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Fierljeppen: pole vaulting for distance

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Abstract

Pole vaulting, the aim of which is to jump over a crossbar with the help of a long flexible pole, is considered to be one of the most complicated and technically demanding motions in track and field athletics. Pole vault performance is basically influenced by the energy exchange between the vaulter and pole. It depends on the sprinting, jumping and acrobatic abilities of the athlete. Less well-known, but just as exciting and fascinating, is pole vaulting for distance. The aim in this sports event, named fierljeppen in the Frisian language, is to jump as far as possible with the help of a long pole. Athletes reach heights of 11.5 m and the record jumping distance is currently 22.21 m. In this paper, we present a simplified mathematical model of the motion of vaulter and pole based on classical mechanics. This model helps us understand the dynamics of pole vaulting for distance and optimize the athlete's performance by looking for the optimal speed of running-up toward the pole and the optimal release angle.

Keywords: classical mechanics, pole vaulting for distance, mathematical modelling

1. Fierljeppen: pole vaulting for distance

Pole vaulting is a sport with a long history [1, 2]. Best known is the pole vault as an event performed in track and field athletics, wherein the athlete uses a flexible pole to clear a crossbar resting on two pegs supported by two standards.

The vaulter's aim is then to attain the greatest height and for this purpose (s)he must apply great sprinting, jumping and acrobatic abilities.

The origin of pole vaulting is historically the use of a spear by a soldier to vault onto horseback and the use of a pole to leap over obstacles. The most common obstacle is not a hedge, a wall or another type of barrier in height, but a ditch, river or canal. Canal jumping using a pole can be traced back to the marshy provinces around the North Sea and the Fens of Cambridgeshire, Lincolnshire and Norfolk. This gave rise to the sports event of leaping with a pole for distance instead



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of height. Leaping for height became an Olympic event in track and field athletics. Leaping for distance turned into a folkloric event and only in the Netherlands it remained a traditional sport called fierljeppen (a West-Frisian compound of fier—far, ljeppen—leaping). Fierljeppen was officially recognised as a part of the Frisian cultural heritage by UNESCO in 2018.

If you find an athlete vaulting over a crossbar at a height a spectacular view, imagine a vaulter using a 13 m long pole of about 20 kg to reach a height of 11.5 m and attain a distance 20 m or more. Much courage and technique is needed for such a performance. Of course, the fierljepper does not run with such a long heavy pole on a runway to place one end of the pole in a plant-box, as in pole vaulting for height. To get a good impression of how a pole vault for distance is arranged in a fierljep sport event, you can watch on YouTube the jump of Jaco de Groot, leading to the record distance of 22.21 m (just enter the keywords ‘world record’ and ‘fierljeppen’ to find a video clip of this jump). Imagine:

- the speed of about 30 km h^{-1} at which this athlete is sprinting on the runway,
- the fearless jump against the pole, which has been placed beforehand with one end firmly in a gravel bank underneath the water surface and kept at a preferred distance away from the take-off point on the runway by the pole holder using a pitchfork,
- the precision, strength, and technique that he uses to grab the 10 cm thick pole and get the pole between his legs, while keeping his body posture such that he and the pole do not deviate to the left or right,
- the muscle power and motion control he needs to climb to the end of the pole in a short time and reach a height of about 11.5 m while controlling the tipping motion of the pole in the straightforward direction and restricting the lateral movement of the pole,
- the technique to push himself off from the pole in order to leap extra decimetres, and
- the courage to land preferably sideways in order to break the fall with his body and avoid injuries of arms and legs.

In figure 1, we visually present the different phases of the jump and add some details based on regulations of official sport events [3]. The

jumping distance is measured from the end of the runway to the nearest break in the sand. Not shown in the drawing is the trainer/coach who runs behind the vaulter on the runway to encourage and coach the athlete. The pole holder keeps the pole at a distance from the end of the runway that is preferred by the vaulter so that (s)he can grab the pole at the highest point, in order to minimize the distance to climb to the top of the pole, while still being able to let the pole tip over to the other side of the water. The pole holder pulls back the pitchfork that holds the pole as soon as the vaulter catches the pole. The pole holder also warns for danger so that the vaulter can release the pole in time and fall into the water without any harm done.

The performance of pole vaulting for height is basically influenced by the energy exchange between the vaulter and pole. It depends on the sprinting, jumping and acrobatic abilities of the athlete [4–6]. Similarly, the performance of pole vaulting for distance is influenced by the exchange of energy between the vaulter and pole, and it depends on the sprinting, jumping, climbing and motion control abilities of the athlete.

We present in this paper a mathematical model of the motion of vaulter and pole based on classical mechanics. We use this model to create computer simulations of the vaulter’s motion under various conditions. These simulations help us understand the dynamics of pole vaulting for distance and give advice to optimize the athlete’s performance. We discuss what would be the optimal speed of running-up toward the pole: not too fast because the vaulter then does not have enough time to climb up the pole; not too slow because the pole then does not pass its tipping point to move across the water and the athlete must let go and fall into the water. We also use the computer model to predict the optimal angle of the pole at which the athlete pushes off against the pole to land furthest in the sand bed. The modelling activities show how physics can help study such kinds of biomechanical problems and inform trainers and athletes.

2. Modelling fierljeppen

The modelling process starts with (1) having a close look at the conditions under which the sport event takes place and how they affect the motion

Fierljeppen: pole vaulting for distance

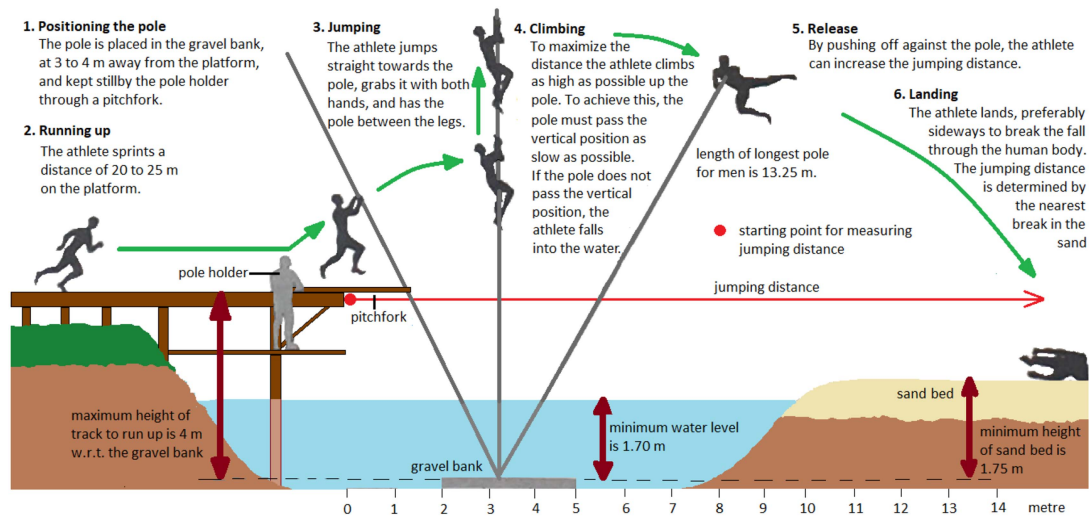


Figure 1. Six phases in pole vaulting for distance under conditions (height of runway, water level, height of sand bed, length of the pole, etc) allowed according to game regulations and favourable for attaining long distances.

of the vaulter and pole; (2) making sensible assumptions so that a mathematical model can be constructed that is simple and yet provides useful results for understanding the athlete's movements, and for giving advice on how to optimize the performance; and (3) lingering upon and deciding which quantities are relevant to include in the mathematical model, so that they can be used to answer the biomechanical questions raised.

2.1. Dimensions of the fierljepp accommodation, the pole and the athlete

The game regulations of the Dutch Fierljepp Association [3], with about 600 active athletes, prescribe the dimensions of the accommodation for non-recreative fierljeppen. Information about fierljeppen can also be found in the periodical published by the association (see for example [7]). The height of the runway is between 3.60 and 4.00 m, measured with respect to the upper level of the gravel bank in which the pole is firmly placed before the jump takes place and which is 2–6 metres away from the end of the runway. The maximum length of the pole is 13.25 m and the maximum diameter above the bottom part is 12.5 cm. Although the material choice for the pole is free, top athletes use a carbon pole with a mass of about 20 kg. The water level above the gravel

bank is between 1.70 and 2.00 m. The sand bed is at least 5 cm above the water level. The best conditions for attaining long jumping distances are accommodations with the highest runway, the lowest sand bed, and the lowest water level (also for reduction of the water resistance), and the use of the longest, rather rigid pole (see figure 1).

Before the jump, the vaulter decides how to place the pole and at what pole height, i.e. the distance from the bottom of the pole, (s)he intends to grab the pole. A larger pole height means that (s)he has to climb less to reach the top of the pole. But there is a catch: a larger pole height also means that the angle between the pole and the horizontal gravel bank gets smaller so that it becomes more difficult, that is, greater running speed and a larger take-off impulse are required from the vaulter to let the pole tip over to the other side of the water.

When the pole holder places the pole about 2–3 metres away from the end of the runway, the pole is less slanted, that is, the angle between the pole and the horizontal gravel bank is larger so that the pole will tip over more easily. In this case, the athlete jumps towards the pole, grabs it with both hands and via a pendular motion of the rest of the body gets close to the pole and places it between their legs. Hereafter, (s)he can start climbing towards the top of the pole. The

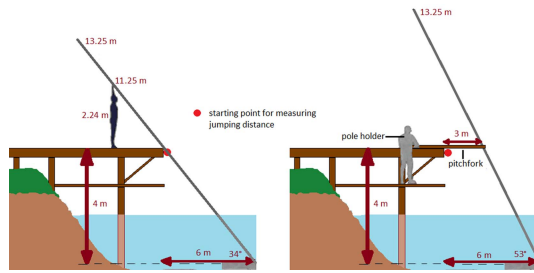


Figure 2. Visualization (not drawn to scale) of the change of the pole angle when a pole holder keeps the pole away from the end of the running platform with a pitchfork.

main advantage of reduction of kinetic energy that can be transformed into rotational energy is not at the expense of the jumping distance because it is still measured from the end of the runway with the bottom part of the pole at the same position in the gravel bank as before. A drawback is of course that the jumping technique becomes more complicated and requires more muscle power and motor skills from the athlete to keep control over the quality of the jump.

To get a quantitative view on the effect of positioning the pole away from the end of the runway at a height of 4 m, we consider a vaulter who reaches upright with his hands at a height of 2.24 m to the pole resting against the end of the runway and touches it at a pre-determined pole height of 11.25 m; see the left-hand side of figure 2. Using the similarity of triangles and Pythagoras' theorem, it is a secondary school exercise to compute the horizontal distance between the end of the runway and the bottom of the pole (see section 4.1). Trigonometry and use of a calculator give a pole angle of 34° . When a pole holder keeps the pole 3 m away from the end of the runway, the pole angle becomes 53° ; see the right-hand side of figure 2.

2.2. Minimum sprinting speed without climbing

By using basic principles of physics and simple models of the vaulter-pole system, we can already analyse the fierljepp jump. For example, we can determine a lower bound of the sprinting speed of the vaulter needed to let the vaulter-pole system

tip over to the other side, by applying the law of conservation of energy.

The modelling process starts with understanding the real situation of a fierljepp jump by observing several video clips of jumps so that we get a mental model of the real situation. This leads to the idea that, for finding this lower bound, we can focus on the motion between the moment that the vaulter has grabbed the pole and placed it between the legs, and the moment that the vaulter-pole system has reached a vertical orientation.

The next modelling phase consists of structuring, making assumptions, and simplifying to turn the situational model into a real model. For example, we can consider a model of the vaulter as a point mass or as a one-segment body consisting of a slim, uniform cylinder. Note that these two models of the vaulter are merely mathematical models, which have little resemblance to reality, but nevertheless serve the purpose of analysing the motion of the vaulter-pole system. The pole is modelled as a long, perfectly rigid rod with uniform weight distribution. For determining a lower bound of the vaulter's sprinting speed, we may ignore friction in air and water, and assume that the vaulter does not climb towards the top of the pole, because physics of rotational motion informs us that more kinetic energy is needed to let the vaulter-pole system reach a vertical orientation. In other words, we focus on the first phase of an inverted pendular motion of a rigid body, as shown in figure 3.

We move on to use basic concepts of mathematics and physics to mathematize and transform the real model into a mathematical model. Under the assumptions made, we can determine a lower bound of the speed of the vaulter needed to let the jumper-pole system tip over in the form of a formula in the following variables: mass of the jumper (M), mass of the pole (m), the initial pole angle (θ_0), pole length (L), pole height of the centre of mass of the jumper (r_0) measured from the bottom end of the pole, speed of the jumper (v) perpendicular to the pole when (s)he has grabbed the pole and placed it between the legs. We can focus on the speed of the jumper perpendicular to the pole at the start of the inverted pendular motion, because this one is only relevant for the rotational motion (the gravel bank

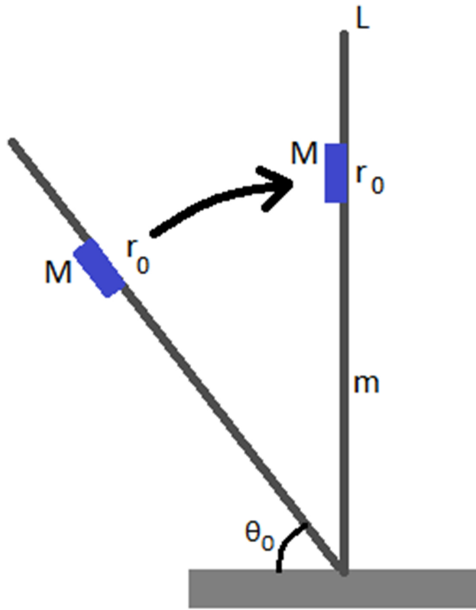


Figure 3. Side-view schematic drawing of the mathematical model. The vaulter is modelled as a one-segment body consisting of a slim, uniform cylinder of mass M that does not move to the top of the pole, but sticks to its original position at pole height r_0 during the forward motion toward the vertical position. The pole is modelled as a perfectly rigid rod of length L and weight m .

dissipated the kinetic energy associated with the velocity component parallel to the pole) and the sprinting speed needed will always be greater than this.

Assuming a 100% efficiency of energy transfer, we can apply the law of conservation of energy and set the total energy of the vaulter-pole system at the initial pole angle θ_0 equal to that at the pole angle of 90° when the system has reached the vertical orientation with hardly any forward motion left. In these two situations, we only have to consider kinetic energy and potential energy due to gravity. We set the gravitational potential energy to zero at the bottom end of the pole placed in the gravel bank. At the start, when the pole angle is θ_0 , the total energy consists of the kinetic energy of the jumper and the potential energy of the jumper and the pole. Ignoring that the vaulter's centre of mass is not exactly located on the pole, we can derive the following expression for the total energy at pole angle

$$\theta_0 : \frac{1}{2}Mv^2 + Mgr_0 \sin \theta_0 + \frac{1}{2}mgL \sin \theta_0$$

When the vaulter-pole system has reached the vertical orientation with hardly any forward motion left, we only have to take into account the potential energy of the vaulter and the pole in the expression for the total energy at pole angle

$$90^\circ : Mgr_0 + \frac{1}{2}mgL$$

Conservation of energy leads to the following equation:

$$\frac{1}{2}Mv^2 + \left(Mr_0 + \frac{1}{2}mL\right) g \sin \theta = \left(Mr_0 + \frac{1}{2}mL\right) g$$

This can be rewritten as

$$v^2 = \left(2r_0 + \frac{m}{M}L\right) g (1 - \sin \theta).$$

So, the speed of the vaulter perpendicular to the pole needed to get the vaulter-pole system in upright position is

$$v_{\min} = \sqrt{\left(2r_0 + \frac{m}{M}L\right) g (1 - \sin \theta)}.$$

Substitution of reasonable values (in SI units) of the variables for a professional vaulter, say

$$r_0=8, m=20, M=75, L=13.25, g=9.81, \theta=50^\circ,$$

gives

$$v_{\min} \approx 6.70 \text{ m s}^{-1} \approx 24.1 \text{ km h}^{-1}.$$

Thus, the sprinting speed of the vaulter on the platform is expected to be at least 24.1 km h^{-1} at the end of the runway. It is in reality probably closer to 30 km h^{-1} . Quite a performance!

From the above formula for the minimum angular speed, we can also draw the following conclusions: more speed is needed when the pole is longer or heavier, when the pole angle is smaller, when the vaulter weighs less, and when the vaulter grabs the pole at a larger pole height. Pole vaulting is easier on the moon when you only take the acceleration of gravity into account.

2.3. Dynamics when a vaulter climbs to the top of the pole

The modelling process of the fierljeep jump discussed in the previous subsection has increased our confidence in this way of mathematical modelling, but it lacks the power to describe the dynamics of the motion and in particular what happens when the vaulter starts climbing. In other words, we must derive the equations of motion that correspond with an extended mathematical model. Rotation mechanics of a rigid body puts us here on the right track, just as in rigid-pole models of the take-off phase in pole vaulting for height [8, 9]. In order to keep the mathematical formulas in the model as simple as possible, we henceforth use radian instead of degree as the unit for angle of rotation.

We extend our mathematical model of the previous subsection as follows. For the vaulter-pole system, once the vaulter has a firm grip with arms and legs on the pole, we focus on the angle of rotation (θ) of the system, which is considered equal to the pole angle, the angular velocity ($\omega = \theta'$), and the moment of inertia (I). At the start of the motion, the angle of rotation and angular velocity are θ_0 and ω_0 , respectively, and the vaulter's centre of mass is at the initial pole height r_0 . When the angle of rotation is between $\frac{1}{2}\pi - \beta$ and $\frac{1}{2}\pi + \beta$, for some positive angle $\beta \leq \frac{1}{2}\pi - \theta_0$, then the vaulter climbs to the top of the pole at a constant velocity u along the pole. The vaulter stops when (s)he has reached the top of the pole or when the angle of rotation reaches the value $\frac{1}{2}\pi + \beta$. The vaulter is modelled as a one-segment body consisting of a slim, uniform cylinder of length l and diameter d positioned at distance r from the bottom of the pole and having weight M . Figure 4 is a schematic drawing of what goes on in the motion.

In this case, it is best to start Newton's law of rotational motion, which states that the derivative of the angular momentum equals the torque:

$$(I \cdot \omega)' = \tau,$$

where τ is the sum of all relevant torques for the vaulter-pole system. Because the vaulter climbs to the top of the pole, the moment of inertia I is not constant. Thus, the above equation can be rewritten as:

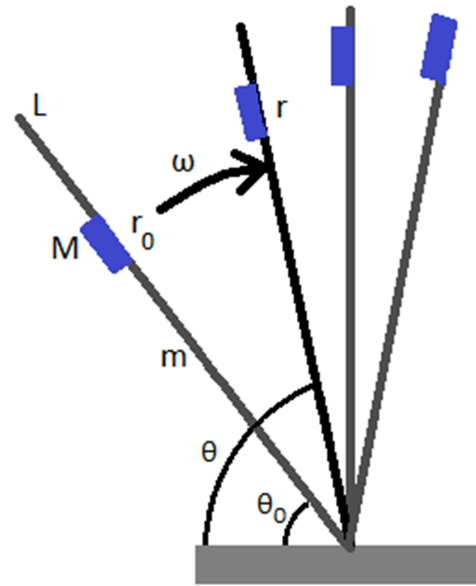


Figure 4. Side-view schematic drawing of the mathematical model for describing the dynamics of the fierljeep jump while the vaulter is still in touch with the pole. The vaulter is modelled as a one-segment body consisting of a slim, uniform cylinder of mass M at pole height r during the forward motion that moves to the top of the pole. The pole is modelled as a perfectly rigid rod of length L and weight m .

$$I' \cdot \omega + I \cdot \omega' = \tau,$$

and in terms of angle of rotation θ as

$$I' \cdot \theta' + I \cdot \theta'' = \tau.$$

The torque, being the product of the force acting on a body and the perpendicular distance of the axis of rotation from the line of action of the force, consists of two components associated with the vaulter and the pole at which gravitational force acts:

$$\begin{aligned} \tau &= \tau_{\text{vaulter}} + \tau_{\text{pole}} \\ &= -Mgr \cos(\theta) - mg \left(\frac{L}{2}\right) \cos(\theta) \\ &= -\left(Mr + \frac{1}{2}mL\right) g \cos(\theta) \end{aligned}$$

The moment of inertia I is the sum of the moment of inertia of the vaulter (I_{vaulter}) and the moment of inertia of the pole (I_{pole}) about the bottom end of the pole placed in the gravel bank. Because the pole is modelled as a thin rigid rod of length L

pivoting about one of its end, the moment of inertia of the pole (look it up in a classical mechanics textbook, or calculate it) is

$$I_{\text{pole}} = \frac{1}{3}mL^2.$$

The moment of inertia of a cylinder of mass M , length l and diameter d about the centre of mass is given by

$$I_{CM} = \frac{1}{48}M(3d^2 + 4l^2).$$

Assuming that the vaulter's model as a cylinder is mostly determined by the head and torso, we can take $d \approx 0.29\text{m}$ and $l \approx 0.92\text{m}$ for a 1.80m tall person [10]. Then the first term is small compared to the second one (7% contribution to the total value) and the formula can be approximated by:

$$I_{CM} \approx \frac{1}{12}Ml^2,$$

which is actually the formula for the moment of inertia of a thin rigid rod of mass M and length l about the centre of mass. The parallel axis theorem gives the moment of inertia at a pole height r :

$$I_{\text{vaulter}} = \frac{1}{12}Ml^2 + Mr^2 = M\left(\frac{1}{12}l^2 + r^2\right)$$

Knowing that the pole height r is 10 metres or more for a professional vaulter, it is clear that we can approximate the moment of inertia of the vaulter by

$$I_{\text{vaulter}} \approx Mr^2.$$

In other words, we could have modelled the vaulter as a point mass, but this conclusion follows from our reasoning during the modelling process and could not so easily have been made in advance.

Combining the formulas, we get the following formula for the moment of inertia I of the vaulter-pole system:

$$I = \frac{1}{3}mL^2 + Mr^2.$$

The derivative of I is given by

$$I' = 2Mr \cdot r' = 2Mr u,$$

provided that $\frac{1}{2}\pi - \beta \leq \theta \leq \frac{1}{2}\pi + \beta$, $r \leq L$, and u is the climbing speed of the vaulter.

So, the derived equation of motion is the following boundary problem:

$$I' \cdot \theta' + I \cdot \theta'' = -\left(Mr + \frac{1}{2}mL\right)g \cos \theta,$$

$$\theta(0) = \theta_0, \theta'(0) = \omega_0,$$

where

$$I = \frac{1}{3}mL^2 + Mr^2, h(0) = h_0$$

and

$$I' = \begin{cases} 2Mr u & \text{when } \frac{1}{2}\pi - \beta \leq \theta \leq \frac{1}{2}\pi + \beta, r \leq L \\ 0 & \text{otherwise} \end{cases}$$

3. Computer simulation of the motion of the vaulter-pole system

The equation of motion of the vaulter-pole system derived in subsection 2.3 cannot be solved analytically. But there are alternatives: one can implement the forward Euler solution method in a spreadsheet program, solve the differential equation numerically in a mathematical software system, or use more specifically a graphical, system dynamics-based modelling environment. We have chosen the third option, using the graphical modelling tool of COACH [10, 11], because this is one of the most convenient options at a secondary school level.

In a graphical model, variables, parameters, and relationships between them are represented by means of a system of icons in a diagrammatic picture. The system of differential equations

$$\begin{cases} \theta' = \omega \\ \omega' = \alpha \end{cases}$$

where the angular acceleration α is equal to $(\tau - I'\omega)/I$, is, for example, represented in a graphical model by the combination of icons shown in figure 5.

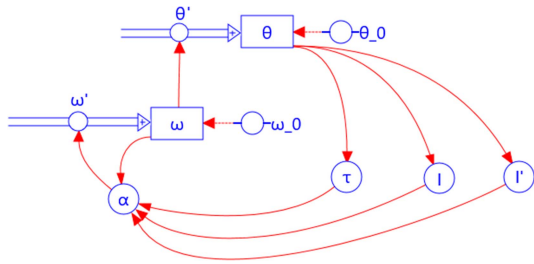


Figure 5. The skeleton of the graphical representation of the system of differential equations that describes the motion of the vaulter-pole system.

The direct relations are not directly visible in figure 5: the connectors (thin arrows) indicate the variables on which a quantity depends, but the modeller has to enter the formula explicitly via a formula editor. For example, the arrow from the rectangle ω to the double arrow θ' represents the identity $\theta' = \omega$ and the arrow from the auxiliary variable α to the double arrow ω' represents the identity $\omega' = \alpha$. The arrows incoming at the auxiliary variable α indicate that α depends on the variable ω, τ, I , and I' . In fact, $\alpha = (\tau - I'\omega) / I$.

A major advantage of a graphical model is that it provides a clear overview of the main structure of the mathematical model. Further explanation of graphical modelling and discussion of its use in education is beyond the scope of this paper. The interested reader is referred to other publications [11–14].

The complete graphical model of the vaulter-pole system that includes all parameters is shown in figure 6. It can be used in computer simulations to explore, for example, how the motion depends on the initial angular speed ω_0 for a given jump at a given initial pole height r_0 , how climbing of the vaulter affects this motion, and whether the vaulter has enough time to climb to the top of the pole for a given initial angular speed, initial pole height, and climbing velocity u . In all computer simulations, we assume a pole of maximum length $L = 13.25\text{ m}$ and weight $m = 20\text{ kg}$, and a vaulter of weight $M = 75\text{ kg}$ jumping at the initial pole height $r_0 = 8\text{ m}$.

We first use the computer model to experimentally find a minimum initial angular speed ω_0 needed to reach the vertical orientation for a jump at pole height r_0 without climbing, i.e. $u = 0$, and with the pole initially at the angle $\theta_0 = 50^\circ$.

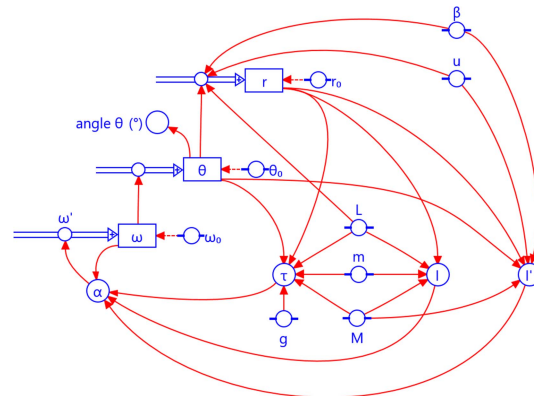


Figure 6. The complete graphical model of the vaulter-pole system used in computer simulations.

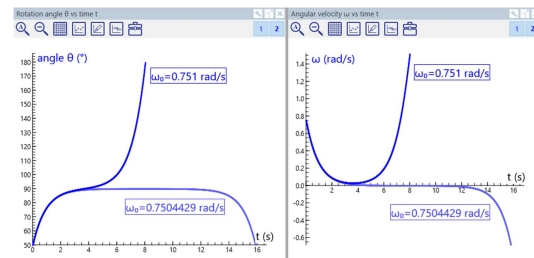


Figure 7. Model results for two nearby values of the initial angular speed ω_0 , one leading to rotating backwards and one for a successful tipping over.

It turns out that $\omega_0 = 0.7504429\text{ rad s}^{-1}$ leads to a vaulter-pole system that stays long in a vertical position and eventually does not tip over, but rotates backwards (see figure 7). Of course, we are curious how this result compares with the earlier computed speed of the vaulter perpendicular to the pole of $v_{\min} \approx 6.7\text{ m s}^{-1}$. The law of conservation of energy allows us to relate these two quantities: the kinetic energy of the vaulter $\frac{1}{2}Mv^2$ is converted completely into rotational energy of the vaulter-pole system $\frac{1}{2}I\omega^2$, where $I = \frac{1}{3}mL^2 + Mr^2$ is the moment of inertia of the system. Simple algebra leads to the following relation

$$v_{\min} = \omega_0 \cdot \sqrt{r_0^2 + \frac{1}{3} \left(\frac{m}{M}\right) L^2}$$

Plugging in the values of the parameters leads to

$$v_{\min} \approx 8.92\omega_0.$$

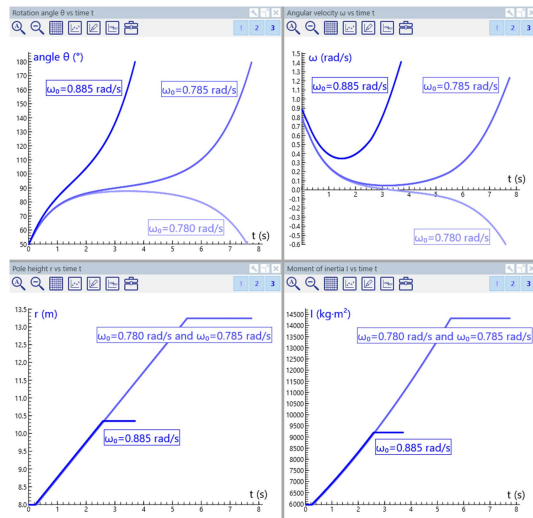


Figure 8. Model results for three values of the initial angular speed ω_0 : one leads to a backward rotation and two other values are high enough for a successful tipping over. One of the latter values is too high so that the vaulter does not have enough time to climb to the top of pole.

The speed of the vaulter computed from the initial angular speed by this formula turns out to be in perfect agreement with the result derived in section 2.2. A slightly higher initial angular speed of $\omega_0 = 0.751 \text{ rad s}^{-1}$ leads to a jump in which the vaulter-pole system tips over; see figure 7.

In the next experiment, we allow the vaulter to climb in the direction of the top of the pole with speed $u = 1 \text{ m s}^{-1}$ as long as the pole angle is between 60° and 120° , i.e. we take in the model $\beta = 30^\circ = \frac{1}{6}\pi$. In figure 8, the time-profiles of the pole angle θ (in degrees) are shown, the angular speed ω , the pole height r , and the moment of inertia I for varying values of the initial angular speed ω_0 . When $\omega_0 = 0.78 \text{ rad s}^{-1}$, climbing to the top of the pole results in this case in rotating backwards because the moment of inertia increases so fast that the torsion stops the forward motion of the vaulter-pole system and lets it rotate backwards. A small increase in initial angular speed, i.e. taking $\omega_0 = 0.785 \text{ rad s}^{-1}$, suffices to have a successful tipping over. When the initial angular speed ω_0 is too high, say $\omega_0 = 0.885 \text{ rad s}^{-1}$, then the vaulter does not have enough time to climb all the way to the top of the pole; he can

only climb a distance of 2.36 m. This shows how important an accurate initial angular speed is for a successful and long distance jump and how small the margin of error in angular speed actually is.

4. Extension of the model: inclusion of the release of the vaulter and computation of the jumping distance

In the end, it is the jumping distance that counts in a fierljeep event. For this purpose, we extend the previous mathematical model for a fierljeep accommodation that has been designed for attaining long jumping distances and the longest pole. As was noted in section 2.1, this means that we have a runway at height 4 m above the gravel bank, a sand bed at height 1.75 m above the gravel bank, and the lowest allowed water level and that we assume that the longest pole of size 13.25 m is used. The jumping distance depends on many variables, but in order to be able to compute it through some mathematical model, we first need to determine at what horizontal distance from the end point of the runway the pole is placed in the gravel bank. Hereafter, we can extend the model of the rotating vaulter-pole system described in the previous section that computes the distance covered after the vaulter releases the pole to land in the sand bed.

4.1. Horizontal distance of the bottom end of the pole

In section 2.1, we have explained by example how a vaulter places the pole in the gravel bank: (s)he places the pole such that (s)he touches the pole at a predetermined pole height h_0 when standing upright with stretched out arms at height l_0 . We can determine a formula for the horizontal distance of the bottom end of the pole depending on l_0 and h_0 . For this, we only need to apply basic geometry and algebra.

Figure 9 illustrates the initial positioning of the pole at distance d_{pole} , before the pole holder pushes it away from the end point of the runway with platform height h_p . We want to express d_{pole} in l_0 and h_0 .

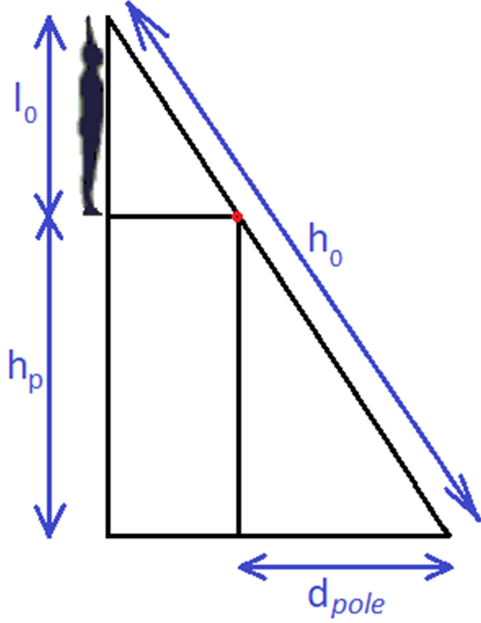


Figure 9. Geometrical representation of the initial placement of the pole.

The length of the hypotenuse of the lower-right rectangular triangle in figure 9 can be computed via Pythagoras' theorem and is equal to $\sqrt{d_{\text{pole}}^2 + h_p^2}$. This triangle is similar to the large rectangular triangle with the vertical side of length $l_0 + h_p$ and hypotenuse of length h_0 . Therefore we have the following equation of ratios:

$$\frac{\sqrt{d_{\text{pole}}^2 + h_p^2}}{h_p} = \frac{h_0}{l_0 + h_p}.$$

Simple algebraic manipulation leads to:

$$d_{\text{pole}} = h_p \cdot \sqrt{\frac{h_0^2}{(l_0 + h_p)^2} - 1}.$$

4.2. Motion after releasing the pole

Just as we have done in section 2.2, where we looked at a special case of the dynamics of the rotating vaulter-pole system, we first explore a special case to come to grips with motion after the vaulter releases the pole. Hereafter, we extend the graphical model of section 3 to include the release

of the vaulter and the calculation of the jumping distance.

4.2.1. Release when a vaulter passes the tipping point of the vaulter-pole system at maximum pole height. We consider the special case in which a vaulter performs such that (s)he passes the tipping point of the vaulter-pole system with an angular speed $\omega_{\frac{1}{2}\pi}$ at maximum height L , and releases the pole at angle φ , while pushing off against the pole with an impulse J . In this special case, we apply basic physics principles and simple mathematics to compute the distance covered by the vaulter after passing the tipping point under the assumption that friction does not play any role.

First, we apply the law of conservation of energy for the motion of vaulter-pole system to determine the angular speed ω_φ at some release angle $\frac{1}{2}\pi$. When the vaulter-pole system passes the tipping point, the total energy is the sum of the gravitational potential energy (set zero at the bottom end of the pole) and the rotational energy of the system:

$$\frac{1}{2}mgL + MgL + \frac{1}{2}I\omega_{\frac{1}{2}\pi}^2,$$

where

$$I = I_{\text{pole}} + I_{\text{vaulter}} = \frac{1}{3}mL^2 + ML^2.$$

The vaulter releases the pole at angle φ and thus at height $h = L \sin \varphi$. At that moment, the angular speed ω_φ has been reached and we can determine the total energy at release:

$$\frac{1}{2}mgh + Mgh + \frac{1}{2}I\omega_\varphi^2$$

Equality of both total energies leads to an equation in which the angular speed ω_φ can be isolated via simple algebra as

$$\omega_\varphi = \sqrt{\omega_{\frac{1}{2}\pi}^2 + \frac{gL(m+2M)(1-\sin\varphi)}{I}}$$

Once we know the angular speed ω_φ at release, we can also determine the horizontal and vertical components (v_x and v_y , respectively) of the vaulter's speed at release:

$$v_x = \omega_\varphi L \sin \varphi - \frac{J}{M} \cos \varphi$$

and

$$v_y = \omega_\varphi L \cos \varphi + \frac{J}{M} \sin \varphi.$$

The second term in the horizontal and vertical speed is the contribution of the impulse J of pushing-off against the pole at release. We will use the value of 120 Ns, reported in [7] for the impulse of a professional vaulter, in calculations. This means that a vaulter weighing 75 kg and pushing off delivers an extra radial velocity of 1.6 m s^{-1} .

The vertical speed v_y at release can be used to compute the time t_{fall} needed for the vaulter to land at the sand bed at height h_{sb} on the basis of free fall formulas:

$$h_{\text{sb}} = -\frac{1}{2}g \cdot t_{\text{fall}}^2 + v_y \cdot t_{\text{fall}} + h,$$

which can be rewritten as quadratic equation

$$\frac{1}{2}g \cdot t_{\text{fall}}^2 - v_y \cdot t_{\text{fall}} - (h - h_{\text{sb}}) = 0.$$

Then the quadratic formula leads to the following t_{fall} :

$$t_{\text{fall}} = \frac{v_y + \sqrt{v_y^2 + 2g(h - h_{\text{sb}})}}{g},$$

where all variables on the right-hand side can be determined from earlier formulas.

Knowing the time t_{fall} between the release and landing of the vaulter, the horizontal distance covered during this part of the jump can be easily computed as $v_x \cdot t_{\text{fall}}$. Adding it all up, we can write down a formula for the jumping distance jump:

$$d_{\text{jump}} = d_{\text{pole}} - L \cos \varphi + v_x \cdot t_{\text{fall}},$$

where the horizontal distance of the bottom end of the pole has already been determined in section 4.1 as

$$d_{\text{pole}} = h_p \cdot \sqrt{\frac{h_0^2}{(l_0 + h_p)^2} - 1}.$$

The second term is the horizontal distance covered by the vaulter-pole system when the pole angle increases from 90° to the release angle φ , and the third term is the contribution to the jumping

distance of the motion of the jumper after release. For the latter part of the formula of the jumping distance, we can use all of the mathematical formulas derived in this section.

We can use the formulas to explore the effect of the release angle on the jumping distance. We do this by example and assume a platform at height $h_p = 4 \text{ m}$; a sand bed at height $h_{\text{sb}} = 1.75 \text{ m}$; a pole of length $L = 13.25 \text{ m}$ and mass $m = 20 \text{ kg}$; a vaulter with mass $M = 75 \text{ kg}$ and vertical reach $l_0 = 2.24 \text{ m}$, choosing a predetermined pole height $h_0 = 11.25 \text{ m}$ for pole placement, and an initial pole height $r_0 = 8 \text{ m}$, and who delivers an impulse $J = 120 \text{ Ns}$ when pushing-off against the pole at release; the acceleration of gravity $g = 9.81 \text{ ms}^{-2}$; and an angular speed when the vaulter-pole system tips over $\omega_{\frac{1}{2}\pi} = 0.01 \text{ rad s}^{-1}$. For every admissible release angle, say between 90 and 170° , we can now compute the jumping distance.

The red graph in figure 10 has been computed in case the vaulter releases the pole without pushing off ($J = 0 \text{ Ns}$). The graph shows that releasing the pole at maximum height at a release angle of 90° actually makes no sense as you land in the water at a distance of 6.2 m. When the vaulter sticks to the pole and releases it when (s)he is close to the sand bed, say at a release angle of 170° , then the jumping distance is already 19.15 m. The optimal release angle is in this example equal to 137.7° and it leads to the maximum jumping distance of 20.18 m. Note that the release angle is greater than 135° , which means that the vaulter is advised to release somewhat later than halfway.

The blue graph in figure 10 has been computed in case the vaulter releases the pole pushing off with maximum impulse ($J = 120 \text{ Ns}$). The graph shows that releasing the pole at maximum height at a release angle of 90° leads to a computed distance almost the same as before, namely 6.23 m. When the vaulter sticks to the pole and releases it at the latest moment, say at a release angle of 170° , then the jumping distance is again almost the same as before, namely 19.23 m. The optimal release angle is in this example equal to 133.7° and it leads to the maximum jumping distance of 21.52 m, which is substantially larger than before. Note that, contrary to the earlier case with the red graph, the release angle is less than 135° , which means that the vaulter is advised to

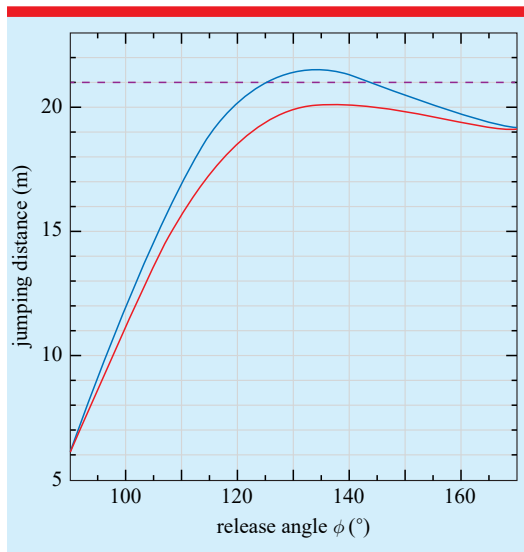


Figure 10. Graphs of jumping distance versus release angle φ when the vaulter does not push off at release (red graph) and when the vaulter does push off at release (blue graph). In the latter case, the optimal release angle is 133.7° and leads to a distance of 21.51 m. The orange dashed line indicates that all jumps are further than 21 m for a release angles between 125.0° and 144.7° .

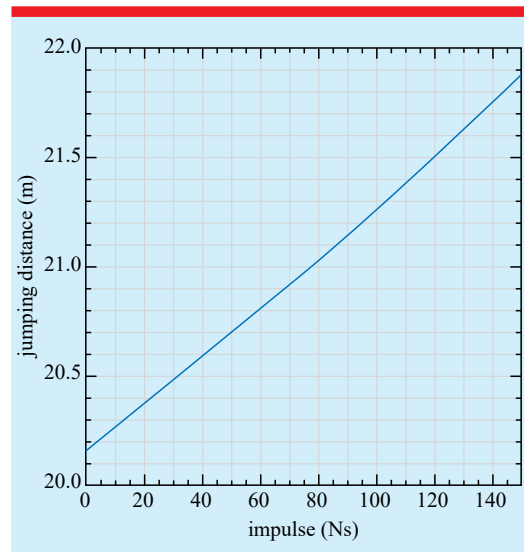


Figure 11. Graph of jumping distance versus impulse of the vaulter pushing off at release angle 135° .

release somewhat earlier than halfway. The margin of error in the release angle when a jumping distance of 21 m is strived for is interesting. In this case, the release angle must be between 125.0° and 144.7° , as indicated in figure 10 by the orange dashed line. This large margin of error exists because the jumping distance performance benefits from releasing at a high point, as well as from sticking to the pole and increasing the angular speed.

Another exploration of the problem situation is the effect of the impulse of pushing off the pole at release on the jumping distance. We use the above conditions with release angle $\varphi = 135^\circ$ and vary the impulse. Figure 11 shows the model results and it is clear that investing in muscle power of the arms to push off harder certainly pays off. A very weak impulse of 20Ns or a strong impulse of 121 Ns makes a difference in jumping distance of 1.10m.

4.2.2. Extension of the computer model. When you watch some video clips of fierljepp jumps, you will quickly notice that the assumptions made in

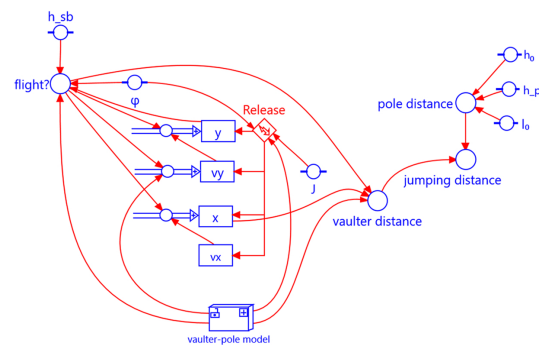


Figure 12. The graphical model of the motion of a vaulter from the moment that (s)he grabs the pole and gets the pole between the legs until the landing in the sand bed.

the section 4.2.1 are hardly ever met: vaulters in most jumps do not have enough time to climb to the top of the pole before the tipping point is reached and therefore they continue climbing towards the top of the pile. How interesting the theoretical interlude of section 4.2.1 may be—it shows what can be achieved by basic concepts of physics and mathematics and gives some insight in the problem situation—we cannot avoid extending our computer model of the rotating vaulter-pole system described in section 3. But in a modern graphical modelling environment such

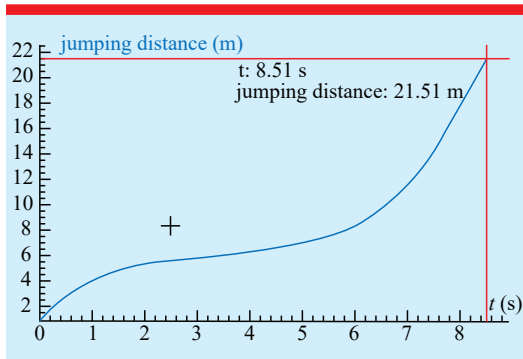


Figure 13. The distance-time graph of the fierljeep jump from the moment that the vaulter grabs the pole and gets the pole between their legs until the landing in the sand bed.

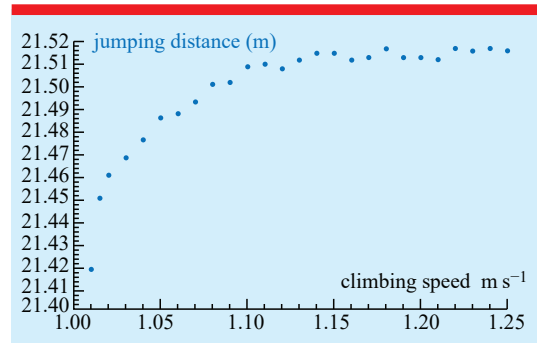


Figure 14. The distance-climbing speed graph of the fierljeep jump for climbing speeds fast enough to bring the vaulter to the top of the pole and small enough to let the vaulter-pole system still pass the tipping point for the given initial angular speed.

as Coach [10, 11], this turns out to be surprisingly easy.

In essence, all we have to do is to construct a graphical model of a frictionless flight starting at height $h = r \sin \varphi$ with an initial horizontal speed $v_x = \omega_\varphi r \sin \varphi - J/M \cos \varphi$ and with an initial vertical speed $v_y = \omega_\varphi r \cos \varphi + J/M \sin \varphi$, where r and ω_φ are the pole height and the angular speed at release angle φ , respectively. The values of r and ω_φ are computed by the already constructed pole-vaulter model. Then we simulate the extended model until the y -coordinate of the vaulter in free fall reaches the height of the sand bed. The x -coordinate is at that moment the horizontal distance covered by the vaulter during the free fall. Knowing the horizontal distance of the bottom of the pole and the pole height of the vaulter at all angles, including the release angle, we can compute the jumping distance at any stage of the motion.

The extended graphical model is shown in figure 12. It contains new graphical elements. The vaulter-pole model box contains the subsystem that models the motion when the vaulter is in contact with the pole and is in fact the graphical model shown in figure 6. In teaching and learning modelling, the use of subsystems is convenient when students are step by step introduced to the details of a model or when they gradually extend themselves a model while trying to keep a good overview of the whole system. This is what we do here too: using the subsystem, we can focus on the

graphical model for the free fall of the vaulter after release.

The second new graphical element is the event icon (the thunderbolt icon labelled ‘Release’). At the discrete time event when the pole angle becomes greater than the release angle, the variables connected with the free fall motion of the vaulter (horizontal and vertical positions and velocities) are instantly updated to their initial values for the free fall. For example, the variables x and v_x are instantly changed from 0 to $-r \cos \varphi$ and $\omega_\varphi r \sin \varphi - J/M \cos \varphi$, respectively. These are once-only actions, after which the programmed dynamics of the free fall takes over. The flight phase is indicated in the model by the Boolean variable ‘flight?’, with a value equal to the evaluation of the expression $(\theta > \varphi) \wedge (y > y_{sb})$. During the flight phase, the change in the vertical velocity, i.e. the vertical acceleration, is equal to the negative of the acceleration of gravity. With no air friction, the horizontal velocity is constant.

During flight, the distance covered by the vaulter is equal to the value of the variable x . While the vaulter is in touch with the pole, the horizontal distance covered by the vaulter with respect to the horizontal position of the bottom of the pole is equal to the value of the expression $-r \cos \theta$, where r and θ are the pole height and pole angle, respectively, during the vaulter’s contact with the pole. In order to compute the jumping distance during the motion, we only need to implement the formula derived for the horizontal distance of the bottom of the pole.

This computer model can be used to simulate a jump. In figure 13 the distance-time graph for the following parameter choices is shown: initial angular speed $\omega_0 = 0.79$ rad/s, climbing speed $u = 1.2$ m s⁻¹, angle $\beta = 30^\circ = \frac{1}{6}\pi$ rad, and release angle $\varphi = 133^\circ = \frac{133}{180}\pi$ rad. All other parameters have values chosen before. In this case, the pole vaulter reaches the top of the pole just after it has passed the tipping point ($\varphi = 93^\circ$). We can read off the final jumping distance of 21.51 m, which is very close to the value computed in the simpler model of section 4.2.1.

The extended model offers a larger space for experimentation with jumping conditions. For example, we can explore the effect of climbing speed while all other conditions are kept the same as in the computer simulation discussed above. When the climbing speed is $u = 1.01$ m s⁻¹, the vaulter just misses the top of the pole for 2 cm, but the jumping distance is 10 cm less than the longest jump for climbing speeds between 1.22 and 1.25 m s⁻¹. When the climbing speed is 1.26 m s⁻¹ or more, the vaulter-pole system does not pass the tipping point and the jump is unsuccessful. In figure 14, the computed jumping distance has been plotted against the climbing speed for which the vaulter reaches the top of the pole. From the graph, it is clear that the vaulter better climbs as fast as possible to the top of the pole under the restriction that the climbing speed does not exceed some value at which the vaulter-pole system does not tip over anymore. Once the climbing speed passes some threshold (here around 1.1 m s⁻¹), the jumping distance remains almost the same.

5. Discussion

In this article, we carried out a biomechanical analysis of vaulting for distance. To this end, we have first reduced the problem situation of a fierljepp jump to a manageable problem that can be studied via fundamental concepts of physics and mathematics. Some of our models provided explicit formulas for jumping distance, while others gave us an equation of motion to be solved numerically via some tool, in our case the graphical system dynamics-based modelling tool of the Coach environment for mathematics and science. How interesting the biomechanical analysis

and the obtained results are—it is fair to ask how relevant the presented work is to physics education, in particular at secondary level, and whether it is not too difficult for secondary physics students.

The answer to the first question depends on the vision on physics education. Internationally, models and modelling occupy an important place in physics curricula programs because of the increasing importance of models and modelling in science and technology. Modelling as competence encompasses both the cognitive component of thinking in models and the practical skill of modelling. Computer modelling, an important ingredient of scientific research, is in this vision a concrete form of applying a modelling approach to problems and should be part of physics curricula (and, in fact, is a mandatory part of the Dutch physics curriculum since 2016). A GIREP conference was dedicated entirely to modelling in physics and physics education [15].

In this paper, we have illustrated that pole vaulting for distance is a subject that offers opportunities to do modelling and use models for understanding of a sports motion. We have gone through various phases of a modelling cycle and have constructed more than one model to explore (part of) the problem situation. All models were based on fundamental physics concepts such as gravity, Newton's laws of motion, conservation of energy, impulse, and angular kinetics. Regarding these aspects, the biomechanical analysis serves students as an interesting example of how physics helps understand complex motions. Hopefully it makes them realize that the modelling process, the underlying thinking processes, and the discussions with fellow students during their work are as important as the obtained results. Yet it is gratifying when experiment, observation, model and theory are in agreement, as is the case in our study of pole vaulting for distance.

Sport remains an interesting topic for students as a subject of individual or team work. The combination of sports and physics offers several attractive ingredients for teaching and learning physics at all levels of education, ranging from primary to university level. These cover topics like sports activities as experiments,

video analysis, and modelling. Literature about the combination of physics and sports is vast: see for example [16] and references herein, and the many books entitled ‘Physics and ...’. Do a literature search on IOPscience (<https://iopscience.iop.org>) applying the filter ‘Physics Education’ and using the keyword ‘sport’: you will find beautiful examples. The authors of this paper have also written several papers on this subject [17–21].

Let us linger upon the difficulty level for secondary physics students. We believe that in particular the energy exchanges between vaulter and pole are challenging but not too complicated for students. We are not alone: the Dutch national physics examination at lower vocational level in 2018 contains the following task about a fierljepp jump. Assuming that the vaulter with a mass of 72 kg climbs a distance of 240 cm to the top of the pole while this pole is in vertical position, calculate the minimum work done by the vaulter. Other questions had to do with the landing phase when the vaulter bends the knees. For example, in a multiple choice question, students were asked with what safety measure in a car the function of bending the knees at landing corresponds: the head rest, the cage, or the crushable zone?

Models and modelling also offer great opportunities in computer exams [22]. Given a computer model and running simulations, it is important that students are able to assess qualitatively whether the applied modelling approximations are justified or yield an oversimplification of the problem. Students can be asked to use models to answer specific questions, to complete the model by specifying extra lines of code, or to extend a computer model by applying their physics knowledge and extra graphical modelling. In our case of pole vaulting for distance, students could be asked to give boundary or stop conditions at different stages of the pole vault, or to extend one of the presented models so that the effect of head or tail wind on the jumping performance is included.

But what is maybe most important and should not be forgotten is that combining physics and sports is fun for many students. It often has a positive effect on their motivation because they can act as practitioners in the field of sports biomechanics in rather authentic activities. Physics teachers can

join in at these activities in several ways: as a source of physics knowledge, as a guide to students, as a project manager, as a client with a sports-related question, and as a participant. These might be welcome changes in the traditional roles of a physics teacher.

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