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# Most, but not more than half, is proportion-dependent and sensitive to individual differences ${ }^{1}$ 

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#### Abstract

In this study we test individual differences in the meaning representations of two natural language quantifiers - most and more than half - in a novel, purely linguistic task. We operationalized differences in meaning representations as differences in individual thresholds which were estimated using logistic regression. We show that the representation of most varies across subjects and its verification depends on proportion. Moreover, the choice of the representation of most affects the verification process. These effects are not present for more than half. The study demonstrates the cognitive differences between most and more than half and individual variation in meaning representations.


Keywords: generalized quantifiers, most, more than half, individual thresholds, meaning representations, verification strategies.

## 1. Introduction

Imagine that there are three candidates in an upcoming election: candidates A, B and C. To win the election the candidate needs most of the votes. How would you check if the sentence "Most of the people voted for candidate A" is true? You can represent this sentence in many ways. You can, for example, think that the number of people who voted for candidate A is greater than half of all of the votes or alternatively that the number of people who voted for candidate A is greater than the number of people who cast for their votes on other candidates. In the literature (e.g. Lidz, Pietroski, Halberda, \& Hunter, 2011; Pietroski, Lidz, Hunter, \& Halberda, 2009; Tomaszewicz, 2013) there are several proposals how the meaning of most can be represented:
(1) Representations of most
a. $\operatorname{most}($ votes in election, votes on A$) \Leftrightarrow \mid$ votes on $\mathrm{A}|>1 / 2|$ votes in election
b. $\operatorname{most}($ votes in election, votes on A$) \Leftrightarrow \mid$ votes on $\mathrm{A}|>|$ votes on not-A $\mid$
c. $\operatorname{most}($ votes in election, votes on A$) \Leftrightarrow$ OneToOnePlus(votes on A, votes on not-A)
d. most $($ votes in election, votes on A$) \Leftrightarrow \mid$ votes on $\mathrm{A}|>|$ votes on $\mathrm{B}|+|$ votes on $\mathrm{C} \mid$
e. $\operatorname{most}($ votes in election, votes on A$) \Leftrightarrow \mid$ votes on $\mathrm{A}|>|$ votes in election $|-|$ votes on $\mathrm{A} \mid$

[^0]For example, if you represent the meaning of most as in (1c), you have to pair each vote cast for candidate A both with votes for either candidate B or C. If you find at least one vote for candidate A left unpaired, then candidate A will win.

According to the Interface Transparency Thesis (Lidz et al, 2011: 233): "The verification procedures employed in understanding a declarative sentence are biased towards algorithms that directly compute the relations and operations expressed by the semantic representation of that sentence". Although some studies (Pietroski et al., 2009; Lidz et al., 2011) support the Interface Transparency Thesis, there is also evidence that people might prefer different verification strategies (Steinert-Threlkeld, Munneke, \& Szymanik, 2015; Talmina, Kochari, \& Szymanik, 2017; see for overview: Szymanik, 2016). In this paper, we will present findings demonstrating that there are individual differences in the representation of natural language quantifiers (expressions like: most, more than half, fewer than half, many, few, some, all, at least). We will show evidence for individual differences in meaning representations with a special focus on two natural language quantifiers: most and more than half.

### 1.1. Most and more than half

Most and more than half are examples of truth-conditionally equivalent quantifiers, that differ in many other aspects, e.g., they seem to trigger different verification strategies (Hackl, 2009) and have different pragmatic associations (Solt, 2016). Generalized Quantifier Theory (GQT, Mostowski, 1957; Barwise \& Cooper, 1981; Peters \& Westerståhl, 2008; Szymanik, 2016) is not able to distinguish between expressions that are logically equivalent, but generate different linguistic intuitions.

According to Hackl's (2009) linguistic analysis, more than half is a comparative expression, while most is the superlative form of many (i.e. MANY+EST). However, under this analysis most also satisfies proportional truth-conditions. In contrast, the opposite quantifier to most fewest - has only a superlative reading. The lack of proportional reading for fewest cannot be explained on the grounds of GQT but falls out naturally from Hackl's analysis.
According to Hackl (2009) the linguistic differences between most and more than half are reflected in different basic logical representations of these quantifiers.
(2) Logical representations of most and more than half
a. $\quad \operatorname{most}(\mathrm{A}, \mathrm{B}) \Leftrightarrow|\mathrm{A} \cap \mathrm{B}|>|\mathrm{A}-\mathrm{B}|$
b. more than half $(\mathrm{A}, \mathrm{B}) \Leftrightarrow|\mathrm{A} \cap \mathrm{B}|>1 / 2|\mathrm{~A}|$

Although both logical forms satisfy the same truth-conditions and thus are indistinguishable from the perspective of GQT, they may trigger different cognitive verification strategies. The verification of more than half requires the comparison of cardinality of the target set $(|\mathrm{A} \cap \mathrm{B}|)$ to half of the size of $\mathrm{A}(1 / 2|\mathrm{~A}|)$, while the verification of most requires comparison between the cardinality of the target set $(|A \cap B|)$ and the cardinality of the complement set $(|A-B|)^{2}$.

[^1]Hackl (2009) supported his linguistic analysis with experimental data. Using a novel paradigm - Self Paced Counting - he argued that most is verified using a vote-counting strategy. In this experiment, he did not find a difference in overall reaction times and accuracy between most and more than half. Hackl (2009) argued that this lack of differences is evidence that participants treated most and more than half as equivalent expressions. To summarize, Hackl (2009) argued that most and more than half are verified using different strategies, but that these two quantifiers are truth-conditionally equivalent and therefore they are both true above $50 \%$ proportion and false below.

In contrast to Hackl's (2009) findings, other studies (Talmina et al., 2017; Kotek, Sudo, \& Hackl, 2015) showed that participants might not treat most and more than half as equivalent quantifiers. Firstly, in a replication study, Talmina et al. (2017) found that more than half is verified slower than most. This finding questions Hackl's (2009) argument that participants treated most and more than half as equivalent quantifiers. Moreover, Talmina et al. (2017) suggested that subjects might have used various verification strategies for both quantifiers. Talmina et al.'s (2017) findings suggest a more complex picture, showing that people might differ in their representation of quantifiers.

Secondly, Kotek et al. (2015) support the hypothesis that most and more than half have different meaning representations. They found a difference between most and more than half in terms of their sensitivity to proportion. While more than half was judged equally likely as false for proportions below $50 \%$ and true for proportions above $50 \%$, most exhibited an asymmetry. It was judged true for proportions above $50 \%$ less often than more than half. Kotek et al. (2015) concluded that the asymmetry between most and more than half for proportions above $50 \%$ might be explained by pragmatic associations of these quantifiers (Solt, 2016).

In particular, Solt (2016) explained the differences between most and more than half in terms of their scale structure requirements. More than half requires precise comparison, which is only possible on a ratio scale. Most has lower scale requirements and can be verified on a semi-ordered scale. On a semi-ordered scale, one of the two proportions to be compared is greater than another, when it is greater by some value. The semi-ordered scale allows only for imprecise, approximate comparisons. As a consequence, most has a preferred interpretation of "significantly greater than more than half".

The differences in required scale structure for most and more than half are reflected in their pragmatics (Solt, 2016). Solt (2016) found, in corpus data, that most is used with higher proportions or in the context, in which the precise comparison is not possible. More than half, in turn, expresses proportions slightly above $50 \%$ and occurs in the context, in which the precise data are available. Although Solt (2016) found clear differences in usage of most and more than half, she used corpus data that does not provide evidence for differences in processing and verification of these quantifiers. Therefore, based on her findings, it is not possible to know whether the differences between most and more than half should be attributed to semantics or rather to the pragmatics of these quantifiers.

Solt's (2016) claim that most has a strong "significantly more than half" interpretation was supported by other studies (Ariel, 2003; Pezzelle, Bernardi, \& Piazza, 2018). For example, Ariel (2003) found a similar pragmatic tendency to use most with the higher proportions than more than half in a questionnaire study. Moreover, she argued that most and more than half are semantically different. Most is an upper-bounded quantifier, while more than half has no upper-bound. In addition, Pezzelle et al. (2018) investigated the meaning boundaries of several quantifiers, among others most. They asked the subjects to select, from a restricted choice, a quantifier that best describe a given scene. They found that most was used for proportions between $40 \%$ and $100 \%$ with a peak around $70 \%$. Its usage highly overlapped with many, however most was chosen more often. Unfortunately, Pezzelle et al. (2018) have not studied more than half so the direct comparison between these two quantifiers on the selection task is not available.

To summarize, the strong preferences to use most with higher proportions stands in conflict with the treatment of most as a quantifier with a $50 \%$ threshold. It also raises a question if most has only one possible representation - truth-conditionally equivalent to more than half. The existing evidence suggests that there are differences between most and more than half, which might result in the differences in thresholds in these quantifiers. While more than half has a clear threshold, the threshold for most might vary between $50 \%$ and higher proportions. According to the truth conditions, most should have the same threshold as more than half. However, experimental evidence (Kotek et al, 2015) and corpus data (Solt, 2016) suggest that most can also have a higher threshold. The fact that most has two possible interpretations raises the question of whether this quantifier is represented in the same way by all language users. Only a few studies (e.g. Yildirim, Degen, Tanenhaus, \& Jaeger, 2016; Talmina et al, 2017) investigated individual differences in quantifiers. Because quantifiers like most are sensitive to different interpretations, it might be also possible that people differ in how they represent quantifiers.

Thus, the question arises: are the differences between most and more than half outlined above reflected in individual differences in thresholds? Before presenting our methods for answering this question, in the next section we review studies showing that individual differences in natural language are widespread.

### 1.2. Individual differences in natural language

Individual differences in natural language are exhibited in many phenomena related to variation in performance of cognitive functions such as working memory and executive function (Kidd, Donnelly, \& Christiansen, 2018), environmental variables (Kidd et al., 2018) or efficiency in updating predictions (Reuter, Emberson, Romberg, \& Lew-Williams, 2018). They are present in many domains of language processing: morphosyntactic processing (Tanner \& Van Hell, 2014), language production (Barlow, 2013), representation of words in context (Halff, Ortony, \& Anderson, 1976), understanding of grammar constriction (passive voice) and universal quantification (Street \& Dabrowska, 2010), among others. Individual differences are also characteristic for language disorders like dyslexia (Heim et al., 2008) or dysgraphia (Döhla, Willmes, \& Heim, 2018).

In contrast, only a few studies have investigated individual differences in meaning representations. Talmina et al. (2017) found that some people use precise verification strategies for quantifiers, while others use estimation-based strategies. Furthermore, Yildirim et al. (2016) showed individual differences in listeners' expectations about the speaker's interpretation of quantifiers. Speakers can also adjust their representation of the quantifier meaning to the listener (Yildirim et al., 2016) or learn a new representation of a quantifier (Heim et al., 2015). Heim et al. (2015) showed that change in representation of one quantifier adjusts other quantifier representations: for example, a change in the representation of many affects the representation of few.

In addition, studies investigating the scalar implicature some-not all (e.g. Bott, Bailey, \& Grodner, 2012; Spychalska, Kontinen, \& Werning, 2016) show that people can be grouped with regards to their preferences in interpretation of natural language quantifiers into socalled pragmatic or logical responders. The logical responders tend to interpret some according to its semantic, literal meaning: some As are B iff the number of As than are B is greater than zero. This interpretation includes also the possibility that all As are B. The pragmatic responders, in turn, judge sentence some As are B as false if in fact all As are B. This division is also reflected in differences in ERPs N400 and late positivity between two groups of responders (Spychalska et al., 2016).

### 1.3. Current study

The current study tests the effect of individual differences in representations of the quantifiers most and more than half. We operationalized the individual differences in quantifier representation as individual thresholds. We asked participants to verify a sentence with quantifiers based on proportion, given as a percentage. We used quantifiers that intuitively varied in sharpness of their meaning boundaries: more than half, fewer than half, most, many and few. We used proportions given as percentage in order to force a proportional reading for all quantifiers. We formulated the following predictions.

According to GQT, most and more than half are truth-conditionally equivalent and therefore, should have the same threshold: $50 \%$. Moreover, there should be no difference in the interpretation of these quantifiers between participants. In contrast to GQT, previous studies (Solt, 2016; Ariel, 2003; Kotek et al., 2015) showed most has also the "significantly greater than more than half" interpretation and it is dispreferred with proportions around $50 \%$. These findings give a prediction that the threshold for most should be higher than the threshold for more than half. Finally, the number of studies (Yildirim et al., 2016; Talmina et al., 2017) showed that quantifiers, like other natural language expressions, are sensitive to individual differences in representation. We hypothesized that participants might vary in terms of which reading of most they prefer. Therefore, we predicted that:
(H1) Participants will have different representations for most and more than half.
Following Hackl (2009) we assumed that the choice of the verification strategy depends on the cognitive representation of the quantifier. Moreover, according to Solt (2016), most is verified using an imprecise, estimation-based strategy. Pietroski et al. (2009) and Lidz et al.
(2011) showed that the usage of the estimation-based strategy results in proportion-dependent performance. However, they did not contrast the proportion-dependent performance of most with more than half and so we cannot conclude that the effect they found was a consequence of linguistic properties of most rather than the task design. We directly compared the effect of proportion on speed of verification (reaction times) between most and more than half. Following Pietroski et al (2009) we hypothesized that when the proportion is close to $50 \%$ the verification of most should be more difficult. We predicted that:
(H2a) The verification of most, but not more than half, is proportion-dependent.
We also aimed to see if we can capture the effect of variation in representations between participants in their reaction times. We assumed that if participants have different representations of quantifiers, they also use different verification strategies. Therefore, we predicted that:
(H2b) Differences in representation will be reflected in differences in verification speed.

## 2. Methods

### 2.1. Participants

We collected data from 90 subjects. After exclusion criteria were applied, the final sample consisted of 47 male (age: $M=35, S D=11$, range: $22-59$ ) and 24 female participants ( $M=$ $34.5, S D=10$, range: $22-59$ ). 6 female and 18 male participants graduated high school, 6 female and 15 male subjects finished high school education and started college, 12 female and 14 male participants graduated college or obtained higher degree. Each participant received 4 US\$ for participation. The study was a part of the project that received European Research Council and University of Amsterdam, Faculty of Humanities Ethics Committee ethical approvals.

### 2.2. Design

Participants were presented two sentences. The first sentence was of the form "Q of the As are B ", where Q was one of the quantifiers: most, more than half, many, fewer than half, few and As and Bs were pseudowords generated with the Wuggy software (Keuleers \& Brysbaert, 2010) from English 6 letters adjectives and nouns. An English native speaker assessed pseudowords; we excluded them if they were too close to real English words or did not sound like plausible English words. 50 pseudo-adjectives and 50 pseudo-nouns were chosen and randomly paired. We checked frequency (Zipf value) of the original adjectives and nouns in SUBTLEX-US database (van Heuven, Mandera, Keuleers, \& Brysbaert, 2014). The Zipf value of final lists were both 4.06. Each quantifier occurred with each pair of pseudowords only once and in a random order.

The second sentence presented to participants was of the form " $p \%$ of the As are B", where As and Bs were the same pseudowords as in the first sentence and $p \%$ was a randomly
generated proportion form $1 \%$ to $99 \%$, excluding $50 \%$. In the case of most, more than half and fewer than half, the proportions above and below $50 \%$ were counterbalanced within participants. Because most does not have a clear upper boundary (Ariel, 2003) we did not include the proportion $100 \%$.

### 2.3 Procedure

Our experiment was conducted on Amazon Mechanical Turk. Participants had to decide if the first sentence is true based on information from the second sentence. They were presented 250 pairs of sentences, 50 per each quantifier. Firstly, they had to press the arrow down button and keep it pressed as long as they wanted to see the first sentence on the screen. Secondly, they had to press arrow down button again to read the second sentence with proportion. Finally, they had to choose the arrow left or arrow right buttons for true or false response. The response buttons were balanced between-subjects.

Before the proper experiment started, participants practiced the procedure for 8 trials in a training block. In the training block we used the quantifiers some, all, and none in the first sentence. At the end of the experiment participants were asked to provide basic demographic information (e.g., gender, age, education background).

### 2.4. Preprocessing reaction times (RT) data

Before we estimated individual thresholds we excluded reaction times shorter than 300 ms and longer than mean +2 SD for each quantifier and true/false responses separately.

### 2.5. Logistic regression model

In order to estimate participants' individual thresholds we applied logistic regression using R $n l s$ self-starting function (Bates \& Chambers, 1992):
(3)

$$
\mathrm{P}(\mathrm{~T}) \sim \frac{1}{1+e^{\left(p_{0}-p\right) / s}},
$$

with starting values: $\mathrm{p}_{0}=50, \mathrm{~s}=4$ for most, more than half and many, and $\mathrm{p}_{0}=41, \mathrm{~s}=-5$ for few and fewer than half.
$\mathrm{P}(\mathrm{T})$ indicates the probability that a participant provided a "true" response, and $p$ the percentage introduced on every trial. The estimated parameters were $p_{0}-$ participant individual threshold - and $s$ - the steepness of logistic regression curve.

The individual threshold could not be estimated using the $n l s$ function if a participant's "true" and "false" responses did not overlap. In those cases, we computed thresholds as the average of the highest proportion for which a participant responded "false" and lowest proportion for which he or she responded "true" (vice versa for few and fewer than half).

## 3. Results

### 3.1. Excluded participants

We excluded 11 participants, who had $50 \%$ or more responses below 300 ms . Additionally, we ran the glmer function in the R package lemrTest (Kuznetsova, Brockhoff, \& Christensen, 2017) separately for each quantifier, with random slope for each participant. The random slopes indicate whether the probability of response "true" increases or decreases with increasing proportion. We assumed that the random slope for quantifier most, more than half and many should be positive and for fewer than half and few negative. We excluded 6 participants, who did not meet this criterion. Finally, we excluded two participants from further analysis because their estimated threshold was higher than $100 \%$ or lower than $0 \%$.

### 3.2. Individual thresholds

We estimated individual thresholds for each quantifier. Figure 1 presents individual thresholds distributions among quantifiers and summarizes descriptive statistics of thresholds. The mean accuracy for all quantifiers above and below threshold was high: many $95 \%$, most $96 \%$, more than half $97 \%$ both above and below threshold, and few and fewer than half $94 \%$ above threshold and $90 \%$ below threshold. The mean reaction times above thresholds were: many $991.88 \mathrm{~ms}(\mathrm{sd}=384.51)$, most $1025.06 \mathrm{~ms}(\mathrm{sd}=502.67)$, more than half $925.28 \mathrm{~ms}(\mathrm{sd}=342.48)$, few $1081.76(\mathrm{sd}=421.89)$ and fewer than half $1068.96(\mathrm{sd}=$ 374.52). The mean reaction times below thresholds were: many 1097.24 (sd $=421.21$ ), most $1035.28(\mathrm{sd}=434.23)$, more than half $942.25(\mathrm{sd}=306.84)$, few $1181.70(\mathrm{sd}=425.60)$, fewer than half $1172.03(\mathrm{sd}=475.01)$.

We tested if there are differences in mean individual thresholds between quantifiers. We found a significant main effect of threshold ( $F_{4,345}=9.21, p<0.001$ ). After applying Bonferroni correction on the significance level, we found that the mean threshold for few was lower than the threshold for the other quantifiers; the mean threshold for many was lower than for most, more than half and almost significantly lower $(p=0.056)$ than for fewer than half; and the threshold for fewer than half was lower than the threshold for most. Importantly, the mean threshold for most was higher than the threshold for more than half.

A Kolmogorov-Smirnov test revealed that the distribution of thresholds is different for most and more than half $(D=0.30 ; p<0.01)$.

Taken together the results show that most has a higher threshold than more than half, but also that participants differ in their representation of most. While in the case of more than half almost all participants had a threshold of $50 \%$, in the case of most some participants had a threshold between 50-70\%.


Figure 1: Histograms present individual thresholds distribution in each quantifier. The dashed lines indicate $50 \%$, the solid lines indicate mean individual threshold. The mean threshold for many is $44 \%(\mathrm{sd}=10)$, few $39 \%(\mathrm{sd}=8)$, fewer than half $(\mathrm{FTH}) 48 \%(\mathrm{sd}=7)$, most $53 \%$ (sd $=6)$, more than half $(\mathrm{MTH}) 50 \%(\mathrm{sd}=4)$.

### 3.3. Proportion effect on reaction times

To understand the effect of proportion on reaction times, we re-coded all responses relative to individuals' thresholds. We coded "true" responses that are above the individual threshold and "false" responses that are below the threshold as correct responses. We ran a mixed effect regression model (R package lmerTest; Kuznetsova et al., 2017) with reaction times as dependent variable and quantifiers (most, more than half), proportion ( $z$-scored) and responses (true/false) and their interactions as predictors. Firstly, we tested the random effects structure. Following Barr, Levy, Scheepers, \& Tily (2013), we tried to keep the random structure maximal until the model converged. We used the best path forward algorithm and included random slopes that significantly improved the model (tested by anova function in R; see Appendix A). If two random slopes were significant, we included the one that had lower $p$-value. To this model we included by-subject random intercept and bysubject random slope for proportion. We used more than half as baseline.

Secondly, we tested the significance of the fixed effects. Table 1 and Figure 2 summarize the effects. Here we focus on the most important one. We found no significant effect of proportion ( $\beta=-22.16 ; t=-0.96 ; p=0.34$ ), but a significant quantifier-proportion interaction ( $\beta=-133.18 ; t=-3.99 ; p<0.001$ ), meaning that the proportion had greater effect on RTs in case of most than more than half. Additionally, we found a significant main effect of quantifier ( $\beta=216.82 ; t=6.43 ; p<0.001$ ).

This finding shows that, in contrast to more than half, the verification of most is dependent on proportion, meaning that the verification is slower when the proportion is close to $50 \%$.

Table 1: The summary of regression models comparing the effect of proportion between most and more than half.

| Effect | estimates | $t$ value | $p$ vale |
| :--- | :--- | :--- | :--- |
| Intercept | 947.15 | 26.28 | $<0.001$ |
| Prop | -22.16 | -0.96 | 0.34 |
| Quant | 216.82 | 6.43 | $<0.001$ |
| Resp | 48.86 | 1.52 | 0.13 |
| Prop:quant | -133.18 | -3.99 | $<0.001$ |
| Prop:resp | 82.96 | 2.59 | $<0.01$ |
| Quant:resp | -42.41 | -0.94 | 0.35 |
| Prop:quant:resp | 237.40 | 5.29 | $<0.001$ |

Notes: Prop. - main effect of proportion; Quant. - main effect of threshold; Resp. - main effect of response; Prop:quant proportion threshold interaction; Prop:Resp - proportion response interaction; Quant:resp - threshold response interaction; Prop:quant:resp - three way interaction.


Figure 2: The figure presents mean reaction times for each proportion and each quantifier. The triangles correspond to responses below threshold and circles to responses above threshold. The dashed lines illustrate the model predictions for responses below threshold and solid lines for responses above threshold. The red lines for most are steeper indicating the proportion effect for this quantifier. For clarity of the figure we constrained the $y$-axis to 5001800 ms .

### 3.3. Individual threshold as predictor of reaction times

In the next step we tested if the individual thresholds predict the speed of the verification process. In the regression model we used reaction times only for correct responses. We $z$ scored the proportion and threshold variables. We tested each quantifier separately using linear mixed effect regression model in R package lmerTest (Kuznetsova et al., 2017). We used reaction times as dependent variable and proportion, individual threshold, response type (true/false) and their interactions as predictors. We set true responses as the baseline level. We used the same procedure to include random slopes as in model comparing proportion effect for most and more than half.

To test the fixed-effect structure, we used the same procedure as above: we started with the maximal model and excluded those effects that did not improve model by using anova function in R (see Appendix A). We summarize all regression models' coefficients in Table 2 and included detailed description of results for fewer than half, many and few in Appendix B.

Table 2: The summary of regression models estimates with significance level (. $<0.1 ; *<.5$; $* *<.01 ; * * *<.001$ ) for all quantifiers.

| Effect | More than <br> half | Fewer than <br> half | Most | Many | Few |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Intercept | $941.55^{* * *}$ | $1136.85^{* * *}$ | $1238.77^{* * *}$ | $1110.22^{* * *}$ | $1541.56^{* * *}$ |
| Prop. | -18.26 | -39.45. | $-216.17^{* * *}$ | $-158.04^{* * *}$ | $316.89 * * *$ |
| Thr. | -13.99 | 15.51 | $216.98^{* * *}$ | -8.13 | $-237.56^{* * *}$ |
| Resp. | $53.20^{*}$ | -33.30 | -67.63 | $208.57^{* * *}$ | $-368.56^{* * *}$ |
| Prop:Thr |  |  | $-111.36^{* *}$ | -9.46 | $-114.40^{* *}$ |
| Prop:Resp | $78.46^{* *}$ |  | $381.99^{* * *}$ | $352.29^{* * *}$ | $-462.63^{* * *}$ |
| Thr:Resp | $24.47^{*}$ |  |  | $-170.40^{* * *}$ | $-217.42^{* * *}$ |
| Prop:Thr:Resp |  |  | $132.66^{* *}$ | $-105.31^{* *}$ | $157.08^{* * *}$ |

Notes: Prop. - main effect of proportion; Thr. - main effect of threshold; Resp. - main effect of response; Prop:Thr proportion threshold interaction; Prop:Resp - proportion response interaction; Thr:Resp - threshold response interaction; Prop:Thr:Resp - three way interaction.

More than half We predicted that the individual thresholds should not influence the reaction times during verification of more than half. We included by-subject random intercept. We used model comparison to determine the best model. The best model did not include threeway interaction between proportion, response and threshold $\left(\chi^{2}(1)=0.39 ; p=0.53\right)$ and interaction between proportion and threshold $\left(\chi^{2}(1)=2.23 ; p=0.14\right)$. As predicted, we did not find a main effect of threshold for more than half ( $\beta=-13.99 ; t=-0.60 ; p=0.55$ ).

Most We hypothesize that the individual threshold should predict the reaction times during verification of most. We found that the best random structure of most includes by-subject random intercept and by-subject random slope for proportion. We found a main effect of threshold ( $\beta=216.98 ; t=4.20 ; p<0.001$ ) and a significant interaction between threshold and response type ( $\beta=-170.40 ; t=-3.61 ; p<0.001$ ), indicating that the threshold effect was smaller for false responses. Finally, we also found a significant interaction between threshold and proportion ( $\beta=-111.36 ; t=-3.03 ; p<0.01$ ) and a three-way interaction between
proportion, response and threshold $(\beta=132.66 ; t=3.17 ; p<0.01)$, meaning that the threshold affected the proportion effect, but only for true responses.

All together, these findings show that the individual thresholds affect the speed of the verification process in vague quantifiers e.g. most, but not in quantifiers that have a clear threshold like more than half. It is worth to mention that the effect of threshold for most was asymmetric and present only for responses above threshold.

## 4. General discussion

The main goal of this paper was to investigate variability of meaning representations between subjects and assessing whether most and more than half are truly equivalent. We tested differences in meaning representations by estimating individual thresholds for quantifiers. We also tested for differences in the verification process of most and more than half by looking into participants' reaction times.

According to Hackl's (2009) linguistic analysis, most and more than half are verified using different strategies. Following Hackl's (2009) findings, Solt (2016) postulated that most is verified using approximate strategy. As a consequence, most should have a "significantly greater than more than half" interpretation. Solt (2016) found supporting evidence for her theory in corpus data.

Following Solt's (2016) findings we considered most as a vague quantifier, which can have a literal interpretation, equivalent to more than half, and a "significantly greater than more than half" interpretation. To test the first hypothesis, we estimated individual thresholds using logistic regression for most and more than half and three other quantifiers: few, fewer than half and many. We found that the threshold for more than half is $50 \%$, while in the case of most, there is higher variation in thresholds. Moreover, the mean threshold for most was higher than mean threshold for more than half. This finding clearly suggest that most is more sensitive to individual interpretation.

In contrast to our finding, Pietroski, Lidz, Hunter, Odic, \& Halberda (2011) conducted additional analyses on Pietroski et al.'s (2009) data to support their claim that subjects had a $50 \%$-threshold for most in their experiment. They investigated the deviation of accuracy from the Approximate Number System model predictions and concluded that the deviation did not increase when the ratio approached 1. The disparity between our and Pietroski et al. (2011) findings might be explained in different ways. Firstly, Pietroski et al.'s (2011) analysis is indirect and specifies only the deviation from the model predictions. In our analysis we estimated individual thresholds directly form participants responses. Therefore, our analysis does not require any additional assumptions about the correctness of the model. Secondly, Pietroski et al. $(2009,2011)$ ran a visual stimuli task design in a way that forced ANS performance. We, instead, gave our participants a purely linguistic task with unlimited time to provide responses. Therefore, our task is able to detect subtle differences in natural language quantifiers' representation, while Pietroski et al. (2009, 2011)'s task confounds the linguistic effects with the influence of visual and number cognition.

The disparity between our and Pietroski et al.'s (2011) results shows the advantage of using a novel purely linguistic task. Verification processes of quantifiers are often studied using
visual stimuli (e.g. Pietroski et al., 2009; Bott, Augurzky, Sternefeld, \& Ulrich, 2017; Deschamps, Agmon, Loewenstein, \& Grodzinsky, 2015; Zajenkowski \& Szymanik, 2013; Szymanik, 2016). For example, Pietroski et al. (2009) and Lidz et al. (2011) used a number cognition model - the Approximate Number System model (Dehaene, 1997) - to test the verification of most. We decided to use a purely linguistic paradigm, because the verification process of quantifiers against visual scene can be affected by many non-linguistic factors. For example, if the verification of quantifiers is based on ANS, then factors like type of task (Gilmore, Attridge, \& Inglis, 2011; see for review: Dietrich, Huber, \& Nuerk, 2015), duration of display (Cheyette \& Piantadosi, 2019; Inglis \& Gilmore, 2013), and set size (Dietrich, Nuerk, Klein, Moeller, \& Huber, 2019) will affect the verification process regardless of quantifier representation. Therefore, we think that the verification of quantifiers should be studied also in purely linguistic tasks to test to what extent the effects found in picture tasks can be attributed exclusively to semantic processing.

It is worth stressing that although we found differences in the interpretation of most and more than half, they are not completely in line with the Solt (2016) and Ariel (2003) findings. Solt (2016) and Ariel (2003) found that most is preferred for proportions above $\sim 65 \%-70 \%$. We found that some participants had thresholds above $60 \%$ for most, but the majority of participants had a threshold lower than $60 \%$. This might mean that Solt (2016) and Ariel (2003) captured some additional pragmatic effects on most, that pushed the threshold of this quantifier higher. In contrast, our task was very abstract (e.g., we used pseudowords) which mitigates the influence of a pragmatic interpretation on most. Moreover, Ariel (2003) tested Hebrew rov for most, while we tested English most. We cannot exclude the possibility that the differences in findings might be explained by differences in languages.

Secondly, we found that the verification of most is proportion-dependent in terms of reaction times. The verification of most takes longer when the given proportion is close to $50 \%$. No such effect was found for more than half. These findings extend the previous studies. Pietroski et al. (2009) showed that the verification of most is dependent on proportion in terms of accuracy by using an ANS model. However, they (Pietroski et al., 2009) did not contrast most with more than half to show that these quantifiers differ in verification process.

There are at least two possible explanations of the proportion effect for most. Firstly, it might be a consequence of a difference in verification strategy. More than half is verified using a precise strategy, comparing the given proportion to $50 \%$. In the case of most, participants had to compute the proportion of As that are not B given the proportion of As that are B. They computed the number of As that are not B approximately, which results in greater proportiondependent performance. Although we used a purely linguistic task, it is possible that participants engage the Approximate Number System into the verification process. Previous studies (e.g. Moyer \& Landauer, 1967; Hinrichs, Yurko \& Hu, Psychology, \& 1981) show that ANS effects, e.g. distance effect, can be found even in a symbolic number comparison task.

According to the second possible explanation, the proportion effect of most is a result of the pragmatic strengthening. On the one hand, participants represented most as more than half; on the other hand, they had a strong pragmatic preference towards using most for higher
proportions. Before they made a decision, they had to choose between these representations. Future studies need to shed light on disentangling these two competing explanations.

In addition to the proportion effect, we tested if the differences in thresholds in vague quantifiers (most, many, few) will affect the verification process. We found an effect of threshold on reaction times in vague quantifiers, but not in quantifiers with sharp meaning boundaries (more than half and fewer than half). The lack of threshold effect for many was one deviation from this result.

The results presented in this paper clearly suggest that more than half and fewer than half have unequivocal thresholds. In contrast, most, many and few have varied thresholds. The literature about many and few (e.g. Partee, 1988) consistently claims that these two quantifiers are highly context dependent and that they can have various interpretations. Our findings suggest that most exhibits similar effects. Further experimental studies are needed to explain how these meanings change and are selected in the context. It is possible that the specific context will trigger pragmatic reasoning about most and push the thresholds even higher.

Our study fits with an increasing number of findings on individual differences in natural language. Although many studies (e.g. Newstead \& Coventry, 2000) investigated how the interpretation of vague quantifiers depends on contextual features like set size (Newstead, Pollard, \& Riezebos, 1987; Newstead \& Coventry, 2000), size of the stimuli and its position with relation object that creates context (Newstead \& Coventry, 2000) or the number of nontarget objects (Coventry, Cangelosi, Newstead, \& Bugmann, 2010), little attention has been paid to individual differences in meaning representations. We aimed to bridge this gap by finding individual differences in natural language on example of quantifiers.

Our study also has several limitations. Firstly, the task was very abstract. On the one hand, this can be considered as an advantage, because abstract tasks limit pragmatic reasoning and allow us to test semantic differences between quantifiers. On the other hand, it makes quantifiers like many and few hard to interpret. Secondly, we tested a wide range of proportions, which means that we had only a limited number of trials per proportion and participant. We tried to compensate for this problem by including a large number of participants into our study (Rouder \& Haaf, 2018) and excluding the outliers’ responses. Thirdly, in our study all five quantifiers were a within-subject variable. It is therefore possible that estimated thresholds are affected by interaction between quantifiers. For example, some participants might have used the same $50 \%$ threshold for most and more than half to simplify the task (assuming that it is easier to perform the task, when participants have to remember only one threshold instead two). It would be worth testing if the same, or even stronger results, can be observed in a between-subject design. Finally, the logistic regression method, which we used to estimate the thresholds, was not always successful. In future work, we hope to overcome this difficulty by applying more complex methods to estimate the underlying properties of the verification process, such as evidence accumulation modeling (Anders et al, 2015; Ratcliff \& McKoon, 2018).

This study contributed to the discussion about differences between most and more than half by showing that most exhibits more sensitivity to individual differences and is proportion-
dependent. In this way, we showed that truth-conditionally equivalent expressions differ in meaning and that most is a vague quantifier with various meaning representations. We showed differences between most and more than half in a novel, purely linguistic task. By using this task, we avoided confounds between semantic meaning of the expression and other cognitive systems and we were able to directly compare most with more than half. Finally, we presented a new method to investigate individual differences in meaning representations.

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## Appendix A

## A.1. Proportion effect on reaction times - random structure

Tests for by-subject random effects: model with only intercept vs. model with random slope for proportion ( $\chi^{2}(2)=16.12 ; p=0.0003$ ), model with only intercept vs. model with random slope for response $\left(\chi^{2}(2)=13.04 ; \mathrm{p}=0.001\right)$, model with random slope for quantifier had singular fit, model with random slope for proportion vs. model with random slope for proportion and response $\left(\chi^{2}(3)=3.29 ; p=0.35\right)$.
A.2. Individual threshold as predictor of reaction times - random structure

More than half: by-subject random slopes for proportion and response gave singular fit; Most: model with only by-subject intercept vs. model with by-subject random slope for proportion $\left(\chi^{2}(2)=12.71 ; p=0.002\right)$, model with only by-subject intercept vs. model with by-subject random slope for response ( $\chi^{2}(2)=11.40 ; p=0.003$ ); model with both random slopes gave singular fit; Many: model with only by-subject intercept vs. model with bysubject random slope for proportion $\left(\chi^{2}(2)=6.43 ; p=0.04\right)$, model with only by-subject intercept vs. model with by-subject random slope for response $\left(\chi^{2}(2)=3.25 ; p=0.2\right)$; Few: model with only by-subject intercept vs. model with by-subject random slope for proportion $\left(\chi^{2}(2)=7.81 ; p=0.02\right)$, model with only by-subject intercept vs. model with by-subject random slope for response ( $\chi^{2}(2)=8.86 ; p=0.01$ ); model with two by-subject slopes did not improve fit $\left(\chi^{2}(3)=4.74 ; p=0.19\right)$; Fewer than half: model with only by-subject intercept vs. model with by-subject random slope for proportion $\left(\chi^{2}(2)=14.80 ; p=0.0006\right)$, model with only by-subject intercept vs. model with by-subject random slope for response $\left(\chi^{2}(2)=\right.$ 18.79; $\mathrm{p}<0.0001$ ); model with two by-subject slopes improved fit ( $\left.\chi^{2}(3)=7.89 ; \mathrm{p}=0.04\right)$.

## Appendix B

Fewer than half We predicted that the individual thresholds should not influence the reaction times during verification of fewer than half as in case of more than half. We included by subject random intercept and by-subject random slope for percent and response type. By using model comparison, we excluded three-way interaction $\left(\chi^{2}(1)=0.22 ; \mathrm{p}=0.64\right)$, threshold-response interaction $\left(\chi^{2}(1)=0.15 ; p=0.7\right)$, threshold-proportions interaction ( $\chi^{2}(1)$ $=1.62 ; \mathrm{p}=0.20)$ and proportion-response interaction $\left(\chi^{2}(1)=1.98 ; \mathrm{p}=0.16\right)$. The final model for fewer than half included only three main effects. The effect of threshold was not significant ( $\beta=15.51 ; t=0.62 ; p=0.54$ ).

Many and few Finally we also predicted that the verification time of many and few should be threshold-dependent. We included by-subject random intercept for both quantifiers and bysubject random slope for proportion for many and by-subject random slope for response type for few. For many we did not find a significant main effect of threshold ( $\beta=-8.13 ; t=-0.27$; $p=0.78$ ) but did find a significant threshold-response type interaction ( $\beta=-217.42 ; t=-$ 4.53; $p<0.001$ ), meaning that the effect of threshold was greater for false responses. We also did not find a significant threshold-proportion interaction ( $\beta=-9.46 ; t=-0.68 ; p=0.50$ ), but did find a significant three-way interaction between proportion, response and threshold ( $\beta=-$ 105.31; $t=-2.95 ; p<0.01$ ), meaning that for responses false there was a threshold-proportion interaction.

For few, we found a main effect of threshold ( $\beta=-237.12 ; t=-3.86 ; p<0.001$ ), a significant interaction between threshold and response type ( $\beta=197.08 ; t=3.55 ; p<0.001$ ), a significant interaction between threshold and proportion ( $\beta=-114.40 ; t=-2.84 ; p<0.01$ ) and a significant three-way interaction between proportion, response and threshold ( $\beta=152.98 ; t$ $=3.59 ; p<0.001$ ), meaning that the effect of threshold was stronger for true responses than for false responses and that it influenced the proportion effect stronger for true responses, than false responses.


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[^1]:    2 Pietroski et al (2009) and Lidz et al (2011) provided evidence that the verification strategy for most should be $\operatorname{most}(\mathrm{A}, \mathrm{B}) \Leftrightarrow|\mathrm{A} \cap \mathrm{B}|>|\mathrm{A}|-|\mathrm{A} \cap \mathrm{B}|$.

