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Dynamical Behaviour of Rigid Block Structures Subjected to Earthquake Motion

Fernando Peña, Francisco Prieto and Paulo B. Lourenço
University of Minho, Department of Civil Engineering, Guimarães, Portugal

Alfredo Campos-Costa
Laboratório Nacional de Engenharia Civil, Lisbon, Portugal

ABSTRACT: This paper studies the dynamic behaviour of rigid-block structures under seismic behaviour. For this purpose, an experimental investigation has been carried out on the rocking response of single blocks, and ensembles of two and three blocks structures. These structures were subjected to three different base motions: free rocking, harmonic and random excitations. The tests were performed at the seismic table of the National Laboratory of Civil Engineering (LNEC) of Portugal.

1 INTRODUCTION

Historical constructions formed by large stone blocks (i.e. columns, sculptures, arches, Greek temples, etc.) have no tensile strength and stability is ensured if the line of pressure due to their own weight falls inside the structure. These structures are particularly vulnerable objects under lateral seismic loading. However, this behaviour is typical of most masonry constructions, which often fail forming large macro-blocks under seismic loading. In this way, the study based upon the assumption of continuum structures would not be realistic for many cases. On the other hand, models based on rigid block assemblies provide a suitable framework for understanding their dynamic behaviour under seismic actions. So that the problem is primarily concerned with Rocking Motion (RM) dynamics (Augusti and Sinopoli 1992).

Rocking motion is defined as the oscillation of the rigid bodies (RB) present in a structure when centre of rotation instantly change from one point to another one; this instantaneous change produces a loss of energy due to an impulsive force.

Studies on RM have been carried out for a long time. However, the reference analytical frame for the study of RM dynamics remains based on the formulation introduced by Housner (1963), which will be referred as *classical theory* in this study. Housner obtained the equation for the period of the system; which depends on the amplitude of rocking, and the equation for the restitution coefficient.

After Housner, several experimental and analytical studies have been made (Aslam et al. 1980, Tso and Wong 1989, Spanos et al. 2001) on the RB under different conditions. Although all these researches, the study of this type of systems continues to be a relevant task. The main reasons are the need to propose adequate criteria for the seismic evaluation of masonry structures and the lack of comprehensive experimental data. Therefore, an extensive experimental investigation on the rocking response of single blocks and an ensemble of two and three blocks structures has been carried out. The tests were performed at the shaking table of the National Laboratory of Civil Engineering (LNEC) of Portugal on four single RB (blue granite stones), an ensemble of two blocks (referred as bi-block structure) and an ensemble of three blocks (referred as dolmen) submitted to different types of base motion. Particular attention was given to the experimental data acquisition, repeatability of the results and stability of the RB.

2 EXPERIMENTAL SET-UP

In this section a brief review of the experimental tests is made. The complete description can be found in Peña et al. (sub.) and Peña (2006).

2.1 *Characteristics of the specimens*

The experimental tests were carried out on four single blue granite stones and ensembles of two and three blue granite stones (Fig. 1). Each stone has different geometrical dimensions (Table 1). The dimensions of the single specimens 1, 2 and 3 were fixed to achieve a Height-Width ratio (*h/b*) of 4, 6 and 8, respectively. In addition, single specimen number 4 was specifically designed with a different geometry than the others specimens, in order to compare its performance with the rest of the stones. It has 45 degree cut (40 mm) at the base. Moreover, a foundation of the same material was used as the base where the blocks are free to rock. This foundation was fixed to the shaking table by means of four steel bolts.

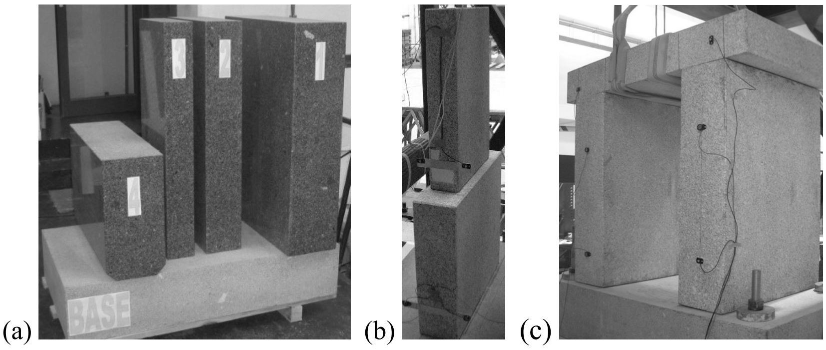


Figure 1 : Test specimens; (a) Single blocks, (b) Bi-block structure, (c) Dolmen.

Table 1 : Test specimens dimensions

Specimen	Width *2b* (m)	Height *2h* (m)	Thickness *2t* (m)	Mass *M* (kg)
1	0.25	1.000	0.754	503
2	0.17	1.000	0.502	228
3	0.12	1.000	0.375	120
4	0.16	0.457	0.750	245
Base	1.00	0.250	0.750	500
Bi-block				
Top	0.15	0.60	0.40	97
Bottom	0.20	0.60	0.55	178
Dolmen				
Pillars	0.22	0.80	0.65	305
Lintel	1.02	0.15	0.65	265

2.2 *Test set-up*

The data acquisition system was designed to describe the position of the specimens at each instant of the test and, simultaneously, to avoid the possibility that the system influences the response of the specimens. In this context, the data acquisition is based upon monitoring Light Emission diode Systems (LEDS) by means of high resolution cameras. This eliminates noise errors and enables accurate position measurement.

The main data obtained were: rotations around *Y* and *Z* axes and linear displacements *X* and *Y* (see Fig. 2 for the system coordinate). Rotations *Y* and *Z* were directly measured by means of a mirror linked to the blocks surface on the West face of the specimens. Two accelerometers were placed at the top of each block. One triaxial accelerometer was located in the North face and one biaxial accelerometer was located in the South face. The displacements and accelerations of the shaking table were also measured.

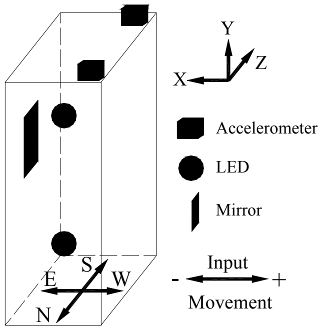


Figure 2 : Reference system of the data acquisition system and typical location of LEDs in the RB.

2.3 *Types of base motion*

In order to study the dynamic behaviour of the RB, three different tests were made: a) Free rocking motion, b) Harmonic motion, and c) Random motion. The first type of test allows to identify the parameters used in the classical theory and at the same time to calibrate the analytical models (Prieto et al. 2006). On one hand, the harmonic tests allow to study, in a simple way, the dynamic behaviour of single blocks undergoing RM regime; while on the other hand, the behaviour of the RB under earthquake conditions was studied with the random test.

Thirty synthetic earthquakes compatibles with the design spectrum proposed by the Eurocode 8 (2004) were generated. In order to identify them, they were named consecutively with the number of generating. The constant branch of the spectrum is located between 0.1 and 0.3 seconds, with a spectral acceleration of 7 m/s^2 while the maximum ground acceleration is 2.8 m/s^2. The main aim of the study is to address stability of RM under random motion.

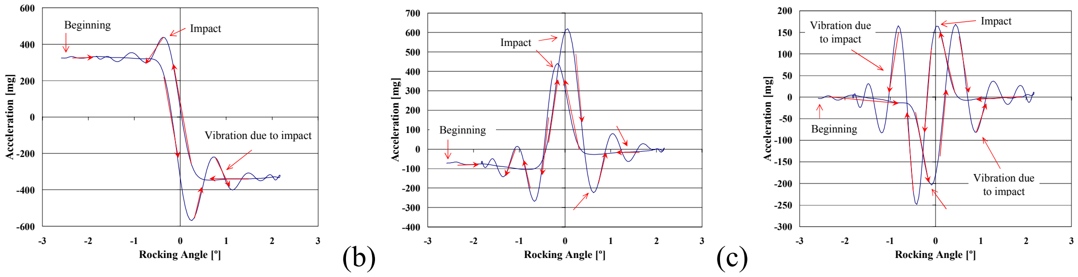
3 SINGLE BLOCKS

3.1 *Free rocking*

In order to understand the dynamics of the free rocking motion, one cycle is detailed in terms of acceleration (measured at the top of the block) and rocking angle, as shown in Fig. 3. When the free rocking motion begins (with an initial angle different than zero), the E-W acceleration is different than zero if the side where the accelerometer is located, is the rotation point; otherwise, it will be different than zero due to the gravity loads. The accelerations remain almost constant until the impact occurs. In this moment an impulsive acceleration (force) appears in the three directions. Due to the impulsive acceleration a variation of the acceleration (vibration) appear after the impact. The E-W acceleration changes sign and will tend to remain constant with the same value of the initial acceleration. The N-S and Vertical accelerations will tend to zero. The maximum impulsive accelerations in E-W and Vertical directions are in the same order of magnitude, while in the N-S direction the maximum value is around the half. This impulsive out-of-plane acceleration causes 3D effects in the RB. For one cycle, the vertical acceleration has a shape of a Dirac-δ, while the N-S acceleration shape is a semi Dirac-δ. The E-W acceleration presents the typical Z shape of rocking systems.

3.2 *Harmonic motion*

The response of a RB under harmonic motion depends on the frequency and the amplitude of the harmonic motion. To illustrate this fact, Table 2 shows typical frequencies and amplitudes at which a specimen shows rocking. A clear relationship between the harmonic motion and the rocking motion exists. It is clear that if the amplitude increases while the frequency remains constant, the amplitude of the rocking increases too. On the other hand, if the frequency of the motion increases, while the amplitude of the load remains constant, the rocking angle decreases. Thus, there are some couples of frequencies and amplitudes of harmonic motion with which the RB will experience rocking. The set of these couples is called rocking motion space.



(a) (b) (c)

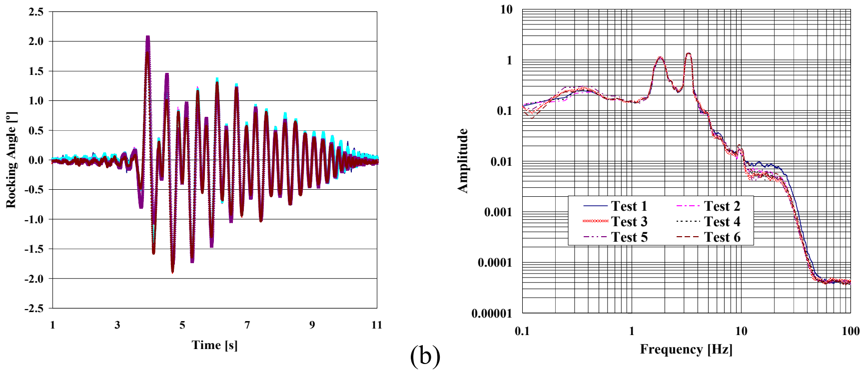
Figure 3 : Typical acceleration vs. rocking angle for one cycle (specimen 1); (a) Horizontal (E-W), (b) Vertical, (c) Out-of-plane (N-S).

Table 2 : Typical rocking motion space and maximum rocking angle (º) for harmonic motion (specimen 2).

Constant Sine	Amplitude (mm)						
(Hz)	03	04	05	06	08	10	12
0.5	N-R						N-R
1.0							N-R
1.5							6.08
2.0		N-R	N-R	N-R	3.04	3.93	
2.5	N-R	N-R	2.27	2.33	2.63		
3.0	1.36	1.49	1.85	1.87	2.15		
3.3	1.35	1.49	1.57	1.42	2.03		
5.0		0.95	1.14	1.23			

N-R = No rocking

Some tests were repeated in order to study the repeatability of the response of the RB under forced excitation. These tests show that the repeatability in the harmonic motion exists. The six test performed with specimen 1 subjected to Hanning sine of 3.3Hz and amplitude of 7 mm are depicted in Fig. 4a. The response of all six tests is similar. Taking the first test as reference, the differences (in percentage) of the values of the maximum rocking angle is below 3% except for test number 5 that presents an error of 10%. Fig. 4b shows the Fourier spectra of the rocking angle. It can see that all six spectra are practically the same and they show little differences on frequencies up to 10 Hz. The Fourier spectrum shows two peaks. One is related with the load frequency (3.3 Hz), while the other (1.9 Hz) is associated to the transient response.



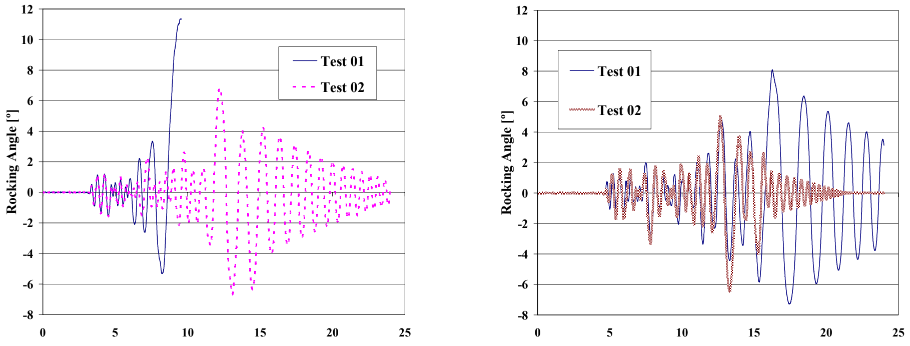
(a) (b)

Figure 4 : Typical repeatability test carried out with Hanning sine with frequency of 3.3 Hz and amplitude of 7 mm (specimen 1); (a) Rocking angle, (b) Fourier spectra.

3.3 *Random motion*

Specimen 1 was damaged after the set of test described above. Therefore, only specimens 2 to 4 were subjected to random motion in order to study the dynamic behaviour under earthquake motions.

A repeatability test was carried out with the generated random motion, under increasing load factors that multiply the original accelerogram. Each test was repeated at leas two times. Typical responses are shown in Fig. 5 in terms of rocking angle curves for specimen 2 loaded with earthquake 20 and load factors of 1.1 and 1.3. With a load factor 1.1, the specimen has a maximum rocking angle of 6.8º for one test while the other test the specimen overturns. It is worth notice that during the first six seconds the behaviour of the RB is similar for the two tests. Two tests were carried out with the load factor of 1.3. In this case, the RB did not overturn for any test and both behaviours are similar during the first seconds. However, at the end of the load the behaviours became rather different, especially during the free rocking motion. This is due mainly to two causes. The first cause is the small changes in the initial position that always exist in the rocking motion and that cause a change of boundary conditions. The second cause is the high frequencies (up to 10 Hz) that random records contain. The combination of these two phenomena leads to large changes in the dynamic behaviour of the blocks.



(a) (b)

Figure 5 : Typical repeatability test carried out with earthquake record 20 (specimen 2); load factor of: (a) 1.1, (b) 1.3.

Table 3 shows the average critical angle obtained from the tests and from the theory (static stability only). In general, the theoretical angle is lower than the experimental one. This should be expected due to the beneficial effect of the inertial forces. Specimen 3 is very slender (*h/b* ratio equal to 8) and the effect of the inertial forces seems of less relevance for stability purposes.

Table 3 : Critical angle.

Specimen	Critical angle (º)		
	Theoretical	Experimental	Difference (%)
2	9.6	11.2	16
3	6.8	6.9	2
4	18.0	20.8	15

4 BI-BLOCKS STRUCTURE

4.1 Harmonic motion

Fig. 6 shows the four possible patterns of rocking motion that a bi-block structure may exhibit (Spanos et al. 2001). They can be divided in two main groups. The first one includes the pattern 1 and 2 and they are equivalent to a two degree of freedom system response, in which the two blocks rotate in the same or opposite direction. The second group (patterns 3 and 4) corresponds to a single degree of freedom system response. In particular, pattern 3 is equivalent to one rigid structure, while pattern 4 is the case where only the top block experiences rotation (Spanos et al. 2001).

From the experimental test, the principal patterns that the bi-block structure tested exhibits were 2 and 4. These patterns are associated with the amplitude and frequency of the loads. Tables 4 and 5 show the couples of frequency and amplitude with which both patterns are presented. A clear relationship between the harmonic motion and the rocking motion exists. There are some couples of frequencies and amplitudes of harmonic motion with which the bi-block structure will experience pattern 2 or 4. In this particular case, the pattern 2 was recorded with higher frequencies and/or higher amplitudes, while pattern 4 was recorded with lower frequencies and lower amplitudes.

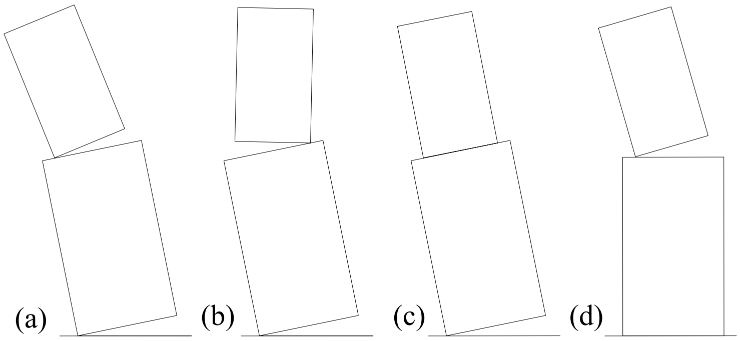


Figure 6 : Classification of rocking patterns for a bi-block structure; (a) Pattern 1, (b) Pattern 2, (c) Pattern 3, (d) Pattern 4 (Spanos et al. 2001).

Table 4: Typical rocking motion space and maximum rocking angle (°) for harmonic motion, top block.

Constant Sine	Amplitude (mm)				
(Hz)	01	02	03	04	05
3.0		2.097	3.159		
3.3		2.105	3.021	4.145	
4.0	1.422	1.990	3.563	3.128	Collapse
5.0		2.357	1.852	Collapse	
6.5	0.880	3.472			

Table 5: Typical rocking motion space and maximum rocking angle (°) for harmonic motion, bottom block.

Constant Sine	Amplitude (mm)				
(Hz)	01	02	03	04	05
3.0		N-R	N-R		
3.3		N-R	N-R	0.674	
4.0	N-R	N-R	0.557	0.361	5.278
5.0		0.419	0.285	9.704	
6.5	0.132	1.794			

N-R = No rocking

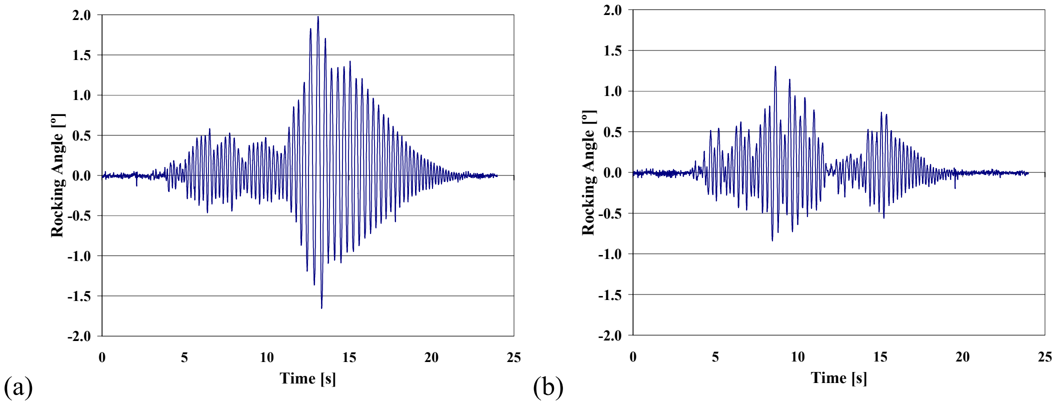


Figure 7 : Typical results obtained with earthquake record 21 (top block); load factor of: (a) 0.2, (b) 0.3.

4.2 Random motion

Due to the characteristics of the random motion and the dimensions of the blocks only the pattern 4 was recorded. Fig. 7 shows typical rocking motion of the top block. In general, chaotic behaviour is found in the response of the bi-block structure under random motion. For example, in the case illustrated (earthquake record 21), the maximum amplitude of the rocking motion is greater with a lower load factor. In the case of the load factor 0.2, the maximum rocking angle is equal to 2º, while with load factor 0.3, the maximum rocking angle is equal to 1.3. This type of response was found for the single blocks too. Similar results can be found elsewhere (Yim et al. 1980, Tso and Wong 1989).

5 DOLMEN STRUCTURE

In this section preliminary results of the bi-block model are presented. Fig. 8 shows the final displacement patterns that the dolmen exhibited during the constant sine base motion. Each pattern is associated to a particular frequency and amplitude. It can be seen that a highly three dimensional behaviour is presented, especially for higher frequencies. In general, it is possible to classify the different patterns in two main groups. The first one correspond a lateral motion of all the structure with sliding of the pillars (Fig.s 8b,c,d), that corresponds to a range of frequencies between 1 to 3.3 Hz. The second group corresponds to torsional motion of the lintel with sliding of the pillars (Fig.s 8e,f). It most be highlighted that the relative motion among elements modify the stability of the construction even with small relative displacements; because the elements lose the perfect contact between them (Fig. 9).

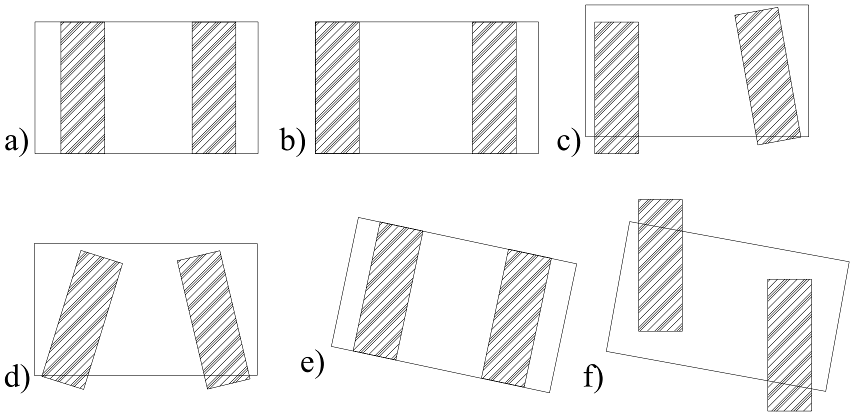


Figure 8 : Final displacement patterns for the dolmen subjected to constant sine (plan view); (a) Initial configuration, (b) 1 - 1.5 Hz, (c) 1.7 Hz, (d) 3.3 Hz, (e) 5 Hz, (f) 6.5 Hz.

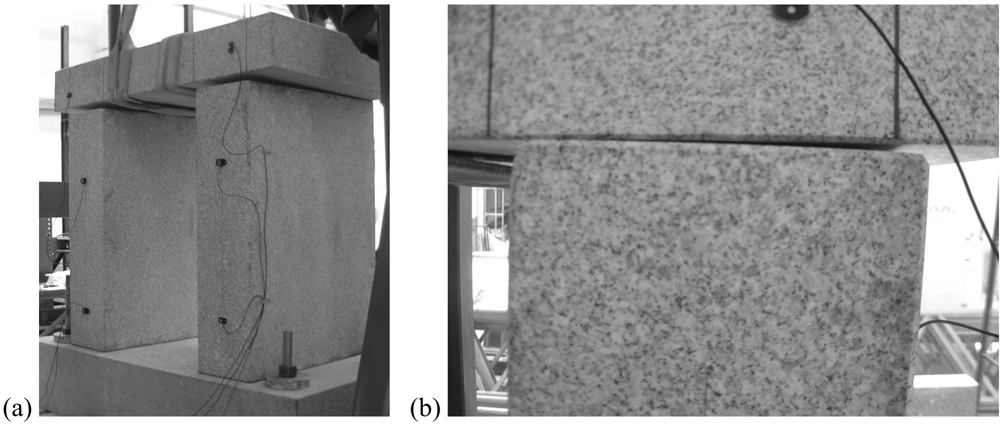


Figure 9 : Details of the final displacement patterns for the dolmen subjected to constant sine.

6 CONCLUSIONS

This paper presents a comprehensive experimental program carried out on four single blocks (blue granite stones) and ensemble of two and three block structures (bi-block and dolmen structures) under dynamic loading. The experimental critical angle that defines stability of the rocking block is always greater than the theoretical. Thus, the theoretical angle is a conservative value for the design process.

Repeatability of the tests with harmonic ground motion exists. However, it does not exist with random motion, where very large variations are found in the response. This is due to the combination of high frequencies and the small changes in the boundary conditions.

The amplitude of rocking depends on the frequency and amplitude of load. The rocking motion will be greater with low frequencies.

The impact produces impulsive accelerations in the block. These accelerations were recorded in three orthogonal directions (horizontally, parallel and transversal to the movement, and vertically). Thus, at each impact an impulsive out-of-plane acceleration produces three-dimensional behaviour.

The bi-block structure presents mainly two patterns of motion. The first one corresponds to an equivalent two degree of freedom system response, in which the two blocks rotate in the opposite direction. The second pattern of motion corresponds to a single degree of freedom system response in which only the top block experiences rotation.

The dolmen structure presents a highly three-dimensional behaviour that cannot be neglected. The pillars and lintel present relative motion among themselves.

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REFERENCES

Aslam, M, Godden W.G. and Scalise D.T. 1980. Earthquake rocking response of rigid bodies. *J. of Engrg. Mech. Div. ASCE* 106, p. 377-392.

Augusti, G. and Sinopoli, A. 1992. Modelling the dynamics of large block structures. *Meccanica* 27, p. 195-211.

Eurocode 8 2004. *Design provisions for earthquake resistance of structures. Part 1-1*. EN 1998-1, CEN, Bruselas.

Housner, G. 1963. The behavior of inverted pendulum structures during earthquakes. *Bulletin of the Seismological Society of America,* 53(2), p. 403-417.

Peña, F., Prieto, F., Lourenço, P. and Campos Costa, A. Sub. On the dynamics of single rigid-block structures. Part I: Experimental Testing. *Earthquake Engineering and Structural Dynamics,* submitted for possible publication.

Peña, F. 2006. *Dinámica de estructuras multibloque: experimentación y simulación numérica*. Report 06-DEC/E-03, Departamento de Engenharia Civil, Universidade do Minho, Portugal.

Prieto, F., Peña, F., Lourenço, P. and Lemos, J. 2006. Numerical simulation of rigid blocks subjected to rocking motion. In P.B. Lourenço, P. Roca, C. Modena and S. Agrawal (eds), *Historical constructions*. New Delhi.

Spanos, P., Roussis, P. and Politis, N. 2001. Dynamic analysis of stacked rigid blocks. *Soil Dynamics and Earthquake Engineering,* 21, p. 559-578.

Tso, W.K. and Wong, C.M. 1989. Steady state rocking response of rigid blocks to earthquake. Part 1: Analysis. Part 2: Experiment. *Earthquake Engineering and Structural Dynamics* 18, p. 89-120.

Yim, C., Chopra, A. And Penzien, J. 1980. *Rocking response of rigid blocks to earthquakes*. Report UCB/EERC-80/02, Earthquake Engineering Research Center, University of California, Berkeley.