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DOI
10.1103/PhysRevB.102.161110

Publication date
2020

Document Version
Final published version

Published in
Physical Review B

Rapid Communications Editors' Suggestion

Generalized hydrodynamics with dephasing noise

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(Received 23 March 2020; accepted 21 September 2020; published 16 October 2020)

We consider the out-of-equilibrium dynamics of an interacting integrable system in the presence of an external dephasing noise. In the limit of large spatial correlation of the noise, we develop an exact description of the dynamics of the system based on a hydrodynamic formulation. This results in an additional term to the standard generalized hydrodynamics theory describing diffusive dynamics in the momentum space of the quasiparticles of the system, with a time- and momentum-dependent diffusion constant. Our analytical predictions are then benchmarked in the classical limit by comparison with a microscopic simulation of the nonlinear Schrödinger equation, showing perfect agreement. In the quantum case, our predictions agree with state-of-the-art numerical simulations of the anisotropic Heisenberg spin in the accessible regime of times and with bosonization predictions in the limit of small dephasing times and temperatures.

DOI: 10.1103/PhysRevB.102.161110

Introduction. Recent advances in controlling and manipulating quantum matter [1–3] have spurred the development of novel methods to study the out-of-equilibrium dynamics of many-body systems. On the one hand, numerical methods based on tensor network algorithms had astonishing achievements, extending their range of applicability to longer times and to a larger class of systems and protocols [4–11]. On the other hand, for one-dimensional exactly solvable models, a new set of analytical tools has been devised to access the long-time stationary state, correlation functions, and entanglement production [12–17]. For homogeneous integrable systems evolving under a time-independent Hamiltonian, it is now completely understood how to express the long-time evolution once the local and quasilocal conserved charges of the model have been classified [18,19]: their expectation values uniquely determine the generalized Gibbs ensemble (GGE) [20] describing the late-time stationary state.

More recently, generalized hydrodynamics (GHD) provided an efficient framework to study integrable systems prepared in inhomogeneous states [21,22]. It progressed at a fast pace leading to several extensions [23–25], analytic results [26], applications [27–33], studies of classical systems [34–37], and even experimental confirmations [38]. Further developments have included diffusive corrections [39–44], predictions beyond integrability [45,46], and quantum fluctuations [47], and have extended its applicability to additional protocols, including space-time dependent forces [48] and interactions [49].

However, a crucial aspect when comparing to real-world experiments is that interaction with the external environment will eventually affect the unitary evolution of the system. Modeling the open dynamics of a quantum system is a notoriously difficult problem [50] as there is not a unique way to incorporate the external degrees of freedom while first-principles constructions often lead to hardly treatable formulations [51]. An important simplification occurs for setups where the correlation time of the bath can be neglected compared with the scale of the system itself. In this case, one can assume that Markovianity and consistency with the laws of quantum mechanics restrict the possible form of open evolution to the so-called Lindblad equation. In practice finding exact solutions for its dynamics is a difficult task, and it constitutes an active subject of research. Notable progress was done in quadratic Fermi systems [52], integrable Linbladians [53–57], and by means of mappings to classical stochastic systems [58–60]. At the leading order, the effect of the environment is to induce phase fluctuations between different portions of the system, without locally exchanging energy or other conserved quantities (although global heating is possible due to the interactions between different regions). This *dephasing* is described by a Lindbladian whose jump operators are Hermitian. In this Rapid Communication, we introduce a general framework where the dynamics of an integrable model subject to the dephasing noise can be studied exactly. In particular, we consider the case where a fluctuating environment is locally coupled to any local operator, focusing primarily on local conserved charges. In the spirit of GHD, we derive, in the long-wavelength limit, a compact evolution equation for the local stationary state, which admits a simple interpretation in terms of diffusion in the momentum space of the relevant quasiparticles. Our approach is particularly suitable for describing cold bosonic atoms trapped in noisy optical lattices or atom chips [61,62] (see Fig. 1) and spin and fermionic chains interacting with phonon modes, whose wavelengths are much larger than that of the system [63].

Noise and dephasing model. We consider generic homogeneous Bethe-Ansatz integrable Hamiltonians $\hat{\boldsymbol{H}}_0$. For definiteness, we focus on lattice systems, with a finite local

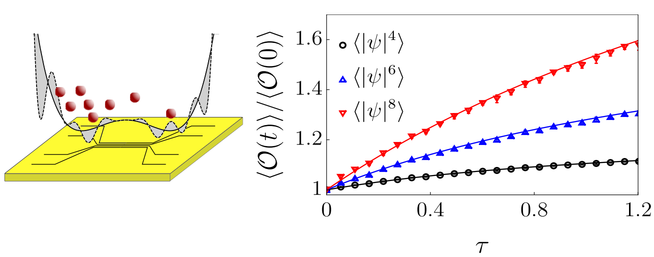

FIG. 1. Left: Pictorial representation of 1d interacting Bose gas trapped on an atom chip, experiencing fluctuations in the confining potential (see main text). Right: Comparison between GHD prediction and *ab initio* numerical simulations in the classical NLS with interaction $c = 1$; the initial state is thermal and homogeneous with inverse temperature $\beta = 1$ and chemical potential $\mu = 2$ (resulting in density $\langle|\psi|^2\rangle = 0.80$). The noise correlation is $F(x) = \ell\sqrt{\pi/2}e^{-x^2/(2\ell^2)}$: Agreement with the theoretical prediction is achieved with $\ell = 4$ and $\gamma = 0.1$. We focus on the time evolution of the density moments $\mathcal{O} = \{|\psi|^4, |\psi|^6, |\psi|^8\}$ as a function of the rescaled time τ in the NLS. Solid lines: Predictions from Eq. (7). Symbols: *Ab initio* numerical simulations.

Hilbert space (e.g., spin chains), although the discussion can be extended to other settings (see below). We assume that the evolution is described by the nonintegrable Hamiltonian

$$\boldsymbol{H}_\eta = \boldsymbol{H}_0 + \sum_j \eta_j(t)\boldsymbol{O}_j, \tag{1}$$

where the second term encodes the dephasing noise, with $\langle\eta_j(t)\eta_{j'}(t')\rangle = \gamma F(j-j')\delta(t-t')$. The parameter γ controls the intensity of the noise, while the function $F(x)$ its spatial correlation. The operator $\boldsymbol{O}_j$ is assumed to have (quasi)local support around the site j. The quantum dynamics of the model is then described as the solution of the Schrödinger equation $d|\psi\rangle/dt = -\imath\boldsymbol{H}_\eta|\psi\rangle$, which is a stochastic differential equation. Note that it involves a multiplicative noise term and the Stratonovich convention is assumed here [64] (see also the Supplemental Material (SM) [65]). The noise-averaged density matrix $\boldsymbol{\varrho} = \overline{|\psi\rangle\langle\psi|}$ satisfies the Lindblad equation

$$\dot{\boldsymbol{\varrho}} = -\imath[\boldsymbol{H}_0, \boldsymbol{\varrho}] - \frac{\gamma}{2}\sum_{j,j'} F(j-j')[\boldsymbol{O}_j, [\boldsymbol{O}_{j'}, \boldsymbol{\varrho}]]. \tag{2}$$

In general, solving Eq. (2) for a many-body system is even harder than its pure dynamics. For short-range noise, i.e., $F(j-j') \to \delta_{j,j'}$, a few solvable cases have recently been discovered: when $\boldsymbol{H}_0$ describes noninteracting spinless fermions and $\boldsymbol{O}_j$ denotes their on-site occupation number, Eq. (2) was shown to be related to the integrable Fermi-Hubbard model [53]; other integrable examples have been classified in Ref. [56]. Moreover the case $\boldsymbol{O} = S^z$ in the *XXZ* spin chain was recently studied in [66,67] in the limit $\gamma \to \infty$ and for δ-correlated noise. Here, instead, we focus on the opposite limit where the correlation $F(j-j')$ is flat within the correlation length ℓ and smoothly decays for $|j-j'| \gg \ell$.

Hydrodynamics description. Let us briefly describe the dynamics for $\gamma = 0$. Since $\boldsymbol{H}_0$ is integrable, there exists an infinite set of conserved quantities $\boldsymbol{Q}^{(\alpha)}$, $\alpha = 1, \ldots,$ commuting with the Hamiltonian $[\boldsymbol{Q}^{(\alpha)}, \boldsymbol{H}_0] = 0$. Starting from an initial density matrix $\boldsymbol{\varrho}_0$, the unitary evolution preserves all the conserved quantities of the system $\boldsymbol{Q}^{(\alpha)}$ and induces equilibration to the GGE pinned down by such initial values [18]. In practice, it is convenient to encode the GGE by introducing the root density of the quasiparticles $\rho(\lambda)$ [68], defined such that $L\rho(\lambda)d\lambda$ equals the number of quasiparticles with rapidities $\in [\lambda, \lambda + d\lambda)$. Quasiparticles are conserved modes and their dynamics is fully encoded in the scattering shift $T(\lambda, \lambda')$ for any integrable system. For simplicity, here we consider a single quasiparticle species, the generalizations being straightforward. The rapidity λ parametrizes the state of each quasiparticle, such that, in the thermodynamic limit,

$$\lim_{L\to\infty} \frac{\mathrm{Tr}[\boldsymbol{\varrho}_0\boldsymbol{Q}^{(\alpha)}]}{L} = \int d\lambda\rho(\lambda)q^{(\alpha)}(\lambda) \equiv \langle\rho|\boldsymbol{Q}^{(\alpha)}|\rho\rangle, \tag{3}$$

where the functions $q^{(\alpha)}(\lambda)$ are the single-particle eigenvalues associated with the αth charge. Equation (3) establishes the correspondence between a complete set of charges and the root density. In the last equality, we employed a generalized microcanonic ensemble to select a pure macrostate $|\rho\rangle$ representative of the root density $\rho(\lambda)$ [69].

Now, we turn on the weak dissipative term in Eq. (2) and we assume that the system remains always in a GGE representative state $|\rho(t)\rangle$ which evolves in time. In order to get the evolution equation for the root density, we look at the time variation of the expectation values of the charges. We replace $\boldsymbol{\varrho} \to |\rho(t)\rangle\langle\rho(t)|$ in the right-hand side of Eq. (2) and we obtain

$$\lim_{L\to\infty} \frac{\mathrm{Tr}[\boldsymbol{Q}^{(\alpha)}\dot{\boldsymbol{\varrho}}]}{L} = \gamma\sum_e \Delta Q_e^{(\alpha)}\hat{F}(\Delta P_e)|\langle\rho|\boldsymbol{O}|\rho;e\rangle|^2 + O(\gamma^2), \tag{4}$$

with the higher-order corrections involving extra numbers of excitations on the GGE state. Here, we assume the observable $\boldsymbol{O}$ is number conserving; hence we inserted a sum over the tower of all the possible *particle-hole* excitations $|\rho, e\rangle$ on top of the GGE state $|\rho\rangle$ [69–71]. We denote by $\Delta Q_e^{(\alpha)} = \langle\rho;e|\boldsymbol{Q}^{(\alpha)}|\rho;e\rangle - \langle\rho|\boldsymbol{Q}^{(\alpha)}|\rho\rangle$ the extra charge due to the excitation e on top of $|\rho\rangle$ [72]. Similarly, ΔP_e is the momentum of the excitation e, while the matrix element $\langle\rho|\boldsymbol{O}|\rho;e\rangle$ is a generalized form factor on top of the state $|\rho\rangle$ [71,73]. We also introduced the Fourier transform of the noise correlation $\hat{F}(k) = \sum_j F(j)e^{-\imath kj}$. We are now interested in the limit of smooth noise with finite correlation length. We thus parametrize $F(j) = \ell f(j/\ell)$, where $f(x)$ is an even and smooth function decaying to zero for $x \gg 1$. Expanding $\hat{F}(k)$ for $\ell \gg 1$, we have

$$\frac{\hat{F}(k)}{2\pi} = \ell f(0)\delta(k) + \frac{\kappa_2}{\ell}\delta''(k) + O(\ell^{-3}), \tag{5}$$

where to simplify the notation we set $\kappa_2 = -f''(0)/2 > 0$. Once Eq. (5) is injected in Eq. (4), we observe that only excitations at small exchanged momentum ΔP_e are relevant. In this limit, the form factor is dominated by a single particle-hole excitation [40,73] and one can replace

$$\sum_e \overset{\Delta P_e\to 0}{\longrightarrow} \int dp_{\mathrm{h}}dp_{\mathrm{p}}[1-n(p_{\mathrm{h}})]n(p_{\mathrm{p}}) + \cdots, \tag{6}$$

where the integral runs over the *dressed* momenta p of the particle $p_{\rm p}$ and the hole $p_{\rm h}$, with $\Delta P_e = p_{\rm p} - p_{\rm h}$. The dressed momentum and the rapidities are related via $dp = 2\pi\rho_t(\lambda)d\lambda$, where $\rho_t(\lambda)$ is the total root density, which counts the number of available modes [74]. For noninteracting systems, $\rho_t(\lambda)$ is a fixed function, but in the presence of interactions, it is state-dependent and is related to $\rho(\lambda)$ via integral equations [75]. The filling function is expressed as $n(p(\lambda)) = \rho(\lambda)/\rho_t(\lambda)$ and it fully specifies a stationary state. The right-hand side of Eq. (6) ensures that the momentum $p_{\rm h}$ ($p_{\rm p}$) is unoccupied (occupied). In particular, the leading order in Eq. (5) gives a vanishing contribution as $\Delta Q_e^{(\alpha)} = O(\Delta P_e)$. The second term instead gives a finite result, which can be entirely expressed in terms of the single particle-hole form factor in the limit of vanishing momentum [32,76] $\lim_{p_{\rm p}\to p_{\rm h}} \langle\rho|\boldsymbol{O}|\rho;\{p_{\rm p}, p_{\rm h}\}\rangle = V^{\boldsymbol{O}}(p_{\rm h})$, where $V^{\boldsymbol{O}}$ is related to the expectation value of the operator on a generic stationary state $2\pi V^{\boldsymbol{O}}(p) = \delta\langle\boldsymbol{O}\rangle/\delta n(p)$ (see SM [65]). If the noise is coupled to a conserved charge, one has the simple result $V^{\boldsymbol{q}}(p) = q^{\rm dr}(p)$ [40,70]. In the rapidity space, the dressed single-particle eigenvalue $q^{\rm dr}(\lambda)$ is determined solving the integral equation $(1 + Tn)q^{\rm dr} = q$ (where the scattering shift T is seen here as a linear operator in the space of λ and $[1]_{\lambda,\lambda'} = \delta(\lambda-\lambda')$ is the identity operator). For the sake of simplicity, we make a little abuse of notation using the same symbol $q^{\rm dr}$ to denote both the dependence on rapidities and momenta. We point out that the dressed momentum introduced above is conventionally defined as the integral of the dressed derivative of the bare momentum $p_{\rm bare}$, since $2\pi\rho_t(\lambda) = (\partial_\lambda p_{\rm bare})^{\rm dr}$.

The terms of order ℓ^{-3} in Eq. (5) generate more complicated excitations such as two particle-hole terms in Eq. (6). Restricting ourselves to the first nontrivial term, we can perform the integration over $p_{\rm p}$, and by employing the completeness of the set of charges $\boldsymbol{Q}^{(\alpha)}$, Eq. (4) can be recast into a diffusion equation for the root density $\rho(\lambda)$ or equivalently for the filling function $n(p)$ [65] describing the state at any time t:

$$\partial_t n_t(p) = \frac{\kappa_2\gamma}{\ell}\,\partial_p\big(\big[V_t^{\boldsymbol{O}}(p)\big]^2\partial_p n_t(p)\big) + O\left(\frac{\gamma}{\ell^3}\right) + O(\gamma^2). \tag{7}$$

This final equation has the simple form of diffusion in the space of dressed momenta p and is the main result of our work. The remaining details of the noise can be completely re-absorbed defining a rescaled time $\tau = \kappa_2\gamma t/\ell$. In the case of a generic driving, Eq. (7) holds in the limit $\ell, \gamma^{-1} \gg 1$, while τ is kept constant. However, in the case where $\boldsymbol{O}$ is chosen as a conserved charge, it is expected to hold for arbitrary γ, provided ℓ is chosen large enough. Indeed, for $\ell\to\infty$, driving with a conserved charge leaves the system unscathed; thus the $O(\gamma^2)$ term is absent and all higher-order ones. On the contrary, in the generic case diffusive corrections of order γ^2/ℓ^0 [45,77] are expected. In the following, we will focus on the most relevant case where the operator $\boldsymbol{O}$ is a conserved density as for example a $U(1)$ charge or the Hamiltonian density of the system. Note that the diffusion constant $\propto [V_t^{\boldsymbol{O}}(p)]^2$ is time-dependent, since it depends on the state $n_t(p)$ itself: the resulting equation is highly nonlinear. Additionally, the mapping from momentum to rapidity space (where the dressing is defined) also evolves in time (see SM [65] and Ref. [78] for details about the numerical solutions).

The interacting Bose gas. As a first application of our general findings, we revert to the 1d interacting Bose gas $\boldsymbol{H}_0 = \int dx\,\partial_x\psi^\dagger\partial_x\psi + c\psi^\dagger\psi^\dagger\psi\psi$, which is ubiquitous in describing the state-of-the-art cold-atom experiments [38,79–85]. Atom chips [61] are routinely used to manipulate 1d atomic gases, in view of their ability of creating customizable external traps [86]. However, imperfections in the chip's fabrication [62], and in particular fluctuations in the currents of the device [61], result in spatially smooth and time-uncorrelated fluctuations of the external trap (Fig. 1, left). This setup provides a first example of the proposed dephasing dynamics in a experimentally relevant context. The model is integrable both in its classical [87] and quantum [88,89] formulation.

Continuous quantum models are notoriously hard to simulate with tensor network techniques; hence their hydrodynamic description is a paramount achievement in experiments' simulations [38,90]. Within the weakly interacting regime and at finite temperature, the quantum system is well described by its classical limit [91–96], i.e., the nonlinear Schrödinger model (NLS), which is amenable to efficient *ab initio* numerical simulations [97,98]. For this reason, hereafter we focus on the classical regime in the repulsive phase $c > 0$ (see SM [65] for details). In Fig. 1, we compare GHD predictions with numerical simulations, finding excellent agreement. The system is initialized in a thermal state, which is then allowed to evolve with a noise coupled to the local density $\psi^\dagger(x)\psi(x)$. The choice of this particular dephasing is motivated by the atom chip setup, where changes in the confining trap results in a locally fluctuating chemical potential. For the sake of simplicity and to better isolate the effects of the noise correlation length scale ℓ, we focus on a homogeneous setup. We consider the time evolution of the density moments $\langle|\psi(x)|^{2n}\rangle$ computed in Ref. [99] for arbitrary GGEs (see also Refs. [100,101] for the quantum case). The details of the numerical simulations and further comparisons are left to the SM [65].

Interacting spin chains. The *XXZ* spin chain is given by the Hamiltonian $\boldsymbol{H}_0 = \sum_j \boldsymbol{h}_{j,j+1}$ where $\boldsymbol{h}_{j,j+1} = \boldsymbol{S}_j^x\boldsymbol{S}_{j+1}^x + \boldsymbol{S}_j^x\boldsymbol{S}_{j+1}^x + \Delta(\boldsymbol{S}_j^z\boldsymbol{S}_{j+1}^z - 1/4)$ with $\boldsymbol{S}^{x,y,z}$ being spin-1/2 operators. We focus on the easy-axis regime $\Delta > 1$, where the quasiparticle are labeled by an extra integer index $s = 1, \ldots, \infty$ representing their spin quantum number. Here, we restrict ourselves to the dynamics close to the ground state, so that only quasiparticles with $s = 1$ are relevant. Since no new quasiparticles are generated by the dynamics of Eq. (7), a gapped ground state remains exactly unperturbed by our dynamics in the limit $\ell\to\infty$. However, by adding an external magnetic field $\boldsymbol{H}_0 + B\sum_j \boldsymbol{S}_j^z$, we can choose Δ and B such that the ground state is gapless and the dynamics is nontrivial [46,68]. As an example, we drive the system with the local energy density $\boldsymbol{q}_j = \boldsymbol{h}_{j,j+1}$, which describes the effect of a phononic bath [102,103]. We plot the evolution of the total energy of the system in Fig. 2 and we notice that agreement with our theoretical prediction does indeed improve for large ℓ, no matter the value of γ, as is expected driving with a conserved charge. We observe that Eq. (7) predicts that energy increases up to a plateau on a prethermal stationary state, different from an infinite-temperature state. The latter indeed

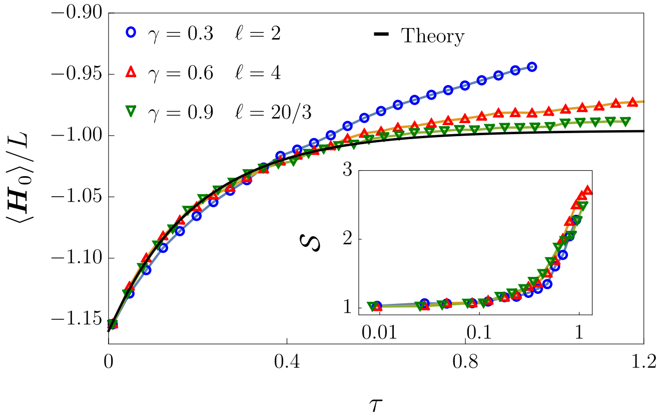

FIG. 2. Plot of the time evolution of the energy density of an *XXZ* chain obtained by the 2-site TDVP algorithm [7] and averaged over 20 realizations with maximally used bond dimension $\chi = 200$ and $L = 300$. The chain is initially prepared in the gapless ground state at $\Delta = \cosh(3/2)$ and $B = 1.75946$, such that $\langle S^z_j \rangle = 1/10$. The noise is coupled with the energy density $\boldsymbol{q}_j = \boldsymbol{h}_{j,j+1}$. The energy density of the infinite-temperature state is given by $-\Delta/4 \simeq -0.59$. Inset: The log plot of the entanglement entropy of the bipartite chain.

cannot be reached by the dynamics given by Eq. (7), as it requires creating quasiparticles with higher spin s, which are not contained in the initial state. Clearly, the corrections in ℓ^{-3} will include terms leading to quasiparticle production that will lead the system to thermalize. However on timescales of order ℓ/γ we observe perfect agreement of the numerical simulation with the evolution (7), proving that it correctly describes the dynamics of the system at such timescales.

It is also interesting to consider the evolution (7) at short times. Starting from the ground state, this implies an initial linear growth in time for the charges; in particular, for the energy we have $\langle \boldsymbol{H}_0 \rangle / L = e_{\rm GS} + [V^{\boldsymbol{O}}(p_F)]^2 v_F\ \tau/\pi + O(\tau^2)$ where $e_{\rm GS}$ is the ground-state energy, v_F is the Fermi velocity of the system, and p_F is the Fermi momentum. In the case of the driving coupled to the local spin $\boldsymbol{q}_j = \boldsymbol{S}^z$, we have $[V^{\boldsymbol{O}}(p_F)]^2 = K$ the Luttinger parameter of the ground state, which recovers the prediction from bosonization [104–107] (see SM [65] for details).

Single noise realization. The dynamics given by Eq. (7) describes the average over several realizations of the noise in the evolution given by Hamiltonian (1) [108,109]. However, in each single realization, the evolution is pure and unitary, and the noise term plays the role of a random force. In the case where the driving is coupled to an external charge, the hydrodynamic equations at first order in the external perturbation are known [48]. However, they are applicable in a regime of weak space-time dependence where the hydrodynamic picture applies. Nevertheless, since the effect of the noise is weak in our regime $\ell \gg 1$, we can assume that in each noise realization the system remains locally close to a quasiequilibrium state. Then, the stochastic evolution of the stochastic filling function $\mathfrak{n}_{x,t} \equiv \mathfrak{n}_{x,t}(\lambda)$ reads [48]

$$\partial_t \mathfrak{n}_{x,t} + v^{\rm eff}_{x,t} \partial_x \mathfrak{n}_{x,t} - (\partial_x U_{x,t}) \left(\frac{q^{\rm dr}_{x,t}}{p'_{x,t}} \right) \partial_\lambda \mathfrak{n}_{x,t} = 0, \quad (8)$$

where $U_{x,t} = \sqrt{\gamma}\eta_x(t)$. The effective velocity of the quasiparticle is given by the *dressed* energy ε, $v^{\rm eff}[\varepsilon] = \partial\varepsilon/\partial p = \partial_\lambda \varepsilon(\lambda)/\partial_\lambda p(\lambda)$, and is also modified by the external force via $\partial_\lambda \varepsilon \to \partial_\lambda \varepsilon + U_{x,t}(\partial_\lambda q)^{\rm dr}$. Note that the noise in Eq. (8) is meant in the Stratonovich convention which makes the noise average nontrivial. Nevertheless, after converting to the Ito formulation, one obtains a translationally invariant filling $n_t(\lambda) \equiv \overline{\mathfrak{n}_{x,t}(\lambda)}$ whose time evolution in the space of momenta matches with Eq. (7).

It is interesting to observe that Eq. (8) does not lead to any entropy production [30]. Starting from a Fermi sea distribution for $\mathfrak{n}_{x,t=0}(\lambda)$, the evolution (8) for a single realization of the noise can be seen as a local (random) boost of the Fermi points so that the state remains a zero-entropy one at all times. It is natural to expect this to result in the suppression of the entanglement entropy production. This is indeed what we observed in the tDMRG simulation at short times, see Fig. 2, where the growth of entanglement entropy is indeed curbed. However, at times of order $t \sim \kappa_2\gamma/\ell$, the entanglement entropy of each realization suddenly starts growing linearly with time, signaling that its dynamics is given by terms that go beyond Eq. (8), as for example diffusive terms of order $O(\partial_x^2 \mathfrak{n}_{x,t})$ [40]. However, Eq. (7) still provides a good description of the averaged evolution of local operators. A similar entropy increasing at intermediate timescales was observed in the studies of classical hard rods in external potentials [30,43].

Discussion and conclusion. We presented an exact hydrodynamic description of the out-of-equilibrium dynamics of integrable systems in the presence of dephasing noise, where the extension to inhomogeneous setups is now obvious [110,111]. For future perspectives, it would be interesting to extend our treatment to include subleading terms, operators which do not conserve particles' number, and to effectively describe three-body losses, one of the leading effects in cold-atom setups involving quantum gases [38,100]. Another exciting direction is the inclusion of a genuinely quantum noise term, as modeled by the coupling to an ensemble of bosonic/fermionic quantum oscillators [112,113].

Acknowledgments. The MPS-based TDVP simulations were performed using the ITensor Library [114]. We thank A. Nahum and D. Bernard for useful discussions. A.B. acknowledges support from the European Research Council (ERC) under ERC Advanced Grant No. 743032 DYNAMINT. J.D.N. is supported by the Research Foundation Flanders (FWO).

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