A New Adaptive Scheme for the
Adaptive Linearizing Control of Bioprocesses

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Abstract

This work deals with the development of model-based adaptive control algorithms for bioprocess operation. Non-linear adaptive control laws are proposed for single input single output regulation. Parameters are continuously adapted following a new adaptive scheme which ensures second-order dynamics of the parameter error system. A computational study is presented of the application of this theory to baker's yeast fermentation. Results put in evidence the efficient performance both of the adaptive scheme and of the related control laws.

Keywords: Adaptive control, linearizing control, 2nd order convergence, fermentation process, baker's yeast

1. Introduction

Biological processes, namely fermentation processes, are generally characterised as exhibiting non-linear behaviour, with a strong variability of their characteristic parameters. The efficient, let alone the optimal, control of bioreactors is by no means a simple task and still today represents a major challenge to theoreticians and practitioners alike.

The exploitation of the basic knowledge of the non-linear structure of bioprocesses, in conjunction with adaptive schemes for process parameters, represents a recent methodology, more and more adopted for the development of efficient bioprocess control algorithms [2, 6, 7].

Recently, so called advanced techniques of differential geometry [5] have been proposed for the control of non-linear systems. Essentially, the idea is to find and apply non-linear transformations of the state and/or the input variables, such that when applied to non-linear systems, a closed-loop feedback system results which exhibits a linear dynamic behaviour.

In the present work the methodology for the synthesis of the non-linear control law follows the work of Bastin and Dochain [2] who employ linearizing control with state feedback for the control of biological reactors. This method leads to a feedback control law which *lato sensu* results in a linear behaviour of the input-output closed loop system.

The structure of fermentation processes

A *general dynamic model of biological reactors* is accepted as representing the dynamics of bioprocesses:

$$\frac{d\xi}{dt} = K\varphi(\xi, t) - D\xi + F - Q \tag{1}$$

where $\xi \in R^n$ represents the n state components; $K\varphi \in R^n$ represents the kinetic structure, assumed unknown; the vectors F and Q represent respectively the process inputs and outputs, known from on-line measurements; and, finally, D represents the dilution rate (ratio feed rate / volume) which will be the manipulated variable in the theoretical development.

In the single output problem, the objective will be to control a scalar *y* which will be taken as a linear combination of the state variables, of the form:

$$y = \sum_{i=1}^{n} L_i \xi_i = L^T \xi \tag{2}$$

where $L^T = [L_1, L_2, ..., L_n]$ is a vector of known constants. Combining model equations (1) with the transformation (2), the following input-output model is readily obtained:

$$\frac{dy}{dt} = -Dy + K_y \varphi + F_y - Q_y \tag{3}$$

where the index y in K, F and Q is employed to represent the transformed matrix and vectors.

Given that the kinetic structure is unknown, a procedure of on-line parameter estimation will have to be employed, which however will have to be based on a reformulated model free of the kinetic terms. In those cases where a number of fast dynamic state components are recognised, this number being equal to or larger than the number of unknown kinetic terms, it is possible to assume that such components are in pseudo steady-state and, through the single perturbation method, to reformulate the problem in terms of a reduced model of the general form:

$$\frac{dy}{dt} = \Phi(\xi) + \theta^T \phi(F, Q) + D(t)\Psi(\xi) \tag{4}$$

where Φ, Ψ and ϕ are known functions of ξ, F and Q, and θ represents the matrix of unknown parameters which characterise the kinetics. The kinetics has been taken as unknown, but even if this were not the case, the fact that model (4) is the result of some simplifying hypotheses means that θ is in all cases a vector of unknown parameters, which necessarily will have to be adapted on-line. From this point of view, ϕ represents the corresponding regression vector associated to θ.

2. Adaptive control

The feedback control objective is that the control variable follows a reference value represented by $y^*(t)$. The linearizing control problem consists of deducing and implementing a non-linear law $D(\xi, Q, y^*)$ such that the controller convergence error $\tilde{y} = (y^* - y)$ be governed by a pre-specified stable linear differential equation, known as reference model. In the single output problem, the control objective will consist of imposing a stable first order closed-loop dynamics of the form:

$$\frac{d}{dt}\left(y^* - y\right) + \lambda\left(y^* - y\right) = 0, \quad \lambda > 0 \tag{5}$$

Combining the input-output reduced model (4) with the reference model (5), will easily lead to the following linearizing control law:

$$D(t) = \Psi(\xi)^{-1}\left\{\lambda\left(y^* - y\right) + \frac{dy^*}{dt} - \Phi(\xi) - \hat{\theta}^T \phi(\xi)\right\} \tag{6}$$

Employing the reduced model, will lead to concentrate all the unknowns in the parameter matrix θ, which will be estimated on-line. Hence, in eqn. (6) the estimates $\hat{\theta}$ will be employed rather than the true values θ.

The dynamics of the control error $\tilde{y} = (y^* - y)$, for the situation where estimated $\hat{\theta}$ are employed, is modelled by substituting eqn. (6) in eqn. (4), giving:

$$\frac{d\tilde{y}}{dt} \equiv \frac{dy^*}{dt} - \frac{dy}{dt} = -\lambda\tilde{y} - \tilde{\theta}^T \phi \tag{7}$$

where $\tilde{\theta}$ represents the error in the parameters θ, defined by $\tilde{\theta} \equiv \theta - \hat{\theta}$.

3. Estimation laws

The basis for the method adopted is the so-called Lyaponov design as applied by Bastin and Dochain [2]. For the single output problem, it is assumed that each parameter θ_i is estimated by the following adaptive law:

$$\frac{d\hat{\theta}_i}{dt} = -\Gamma_i \phi_i\left(F, Q\right)\left(y^* - y\right) \tag{8}$$

where $\phi_i(F, Q)$ is the regressor associated to θ_i, function of F and Q, and Γ_i the positive definite estimator gain.

The dynamic model for the error system, composed by the control error $\tilde{y}$ and by the parameter error $\tilde{\theta}_i$, can be deduced from eqns. (6) and (7) and written as

$$\frac{d}{dt}\begin{bmatrix} \tilde{y} \\ \tilde{\theta}_i \end{bmatrix} = \begin{bmatrix} -\lambda & -\phi_i \\ \Gamma_i \phi_i & 0 \end{bmatrix}\begin{bmatrix} \tilde{y} \\ \tilde{\theta}_i \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\frac{d\theta_i}{dt} \tag{9}$$

whose characteristic equation presents the roots

$$0.5\lambda \pm \sqrt{0.25\lambda^2 - \Gamma_i \phi_i^2} \tag{10}$$

The criterion chosen for the error dynamics determines the parameter convergence and as such the quality and even the feasibility of the proposed adaptive scheme.

Choice of a real double pole

Perrier and Dochain [8] proposed to fixing the closed loop dynamics by choosing a real double pole, which leads to the following relationship between controller and estimator parameters:

$$\Gamma_i = 0.25\lambda^2 \phi_i^{-2} \tag{11}$$

Substituting eqn. (11) in (8), gives:

$$\frac{d\hat{\theta}}{dt} = -\gamma\phi_i^{-1}\left(y^* - y\right) \tag{12}$$

with $\gamma \equiv 0.25\lambda^2$.

Generalising for 2nd order convergence

Regression equation (12) will be employed, with a constant but undefined γ. Differentiating, leads to:

$$\frac{d^2\hat{\theta}_i}{dt^2} = -\frac{\gamma}{\phi_i}\frac{d\tilde{y}}{dt} + \frac{\gamma}{\phi_i^2}\frac{d\phi_i}{dt}\tilde{y} \tag{13}$$

Combining with eqn. (9), leads to the following 2nd order equation:

$$\frac{1}{\gamma}\frac{d^2\hat{\theta}_i}{dt^2} + \frac{1}{\gamma}\left(\lambda + \frac{1}{\phi_i}\frac{d\phi_i}{dt}\right)\frac{d\hat{\theta}_i}{dt} + \hat{\theta}_i = \theta_i \tag{14}$$

Defining the characteristic adaptive and control parameters as:

$$\lambda \equiv \frac{2\zeta}{\tau} - \frac{1}{\phi_i}\frac{d\phi_i}{dt} \quad \text{and} \quad \gamma \equiv \frac{1}{\tau^2} \tag{15}, (16)$$

equation (13) can be rewritten as -

$$\tau^2 \frac{d^2\hat{\theta}_i}{dt^2} + 2\zeta\tau\frac{d\hat{\theta}_i}{dt} + \hat{\theta}_i = \theta_i \tag{17}$$

which has the meaning that each $\hat{\theta}_i$ converges for the true θ_i through a 2nd order dynamic trajectory, with a natural period of oscillation τ and a damping coefficient ζ.

In the specific case of the choice of a double pole (i.e. $\gamma \equiv 0.25\lambda^2$) the controller gain is inversely proportional to the corresponding periods of oscillation, as follows:

$$\lambda = 2/\tau \qquad (18)$$

and, in fact, parameters ζ and τ will not be independent, rather being related by:

$$\zeta = 1 + \frac{\tau}{2\phi_i}\frac{d\phi_i}{dt} \qquad (19)$$

This problem formulation based on the 2nd order convergence dynamics has the advantages of linking the controller tuning to the estimator tuning through two parameters (or one in the case of the real double pole criterion) which have a simple and well established physical meaning. In the case where the choice is the tuning of both 2nd order convergence parameters, the controller gain will be a function of the regressor $\phi_i(t)$, hence being time varying.
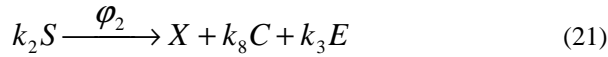
4. Control of baker's yeast production

Baker's yeast production is generally accepted [10] as occurring through three main metabolic pathways, being described by the following verbal scheme -
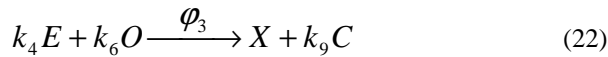
Respiratory growth in glucose (oxidative pathway)

$$k_1 S + k_5 O \xrightarrow{\varphi_1} X + k_7 C \qquad (20)$$
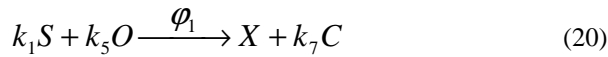
Fermentative growth in glucose (reductive pathway)

$$k_2 S \xrightarrow{\varphi_2} X + k_8 C + k_3 E \qquad (21)$$

Respirative growth in ethanol (oxidative pathway)

$$k_4 E + k_6 O \xrightarrow{\varphi_3} X + k_9 C \qquad (22)$$

where S, O, X, C and E represent respectively glucose, oxygen, biomass, carbon dioxide and ethanol; φ_1, φ_2 and φ_3 represent growth rates and the k_i are yield coefficients.

By reasons of productivity, the fermentation is carried out in semi-batch (fed-batch, in bioengineering terminology) with the controlled addition of substrate (glucose). Theoretical and experimental studies [3, 1, 9] show that the regulation of ethanol concentration corresponds to a good compromise between yield and productivity. The accumulation of ethanol affects negatively both yield and productivity. It is however known that a low level of ethanol will have the desirable effect of inducing the enzymatic system of the fermentative catabolic pathway, in this way avoiding a long adaptation period when using the yeast [9].

In the present work the control problem of concern is that of regulating the ethanol concentration by manipulation of the glucose feed rate (substituted here by the equivalent problem of manipulating the dilution rate). The state model adopted is represented by eqn. (23):

$$\frac{d}{dt}\begin{bmatrix} X \\ S \\ E \\ O \\ C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ -k_1 & -k_2 & 0 \\ 0 & k_3 & -k_4 \\ -k_5 & 0 & -k_6 \\ k_7 & k_8 & k_9 \end{bmatrix}\begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{bmatrix} - D\begin{bmatrix} X \\ S \\ E \\ O \\ C \end{bmatrix} + \begin{bmatrix} 0 \\ DS_{in} \\ 0 \\ OTR \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ CTR \end{bmatrix} \qquad (23)$$

where OTR (oxygen transfer rate) and CTR (carbon dioxide transfer rate) are known functions of O and C respectively. Assuming that S and O exhibit *fast* dynamics relatively to the *slow* limiting dynamics of X, E, and C, and adopting the single perturbation procedure, it is possible to obtain, a reduced model representation of the process: i) recurring to the algebraic equations in S and O, growth rates (φ_1, φ_2 and φ_3) are obtained as functions of the input and output vectors F and Q and of the dilution rate D; ii) substituting in the dynamic equations for X, E and C leads to reduced representation of such dynamics. For the relevant component in this study, ethanol, the following can be obtained:

$$\dot{E} = -DE - \theta_1 CTR - \theta_2 OTR + \theta_3 DS_{in} \qquad (24)$$

In [4] it is shown that the term $\theta_1 CTR$ is null, consequently it will not appear in the sequel. Selecting a first order reference model for the convergence error:

$$\frac{d}{dt}\left(E^* - E\right) + \lambda\left(E^* - E\right) = 0 \qquad (25)$$

the linearizing adaptive control law is readily obtained, by substituting (24) in (25):

$$D = \frac{\lambda\left(E^* - E\right) + \hat{\theta}_2 OTR}{\hat{\theta}_3 S_{in} - E} \qquad (26)$$

In this law the following should be noted:

- except for ethanol, the regulated variable, the on-line measurement of the state variables is not required;

- it is not required to know the micro-organisms growth rate;

- only the oxygen transfer rate, available on-line, is required;

- the law includes a feedforward compensation for variation in the inlet concentration of glucose in the feed (S_{in}).

The adaptive law is based on the on-line estimation of the unknown parameters θ_2 and θ_3, employing respectively ($-OTR$) and (DS_e) as regressors. Three alternative methods are considered for the identification of θ_2 and θ_3: i) the second-order dynamic estimator (SODE) method proposed here (eqns. 12, 15 and 16); ii) recursive least squares (RLS); and (iii) the pole placement design (eqns. 12 and 18). In discretized form the SODE can be written as:

$$\hat{\theta}_{2,k+1} = \hat{\theta}_{2,k} + T(E^* - E_k)/(OTR_k \tau^2) \qquad (27)$$

$$\hat{\theta}_{3,k+1} = \hat{\theta}_{3,k} - T(E^* - E_k)/(D_k S_{in} \tau^2) \qquad (28)$$

and the controller gain computed by:

$$\lambda_k = 2\zeta/\tau - (OTR_k - OTR_{k-1})/(TOTR_k) \qquad (29)$$

For the pole placement method the controller gain is constant, given by eqn. (17).

5. Results

The initial conditions for the case-study are presented in Table 1. Fermentation is stopped when the fermentor limiting volume is attained (5 L) or alternatively for a limiting time of 25 hours. The initial non-zero concentration of ethanol is explained by the fact that normally a fed-batch fermentation (whose start is the time zero for the control study) is preceded by a period of batch production, which leads to some ethanol production.

Table 1 - Initial values for the case-study

$X(0)$	$S(0)$	$E(0)$	$O(0)$	$C(0)$	$V(0)$	$S_{in}(0)$
$g.L^{-1}$	$g.L^{-1}$	$g.L^{-1}$	$g.L^{-1}$	$g.L^{-1}$	L	$g.L^{-1}$
0.30	0.020	0.50	0.0027	0.017	2.0	20

At a sampling rate of 0.1 hours, the simulator supplies the controller with the relevant variables, viz. - ethanol, oxygen and inlet glucose concentrations.

The objective is to keep ethanol concentration at a reference value $E^* = 0.5$ g.L^{-1}, by manipulation of the feed volumetric flowrate.

In the cases where no theoretical relationships are available, controller tuning is performed by trial and error. Controller performance is assessed through the ITAE criterion (integral of the absolute error times the operation time). Table 2 presents the control results for the three methods employed: SODE, RLS and Pole Placement. Slightly better results were obtained with the SODE approach.

Table 2. Comparison of adaptive laws: ITAE and mean error criteria

Adaptive laws	Parameters	ITAE	mean error (%)
SODE	$\tau = 0.10$ h, $\zeta = 2.0$	0.021	0.017
RLS	$\Gamma_{2(o)} = \Gamma_{3(o)} = 100$ $\gamma_2 = \gamma_3 = 0.95$ $\lambda = 10.0$ h^{-1}	0.041	0.038
SODE with double pole (LYAPONOV)	$\tau = 0.13$ h, $(\lambda = 15.0$ h$^{-1})$	0.037	0.028

Figure 1 shows the profile of ethanol concentration. It is visible that the controller with the SODE tuning performs well. The slight initial oscillation is due to the known intrinsic switching of the metabolisms which occur at the beginning of the fed-batch (and which have been programmed in the simulator) which leads to some oscillation in the beginning of the identification procedure. As theoretically expected the control action (b) shows two distinct phases: a first phase, during the

first 8 hours where there is a periodic switching of metabolic pathways, and a second phase with a typical exponential profile, corresponding to the maintenance required by the respiro-fermentative regime.

Figure 2 shows the feedforward effect embedded in the control law, for changes in the feedrate glucose concentration (20 $\pm$ 10 g.L^{-1}) after 4 hours of operation. For both cases the control action is visibly adequate.

Further discussion is out of the scope of this work. A detailed assessment of the performance of the proposed controller can be found elsewhere [4], the study including in particular the analysis of performance when facing further changes in set-point and in loads (inlet concentration of glucose and aeration rates) and also under corrupted noisy measurements.

6. Conclusions

Algorithms were presented for the adaptive control of fermentation processes. The synthesis of the non-linear control laws was performed by applying a model-based procedure which ensures linear behaviour of the closed loop feedback system. A new scheme was proposed for the required on-line parameter identification, which guarantees second-order convergence for the dynamics of the error system.

A case-study consisting of the regulation of ethanol concentration in fed-batch of baker's yeast production was employed for tests by simulation. Two alternative identification methods were also employed for comparison purposes.

The control and adaptive laws based on the reduced model representation of the key component dynamics proved to be adequate for process operation. The scheme based on the second order estimation procedure (SODE) showed better performance, albeit small, as assessed by the ITAE and by the mean error criteria.

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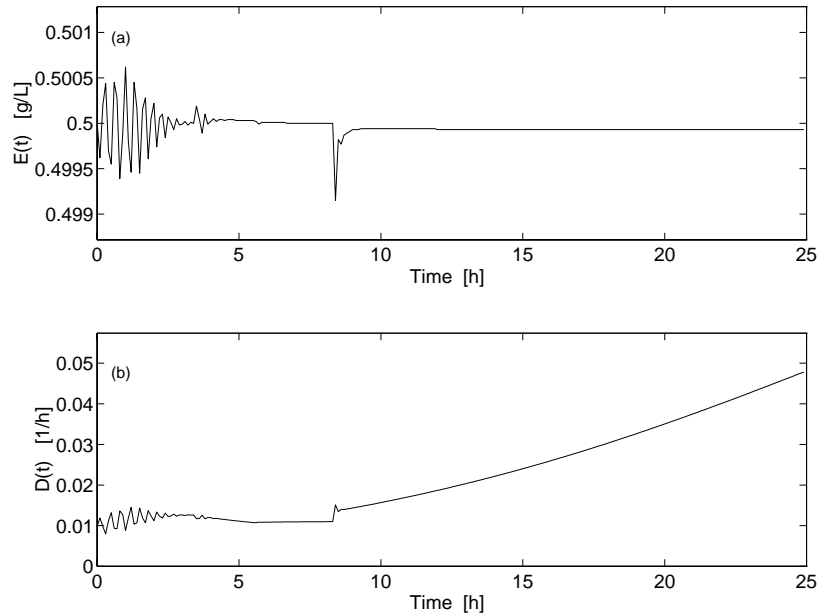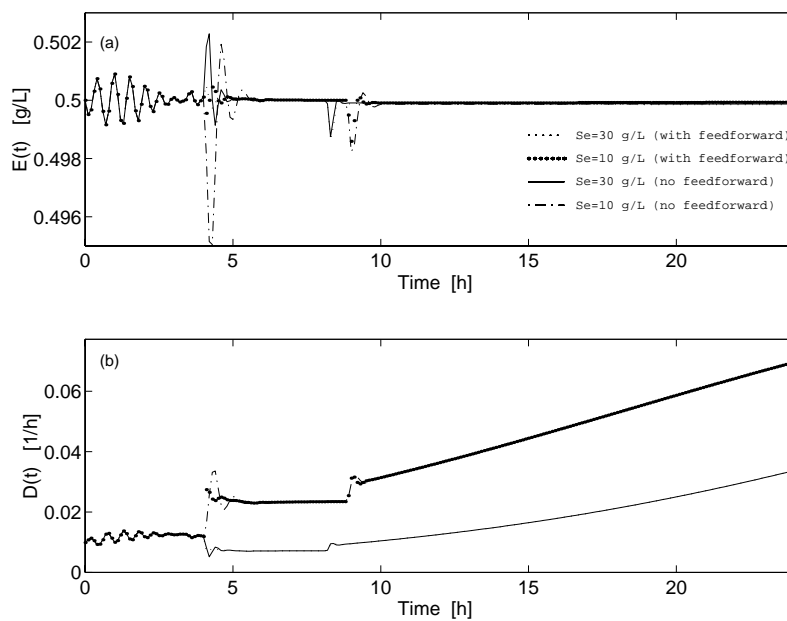
Figure 1 Control of ethanol at $E^* = 0.5$ (a); control action (b)

Figure 2 Feedforward effect.