

## EXACT FUZZY OBSERVER FOR A BAKER'S YEAST FERMENTATION PROCESS

Enrique Herrera<sup>a,1</sup>, Bernardino Castillo<sup>a</sup>, Jesús Ramírez<sup>b</sup> and Eugénio C. Ferreira<sup>c</sup>

<sup>a</sup>Centro de Investigación y de Estudios Avanzados del I.P.N. Unidad Guadalajara,  
Av. Científica 1145, Colonia el bajo C.P. 45010, Zapopan, México

<sup>b</sup>Centro de Investigación y Asistencia en Tecnología y Diseño del Estado de Jalisco A.C.  
Av Normalistas 800, C.P. 44270, Guadalajara, México

<sup>c</sup>IBB Institute for Biotechnology and Bioengineering, Centre of Biological Engineering,  
Universidade do Minho 4710-057, Braga Portugal

[toledo@gdl.cinvestav.mx](mailto:toledo@gdl.cinvestav.mx), {[eherrera](mailto:eherrera@ciatej.net.mx), [jramirez](mailto:jramirez@ciatej.net.mx)}@ciatej.net.mx, [ecferreira@deb.uminho.pt](mailto:ecferreira@deb.uminho.pt)

**Abstract:** The purpose of this work is to design an exact fuzzy observer for a bioprocess switching between two different metabolic states. A continuous baker's yeast culture is divided in two sub-models: a respiro-fermentative with ethanol production and a respirative with ethanol consumption. An exact fuzzy observer model using sector nonlinearity was built for both nonlinear models; the observer gains were designed using Linear Matrix Inequalities (LMI's). The observer premise variables depend on the state variables estimated by the fuzzy observer. *Copyright © 2007 IFAC*

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### 1. INTRODUCTION

In bioindustries producing food or beverages by using bioprocesses, there are strong demands of systems providing optimization and automatic control of those bioprocesses in terms of quality control and cost savings (Horiouchi, *et al.*, 2002). However, the lack of cheap and reliable sensors providing on line measurements of the biological state variables has hampered the application of automatic control to bioprocess (Bastin and Dochain, 1990). This situation encourages the searching of new software sensors in bioprocesses.

Two class of state observers for (bio)chemical processes can be found in the literature (Dochain, 2003). A first class of observer called asymptotic observer, is based on the idea that the uncertainty in

bioprocess models is located in the process kinetics models. A second class is based on the perfect knowledge of the model structure (Luenberger, Kalman Observers and nonlinear observers). When the perfect knowledge of a nonlinear system is available, also a fuzzy model can be used. A first approach can be done using the Takagi-Sugeno (TS) fuzzy model (Takagi and Sugeno, 1985), where the consequent part of the fuzzy rules are replaced by linear systems. This can be attained, for example, linearizing the model around operational points, getting local linear representation of the nonlinear system.

Another way for obtaining TS models can be achieved using the method of sector nonlinearities, which allows constructing an exact fuzzy model from the original nonlinear system by means of linear subsystems (Tanaka and Wang, 2001). From this exact model, a state observer may be designed based on the linear subsystems. Different fuzzy logic applications to bioprocesses can be found in the

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<sup>1</sup>On Ph. D. studies leave from the Centro de Investigación y Asistencia en Tecnología y Diseño del Estado de Jalisco A. C.

scientific literature (Horiouchi, et al., 1998; Azevedo et al., 2004) among others.

In this work a fuzzy state observer based on sector nonlinearities is proposed and applied to a continuous baker's yeast process. An interesting feature of this model is the splitting in two different partial models: a respire-fermentative (RF) model with ethanol production and the respirative (R) model with ethanol consumption. The switching condition depends on whether the process is consuming or producing ethanol. The observer premise variables depend on the estimated variables by the fuzzy observer. The use of fuzzy observers obtained from an exact fuzzy model, applied to a bioprocess described by partial models has not been, to the best author's knowledge, reported in the literature.

## 2. PRELIMINARIES ON FUZZY MODELS

### 2.1 Takagi-Sugeno Fuzzy Model.

The Takagi-Sugeno fuzzy models are used to represent nonlinear dynamics by means of a set of IF-THEN rules. The consequent part of the rules are local linear systems obtained from specific information about the original nonlinear plant. The  $i$ th rule of a continuous fuzzy model has the following form:

Rule  $i$ :

$$\begin{aligned} & \text{IF } z_1(t) \text{ is } M_1^i \text{ and...and } z_p(t) \text{ is } M_p^i \\ & \text{THEN} \begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases} \quad i = 1, \dots, r. \end{aligned} \quad (1)$$

where  $M_j^i$  is a fuzzy set and  $r$  is the number of rules in the fuzzy model;  $x(t) \in R^n$  is the state vector,  $u(t) \in R^m$  is the input vector,  $y(t) \in R^q$  is the output vector,  $A_i \in R^{n \times n}$ ,  $B_i \in R^{n \times m}$ , and  $C_i \in R^{q \times n}$  are suitable matrices, and  $z(t) = [z_1(t), \dots, z_p(t)]$  is a known vector of premise variables which may depend partially on the state  $x(t)$ . Given a pair of  $(x(t), u(t))$  and using a product inference engine, the aggregate TS fuzzy model can be inferred as:

$$\dot{x}(t) = \frac{\sum_{i=1}^r \varpi_i(z(t)) \{A_i x(t) + B_i u(t)\}}{\sum_{i=1}^r \varpi_i(z(t))} \quad (2)$$

$$= \sum_{i=1}^r h_i(z(t)) \{A_i x(t) + B_i u(t)\}, \quad (3)$$

$$y(t) = \frac{\sum_{i=1}^r \varpi(z(t)) C_i x(t)}{\sum_{i=1}^r \varpi(z(t))} \quad (4)$$

$$= \sum_{i=1}^r h_i(z(t)) C_i x(t), \quad (5)$$

where

$$\begin{aligned} \varpi_i(z(t)) &= \prod_{j=1}^p M_j^i(z_j(t)), \\ h_i(z(t)) &= \frac{\varpi_i(z(t))}{\sum_{i=1}^r \varpi_i(z(t))}, \end{aligned} \quad (6)$$

for all  $t$ . The term  $M_j^i(z_j(t))$  is the membership value of  $z_j(t)$  in  $M_j^i$ . Since

$$\varpi_i(z(t)) \geq 0 \text{ and } \sum_{i=1}^r \varpi_i(z(t)) > 0, \quad i = 1, \dots, r,$$

we have that

$$h_i(z(t)) \geq 0 \text{ and } \sum_{i=1}^r h_i(z(t)) = 1, \quad i = 1, \dots, r,$$

for all  $t$ .

### 2.2 Fuzzy Observers

The state of a system is not always fully available, so it is necessary to use an observer to reconstruct, at least partially the states variables of the process. This requires to satisfy that  $\lim_{t \rightarrow 0} (x(t) - \hat{x}(t)) = 0$ , where

$\hat{x}(t)$  denotes the state vector estimated by the fuzzy observer. There are two cases for fuzzy observers design depending on whether or not  $z(t)$  depends on the state variables estimated by a fuzzy observer (Tanaka and Wang, 2001). Given the TS fuzzy model (1), the  $i$ th rule of a continuous fuzzy observer can be constructed as:

Observer Rule  $i$

$$\begin{aligned} & \text{IF } z_1(t) \text{ is } M_1^i \text{ and...and } z_p(t) \text{ is } M_p^i \\ & \text{THEN} \end{aligned} \quad (7)$$

$$\begin{aligned} \hat{x} &= \sum_{i=1}^r h_i(z(t)) \{A_i \hat{x}(t) + B_i u(t) + K_i (y(t) - \hat{y}(t))\}, \\ \hat{y}(t) &= h_i(z(t)) C_i \hat{x}(t). \end{aligned} \quad (8)$$

where  $K_i$  is the observer gain for the  $i$ th subsystem. If  $z(t)$  depends on the estimated state variables, the observer takes the following form:

$$\hat{x} = \sum_{i=1}^r h_i(z(t)) \{A_i \hat{x}(t) + B_i u(t) + K_i (y(t) - \hat{y}(t))\}, \quad (9)$$

$$\hat{y}(t) = h_i(z(t)) C_i \hat{x}(t). \quad (10)$$

## 3. THE BAKER'S YEAST MODEL

A continuous baker's yeast culture can be represented by the following set of differential equations

$$\begin{aligned} \dot{x}_1 &= (\mu_s^o + \mu_s^r + \mu_e^o) x_1 - D x_1 \\ \dot{x}_2 &= (-k_1 \mu_s^o - k_2 \mu_s^r) x_1 - D x_2 + D S_{in} \\ \dot{x}_3 &= (k_3 \mu_s^r - k_4 \mu_e^o) x_1 - D x_3 \\ \dot{x}_4 &= (-k_5 \mu_s^o - k_6 \mu_e^o) x_1 - D x_4 + OTR \end{aligned} \quad (11)$$

Pormelean (1990) suggested a reformulation of model (11) using two partial models: a respiro-fermentative partial model (RF) with ethanol production and a respirative partial model (R) with ethanol consumption. With this reformulation a split process model is obtained switching from the RF partial model to the R partial model and vice versa depending on whether the system is consuming or producing ethanol. To precise the ideas consider a nonlinear system described by the model (11) which can be written as:

$$\begin{aligned} \dot{x}(t) &= f_i(x(t)) + Bu(t) + d(x(t)) \\ y(t) &= h(x(t)) \end{aligned} \quad (12)$$

where  $f_i(x(t))$  describe both the RF and R partial models, namely

$$f_1 = \begin{bmatrix} (\mu_{s\_RF}^o + \mu_{s\_RF}^r)x_1 - Dx_1 \\ (-k_1\mu_{s\_RF}^o - k_2\mu_{s\_RF}^r)x_1 - Dx_2 \\ k_3\mu_{s\_RF}^r x_1 - Dx_3 \\ -k_5\mu_{s\_RF}^o x_1 - Dx_4 \end{bmatrix} := f_{RF} \quad (13)$$

and for the R model

$$f_2 = \begin{bmatrix} (\mu_{s\_R}^o + \mu_{e\_R}^o)x_1 - Dx_1 \\ -k_1\mu_{s\_R}^o x_1 - Dx_2 \\ -k_4\mu_{e\_R}^o x_1 - Dx_3 \\ (-k_5\mu_{s\_R}^o - k_6\mu_{e\_R}^o)x_1 - Dx_4 \end{bmatrix} := f_R, \quad (14)$$

and  $u(t)=S_{in}$ ,  $B=[0 \ D \ 0 \ 0]^T$  and  $h(x(t))=x_3$ . The oxygen transfer rate OTR is assumed to be a measurable and known perturbation, and thus

$$d = [0 \ 0 \ 0 \ OTR]^T \quad (15)$$

The variables and parameter values used in (11-14) are shown in table 1. Yield coefficients values for  $k_I$ ,  $k_2$ ,  $k_3$ ,  $k_4$ ,  $k_5$ ,  $k_6$  and the oxygen transfer rate (OTR) are described in Ferreira, (1995)

Table 1 Parameters and variables used in the baker's yeast model.

Variable / parameter	units
$x_1$ = biomass	g/l
$x_2$ = substrate	g/l
$x_3$ = ethanol	g/l
$x_4$ = dissolved oxygen	mg/l
$S_{in}$ = inlet substrate concentration	g/l
$D$ = dilution rate	$h^{-1}$
$\mu_s^o$ = specific growth rate for respiratory growth on glucose	$h^{-1}$
$\mu_s^r$ = specific growth rate for fermentative growth on glucose	$h^{-1}$
$\mu_e^o$ = specific growth rate for respiratory growth on ethanol.	$h^{-1}$

The specific rates for the RF partial model are given by:

$$\mu_{s\_RF}^o = Y_{O2} \left( q_o^{\max} \frac{x_4}{Ko + x_4} \right) \quad (16)$$

$$\mu_{s\_RF}^r = Y_r \left( q_s^{\max} \frac{x_2}{Ks + x_2} - q_o^{\max} \frac{x_4}{Ko + x_4} \frac{Y_{O2}}{Y_o} \right). \quad (17)$$

For the R partial model the specific rates are given by

$$\mu_{s\_R}^o = Y_o \left( q_s^{\max} \frac{x_2}{Ks + x_2} \right), \quad (18)$$

and  $\mu_{e\_R}^o$ , which can take the following values

$$\mu_{e\_R}^o = \begin{cases} q_{e_1} & \text{IF } q_{e_1} < q_{e_2} \\ q_{e_2} & \text{IF } q_{e_1} \geq q_{e_2} \end{cases} \quad (19)$$

where

$$q_{e_1} = Y_e q_e^{\max} \frac{x_3}{Ke + x_3} \frac{Ki}{Ki + x_2} \quad (20)$$

$$q_{e_2} = Y_{O2} e^{\left( q_o^{\max} \frac{x_4}{Ko + x_4} - \frac{Y_o}{Y_{O2}} \left( q_s^{\max} \frac{x_2}{Ks + x_2} \right) \right)} \quad (21)$$

From equation (20) and (21) it can be inferred that the R partial model given by (14) should be split in two new models: (Rqe1) when  $\mu_{e\_R}^o = q_{e_1}$  and (Rqe2) when  $\mu_{e\_R}^o = q_{e_2}$ . The switching condition between the RF and R partial model is described by Ferreira, (1995). The yield coefficients used in the specific rates (16-21) and in the partial models (13) and (14) are those reported by Sonnleitner and Käppeli (1986). To change between the RF to the R models,  $S_{in}$  was varied according to figure 1, while the dilution rate was set to  $0.2 \text{ h}^{-1}$ .

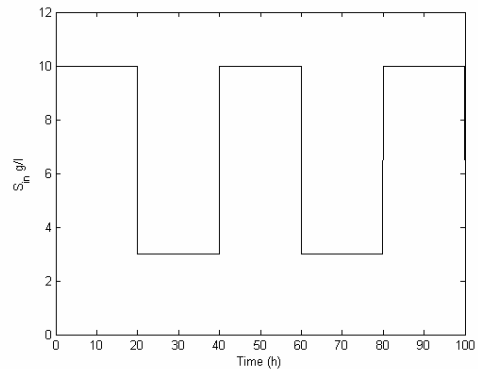


Fig. 1. The inlet substrate concentration signal

#### 4. THE EXACT FUZZY MODEL

When the nonlinear dynamic model for the baker's yeast is known, as well as all their parameters, a fuzzy exact model can be derived from the given nonlinear model. This requires a sector nonlinearity

approach (Tanaka and Wang, 2001). To construct the RF and R exact fuzzy models,  $f_{RF}$  can be expressed as

$$f_{RF} = \begin{bmatrix} \frac{x_4}{Ko+x_4} q_o^{\max} \left( Y_{O2} - Y_r \frac{Y_{O2}}{Y_o} \right) - D & Y_r q_s^{\max} \frac{x_1}{Ks+x_2} & 0 & 0 \\ \frac{x_4}{Ko+x_4} q_o^{\max} \left( -k_1 Y_{O2} + k_2 Y_r \frac{Y_{O2}}{Y_o} \right) & -k_2 Y_r q_s^{\max} \frac{x_1}{Ks+x_2} - D & 0 & 0 \\ -k_3 Y_r q_o^{\max} \frac{Y_{O2}}{Y_o} \frac{x_4}{Ko+x_4} & k_3 Y_r q_s^{\max} \frac{x_1}{Ks+x_2} & -D & 0 \\ -k_3 Y_{O2} q_o^{\max} \frac{x_4}{Ko+x_4} & 0 & 0 & -D \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (22)$$

and according to (20) and (21)  $f_R$  should be split to

$$f_{Rq1} = \begin{bmatrix} Y_r q_s^{\max} Ki \frac{x_3}{(Ke+x_3)(Ki+x_2)} - D & Y_o q_s^{\max} \frac{x_1}{Ks+x_2} & 0 & 0 \\ 0 & -k_1 Y_o q_s^{\max} \frac{x_1}{Ks+x_2} - D & 0 & 0 \\ -k_1 Y_r q_s^{\max} Ki \frac{x_3}{(Ke+x_3)(Ki+x_2)} & 0 & -D & 0 \\ -k_1 Y_r q_s^{\max} Ki \frac{x_3}{(Ke+x_3)(Ki+x_2)} & -k_2 Y_o q_s^{\max} \frac{x_1}{Ks+x_2} & 0 & -D \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (23)$$

$$f_{Rq2} = \begin{bmatrix} Y_{O2} q_o^{\max} \frac{x_4}{Ko+x_4} - D & \frac{x_1}{Ks+x_2} q_s^{\max} \left( Y_o - Y_{O2} e^{\frac{Y_o}{Y_{O2}}} \right) & 0 & 0 \\ 0 & -k_1 Y_o q_s^{\max} \frac{x_1}{Ks+x_2} - D & 0 & 0 \\ -k_4 Y_{O2} q_o^{\max} \frac{x_4}{Ko+x_4} & k_4 Y_{O2} q_s^{\max} \frac{x_1}{Ks+x_2} \frac{Y_o}{Y_{O2}} & -D & 0 \\ -k_6 Y_{O2} q_o^{\max} \frac{x_4}{Ko+x_4} & \frac{x_1}{Ks+x_2} q_s^{\max} \left( -k_5 Y_o + k_4 Y_{O2} e^{\frac{Y_o}{Y_{O2}}} \right) & 0 & -D \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (24)$$

From (22-24) the fuzzy exact model using sector nonlinearities can be constructed. The premise variables are chosen as:

$$z_1(t) = \frac{x_4}{Ko+x_4} \quad (25) \quad z_2(t) = \frac{x_1}{Ks+x_2} \quad (26)$$

$$z_3(t) = \frac{x_3}{(Ke+x_3)(Ki+x_2)}. \quad (27)$$

To construct the membership functions we use

$$z_1(t) = \sum_{i=1}^2 M_i(z_1(t)) a_i \quad (28)$$

where the property  $M_1(z_1(t)) + M_2(z_1(t)) = 1$  must be accomplished (Tanaka and Wang, 2001). The membership functions for the RF partial model can be built from the premise variables (25–26) assuming the ranges  $x_1(t) \in [0, 10]$ ,  $x_2(t) \in [0, 1]$ ,  $x_3(t) \in [0, 10]$  and  $x_4(t) \in [0, 0.007]$

$$M_1(z_1(t)) = \frac{z_1(t) - a_2}{a_1 - a_2}, \quad M_2(z_1(t)) = \frac{-z_1(t) + a_1}{a_1 - a_2}, \quad (29)$$

$$N_1(z_2(t)) = \frac{z_2(t) - b_2}{b_1 - b_2}, \quad N_2(z_2(t)) = \frac{-z_2(t) + b_1}{b_1 - b_2}, \quad (30)$$

where

$$a_1 = \max_{x_4(t)} z_1(t) = 0.9859, \quad a_2 = \min_{x_4(t)} z_1(t) = -0.01,$$

$$b_1 = \max_{x_1(t), x_2(t)} z_2(t) = 50, \quad b_2 = \min_{x_1(t), x_2(t)} z_2(t) = -0.01, \\ c_1 = \max_{x_2(t), x_3(t)} z_3(t) = 9.8039, \quad c_2 = \min_{x_2(t), x_3(t)} z_3(t) = -0.01.$$

The fuzzy rules for the RF partial model are then:

IF  $z_1(t)$  is “ $M_1(z_1(t))$ ” and  $z_2(t)$  is “ $N_1(z_2(t))$ ”  
THEN  $\dot{x}_{11}^{RF}(t) = A_{11}^{RF} x(t) + Bu(t) + d$   
IF  $z_1(t)$  is “ $M_1(z_1(t))$ ” and  $z_2(t)$  is “ $N_2(z_2(t))$ ”  
THEN  $\dot{x}_{12}^{RF}(t) = A_{12}^{RF} x(t) + Bu(t) + d$   
IF  $z_1(t)$  is “ $M_2(z_1(t))$ ” and  $z_2(t)$  is “ $N_1(z_2(t))$ ”  
THEN  $\dot{x}_{21}^{RF}(t) = A_{21}^{RF} x(t) + Bu(t) + d$   
IF  $z_1(t)$  is “ $M_2(z_1(t))$ ” and  $z_2(t)$  is “ $N_2(z_2(t))$ ”  
THEN  $\dot{x}_{22}^{RF}(t) = A_{22}^{RF} x(t) + Bu(t) + d,$

where the linear subsystems  $A_{11}^{RF}, A_{12}^{RF}, A_{21}^{RF}, A_{22}^{RF}$  are derived from

$$A_{ij}^{RF} = \begin{bmatrix} a_i q_o^{\max} \left( Y_{O2} - Y_r \frac{Y_{O2}}{Y_o} \right) - D & Y_r q_s^{\max} b_j & 0 & 0 \\ a_i q_o^{\max} \left( -k_1 Y_{O2} + k_2 Y_r \frac{Y_{O2}}{Y_o} \right) & -k_2 Y_r q_s^{\max} b_j - D & 0 & 0 \\ -k_3 a_i Y_r q_o^{\max} \frac{Y_{O2}}{Y_o} & k_3 Y_r q_s^{\max} b_j & -D & 0 \\ -k_3 a_i Y_{O2} q_o^{\max} & 0 & 0 & -D \end{bmatrix} \quad (31) \\ i, j = 1, 2$$

The final aggregated model turns to be:

$$\dot{x}^{RF}(t) = \sum_{i=1}^2 \sum_{j=1}^2 M_i(z_1(t)) N_j(z_2(t)) \{ A_{ij}^{RF} x(t) + Bu(t) + d \} \\ y^{RF}(t) = \sum_{i=1}^2 \sum_{j=1}^2 M_i(z_1(t)) N_j(z_2(t)) Cx(t)$$

This fuzzy model exactly represents the RF partial nonlinear model in the region  $x_1(t) \in [0, 10]$ ,  $x_2(t) \in [0, 1]$ ,  $x_3(t) \in [0, 10]$  and  $x_4(t) \in [0, 0.007]$ . The fuzzy exact model for the Rq1 and Rq2 models were constructed following the same procedure.

## 5. THE EXACT FUZZY OBSERVER

After an exact fuzzy model for the nonlinear baker's yeast partial model has been obtained, a fuzzy observer can now be designed. The following assumptions are made:

H1. The yield coefficients  $k_1, k_2, k_3, k_4, k_5$  and  $k_6$  are constant and known.

H2. The ethanol, the dissolved oxygen concentration and the OTR are known.

When the output of the RF partial model is the ethanol and the subsystems given by (31) are used observability full rank cannot be achieved and only is possible to estimate biomass, substrate and ethanol. With this approach dissolved oxygen can not be estimated. However, the observer can still be designed, due to the fact that dissolved oxygen and OTR are usually available on line. The same

situation applies with the Rq1 and Rqe2 partial models. Since the dissolved oxygen is measurable, the premise variable  $z_1(t)$  is taken as in (25). However for  $z_2(t)$  and  $z_3(t)$ , we have to consider the estimates, namely

$$\hat{z}_2(t) = \frac{\hat{x}_1}{Ks + \hat{x}_2} \quad (32)$$

$$\hat{z}_3(t) = \frac{x_3}{(Ke + \hat{x}_2)(Ki + x_3)} \quad (33)$$

To construct the fuzzy observer for the RF partial model the procedure described by Tanaka and Wang (2001) is followed. The membership function for the premise variable  $z_1(t)$  is once again given by (29); however, for  $\hat{z}_2(t)$  the membership function is described by

$$N_1(\hat{z}_2(t)) = \frac{\hat{z}_2(t) - b_2}{b_1 - b_2}, \quad N_2(\hat{z}_2(t)) = \frac{-\hat{z}_2(t) + b_1}{b_1 - b_2}, \quad (34)$$

To construct the RF observer the linear subsystems  $A_{11}^{RF}, A_{12}^{RF}, A_{21}^{RF}, A_{22}^{RF}$  are derived from

$$A_{ij}^{RF} = \begin{bmatrix} a_i q_o^{\max} \left( Y_{O_2} - Y_r \frac{Y_{O_2}}{Y_o} \right) - D & b_j Y_r q_s^{\max} & 0 \\ a_i q_o^{\max} \left( -k_i Y_{O_2} + k_2 Y_r \frac{Y_{O_2}}{Y_o} \right) & -k_2 b_j Y_r q_s^{\max} - D & 0 \\ -k_3 a_i Y_r q_o^{\max} \frac{Y_{O_2}}{Y_o} & k_3 b_j Y_r q_s^{\max} & -D \end{bmatrix} \quad (35)$$

$i, j = 1, 2$

The fuzzy rules for the RF partial model observer are stated as

IF  $z_1(t)$  is " $M_1(z_1(t))$ " and  $z_2(t)$  is " $N_1(\hat{z}_2(t))$ "

THEN

$$\hat{x}_{11}^{RF}(t) = A_{11}^{RF} \hat{x}(t) + Bu(t) + K_{1\_RF}(y(t) - \hat{y}(t))$$

IF  $z_1(t)$  is " $M_1(z_1(t))$ " and  $z_2(t)$  is " $N_2(\hat{z}_2(t))$ "

THEN

$$\hat{x}_{12}^{RF}(t) = A_{12}^{RF} \hat{x}(t) + Bu(t) + K_{2\_RF}(y(t) - \hat{y}(t))$$

IF  $z_1(t)$  is " $M_2(z_1(t))$ " and  $z_2(t)$  is " $N_1(\hat{z}_2(t))$ "

THEN

$$\hat{x}_{21}^{RF}(t) = A_{21}^{RF} \hat{x}(t) + Bu(t) + K_{3\_RF}(y(t) - \hat{y}(t))$$

IF  $z_1(t)$  is " $M_2(z_1(t))$ " and  $z_2(t)$  is " $N_2(\hat{z}_2(t))$ "

THEN

$$\hat{x}_{22}^{RF}(t) = A_{22}^{RF} \hat{x}(t) + Bu(t) + K_{4\_RF}(y(t) - \hat{y}(t)),$$

The aggregated fuzzy observer for the RF model turns to be

$$\hat{x}^{RF}(t) = \sum_{i=1}^2 \sum_{j=1}^2 M_i(z_1(t)) N_j(\hat{z}_2(t)) \times \left\{ A_{ij}^{RF} \hat{x}(t) + Bu(t) + K_{i\_RF}(y(t) - \hat{y}(t)) \right\}$$

$$\hat{y}^{RF}(t) = \sum_{i=1}^2 \sum_{j=1}^2 M_i(z_1(t)) N_j(\hat{z}_2(t)) C \hat{x}(t)$$

The fuzzy observers for the Rq1 and Rqe2 partial models were derived using the same procedure. The observer gains were calculated by means of the Linear Matrix Inequalities (LMI's) given by (36) and solved using the MATLAB™ Linear Matrix Inequalities toolbox. The observer gains for the RF, Rq1 and Rqe2 partial models are shown in table 2.

$$X > 0$$

$$\begin{bmatrix} X & A_i^T X - C_i^T N_i^T \\ X A_i^T - N_i^T C_i & X \end{bmatrix} > 0 \quad (36)$$

where  $X=P^{-1}$ ,  $N=PK$ ,  $P$  is a common definite positive matrix and  $K$  is the observer gain.

Table 2 Fuzzy observer gains for the RF, Rq1, and Rqe2 partial models.

	$x_1$	$x_2$	$x_3$
$K_{1\_RF}$	-3009	6372	-57.87
$K_{2\_RF}$	-5.335	8.685	0.679
$K_{3\_RF}$	-3004	6361	-58.04
$K_{4\_RF}$	0.167	-1.98	0.511
$K_{1\_Rqe1}$	-1022	1996	79.14
$K_{2\_Rqe1}$	-1006	1966	77.83
$K_{3\_Rqe1}$	-19.32	36.48	1.808
$K_{4\_Rqe1}$	-3.716	6.509	0.4972
$K_{1\_Rqe2}$	-2093	4460	-56.08
$K_{2\_Rqe2}$	-3.532	5.744	0.5116
$K_{3\_Rqe2}$	-2089	4452	-56.14
$K_{4\_Rqe2}$	-0.235	-1.72	0.4531

Also, common positive definite matrices that guarantees global asymptotic stability (Tanaka and Wang, 2001), were found for each partial model, namely

$$P_{RF} = \begin{bmatrix} 8.837 \times 10^{-5} & -1.7137 \times 10^{-4} & -2.6913 \times 10^{-6} \\ -1.7137 \times 10^{-4} & 0.36302 \times 10^{-4} & -3.3403 \times 10^{-6} \\ -2.6913 \times 10^{-6} & -3.3403 \times 10^{-6} & 4.962 \times 10^{-6} \end{bmatrix}$$

$$P_{Rqe1} = \begin{bmatrix} 5.3024 \times 10^{-5} & -1.0182 \times 10^{-4} & -4.4542 \times 10^{-6} \\ -1.0182 \times 10^{-4} & 1.99 \times 10^{-4} & 7.8516 \times 10^{-6} \\ -4.4542 \times 10^{-6} & 7.8516 \times 10^{-6} & 1.0353 \times 10^{-6} \end{bmatrix}$$

$$P_{Rqe2} = \begin{bmatrix} 2.5022 \times 10^{-3} & -4.9634 \times 10^{-3} & -3.889 \times 10^{-5} \\ -4.9634 \times 10^{-3} & 1.0578 \times 10^{-2} & -1.344 \times 10^{-4} \\ -3.889 \times 10^{-5} & -1.344 \times 10^{-4} & 1.4845 \times 10^{-4} \end{bmatrix}$$

## 6. SIMULATION RESULTS

The application of the proposed observer scheme was simulated using MATLAB™. The fuzzy observers were tested using the continuous RF and the R baker's yeast partial models given above. The inlet substrate concentration was varied between 3 g/l and 10 g/l in order to force the switching between both models. The initial conditions were chosen as

$x_1(0)=2.0$  g/l,  $x_2(0)=0.02$  g/l,  $x_3(0)=0.15$  g/l and  $x_4(0)=0.0066$  mg/l. A white noise signal was added to the ethanol and the dissolved oxygen on line measurements, see figure 2. The behavior of the fuzzy observer for biomass estimation is shown in figure 3. It can be noticed that the dynamics of the baker's yeast switch through the RF, Rqe1 and Rqe2 partial models. The observer converges to the real biomass values. The estimated substrate is shown in figure 4, as it can be seen in this case the substrate observer is more sensitive to the white noise signal; however the observer performance is acceptable for the range of chosen values. The rate of convergence could be improved by convenient closed loop eigenvalues relocation.

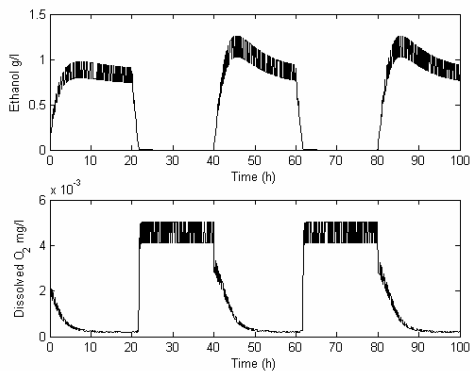


Fig.2. Ethanol and dissolved oxygen with a white noise signal of  $\pm 0.1$ .

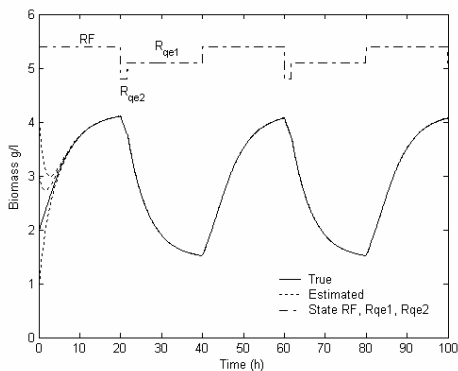


Fig. 3. Biomass estimation for three initial conditions  $\hat{x}_1(0) = 1.0, 3.0$  and  $4.0$  g/l.

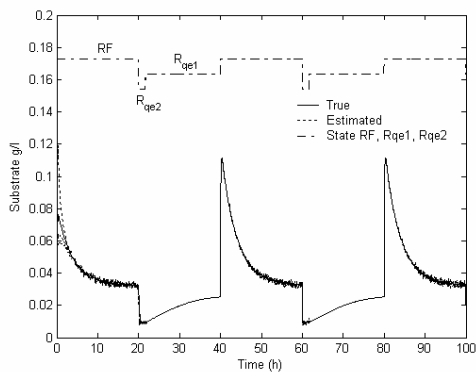


Fig.4. Substrate estimation for three initial conditions  $\hat{x}_2(0) = 0.01, 0.03$  and  $0.06$  g/l.

## 7. CONCLUSIONS

Based on the idea of splitting the baker's yeast model, a novel TS fuzzy model was proposed using the sector nonlinearities method, giving an exact representation of the original nonlinear plant. Moreover, an observer for each partial model was constructed. It is worth noting that the observer was capable of switching along the partial models, without performance degradation. Therefore, the approach presented here may be considered a valid method to design an observer. Future work will include the experimental validation of the fuzzy observer, as well as the implementation of the fuzzy exact observer for a baker's yeast fed batch culture.

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