Inflation targeting and liquidity traps under endogenous credibility

Hommes, C.; Lustenhouwer, J.

DOI
10.1016/j.jmoneco.2019.01.027

Publication date
2019

Document Version
Final published version

Published in
Journal of Monetary Economics

Journal of Monetary Economics 107 (2019) 48–62

Contents lists available at ScienceDirect

Journal of Monetary Economics

journal homepage: www.elsevier.com/locate/jmoneco

Inflation targeting and liquidity traps under endogenous credibility[☆]

Cars Hommes[a,b,c,*], Joep Lustenhouwer[d]

[a] *CeNDEF, University of Amsterdam, Amsterdam, the Netherlands*
[b] *Tinbergen Institute, Amsterdam, the Netherlands*
[c] *Bank of Canada, Ottawa, Canada*
[d] *Otto-Friedrich-Universität Bamberg, Germany*

ARTICLE INFO

Article history:
Received 16 July 2015
Revised 9 January 2019
Accepted 22 January 2019
Available online 1 August 2019

JEL classification:
E52
E32
C62

Keywords:
Monetary policy
Liquidity traps
Heterogeneous expectations
Bounded rationality

ABSTRACT

Policy implications are derived for an inflation-targeting central bank, whose credibility is endogenous and depends on its past ability to achieve its targets. This is done in a New Keynesian framework with heterogeneous and boundedly rational expectations. We find that the region of allowed policy parameters is strictly larger than under rational expectations. However, when the zero lower bound on the nominal interest rate is accounted for, self-fulfilling deflationary spirals can occur, depending on the credibility of the central bank. Deflationary spirals can be prevented with a high inflation target and aggressive monetary easing.

1. Introduction

Many central banks (CBs) have recently adopted some form of inflation targeting. Besides setting the nominal interest rate, an important aspect of inflation targeting is *managing expectations*. This is e.g. stressed by Woodford (2004). For the inflation targeting to be effective, it is important that the CB has enough *credibility*. If the private sector does not believe the CB when it announces an inflation target, the realized value of inflation will likely not be equal to this target. Whether the CB is likely to be believed furthermore will typically depend on whether it was able to achieve its targets in the past.

Inflation targeting is usually modeled in a New Keynesian setting under the assumption that agents have fully rational expectations. Under this assumption, all agents form the same perfectly model-consistent expectations, which, in the absence of shocks, implies that they have perfect foresight. When rational expectations are assumed, there is no longer a clear role for the credibility of an inflation and output gap target inside the model. Either expectations about inflation and output

☆ The authors gratefully acknowledge the financial support of NWO (Dutch Science Foundation) Project No. 40614011, “Monetary and Fiscal Policy under Bounded Rationality and Heterogeneous Expectations”, and the financial support from the EU FP7 project “Integrated Macro-Financial Modeling for Robust Policy Design” (MACFINROBODS, grant 612796).

* Corresponding author at: University of Amsterdam, Amsterdam School of Economics, Roeteresstraat 11, 1018WB, Amsterdam, the Netherlands.
E-mail address: c.h.hommes@uva.nl (C. Hommes).

https://doi.org/10.1016/j.jmoneco.2019.01.027
0304-3932/

coincide with the targets of the central bank and the CB has full credibility[1], or expectations are not in line with the targets and the announcements of the CB are not credible.

Rational expectations are furthermore an unrealistically strong assumption when inflation and output forecasts by price setters (i.e. the private sector) are concerned. Both surveys of consumers and professional forecasters and laboratory experiments with human subjects show that there is considerable heterogeneity in inflation forecasts (e.g. Mankiw et al. (2003), and Pfajfar and Žakelj (2018)). Assenza et al. (2014) furthermore show that in their laboratory experiments, expectations of subjects can quite accurately (both qualitatively and quantitatively) be described as switching between simple heterogeneous forecasting heuristics based on their relative past performance.

In this paper, we study inflation targeting and liquidity traps under heterogeneous expectations. Instead of assuming rational expectations, a heuristic switching model is used, that allows for *endogenous credibility*. Heuristic switching models were introduced by Brock and Hommes (1997), and have since successfully been used to model heterogeneous expectations in finance and macroeconomics (Hommes, 2013, 2019). In our model, agents switch between two intuitive forecasting heuristics based on *relative performance*. The most important forecasting heuristic can be described as “Trust the central bank”. Followers of this heuristic are called fundamentalists and expect future inflation and output gap to be equal to the targets of the central bank. The fraction of fundamentalists can be interpreted as the *credibility* of the central bank. In contrast with rational expectations models, our model, therefore, involves *endogenous credibility*. We assume that this fundamentalist belief competes with naive expectations, which use the last observation as their best guess for future realizations of inflation and output. The naive heuristic coincides with rational expectations when inflation or output follows a random walk. If inflation or output follows a near unit root process, the naive forecast is nearly rational. Naive agents furthermore add persistence in inflation and output gap to our model in a very simple and intuitive manner, without the need to assume heavily serially correlated shocks. Similar to the naive heuristic, Milani (2007) has stressed that homogeneous adaptive learning also generates high persistence in inflation and output dynamics, especially under constant gain learning (Evans et al., 2009).

Cornea-Madeira et al. (2019) estimate a New Keynesian Phillips curve assuming expectations are formed by a heuristic switching model with fundamentalists and naive agents. Fundamentalists here make use of the forward-looking relation between inflation and marginal costs and use a VAR approach to make inflation forecasts. Cornea-Madeira et al. (2019) find that their model fits the data well, and the endogenous mechanism of switching between the two heuristics based on past performance is thus supported by the data. Moreover, there is quite some time variation in the fractions of fundamentalists and naive agents. Whenever inflation drifts away from its target, either above or below, the fraction of naive agents increases, reinforcing persistent drifts away from the target. Branch (2004, 2007) fits a heuristic switching model with, among others, a naive heuristic and a fundamentalist VAR heuristic to data from the Michigan Survey of Consumer Attitudes and Behavior. Both these papers find clear evidence of switching between heuristics based on past performance. Branch (2004) furthermore finds that both our heuristics are present in the survey data, and Branch (2007) finds that the heuristic switching model better fits the survey data than a static sticky information model. A case study of the Volcker disinflation by Mankiw et al. (2003) furthermore nicely illustrates the presence of our two heuristics in survey data. In their Fig. 12 (Mankiw et al., 2003, p. 46), the evolution of inflation expectations as measured by the Michigan Survey from 1979, up to and including 1982 is plotted. They show that at the start of 1979, expectations were centered around a high inflation value. Over the next eight quarters (during which Paul Volcker was appointed chairman of the Board of Governors of the Federal Reserve Board), the distribution of expectations clearly becomes bimodal, with a fraction of agents still expecting the same high values of inflation and another fraction expecting lower inflation. In terms of our model, this can be interpreted as follows. Before Volcker was appointed, the FED had very little credibility, and most agents expected inflation to remain at the high values that it had been at in the recent past (consistent with the naive heuristic). In the following quarters, the FED gained more credibility, and an increasing fraction of agents started to believe that Volcker would be able to drive down inflation toward its target level (more agents started to follow the fundamentalist heuristic). Furthermore, when in 1982 actual inflation started to decline, the mass on high inflation expectations slowly started to move towards lower inflation. This can be interpreted as backward-looking, naive agents believing that lower observed inflation would also mean lower inflation in the future.

We first abstract from a lower bound on the nominal interest rate and analyze the stability properties of our heuristic switching model in normal times. Here it is shown that the region of policy parameters that leads to a *locally stable* fundamental steady state (with inflation and output gap equal to target) is strictly larger than the region of policy parameters that gives a locally determinate rational expectations equilibrium. Even when the Taylor principle is not satisfied, there could very well be convergence to the fundamental steady state in our model. The intuition for this result is that a central bank that was able to gain credibility in the past has to do less with its current interest rate policy to stabilize expectations than a central bank that does not have credibility.

Next, we introduce the zero lower bound (ZLB) on the nominal interest rate to the above heterogeneous expectations framework, and investigate its effect on inflation and output gap dynamics. It turns out that with the ZLB, expectations-driven liquidity traps can arise. In rational expectations models, shocks to the fundamentals of the economy can lead to a temporary liquidity trap. However, as soon as the sequence of bad shocks is over, the liquidity trap is immediately

[1] Note that we refer to the credibility of the CB's targets, and not to the credibility of its future policy actions. See Section 3 for details.

resolved. In our model, a one-period shock to economic fundamentals can lead to a prolonged liquidity trap due to a loss in credibility of the central bank and low, self-fulfilling expectations. Mertens and Ravn (2014) highlight the distinction between expectations-driven liquidity traps and fundamental liquidity traps. Depending on the magnitude of the shock and the loss in credibility, our expectations-driven liquidity traps can be temporary or take the form of a deflationary spiral with ever decreasing inflation and output gap. Deflationary spirals have recently been observed in laboratory experiments by Arifovic and Petersen (2015), Assenza et al. (2014) and Hommes et al. (2019). When the ZLB is accounted for, the fundamental steady state can no longer be *globally* stable, but only *locally* stable, coexisting with a deflationary spiral region. This deflationary spiral region especially becomes a danger when there is low credibility and beliefs of naive agents create persistence in the economy (sluggish beliefs). In this case, the central bank has a harder time managing aggregate demand, and negative shocks can bring the economy more easily to the ZLB than in the case of rational expectations. In contrast, as long as credibility remains high enough, deflationary spirals do not arise. Finally, we illustrate in simulations that the central bank can prevent deflationary spirals by increasing the inflation target and conducting aggressive monetary easing as soon as a liquidity trap is imminent.

Our model is closely related to that of Branch and McGough (2010) and De Grauwe and Ji (2019), who use heuristic switching models in a monetary policy environment with combinations of forecasting heuristics that are similar to ours. We add to Branch and McGough (2010) by also studying the ZLB on the nominal interest rate, and our contribution differs from De Grauwe and Ji (2019) in that they rely solely on simulations, and do not provide analytical results. Other works with heuristic switching models in a macroeconomic setting include Anufriev et al. (2013); De Grauwe (2011); Massaro (2013) and Agliari et al. (2015).

Closely related to our investigation of liquidity traps under bounded rationality are Eusepi (2010); Evans et al. (2008) and Benhabib et al. (2014). These authors study monetary and fiscal policy under adaptive learning in a New Keynesian framework. They show the existence of multiple equilibria: the target equilibrium, and an equilibrium with low inflation. The existence of this second equilibrium when a ZLB is introduced to the model has been first highlighted by Benhabib et al. (2001a,b). Evans et al. (2008) furthermore show that, in their model, a liquidity trap arises in the form of a deflationary spiral with ever decreasing inflation and output gap. A drawback of these models is their focus on a representative agent with adaptive learning. In our model, we extend the analysis to allow for heterogeneity in expectations and define an explicit measure of endogenous credibility. Moreover, we are able to obtain analytical expressions for dynamics under the ZLB.

In e.g. Orphanides and Williams (2004) and Busetti et al. (2014), an interpretation of endogenously evolving anchoring of expectations can be made, which is related to our concept of credibility. In these papers, agents use constant-gain adaptive learning, with a constant in their perceived law of motion. When this constant starts to deviate from the target of the central bank, this can be seen as deanchoring of expectations. Moreover, such deviations from the target arise as inflation realizations have been farther away from the target in the recent past. Other papers with such perpetual learning in a monetary policy setting include Orphanides and Williams (2005, 2007a, 2007b). This deanchoring of expectations is related to our concept of endogenous credibility. However, our model with heterogeneous agents allows us to define a more formal metric of credibility as the fraction of agents that trusts the targets of the central bank. Under homogeneous expectations, either all agents believe in the targets, or their beliefs deviate from the targets, in which case one can only look at the distance between their belief and the target. While considering this distance is definitely interesting, we believe that our model more naturally and intuitively fits the concept of credibility and its endogenous evolution.

The rest of the paper is organized as follows. In Section 2, the New Keynesian model is presented. Section 3 introduces the heuristic switching model and conducts the analysis without the zero lower bound. In Section 4, the ZLB is added to the model, and liquidity traps are analyzed. Simulations that illustrate how liquidity traps arise and how policy interventions can prevent this are presented in Section 5. Section 6 concludes.

2. Inflation targeting model

In order to facilitate comparison with the rational expectations benchmark, we use a standard New Keynesian model in line with Galí (2002) and Woodford (2003). Micro-foundations of this model under heterogeneous expectations are derived in the online appendix. This derivation is closely related to the micro-foundations in Kurz et al. (2013), but additionally makes use of the properties of our heuristic switching model, defined in Section 3. The New Keynesian IS curve and Phillips curve, describing the output gap and inflation respectively, are given by

$$x_t = \bar{E}_t x_{t+1} - \frac{1}{\sigma}(i_t - \bar{E}_t \pi_{t+1} - \bar{r}) + u_t, \tag{1}$$

$$\pi_t = \beta \bar{E}_t \pi_{t+1} + \kappa x_t + e_t. \tag{2}$$

Here β is the discount factor, and

$$\kappa = \frac{(\sigma + \eta)(1-\omega)(1-\beta\omega)}{\omega}, \tag{3}$$

with σ and η the inverses of respectively the elasticity of intertemporal substitution and the elasticity of labor supply. $(1-\omega)$ is the fraction of firms that can adjust their price in a given period. i_t is the nominal interest rate, which can be freely chosen by the central bank, while $\bar{r} = 1 - \frac{1}{\beta}$ is the steady state real interest rate. $\bar{E}_t$ denotes aggregate expectations of all agents in the economy, and u_t and e_t are shocks to respectively output gap and inflation, which can be interpreted as a productivity or preference shock and a cost-push shock. Throughout the paper, shocks will be white noise and hence have no autocorrelation.

We assume the central bank uses the following forward-looking Taylor rule that replaces contemporaneous values of inflation and output gap by expectations in the rule proposed by Taylor (1993):

$$i_t = \bar{r} + \pi^T + \phi_1(\bar{E}_t\pi_{t+1} - \pi^T) + \phi_2(\bar{E}_t x_{t+1} - x^T). \tag{4}$$

Abstracting from shocks and plugging (4) into (1), gives the following model:

$$x_t = \left(1 - \frac{\phi_2}{\sigma}\right)\bar{E}_t x_{t+1} - \frac{\phi_1 - 1}{\sigma}\left(\bar{E}_t\pi_{t+1} - \pi^T\right) + \frac{\phi_2}{\sigma}x^T, \tag{5}$$

$$\pi_t = \beta\bar{E}_t\pi_{t+1} + \kappa x_t. \tag{6}$$

We assume here that the model parameters are positive, and that the policy parameters of the central bank are nonnegative.

Assumption 1. $\kappa, \sigma > 0,\ \phi_1, \phi_2, \pi^T \geq 0.$

3. Analysis with heuristic switching model

In this section, a heuristic switching model is used to analyze the dynamics of output gap and inflation when expectations are non-rational and heterogeneous. In a heuristic switching model, as in Brock and Hommes (1997), beliefs are formed by a set of simple rules of thumb, or heuristics. The population consists of agents that can switch between those heuristics. As a heuristic performs better in the recent past, the fraction of the population that follows that prediction rule increases. Agents are therefore learning over time by evolutionary selection based upon relative performance. The fractions of agents following the different heuristics evolve according to the following discrete choice model with multinomial logit probabilities (see Manski et al. (1981)):

$$n_{h,t} = \frac{e^{bU_{h,t-1}}}{\sum_{h=1}^{H} e^{bU_{h,t-1}}}. \tag{7}$$

Here $n_{h,t}$ is the fraction of agents that follows heuristic h in period t, and $U_{h,t-1}$ is the fitness measure of heuristic h in period $t-1$, i.e., a measure of how well the heuristic performed in the past. Finally, b is the intensity of choice. The higher the intensity of choice, the more sensitive agents become with respect to the relative performance of the heuristics.

We assume private sector beliefs are formed by two simple, but plausible heuristics: fundamentalist and naive. Followers of the naive heuristic make use of the high persistence in inflation and output gap dynamics and believe future inflation or output gap to be equal to their last observed values. Note that the naive forecast is optimal when inflation and output follow a random walk, and close to optimal when the system contains a near unit root.

Followers of the fundamentalist heuristic, on the other hand, believe inflation and output gap to be equal to the target values of the central bank, π^T and x^T, in every period. In the presence of shocks, the central bank will not be able to achieve these targets in every period. However, since shocks are assumed to be white noise in our model, expectations of fundamentalists coincide with rational expectations when there are no naive agents. Fundamentalists can thus be seen to act as if all agents are rational. They do not take into account that there are other agents in the economy, as they lack the cognitive ability to know exactly the beliefs of other agents or the number of agents with different expectations. Since fundamentalists believe in the long-run targets of the central bank and see these long-run values as the best predictors for short-run future inflation and output gap, we can interpret the fraction of fundamentalist agents in the economy as the credibility of the central bank.

Note that we talk about the credibility of the central bank's target values of inflation and output gap, and not about the credibility of its future policy actions (as credibility is often referred to in the literature). This is in line with the fact that inflation targeting can be seen as a commitment to goals rather than a commitment to future actions and details of the CB's operations. Credibility over targets furthermore implicitly captures both the CB's willingness to take actions to achieve its targets *and* its ability to do so, where the latter is not straightforward in an economy with boundedly rational agents.

Finally, let the fitness measure for both variables be a weighted sum of the negative of the last observed squared prediction error, and the previous value of the fitness measure:

$$U_{t-1}^z = -(1-\rho)(z_{t-1} - \tilde{E}_{t-2}z_{t-1})^2 + \rho U_{t-2}^z, \quad z = \pi, x, \tag{8}$$

where $0 \leq \rho \leq 1$, is the memory parameter, and $\tilde{E}$ represents boundedly rational expectations of either naive agents or fundamentalists. For analytical tractability, ρ is set to 0 for now, and reintroduced in the simulations in Section 5.

To simplify calculations and presentation, we introduce a new variable, which is defined as the difference between the fraction of fundamentalist agents (n_t^z) and the fraction of naive agents (1-n_t^z):[2]

$$m_t^z = n_t^z - (1 - n_t^z) = 2n_t^z - 1, \quad z = \pi, x. \tag{9}$$

When all agents are fundamentalists, the difference in fractions equals 1, and when all agents are naive, the difference in fractions equals -1. Henceforth we will refer to these differences in fractions simply as fractions. These fractions can be interpreted as *endogenous credibility*. When $m_t^x = m_t^\pi = 1$, the central bank has full credibility, and when $m_t^x = m_t^\pi = -1$, the CB has lost all its credibility. This credibility measure will turn out to be very important in determining the effectiveness of monetary policy.

Using (9), aggregate expectations about inflation and output gap can be written as

$$\bar{E}_t \pi_{t+1} = \frac{(1 + m_t^\pi)}{2}\pi^T + \frac{(1 - m_t^\pi)}{2}\pi_{t-1}, \tag{10}$$

$$\bar{E}_t x_{t+1} = \frac{(1 + m_t^x)}{2}x^T + \frac{(1 - m_t^x)}{2}x_{t-1}. \tag{11}$$

The model is now given by (5), (6), (10), (11) and two equations for m_{t+1}^x and m_{t+1}^π that can be obtained by combining (7), (8) and (9):

$$m_{t+1}^x = \text{Tanh}\left(\frac{b}{2}(x_{t-2}^2 - (x^T)^2 - 2(x_{t-2} - x^T)x_t)\right), \tag{12}$$

$$m_{t+1}^\pi = \text{Tanh}\left(\frac{b}{2}(\pi_{t-2}^2 - (\pi^T)^2 - 2(\pi_{t-2} - \pi^T)\pi_t)\right). \tag{13}$$

As discussed in the online appendix, this is a six-dimensional system, whose state vector can be written as

$$\begin{pmatrix} x_t & \pi_t & m_{t+2}^x & m_{t+2}^\pi & m_{t+1}^x & m_{t+1}^\pi \end{pmatrix}. \tag{14}$$

3.1. Steady states and stability

The central bank aims to keep inflation at its target level. It would, therefore, be desirable for our dynamical system to have a steady state with $\pi^* = \pi^T$. Proposition 1 states that such a steady state indeed exists, as long as the central bank chooses an output gap target corresponding to the desired inflation level. The proof of Proposition 1 is given in the online appendix.

Proposition 1. *When the central bank sets $x^T = \frac{1-\beta}{\kappa}\pi^T$, then a steady state with $\pi^* = \pi^T$, $x^* = x^T$, $m^{x*} = 0$, $m^{\pi*} = 0$ exists for any value of π^T.*

From now on, we will assume that the central bank always chooses an output gap target consistent with its inflation target so that the steady state where $\pi^* = \pi^T$ and $x^* = x^T$ exists. Since this steady state coincides with the expectations of fundamentalists, we call this steady state the *fundamental steady state*. Even though in this steady state convergence to fundamentalist expectation *values* has taken place, it is not the case that all agents use the fundamentalist *heuristic*. This is so because the naive heuristic also gives perfect steady state predictions, so that both fundamentalists and naive agents have perfect foresight at the fundamental steady state. The difference in fractions, therefore, equals zero for both variables ($m^{x*} = m^{\pi*} = 0$). An intuition for this is that when the economy is stable around the central bank's targets, all agents have expectations close to this target, but that the reason for these expectations differs across agents. In particular, some agents (the fundamentalists) trust the credibility of the central bank and believe that the central bank will also be able to achieve its targets in the future. Other agents (the naive), however, expect values close to targets only in the short run, because inflation and the output gap were close to the targets in the recent past. When larger shocks start hitting the economy, the first group of agents will keep believing in the targets, while the other group of agents will start to have different expectations farther from target, in line with recent observations in the economy.

The central bank would like to achieve convergence to the fundamental steady state from as wide a range of initial conditions as possible. This requires first of all that the fundamental steady state is *locally stable*. The central bank can try to achieve stability of the fundamental steady state by choosing the right values of the parameters in its monetary policy rule, ϕ_1 and ϕ_2. It turns out that the inflation target, π^T, does not matter for the stability of the fundamental steady state. Proposition 2 describes the conditions for local stability. The results of the proposition are illustrated in Fig. 1 and will be discussed below. The proof of Proposition 2 is given in the online appendix.

[2] Note that we allow the fraction of fundamentalists, n_t^z, to differ between inflation and output gap ($z = \pi, x$). Agents will then learn to use the same heuristic for both variables in times where the time series of inflation and output gap have similar features. However, e.g. in periods of hyperinflation with an output gap close to target, agents will learn to be fundamentalist about the output gap but to use past inflation as their best predictor of future inflation. All results presented in this section continue to hold when it is imposed that n_t^x and n_t^π should evolve together. Results presented in Sections 4 and 5 would also remain valid qualitatively.

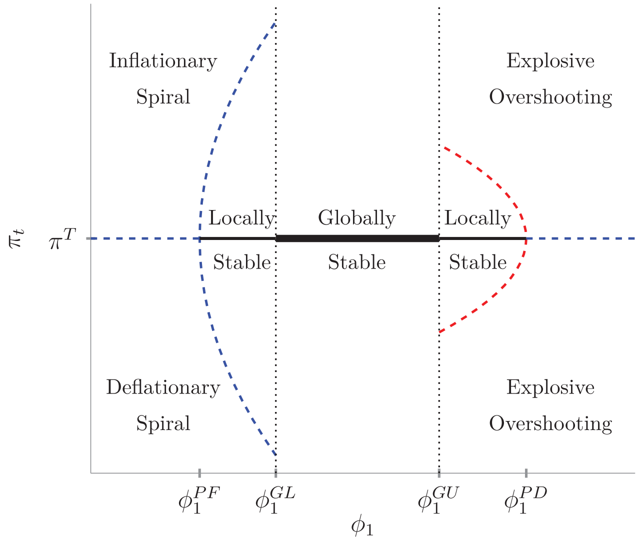

Fig. 1. Bifurcation diagram in ϕ_1 in case of $\phi_2 = \sigma$ and initial output gap at steady state. The locally (globally) stable area of the fundamental steady state is indicated with a solid (thick) black line. Unstable steady states are indicated with blue dashed curves, while the red dashed curves represent an unstable 2-cycle. When the fundamental steady state is not globally stable, explosive overshooting can occur if monetary policy is too aggressive. When monetary policy is too weak, either an inflationary or a deflationary spiral will occur. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Proposition 2. *(See Fig. 1) The fundamental steady state is locally stable if and only if*

$$1 - (2-\beta)\frac{\sigma + \phi_2}{2\kappa} = \phi_1^{PF} < \phi_1 < \phi_1^{PD} = 1 + (2+\beta)\frac{3\sigma - \phi_2}{2\kappa}. \tag{15}$$

At $\phi_1 = \phi_1^{PF}$, a subcritical pitchfork bifurcation takes place (with two unstable, non-fundamental steady states above the bifurcation value), with $\lambda_1 = +1$. At $\phi_1 = \phi_1^{PD}$, a period doubling bifurcation takes place, with $\lambda_2 = -1$. This bifurcation is subcritical (with a 2-cycle below the bifurcation value) if $\phi_2 < 3\sigma$ and supercritical (with a 2-cycle above the bifurcation value) if $\phi_2 > 3\sigma$.

It follows from Proposition 2 that the fundamental steady state can be unstable either because the central bank responds too weakly or because the CB responds too strongly to inflation and output gap expectations. The intuition of instability of the fundamental steady state due to monetary policy that reacts too weakly is the following. If period t expectations about period $t+1$ inflation and/or output gap are high, and the central bank does not respond with a large enough increase in the interest rate, these high expectations will lead to period t realizations of inflation and output gap that are even higher. This will lead expectations about period $t+2$, formed in period $t+1$, to be higher than expectations about period $t+1$, leading to even higher period $t+1$ realizations. This will lead to a loss of credibility for the central bank (more agents become naive), leading to more instability. What follows is a continued rise of both inflation and output gap, together with declining credibility: an inflationary spiral. Analogously, for low initial conditions, a deflationary spiral will occur under weak monetary policy.

If the central bank responds too strongly to expectations, high inflation and/or output gap expectations about period $t+1$ are countered in period t by a very high interest rate. This results in very low inflation and output gap realizations in period t, leading to very low expectations about period $t+2$. The CB then sets the interest rate very low in period $t+1$, leading to even higher inflation and output gap realizations in period $t+1$ than agents had expected. This causes a loss in credibility, and high naive expectations about period $t+3$, resulting in a high interest rate and even lower realizations in period $t+2$ than agents expected. These cyclical dynamics continue, leading inflation and output gap to jump up and down between ever higher and lower values, with ever declining credibility: explosive overshooting.

The results of Propositions 1 and 2 can be combined in a bifurcation diagram. This is done in Fig. 1, with ϕ_1 on the horizontal axis and π_t on the vertical axis. The fundamental steady state is located at $\pi_t = \pi^T$, and the black line between ϕ_1^{PF} and ϕ_1^{PD} indicates the range of ϕ_1 values for which this steady state is locally stable. To the left of ϕ_1^{PF} and to the right of ϕ_1^{PD} the fundamental steady state is unstable, which is indicated by blue dashed lines. In this picture, it is assumed that $\phi_2 = \sigma$, so that the period doubling bifurcation is subcritical. This implies the existence of an unstable 2-cycle to the *left* of ϕ_1^{PD}, which is depicted by the red dashed curves. The blue dashed curves between ϕ_1^{PF} and 1 represent the non-fundamental unstable steady states from Proposition 1 that are created in the subcritical pitchfork bifurcation. As discussed above, explosive cyclical dynamics, due to overshooting, occur to the right of ϕ_1^{PD}. To the left of ϕ_1^{PF}, inflation either monotonically increases or decreases, depending on initial conditions. In Fig. 1, it is assumed that initial output gap is at its target, so that the inflation target is the boundary between inflationary and deflationary spirals.

Table 1
Parameterizations and corresponding stability conditions.

Author(s)	σ	κ	ϕ_1^{PF}	ϕ_1^{PD}	ϕ_1^{GL}	ϕ_1^{GU}
W	0.157	0.024	−5.62	20.56	0.93	14.02
CGG	1	0.3	−2.367	10.97	0.97	7.63
MN	1/0.164	0.3	−19.53	61.77	0.80	41.45

The first two columns of the table display the coefficients of relative risk aversion, σ, and the slope of the Phillips curve, κ, found by Woodford (1999), W, Clarida et al. (2000), CGG, and McCallum and Nelson (1999), MN. The final four columns depict the corresponding boundaries on ϕ_1 (the coefficient on inflation in the Taylor rule) for local stability (ϕ_1^{PF} and ϕ_1^{PD}) and for global stability (ϕ_1^{GL} and ϕ_1^{GU}), that follow from Proposition 2 and 3. Here, $\beta = 0.99$ and it is assumed that $\phi_2 = \sigma$.

The results of Proposition 2 are similar to the conditions required for local determinacy under rational expectations presented by Bullard and Mitra (2002). These authors show that, when the central bank responds to inflation and output gap expectations, determinacy of the rational expectations equilibrium requires both that the well known Taylor principle is satisfied, and that the central bank does not respond too strongly to expectations. More specifically, the authors find that first of all $\phi_1 > 1 - (1-\beta)\frac{\phi_2}{\kappa}$ must hold. However, the condition for local stability that follows from Proposition 2 requires ϕ_1 to be larger than ϕ_1^{PF}, which is (under Assumption 1) strictly smaller than the value found by Bullard and Mitra (2002). The fundamental steady state can, therefore, be locally stable even if the Taylor principle is not satisfied. The second condition for determinacy given by Bullard and Mitra (2002) reduces to $\phi_1 < 1 + (1+\beta)\frac{2\sigma - \phi_2}{\kappa}$. This condition is again strictly stronger than our condition for local stability, which requires $\phi_1 < \phi_1^{PD}$.[3]

It can be concluded that with heterogeneous expectations, the range of policy parameters that are allowed in order to have a locally stable fundamental steady state is strictly larger (in both directions) than the range of parameters allowed under rational expectations in order to have a locally determinate equilibrium. The intuition for this result is that the central bank has some credibility close to the steady state. In particular, local stability of the steady state requires that the system converges when half of the agents believe in the target of the central bank. The central bank then only has to stabilize the expectations of the other half of the agents, which results in weaker conditions on monetary policy. For global stability of the steady state, it is however required that the model converges even when the system is far away from steady state, and the central bank has lost all its credibility (all naive expectations). The conditions for that case coincide with the conditions for a locally determinate rational expectations equilibrium. This is stated in Proposition 3, the proof of which is given in the online appendix.

Proposition 3. *The fundamental steady state is globally stable when the central bank chooses*

$$1 - (1-\beta)\frac{\phi_2}{\kappa} = \phi_1^{GL} < \phi_1 < \phi_1^{GU} = 1 + (1+\beta)\frac{2\sigma - \phi_2}{\kappa}. \tag{16}$$

The global stability region of the fundamental steady state is indicated by the thick black line in Fig. 1. For this region of policy parameters, no unstable steady states or 2-cycles exist.

3.2. Policy implications

The difference between the local and the global stability results of the previous section highlight the importance of the credibility of the central bank in stabilizing the economy. When the central bank is able to retain a substantial amount of credibility, even after a sequence of bad shocks, its conditions on monetary policy will not be very restrictive and lie close to those given in Proposition 2. In our model, this situation would e.g. occur if the intensity of choice is not too high. If, on the other hand, the central bank is likely to lose *all* its credibility after a sequence of bad shocks, the restrictions on monetary policy lie close to those given in Proposition 3. This is in line with the results of the agent-based model of Salle et al. (2013), who find that under low (exogenous) credibility of the central bank's inflation target, conditions on policy parameters are much more restrictive than under high credibility. As in our model, the Taylor principle is furthermore not a necessary condition in the latter case.

To investigate whether the stability conditions are likely to be satisfied in practice, we consider three different parameterizations. The first two columns of Table 1 give the calibrations of σ and κ of Woodford (1999), Clarida et al. (2000), and McCallum and Nelson (1999). Under all calibrations, the discount factor is $\beta = 0.99$.

Columns 3 and 4 of Table 1 state the values of the pitchfork bifurcation (ϕ_1^{PF}) and the period doubling bifurcation (ϕ_1^{PD}), assuming that $\phi_2 = \sigma$. Columns 5 and 6 state the values of the global stability restrictions given in Proposition 3. From

[3] We also analyzed local stability under a more traditional Taylor rule where the central bank responds to contemporaneous values of inflation and output. Here we find that local stability of the fundamental steady state requires that $\phi_1 > \frac{1}{2}(1 - (2-\beta)\frac{\sigma + 2\phi_2}{2\kappa})$, which is a strictly weaker condition than the one found in Proposition 2. Furthermore, just as under rational expectations, there is no upper bound on the monetary policy parameters under a contemporaneous Taylor rule.

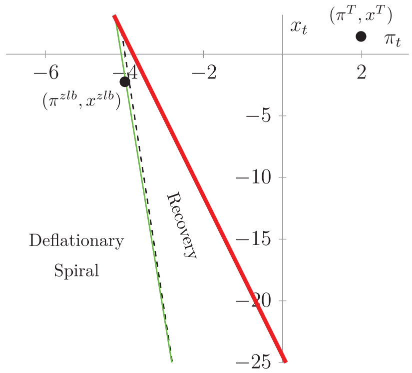

Fig. 2. Conditions for recovery and divergence, presented in Proposition 7, for the Woodford (1999) calibration in the annualized (π, x)-plane. Here $b = +\infty$ and $m^{\pi}_{t+1} = m^{\pi}_{t+2} = m^{x}_{t+1} = m^{x}_{t+2} = -1$. The thick red line indicates the ZLB for naive expectations. The black dot at 2% inflation indicates the target steady state, while the black dot just below the ZLB depicts the unstable saddle steady state. For initial conditions to the left of the solid green and black dashed lines, a deflationary spiral occurs. For initial conditions to the right of these two lines, recovery to the positive interest region occurs. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Column 3 it follows that the pitchfork bifurcation occurs at negative values for all calibrations. This means that, in contrast to the Taylor principle, under these calibrations, the fundamental steady state is locally stable for monetary policy that reacts weakly to inflation ($0 < \phi_1 < 1$) as long as $\phi_2 = \sigma$. This result furthermore turns out to hold for any non-negative choice of ϕ_2. The values of the period doubling bifurcation (ϕ_1^{PD}) given in Column 4 of Table 1 are all unrealistically high. This means that when $\phi_2 = \sigma$, reacting too strongly to inflation will not be a problem for any reasonable value of ϕ_1. When ϕ_2 is increased, the values of ϕ_1^{PD} decline. However, even when ϕ_2 is set to 0.375 (which corresponds to a coefficient of 1.5 when annual rather than quarterly data are used, and which can be considered as very high), the period doubling bifurcation takes place at $\phi_1^{PD} = 7$ under the Woodford (1999) calibration, and even higher values under the other two calibrations. Hence, we can conclude that instability due to the period doubling bifurcation only takes place when monetary policy is extremely aggressive.

4. Zero lower bound on the nominal interest rate

In the previous section, no restrictions were placed on the values that can be taken by the nominal interest rate. In practice, the nominal interest rate will however never be set negative. We will show that if the zero lower bound (ZLB) is accounted for, the global stability result of Proposition 3 no longer holds, but that deflationary spirals with ever decreasing inflation and output gap can arise, under any specification of the Taylor rule. We show this in a sequence of propositions for the limiting case of infinite intensity of choice, i.e., when all agents immediately switch to the best predictor, in Section 4.2. In Section 4.3, we argue that for finite intensity of choice, qualitatively similar dynamics occur.

First, Section 4.1 presents a brief discussion of the existence of an extra steady state under the ZLB under rational expectations. Section 4.2 then moves on to show that also in our model the introduction of the ZLB can lead to the existence of an additional steady state (Proposition 4), and that the presence of this steady state causes divergence to minus infinity for low inflation and output gap in our model, just as in Evans et al. (2008).

Whether such a deflationary spiral occurs, however, not only depends on initial inflation and output gap, but also on the *credibility* of the central bank (i.e., the fractions of fundamentalists). We argue that a liquidity trap can never arise as long as the CB retains full credibility (Proposition 5), and that a self-fulfilling deflationary spiral only arises when naive agents perform better than fundamentalists about both variables (Proposition 6). However, even in a liquidity trap where the CB has lost *all* its credibility, recovery is still possible if inflation and output gap are not too low. The deflationary spiral and recovery regions are illustrated in Fig. 2. The corresponding sufficient conditions for recovery or a deflationary spiral to occur will be presented in Proposition 7.

Our model is capable of describing expectations-driven liquidity traps. Low initial inflation and output gap can be interpreted as having been caused by a negative shock to the fundamentals of the economy. Under rational expectations, the economy would immediately recover from such a shock if it has no persistence. This is not the case in our model with heterogeneous expectations. Here it is likely that the low realizations of inflation and output gap caused by the shock, lead to a loss of credibility of the central bank, i.e., a higher fraction of naive agents. These naive agents expect the low realizations of inflation and output gap, and therefore the liquidity trap, to continue. These low expectations then lead to low realizations of inflation and output gap in the next period, so that the liquidity trap indeed continues, and expectations become self-fulfilling. If the shocks to inflation and output gap were not too large, or if the central bank retained enough credibility, both variables will start to rise again, and recovery to the positive interest rate region occurs. However, if expectations are too low, inflation and output gap start to decline, resulting in more loss of credibility. The economy then ends up in a self-fulfilling deflationary spiral with zero credibility for the central bank.

4.1. Rational expectations

It is well established that under rational expectations, the New Keynesian model with a ZLB on the nominal interest rate has two steady states. In particular, as first highlighted by Benhabib et al. (2001a,b), there are two combinations of a nominal interest rate and an inflation rate that satisfy the Taylor rule with a ZLB as well as the Fisher equation, $i - \pi = \bar{r}$. One of these solutions is the target steady state, while the other steady state has a binding ZLB with $i = 0$ and $\pi = -\bar{r}$. This second steady state, where monetary policy is passive because the nominal interest rate cannot be set negative, is locally indeterminate under rational expectations.

Below, we investigate whether multiple steady states also arise in our model of bounded rationality when the ZLB on the nominal interest rate is introduced, and what this implies for the dynamics.

4.2. Analysis for infinite intensity of choice

When the ZLB is introduced in our model, the nominal interest rate becomes piecewise linear. In normal times the interest rate is still given by Eq. (4) and the model is as in Section 3. However, when (4) implies that the nominal interest rate is negative, it is instead set equal to zero. We will refer to combinations of expectations where $i_t > 0$ as the "positive interest rate region". Combinations of expectations that imply a binding ZLB will be referred to as the "ZLB region". In the ZLB region, the model is described by

$$x_t = \frac{(1+m_t^x)}{2}x^T + \frac{(1-m_t^x)}{2}x_{t-1} + \frac{(1+m_t^\pi)}{2\sigma}\pi^T + \frac{(1-m_t^\pi)}{2\sigma}\pi_{t-1} + \frac{\bar{r}}{\sigma}, \tag{17}$$

$$\pi_t = \beta\frac{(1+m_t^\pi)}{2}\pi^T + \beta\frac{(1-m_t^\pi)}{2}\pi_{t-1} + \kappa x_t, \tag{18}$$

with fractions given by (12) and (13). The steady states of this nonlinear system depend on the fractions of agents following the different heuristics and therefore are, in general, quite difficult to analyze. For this reason, we first consider the limiting case where the intensity of choice equals infinity, and all agents immediately switch to the best performing heuristic. The (six-dimensional) system then becomes piecewise linear.

Proposition 4 states that, when the intensity of choice equals infinity, at most one (unstable) steady state exists in the ZLB region. In this steady state, fundamentalists make persistent prediction errors, implying that *all* agents have switched to naive expectations about both variables. Naive agents do not make prediction errors, so in this steady state, expectations are perfectly self-fulfilling. The proof of Proposition 4 is given in the online appendix.

Proposition 4. *For $b = +\infty$ there exists exactly one steady state in the ZLB region when the Taylor principle is adhered to ($\phi_1 > 1 - (1-\beta)\frac{\phi_2}{\kappa}$). This steady state is given by $\pi = -\bar{r}$, $x = -\frac{1-\beta}{\kappa}\bar{r}$ and $m^\pi = m^x = -1$, and always is an unstable saddle point. When the Taylor principle is not adhered to ($\phi_1 < 1 - (1-\beta)\frac{\phi_2}{\kappa}$), no steady states exist in the ZLB region.*

Initial conditions in the ZLB region typically will not lead to convergence to the steady state of Proposition 4 since it is unstable. Two generic possibilities that can occur for initial conditions in the ZLB region are the following. First of all, it is possible that inflation and output gap start increasing and at some point cause the system to cross the ZLB and enter the positive interest rate region. From then on, the dynamics will be as in Section 3. That is, for monetary policy that satisfies the conditions of Proposition 3 (and under some additional restrictions also policy that satisfies the conditions of Proposition 2) convergence to the fundamental steady state occurs. The other possibility for dynamics in the ZLB region is that inflation and output gap decline towards minus infinity. Such a deflationary spiral can be interpreted as an inescapable liquidity trap. We will refer to the first case as "recovery", and to the second case as "divergence", or as a "deflationary spiral".

When the intensity of choice equals infinity, all fractions are either −1, 1 or 0. Fractions of 0 will typically only occur in a steady state and are not relevant for out-of-steady-state dynamics. This possibility will therefore not be considered below. Four out of six state variables presented in (14) then take on only two different values. These four state variables (fractions for inflation and output gap in the coming two periods) can be represented by a table of 16 different combinations. This is done in Table 2, illustrating which initial conditions lead to recovery (rather than divergence).

In Proposition 5 it is stated that for the four cases in the first column of Table 2, initial fractions are such that the system already is in the positive interest rate region from the very first period onward. For the cases in the first row of Table 2, the system trivially is in the positive interest rate region after one period. Therefore, it is indicated in Table 2 that for these cases recovery "Always" occurs. The intuition behind this result is that the economy can never be in a liquidity trap when the central bank has full credibility. Proof of Proposition 5 is given in the online appendix.

Proposition 5. *If at any point in time all agents are fundamentalist about both inflation and output gap ($m_t^\pi = 1$ and $m_t^x = 1$; full credibility), recovery has occurred.*

For the remaining nine cases of Table 2, recovery or divergence occurs conditional on the initial conditions of the other two state variables: π_t and x_t. For most cases, it is not straightforward to define exactly for which values of π_t and x_t recovery and divergence occur. However, if a deflationary spiral occurs this must be because all agents have (negative) naive

Table 2
Conditions for recovery for different initial conditions when $b = +\infty$.

	$m^{\pi}_{t+1} = 1$		$m^{\pi}_{t+1} = -1$	
	$m^{x}_{t+1} = 1$	$m^{x}_{t+1} = -1$	$m^{x}_{t+1} = 1$	$m^{x}_{t+1} = -1$
$m^{\pi}_{t+2} = 1,\ m^{x}_{t+2} = 1$	Always	Always	Always	Always
$m^{\pi}_{t+2} = 1,\ m^{x}_{t+2} = -1$	Always	-	-	-
$m^{\pi}_{t+2} = -1,\ m^{x}_{t+2} = 1$	Always	-	-	-
$m^{\pi}_{t+2} = -1,\ m^{x}_{t+2} = -1$	Always	-	-	Deflationary spiral case

When $b = +\infty$, the fraction variables m^{π}_{t+1}, m^{x}_{t+1}, m^{π}_{t+2} and m^{x}_{t+2} take on (excluding knife edge cases) only the values -1 or 1. The table displays that for some of the 16 possible combinations of these state variables, recovery to the positive interest region "Always" occurs. The case with $m^{\pi}_{t+1} = m^{x}_{t+1} = m^{\pi}_{t+2} = m^{x}_{t+2} = -1$ is the "Deflationary spiral case", discussed in Proposition 6. Conditions for recovery for this case are presented in Proposition 7.

expectations about both variables after a few periods. That is, as long as the CB retains some credibility, the economy has not (yet) entered a deflationary spiral. This is stated more formally in Proposition 6, the proof of which is given in the online appendix.

Proposition 6. *A necessary condition for a deflationary spiral to occur is that at some point in time, $s \geq t$, all agents are naive with respect to both inflation and output gap for the next two periods ($m^{\pi}_{s+1} = -1$, $m^{x}_{s+1} = -1$, $m^{\pi}_{s+2} = -1$ and $m^{x}_{s+2} = -1$).*

From Proposition 6 it follows that a necessary condition for a deflationary spiral is that the system at some point has moved to the bottom right entry of Table 2, with all naive expectations. This entry is, therefore, the most interesting case when a deflationary spiral is concerned, and hence is labeled the "deflationary spiral case". From any other entry in Table 2 either recovery occurs, or the system moves to the deflationary spiral case, after which the occurrence of recovery or divergence depends on the conditions of that case. We, therefore, do not present individual conditions for all these cases, but instead focus on the deflationary spiral case.

If, in the deflationary spiral case, initial inflation and output gap are too low, expectations will remain naive, and output gap and inflation will keep decreasing without bound. If, however, initial inflation and output gap are high enough, recovery occurs, either because of (positive) naive expectations, or because at some point all agents become fundamentalists.

In Proposition 7, sufficient conditions for both recovery and divergence for the deflationary spiral case are presented. This proposition thereby also proves that it is always possible to find initial conditions that lead to a deflationary spiral in our model. The proof of Proposition 7 is presented in the online appendix, and the corresponding deflationary spiral and recovery regions will be presented in Fig. 2.

Proposition 7. *(See Fig. 2) If all agents' expectations about both inflation and output gap are naive for two consecutive periods ($m^{\pi}_{t+1} = m^{\pi}_{t+2} = m^{x}_{t+1} = m^{x}_{t+2} = -1$) a sufficient condition for recovery is that*

$$x_t > \max(\underline{x}^{ev}, \underline{x}^{out}), \tag{19}$$

and a sufficient condition for a deflationary spiral (i.e. divergence) is that

$$x_t < \min(\bar{x}^{ev}, \bar{x}^{out}, \bar{x}^{inf}), \tag{20}$$

where $\underline{x}^{ev}$, $\underline{x}^{out}$, $\bar{x}^{ev}$, $\bar{x}^{out}$ and $\bar{x}^{inf}$ are defined in the online appendix.

The above implies that for infinite intensity of choice, a deflationary spiral can always occur if initial inflation and output gap are low enough.

The intuition for the conditions that need to be satisfied to provide a sufficient condition for a deflationary spiral is the following. First of all, $x_t < \bar{x}^{ev}$ guarantees that inflation and output gap will keep decreasing as long as expectations about both variables remain naive (initial conditions must lie below the stable eigenvector of the saddle point steady state). Secondly, $x_t < \bar{x}^{out}$ and $x_t < \bar{x}^{inf}$ guarantee that agents do not become fundamentalists about respectively output gap and inflation. Similar intuitions hold for the sufficient condition for recovery (see the proof of Proposition 7 for details).

Fig. 2 plots the conditions from Proposition 7 in the (π, x)-plane for the Woodford (1999) calibration. The thick red line indicates the naive expectations ZLB for our benchmark calibration and an annualized inflation target of 2%. This line separates the positive interest rate region from the ZLB region. Under the above calibration, $x_t < \bar{x}^{inf}$ is always satisfied when $x_t < \bar{x}^{ev}$ and $x_t < \bar{x}^{out}$ hold. This condition is, therefore, not plotted in Fig. 2. $\bar{x}^{ev}$ and $\bar{x}^{out}$ are plotted in respectively solid green and dashed black. For initial conditions to the left of these two lines, the sufficient conditions for a deflationary spiral are satisfied, and inflation and output gap will keep decreasing. For initial conditions to the right of these two lines (at least for the range of values considered in the figure), the sufficient conditions for recovery[4] are satisfied, and inflation and output gap will increase towards the positive interest rate region.[5]

From the steepness of the green and dashed lines, and from the difference in scale on the axes of Fig. 2, it can be concluded that under this calibration, inflation expectations are considerably more important than output gap expectations in determining whether recovery or divergence occurs. Similar results are obtained under the Clarida et al. (2000) calibration.

4.3. Finite intensity of choice

Now, turn to the more general case of finite intensity of choice, where most, but not all, agents switch to the best performing rule. Because the system is linear in fractions, consider first the other limiting case where the intensity of choice is zero. Here, always half of the agents are naive, and half are fundamentalists about each variable. Proposition 8 describes the dynamics of this system. Its proof is given in the online appendix.

Proposition 8. *Assume $b = 0$. If $\kappa \leq \frac{(2-\beta)\sigma}{2}$, the system described by (17), (18), (12) and (13) has a unique, stable steady state with inflation and output gap above their targets. If $\kappa > \frac{(2-\beta)\sigma}{2}$, the system has an unstable saddle steady state with inflation and output gap below their targets. Furthermore, the stable eigenvector of the system then has the same slope as that of the system with $b = +\infty$ and all naive expectations.*

In all calibrations we consider, it holds that $\kappa < \frac{(2-\beta)\sigma}{2}$. It then follows from Proposition 8 that with $b = 0$, all initial conditions under the ZLB lead to recovery (they are attracted to a steady state in the positive interest rate region). When the naive heuristic is best performing, the set of initial conditions that lead to recovery is, therefore, strictly larger when $b = 0$ (where half of the agents remain fundamentalists) than when $b = +\infty$.

In line with Proposition 6, the naive heuristic must necessarily be best performing for a deflationary spiral to arise. Focusing on this case, the following can be concluded for finite intensity of choice.[6] Since under finite intensity of choice the system is a convex combination of the systems with $b = 0$ and $b = +\infty$, it follows that a lower intensity of choice leads to a larger region of initial inflation and output gap from which recovery occurs. In that case, the lines in Fig. 2 that separate the deflationary spiral region from the recovery region are moved to the left. The intuition is that a lower intensity of choice results in a significant fraction of fundamentalists (higher credibility), even when inflation and output gap are low for a few periods. These fundamentalists put upward pressure on output gap and inflation, and thereby prevent divergence for some initial conditions where a deflationary spiral would have occurred for infinite intensity of choice. However, for lower inflation and output gap, more and more agents become naive, so that eventually (almost) all agents are naive, just as in Section 4.2, and deflationary spirals still arise.

5. Monetary policy and liquidity traps

Shocks to inflation and output gap can push the economy into a liquidity trap by triggering low self-fulfilling expectations. How can monetary policy prevent these self-fulfilling liquidity traps? In this section, we address this question with stochastic simulations. These simulations serve two purposes. First, they illustrate in an intuitive way how stochastic shocks can push our economy with heterogeneous expectations and a ZLB on the nominal interest rate into an expectations-driven liquidity trap (Section 5.2). Secondly, we study the effectiveness of an increased inflation target and aggressive monetary easing in preventing a liquidity trap (Section 5.3).

5.1. Calibration

Unless stated otherwise, the following parameterization is used. We follow Woodford (1999) with $\kappa = 0.024$, $\sigma = 0.157$ and $\beta = 0.99$. We further use the policy coefficients derived by Evans and Honkapohja (2003), $\phi_1 = 1 + \frac{\sigma\kappa\beta}{\mu+\kappa^2}$, $\phi_2 = \sigma$. Here the weight on output gap in the loss function of the central bank is set to $\mu = 0.25$, following McCallum and Nelson (2004) and Walsh (2003). The policy coefficients then are given by $\phi_1 = 1.015$ and $\phi_2 = 0.157$. The inflation target is set equal to an annualized 2%.

The shocks to output gap (u_t) and to inflation (e_t), presented in Eqs. (1) and (2), are reintroduced in this section. Both u_t and e_t are defined as Gaussian white noise and are calibrated to have an annualized standard deviation of 1.1%. The calibration of the standard deviations of shocks determines the likelihood of the occurrence of periods where the ZLB binds, as well as their severity. With the chosen parameterization, liquidity traps do arise, but they are not so severe that no

---

[4] Our "recovery region" is strongly related to the "corridor of stability" discussed in Benhabib et al. (2014) and Eusepi (2010).

[5] In the online appendix, it can be seen that as long as $\pi_t < \pi^T$ (which is the case for the green and the dashed lines in Fig. 2), it holds that $\underline{x}^{ev} = \bar{x}^{ev}$ and $\underline{x}^{out} = \bar{x}^{out}$.

[6] Note that when the system is not in the deflationary spiral case, it may be that at some point in time fundamentalist expectations perform better than naive expectations. In this case, a lower intensity of choice leads to less fundamentalists. It may then be that for some initial conditions recovery is assured in the infinite intensity of choice case, but divergence occurs for finite intensity of choice. This is, however, a practically less relevant case.

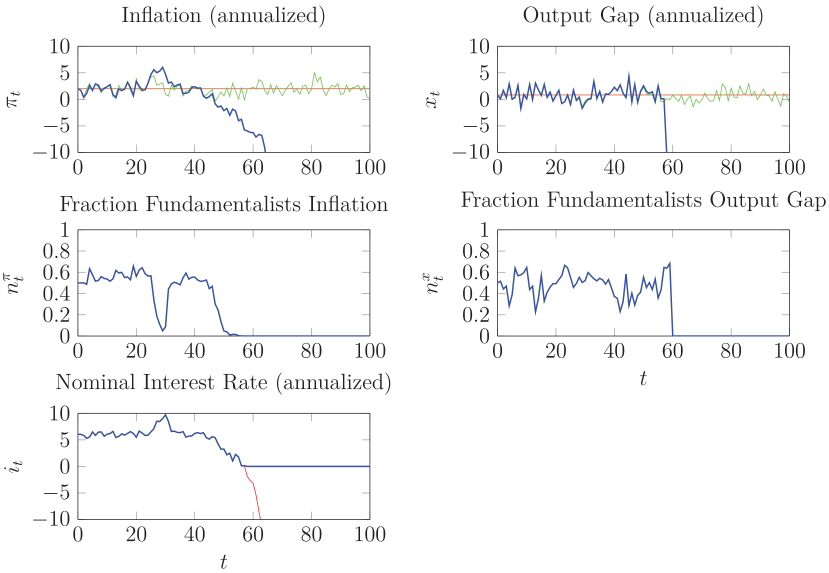

Fig. 3. Simulated time series. In the upper panels, the blue lines depict inflation and output in our model, the thin green lines depict time series that would have occurred under rational expectations, and the horizontal red lines depict the inflation and output gap targets. The bottom panel depicts both the actual interest rate (blue) and the rate prescribed by Eq. (4) (thin red). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

reasonable policy measure can prevent them. We initialize the simulations at the target steady state.[7] The same random seed will be used throughout this section.

Finally, the parameters of the heuristic switching model need to be calibrated. The memory parameter in the fitness measure, (8), is set to $\rho = 0.5$, allowing agents to update their evaluation of the heuristics significantly when new information arises, but also to put considerable weight on the past.[8] The intensity of choice is set to $b = 40.000$, so that it is possible that almost all agents switch to the same heuristic, but that typically both fundamentalists and naive agents will be present.[9]

5.2. The effect of the zero lower bound

Because of the presence of shocks in the model, inflation and output gap no longer exactly converge to a steady state, but instead keep fluctuating around it. First, the model is simulated for 100 periods assuming there is no ZLB on the nominal interest rate. In Fig. 3, the time series of annualized inflation (upper left panel) and annualized output gap (upper right panel) are plotted in blue, together with the fractions of fundamentalists (Credibility) for both inflation (middle left panel) and output gap (middle right panel). The bottom panel depicts the annualized nominal interest rate in blue, and the value that the Taylor rule would prescribe in the absence of the ZLB in thin red. In the upper two panels, the horizontal red lines indicate the inflation and output gap targets, and the thin green curves depict inflation and output gap time series that would have occurred under rational expectations.

Fig. 3 illustrates that there are periods where inflation fluctuates closely around the target, and periods where inflation drifts away (e.g. around period 30). The intuition behind upward drifts in inflation is the following. When shocks lead to inflation above target for a few consecutive periods, the central bank loses credibility, and most agents become naive with respect to inflation (as can be seen in the middle left panel of Fig. 3, where the fraction of fundamentalists about inflation moves towards 0). The high expectations of naive agents put upward pressure on inflation and become self-fulfilling. Meanwhile, the central bank tries to control inflation by increasing the interest rate, but does not immediately succeed. This is so because the CB also cares about the output gap, and does not want to react too strongly to inflation expectations in order to limit variations in the output gap. Indeed, it can be seen that the output gap stays very close to its target during all periods, and eventually inflation returns to its target as well.

A similar drift in self-fulfilling naive expectations, but then downward, arises around period 50. However, now the nominal interest rate hits the ZLB around period 60, so that it is set sub-optimally high. This implies a high real interest rate,

[7] Different initial conditions would change the blue curves for only the first four periods or so, and the thin green curves only in the first period.

[8] We also ran all simulations in this section with $\rho = 0$. This only changes result quantitatively. The policy measures presented in Section 5.3 still work to prevent liquidity traps with a lower memory parameter, but the magnitude of the policy change needed to achieve this is larger in that case.

[9] Note that the calibration of the intensity of choice depends on the unit of measurement of the fitness measure. Since a 1% deviation of quarterly inflation from steady state is measured as 0.01 and results in a squared forecast error of 0.0001, an intensity of choice of 40.000 should not be considered particularly large.

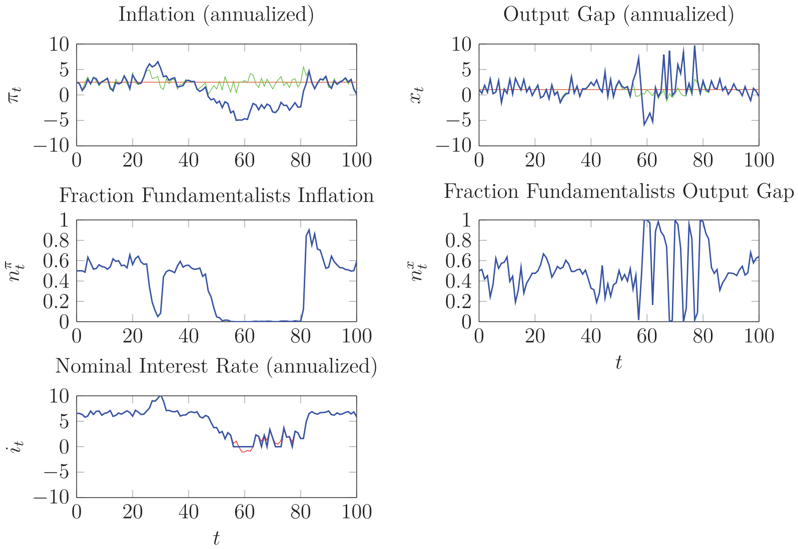

Fig. 4. Simulated time series in case of an increased inflation target of 2.5% and aggressive monetary easing: the interest rate is set to 0 when it falls below the threshold of 1.5%. In the upper panels, the blue lines depict inflation and output in our model, the thin green lines depict time series that would have occurred under rational expectations, and the horizontal red lines depict the inflation and output gap targets. The bottom panel depicts both the actual interest rate (blue) and the rate prescribed by Eq. (4) (thin red). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

which depresses the output gap. Therefore, the economy enters a recession. Furthermore, the CB loses credibility with respect to output gap as well as inflation, implying that the economy is in the deflationary spiral case of Table 2 in Section 4.2. Moreover, inflation and output gap have become so low, that the economy is in the deflationary spiral region of Fig. 2, and inflation and output gap keep decreasing while expectations remain naive. Hence, the system has entered a self-fulfilling deflationary spiral.

Comparing the blue curves in the top panels with the thin green curves, it can be concluded that the drifts in inflation indeed arise because of expectations and a loss in credibility and not just because of shocks. With the same shock sequence, rational expectations would lead both inflation and output gap to always stay close to their targets, and the ZLB would not become binding. In the online appendix it is shown that when the intensity of choice is set so low that credibility does not significantly change after a sequence of positive or negative shocks, inflation and output gap remain relatively stable and close to the rational expectations benchmark. We consider this however to be a less realistic calibration, because Cornea-Madeira et al. (2019) and Branch (2004, 2007) find considerable switching between our heuristics in inflation and survey data. In the online appendix it is further shown that for higher values of the intensity of choice parameter, where there are larger fluctuations in credibility, similar dynamics of inflation, output gap and the interest rate arise as in Fig. 3, indicating robustness of the above simulation.

5.3. Preventing deflationary spirals

How can the central bank prevent such a deflationary spiral? One possible solution would be to respond with aggressive monetary easing (ME) as soon as a liquidity trap is imminent. The central bank could set the interest rate as low as possible (i.e. zero) as soon as it would otherwise have set the interest rate below some threshold. That is,

$$i_t^{ME} = \begin{cases} i_t & : i_t \geq TR \\ 0 & : i_t < TR \end{cases} \tag{21}$$

The threshold, TR, indicates a danger zone of a low interest rates that threaten to fall below zero.

Another possibility is increasing the inflation target, which would make it less likely that the ZLB becomes binding and might limit how low inflation and output gap become when it does become binding. This could guarantee that the system remains in the recovery region of Fig. 2 and a deflationary spiral does not occur.

It turns out that both an increased inflation target and aggressive monetary easing can indeed prevent deflationary spirals in our simulations. Furthermore, the two measures are complements. When there is aggressive monetary easing in place, the inflation target needs to be raised by less to prevent a particular deflationary spiral, and vice versa.

Fig. 4 illustrates that a combination of an increased inflation target and aggressive monetary easing indeed can prevent the deflationary spiral that occurred in Fig. 3. The inflation target is here set to an annualized 2.5%, and the central bank conducts aggressive monetary easing as soon as the annualized interest rate would have fallen below 1.5% ($TR = 0.015/4$).

During the period of low inflation from period 50 onward, the higher inflation target and the aggressive monetary easing work together to prevent a deflationary spiral. Inflation still falls and inflation expectations still become naive, but due to the increased inflation target, there is more room for inflation to fall before the ZLB is hit. However, inflation does fall enough for interest rates to drop below 1.5%. This induces the central bank to start with aggressive monetary easing by setting the interest rate to 0%, which leads to a lower real interest rate than would otherwise have occurred. As a consequence, the output gap turns positive, which puts upward pressure on inflation, limiting the decline of inflation even more. When, in period 58, the interest rate finally reaches the ZLB, inflation and output gap are now high enough for the system to remain in the recovery region of Fig. 2 and a deflationary spiral does not arise. Even though inflation expectations remain naive, inflation starts to rise slowly. During the subsequent recovery phase where the ZLB is no longer binding, the central bank keeps regularly applying aggressive monetary easing to let inflation increase towards its target more rapidly. Eventually, both inflation and output gap are brought back to their targets, and the central bank regains its credibility.

It must be noted, however, that the aggressive monetary easing leads to large output gap fluctuations, which is obviously not desirable from a welfare perspective. Moreover, an increased inflation target is generally considered to be costly as well. However, if the alternative is a long-lasting liquidity trap and/or deflationary spiral, then these costs may be worth paying. In the online appendix, it is illustrated though, that there are also other measures that can help in preventing deflationary spirals, such as increasing the coefficient on inflation in the Taylor rule.

6. Conclusion

A New Keynesian model is used to study inflation targeting and liquidity traps. Instead of assuming rational expectations, we allow expectations to be formed heterogeneously by using a model where agents switch between heuristics based on relative performance. In our model, fundamentalists, who trust the central bank, compete with naive agents, who base their forecast on past information. The fraction of fundamentalist agents can, therefore, be interpreted as the credibility of the central bank. Unlike in rational expectations models, this allows us to *endogenously* model the central bank's credibility, which is of crucial importance in understanding liquidity traps.

Our first finding is that a nominal interest rate that responds too weakly or too strongly to output gap expectations leads to instability of the fundamental steady state. In this steady state, both inflation and output gap are equal to the targets set by the central bank. The region of policy parameters that lead to local stability of the fundamental steady state is however strictly larger than the region of policy parameters that result in a locally determinate equilibrium when rational expectations are assumed. In fact, we find that the well known Taylor principle is not a necessary condition for local stability in our heterogeneous expectations model.

When the ZLB is introduced in our model, we find that expectations-driven liquidity traps can occur, for all parameterizations of the policy rule. In such a liquidity trap, the central bank has lost some, or all, of its credibility, and low, naive expectations make the ZLB constraint binding. Whether the economy can recover from such a liquidity trap, or whether a deflationary spiral with ever decreasing inflation and output gap occurs, depends critically both on how low inflation and output gap have become, and on how much credibility the central bank is able to retain. If the central bank has lost too much credibility, and inflation and output gap are too low, more and more agents start to coordinate on low, naive expectations, resulting in a self-fulfilling deflationary spiral. Coordination on naive expectations or on some other form of adaptive expectations is an empirically relevant and plausible situation that is encountered e.g. in Assenza et al. (2014) and other laboratory experiments.

With stochastic simulations, it is shown that small shocks to the economy can lead to coordination on low naive expectations, and that this can result in deflationary spirals. It is furthermore shown that a central bank can prevent deflationary spirals by increasing the inflation target and applying aggressive monetary easing when the interest rate becomes too low. These policy measures come, however, with their own disadvantages and costs to the economy. Therefore, a well balanced combination of different measures may be the best way to proceed.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.jmoneco.2019.01.027.

References

Agliari, A., Pecora, N., Spelta, A., 2015. Coexistence of equilibria in a new keynesian model with heterogeneous beliefs. Chaos, Solitons & Fractals 79, 83–95.

Anufriev, M., Assenza, T., Hommes, C., Massaro, D., 2013. Interest rate rules and macroeconomic stability under heterogeneous expectations. Macroecon. Dyn. 17, 1574–1604.

Arifovic, J., Petersen, L., 2015. Escaping expectations-driven liquidity traps: Experimental evidence. Manuscript, Simon Fraser University.

Assenza, T., Heemeijer, P., Hommes, C., Massaro, D., 2014. Managing self-organization of expectations through monetary policy: a macro experiment. Technical Report. CeNDEF Working Paper, University of Amsterdam.

Benhabib, J., Evans, G.W., Honkapohja, S., 2014. Liquidity traps and expectation dynamics: fiscal stimulus or fiscal austerity? J. Econ. Dyn. Control 45, 220–238.

Benhabib, J., Schmitt-Grohé, S., Uribe, M., 2001. Monetary policy and multiple equilibria. Am. Econ. Rev. 91 (1), 167–186.

Benhabib, J., Schmitt-Grohé, S., Uribe, M., 2001. The perils of taylor rules. J. Econ. Theo. 96 (1), 40–69.

Branch, W.A., 2004. The theory of rationally heterogeneous expectations: evidence from survey data on inflation expectations. Econ. J. 114 (497), 592–621.

Branch, W.A., 2007. Sticky information and model uncertainty in survey data on inflation expectations. J. Econ. Dyn. Control 31 (1), 245–276.
Branch, W.A., McGough, B., 2010. Dynamic predictor selection in a new keynesian model with heterogeneous expectations. J. Econ. Dyn. Control 34 (8), 1492–1508.
Brock, W.A., Hommes, C.H., 1997. A rational route to randomness. Econometrica J. Econ. Soc. 65, 1059–1095.
Bullard, J., Mitra, K., 2002. Learning about monetary policy rules. J. Monet. Econ. 49 (6), 1105–1129.
Busetti, F., Ferrero, G., Gerali, A., Locarno, A., et al., 2014. Deflationary shocks and de-anchoring of inflation expectations. Technical Report. Bank of Italy, Economic Research and International Relations Area.
Clarida, R., Galí, J., Gertler, M., 2000. Monetary policy rules and macroeconomic stability: evidence and some theory. Quart. J. Econ. 115, 147–180.
Cornea-Madeira, A., Hommes, C., Massaro, D., 2019. Behavioral heterogeneity in u.s. inflation dynamics. J. Busin. Econ. Stat. 37, 288–300.
De Grauwe, P., 2011. Animal spirits and monetary policy. Econ. Theo. 47 (2–3), 423–457.
De Grauwe, P., Ji, Y., 2019. Inflation targets and the zero lower bound in a behavioral macroeconomic model. Economica 86 (342), 262–299.
Eusepi, S., 2010. Central bank communication and the liquidity trap. J. Money Credit Bank. 42 (2–3), 373–397.
Evans, G.W., Guse, E., Honkapohja, S., 2008. Liquidity traps, learning and stagnation. Eur. Econ. Rev. 52 (8), 1438–1463.
Evans, G.W., Honkapohja, S., 2003. Expectations and the stability problem for optimal monetary policies. Rev. Econ. Stud. 70 (4), 807–824.
Evans, G.W., Honkapohja, S., et al., 2009. Learning and macroeconomics. Annu. Rev. Econ. 1 (1), 421–451.
Galí, J., 2002. New perspectives on monetary policy, inflation, and the business cycle. Technical Report. National Bureau of Economic Research.
Hommes, C.H., 2013. Behavioral rationality and heterogeneous expectations in complex economic systems. Cambridge University Press.
Hommes, C.H., 2019. Behavioral & experimental macroeconomics and policy analysis: a complex systems approach. J. Econ. Lit., forthcoming.
Hommes, C., Massaro, D., Salle, I., 2019. Monetary and fiscal policy design at the zero lower bound: evidence from the lab. Econ. Inq. doi:10.1111/ecin.12741.
Kurz, M., Piccillo, G., Wu, H., 2013. Modeling diverse expectations in an aggregated new keynesian model. J. Econ. Dyn. Control 37 (8), 1403–1433.
Mankiw, N.G., Reis, R., Wolfers, J., 2003. Disagreement about inflation expectations. Technical Report. National Bureau of Economic Research.
Manski, C.F., McFadden, D., et al., 1981. Structural analysis of discrete data with econometric applications. Mit Press Cambridge, MA.
Massaro, D., 2013. Heterogeneous expectations in monetary DSGE models. J. Econ. Dyn. Control 37 (3), 680–692.
McCallum, B.T., Nelson, E., 1999. Performance of Operational Policy Rules in an Estimated Semiclassical Structural Model. Monetary policy rules. University of Chicago Press.
McCallum, B.T., Nelson, E., 2004. Timeless perspectives vs. discretionary monetary policy in forward-looking models. Federal Reserve Bank of St. Louis Review 86 (2), 43–56.
Mertens, K.R., Ravn, M.O., 2014. Fiscal policy in an expectations-driven liquidity trap. Rev. Econ. Stud. 81 (4), 1637–1667.
Milani, F., 2007. Expectations, learning and macroeconomic persistence. J. Monet. Econ. 54 (7), 2065–2082.
Orphanides, A., Williams, J., 2004. Imperfect Knowledge, Inflation Expectations, and Monetary Policy. In: The inflation-targeting debate. University of Chicago Press, pp. 201–246.
Orphanides, A., Williams, J.C., 2005. Inflation scares and forecast-based monetary policy. Rev. Econ. Dyn. 8 (2), 498–527.
Orphanides, A., Williams, J.C., 2007. Inflation targeting under imperfect knowledge. Economic Review-Federal Reserve Bank of San Francisco 1.
Orphanides, A., Williams, J.C., 2007. Robust monetary policy with imperfect knowledge. J. Monet. Econ. 54 (5), 1406–1435.
Pfajfar, D., Žakelj, B., 2018. Inflation expectations and monetary policy design: evidence from the laboratory. Macroecon. Dyn. 22 (4), 1035–1075.
Salle, I., Yıldızoğlu, M., Sénégas, M.-A., 2013. Inflation targeting in a learning economy: an ABM perspective. Econ. Model. 34, 114–128.
Taylor, J.B., 1993. Discretion versus policy rules in practice. In: Carnegie-Rochester conference series on public policy, Vol. 39. Elsevier, pp. 195–214.
Walsh, C.E., 2003. Speed limit policies: the output gap and optimal monetary policy. Am. Econ. Rev. 93, 265–278.
Woodford, M., 1999. Optimal monetary policy inertia. The Manchester School 67 (s1), 1–35.
Woodford, M., 2003. Interest and prices: Foundations of a theory of monetary policy. Princeton University Press.
Woodford, M., 2004. Inflation targeting and optimal monetary policy. Federal Reserve Bank of St. Louis Economic Review 86 (4), 15–41.