Topological Spatial Relations between a Spatially Extended Point and a Line for Predicting Movement in Space

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ABSTRACT

Location is an important dimension of contextual information for mobile systems, playing a key role in the development of context-aware and location-based applications. The identification of a specific location is well addressed by several existing technologies such as, for example, GPS (Global Positioning System). Moreover, the prediction of the next position of a mobile user is a valuable enabler for the development of pro-active location-based applications. Based on this knowledge, those applications become able to provide useful services for the users before they explicitly ask for them. As a step towards the prediction of the next position of a mobile user, this paper presents the identification of the topological spatial relations that can exist between a spatially extended point (representing the uncertainty on the position of a mobile user) and a line (representing objects in which movement in space is possible). Using a 4x3 intersection matrix we identified 38 topological spatial relations that can exist between the objects in analysis (spatially extended points and lines). The geometric realization of the 38 topological spatial relations was done through the analysis of each one of the identified valid matrices. The validation of the existence of the identified topological relations was verified from their geometric realization.

INTRODUCTION

The increasing availability of mobile networks and mobile devices motivate the emergence of new paradigms of interaction between users and between a user and the surrounding environment. In particular, when the position of a user is known, applications running on the local infrastructure or on mobile devices can adapt their behaviour accordingly to his/her current position. These applications are known as location-aware applications. In these applications, the position information is obtained from a set of positioning sensors, or from network-based location services, and is often used directly, without any further processing, as a parameter in the adaptation process. Besides this usual behaviour of the applications, location-aware applications can be pro-active, providing services that can be useful for its user before he/she asks for them. For this kind of actions, location-aware applications need to anticipate the user position, predicting where the user is going to be in the future.

The work presented in this paper is part of a project[1] that aims to contribute to the broad objective of enhancing location-aware applications by addressing the issue of the prediction of the user's future position. The project aims the prediction of the user's position based on topological constrains present in the geographic space in which the user is located, and not based on the user previous behaviour. At this stage of the project, the objective is to predict movement of vehicles in a road network.

As part of the work addressed so far this paper presents the definition of a conceptual framework that allows the identification of the topological spatial relations that can exist between a spatially extended point (representing the position of a mobile user) and a line (representing objects in which

[1] The GUESS Project: Prediction of the User Position in a Context-aware Mobile Environment through Qualitative Spatial Reasoning.

movement in space is possible). As the result of the work undertaken, 38 topological spatial relations were identified and their geometric realization was defined. These will be used as the input to a conceptual neighbourhood graph[2] that will support the prediction task (not described in this paper).

This paper is organized as follows: next section presents the motivation for this work and states its importance to the domain of location-aware applications. The conceptual framework adopted for the identification of the topological spatial relations is then presented, describing the conditions adopted and the identified 38 topological spatial relations. Afterwards, it is presented the geometric realization of the several recognised topological relations. This paper is concluded with a summary of the work undertaken and some guidelines for future work.

LOCATION-AWARE APPLICATIONS AND THE PREDICTION OF THE USER'S POSITION

The wide availability of wireless local area networks and mobile cellular networks, as well as the improved capabilities of the current mobile devices, are enabling the development of new and advanced context-aware applications. Within this class of applications, the location-aware applications are those that are able to adapt their behaviour accordingly to the location of the user (Dey, 2001). A key aspect of context-aware applications is the easy access to the context of their users. One simple way of describing that context is to describe the place where the mobile user is in a particular instant of time. The most usual and elementary form of context is the current position of the user, described as a pair of geographic coordinates, or the location, described as an address or other symbolic reference.

The information about the user's location is used by these applications to make available a wide range of services that are, hopefully, useful to the users. The services that are provided depend upon the user location and the geographic context in which the user is mapped by the location-aware application. Actions can only be started, e.g. making a specific service available, after the contextualization of the user.

Another potential contribution to the location context of a user is through the use of predictive models to estimate future movements of a person (Ashbrook and Starner, 2003). Based on the current user's location (or the previous available locations of the user), several studies have been carried out in order to predict the next user's location. Laasonen (Laasonen, 2005) proposed an algorithm for predicting movement from cell-based location data. In that approach, location data consists of transitions between cells, with no regard to physical locations or topology.

Marmasse and Schmandt (Marmasse and Schmandt, 2002) presented a location model that uses a set of learned places (destinations of the users), expressed in latitude/longitude coordinates, in which the user was categorized. The model also uses knowledge of the routes between the destinations and the time it takes to travel them. The proposed model is based on user experience (his/her pattern of mobility), so models differ among users.

Ashbrook and Starner (Ashbrook and Starner, 2003) developed a system that automatically clusters GPS data in order to identify locations. The system collets data over an extended period and allows the creation and querying of location models.

As can be noted from previous paragraphs, the most common approaches used to estimate the user's position in a mobile environment follow quantitative approaches, in which the previous positions of the user and the estimation of his/her moving velocity allow the prediction of his/her next

[2] In those graphs an edge represents a transition from one relation to another and, in our application domain, from one place to another.

position. There are also other approaches in which the systems use the previous behaviour of the user to predict the most probable next location.

The use of the user's position and his/her past behaviour requires the collection of data over a certain period of time, in order to be possible to establish his/her movement pattern. Besides the fact the user needs to be tracked, and privacy issues can arise, some learning time is required. Instead of tracking the user to predict his/her next position, the GUESS project proposes the prediction of such position based on topological constrains present in the geographical space in which the user is located. To be possible, the topological spatial relations that can exist between a spatially extended point and a line must be identified. To the best knowledge of the authors of this paper, the identification of those relations was not addressed by other researchers.

Models of spatial relations need to deal with two different aspects: the formalization of the spatial relations and the cognitive assessment whether the formalization reflects human perception. The identification of the spatial relations deals with the formalization of spatial relations, whereas the cognitive assessment is concerned with the definition of conceptual neighbourhood graphs as a mean to establish a bridge between formalization and spatial cognition. These graphs model possible transitions between spatial relations in situations of continuous change (Reis, 2006).

In a conceptual neighbourhood graph nodes represent spatial relations between objects and edges represent possible transitions from one relation to other possible relations. These graphs (examples can be found in (Egenhofer and Mark, 1995)) are useful for reducing the search space when looking for the next possible situations that can occur in a process of continuous change.

The topological relations identified in this work, and presented in this paper, will be evaluated in order to verify its applicability to the application domain of future position prediction, and after that, a conceptual neighbourhood graph will be defined to predict the next position of a mobile user taking into consideration topological constraints present in the space.

DEFINING TOPOLOGICAL SPATIAL RELATIONS BETWEEN A SPATIALLY EXTENDED POINT AND A LINE

Topological relations are those relationships that are invariant under continuous transformations of space such as rotation or scaling. The topological relations that can exist between objects need to be identified and this is called the problem of existence. It consists in the identification of a set of mutually exclusive and pair-wise disjoint relations that covers all possible configurations (Reis, 2006). Looking at two objects, a qualitative change occurs when the movement of an object affects the topological relationship with respect to another object.

This section presents the identification of the topological relations that can exist between a spatially extended point and a line. A spatially extended point can be considered as a region-like concept. A spatially extended point has its own interior, boundary and exterior to represent its scope of influence. While it shares the same concepts of interior, boundary and exterior of a region, the spatially extended point is distinguished from a general region by the identification of a point within the interior called the pivot. The pivot is conceptually similar to a 0-dimension object. A major difference between a usual point and a pivot is that a pivot is a standard to know which region is affecting the spatially extended point substantially (Lee and Flewelling, 2004).

In this work, the region of influence of the pivot represents a certain degree of uncertainty about the position of a mobile user (Figure 1).

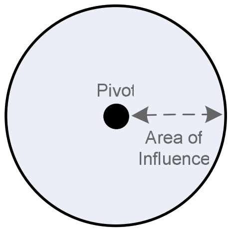

Figure 1: A Spatially Extended Point

The formalism used in this work for the identification of the spatial relations that can exist between a spatially extended point and a line is based on the algebraic approach proposed by Egenhofer (the 4- and 9-intersections models) (Egenhofer and Herring, 1991). Based on this approach, several topological relations were already identified: between regions (Egenhofer and Herring, 1991), between lines (Egenhofer and Herring, 1991), between regions and lines (Egenhofer and Herring, 1991; Egenhofer and Mark, 1995), between a spatially extended point and a region (Lee and Flewelling, 2004), between broad lines (Reis, 2006), between lines with broad boundaries (Reis, 2006), among others. However, until now, the study of the topological relations that can exist between spatially extended points and lines has not been addressed by the scientific community.

In this work, the topological relations were identified using a 4x3 matrix like the one proposed in (Lee and Flewelling, 2004). In this matrix, the intersections ($\bigcap$) between the pivot ($P^{\bullet}$), interior (P°), boundary (∂P) and exterior (P^{-}) of the spatially extended point P and the interior (L°), boundary (∂L) and exterior (L^{-}) of the line L are analysed. Each relation *R* is characterized by 12 intersections with empty ($\varnothing$) or non-empty ($\neg\varnothing$) values.

$$R(P,L)=\begin{bmatrix} P^{\bullet}\bigcap L^{\circ} & P^{\bullet}\bigcap \partial L & P^{\bullet}\bigcap L^{-} \\ P^{\circ}\bigcap L^{\circ} & P^{\circ}\bigcap \partial L & P^{\circ}\bigcap L^{-} \\ \partial P\bigcap L^{\circ} & \partial P\bigcap \partial L & \partial P\bigcap L^{-} \\ P^{-}\bigcap L^{\circ} & P^{-}\bigcap \partial L & P^{-}\bigcap L^{-} \end{bmatrix}$$

The several conditions proposed by Egenhofer and Harring (Egenhofer and Herring, 1991) for the definition of the topological relations that can exist between regions, lines and points in a Geographic Database were analysed. Following the suggestions of these authors, 9 conditions were applied. These conditions are associated with the definition of the topological spatial relations that can exist: between regions; between a region and a line; and between a non-point object (a region or a line) and a point. These are referred in this work as conditions 1 to 9. Additional conditions were defined attending to the particular case of the definition of the topological relations that can exist between a spatially extended point and a line. These conditions are named as condition 10 to condition 14. All these conditions are described next.

Condition 1. The exteriors of the two objects (P and L) intersect each other.

$$R(P,L)\neq\begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \\ - & - & \phi \end{bmatrix}$$

Condition 2. If P's interior is a subset of L's closure[3] then P's boundary must be a subset of L's closure as well, and *vice-versa* ($\vee$ means *or*).

$$R(P,L) \neq \begin{bmatrix} - & - & - \\ - & - & \phi \\ - & - & \neg\phi \\ - & - & - \end{bmatrix} \vee \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \\ \phi & \neg\phi & - \end{bmatrix}$$

Condition 3. P's boundary intersects with at least one part of L and *vice-versa*.

$$R(P,L) \neq \begin{bmatrix} - & - & - \\ - & - & - \\ \phi & \phi & \phi \\ - & - & - \end{bmatrix} \vee \begin{bmatrix} - & - & - \\ - & \phi & - \\ - & \phi & - \\ - & \phi & - \end{bmatrix}$$

Condition 4. If both interiors are disjoint then P's interior cannot intersect with L's boundary.

$$R(P,L) \neq \begin{bmatrix} - & - & - \\ \phi & \neg\phi & - \\ - & - & - \\ - & - & - \end{bmatrix}$$

Condition 5. If L's interior intersects with P's interior and exterior, then it must also intersect with P's boundary.

$$R(P,L) \neq \begin{bmatrix} - & - & - \\ \neg\phi & - & - \\ \phi & - & - \\ \neg\phi & - & - \end{bmatrix}$$

Condition 6. The interior of P always intersects with the exterior of L.

$$R(P,L) \neq \begin{bmatrix} - & - & - \\ - & - & \phi \\ - & - & - \\ - & - & - \end{bmatrix}$$

[3] The closure of a line is the union of its interior and its boundary. The exterior of a line is the difference between the embedding space and the closure of the line (Egenhofer and Herring, 1991).

Condition 7. The boundary of P always intersects with the exterior of L.

$$R(P,L) \neq \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & \phi \\ - & - & - \end{bmatrix}$$

Condition 8. The interior of L must intersect with at least one of the four parts of P.

$$R(P,L) \neq \begin{bmatrix} \phi & - & - \\ \phi & - & - \\ \phi & - & - \\ \phi & - & - \end{bmatrix}$$

Condition 9. The pivot of P can only intersect with a single part of L.

$$R(P,L) \neq \begin{bmatrix} \neg\phi & \neg\phi & - \\ - & - & - \\ - & - & - \\ - & - & - \end{bmatrix} \vee \begin{bmatrix} - & \neg\phi & \neg\phi \\ - & - & - \\ - & - & - \\ - & - & - \end{bmatrix} \vee \begin{bmatrix} \neg\phi & - & \neg\phi \\ - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}$$

Condition 10. The pivot of P must intersect with at least one part of L.

$$R(P,L) \neq \begin{bmatrix} \phi & \phi & \phi \\ - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}$$

Condition 11. If the interior of P intersects with the interior of L, and the exterior of P intersects with the boundary of L, then the boundary of P must intersect with the interior of L.

$$R(P,L) \neq \begin{bmatrix} - & - & - \\ \neg\phi & - & - \\ \phi & - & - \\ - & \neg\phi & - \end{bmatrix}$$

Condition 12. The boundary of a simple line[4] L can only intersect with at most two parts of P.

$$R(P,L) \neq \begin{bmatrix} - & \neg\phi & - \\ - & \neg\phi & - \\ - & \neg\phi & - \\ - & - & - \end{bmatrix} \vee \begin{bmatrix} - & - & - \\ - & \neg\phi & - \\ - & \neg\phi & - \\ - & \neg\phi & - \end{bmatrix} \vee \begin{bmatrix} - & \neg\phi & - \\ - & - & - \\ - & \neg\phi & - \\ - & \neg\phi & - \end{bmatrix} \vee \begin{bmatrix} - & \neg\phi & - \\ - & \neg\phi & - \\ - & - & - \\ - & \neg\phi & - \end{bmatrix}$$

Condition 13. If the boundary of L intersects the pivot of P, then the interior of P must intersect the interior of L.

$$R(P,L) \neq \begin{bmatrix} - & \neg\phi & - \\ \phi & - & - \\ - & - & - \\ - & - & - \end{bmatrix}$$

Condition 14. If the interior of L intersects the pivot of P, then the interior of P must intersect the interior of L.

$$R(P,L) \neq \begin{bmatrix} \neg\phi & - & - \\ \phi & - & - \\ - & - & - \\ - & - & - \end{bmatrix}$$

The adoption of a 4x3 matrix for the definition of the intersections that can exist between the pivot, interior, boundary and exterior of P, and the interior, boundary and exterior of L, results in the identification of 4096 (2^{12}) different matrices. In this huge set, with a large number of possible combinations, only a reduced number of matrices represent valid topological relations for the objects in analysis. In order to support the process of generation of the 4096 different hypothesis and the elimination of the invalid ones, a Mathematica (Mathematica, 2006) notebook was implemented. This implementation allowed the identification of the 4096 matrices, the definition of the several conditions (identified above: condition 1 to condition 14) and the automatic elimination of the invalid patterns associated with those conditions.

After the application of the 14 conditions, 38 matrices left as possible ones. Each one of these matrices was manually analysed in order to identify if it is effectively valid, or not. The analysis undertaken allowed the geometric realization of the 38 different topological spatial relations, which are presented in next section.

GEOMETRIC REALIZATION OF THE IDENTIFIED TOPOLOGICAL SPATIAL RELATIONS

The geometric realization of the 38 topological spatial relations was manually done, analysing each one of the identified valid matrices. The geometric realization of the several patterns allowed the validation of the identified relations in terms of their existence. Figure 2 and Figure 3 present the 38

[4] A simple line has a boundary with two points, each of which has no extend (Egenhofer and Herring, 1991).

geometries and the respective intersection matrices, using the notation 0 (∅) for non-intersection and 1 (¬∅) for intersection.

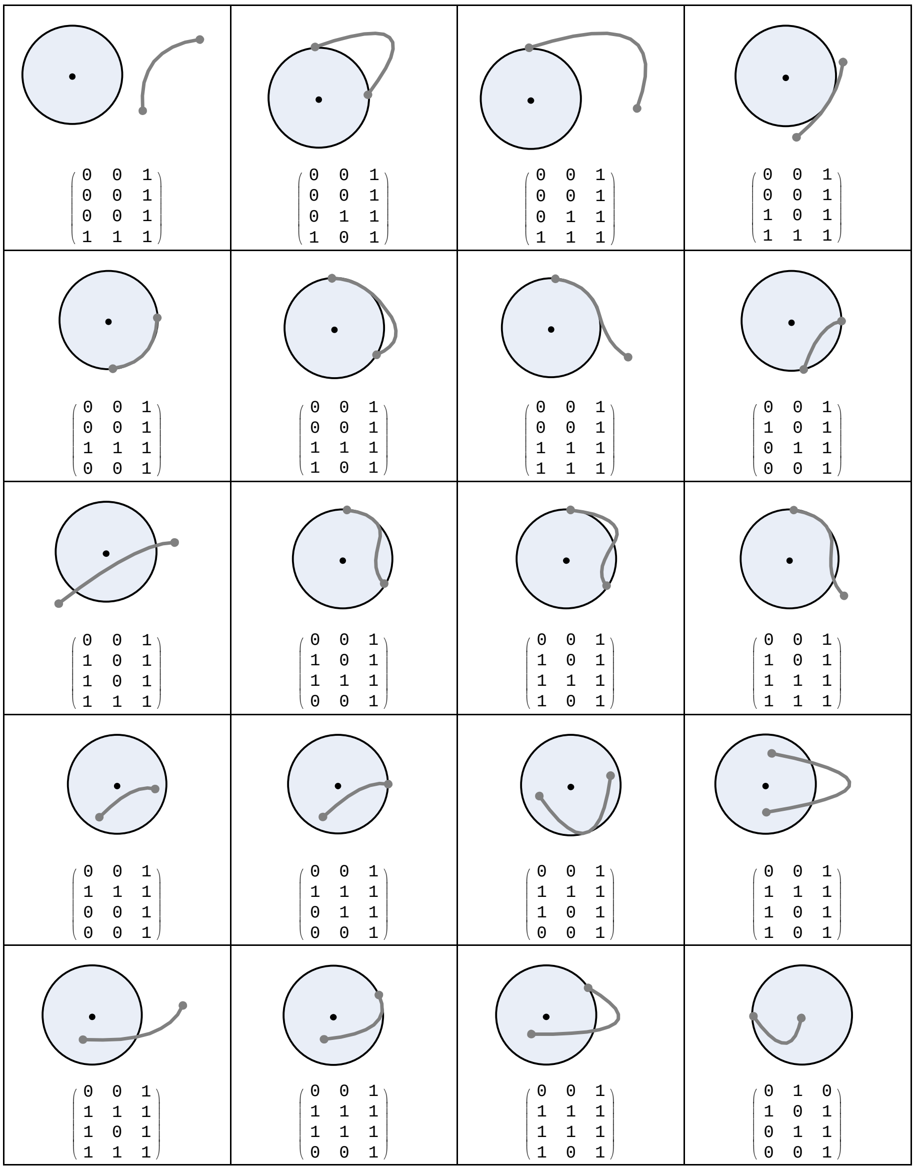

Figure 2: Geometric realization of the topological spatial relations (part I)

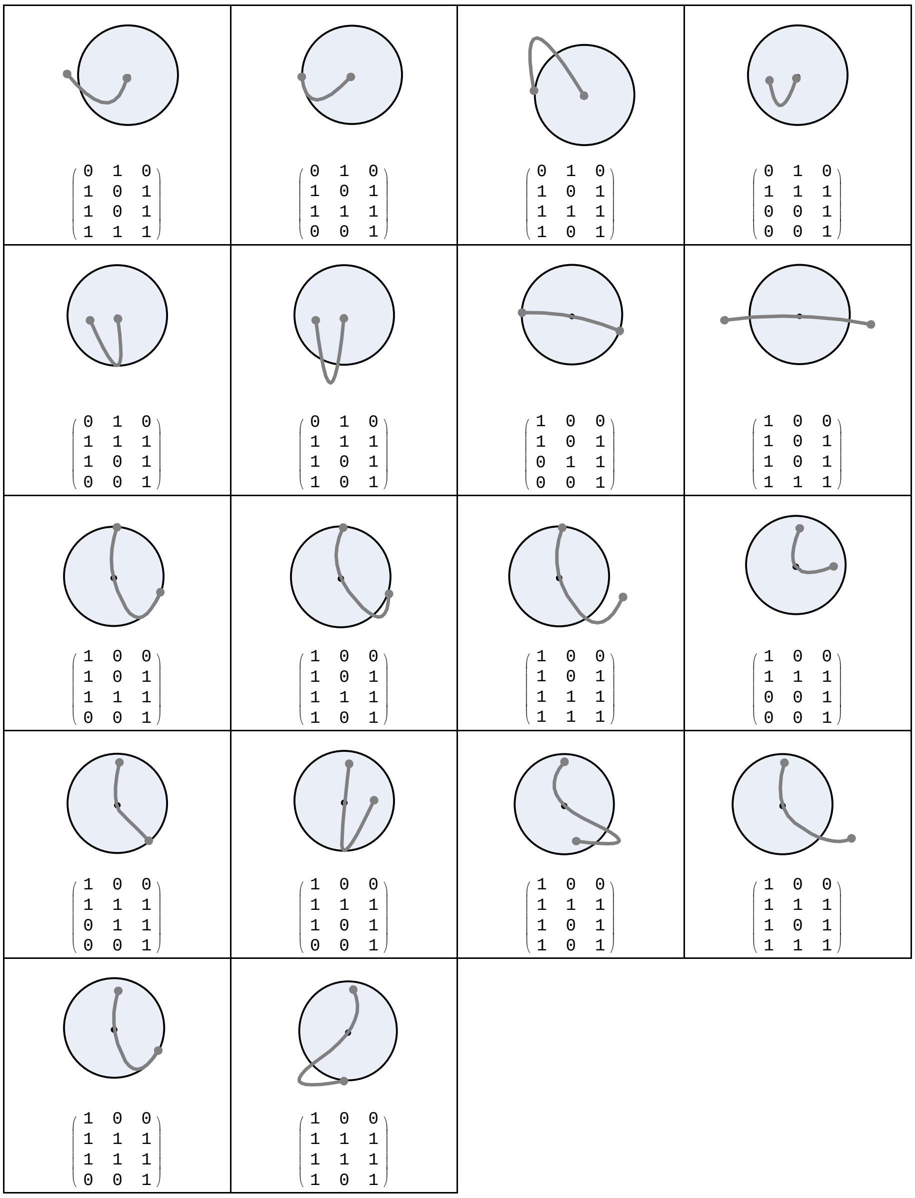

Figure 3: Geometric realization of the topological spatial relations (part II)

CONCLUSIONS AND FUTURE WORK

This paper presented the identification of the topological spatial relations that exist between a spatially extended point and a line. For the identification of the valid relations, a 4x3 matrix was used, allowing the verification of the intersections that exist between the object's parts in analysis: the pivot ($P^{\bullet}$), interior (P°), boundary (∂P) and exterior (P^{-}) of a spatially extended point, and the interior (L°), boundary (∂L) and exterior (L^{-}) of a line. For the identified 38 valid patterns, the geometric realization of the relations was undertaken, proving the existence of such relations.

The work presented in this paper in part of a broader project, the GUESS project, which aims the prediction of a mobile user's future position taking into consideration topological constraints present in the geographical space in which the user is located. The prediction task will be facilitated by the use of qualitative spatial reasoning strategies that are going to manipulate the identified topological spatial relations.

As future work, we point out that the identified topological spatial relations will be analysed in order to evaluate their applicability to the domain of location-aware applications. After this evaluation, a conceptual neighbourhood graph will be defined to be used as input in the process of prediction the user's future position in a mobile environment.

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