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Computing Compliance

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Abstract. Inquisitive semantics (cf. Groenendijk, 2008) provides a formal framework for reasoning about information exchange. The central logical notion that the semantics gives rise to is *compliance*. This paper presents an algorithm that computes the set of compliant responses to a given initiative. The algorithm is sound and complete. The implementation is accessible online via `www.illc.uva.nl/inquisitive-semantics`.

1 Introduction

Traditionally, logic is concerned with argumentation. As a consequence, the meaning of a sentence is traditionally identified with its *informative* content. In much recent work, this notion is given a dynamic twist, and the meaning of a sentence is taken to be its potential to change the 'common ground' of a conversation. The most basic way to formalize this idea is to think of the common ground as a set of possible worlds, and of a sentence as providing information by eliminating some of these possible worlds.

Of course, this picture is limited in several ways. First, when exchanging information sentences are not only used to provide information, but also—crucially—to *raise issues*, that is, to indicate which kind of information is desired. Second, the given picture does not take into account that updating the common ground is a *cooperative* process. One conversational participant cannot simply change the common ground all by herself. All she can do is *propose* a certain change. Other participants may react to such a proposal in several ways. Changes of the common ground come about by mutual agreement.

In order to overcome these limitations, *inquisitive semantics* (cf. Ciardelli and Roelofsen, 2009; Groenendijk, 2008; Groenendijk and Roelofsen, 2009; Mascarenhas, 2008) starts with an altogether different picture. It views propositions as proposals to enhance the common ground. These proposals do not always specify just one way of enhancing the common ground. They may suggest alternative ways of doing so, among which the responder is then invited to choose. Formally, a proposition consists of one or more *possibilities*. Each possibility is a set of possible worlds and embodies a possible way to enhance the common ground. If a proposition consists of two or more possibilities, it is *inquisitive*: it invites the other participants to respond in a way that will lead to a cooperative choice between the proposed alternatives. Inquisitive propositions raise an issue. They indicate which kind of information is desired. Thus, inquisitive semantics directly reflects the idea that information exchange consists in a cooperative dynamic process of raising and resolving issues.

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Traditional semantics gives rise to the logical notion of entailment, which judges the validity of argumentation. Inquisitive semantics gives rise to the logical notion of *compliance*, which judges whether or not a sentence makes a significant contribution towards resolving a given issue. Extensive motivation for the precise formulation of compliance can be found in (Groenendijk and Roelofsen, 2009). The aim of the present paper is to devise an algorithm that computes the set of compliant responses to a given initiative. Such an algorithm will form the basis for practical applications of inquisitive semantics.

The paper is organized as follows. Section 2 reviews the basic notions of inquisitive semantics and some basic properties of the system, section 3 discusses and illustrates the definition of compliance, and section 4 presents a sound and complete algorithm for computing compliant responses.

2 Inquisitive Semantics

Definition 1 (Language). Let $\mathcal{P}$ be a finite set of proposition letters that we will consider fixed throughout the paper. We denote by $\mathcal{L}_\mathcal{P}$ the set of formulas built up from letters in $\mathcal{P}$ and $\bot$ using the binary connectives $\wedge, \vee$ and $\rightarrow$. We will refer to $\mathcal{L}_\mathcal{P}$ as the propositional language based on $\mathcal{P}$.

We will also make use of the following abbreviations: $\neg\varphi$ for $\varphi \rightarrow \bot$, $!\varphi$ for $\neg\neg\varphi$, and $?\varphi$ for $\varphi \vee \neg\varphi$.

Definition 2 (Indices). An *index* is a function from $\mathcal{P}$ to $\{0, 1\}$. We denote by ω the set of all indices.

Definition 3 (States). A *state* is a set of indices. We denote by $\mathcal{S}$ the set of all states.

Definition 4 (Support)

1. $s \models p \quad \text{iff} \quad \forall w \in s : w(p) = 1$
2. $s \models \bot \quad \text{iff} \quad s = \emptyset$
3. $s \models \varphi \wedge \psi \quad \text{iff} \quad s \models \varphi$ and $s \models \psi$
4. $s \models \varphi \vee \psi \quad \text{iff} \quad s \models \varphi$ or $s \models \psi$
5. $s \models \varphi \rightarrow \psi \quad \text{iff} \quad \forall t \subseteq s :$ if $t \models \varphi$ then $t \models \psi$

It follows from the above definition that the empty state supports any formula φ. Thus, we may think of $\emptyset$ as the *inconsistent* state.

Fact 5 (Persistence). If $s \models \varphi$ then for every $t \subseteq s$: $t \models \varphi$

Fact 6 (Singleton states behave classically). For any index w and formula φ:

$$\{w\} \models \varphi \iff \varphi \text{ is classically true under the valuation } w$$

In particular, $\{w\} \models \varphi$ or $\{w\} \models \neg\varphi$ for any formula φ.

It follows from definition 4 that the support-conditions for $\neg\varphi$ and $!\varphi$ are as follows.

Fact 7 (Support for negation)

1. $s \models \neg\varphi \quad \text{iff} \quad \forall w \in s : w \models \neg\varphi$
2. $s \models !\varphi \quad \text{iff} \quad \forall w \in s : w \models \varphi$

In terms of support, we define the *possibilities* for a sentence φ and the *proposition* expressed by φ. We also define the *truth-set* of φ, which is the meaning that would be associated with φ in a classical setting.

Definition 8 (Truth sets, possibilities, propositions). Let φ be a formula.

1. A *possibility* for φ is a maximal state supporting φ, that is, a state that supports φ and is not properly included in any other state supporting φ.
2. The *proposition* expressed by φ, denoted by $[\varphi]$, is the set of possibilities for φ.
3. The *truth set* of φ, denoted by $|\varphi|$, is the set of indices where φ is classically true.

Notice that $|\varphi|$ is a state, while $[\varphi]$ is a set of states. The following result guarantees that the proposition expressed by a formula completely determines which states support that formula, and vice versa.

Fact 9 (Support and Possibilities). For any state s and any formula φ:

$$s \models \varphi \quad \Longleftrightarrow \quad s \text{ is contained in a possibility for } \varphi$$

Example 10 (Disjunction). Inquisitive semantics crucially differs from classical semantics in its treatment of disjunction. To see this, consider figures 1(a) and 1(b). In these figures, it is assumed that $\mathcal{P} = \{p, q\}$; index 11 makes both p and q true, index 10 makes p true and q false, etcetera. Figure 1(a) depicts the truth set—that is, the classical meaning—of $p \vee q$: the set of all indices that make either p or q, or both, true. Figure 1(b) depicts the proposition associated with $p \vee q$ in inquisitive semantics. It consists of two possibilities. One possibility is made up of all indices that make p true, and the other of all indices that make q true. So, as in the classical setting, $p \vee q$ is *informative*, in that it proposes to eliminate the index where both p and q are false. But it is also *inquisitive*, in that it proposes two alternative ways of enhancing the common ground, and invites a response that is directed at chosing between these two alternatives. This inquisitive aspect of meaning is not captured in a classical setting.

Definition 11 (Inquisitiveness and informativeness)

– φ is *inquisitive* iff $[\varphi]$ contains at least two possibilities;
– φ is *informative* iff φ proposes to eliminate at least one index, that is, iff $\bigcup[\varphi] \neq \omega$

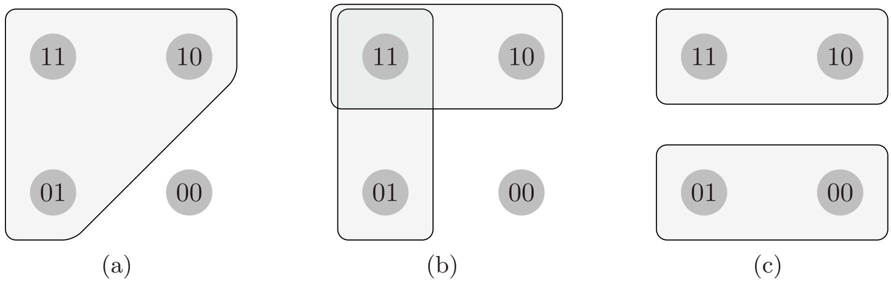


Fig. 1. (a) the classical picture of $p \vee q$, (b) the inquisitive picture of $p \vee q$, and (c) the inquisitive picture of the polar question $?p$

Definition 12 (Questions, assertions, and hybrids)

- φ is a *question* iff it is not informative;
- φ is an *assertion* iff it is not inquisitive;
- φ is a *hybrid* iff it is both informative and inquisitive.

Example 13 (Questions, assertions, and hybrids). We have already seen that $p \vee q$ is both informative and inquisitive, i.e., hybrid. The proposition depicted in figure 1(a) is expressed by $!(p \vee q)$. This proposition consists of exactly one possibility. So $!(p \vee q)$ is an assertion. The proposition depicted in figure 1(c) is expressed by $?p$. This proposition covers the entire logical space, so $?p$ does not propose to eliminate any index. That is, $?p$ is a question.[1]

The following result gives some sufficient syntactic conditions for a formula to be an assertion.

Fact 14. For any proposition letter p and formulas φ, ψ:

1. p is an assertion;
2. $\bot$ is an assertion;
3. if φ, ψ are assertions, then $\varphi \wedge \psi$ is an assertion;
4. if ψ is an assertion, then $\varphi \to \psi$ is an assertion.

Note that items 2 and 4 imply that any negation is an assertion. In particular, $!\varphi$ is always an assertion. In fact, as a consequence of proposition 7, the possibilities for $\neg\varphi$ and $!\varphi$ can be characterized as follows.

[1] Notice that questions do not have to be inquisitive, and assertions do not have to be informative. For instance, the tautology $!(p \vee \neg p)$ is both a question and an assertion, even though (or rather *because*) it is neither inquisitive nor informative. Groenendijk and Roelofsen (2009) give a slightly more involved definition of questions and assertions, which makes sure that the two notions are strictly disjoint. This may be more desirable from a linguistic point of view, but the additional complexity is not quite relevant in the present setting, and is therefore avoided.

Fact 15 (Negation)

1. $[\neg\varphi] = \{|\neg\varphi|\}$
2. $[!\varphi] = \{|\varphi|\}$

Using fact 14 inductively we obtain the following corollary showing that disjunction is the only source of inquisitiveness in our propositional language.

Corollary 16. *Any disjunction-free formula is an assertion.*

In inquisitive semantics, the informative content of a formula φ is captured by the union $\bigcup[\varphi]$ of all the possibilities for φ. For φ proposes to eliminate all indices that are not in $\bigcup[\varphi]$. In a classical setting, the informative content of φ is captured by $|\varphi|$. The following result says that, as far as informative content goes, inquisitive semantics does not diverge from classical semantics. In this sense, inquisitive semantics is a 'conservative extension' of classical semantics.

Fact 17. For any formula φ: $\bigcup[\varphi] = |\varphi|$

We end this section with a definition of inquisitive equivalence.

Definition 18 (Equivalence)
Two formulas φ and ψ are *equivalent*, $\varphi \equiv \psi$, iff $[\varphi] = [\psi]$.

It follows immediately from fact 9 that $\varphi \equiv \psi$ just in case φ and ψ are supported by the same states.

3 Compliance

The notion of compliance judges whether a certain conversational move makes a significant contribution to resolving a given issue. Before stating the formal definition, let us first review some of the basic logico-pragmatical intuitions behind it.

Basic intuitions. Consider a situation where a sentence φ is a *response* to an *initiative* ψ. We are mainly interested in the case where the initiative ψ is inquisitive, and hence proposes several alternatives. In this case, we consider φ to be an optimally compliant response just in case it picks out exactly one of the alternatives proposed by ψ. Such an optimally compliant response is an assertion φ such that the unique possibility α for φ equals one of the possibilities for ψ: $\lfloor\varphi\rfloor = \{\alpha\}$ and $\alpha \in \lfloor\psi\rfloor$. Of course, the responder will not always be able to give such an optimally compliant response. It may still be possible in this case to give a compliant informative response, not by picking out *one* of the alternatives proposed by ψ, but by selecting some of them, and excluding others. The informative content of such a response must correspond with the union of some but not all of the alternatives proposed by ψ. That is, $|\varphi|$ must coincide with the union of a proper non-empty subset of $\lfloor\psi\rfloor$.

If such an informative compliant response cannot be given either, it may still be possible to make a significant compliant move, namely by responding with an inquisitive sentence, replacing the issue raised by ψ with an easier to answer sub-issue. The rationale behind such an inquisitive move is that, if part of the original issue posed by ψ were resolved, it might become possible to subsequently resolve the remaining issue as well.

Summing up, there are basically two ways in which φ may be compliant with ψ:

(a) φ may partially *resolve* the issue raised by ψ;
(b) φ may *replace* the issue raised by ψ by an easier to answer sub-issue.

Combinations are also possible: φ may partially resolve the issue raised by ψ and at the same time replace the remaining issue with an easier to answer sub-issue. What is important is that φ should do nothing *more* than this: it should not provide any information that is not strictly related to the given issue, and it should not raise any issues that are not strictly related to the given issue, or issues that are more difficult to resolve. This means, in particular, that over-informative answers are not compliant. For instance, $p \wedge q$ is not a compliant response to $?p$, because it does not resolve the issue any more than the less informative answer p would do.

These considerations are captured by the following definition:

Definition 19 (Compliance). φ is compliant with ψ, $\varphi \propto \psi$, iff

1. every possibility in $[\varphi]$ is the union of a non-empty set of possibilities in $[\psi]$
2. every possibility in $[\psi]$ restricted to $|\varphi|$ is contained in a possibility in $[\varphi]$

Here, the *restriction* of $\alpha \in [\psi]$ to $|\varphi|$ is defined to be the intersection $\alpha \cap |\varphi|$. To explain the workings of the definition, we will distinguish several cases, depending on whether ψ and φ are assertions, questions or hybrids.

First, consider the case where ψ is an assertion. Then the first clause says that every possibility for φ should coincide with the unique possibility for ψ. This can only be the case if φ is equivalent to ψ. In this case, the second clause is trivially met. Thus, the only way to compliantly respond to an assertion is to confirm it.

Fact 20. If ψ is an assertion, then $\varphi \propto \psi$ iff $[\varphi] = [\psi]$.

If φ is an assertion, then the first clause in the definition of compliance requires that $|\varphi|$ coincides with the union of a set of possibilities for ψ. The second clause is trivially met in this case.

Fact 21. If φ an assertion, then $\varphi \propto \psi$ iff $|\varphi|$ coincides with the union of a non-empty set of possibilities for ψ.

In particular, if φ is an assertion and ψ is *inquisitive*, then fact 21 tells us that φ is compliant with ψ just in case φ partially resolves the issue raised by ψ, without being over-informative. Thus, compliance embodies a strict notion of partial answerhood.[2]

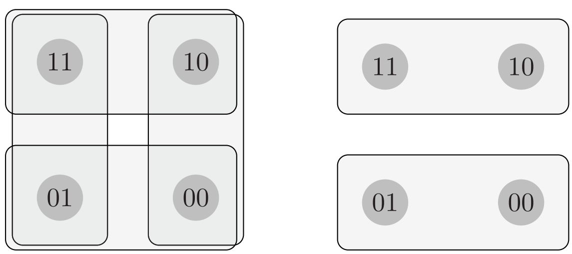
Next, consider the case where φ is a question. Then the first clause in the definition of compliance requires that ψ is a question as well. Moreover, the first clause also requires that every complete answer to φ is at least a partial answer to ψ.

The second clause also plays a role in this case. However, since φ is assumed to be a question, and since questions are not informative, the second clause can be simplified: the restriction of the possibilities for ψ to $|\varphi|$ does not have any effect, because $|\varphi| = \omega$. Hence,the second clause simply requires that every possibility for ψ is contained in a possibility for φ.

Fact 22. If φ is a question, then $\varphi \propto \psi$ iff

1. every possibility in $[\varphi]$ is the union of a non-empty set of possibilities in $[\psi]$
2. every possibility in $[\psi]$ is contained in a possibility in $[\varphi]$

The second constraint prevents φ from being more difficult to answer than ψ. Let us illustrate this with an example. Consider the case where $\psi \equiv ?p \vee ?q$ and $\varphi \equiv ?p$. The propositions expressed by $?p \vee ?q$ and $?p$ are depicted in figure 2.



(a) $?p \vee ?q$ (b) $?p$

Fig. 2. Choice question and polar question

Intuitively, $?p \vee ?q$ is a *choice question*. To resolve it, one may either provide an answer to the question $?p$ or to the question $?q$. Thus, there are four possibilities, each corresponding to an optimally compliant response: p, $\neg p$, q and $\neg q$. The question $?p$ is more demanding: there are only two possibilities and thus only two optimally compliant responses, p and $\neg p$. Hence, $?p$ is more difficult to answer than $?p \vee ?q$, and should therefore not count as compliant with it. This is

[2] Earlier formal analyses of questions (cf. Groenendijk and Stokhof, 1984) usually characterize partial answerhood in terms of entailment. Such characterizations are satisfactory as long as questions are assumed to *partition* logical space. In inquisitive semantics, questions are no longer associated with partitions: possibilities may overlap. As a consequence, partial answerhood cannot be characterized in terms of entailment anymore (cf. Groenendijk and Roelofsen, 2009).

not taken care of by the first clause in the definition of compliance, since every possibility for $?p$ is also a possibility for $?p \vee ?q$. So the second clause is essential in this case: it says that $?p$ is not compliant with $?p \vee ?q$ because two of the possibilities for $?p \vee ?q$ are not contained in any possibility for $?p$. The fact that these possibilities are, as it were, 'ignored' by $?p$ is the reason that $?p$ is more difficult to answer than $?p \vee ?q$.[3]

Notice that the second clause in the definition of compliance only plays a role in case both φ and ψ are inquisitive. Moreover, the *restriction* of the possibilities for ψ to $|\varphi|$ can only play a role if $|\varphi| \subset |\psi|$, which is possible only if φ is informative. Thus, the second clause can only play a role in its unsimplified form if φ is both inquisitive and informative, i.e., hybrid. If φ is hybrid, just as when φ is a question, the second clause forbids that a possibility for ψ is ignored by φ. But now it also applies to cases where a possibility for ψ is partly excluded by φ. The part that remains should then be fully included in one of the possibilities for φ.

As an example where this condition applies, consider $p \vee q$ as a response to $p \vee q \vee r$. One of the possibilities for $p \vee q \vee r$, namely $|r|$, is ignored by $p \vee q$: the restriction of $|r|$ to $|p \vee q|$ is not contained in any possibility for $p \vee q$. Again, this reflects the fact that the issue raised by $p \vee q$ is more difficult to resolve than the issue raised by $p \vee q \vee r$.

A general characterization of what the second clause says, then, is that φ may only remove possibilities for ψ by providing information. A possibility for ψ must either be excluded altogether, or it must be preserved: its restriction to $|\varphi|$ must be contained in some possibility for φ.

4 Computing Compliance

In this section, we specify an algorithm which computes, for a given sentence ψ, all sentences (up to logical equivalence) that are compliant with ψ. In order to do so, we first introduce a procedure DNF, which determines, for any formula ψ, an equivalent formula $\text{DNF}(\psi)$ which is a disjunction of assertions (a *disjunctive normal form*).

Definition 23. $\text{DNF}(\psi)$ is recursively defined as follows:

1. $\text{DNF}(p) = p$
2. $\text{DNF}(\bot) = \bot$
3. $\text{DNF}(\neg\psi) = \neg\psi$
4. $\text{DNF}(\psi \vee \chi) = \text{DNF}(\psi) \vee \text{DNF}(\chi)$
5. $\text{DNF}(\psi \wedge \chi) = \bigvee_{i,j} (\psi_i \wedge \chi_j)$

   where:
   - $\text{DNF}(\psi) = \psi_1 \vee \ldots \vee \psi_n$
   - $\text{DNF}(\chi) = \chi_1 \vee \ldots \vee \chi_m$

[3] Notice that compliance does not hold in the other direction either. That is, $?p \vee ?q$ is not compliant with $?p$ (in this case, the first clause is not satisfied).

  - i ranges over $\{1, \ldots, n\}$
  - j ranges over $\{1, \ldots, m\}$

6. $\textsc{dnf}(\psi \to \chi) = \bigvee_{k_1,\ldots,k_n} \bigwedge_i (\psi_i \to \chi_{k_i})$

   where:
   - $\textsc{dnf}(\psi) = \psi_1 \vee \ldots \vee \psi_n$
   - $\textsc{dnf}(\chi) = \chi_1 \vee \ldots \vee \chi_m$
   - i ranges over $\{1, \ldots, n\}$
   - $k_1, \ldots, k_n$ all range over $\{1, \ldots, m\}$

Proposition 24. For all ψ, $\textsc{dnf}(\psi)$ is a disjunction of assertions.

Proposition 25. For all ψ, $\textsc{dnf}(\psi) \equiv \psi$

There is a close correspondence between $\textsc{dnf}(\psi)$ and the possibilities for ψ.

Proposition 26. If π is a possibility for ψ then π is a possibility (the unique possibility) for some disjunct of $\textsc{dnf}(\psi)$.

The converse, however, is not true. This is because some disjuncts of $\textsc{dnf}(\psi)$ may be entailed by others. If one disjunct α entails another β, then $|\alpha|$ is *contained* in a possibility for ψ, but it is not identical to any such possibility. To get a full correspondence between the possibilities for ψ and the disjuncts of $\textsc{dnf}(\psi)$, we must eliminate those disjuncts that entail others. This operation preserves logical equivalence. We call the resulting formula the *clean disjunctive normal form* of ψ, $\textsc{cdnf}(\psi)$.

Definition 27. $\textsc{cdnf}(\psi)$ is obtained from $\textsc{dnf}(\psi)$ by removing any disjunct that classically entails any other disjunct.

Proposition 28. For all ψ, $\textsc{cdnf}(\psi)$ is a disjunction of assertions.

Proposition 29. For all ψ, $\textsc{cdnf}(\psi) \equiv \psi$.

Proposition 30. π is a possibility for ψ *if and only if* π is a possibility (the unique possibility) for some disjunct of $\textsc{cdnf}(\psi)$.

So $\textsc{cdnf}(\psi)$ gives us, as it were, a syntactic representation of the possibilities for ψ. This is exactly what we need to compute compliant responses. We are now ready to define an algorithm that takes a sentence ψ as its input, and yields a set $\textsc{comp}(\psi)$ of sentences that are compliant responses to ψ.

Definition 31 (Algorithm)

1. The algorithm takes as its input a sentence ψ. It first computes $\textsc{cdnf}(\psi)$. If $\textsc{cdnf}(\psi)$ consist of a single disjunct, then ψ is an assertion. Then, a sentence is compliant with ψ iff it is equivalent with ψ. So we output $\textsc{comp}(\psi) = \{\psi\}$ in this case.

2. If ψ is not an assertion, we first use $\textsc{cdnf}(\psi) = \psi_1 \vee \ldots \vee \psi_n$ to compute the set $\textsc{ca}(\psi)$:

$$\textsc{ca}(\psi) = \{!(\psi_{i_1} \vee \ldots \vee \psi_{i_m}) \mid i_1, \ldots i_m \in \{1, \ldots, n\},\ m \geq 1\}$$

   $\textsc{ca}(\psi)$ consists of all formulas that are obtained from $\textsc{cdnf}(\psi)$ by removing some (possibly zero, but not all) disjuncts, and then turning the remaining disjunction into an assertion using the ! operator. The unique possibility for such a formula always coincides with the union of a non-empty set of possibilities for ψ. So all formulas in $\textsc{ca}(\psi)$ satisfy the first condition in the definition of compliance. Moreover, by fact 21, the second condition does not play a role for assertive responses. So all formulas in $\textsc{ca}(\psi)$ are compliant with ψ (hence the name $\textsc{ca}$, short for 'compliant assertions').
3. Oc course, a compliant response to ψ does not have to be an assertion. It may very well be inquisitive. Any inquisitive compliant response, however, must be equivalent with a disjunction of compliant assertions. Thus, we compute the set of potentially compliant responses, $\textsc{pcr}$, as follows:

$$\textsc{pcr}(\psi) = \{\chi_1 \vee \ldots \vee \chi_n \mid 1 \leq n \leq |\textsc{ca}(\psi)| \text{ and } \chi_1 \ldots \chi_n \in \textsc{ca}(\psi)\}$$

   All formulas in $\textsc{pcr}$ satisfy the first condition in the definition of compliance, and vice versa, every formula that satisfies this condition is equivalent with some formula in $\textsc{pcr}(\psi)$.
4. What remains to be done is to filter out those formulas in $\textsc{pcr}(\psi)$ that do not satisfy the second condition in the definition of compliance. To do so, we proceed as follows.
   Take a sentence $\chi \in \textsc{pcr}(\psi)$. We know that $\chi = \chi_1 \vee \ldots \vee \chi_n$, where all χ_i's are assertions. We have to check that every possibility for ψ, when restricted to $|\chi|$, is contained in some possibility for χ.
   To do so, take a disjunct ψ_j of $\textsc{cdnf}(\psi)$, and check if $\psi_j \wedge !\chi$ classically entails one of the disjuncts of χ. If this works for all ψ_j, then χ is compliant with ψ, otherwise it is not.
   Carrying out this procedure for all $\chi \in \textsc{pcr}(\psi)$ yields the desired set of sentences $\textsc{comp}(\psi)$.
5. Finally, there is some optional 'cleaning up' to do. The formulas in $\textsc{comp}(\psi)$ are all disjunctions of assertions, that is, formulas in disjunctive normal form. To allow for a more intelligible output, we would like to bring these formulas into *clean* disjunctive normal form. To do so, we simply apply $\textsc{cdnf}$ to every formula in $\textsc{comp}(\psi)$.

We are now ready to state our main result: $\textsc{comp}(\psi)$ does not just contain *some* sentences that are compliant with ψ, it actually contains *all* such sentences (up to logical equivalence). The proof of this result is suppressed here for reasons of space. The interested reader is referred to (Cornelisse, 2009).

Theorem 1 (Soundness and Completeness of the Algorithm)
φ is compliant with ψ iff φ is logically equivalent with some sentence in $\textsc{comp}(\psi)$.

We end with a remark regarding the implementation of the algorithm. Notice that most of the operations that have to be carried out consist in syntactic manipulation of formulas. The only 'reasoning' steps consist in checking classical entailment. Existing entailment/satisfiability checking algorithms can be used to carry out this task. The algorithm has been implemented, and is accessible through a graphical user interface at `www.illc.uva.nl/inquisitive-semantics`.

5 Conclusion and Outlook

The established algorithm could serve as the basis for practical applications of inquisitive semantics, and will also aid in further developing and imparting the theoretical framework. Several extensions suggest themselves. In particular, Groenendijk and Roelofsen (2009) discuss some general criteria for *preferring* certain compliant responses over others. Thus, one natural step to take would be to develop an algorithm that, given an initiative ψ and an agent A with information state s_A, determines the *most* compliant response(s) to ψ that A may truthfully utter.

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