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Has Portugal gone wireless?
Looking back, looking ahead

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Has Portugal gone wireless?
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Abstract

This paper analyses the pattern and rate of adoption of mobile telephones by the Portuguese population. It is shown that the pattern of diffusion is S-shaped and is consistent with a logistic function, which describes a symmetrical growth process. Furthermore, it is found that about 67 percent of the population will likely adopt mobile phones, and that the levelling-off process in the diffusion of mobile phones has already begun. The analysis is intended to inform the larger discussion of managing the communications service, as well as to assist analysts concerned about assessing the impact of public policies in the evolution of communications sectors.

Keywords: technology diffusion, mobile telecommunications
JEL classification: C53, L96, O30

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1. Introduction

The mobile telecommunications industry has evolved at an astonishing rate within the countries of the European Union. Since the mid-1980s, when cellular phones began to catch on, rapidly increasing capabilities and plummeting prices have caused an explosive rise in the number of cellular phone subscribers. At last count, the European Union was registering an average penetration rate of approximately 50 cellular phone subscribers per 100 inhabitants, ranging from 38 percent in Belgium to 68 percent in Finland.[1]

An effective management of this diffusion process requires an understanding of the factors that underlie the evolution of the market. Factors such as market potential, timing and speed of adoption have important implications for capacity planning, pricing and implementation strategies for cellular phone operators, as well as for the production strategies of telecommunications equipment suppliers. As the growth in the industry continues throughout the European Union, understanding the diffusion of cellular phones has become increasingly important. Consequently, the evolution of cellular phone markets, as well as their likely future trends, has been the subject of extensive research and analyses.

Recently, Gruber and Verboven (1999) have estimated a total market potential, or saturation level, of about 60 cellular phone subscribers per 100 inhabitants in the fifteen member countries of the European Union. Questioning the validity of this estimate for some member countries, Barros and Cadima (2000) have estimated a market potential of 70

[1] The data on the penetration rates of cellular phones in all countries of the European Union were obtained from *ICP – Instituto das Comunicações de Portugal* (http://www.icp.pt).

cellular phone subscribers per 100 inhabitants in Portugal. These authors, however, relied solely on subjective inputs to arrive at their estimate of market potential in Portugal.[2]

In this paper we use a more objective, statistically based method to estimate the market potential for cellular phones in the case of Portugal. The method used allows us to generate confidence intervals about the estimated market potential, and compare it with the subjective estimate made by Barros and Cadima (2000). More specifically, we use time-series data on the number of subscribers to examine the diffusion pattern of cellular phones in Portugal. In addition to allowing us to empirically derive the expected market potential, and to determine the current stage of the market with respect to this saturation level, the analysis also allows us to make specific observations about the underlying growth process. For example, it allows us to verify when, if at all, the levelling-off process in the diffusion of cellular phones has begun.

This paper proceeds as follows. Section 2 sets out the diffusion model under investigation. Section 3 presents the data. Empirical results are provided in Section 4, and Section 5 contains concluding remarks.

2. Diffusion Model

The innovation diffusion literature focuses on the "process by which an innovation is communicated through certain channels over time among the members of a social system" (Rogers (1983)). This literature has established that the spread of a successful innovation over time typically follows a sigmoid or S-shaped curve.

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[2] Similarly, due to problems of convergence in the estimation procedure, Gruber and Verboven (2000) have assumed the saturation level to be a *known* proportion of the population in some countries. Both the proportionality values and the list of countries for which the saturation level was not estimated but exogenously fixed are not reported in their study, though.

Several behavioural theories have traditionally been set forth to explain the S-shaped nature of diffusion processes. For example, Griliches (1957) proposed an "epidemic", demand-induced explanation for the emergence of a S-shaped diffusion curve; Mansfield (1961) sought to explain the observed patterns of diffusion in terms of the expected profitability of the innovation, and the dissemination of information about its technical and economic characteristics; Rogers (1983) employed a communications-based model for explaining diffusion patterns; Artle and Averous (1973), analysing the telephone system, offered a "network consumption externality" explanation wherein the value of the network for a subscriber increases with the number of adopters of the system.[3]

Irrespective of the particular account of the diffusion process, the stylised diffusion path of most innovations results from the fact that initially, during an embryonic phase, only a few members of the social system adopt the innovation. Over time, though, an increasing flow of new adopters is observed as the diffusion process unfolds. This is the phase of rapid market growth. Finally, during a maturing phase, the trajectory of the diffusion curve gradually slows down, and eventually reaches an upper asymptote or saturation level.

Consistent with these theories and research findings, we hypothesize that the cumulative number of cellular phone subscribers in Portugal grows over time according to an S-curve. Such evolutionary pattern has been typically expressed as the differential equation[4]

$$\frac{dy(t)}{dt} = \gamma y(t)\left[y^{*} - y(t)\right] \qquad (1)$$

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[3] For a recent survey of alternative explanations for the emergence of a S-shaped diffusion curve, see Geroski (1999) and references therein.

[4] See, for example, Mahajan and Peterson (1985) for a survey of modelling techniques used in diffusion research.

where y(t) is the cumulative number of subscribers at time t, y^* is the saturation level, and γ is the coefficient of diffusion.[5] Embedded in this model formulation are the assumptions that the rate of growth in the number of subscribers is positively influenced by the number of existing subscribers, as well as by the difference between the saturation level and the number of existing subscribers. It follows that the growth rate declines as the number of potential subscribers decreases.[6]

Although the diffusion path of most innovations can be represented in the general S-shaped fashion presented in Equation (1), different types of innovations can result in different specific evolutionary patterns. Hence, the exact form of the curves, including the slope and the asymptote, may be different for each particular innovation. For example, the slope may be very steep during early phases, indicating rapid diffusion, or it may be gradual suggesting a slow and hesitant start. The two major functional forms of S-curves representing different evolutionary patterns are the logistic, or Pearl-Reed, and the Gompertz functions.

The Gompertz functional form can be expressed as

$$y_t = ke^{-\alpha e^{-\beta t}} \tag{2}$$

[5] Widely cited applications of this model are those of Chow (1967), Griliches (1957), and Mansfield (1961).

[6] Note that, within this model formulation, the saturation level is assumed to be constant. While in general the saturation level of the telecommunications system may change over time (see, for example, Chaddha and Chitgopekar (1971)), the potential subscriber base in our application is the Portuguese population, which has remained roughly constant over the years. It is therefore reasonable to assume that the number of potential subscribers is constant in our application. Given the regulatory and technological developments (briefly described in Section 3) registered in the Portuguese telecommunications sector, however, future research might want to confront the predictions generated by this model with those resulting from a formulation that allows for multiple adoptions by the members of the population or that treats successive generations of cellular phones as distinct products.

where y_t is the number of existing subscribers at time t, and k, α, and β are parameters to be estimated. The Gompertz function ranges from a lower asymptote of zero to the upper bound k as t ranges from $-\infty$ to $+\infty$, and the parameters α and β determine the location and the shape of the curve, respectively. The maximum growth rate is achieved when $y_t=k/e$, that is, when y_t reaches about 37 percent of its upper bound.

The logistic functional form can be written as

$$y_t = \frac{k}{1+\alpha e^{-\beta t}} \tag{3}$$

As with the Gompertz function, the upper bound of y_t for the logistic function is designated k, and the parameters α and β model the location and shape of the curve, respectively. The logistic curve reaches its maximum growth rate when $y_t=k/2$, that is, at half the saturation level of y_t. Thus, unlike the Gompertz, the logistic curve is symmetrical about its inflection point.

The mathematical properties of inflection point (the maximum rate of diffusion), and symmetry of the diffusion model are crucial to its usefulness both as a descriptive and as a forecasting method. If the dynamics of the diffusion process is such that growth is quite rapid at an early phase, and relatively slow when approaching the saturation level, then the Gompertz function is the best method because it attains its maximum rate of growth at an earlier phase than the logistic, which is symmetric. If, on the other hand, the dynamics of the diffusion process is such that growth is initially slow, and relatively rapid during the maturing phases, then the logistic is a superior forecasting method because it grows more rapidly toward the maximum than the Gompertz. In fact, research has shown that models based on initial data for a growth curve are quite valid in later forecasts if the correct functional form has been identified.[7] We therefore test the S-curve hypothesis by testing the two major functional forms of S-curves – the Gompertz and the logistic.

Because an S-curve specification may also be approximated by an exponential curve when an innovation is in its early phases, we also test an exponential growth pattern. The equation for the exponential growth model is given by

$$\ln y_t = a + bt \qquad (4)$$

where ln y_t is the natural logarithm of the number of existing subscribers at time t, and the slope coefficient b measures the constant proportional change in the variable y.

We test our hypotheses on data collected on the number of cellular phone subscribers in Portugal. A brief description of major events in the Portuguese mobile telecommunications sector is presented below, along with the data.

3. Data

The Portuguese cellular telecommunications sector has experienced a number of technological and regulatory developments over the last decade. As in most other European Union countries, the first cellular technology adopted in Portugal was the analogue technology using portions of the spectrum around the 450 MHz frequency.[8] The incumbent

[7] For example, Griliches (1957), adopted the logistic model to investigate the diffusion of hybrid seed corn in the United States. Dixon (1980) applied the Gompertz model to the same data, and concluded that overall it fitted the data better.

[8] The only exception is Greece where mobile telecommunications services were first introduced during the mid-1990s using the digital technology. For a distinction between the analogue and digital technologies, and how it relates to an efficient exploitation of spectrum capacity, see Gruber and Verboven (1999, 2000).

fixed line operator was granted the single license for the C-450 analogue standard in January of 1989.[9]

The digital technology, which uses the spectrum much more efficiently and is capable of accommodating many more customers than its analogue counterpart, was first introduced in the European Union in 1992. Following the EU Directive 91/287 instructing the member countries to adopt the digital GSM (Global System of Mobile Communications) standard, the Portuguese government granted two GSM licenses (in the 900 MHz band of the spectrum) in October of 1992.

The second generation digital system, working at the 1800 MHz frequency, was introduced in Portugal in September of 1998, when one more operator entered the market. This system, known as GSM/DCS 1800, is able to accommodate more customers than the GSM 900 system, but keeps compatibility with the latter. In its licensing policy, the Portuguese government followed the EU Directive 96/2 that instructed the member countries to grant at least two licenses for the GSM 900 system, and allow further operators to enter the GSM/DCS 1800 system.

During this period of technological and regulatory developments in the mobile telecommunications sector, the number of cellular phone subscribers in Portugal registered a tremendous growth.[10] The data for the total number of cellular phone subscribers from 1989 to the second quarter of 2000 are given in Table 1. The data was obtained from *ICP-Instituto das Comunicações de Portugal*, the regulatory body of the Portuguese

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[9] Within the European Union, only Portugal and Germany (the home country) adopted the C-450 analogue standard. See Gruber and Verboven (2000) for a description of the technological features of various analogue and digital standards.

[10] How each of these developments impacts the growth of mobile diffusion is not analysed here. See, for example, Parker and Röller (1997), and Gruber and Verboven (1999, 2000) for assessments of the effects of competition and standard-setting on mobile phone diffusion.

communications sector. Quarterly data was obtained for the period 1996 to the second quarter of 2000. Prior to 1996, only data on the number of subscribers at the end of each year is available to us.

TABLE 1 ABOUT HERE

The cumulative number of subscribers is plotted against time in Figure 1. It is apparent from the figure that the diffusion of cellular phones takes off essentially after 1992, when the digital technology was adopted in Portugal. The following years saw a continuing, extraordinary rise in the number of subscribers, pushing it from 101 thousand in 1993 to 5 million in the second quarter of 2000. Although the overall picture reveals a phenomenal growth, the more recent observations show a slowdown in the rise in the number of subscribers. High growth rates may gather momentum again in 2002 when a new cellular standard will be adopted in Portugal. In fact, the Portuguese government has announced that four licenses for the UMTS (Universal Mobile Telecommunication System) standard will be granted during the first quarter of 2001. One license is reserved for a new entrant, and UMTS services are expected to start in 2002.

FIGURE 1 ABOUT HERE

4. Empirical Results

After adding a disturbance term to each equation set forth in Section 2, we estimated the exponential, Gompertz, and logistic growth models using the available quarterly data on the number of cellular phone subscribers. Given that the data for the whole 1989-2000 period are on different frequency, we elected to use only the quarterly data from the fourth quarter of 1995 until the second quarter of 2000. Since estimating the saturation level of the market is of considerable interest in our analysis, it is the more recent quarterly observations that are of utmost importance to us. Moreover, inclusion of the annual observations does not alter any inferences we draw, and by discarding them for estimation purposes we avoided making unduly assumptions concerning the quarterly evolution of the market during the sub-period that goes from 1989 until the last quarter of 1995.[11]

Our analysis begins with a test of the exponential growth model. The parameters of this model were estimated by ordinary least squares. The results are presented in Table 2, and Figure 2 includes a graphical depiction of the fit.

TABLE 2 ABOUT HERE

As shown in Table 2, the exponential growth model fits the data very well, having a high coefficient of determination and extremely high *t*-statistics for each of the estimated parameters. However, the model predicts that the number of cellular phone subscribers is growing at the rate of 18 percent per quarter,[12] which is a clear overestimate of the more recent growth of the number of cellular phone subscribers. This suggests that a constant growth curve does not provide a realistic description of the cellular phone diffusion path in Portugal. Thus, S-curve models, which incorporate decreasing growth rates, are called for.

[11] All our results were generated using the Windows 98/95/NT 6.0 version of the Intercooled Stata software. The codes for the Gompertz and logistic functions were developed by the authors, and the estimation proceeded using an iterative (modified Gauss-Newton) algorithm. A machine-readable copy of all the command files is available from the authors for anybody wishing to replicate our results. Contact Botelho for details.

[12] The coefficient estimate .1654817 shown in Table 2 is the estimated instantaneous growth rate. The corresponding quarterly growth rate is computed as $(e^{.1654817}-1)=.1799613$.

FIGURE 2 ABOUT HERE

The results of the estimation of the Gompertz curve are displayed in Table 3. Because of its nonlinearity, the parameters of the Gompertz specification were estimated by nonlinear least squares.

TABLE 3 ABOUT HERE

As can be gleaned from Table 3, the Gompertz model estimates 12,528 million cellular phones at saturation, a seemingly unreasonable figure for a population of approximately 10 million people. Moreover, a visual examination of the fit depicted in Figure 3 reveals that the Gompertz specification overestimates the most recent growth of the number of cellular phone subscribers, suggesting that this model does not provide accurate forecasts of the growth phenomenon.

This finding sits well with results reported by Barros and Cadima (2000). In an attempt to estimate the saturation level of the Portuguese cellular phone market using a statistically based method, the authors applied the Gompertz model to data on the number of subscribers from 1993 up to the third quarter of 1999. The result was a saturation level in the range of 13 million to 173 million cellular phone subscribers. The authors rejected these values, pointing out that the Gompertz model was capturing essentially the exponential phase of the diffusion process, and conceded that the high-growth phase of this process had just been finished.

Given the failure of the Gompertz model, Barros and Cadima (2000) turned to qualitative, or judgmental, methods to arrive at their estimate of 70 cellular phone subscribers per 100 inhabitants in Portugal. Our own findings also lead us to reject the Gompertz hypothesis, but we next tested the logistic hypothesis.

FIGURE 3 ABOUT HERE

Nonlinear least squares estimates of the parameters of the logistic specification are reported in Table 4, and actual versus fitted values are depicted in Figure 4. The results show that the logistic model provides an excellent fit with the data, having a high coefficient of determination[13] and high *t*-statistics for each of the estimated parameters.[14]

TABLE 4 ABOUT HERE

An examination of Figure 4 further indicates that the logistic model outperforms the Gompertz model. In fact, the graph shows that, in contrast to the Gompertz, the logistic specification accurately estimates the most recent growth of the number of cellular phone subscribers. The analysis of the estimated logistic curve also reveals that the inflection point of the curve occurred in the first quarter of 1999. This means that the rate of growth

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[13] Although the value of the coefficient of determination is not bound to lie in the 0-1 interval when nonlinear least squares is used, it still provides a useful descriptive measure of the goodness of fit. See, for example, Greene (2000).

[14] The nonparametric Geary test was conducted to test for serial correlation among the disturbances. We found ten positive and nine negative residuals, and eight runs in the sequence. Thus, we do not reject the hypothesis of no serial correlation in the regression residuals at the .05 significance level (see Siegel and Castellan (1988) for a description of this test). A Shapiro-Wilk test was also conducted to test the null hypothesis that the errors are normally distributed. The resulting test statistic is z = -.342 (p = .634), and we do not reject the null. This conclusion is further corroborated by the Shapiro-Francia test for normality, which yielded the test statistic z = -.599 (p = .725). See Shapiro and Wilk (1965), and Shapiro and Francia (1972) for the description of these tests. Note that, in the case of normally distributed disturbances, the nonlinear least squares estimator is efficient, and is also the maximum likelihood estimator. A complete discussion of the subject can be found in Greene (2000).

of the number of cellular phone subscribers was increasing until this time, when it began to decline.

FIGURE 4 ABOUT HERE

As shown in Table 4, the logistic model predicts a saturation level of 6,738 million subscribers, corresponding to a penetration rate of 67,4 percent, an estimate that appears to be a reasonable upper bound for the size of the cellular phone market in Portugal. Moreover, an interval estimation yields an expected penetration rate in the range of 61,4 to 73,4 percent with a confidence coefficient of .95. These forecasts compare well with the subjective forecast of Barros and Cadima (2000).

Using the estimated coefficients of the logistic curve, we also generated a forecast of the growth of the number of cellular phone subscribers. This is depicted in Figure 5. It is seen that the diffusion of cellular phones is rather close to the upper asymptote, and that convergence to the saturation level from the most recent observations occurs at a very slow pace. In fact, the penetration rate is expected to reach 67,4 percent sometime near the end of year 2008. Figure 5 also shows that a saturation level, defined as 95 percent of the estimated maximum, will be achieved in 2002. Currently, the number of subscribers is at 77 percent of the estimated saturation level.

FIGURE 5 ABOUT HERE

5. Concluding Remarks

In this paper, we have analysed the growth of the cellular phone market in Portugal. Our research has provided strong support for the hypothesis that the adoption of cellular phones in Portugal follows an S-curve pattern over time. As noted earlier, this hypothesis can be generated by economic theory in several ways. Interestingly enough, the results presented here show that, although both the Gompertz and logistic models describe a sigmoid diffusion curve, only the logistic model provides an adequate description of the cellular phone diffusion path in Portugal.

Recall that these two models assume different distribution functions for the threshold values of cellular phone subscriptions. While the logistic model is based upon a symmetric frequency distribution, the Gompertz model is derived from a skewed frequency distribution. This distinction has important implications for capacity planners. The results show that the rate of diffusion in the case of the Portuguese cellular phone market is closer to symmetric, as implied by the logistic model, rather than attaining its maximum growth at an earlier phase as the Gompertz model would suggests. In fact, as seen earlier, several factors, such as technological improvements, have assisted in maintaining improvements in the growth process as the number of subscribers grows toward the maximum.

Our analysis also suggest that the high-growth phase of the diffusion of cellular phones ended in the first quarter of 1999, when the rate of growth of the number of subscribers began to decline. Additionally, the results indicate that about 67,4 percent of the Portuguese population will likely adopt cellular phones. These findings should be of interest both to telecommunications professionals and researchers interested in assessing the role of entry regulation, standards setting, interaction with the fixed-link network, country characteristics, or other factors in the diffusion of cellular phones.

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Table 1—Cumulative number of cellular phone subscribers in Portugal, 1989-2000

Time Period	Number of Subscribers (in thousands)	Time Period	Number of Subscribers (in thousands)
1989	2.8	1997 Q1	731.9
1990	6.5	Q2	887.2
1991	12.6	Q3	1136.3
1992	37.3	Q4	1507.0
1993	101.2	1998 Q1	1715.2
1994	173.5	Q2	1983.7
1995 Q4	340.8	Q3	2364.0
1996 Q1	385.6	Q4	3074.6
Q2	442.6	1999 Q1	3453.9
Q3	516.1	Q2	3752.3
Q4	663.7	Q3	4220.3
		Q4	4671.5
		2000 Q1	4930.9
		Q2	5193.5

Source: ICP – Instituto das Comunicações de Portugal (www.icp.pt)

Table 2—Results of OLS Estimation of the Exponential Model

Dependent Variable $\ln y_t$—Estimation by ordinary least squares				
Number of observations		19		
R-squared		.9843		
Adj R-squared		.9834		
Regression F(1,17)		1068.96		
Significance level of F		.0000		
Coefficient	*Estimate*	*Std Error*	*t-Stat*	*Signif*
a	5.68384	.0577087	98.492	.000
b	.1654817	.0050614	32.695	.000

Table 3—Results of NLS Estimation of the Gompertz Model

Dependent Variable y_t—Estimation by nonlinear least squares				
Number of observations		19		
R-squared		.9980		
Adj R-squared		.9976		
Regression F(3,16)		2598.85		
Significance level of F		.0000		
Coefficient	*Estimate*	*Std Error*	*t-Stat*	*Signif*
K	12527.97	2346.99	5.338	.000
α	4.806503	.1698393	28.300	.000
β	.0914657	.0118354	7.728	.000

Table 4—Results of NLS Estimation of the Logistic Model

Dependent Variable y_t—Estimation by nonlinear least squares				
Number of observations		19		
R-squared		.9991		
Adj R-squared		.9989		
Regression F(3,16)		6004.04		
Significance level of F		.0000		
Coefficient	*Estimate*	*Std Error*	*t-Stat*	*Signif*
K	6738.023	282.8562	23.821	.000
α	36.68252	2.761275	13.285	.000
β	.2566328	.0107056	23.972	.000

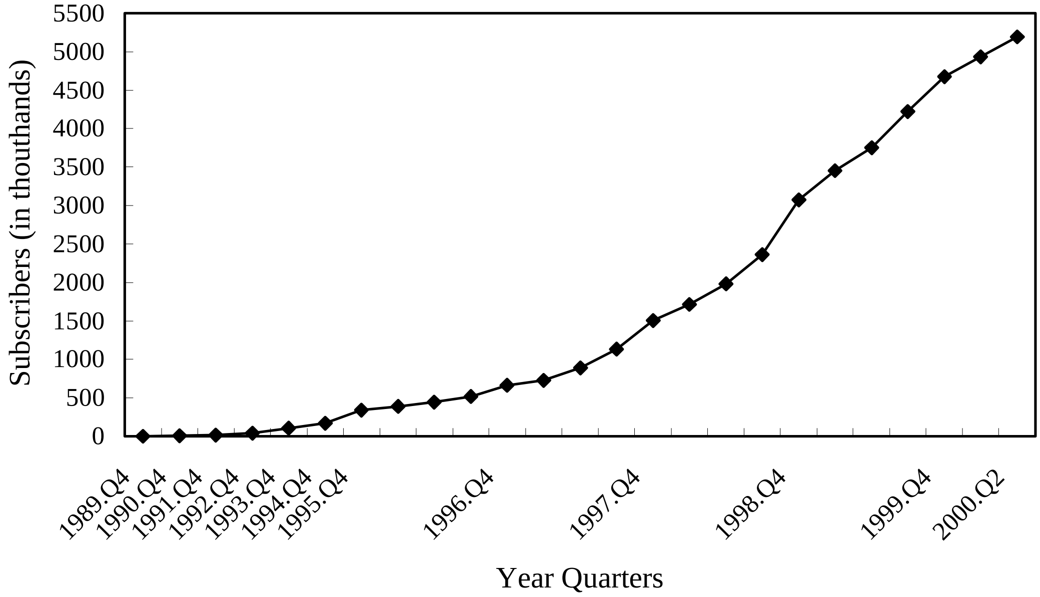

Figure 1—Cumulative number of cellular phone subscribers in Portugal, 1989-2000

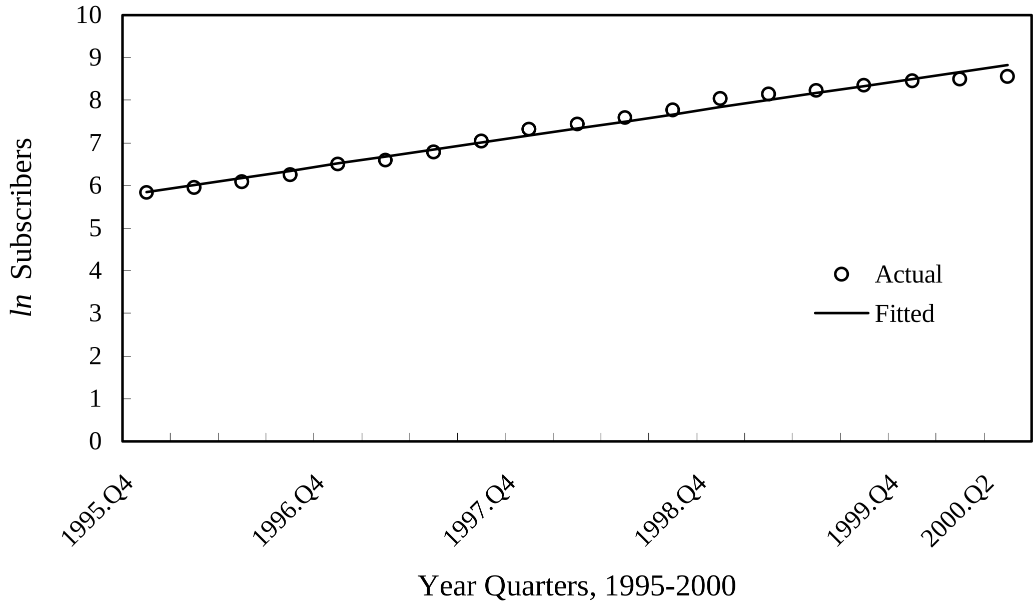

Figure 2—Exponential Fit

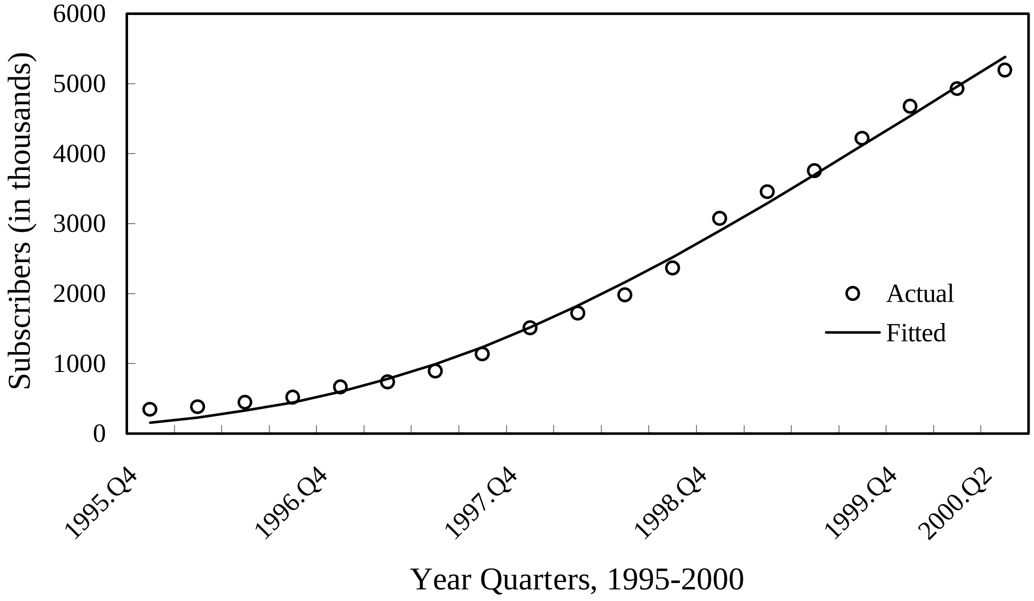

Figure 3—Gompertz Fit

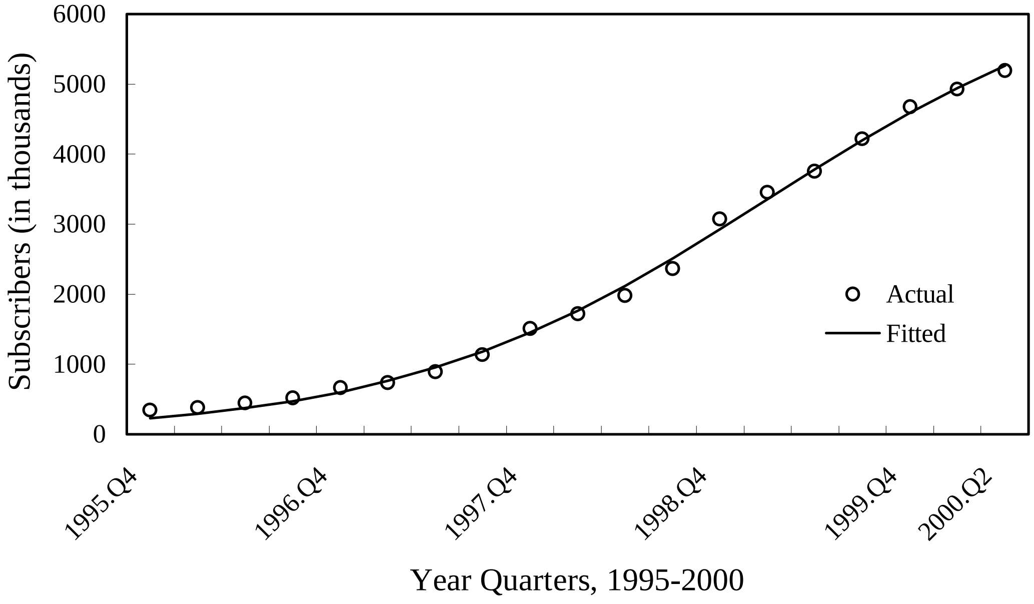

Figure 4—Logistic Fit

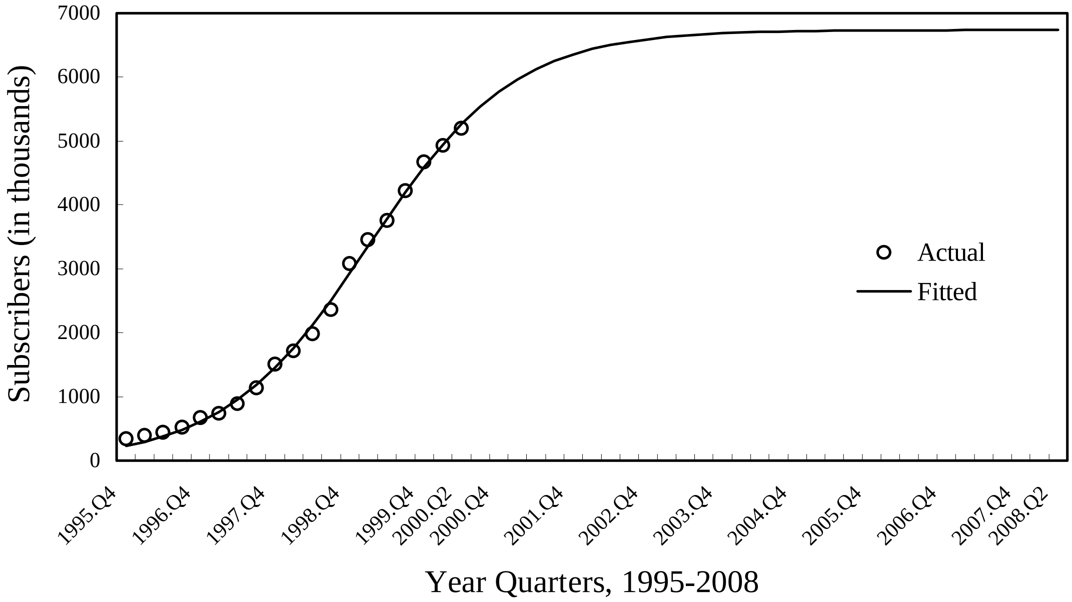

Figure 5—Logistic Forecast