DYNAMIC IDENTIFICATION OF THE QUTB MINAR, NEW DELHI, INDIA

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Abstract

In the framework of the Eu-India Economic Cross Cultural Programme "Improving the Seismic Resistance of Cultural Heritage Buildings", aimed at the preservation of ancient masonry structures with regard to the seismic risk, dynamic identification tests were applied to the Qutb Minar, New Delhi, India, in September 2005. The paper describes the dynamic identification works, intended to define the dynamic response of the tower. For the dynamic modal identification analysis different test equipments were used, in order to compare the data and to have more reliable results. The dynamic parameters resulted from the acquisition campaigns will be used to estimate the mechanical properties of the masonry walls and the boundary conditions of the structure, to be considered in successive seismic nonlinear analyses of the Qutb Minar, aimed at the assessment of the safety level of the construction.

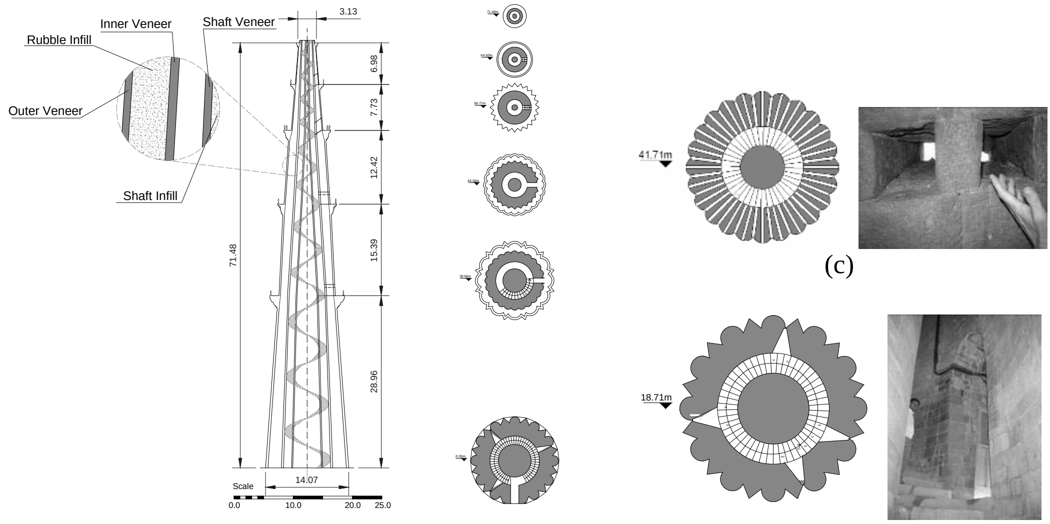
1 Introduction

The Qutb Minar, is the highest monument of India and one of the tallest stone masonry towers in the world. Inside, a helical staircase with 379 steps communicates to five balconies, where the Mu'adhdhin (muezzin) called to prayer. The minaret has also a symbolic function, being a sign to glorify the victory of Islam against idolatry. The construction began during the reign of Qutb-ud-din around 1202, but the erection stopped at the first storey. The next ruler, Iltutmish, added the next three storeys. The tower was damaged by lightning in 1326 and again in 1368. In 1503 Sikandar Lodi carried out some restoration and enlargement of the upper storeys [1]. As a reference to the importance of the monument, the Qutb Minar has been inscribed in the world heritage monument list since 1993.

Figure 1 General views of the Qutb Minar

2 Structural and Damage Survey

The Qutb Minar directly rests on a 1.7 m deep square ashlar masonry platform with sides of approximately 16.5 m, which in turn overlies a 7.6 m deep lime mortar rubble masonry layer, also square, with sides of approximately 18.6 m. The bedrock is located around 50-65 m below the ground level. The Minar cross-section is circular/polilobed, being the base diameter equal to 14.07 m and tapering off to a diameter of 3.13 m at the top, over a height of 72.45 m. The tower is composed by an external shell corresponding to a three leaf masonry wall and a cylindrical central core (Figure 2a).



(a) (b) (d)

Figure 2 Geometrical survey: (a) vertical section of the Qutb Minar; (b) cross sections at different levels; (c) small size ventilation openings; (d) tapered windows

The core and the external shell are connected by a helical stairway and by 27 “bracings” composed by stone units with an average cross section of $0.40 \times 0.40\ m^2$. The stairway is spiral, disposed around the central masonry shaft, and it is made of Delhi quartzite stone. Each storey has a balcony and the uppermost storey finishes with a platform.

The Minar is also provided with diffuse ventilation openings that can be divided into two groups: some smaller openings on three levels and larger openings (windows) respectively presented in Figure 2c and Figure 2d. In correspondence of the second and third levels of the smaller openings the cross section of the tower decreases almost to 50% of the total.

The Minar outer shell is composed by a three leaf masonry wall. In the first three storeys the external veneer is made of ashlars of red and buff coloured sandstone whereas the internal is composed by Delhi quartzite ashlars. In the two upper storeys the external veneer is made of white marble stones and the internal of red sandstone. The infill is composed by rubble stone masonry, mainly with stones taken from the destroyed temples during the Islamic dominion.

3 Dynamic Identification Analysis

Preliminary analyses, carried out by the Indian Institute of Technology, Madras, Chennai, India [1], were taken into account for the test planning definition. The natural frequencies of the Qutb Minar were estimated by two numerical models with different boundary conditions: a) stiff and b) flexible base restraints. In both cases the masonry was considered to be isotropic, with an elastic modulus of 1.0 GPa in the first model and 0.6 GPa in the second. Table 1 presents the first five natural frequency values related to the observed mode shapes.

According to these first numerical estimations the first five expected frequencies of the tower range between 0.6 and 8.1 Hz. The tower behaves as a cantilever beam with mainly bending mode shapes.

Table 1 First five natural frequency values

Mode shape	Stiffer end Hz	Flexible end Hz	Comment
1	1.06	0.58	1st Bending
2	3.26	1.70	2nd Bending
3	6.47	3.39	3rd Bending
4	7.69	4.04	Torsion
5	8.12	4.25	Axial

3.1 First Identification Analysis

An acquisition chain composed of 8 uniaxial piezoelectric accelerometers, with a bandwidth ranging from 0.15 to 1000 Hz , a dynamic range ±0.5 g and a sensitivity of 10 V/g, connected to a data acquisition system with 16 bit A/D converter, provided with anti-aliasing filters in the amplification cards was considered for the first modal identification, performed by the University of Minho. As the digital band was configured to digitalize between ±0.05 g the system resolution was equal to 8 μg (equal to the transducer resolution).

1.3.1 Test Planning

To measure the dynamic response of the Qutb Minar to ambient vibrations, 20 points on five levels of the structure were selected. The sensors were placed in correspondence of the five existing balconies of the tower. The measuring points and directions are presented in Figure 3. As the experimental tests were based on the measurement of ambient vibrations with sequential data sets, one point at the top of the Minar was selected to be the reference point in the *x* (E-W direction), *y* (N-S direction) and *z* (vertical) directions.

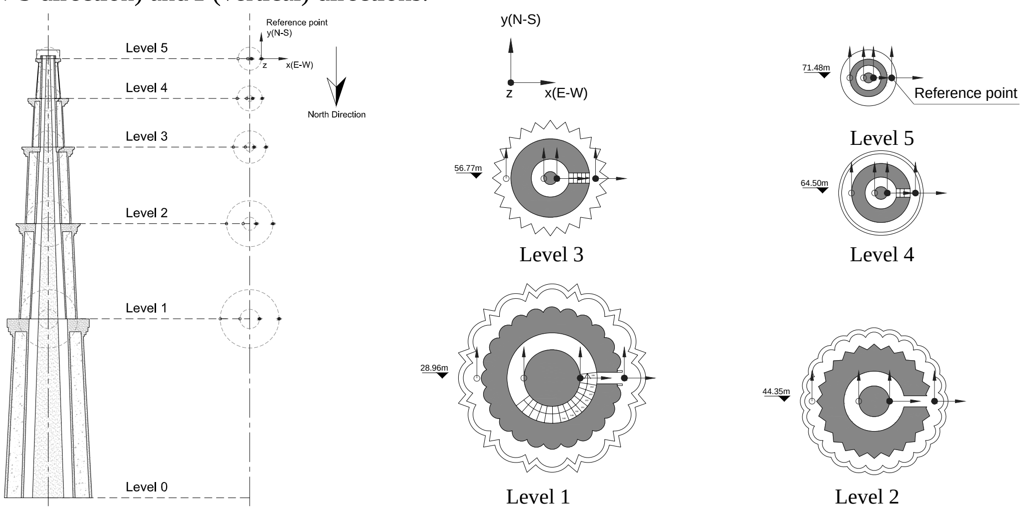


Figure 3 Measuring points and directions

On each balcony the transducers were positioned in four points, two in triaxial (*x*, *y* and *z*) configuration and two in the *y* direction. The four points were positioned in the same line along the *x* direction; two points were located in the external shell and the others two in the internal core. Such test layout was chosen in order to evaluate the degree of connection between the external shell and the central core, since the two structural parts could have manifested a different dynamic response. The ambient vibrations were measured in all of the 20 points with 9 sequential setups, with a sampling frequency of 100 Hz and a total sampling of 20 min, approximately 1000 times the minimal expected period.

2.3.1 Experimental Results

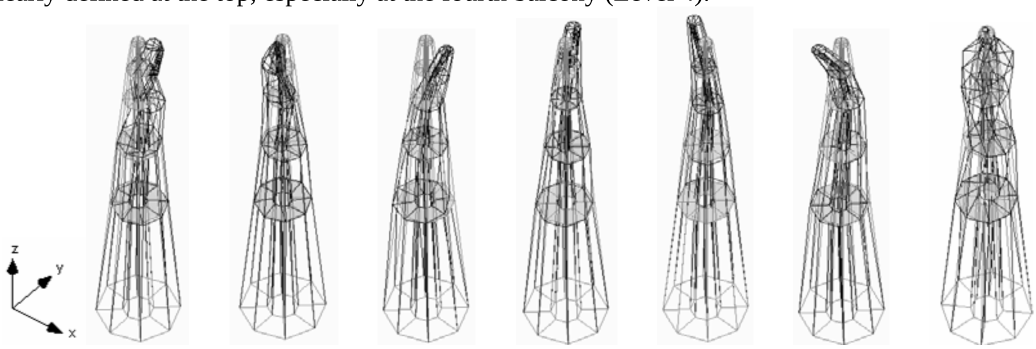
Output-only modal identifications techniques were used to estimate the modal parameters: resonant frequencies, mode shapes and damping coefficients. These techniques are based on the dynamic response measurements of a virtual system under natural (ambient or operational) conditions, and they are based on the assumption that the excitation is reasonably random in time and in the physical space of the structure [2] [3]. In the case of the Minar, the ambient vibrations were mainly induced by the wind.

The software ARTeMIS Extractor Pro was used for the signal processing. Two different techniques were considered to estimate the dynamic parameters, in order to have more reliable results. Both of the techniques operate in the time domain and are based on the Stochastic Sub-space Identification (SSI) method: Unweight Principal Component (UPC) and Principal Component (PC). These techniques were selected because they are robust and allow modal parameters estimation with high frequency resolution [4]. In the different analyses, pairs of closely spaced frequencies were noticed, especially concerning the first two modes. This is due to the axisymmetric cross section of the structure, fact that generally leads to near pairs of bending mode shapes. For this reason SSI methods were selected for the modal identification, since they are between the most adequate in similar cases, given the difficulty to estimate closely spaced modes with frequency domain methods.

Table 2 Dynamic results

Mode shape	UPC Hz	PC Hz	Error %	MAC	Aver. Damping %	Comment
1	0.793	0.785	1.003	0.777	3.253	1st Bending
2	0.814	0.814	0.025	0.443	2.555	2nd Bending
3	1.955	1.953	0.079	0.988	1.126	3rd Bending
4	2.010	2.009	0.080	0.994	0.749	4th Bending
5	3.741	3.741	0.001	0.992	1.394	5th Bending
6	3.861	3.864	0.061	0.988	0.884	6th Bending
7	4.400	4.484	1.889	0.309	3.570	1st Torsion
8	6.006	5.966	0.671	0.743	1.653	7th Bending
9	6.146	6.073	1.186	0.832	1.347	8th Bending
10	6.282	6.261	0.330	0.907	1.270	Axial
11	6.977	6.968	0.137	0.865	0.972	2nd Torsion
12	8.090	8.174	1.033	0.690	2.131	Undefined
13	8.525	8.530	0.065	0.788	2.247	9th Bending
14	8.669	8.663	0.064	0.879	2.471	10th Bending

The maximum level of vibration measured on the top of the Minar was lower than 2.5 mg. The 9 data series acquired at 100 SPS (Samples Per Second) were then processed by a decimation of 5 (Nyquist frequency of 10 Hz), with segment length of 516 points and 66.67% window overlap, with 3 projection channels for the subspace identification. 20 structural modes and 30 noise modes were considered for the stochastic estimation of the models. Concerning the frequency results, Table 2 summarizes the experimental values for both of the analyses. The estimated corresponding frequencies, arising from the two methods, are very similar, with an error less than 2%. The closest frequencies are related to the first two modes, with values of 0.79 and 0.81 Hz. Figure 4 shows a perspective view of the defined mode shapes. Ten bending, two torsion, one axial and one undefined mode shapes were estimated. It is stressed that the two first bending modes were not clearly defined at the top, especially at the fourth balcony (Level 4).



Mode 1 Mode 2 Mode 3 Mode 4 Mode 5 Mode 6 Mode 7

Figure 4 Perspectives of the first seven mode shapes (UPC method)

Figure 5 shows the estimated bending modes, from above. As it can be observed, the bending modes directions are almost perpendicular for the closely spaced pairs of frequencies. This is due to the axisymmetric cross section of the tower.

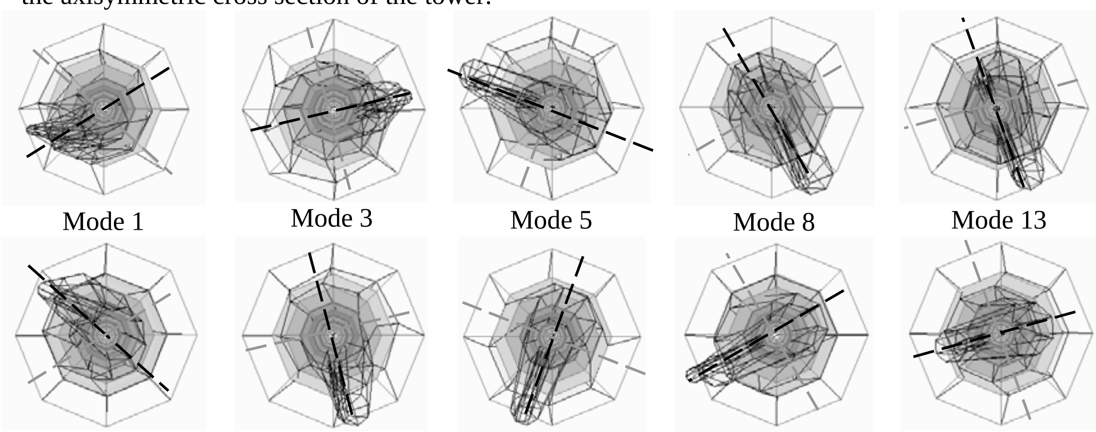


Figure 5 Bending main directions for bending mode shapes (UPC method)

The Modal Assurance Criterion [5], the well known procedure to evaluate the correlation between two sets of mode shape vectors (the results vary from 0 to 1, i.e. from bad to good correlation), was used to compare the results emerged from the two modal identification procedures considered. Results are presented in Table 2. Only for the second and the seventh mode the MAC value is lower than 0.70, which means that, globally, the analysis can be considered accurate.

In order to improve the definition of the first two modes shapes in the upper part of the tower, an additional modal estimation was performed. In this further analysis the records acquired in the fourth balcony (Level 4, see Figure 3) were removed from the data processing since they seemed to worsen the modal estimation. The results obtained from the first and the second analysis are summarized and compared in Table 3 and Figure 7. The fourth level data removal does not sensibly change the results in the others levels, this fact being a possible indicator of an accurate modal estimation. Moreover, the Coefficient of Variation (CV), is lower than 2% for all of the modes estimated with the two different analyses.

Table 3 Comparison results between different analyses

Mode	UPC		PC		Average Hz	CV %	Comment
	5 Levels Hz	4 Levels Hz	5 Levels Hz	4 Levels Hz			
1	0.793	0.783	0.785	0.779	0.785	0.71%	1st Bending
2	0.814	0.812	0.814	0.822	0.816	0.51%	2nd Bending
3	1.955	1.949	1.953	1.952	1.952	0.11%	3rd Bending
4	2.010	2.012	2.009	2.010	2.010	0.07%	4th Bending
5	3.741	3.748	3.741	3.744	3.744	0.09%	5th Bending
6	3.861	3.871	3.864	3.868	3.866	0.11%	6th Bending
7	4.400	4.323	4.484	4.500	4.427	1.84%	1st Torsion
8	6.006	5.985	5.966	5.984	5.985	0.28%	7th Bending
9	6.146	6.115	6.073	6.122	6.114	0.50%	8th Bending
10	6.282	6.266	6.261	6.256	6.266	0.18%	Axial
11	6.977	6.968	6.968	6.975	6.972	0.07%	2nd Torsion
12	8.090	8.178	8.174	8.058	8.125	0.74%	Undefined
13	8.525	8.534	8.530	8.532	8.530	0.05%	9th Bending
14	8.669	8.679	8.663	8.664	8.669	0.08%	10th Bending

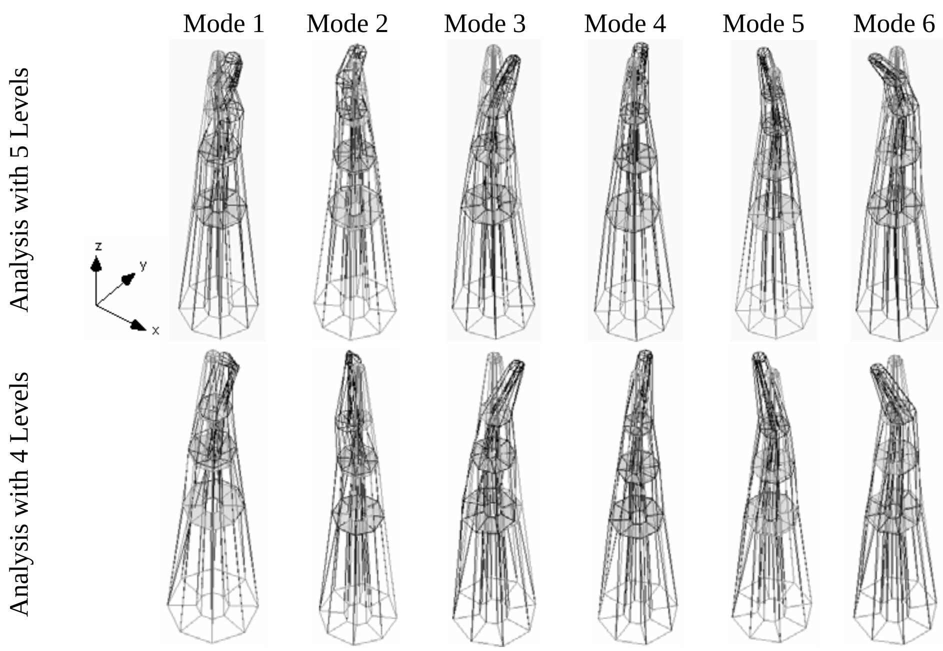
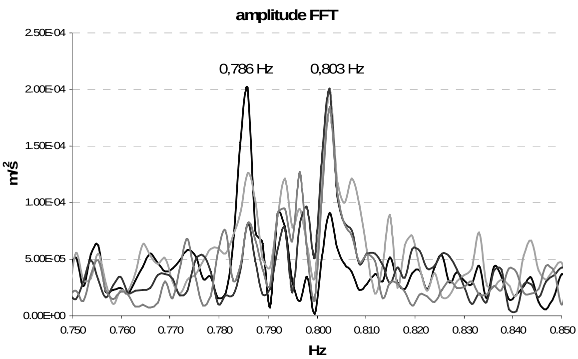


Figure 6 Comparison between the two analysis (UPC method)

3.2 Second Identification Analysis

By using the sensors disposition reported in Figure 3 at the top (fifth) level of the Minar, Ambient Vibration Tests were also carried out by the University of Padua, by using a different acquisition system. Eight piezoelectric acceleration transducers with the same characteristics respect the first acquisitions were used, connected to a module provided with 24 bit A/D converter acquisition card. A sampling rate of 100 SPS was considered, with a total of 65,536 points acquired. In this case, a single amplitude peak FFT on the complete set of points was performed. The visualization of the FFT function in correspondence of the first resonant peaks and comparative results in the frequency domain respect the first 9 frequencies identified by University of Minho (PC method) are provided in Figure 7. A good match between the obtained frequencies can be noticed (maximum error equal to 2.7%).



Mode	FFT Hz	PC Hz	Error %
1	0.786	0.785	0.127
2	0.803	0.814	1.351
3	1.950	1.953	0.154
4	2.008	2.009	0.050
5	3.755	3.741	0.373
6	3.866	3.864	0.052
7	---	4.484	---
8	5.992	5.966	2.600
9	6.046	6.073	2.700

Figure 7 left: amplitude FFT, first resonant peaks, related to the two first mode shapes individuated; right: comparative frequencies results, different acquisition systems

4 Conclusions

Concerning the preservation of cultural heritage historical buildings, the first step in the structural analysis is the attainment of a sound knowledge of the structure and of the composing materials, in order to proceed in the assessment of the building with “correct” bases. With this perspective and aiming at the seismic protection of the building, the tests were carried out by different research groups on the structure of the Qutb Minar in Delhi, India, to evaluate its structural behavior.

Dynamic tests were carried out considering several test positions, at different heights, in order to proceed with the modal identification of the tower. To double-check the obtained results, different acquisition system were employed. Tests carried out by the University of Minho, Portugal, were aimed at the definition of the modal parameters (natural frequencies, mode shapes and damping coefficients) of the Minar, also to evaluate the degree of connection between the central shaft of the tower and the external shell.

The modal identification procedures considered gave satisfactory results. Several natural frequencies and corresponding modes were defined, with reduced errors in the frequency values and a general high correlation (MAC values) between the mode shapes defined with the two time domain methods used. An additional modal estimation carried out removing unsatisfactory data recorded at the fourth level of the Minar improved the mode shapes definition, comporting negligible variations in terms of estimated frequencies.

The comparison with the results obtained by the University of Padua indicated a good match between the obtained frequencies, with reduced errors.

Difficulties in obtaining a clear definition of the two first mode shapes in the upper levels (the central shaft seemed to present higher displacements respect the external shell) can be possibly correlated to a decreased degree of connection between the external and the internal parts of the structure, at the top of the tower. To ascertain this observation, a more refined net of sensors would however be required.

The obtained data will be used for the validation of the preliminary numerical models considered for the definition of the dynamic test layout, in order to have reliable tools for the seismic assessment of the Qutb Minar.

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