Tortuosity in Bioseparations and its Application to Food Processes

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1. Introduction

There are several ways of describing porous media and the associated mass transfer phenomena occuring in them. In general the two major properties considered are the permeability coefficient (flow phenomena) and the effective diffusion coefficient (mass transfer phenomena)[1]. Both coefficients, in turn, are functions of the characteristics of the porous media, namely porosity and tortuosity.

The effective diffusion coefficient, D_e, which characterizes mass transfer in porous media, is written as:

$$D = D_o\ (\varepsilon / T) \qquad (1)$$

where D_o is the diffusion coefficient in the bulk medium, ε the porosity and T the tortuosity.

Tortuosity is classically defined as

$$T = Le / L \qquad (2)$$

where Le stands for the average length of the streamlines in the void space of a porous medium of thickness L [1].

A more detailed analysis shows that this definition does not apply to all cases since many paths of the same total length but different tortuosity may be found.

The following examples

A **B** **C** **D**

illustrate what we have just mentioned.

As a matter of fact, considering the 4 paths outlined above, it is easy to understand that each path, from A to D, has the same total length Le, as well as the same medium thickness L (the inlet and outlet points le in the same position in every case). However, the number of bends increases from 2 (path A) to 7 (path D), thereby giving rise to an increase in the tortuosity from left to right.

On the other hand, for the same porosity, one may show the existence of many packed beds of a rather different tortuosity which may be achieved, for instance, by using bi- or tri-dispersed spherical beds [2]. Furthermore, experimental results are quite disperse, and, for basically the same kind of porous media, tortuosity values may range from 1.7 to 4, depending on many factors such as packing arrangement, media homogeneity, channel shape, kind of material to be transferred [3-5].

This raises the hypothesis that the permeability and the tortuosity of a packed bed may be modulated through the control of a) the size of the particles constituting the packed bed and b) the proportion of large and small spherical particles in the bed.

The aim of this work was to study how the packing arrangement and compostion of spherical particle mixed beds could influence the tortuosity. It was also intended to demonstrate how these concepts could be used in practical applications.

2. Experimental

The following types of glass beads were used for mixtures:

- Beads of *Glen Mills Inc.*, code 4512, diameter 3.3 - 3.6 mm and code 4504, diameter 1.0 - 1.25 mm;
- Beads of *Sigmund Lindner*, code 4508, diameter 2.0 - 2.3 mm;
- Beads of *Sovitec Iberica, s.a.,* Microperl, diameter 0.5 - 0.84 mm;
- Beads of *Potters-Ballotini, s.a.*, Visibead, diameter 0.25 - 0.425 mm.

Samples of beads were kindly provided by the above mentioned companies. For all samples 80-90% of particles correspond to the average particle size. Particle samples listed above were chosen in order to avoid overlapping sizes.

A cylinder of 45 mm diameter was used to form a bed of particles for which the ratio (cylinder diameter)/(largest particles diameter) was above 10 to minimise wall effects. Samples of beads were kindly provided by the above mentioned companies.

Beads with average diameter d = 0.3375 mm were used as small particles for preparing mixed beds. Particle ratio D/d = 24 was reached by mixing the above beads with other beads with diameter 8.1 mm; D/d = 10.22 – with 3.45 mm beads; D/d = 6.37 – with 2.15 mm beads, D/d = 3.33 – with 1.125 mm beads and D/d = 1.985 – with 0.67 mm beads, respectively.

Porosity. To avoid particle seggregation, the following procedure was used for preparing mixtures: 1) a cylinder was filled with large particles to build the bed skeleton; 2) small particles were then added to the bed by spraying them over the bed top surface.

Beads with d = 0.3375 mm were used as small particles for preparing all mixed beds. A particle ratio D/d = 10.22 was achieved by mixing them with a fraction of 3.45 mm diameter, D/d = 6.37 – with 215 mm beads, D/d = 3.33 – with 1.125 mm beads, and D/d = 1.985 – with 0.67 mm beads, respectively.

The glass cylinder was used to build the bed of particles. Distilled water was used to measure the void volume. Porosity was measured by two methods: 1) measuring the volume of water occupying the bed void and 2) weighing the water used. The 2 methods gave good agreement (less than 10% of relative deviation).

Filtration. A filter made with glass fibres for qualitative analysis (type MN, *Macherey-Nagel GmbH & Co.*) was used as layer support. The cylinder was filled with water for filtration run. To protect the bed surface during the filling procedure a wire mesh was put on the top of the bed. The bed thickness was in the range of 2.5 – 3 cm. Vacuum was used to create a pressure drop across the mixed bed. The filtration occurred under a constant pressure drop of Δp = 13 kPa to avoid bed compression effects. During filtration, the flow velocity $u = V/(F \cdot t)$ was measured for further calculation of the bed permeability. Here V is the filtrate volume, F is the filtration area, and t is the volume filtration time. Hydraulic resistance of the layer support was checked before and after each filtration test. For each bed a new support layer was used.

The calculation of the mixed bed permeability was based upon the Kozeny-Carman model, as described below.

Pressure drop in packed beds is usually described by Kozeny-Carman model:

$$\frac{\Delta p}{L} = mK\frac{a^2}{\boldsymbol{e}^3}u\ , \tag{3}$$

where *Dp* is the pressure drop through packed bed, L is the bed thickness, $K = K_0 T^2$ is the Kozeny´s coefficient, K_0 is a constant, usually K_0 = 2, T is the tortuosity, *m* is the liquid viscosity, *a* is the specific area of the bed, and *u* is the flow velocity.

For packed beds of mono-size spherical particles T ~ 1.5 (***e*** = 0.4). Hence, in this case

$$K = K_0 T^2 = 4.5 \tag{4}$$

If we introduce a specific particle area of spherical particle diameter d_p, as a_0 = a/(1 - ε)=6/dp, equation (3) becomes:

$$\frac{\Delta p}{L} = K\frac{36(1-\boldsymbol{e})^2}{\boldsymbol{e}^3 d_p^2}mu, \tag{5}$$

or, in the case of application of an equivalent pore diameter $d_e = \frac{2}{3} d_p \boldsymbol{e}/(1-\boldsymbol{e})$

$$\frac{\Delta p}{L} = K\frac{16}{\boldsymbol{e}\cdot d_e^2}mu, \tag{6}$$

In filtration with a constant pressure to control the filtration velocity, *u*, equation (5) can be rewritten as

$$u = \frac{\boldsymbol{e}^3 d_p^2}{36K(1-\boldsymbol{e})^2}\cdot\frac{\Delta p}{mL} \tag{7}$$

The complex

$$k = \boldsymbol{e}^3 d_p^2 / 36K(1-\boldsymbol{e})^2 = \left(\frac{\boldsymbol{e}}{T}\right)^2 \frac{\boldsymbol{e}\cdot d_p^2}{36(1-\boldsymbol{e})^2 K_0} \tag{8}$$

is defined as a permeability, k [m^2]. The particle diameter d_p is considered as the average particle diameter of a binary mixture and was calculated as $1/d_p = x_D/D + (1-x_D)/d$. The permeability was calculated based on equation (7) and K defined from equation (9): $K = K_0 T^2$. Then, the average tortuosity was determined as

$$T = \sqrt{K/K_0} \tag{10}$$

where K_0 was assumed to be 2.0.

Based on this methodology, k, K, and T values were experimentally determined, for a set of mixed beds of spherical glass beads. The effect of x_D, the volume fraction of the largest particles, and ***e***, the porosity of the mixed bed, on the permeability and the tortuosity could thus be evaluated.

3. Results

The dependence of permeability on the volume fraction of large particles in the mixture is shown on Fig. 1. A sudden increase of 1-2 orders of magnitude is observed for permeability when x_D > 0.7.

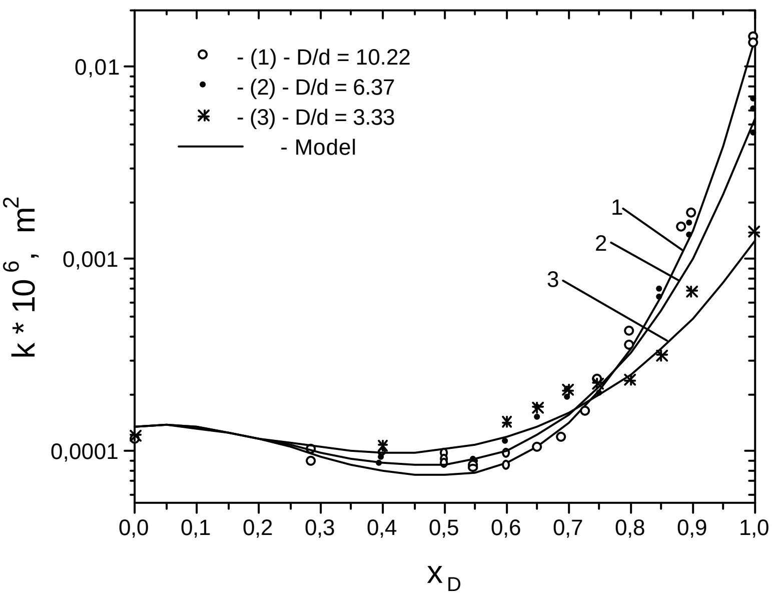

Figure 1. Permeability k vs. x_D .

Based on the Kozeny´s coefficient K, the tortuosity was calculated for several particle size ratios. The tortuosity for D/d = 10.22 is shown on Fig. 2. The range of T variation corresponds to published data for granular beds: Riley et al. (1996), Zhang and Bishop (1994) Bear (1972) Dullien (1975), and Suzuki (1990). The polynomial fit, Fig. 2, (dotted curve), is

$$T = 1.47157 + 0.16565x_D - 0.93301x_D^2 + 3.41422\, x_D^3 - 2.62552\, x_D^4 \qquad (11)$$

with a correlation coefficient of R = 0.919.

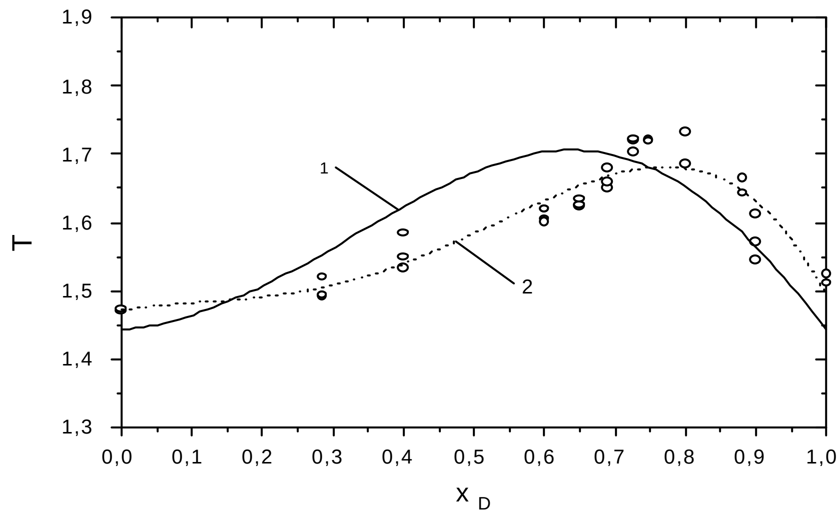

Figure 2. Experimental tortuosity (circles) T vs. volume fraction of large particles x_D for a binary mixture of D/d = 10.22. 1 – tortuosity calculated by $T=1/\varepsilon^{0.4}$, 2 – tortuosity calculated by the polynomial function (11).

Tortuosity may be related to porosity by a functional relationship of the type $T \sim 1/\varepsilon^{\alpha}$, where α usually has a value between $0 < \alpha \leq 1.0$ [4]. For the used mixtures, the best fit gave $\alpha = 0.4$, which gives for the tortuosity the following expression

$$T = \frac{1}{\varepsilon^{0.4}} \tag{12}$$

As was mentioned above, the tortuosity defined by experimentation through Kozeny´s coefficient $K = K_0 T^2$, differs from the calculated by the polynomial model, equation (11), when $T=1/\varepsilon^{0.4}$ is used. However, differences in prediction by the model tortuosity (12) and experimental values of T/T_0, Fig. 2, in all range of x_D do not exceed 10%. Hence, we can expect good estimations with this model. The permeability becomes then

$$k_{0.4} = \frac{\varepsilon^{3.8} \cdot [d(x_D)]^2}{36 \cdot K_0 (1-\varepsilon)^2} = \frac{\varepsilon^{3.8} \cdot [d(x_D)]^2}{72(1-\varepsilon)^2}, \tag{13}$$

Using equation (13), values for the permeability may be estimated for different x_D and several diameter ratios. These estimations are displayed in Figure 3.

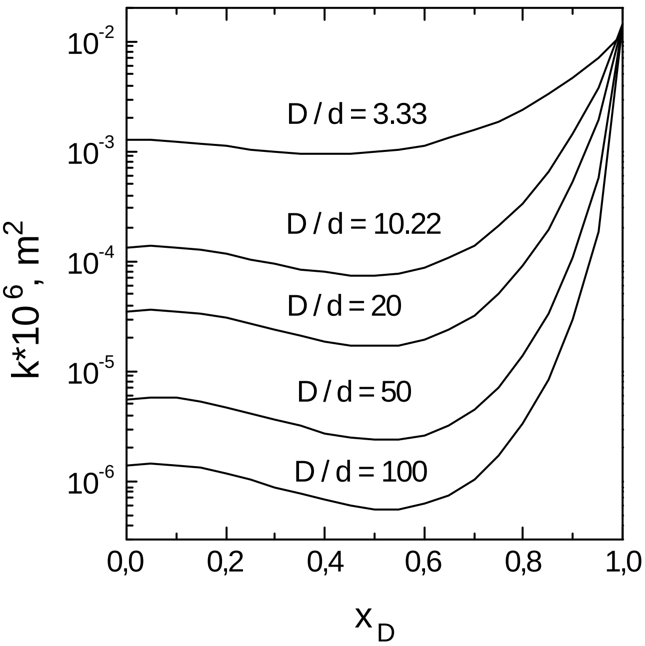

Figure 3. Dependence of the permeability k vs. x_D by the model, equation (1), for different *D/d*. Size of large particles, D = 3.45 mm.

Finally, the function $(\varepsilon/T)^2$ included in permeability was calculated by the model (8) (see Fig. 4). For comparison, dotted curves are shown for the case of a constant tortuosity (independent on porosity) of T = 1.45. As expected, diffusivity is more sensitive to tortuosity than permeability. Hence, the impact of tortuosity variation due to binary particle beds must be taken in account when modelling transport phenomena in granular beds. Comparing these results with the ones obtained for diffusivity in binary beds in a previous work (Mota *et al*., 1998), it may be seen that diffusivity is minimal for x_D around 0.9, whereas the tortuosity is maximum for x_D around 0.8. This raises the hypothesis of dissociating tortuosity from diffusivity, if the appropriate value for x_D is chosen.

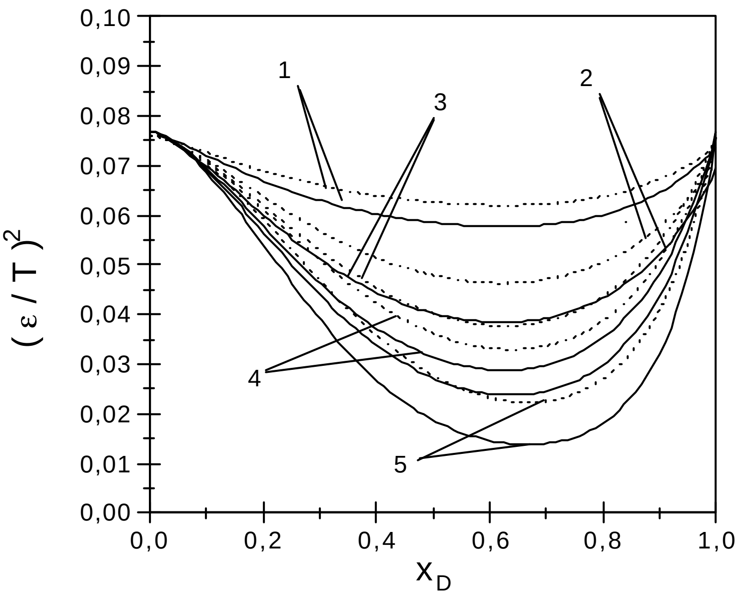

Figure 4. Function $(e/T)^2$ vs. x_D, solid curves. 1-- D/d = 1.985, 2 -- D/d = 3.33, 3 -- D/d = 6.37, 4 -- D/d = 10.22, and 5 -- $D/d \rightarrow \infty$. Dotted curves correspond to T = 1.45.

4. Conclusion

After having obtained this set of results, the question was to know whether they were useful. One immediate hypothesis was to use binary mixtures in filtration media, where the proportion of big particles could increase to 0.7 without significant variation in permeability, as pointed out by Fig. (3). A particular type of filtration was developed. Mixed beds of about 4 cm thickness were formed by a skeleton of glass beads (average diameter 0.3375 mm) and different kieselguhr slurries (from 12 to 50 microns). A bakers'yeast suspension was used. A concentration of 6g/L of cell dry weight was chosen since it is the typical value found in alcoholic beverages – wine, beer, cider. The suspension was filtered through the filtration medium at a constant pressure of 80 kPa. The filtrate was checked for the presence of yeast cells both directly by microscopy and by cultivation in malt-agar. No cells were detected. The filtration effectiveness was also compared with a traditional kieselguhr cake filtration. No differences were detected. The glass beads were back-washed, fluidised and reused 50 times – the total number of runs – without being damaged. The amount of kieselguhr was 5 times less than the usual, with evident advantages in terms of filter aid savings and lowering of pollution levels.

5. References

[1] Bear, J. *Dynamics of Fluids in Porous Media*, Dover Pub., 1972.
[2] Mota, M., Teixeira, J. and Yelshin, A., Image Analysis of Packed Beds of Spherical Particles of Different Sizes, *Separation and Purification Technology*, in press, 1998.
[3] Dullien, F.A.L., Single Phase Flow through Porous Media and Pore Structure, *Chem. Eng. J.*, 10, 1-34, 1975.
[4] Suzuki, M., *Adsorption Engineering*, Elsevier, 1990.
[5] Yu, A.B., Zou, R.P. and Standish, N., Modifying the Linear Packing Model for Predicting the Porosity of Non-spherical Particle Mixtures, *Ind. Chem. Res.*, 35, 3730-3741, 1996.