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Cointegration and the joint confirmation hypothesis

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Abstract

Recent papers by Charemza and Syczewska (1998) and Carrion, Sansó and Ortuño (2001) focused on the joint use of unit root and stationarity tests. In this paper, the discussion is extended to the case of cointegration. Critical values for testing the joint confirmation hypothesis of no cointegration are computed and a small Monte Carlo experiment evaluates the relative performance of this procedure.

Key Words: Cointegration; Joint confirmation hypothesis; Monte Carlo simulations.

JEL Classification: C12; C15; C22.

1 Introduction

The issues of unit roots and cointegration have generated a vast literature in the past few years. More recently, it has been argued that confirmatory analysis (i.e., applying unit root tests in conjunction with stationarity tests) may in some cases lead to a better description of the series, improving upon the separate use of each type of test (see, for example, Amano and Van Norden, 1992 and the discussion in Maddala and Kim, 1998). If the two approaches give consistent results, i.e. there is an acceptance *and* a rejection of the nulls, one may conclude whether a given series is stationary or not. On the other hand, if both tests either reject or accept their respective null hypotheses, the results are inconclusive.

Some practical aspects concerning the joint use of unit root and stationarity tests have been addressed by Charemza and Syczewska (1998) and Carrion, Sansó and Ortuño (2001). The first authors suggest that instead of conventional individual critical values for each type of test, one should use symmetric critical power values. These correspond to the probability of type I error for one type of test and power for the other test when both cumulative marginal distributions are equal. Thus, Charemza and Syczewska

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(1998), using Monte Carlo methods, tabulate the new critical values needed for the joint confirmation hypothesis (JCH) of stationarity when the Augmented Dickey-Fuller (ADF) test and the Kwiatkowski, Phillips, Schmidt and Shin (1992, KPSS henceforth) are to be used. However, this approach depends on the parameterization of the autocorrelation in the errors. Hence, Carrion *et al.* (2001) recommend that the JCH of a unit root should be tested instead, providing a new set of critical values.

In this paper, we study the application of this methodology to cointegration testing. Following Charemza and Syczewska (1998) and Carrion *et al.* (2001), we show how the testing procedure may be implemented and the related critical values obtained for tests with null hypothesis of no cointegration, as well as null of cointegration. We address the cases where Engle-Granger's ADF and Phillips-Ouliaris Z_α and Z_t tests are used in conjunction with the KPSS-type test for the null hypothesis of cointegration developed by McCabe, Leybourne and Shin (1997) (see Gabriel, 2001 for a comparative study of the properties of null of cointegration tests). Furthermore, the application of the joint confirmation procedure is assessed by means of a set of Monte Carlo experiments, establishing some comparisons with the separate use of each type of test. This is of great interest, since joint testing will be an alternative approach only if it is able to produce better results that individual testing.

The paper proceeds as follows. The next section establishes the notation for the JCH in the context of cointegration, while section 3 presents the critical values for the JCH of no cointegration. The Monte Carlo study is undertaken in section 4 and section 5 concludes.

2 Joint Confirmation Hypothesis and Cointegration

The first step in order to implement the joint use of null of no cointegration and null of cointegration tests is to decide whether one wishes to test the JCH of cointegration or no cointegration. A simple cointegrated model is generally formulated as

$$y_t = x_t^{'}\beta + u_t, \tag{1}$$

where y_t is a scalar $I(1)$ process and x_t is a vector $I(1)$ process of dimension k. The variables y_t and x_t are said to be cointegrated if u_t is $I(0)$, whereas if u_t is $I(1)$ there is no long run equilibrium relationship between y_t and x_t.

A common parameterization for the error process is to assume that u_t is an autoregressive process $u_t = \rho u_{t-1} + \omega_t$, $\omega_t \sim n.i.d.(0, \sigma_\omega)$, with $|\rho| < 1$ in the case of cointegration and $\rho = 1$ when there is no cointegration. Another possibility is to assume that under the hypothesis of no cointegration the disturbance u_t may be decomposed into the sum of a random walk and stationary component,

$$u_t = \gamma_t + \varepsilon_t, \tag{2}$$

where the random walk is $\gamma_t = \gamma_{t-1} + \eta_t$, with $\gamma_0 = 0$ and η_t distributed as $i.i.d.(0, \sigma_\eta^2)$, while the stationary part ε_t is distributed as $i.i.d.(0, \sigma_\varepsilon^2)$ and is assumed independent of η_t. Cointegration stems from this formulation when $\sigma_\eta^2 = 0$, so that $\gamma_t = 0$ and no longer is a random walk. Note that in this case the *i.i.d.* assumption of the errors ($u_t = \varepsilon_t$) is not very realistic, since in empirical applications we should expect some degree of serial correlation. Thus, we may relax this assumption and assume that $\varepsilon_t = \pi\varepsilon_{t-1} + \zeta_t$, ζ_t being $i.i.d.(0, \sigma_\zeta^2)$.

If one chooses to test the JCH of cointegration (meaning $I(0)$ errors), the critical values would always depend on the value autoregressive parameter of the error term, be it ρ if we specify the null hypothesis of cointegration as H_0: $|\rho| < 1$, or π if H_0: $\sigma_\eta^2 = 0$, allowing for autocorrelation in ε_t. It would involve extensive tabulations for a few particular values of ρ (or π), very likely to be different from the actual, unknown value in the empirical situation the researcher is dealing with. Note that this is a similar problem to that pointed out by Carrion *et al.* (2001) for the univariate case. Therefore, a way to circumvent this obstacle is to specify the JCH of no cointegration.

We closely follow the notation of Charemza and Syczewska (1998) and Carrion *et al.* (2001) by defining the probability of joint confirmation (PJC) of the null hypothesis of no cointegration as

$$\int_{\tilde{z}_D^{PJC}}^{\infty} \int_{\tilde{z}_K^{PJC}}^{\infty} f_{D,K}(z_D, z_K; \Theta, T | H_0^D, H_1^K) dz_D \ dz_K = \text{PJC}. \tag{3}$$

Here, z_j ($j = D, K$) represents the test statistics (in which we maintain the original notation), D for the ADF t-statistic and K for the KPSS cointegration version of McCabe *et al.* (1997). The vector of DGP parameters is denoted as Θ, T is the sample size, $f_{D,K}$ is the joint density function, while $\tilde{z}_j^{PJC}$ are the critical values from the joint distribution for a given PJC significance level. As discussed in the above mentioned papers, for each PJC significance level the number of possible critical values is infinite. However, if we impose the restriction that the marginal probabilities (MPr) should be equal, then there is a unique pair $(\tilde{z}_D^{PJC}, \tilde{z}_K^{PJC})$ satisfying

$$\int_{\tilde{z}_D^{PJC}}^{\infty} f_D(z_D; \Theta, T | H_0^D) dz_D = \int_{\tilde{z}_K^{PJC}}^{\infty} f_K(z_K; \Theta, T | H_1^K) dz_K = \text{MPr}. \tag{4}$$

This restriction means that the probability of deciding wrongly when applying each statistic is equal, that is, when the ADF statistic does not reject the null of no cointegration (type II error) and the KPSS-type test rejects a true null of cointegration (type I error). Such pairs $(\tilde{z}_D^{PJC}, \tilde{z}_K^{PJC})$ are dubbed symmetric critical power values (SCPV). Therefore, we find cointegration at a PJC significance level if the joint ADF-KPSS statistic is in the interval $\{(-\infty, \tilde{z}_D^{PJC}), (0, \tilde{z}_K^{PJC})\}$, whereas the converse situation leads to a non-rejection of the JCH of no cointegration. In principle, this strategy would avoid prioritizing either the cointegration or no cointegration hypotheses, although in practice this may be questionable, given the simulation results in Carrion *et al.* (2001). We will return to this below.

Also note that we may also consider the JCH with the other pairs of tests, changing the notation conformably. In fact, we will also consider the joint application of the Phillips-Ouliaris Z_α and Z_t tests, and KPSS-type test. In the next section, critical values for these cases are presented.

3 Critical Values for the JCH of No Cointegration

As known, critical values for cointegration testing depend not only on the number of regressors k, but also on the deterministic components that may be present in the cointegration space. We will restrict our attention to single equation models with a single cointegration vector. Generalizing (1) as

$$y_t = \alpha + \delta t + x_t^{'}\beta + u_t, \tag{5}$$

where t denotes a time trend, we consider three cases: no constant ($\alpha = \delta = 0$), constant with no trend ($\alpha \neq 0, \delta = 0$) and the model with trend component ($\alpha \neq 0, \delta \neq 0$), up to $k = 5$. Since we are considering the JCH of no cointegration, $u_t = u_{t-1} + \omega_t$ ($\rho = 1$), ω_t is assumed to be $n.i.d.(0, 1)$ and $u_0 = 0$. We also set $\alpha = 1$ and $\delta = 1$ for the relevant cases. After generating $n = 50000$ replications for sample sizes $T = 50, 100$ and 250, pairs of ADF-KPSS, Z_α-KPSS and Z_t-KPSS tests are computed. Using OLS, an appropriate lag length for the ADF test is obtained with a t-test downward selection procedure, by setting the maximum lag equal to 6 and then testing downward until a significant last lag is found, at the 5% level. Concerning Z_α and Z_t, the long run variance is estimated by means of a prewhitened quadratic spectral kernel with an automatically selected bandwidth estimator, using a first-order autoregression as a prewhitening filter, as recommended by Andrews and Monahan (1992). As for the KPSS cointegration statistic, we use Saikkonen's (1991) dynamic least squares estimator and filter the residuals with an ARIMA$(p, 1, 1)$ model, then using the variance estimator suggested by Leybourne and McCabe (1999) (see McCabe *et al.,* 1997 and Gabriel, 2001 for more details on the computation of the statistic).

Again, we follow the methodology of Charemza and Syczewska (1998) and Carrion *et al.* (2001) to obtain the critical values. Thus, the n pairs of observations are sorted according to the ADF (or Z-type) test and then 250 fractiles are computed. For each of these ADF (Z-type) fractiles, another 250 fractiles were obtained for the KPSS statistic, which means that we get a 250×250 table of empirical joint frequencies. After cumulating these frequencies and thus obtaining the joint distribution function, we may tabulate critical values for the desired significance levels. These are shown in Table 1. The computer routine to obtain these critical values was written in GAUSS[1] and is an adaptation of the program used by Charemza and Syczewska (1998). Since the ADF and Z_t share the same (marginal) asymptotic distribution and given that the results obtained in the simulations for these two tests are practically the same, we only show the critical values for the ADF test.

4 Monte Carlo Experiment

In order to assess the performance of the JCH of no cointegration in terms of classifying the model as cointegrated or not, we devised a set of Monte Carlo simulations. The DGP similar to the one in Carrion *et al.* (2001) and is practically the same as in the previous section, although the errors are allowed to

[1] Available upon request.

follow an ARMA(1,1) model of the form

$$u_t = \rho u_{t-1} + \omega_t + \theta\omega_{t-1}, \tag{6}$$

where ρ takes the values $\{0.5, 0.9, 1\}$ and $\theta = \{-0.8, 0\}$. For simplicity, we only consider a model with a single regressor and a constant term, setting the sample size as $T = 100$ and 250, computing 2500 replications.

The results from this simulation exercise are shown in Table 2 and 3 for $T = 100$ and $T = 250$, respectively. We considered different testing approaches. First, computing each test individually[2] and using the respective marginal distributions (i.e. the standard critical values), we gauge the proportion of times that the tests classify a given DGP as being cointegrated (in the line C) or not (NC), at the 5% level of significance. This corresponds to the usual power-size analysis. Secondly, and still resorting to the 5% critical values from the marginal distributions, we count the frequency a realization of the DGP is classified as cointegrated or not in the following way: (i) if tests for the null of no cointegration (ADF, Z_α and Z_t) reject their null and the KPSS null of cointegration test does not, the process is considered to be cointegrated (C); (ii) if tests for the null of no cointegration do not reject their null and the null of cointegration test does, the process is considered *not* to be cointegrated (NC); (iii) if both types of tests reject their nulls (Inconclusive type A) or do not reject the respective nulls (Inconclusive type B), no conclusion is achieved. These joint tests are labeled as *D-K* for ADF and KPSS tests and *Z-K* for Z_a and KPSS tests. Finally, a similar exercise is carried out, this time using the 5% critical values from the joint distribution as displayed in Table 1, with the tests denoted as JU(*D-K*) and JU(*Z-K*).

From the analysis of Tables 2 and 3, we observe that testing the JCH of no cointegration in the latter case leads to a very small number of correct decisions when the errors are stationary. This is also the case for joint testing with standard critical values. Indeed, most of the times an inconclusive response is obtained, namely rejections by both tests (type A inconclusive answers). Moreover, the results do not seem to improve for larger sample sizes, when we compare Table 2 and 3. On the other hand, when the DGP is truly non-cointegrated, the JCH approach is the most accurate in delivering the correct answer, except when a negative MA component is present. Overall, this is in accordance with the results for univariate testing in Carrion *et al.* (2001), although with a much poorer performance. A possible explanation for these disappointing results may lie on the fact that the particular restriction imposed in (4) may not be the most appropriate.

Comparing this performance with that of individual tests, we see that the latter have a much more reliable behaviour in terms of providing the correct decision, both when there is cointegration and when there is not. The performance of the KPSS cointegration test should be highlighted, given its relative robustness to serial correlation and most especially to the introduction of negative MA components in the errors. In fact, the performance of ADF and Z_α tests, as well as that of joint tests, seems to suffer a great deal with a negative MA error structure, which confirms previous results in the literature. On

[2] This alternative was not considered in Carrion *et al.* (2001) for the univariate case.

the other hand, individual tests also have the advantage of not pointing to inconclusive answers, as it happens with the JCH methodology.

Given these results, it would also be interesting to investigate what the outcome would be if one tested the JCH of cointegration. As explained earlier, there is the problem with the critical values depending on the degree of correlation of the errors. However, the researcher could choose an intermediate, though non-optimal, approximation by fixing ρ at an empirically plausible value and use the corresponding critical values. Such a value could be $\rho = 0.75$, which is also recommended and tabulated by Charemza and Syczewska (1998). Of course, if the true ρ is larger than 0.75, the critical values would be too conservative, while the converse would lead to overrejecting the JCH of cointegration. Nevertheless, despite the arbitrariness of such a choice, this seems a fairly realistic way to proceed.

Therefore, adapting the methodology discussed in section 2 and 3 to the JCH of cointegration, we computed the 5% critical values for the DGP in this section[3] and evaluated its performance using the same set of simulation experiences. The results are also displayed in Tables 2 and 3, under the columns $JS(D\text{-}K)$ and $JS(Z\text{-}K)$. We observe that this strategy clearly improves upon that of JCH of no cointegration, since a lot more of correct decisions are achieved when the DGP is cointegrated. However, this behaviour is not sustained asymptotically, as the results for $T = 250$ are in general worse. On the other hand, the ability to detect non-cointegrated models improves with the sample size and attains very reasonable levels. Still, this approach does not seem to beat the conventional one, with individual testing.

5 Concluding Remarks

In this paper, we extended the joint confirmation hypothesis approach to the context of cointegration. Following Charemza and Syczewska (1998) and Carrion *et al.* (2001), we tabulated critical values for the JCH of no cointegration. However, our subsequent Monte Carlo simulations question the usefulness of such a methodology. Indeed, our simulation study, despite its limitations, lead us to conclude that the joint application of different types of tests may obscure, rather than clarify, the process of deciding whether a given model is cointegrated or not. In particular, testing the JCH of no cointegration with the critical values derived here is to be avoided, as it mainly leads to inconclusive answers when the DGP is truly cointegrated. By reversing the JCH to be tested (that is, cointegration), slightly better results are achieved. Nevertheless, it seems preferable to use the standard individual testing approach, which consistently gave better (or at least as good) results. Further research is required, however, as there are issues that should be addressed, namely that of the performance of the different types of tests under distinct null hypothesis.

[3] These are -2.482, -17.023 and 155.421 ($T = 100$) and -4.065, -46.76 and 14.712 ($T = 250$) for the ADF, Z_α and KPSS tests, respectively. A more extensive tabulation is available upon request.

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6 Appendix

Table 1 - Critical values

		no constant			*constant*			*trend*		
	PJC	*ADF*	*MLS*	Z_a	*ADF*	*MLS*	Z_a	*ADF*	*MLS*	Z_a
$T = 50$	0.99	−4.054	0.044	−25.213	−4.703	0.032	−29.256	−5.212	0.061	−35.144
$k = 1$	0.95	−3.301	0.073	−18.721	−3.946	0.042	−23.029	−4.498	0.11	−29.081
	0.90	−2.983	0.107	−15.731	−3.592	0.052	−19.869	−4.184	0.186	−26.181
$k = 2$	0.99	−4.588	0.03	−28.964	−5.095	0.024	−34.365	−5.552	0.033	−38.768
	0.95	−3.879	0.037	−22.376	−4.426	0.029	−29.169	−4.885	0.043	−32.735
	0.90	−3.572	0.044	−19.32	−4.062	0.033	−29.914	−4.547	0.051	−29.968
$k = 3$	0.99	−5.041	0.022	−32.315	−5.477	0.018	−38.227	−5.895	0.023	−41.641
	0.95	−4.332	0.026	−26.127	−4.826	0.021	−32.514	−5.216	0.027	−36.267
	0.90	−3.99	0.029	−22.981	−4.496	0.024	−29.367	−4.902	0.031	−33.454
$k = 4$	0.99	−5.454	0.012	−35.28	−5.828	0.02	−40.87	−6.199	0.012	−44.105
	0.95	−4.758	0.015	−29.172	−5.152	0.023	−35.54	−5.491	0.015	−38.913
	0.90	−4.417	0.016	−26.27	−4.818	0.027	−32.602	−5.164	0.016	−36.238
$k = 5$	0.99	−5.843	0.024	−38.036	−6.241	0.016	−44.003	−6.499	0.024	−46.889
	0.95	−5.119	0.028	−31.93	−5.487	0.02	−38.662	−5.808	0.029	−41.482
	0.90	−4.751	0.031	−28.859	−5.139	0.022	−35.764	−5.461	0.033	−38.873
$T = 100$	0.99	−3.76	0.049	−25.977	−4.434	0.03	−31.871	−4.937	0.072	−38.553
$k = 1$	0.95	−3.185	0.109	−19.118	−3.758	0.047	−23.617	−4.312	0.229	−31.059
	0.90	−2.891	0.241	−16.039	−3.452	0.065	−20.091	−4.017	0.522	−27.416
$k = 2$	0.99	−4.336	0.03	−31.086	−4.866	0.023	−38.252	−5.265	0.038	−44.307
	0.95	−3.738	0.046	−23.508	−4.19	0.03	−29.60	−4.662	0.06	−36.002
	0.90	−3.427	0.063	−19.998	−3.893	0.036	−25.665	−4.368	0.088	−32.40
$k = 3$	0.99	−4.791	0.023	−35.89	−5.244	0.018	−43.747	−5.565	0.026	−48.861
	0.95	−4.177	0.03	−27.931	−4.61	0.022	−35.036	−4.971	0.034	−40.891
	0.90	−3.857	0.036	−24.342	−4.286	0.025	−31.219	−4.699	0.042	−36.914
$k = 4$	0.99	−5.176	0.017	−40.409	−5.561	0.018	−48.901	−5.897	0.02	−53.931
	0.95	−4.562	0.022	−32.02	−4.97	0.023	−40.519	−5.295	0.024	−45.324
	0.90	−4.258	0.025	−28.372	−4.65	0.026	−36.185	−5.011	0.028	−41.519
$k = 5$	0.99	−5.569	0.02	−45.232	−5.926	0.016	−53.681	−6.198	0.025	−57.833
	0.95	−4.917	0.025	−36.20	−5.293	0.02	−45.041	−5.592	0.033	−50.158
	0.90	−4.596	0.028	−32.285	−4.963	0.023	−40.649	−5.306	0.042	−45.057

Table 1 (continued)

		no constant			*constant*			*trend*		
	PJC	*ADF*	*MLS*	Z_a	*ADF*	*MLS*	Z_a	*ADF*	*MLS*	Z_a
$T = 250$	0.99	−3.689	0.065	−26.856	−4.255	0.035	−32.179	−4.709	0.136	−40.002
$k = 1$	0.95	−3.118	0.37	−19.473	−3.661	0.073	−23.887	−4.161	0.929	−31.275
	0.90	−2.841	1.262	−16.111	−3.371	0.141	−19.99	−3.894	1.73	−27.549
$k = 2$	0.99	−4.204	0.036	−31.956	−4.717	0.023	−39.71	−5.107	0.053	−47.115
	0.95	−3.631	0.082	−23.857	−4.064	0.039	−29.998	−4.533	0.153	−37.605
	0.90	−3.347	0.221	−20.333	−3.773	0.057	−25.772	−4.234	0.544	−32.874
$k = 3$	0.99	−4.652	0.025	−37.666	−5.05	0.018	−45.734	−5.43	0.034	−52.589
	0.95	−4.085	0.042	−28.826	−4.438	0.026	−35.907	−4.827	0.06	−42.496
	0.90	−3.779	0.07	−24.863	−4.157	0.035	−31.477	−4.566	0.105	−38.156
$k = 4$	0.99	−5.025	0.019	−43.11	−5.398	0.017	−51.947	−5.70	0.024	−58.243
	0.95	−4.432	0.029	−33.709	−4.802	0.025	−41.802	−5.13	0.036	−48.041
	0.90	−4.159	0.039	−29.494	−4.504	0.031	−37.198	−4.846	0.05	−43.333
$k = 5$	0.99	−5.342	0.019	−47.301	−5.697	0.015	−57.523	−5.98	0.024	−63.211
	0.95	−4.783	0.029	−38.699	−5.117	0.02	−47.442	−5.44	0.038	−53.378
	0.90	−4.498	0.039	−34.129	−4.832	0.025	−42.828	−5.161	0.058	−48.629

Table 2 - Monte Carlo results for ADF, Z_α and KPSS tests ($T = 100$)

(ρ, θ)		*ADF*	Z_α	*KPSS*	*D-K*	*Z-K*	*JU(D-K)*	*JU(Z-K)*	*JS(D-K)*	*JS(Z-K)*
$(0.5, 0)$	*C*	0.963	1.00	0.793	0.199	0.207	0.04	0.041	0.986	0.994
	NC	0.037	0.00	0.207	0.029	0.00	0.057	0.00	0.00	0.00
	Inc. A	0.00	0.00	0.00	0.764	0.793	0.902	0.959	0.006	0.006
	Inc. B	0.00	0.00	0.00	0.008	0.00	0.001	0.00	0.009	0.00
$(0.5, -0.8)$	*C*	1.00	1.00	0.97	0.03	0.03	0.001	0.001	1.00	1.00
	NC	0.00	0.00	0.03	0.00	0.00	0.004	0.00	0.00	0.00
	Inc. A	0.00	0.00	0.00	0.97	0.97	0.995	0.999	0.00	0.00
	Inc. B	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(0.9, 0)$	*C*	0.29	0.282	0.624	0.102	0.11	0.014	0.017	0.648	0.40
	NC	0.71	0.718	0.376	0.514	0.53	0.832	0.797	0.054	0.096
	Inc. A	0.00	0.00	0.00	0.188	0.172	0.126	0.16	0.082	0.04
	Inc. B	0.00	0.00	0.00	0.196	0.188	0.029	0.026	0.216	0.464
$(0.9, -0.8)$	*C*	0.927	1.00	0.938	0.06	0.062	0.007	0.008	0.976	0.994
	NC	0.073	0.00	0.062	0.07	0.00	0.106	0.00	0.00	0.00
	Inc. A	0.00	0.00	0.00	0.868	0.938	0.886	0.992	0.006	0.006
	Inc. B	0.00	0.00	0.00	0.002	0.00	0.001	0.00	0.018	0.00
$(1, 0)$	*C*	0.072	0.061	0.062	0.008	0.007	0.00	0.00	0.138	0.054
	NC	0.928	0.939	0.938	0.874	0.883	0.971	0.97	0.405	0.534
	Inc. A	0.00	0.00	0.00	0.063	0.054	0.027	0.028	0.187	0.058
	Inc. B	0.00	0.00	0.00	0.054	0.056	0.002	0.002	0.27	0.354
$(1, -0.8)$	*C*	0.595	0.977	0.061	0.041	0.06	0.004	0.006	0.588	0.76
	NC	0.405	0.023	0.939	0.385	0.022	0.475	0.024	0.053	0.003
	Inc. A	0.00	0.00	0.00	0.554	0.917	0.519	0.969	0.18	0.23
	Inc. B	0.00	0.00	0.00	0.02	0.00	0.002	0.00	0.179	0.007

Table 3 - Monte Carlo results for ADF, Z_α and KPSS tests ($T = 250$)

(ρ, θ)		*ADF*	Z_α	*KPSS*	*D-K*	*Z-K*	*JU(D-K)*	*JU(Z-K)*	*JS(D-K)*	*JS(Z-K)*
$(0.5, 0)$	*C*	1.00	1.00	0.951	0.049	0.049	0.011	0.011	0.806	0.808
	NC	0.00	0.00	0.049	0.00	0.00	0.00	0.00	0.00	0.00
	Inc. A	0.00	0.00	0.00	0.951	0.951	0.989	0.989	0.192	0.192
	Inc. B	0.00	0.00	0.00	0.00	0.00	0.001	0.00	0.002	0.00
$(0.5, -0.8)$	*C*	1.00	1.00	0.998	0.002	0.002	0.00	0.00	0.264	0.264
	NC	0.00	0.00	0.002	0.00	0.00	0.00	0.00	0.00	0.00
	Inc. A	0.00	0.00	0.00	0.998	0.998	1.00	1.00	0.736	0.736
	Inc. B	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(0.9, 0)$	*C*	0.858	0.92	0.702	0.336	0.36	0.121	0.139	0.369	0.061
	NC	0.142	0.08	0.298	0.102	0.064	0.262	0.171	0.194	0.264
	Inc. A	0.00	0.00	0.00	0.522	0.56	0.581	0.672	0.073	0.004
	Inc. B	0.00	0.00	0.00	0.04	0.016	0.036	0.018	0.364	0.671
$(0.9, -0.8)$	*C*	0.997	1.00	0.991	0.009	0.009	0.00	0.00	0.381	0.389
	NC	0.003	0.00	0.009	0.003	0.00	0.01	0.00	0.019	0.00
	Inc. A	0.00	0.00	0.00	0.988	0.991	0.99	1.00	0.592	0.611
	Inc. B	0.00	0.00	0.00	0.002	0.00	0.001	0.00	0.008	0.00
$(1, 0)$	*C*	0.068	0.06	0.018	0.002	0.002	0.00	0.00	0.002	0.00
	NC	0.932	0.94	0.982	0.916	0.924	0.969	0.967	0.90	0.91
	Inc. A	0.00	0.00	0.00	0.066	0.058	0.03	0.032	0.01	0.00
	Inc. B	0.00	0.00	0.00	0.016	0.016	0.002	0.001	0.088	0.09
$(1, -0.8)$	*C*	0.40	0.98	0.004	0.002	0.004	0.001	0.001	0.053	0.157
	NC	0.60	0.02	0.996	0.598	0.02	0.679	0.03	0.65	0.126
	Inc. A	0.00	0.00	0.00	0.399	0.976	0.32	0.969	0.181	0.705
	Inc. B	0.00	0.00	0.00	0.001	0.00	0.00	0.00	0.116	0.012