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HOMOGENIZATION OF MASONRY USING A MICRO-MECHANICAL MODEL: COMPRESSIVE BEHAVIOUR

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Abstract. *Despite considerable experimental and analytical research in the past, modern regulations still adopt very conservative simplified formulas for the compressive strength of masonry. The present paper contributes to the understanding of masonry under compression, using a novel non-linear homogenisation tool that includes the possibility of tensile and compressive progressive damage, both in the unit and mortar. The simplified homogenised model uses an iterative procedure and a few ingenious micro-deformation mechanisms, being able to accurately reproduce complex simulations carried out with non-linear continuum finite element analysis, at a marginal cost of CPU time and with no convergence difficulties.*

1 INTRODUCTION

The mechanical behaviour of different unreinforced masonry types is generally characterised by the same common feature: a very low tensile strength. This property is so important that it has determined the shape of ancient constructions until the 19th century. For over ten thousand years, masonry structures have been used only in compression, being this still a normal practice unless reinforced or prestressed masonry is used. Therefore, the compressive strength of masonry in the direction normal to the bed joints has been traditionally regarded as the sole relevant structural material property, at least until the recent introduction of numerical methods for masonry structures.

It has been generally accepted that the difference in elastic properties of the unit and mortar is the precursor of failure. Uniaxial compression of masonry leads to a state of tri-axial compression in the mortar and of compression/biaxial tension in the unit. Recently, sophisticated non-linear analyses of masonry under compression have been carried out using continuum finite element models. The problem of reproducing the experimental response of the masonry composite from the behaviour of masonry components is rather difficult due to the number of influencing parameters and the complex micro-structure. Models such as the one proposed in the paper allow to better understand the failure of masonry under compression.

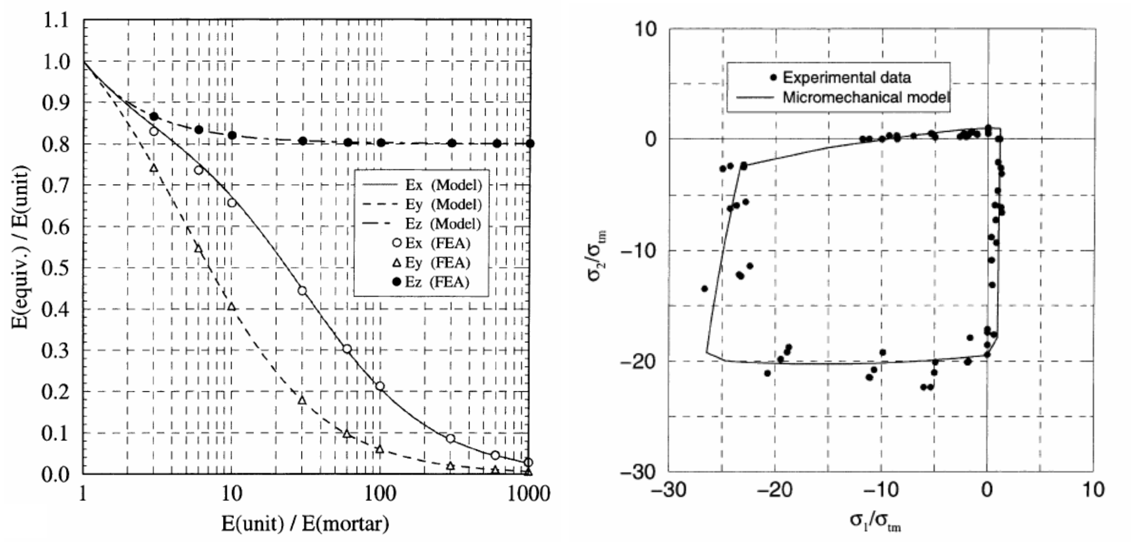
The present paper aims at further discussing the mechanics of masonry under compression and at proposing a homogenisation tool that is able to reproduce the results of advanced non-linear finite element computations, at a marginal fraction of the cost. For this purpose, a homogenisation approach previously developed by the authors [1,2], is extended to the case of masonry under compression. It is much relevant that the proposed simplified approach provides results almost equal to very complex non-linear finite element analysis of a masonry representative volume.

2 FORMULATION OF THE MODEL

2.1 General

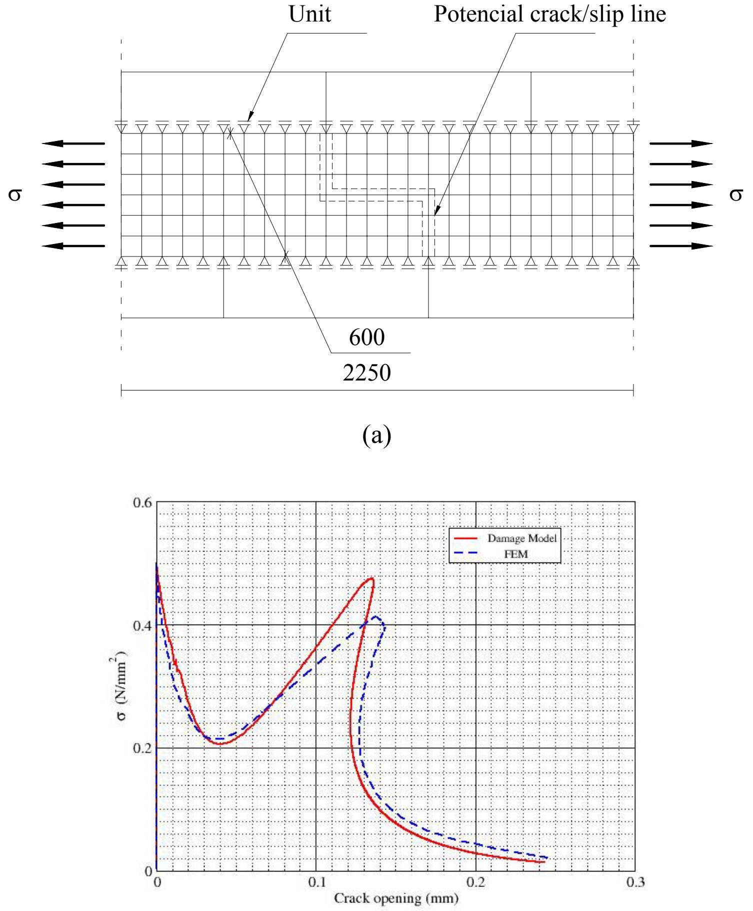
Zucchini and Lourenço [1] have shown that the elastic mechanical properties of an orthotropic material equivalent to a basic masonry cell can be derived from a suitable micro-mechanical model with appropriate deformation mechanisms, which take into account the staggered alignment of the units in a masonry wall. The unknown internal stresses and strains can be found from equilibrium equations at the interfaces between the basic cell components, from a few ingenuous assumptions on the kinematics of the basic cell deformation and by forcing the macro-deformations of the model and of the homogeneous material to contain the same strain energy. Fig. 1 illustrates typical linear elastic results of the proposed homogenisation approach, in terms of Young's moduli and failure criterion. It can be observed that the model provides excellent results in both cases.

This homogenisation model has already been extended with good results to non-linear problems in the case of a masonry cell failure under tensile loading parallel to the bed joint, [2]. The simulation has been accomplished by coupling the elastic micro-mechanical model with a damage model for joints and units by means of an iterative solution procedure to calculate the damage coefficients. A simple isotropic damage model with only one single parameter has been utilized, because the discrete internal structure of the cell, and implicitly its global anisotropic behaviour, is taken into account by the three-dimensional micromechanical model. Fig. 2 illustrates the results of the non-linear homogenization approach, for uniaxial tensile loading parallel to the bed joints. It can be observed that the model provides again excellent results, when compared with advanced non-linear finite elements analysis.



(a) (b)

Figure 1: Results from micro-mechanical homogenization model [1]: (a) comparison for Young's moduli between model and FEA results for different stiffness ratios; (b) failure criterion for masonry under biaxial in-plane loading (principal axes coincident with material axes).



(a)

(b)

Figure 2: Infinitely long masonry wall under tensile loading parallel to the bed joints [2]: (a) geometry and loading; (b) comparison between FEM results and the homogenisation model for stress/crack opening diagram.

2.2 Adopted coupled homogenisation-damage model

This paper addresses the problem of a basic masonry cell under compressive loading perpendicular to the bed joint. The geometry for the basic masonry cell and its components is shown in Fig. 3, where it can be seen that the complex geometry is replaced by four components, namely unit, bed joint, head joint and cross joint. When the basic cell is loaded only with normal stresses, the micromechanical model of Zucchini and Lourenço [1] assumes that all shear stresses and strains inside the basic cell can be neglected, except the in-plane shear stress and strain (σ_{xy} and ε_{xy}) in the bed joint and in the unit. The non-zero stresses and strains in the bed joint, head joint and unit are assumed to be constant, with the exception of the normal stress σ_{xx} in the unit, which is a linear function of x and accounts for the effect of the shear σ_{xy} in the bed joint, and with the exception of the shear stress σ_{xy} in the unit, which is linear in y.

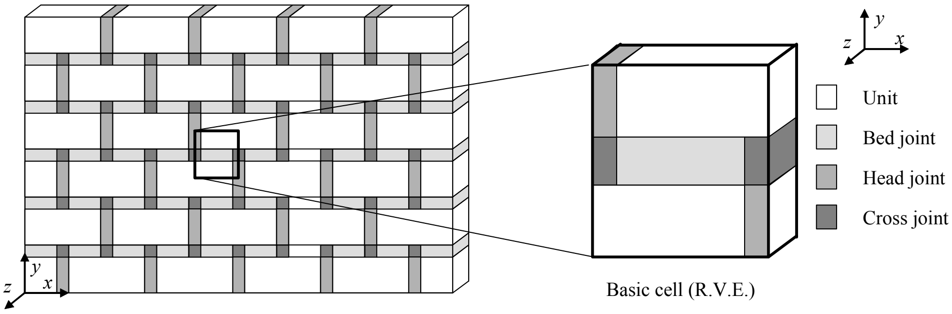


Figure 3: Basic cell for masonry and homogenisation process.

The coupling of this model with a material damage model in tension [2], leads to an iterative algorithm, in which at each cycle a system of equilibrium equations is solved to obtain the unknown effective stresses and strains, making use of the damage coefficients from the previous iteration. The damage coefficients can then be updated, by means of the damage model, from the new stresses and the process is iterated until convergence of the coefficients, within an input tolerance. Finally, the damaged internal stresses in the cell components and the unknown homogenized stresses and strains can be derived from the values of the converged internal stresses.

The governing linear system of twenty equilibrium equations [1] in the unknown internal stresses and strains of the masonry cell, to be solved at each iteration, can be rewritten for a strain driven compression in y, as:

$$r^2\sigma^2_{xx} = r^b\bar{\sigma}^b_{xx} - \frac{l-t}{2h} r^1\sigma^1_{xy} \tag{1}$$

$$r^b\sigma^b_{yy} = r^1\sigma^1_{yy} \tag{2}$$

$$hr^2\sigma^2_{xx} + 2tr^1\sigma^1_{xx} + hr^b\bar{\sigma}^b_{xx} + (l-t)r^1\sigma^1_{xy} = 0 \tag{3}$$

$$(h + 4t\frac{r^2}{r^1 + r^2})\,\varepsilon^2_{yy} + h\varepsilon^b_{yy} = 2(h+t)\varepsilon^0_{yy} \tag{4}$$

$$thr^2\sigma^2_{zz} + (l - t + t\frac{r^1 + r^2}{r^1})tr^1\sigma^1_{zz} + lhr^b\sigma^b_{zz} = 0 \tag{5}$$

$$2t\varepsilon_{yy}^{1}+h\varepsilon_{yy}^{b}=(4t\frac{r^{2}}{r^{1}+r^{2}}+h)\varepsilon_{yy}^{2} \tag{6}$$

$$t\varepsilon_{xx}^{2}+l\bar{\varepsilon}_{xx}^{b}=(l-t+4t\frac{r^{1}}{r^{1}+r^{2}})\varepsilon_{xx}^{1} \tag{7}$$

$$\varepsilon_{zz}^{b}=\varepsilon_{zz}^{1} \quad , \quad \varepsilon_{zz}^{b}=\varepsilon_{zz}^{2} \tag{8}$$

$$\begin{aligned}
\varepsilon_{xx}^{k} &= \frac{1}{E^{k}}\left[\sigma_{xx}^{k}-\nu^{k}\left(\sigma_{yy}^{k}+\sigma_{zz}^{k}\right)\right] \\
\varepsilon_{yy}^{k} &= \frac{1}{E^{k}}\left[\sigma_{yy}^{k}-\nu^{k}\left(\sigma_{xx}^{k}+\sigma_{zz}^{k}\right)\right], \; k=b,1,2 \\
\varepsilon_{zz}^{k} &= \frac{1}{E^{k}}\left[\sigma_{zz}^{k}-\nu^{k}\left(\sigma_{xx}^{k}+\sigma_{yy}^{k}\right)\right]
\end{aligned} \tag{9}$$

$$\varepsilon_{xy}^{1}=\frac{\varepsilon_{xx}^{2}-\bar{\varepsilon}_{xx}^{b}}{4}-(\frac{l-t}{8hE^{b}}+\frac{h}{6tG^{b}})\frac{r^{1}}{r^{b}}\sigma_{xy}^{1} \tag{10}$$

$$\sigma_{xy}^{1}=2G^{1}\varepsilon_{xy}^{1} \tag{11}$$

As shown in Fig. 4, l is half of the unit length, h is half of the unit height and t is half of the bed joint width. Here also, E is the Young modulus, G is the shear modulus, ν is the Poisson coefficient, ε_{ij} is the strain component and σij is the stress component. Unit, bed joint, head joint and cross joint variables are indicated throughout this paper, respectively by the superscripts b, 1, 2 and 3, according to Fig. 4. $\bar{\sigma}_{xx}^{b}$ and $\bar{\varepsilon}_{xx}^{b}$ are the mean value of the (non-constant) normal stress and of the (non-constant) normal strain in the unit, respectively. ε_{yy}^{0} is the uniform normal (macro) strain, perpendicular to the bed joint, on the faces of the homogenised basic cell. Finally, $r=1-d$, where d is the scalar damage coefficient, ranging from 0 to 1 and representing a measure of the material damage. The damaged σ_d and undamaged (or effective) stresses σ are correlated by the relation:

$$\boldsymbol{\sigma}_{d}=\left(1-d\right)\mathbf{D}\boldsymbol{\varepsilon}=(1-d)\boldsymbol{\sigma} \tag{12}$$

where **D** is the elastic operator.

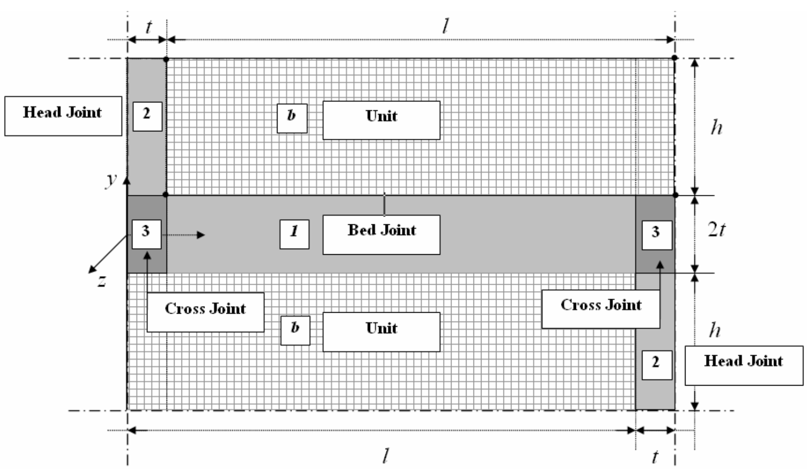


Figure 4: Definition of masonry axes and masonry components considered in the adopted formulation: unit, head joint, bed joint and cross joint.

The adopted damage model in tension [2] is a simple scalar isotropic model, with a Rankine type damage surface:

$$\sigma_p = \sigma_t \tag{13}$$

where σ_p is the maximum effective principal stress and σ_t the tensile strength of the given cell component. In the unit, where the normal stress σ^0_{xx} varies linearly in the x direction, the damage is controlled by the maximum principal stress in the entire unit and not by the maximum principal stress obtained with the average value $\bar{\sigma}^0_{xx}$.

The damage can only increase monotonically with the evolution law:

$$d = 1 - \frac{\sigma_t}{\sigma_p} e^{A\left(1-\frac{\sigma_p}{\sigma_t}\right)}, \qquad \sigma_t \le \sigma_p \le \infty \tag{14}$$

The parameter A is related to the mode I or mode II fracture energies (G^I and G^I) and strengths (σ_t and σ_s) of the material respectively by

$$A_t = \left(\frac{G^I E}{l\sigma_t^2} - \frac{1}{2}\right)^{-1} \qquad A_s = \left(\frac{G^{II} G}{l\sigma_s^2} - \frac{1}{2}\right)^{-1} \tag{15}$$

where E and G are the Young and shear moduli and l is the characteristic internal length of fracture [3], which is assumed here to be the material dimension in the direction of the load.

2.3 Extension of the formulation with a plasticity model in compression

In a basic masonry cell under vertical compressive load, the elastic mismatch between the unit and the bed joint is responsible for a tension-compression state, in the plane perpendicular to the loading, of these two components, with the stiffer one in tension. Failure or degradation of the cell properties, under increasing load, can be caused by either high tension or compression stresses in the component materials. The study of the inelastic behaviour of the basic cell in compression up to failure requires therefore the introduction of a non-linear constitutive model in compression. A Drucker-Prager model has been adopted for the simulation of the plastic deformation of each cell components. Its classic formulation, [4], reads as:

$$3k_1\sigma_m + \bar{\sigma} - k_2 = 0 \tag{16}$$

where

$$\sigma_m = \frac{\sigma_{ii}}{3} = -p, \quad \bar{\sigma} = \sqrt{\frac{1}{2}\sigma'_{ij}\sigma'_{ij}} = \frac{q}{\sqrt{3}} \tag{17}$$

$$k_1 = \frac{2\sin\phi_f}{\sqrt{3}(3-\sin\phi_f)}, \quad k_2 = \frac{6\cos\phi_f}{\sqrt{3}(3-\sin\phi_f)}c \tag{18}$$

The unknown plastic strains of the Drucker-Prager model are assumed to be constant in each cell component and can be derived from the total (elastic + plastic) strains $\boldsymbol{\varepsilon}_t$ with the return mapping algorithm, i.e. by integration over the loading path of the following system of incremental elasto-plastic equations from stage *n*–1 to stage *n*, e.g. [5] :

$$\begin{cases} \boldsymbol{\sigma}^n = \boldsymbol{\sigma}^* - \mathbf{D}\Delta\boldsymbol{\varepsilon}_p^n & \textit{plastic corrector} \\ \Delta\boldsymbol{\varepsilon}_p^n = \Delta\lambda^n \dfrac{\partial g(\boldsymbol{\sigma}^n, \varepsilon_{p,eq}^n)}{\partial \boldsymbol{\sigma}^n} & \textit{flow rule} \\ f(\boldsymbol{\sigma}^n, \varepsilon_{p,eq}^n) = t^n - p^n \tan\phi_f(\varepsilon_{p,eq}^n) - c(\varepsilon_{p,eq}^n) = 0 & \textit{yield surface} \end{cases} \tag{19}$$

Here the vector notation for stress and strains is used, being ***D*** the elastic stiffness matrix, $\boldsymbol{\sigma}^*$ the elastic predictor and g the non-associated plastic potential with a dilatancy angle $\phi_d \neq \phi_f$ and $\varepsilon_{p,eq}^n$ the equivalent plastic strain, such that

$$\boldsymbol{\sigma}^* = \boldsymbol{\sigma}^{n-1} + \mathbf{D}\Delta\boldsymbol{\varepsilon}_t^n \tag{20}$$

$$\Delta\varepsilon_{p,eq}^n = \sqrt{\frac{2}{3}(\Delta\bar{\boldsymbol{\varepsilon}}_p^n)^T(\Delta\bar{\boldsymbol{\varepsilon}}_p^n)} \tag{21}$$

where the notation $\bar{\boldsymbol{\varepsilon}}_p$ means the vector $\bar{\boldsymbol{\varepsilon}}_p^T = \{\varepsilon_{p1}, \varepsilon_{p2}, \varepsilon_{p3}, \sqrt{2}\varepsilon_{p4}, \sqrt{2}\varepsilon_{p5}, \sqrt{2}\varepsilon_{p6}\}$. Here, parabolic hardening, followed by parabolic softening is assumed for the compressive failure.

2.4 Considerations on compression-tension model coupling

With the Drucker-Prager model it is possible now to take into account the degradation of the mechanical properties of the cell components due not only to damage in tension, but also to plastic flow of the materials and to hardening-softening of their strengths with increasing deformations. The homogenisation-damage model of the cell internal structure can be coupled with the plasticity model of cell components in the algorithm shown in Fig. 5. In the original basic equilibrium system of the cell, Eqs.(1-11), the strain variables are the elastic strains in the stress-strain relations, Eqs.(9), and the total strains in the other equations, derived from geometric considerations on the deformation modes of the model. In [2] elastic and total strains coincide, because plastic deformations were not taken into account. With the introduction of plasticity and of the plastic strains as new additional variables, the total strains are chosen as master variables in all system equations. The system can immediately be applied to the new coupled model, because the stress equilibrium equations are not affected by the introduction of plastic strains, while most of the strain equations are unchanged, being already formulated in terms of total strains. Only Eqs.(9), which represent the elastic stress-strain relations, have obviously to be replaced by the usual decomposition of elastic plus plastic strain, resulting in :

$$\begin{aligned} \varepsilon_{xx}^k &= \frac{1}{E_k}\left[\sigma_{xx}^k - \nu_k\left(\sigma_{yy}^k + \sigma_{zz}^k\right)\right] + \varepsilon_{p,xx}^k \\ \varepsilon_{yy}^k &= \frac{1}{E_k}\left[\sigma_{yy}^k - \nu_k\left(\sigma_{xx}^k + \sigma_{zz}^k\right)\right] + \varepsilon_{p,yy}^k, \quad k = b,1,2 \\ \varepsilon_{zz}^k &= \frac{1}{E_k}\left[\sigma_{zz}^k - \nu_k\left(\sigma_{xx}^k + \sigma_{yy}^k\right)\right] + \varepsilon_{p,zz}^k \end{aligned} \tag{22}$$

The inner loop in Fig. 5 is an iterative process, in which at each cycle the system of equilibrium equations is solved to obtain the unknown stresses in each masonry component and total strains, making use of the damage coefficients and of the plastic strains of unit and joints from the previous iteration. Both damage and plasticity of each cell component are checked at each loading step. Coupling of damage and plasticity is straightforward, based on the effective stress concept, used e.g. in [6], according to which the plastic deformation is driven by the undamaged stresses. The damage coefficients and the plastic strains can then be updated respectively with the new undamaged stresses, by means of the damage model, and with the new total strains, by means of the plasticity model. The process is iterated until convergence of the coefficients and of the strains, within an input tolerance.

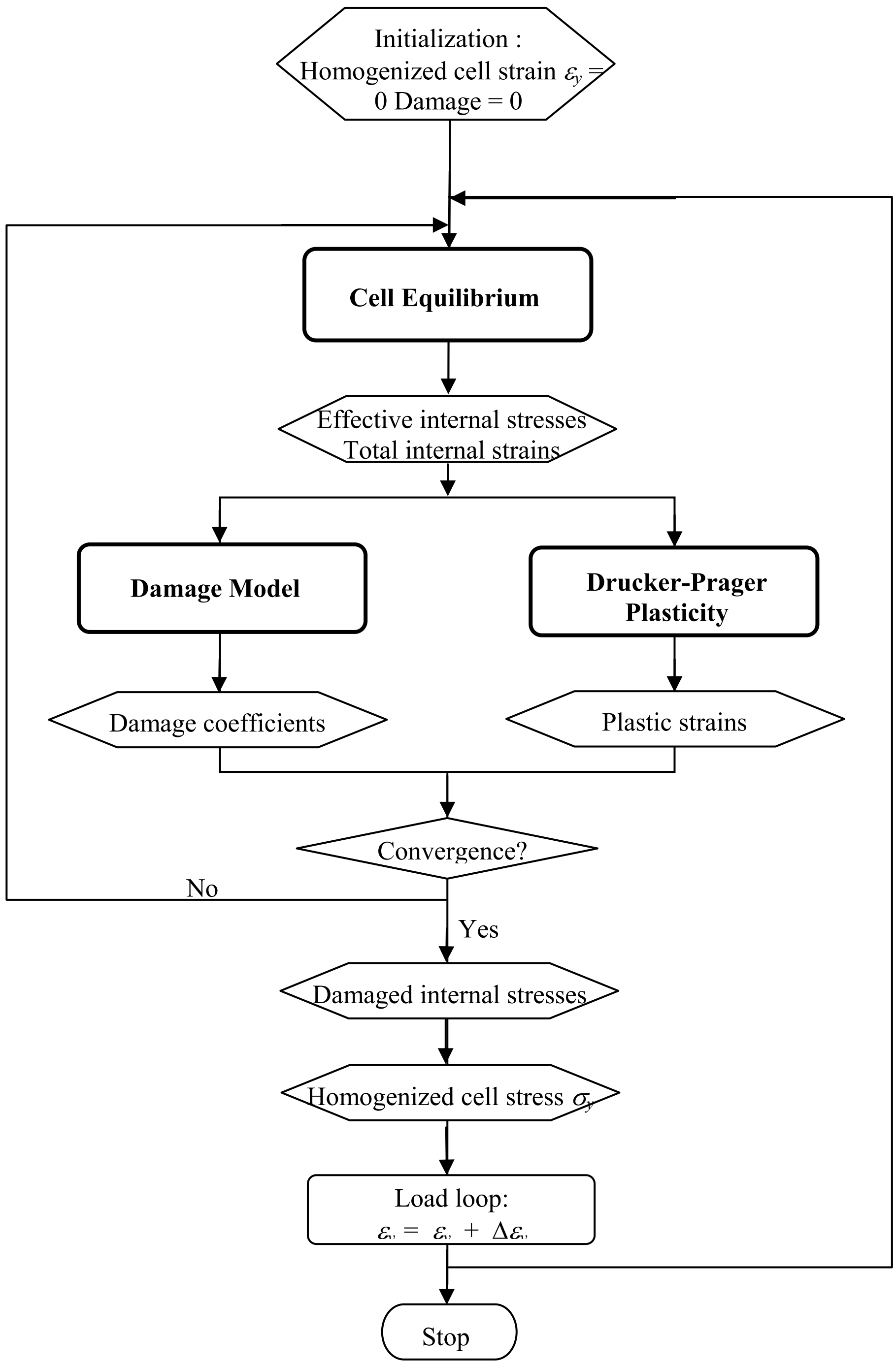


Figure 5: Iterative procedure for the non-linear homogenisation of a masonry basic cell with tensile damage and compressive plastic behaviour.

The outer loop is the cycle related to the incremental loading steps, in which ε_{yy}^{0}, the normal cell strain perpendicular to the bed joint, is increased, as usual in displacement driven experimental tests. The damaged stresses in the cell components at each loading step can be derived from the values of the converged effective internal stresses. The unknown homogenized cell stress σ_{yy}^{0}, perpendicular to the bed joint, can finally be obtained as:

$$\sigma_{yy}^{0} = \frac{l r_b}{(l+t)} \sigma_{yy}^{b} + \frac{t r_2}{(l+t)} \sigma_{yy}^{2} \tag{23}$$

3 MODEL VALIDATION WITH FEM RESULTS

The homogenisation model, with tension damage and compression plasticity, and the algorithm described in the previous section have been implemented in a computer program for the simulation up to failure of a basic masonry cell under axial compressive loading perpendicular to the bed joint. For this problem numerical results are available from the accurate FEM calculations of Pina-Henriques and Lourenço [7] in the case of a masonry cell with solid soft-mud bricks of dimensions 250×120×55 mm^3 and mortar joint thickness of 10 mm. These FEM analyses have been carried out with very detailed meshes (see Fig. 6) either in plane stress, plane strain and enhanced plane strain with constant but non-zero normal strains in the out-of-plane direction, being the latter considered the closest possible plane representation of the three-dimensional behaviour.

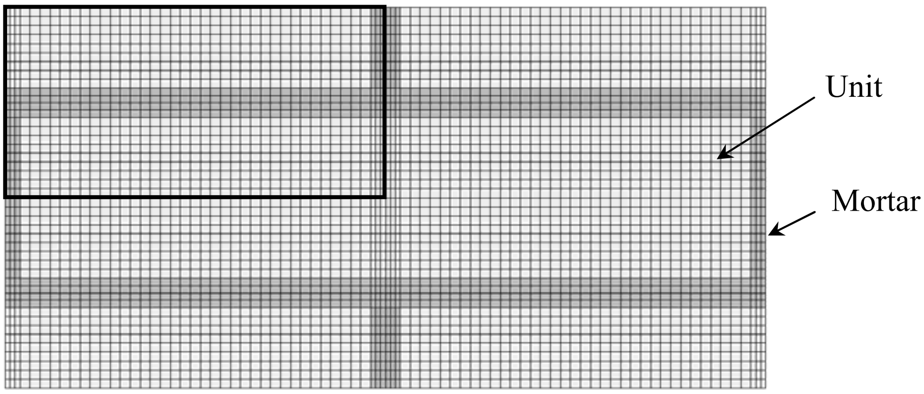
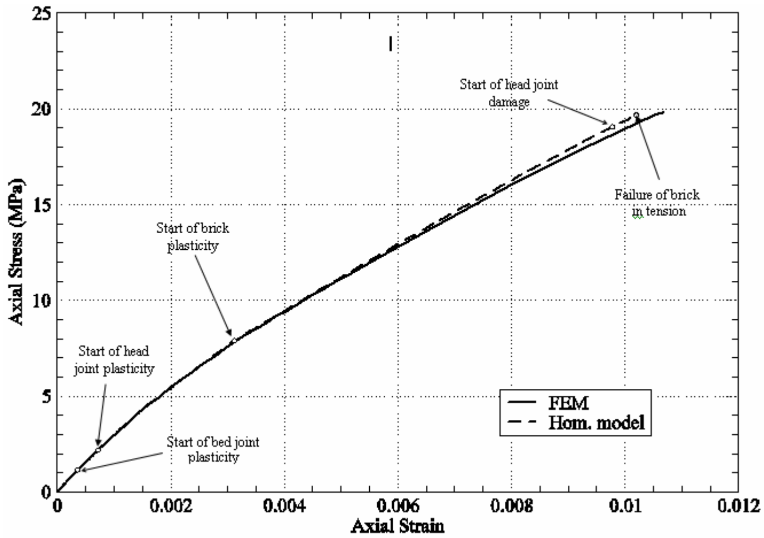


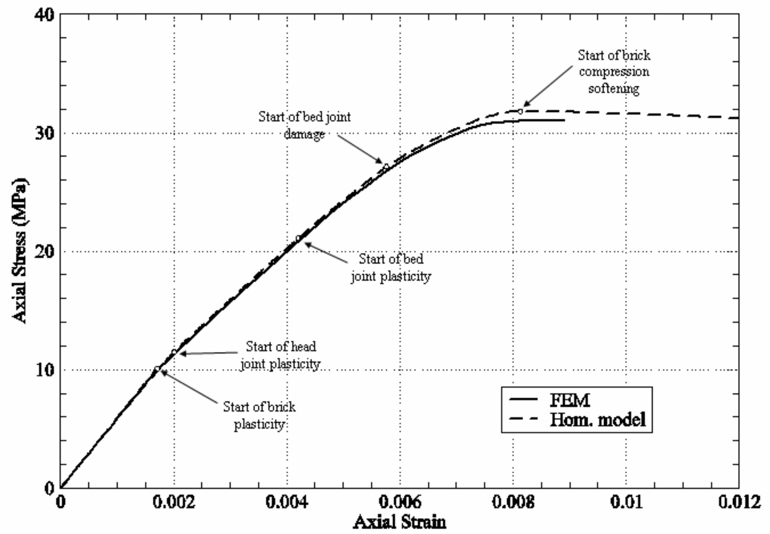
Figure 6: Model used in the finite element simulations (only the quarter indicated was simulated, assuming symmetry conditions).

Considering the symmetry of the cell in Fig.6, only the upper left quarter (corresponding to the basic cell of the micro-mechanical model) was modelled in the FEM study and the total number of degrees of freedom was around 7500. Symmetry boundary conditions were assumed for the two sides along the symmetry axes and periodicity conditions for the two sides on the external boundary of the cell. The non-linear behaviour of the cell components has been simulated by means of Drucker-Prager plasticity in compression and Rankine model or cracking in tension. Two different types of mortar were taken into consideration [7]. Mortar 1 is a weak mortar and mortar 3 is stiffer and stronger than the brick. The material data used by the homogenisation model are exactly the same as in [7].

The axial stress vs. axial strain curves of both the micromechanical model and the FEM analysis, for masonry prisms with mortar type 1 and 3 (identified respectively by MU1 and MU3), are given in Fig. 7. The curves obtained with the homogenisation model almost coincide with the corresponding FEM results in enhanced plane strain, with marginal computational effort and no convergence difficulties. For cell MU1 (Fig. 7a) the plastic flow of the mortar joints starts very early in the loading path, while the brick non-linear behaviour begins a little later. The brick is in a tension-compression-tension state, while the mortar is in a tri-axial compression state for the lateral containment effect of the stiffer brick. The head joint suffers some negligible damage in tension just before the complete failure of the brick in tension, which leads to the catastrophic failure of the entire cell. In cell MU3 (Fig. 7b) the plastic flow starts earlier in the brick than in the bed joint, due to the higher strength of the mortar. The inversion of the elastic mismatch between mortar and brick in this case (the mortar is much stiffer than the brick) yields in this case a tension-tension-compression state of the bed joint. A substantial (57%) isotropic damage in tension is reached in the bed joint, but the failure of the masonry cell is driven again by the crushing of the brick. The damage of the mortar in the bed is due to the high tension in the *x* and *z* direction.



(a)



(b)

Figure 7: Axial stress vs. axial strain for prisms: (a) MU1; (b) MU3. Comparison between finite element simulation [7] and non-linear homogenisation model.

4 CONCLUSIONS

The present paper addresses a novel non-linear homogenisation approach for masonry including tensile damage and compressive plasticity. It is shown that the homogenised model can reproduce almost exactly the results of a non-linear finite element calculation, at a marginal fraction of the computational effort.

Nevertheless in [7] significant differences have been found between FEM analysis and experimental values. It is well known that the mechanical properties of mortar inside a composite can be quite different from the properties of specimens cast separately of the same mortar, due to different curing conditions. Another problem is that the volumetric behaviour of triaxially compressed solids requires a cap model for an accurate description of volume change, but a key issue seems to be the general limitation of continuum mechanics approach. Alternative discontinuum modelling approaches that consider the micro-structure of quasi-brittle materials seem needed to study the uniaxial compressive behaviour of masonry.

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