sigma^2} \sim \text{gamma}\left(118.5; \frac{1}{14597.6}\right)$	$\frac{1}{\sigma^2} \sim \text{gamma}\left(118.5; \frac{1}{19.194}\right)$

As the mean is conditional on the variance, prior and posterior estimates for the mean value and standard deviation were obtained by simulation using the Gibbs sampler similarly to the previous example. The main results for the prior and posterior distributions are presented in Tables 8 and 9.

Table 8 – Prior and posterior estimates of the mean value of E (normal distribution)

Parameter	Prior values (GPa)	Posterior values (GPa)
μ	38.5	37.4
$\sigma(\mu)$	2.02	0.73
σ	17.5	11.1
$\sigma(\sigma)$	1.45	0.52
95% CI for the mean	35.2-41.8	36.2-38.6
μ_{pop}	38.4	37.5
σ_{pop}	19.6	11.9
95% CI for the population mean	6.1-70.7	17.9-57.1

Table 9 – Prior and posterior estimates of the mean value of E (lognormal distribution)

Parameter	Prior values (GPa)	Posterior values (GPa)
μ	32.8	35.2
$\sigma(\mu)$	2.47	0.915
σ	1.943	1.498
$\sigma(\sigma)$	0.105	0.028
95% CI for the mean	28.9-37.1	33.6-36.7
μ_{pop}	42.8	38.3
σ_{pop}	36.1	17.3
95% CI for the population mean	9.8-109.2	17.2-71.0

The updated mean value of the mean (μ) underwent a small variation from prior to posterior estimates. In fact this variation was only of about 3% and 7% for the normal and lognormal case, respectively. The initial mean value was already close to the results provided by the LFJ tests. This means that the analytical solutions provided a very good estimate of E.

The most important aspect is the substantial uncertainty reduction at all levels. For the normal distributions case the standard deviation of the mean ($\sigma(\mu)$) has reduced from 2.02 GPa to 0.73 GPa, i.e. only 36% of the initial value. The mean of the standard deviation (σ) underwent a 37% decrease from 17.5 GPa to 11.1 GPa. Finally, the standard deviation of the standard deviation ($\sigma(\sigma)$) was also significantly decreased from 1.45 GPa to 0.51 GPa.

The lognormal distribution follows the same trend of uncertainty reduction. The relative reduction of $\sigma(\mu)$ was very similar to the previous case. In relation to the remaining parameters, σ and $\sigma(\sigma)$, they were reduced in 23% and 73%, respectively.

To illustrate this fact, Figure 13 shows the prior and posterior probability density functions of the mean value of E considering the mean value of its standard deviation. The uncertainty reduction from the prior to the posterior can be clearly observed.

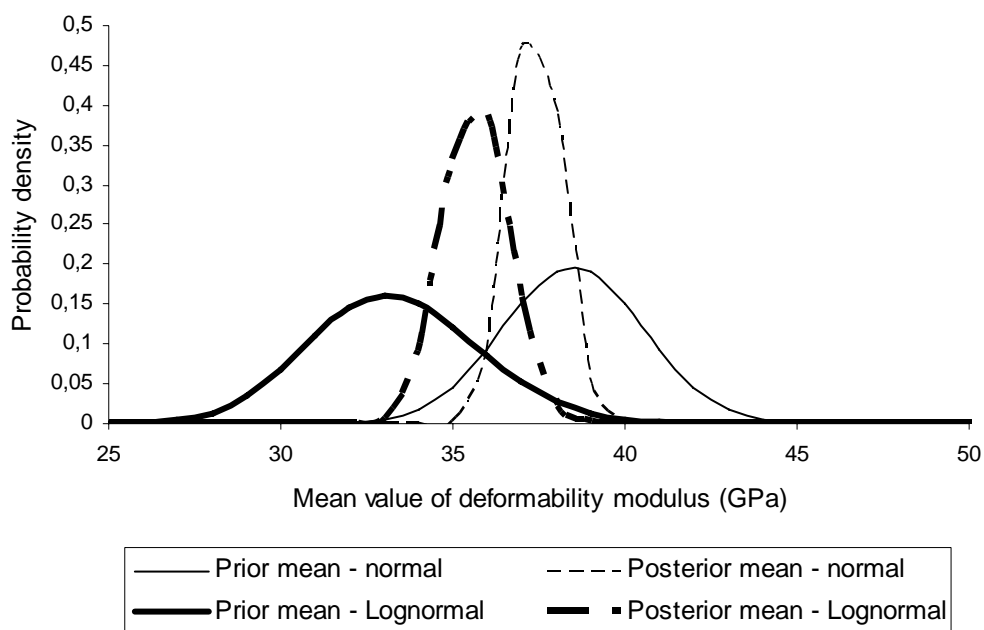


Figure 13 – Prior and posterior probability density functions for the mean value of E

Using simulation it was possible to infer mean and 95% CI for the population. In relation to the mean value the updating process only changed significantly the mean of the lognormal distribution which was reduced in about 11%. For the normal distribution case this value remained almost unchanged.

Also for the population values the updating process allowed a significant reduction on the dispersion measures which means less uncertainty. The standard deviation values were reduced in 39% and 52%, respectively for the normal and lognormal distributions with direct impact on a substantial narrowing of the 95% CI for the mean.

In Figure 14 the prior and posterior probability distributions of E considering the mean values of the mean and standard deviation is presented. The uncertainty about the parameter was clearly reduced using the Bayesian methodology. For the normal distribution case the prior allowed for negative values to have positive probabilities. The updating process corrected this situation. The prior lognormal distribution avoided this situation to happen because it does not allow negative values. The updating enabled to reduce the uncertainty as well as the high skewness of the prior.

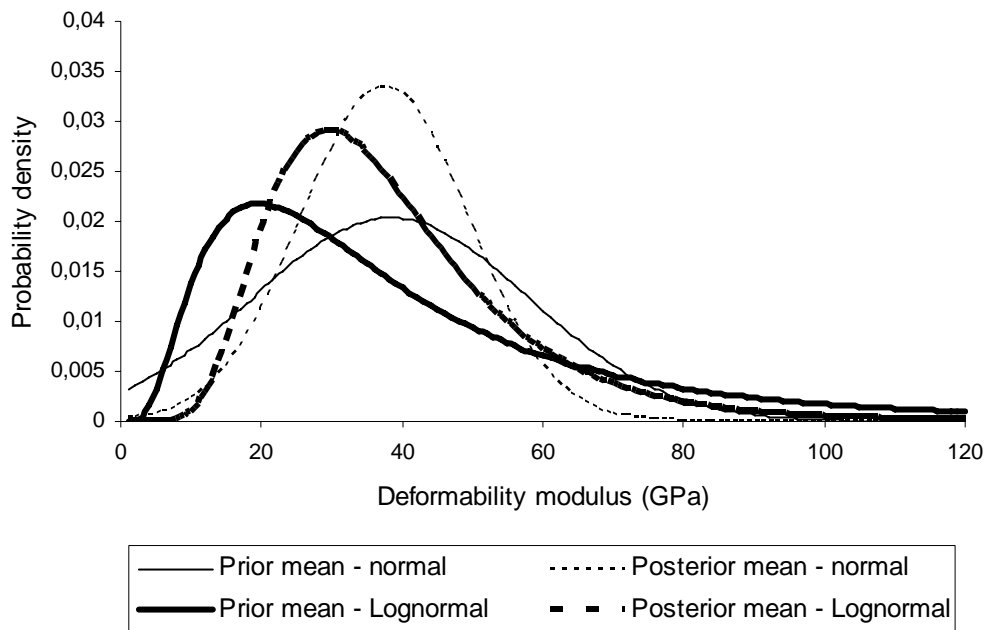


Figure 14 – Prior and posterior probability density functions for E (inferred values for the population)

3. Final remarks

The vast amounts of data which are produced in the different activity fields can not be adequately explored and analysed using classical tools like statistics. Deeper understanding of data and relationships or patterns embedded in highly complex databases urge the need of using “intelligent tools” to uncover them and transform it into useful knowledge. The overall process of the intelligent knowledge discovery in complex databases is called Knowledge Discovery in Databases (KDD). DM is only a step of this process related to the application of the algorithms to induce the models.

In the particular case of geotechnical engineering, vast quantities of data are produced associated for instance to important underground structures. However, and in spite of some very successful applications, its use it is not yet widespread.

Bayesian methods have an inherent flexibility introduced by the incorporation of multiple levels of uncertainty and the resultant ability to combine information from different sources. This methodology allows one to update random variables as new data are collected.

It is believed that the characteristics of the Bayesian data analysis make it well suited to be applied on geotechnical problems where uncertainty is always present at different levels. In geotechnics, the information about the interested formations increases as the project advances for different stages and can be used to update the geotechnical models. Nowadays, this updating is carried out based on empirical knowledge and basic statistic procedures.

The simple examples presented in this work, which used real data, showed how DM and Bayesian techniques can be used in the geomechanical field.

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4. References

- Barton, N., Lien, R., & Lunde, J. 1974. *Engineering Classification of Rock Masses for the Design of Tunnel Support*. Rock Mechanics, Springer-Verlag, Vol. 6, pp. 189-236.
- Bieniawski, Z. T. 1989. *Engineering Rock Mass Classifications*. John Wiley & Sons, 251p.
- Bernardo, J. & Smith, A. 2004. *Bayesian theory*. John Wiley & Sons. 586p.
- Berry, M. J. A., Linoff, G. 2000. *Mastering Data Mining: The Art and Science of Customer Relationships Management*, John Wiley & Sons, Inc., USA.
- Berthold, M. & Hand, D. 2003. *Intelligent Data Analysis: An Introduction*. Springer, Second Edition.
- Brooks, S. 1998. *Markov chain Monte Carlo method and its application*. The Statistician, 47, Part 1, pp. 69-100.
- Cortez, P. 2002. *Models inspired in nature for temporal series prediction*. PhD thesis. University of Minho, Guimarães, Portugal. 214p. (in Portuguese).
- Ditlevsen, O. & Madsen, H. 1996. *Structural reliability methods*. John Wiley & Sons. 372p.
- Einstein, H. 2006. *Use of decision aids for tunnelling. Geotechnical risk in rock tunnels*. Ed. Campos e Matos, Ribeiro e Sousa, Klerberger and Lopes Pinto. Taylor & Francis group, London, pp. 63-74.
- Faber, M. 2005. *Risk and safety in civil, surveying and environmental engineering*. Lecture notes. Swiss Federal Institute of Technology. 394 p.
- Fayyad, U.; Piatetsky-Shapiro, G.; Smyth, P. 1996. *From Data Mining to Knowledge Discovery: An Overview*. In Fayyad et al. (eds) *Advances in Knowledge Discovery and Data Mining*. AAAI Press / The MIT Press, Cambridge MA, pp 471-493.
- Goebel, M. and Gruenwald, L. 1999. *A Survey of Data Mining and Knowledge Discovery Software Tools*. SIGKDD Explorations, 1 (1): 20-33, ACM SIGKDD.
- Hand, D.; Mannila, H.; Smyth, P. 2001. *Principles of Data Mining*. MIT Press, Cambridge, MA.
- Han, J. & Kamber, M. 2000. *Data Mining: Concepts and Techniques*. Morgan Kaufmann Pub., 312p.
- Harrison, T. H. (1998). *Intranet Data Warehouse*, São Paulo, Berkeley. Brasil.
- Kohavi, R. & Provost, F. 1998. *Glossary of Terms, Machine Learning*, 20(2/3):271-274.
- Lee, S. and Siau, K. 2001. *A review of data mining techniques*. Industrial Management & Data Systems, 101 (1): 41-46, MCB University
- LNEC. 1983. *LNEC contribution in the geological-geotechnical studies for the hydraulic circuit of Venda Nova II*. Internal report (in portuguese), 341p.
- Miranda, T.; Gomes Correia, A.; Nogueira, I.; Santos, M.; Cortez, P.; Ribeiro e Sousa, L. 2007 a). *Alternative models for the calculation of the RMR and Q indexes for granite rock masses*. 5th Int. Work. App. Comp. Models in Geo. Eng. Sousa, Fernandes, Vargas Jr. & Azevedo (eds). Taylor & Francis Group. Guimarães, 1-4 April, pp. 151-162.
- Miranda, T.; Gomes Correia, A.; Ribeiro e Sousa, L. 2007 b). *Geomechanical parameters updating in an underground work*. ISRM 2007. Lisbon 9-13 July. In press.
- Sakellariou, M. & Ferentinou, M. 2005. *A study of slope stability prediction using neural networks*. Geotechnical and Geological Engineering 23: 419-445 Springer.
- Santos, M. F. & Azevedo, C. 2005. *Data Mining. Knowledge discovery in databases*. FCA – Editora de Informática, Lda. 194p. (In Portuguese).