



UvA-DARE (Digital Academic Repository)

Animal spirits, heterogeneous expectations and the amplification and duration of crises

Assenza, T.; Brock, W.A.; Hommes, C.H.

DOI

[10.1111/ecin.12367](https://doi.org/10.1111/ecin.12367)

Publication date

2017

Document Version

Final published version

Published in

Economic Inquiry

License

Article 25fa Dutch Copyright Act

[Link to publication](#)

Citation for published version (APA):

Assenza, T., Brock, W. A., & Hommes, C. H. (2017). Animal spirits, heterogeneous expectations and the amplification and duration of crises. *Economic Inquiry*, 55(1), 542-564. <https://doi.org/10.1111/ecin.12367>

General rights

It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations

If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: <https://uba.uva.nl/en/contact>, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.

UvA-DARE is a service provided by the library of the University of Amsterdam (<https://dare.uva.nl>)

ANIMAL SPIRITS, HETEROGENEOUS EXPECTATIONS, AND THE AMPLIFICATION AND DURATION OF CRISES

TIZIANA ASSENZA, WILLIAM A. BROCK and CARS H. HOMMES*

We introduce a simple equilibrium model of a market for loans, where households lend to firms based on heterogeneous expectations about their loan default probability. Agents select endogenously among heterogeneous expectation rules, based upon their relative performance. Due to strong nonlinearities, a small fraction of pessimistic traders already has a large aggregate effect, leading to a crisis characterized by high interest rates for loans and low output. Our stylized model illustrates how animal spirits and heterogeneous expectations and, in particular, how coordination on pessimistic expectations amplifies crises and slows down recovery. (JEL E32, D83, D84)

I. INTRODUCTION

In their book, Akerlof and Shiller (2009) stressed the importance of “animal spirits” for the origin and propagation of a financial-economic crisis, for the subsequent recession, and for the “exit” process from the recession. They discuss recent advances in behavioral economics in order to identify different types of “animal spirits,” with “confidence” being one of the cornerstone animal spirits. Akerlof and Shiller point to an important problem facing economics: “confidence” shares with “financial factors” the fate of being difficult to conceptualize, model, and measure. This article is essentially an attempt to build a dynamic equilibrium model of agents’ confidence in a market for loans. We introduce a simple dynamic equilibrium model for loanable funds, and show how a sudden collapse of confidence may, on the one hand, accelerate and amplify the downturn of an economy after a negative shock, and, on the other hand, slow down the

*Earlier versions of this article have been presented at the conference *Rethinking Economic Policies in a Landscape of Heterogeneous Agents*, Università Cattolica del Sacro Cuore, Milano, October 13–15, 2011; the workshop *Complex Systems: Towards a Better Understanding of Financial Stability and Crises*, De Nederlandsche Bank, Amsterdam, November 3–4, 2011; the workshop of the *International Network on Expectational Coordination (INEXC)*, Paris, June 27–29, 2012; the AEA meetings, San Diego, January 4–6, 2013; the Research Seminar series at the De Nederlandsche Bank, Amsterdam, January 22, 2013; the EEA-ESEM meetings, Gothenburg, August 26–30, 2013; the 19th International Conference *Computing in Economics and Finance*, Vancouver, July 10–12, 2013; and the *OFCE-SKEMA-GREQAM Workshop on Agent-Based Macroeconomics*, Sophia Antipolis, Nice, November 22, 2013. We would like to thank participants for useful comments and discussions. We are also grateful to three anonymous referees and an Editor for detailed comments on an earlier draft, which have led to significant improvements. Financial support from the Netherlands Organization for Scientific Research (NWO), project *Understanding Financial Instability through Complex Systems*; the EU 7th framework projects *Complexity Research Initiative for Systemic Instabilities (CRISIS)*, grant no. 288501, *Macro-Risk Assessment and Stabilization Policies with New Early Warning Signals (RASTANEWS)*, grant no. 320278, and *Integrated Macro-Financial Modelling for Robust Policy Design (MACFINROBODS)*, grant no. 612796; the Unicredit & Universities Post doc Grant project *Animal Spirits and Financial Crises*; University of Wisconsin Department of Economics and from the Vilas Trust are gratefully acknowledged. None of the above are responsible for errors in this article.

Assenza: Associate Professor, Complexity Lab in Economics (CLE), Department of Economics and Finance, Università Cattolica del Sacro Cuore, 20123, Milano, Italy; Amsterdam School of Economics, University of Amsterdam, CeNDEF, 1018 WB Amsterdam, The Netherlands; Phone +390272342483, Fax +390272342781, E-mail tiziana.assenza@unicatt.it

Brock: Professor Emeritus, Department of Economics, University of Wisconsin at Madison, Madison, WI 53706, Department of Economics, University of Missouri,

Columbia, Columbia, MO 65211-6040. Phone 608 263 2989, Fax 608 262 2033, E-mail wbrock@ssc.wisc.edu

*Hommel*s: Full Professor, Amsterdam School of Economics, University of Amsterdam, CeNDEF, 1018 WB Amsterdam, The Netherlands; Tinbergen Institute, 1082 MS Amsterdam, The Netherlands. Phone +31205254246, E-mail C.H.Hommel@uva.nl

ABBREVIATIONS

FOC: First Order Condition
MSE: Mean Squared Error
ZLB: Zero Lower Bound

recovery from an economic crisis. The core ingredient of our model is the crucial role we assign to *expectations' heterogeneity* and, especially how endogenous selection of heterogeneous expectation rules based on their relative performance feeds into the dynamics of wages, output and the dynamics of contracting terms that the lending side of the economy imposes on the borrowing side of the economy in dynamic equilibrium.

It is almost a commonplace that the behavior of a variable in the aggregate—that is, at the macroeconomic level—does not necessarily correspond to the behavior of the same variable as decided at the microeconomic level by a “representative” individual: “Any meaningful model of the macroeconomy must analyze not only the characteristics of the individuals but also the structure of their interactions” (Colander et al. 2008, 237). Arrow also stressed the key role of heterogeneous expectations for modeling the economy: “One of the things that microeconomics teaches you is that individuals are not alike. There is heterogeneity, and probably the most important heterogeneity here is heterogeneity of expectations. If we didn't have heterogeneity, there would be no trade. But developing an analytic model with heterogeneous agents is difficult” (Ken Arrow, in: Colander, Holt, and Rosser 2004, 301). The emergent macro behavior in our model driven by endogenous interactions of heterogeneous expectations includes amplification of persistent crisis episodes.

In behavioral modeling of animal spirits and confidence, bounded rationality plays a key role. In macroeconomics in the last two decades much work has already been done on bounded rationality and adaptive learning; see for example Sargent (1993) and Evans and Honkapohja (2001), for extensive discussions. In the adaptive learning literature, the representative agent assumption is still the workhorse of contemporary models. Moreover most attention has focussed on cases where the learning process ends with the discovery of the “true model” of the economy, thus confirming rational expectations *ex post*. More recently, a number of macro models with heterogeneous expectations have been introduced, for example Assenza and Berardi (2009), Berardi (2007), Brock and de Fontnouvelle (2000), and Evans and Honkapohja (2003, 2006).¹ We will use the heterogeneous expectations framework of

Brock and Hommes (1997, 1998), where agents are boundedly rational and switch between different expectations rules based upon their relative success.² Anufriev et al. (2013a), Brazier et al. (2008), Branch and Evans (2006), Branch and McGough (2009), De Grauwe (2011), and Lines and Westerhoff (2010) have applied this heterogeneous expectations framework in various macroeconomic settings. Cornea, Hommes, and Massaro (2013) estimated a heterogeneous expectations model with forward looking fundamentalists versus backward looking naive expectations to U.S.-inflation data.

There is quite some empirical evidence for the persistence of heterogeneity in expectations, both in survey data and in laboratory experiments. For example, Branch (2004), Mankiw, Reis, and Wolfers (2003), and Pfajfar and Santoro (2010) provided empirical evidence in support of heterogeneous expectations using survey data on inflation expectations. Expectations heterogeneity in experimental data are found, for example, in Adam (2007), Hommes et al. (2005a), and Pfajfar and Zakelj (2014, 2016). Anufriev and Hommes (2012), Assenza et al. (2013), and Roos and Luhan (2012) find evidence for performance-based switching between forecasting rules in laboratory experiments; see Assenza et al. (2014), Duffy (2008), and Hommes (2011) for overviews of experimental work in macro.

In order to model the Akerlof–Shiller “animal spirits” and “confidence,” we apply the Brock–Hommes heterogeneous expectations framework to a dynamic equilibrium model of loanable funds. We abstract from the complexity of the real world contract terms for a loan by using a one-dimensional proxy variable that we call the “contract rate.” The reader should think of a contract rate not only as a measure of the

to expectations can lead to deflationary spirals with falling prices and falling output. To avoid this outcome, Evans, Guse, and Honkapohja (2008) recommend augmenting normal policies with aggressive monetary and fiscal policy guaranteeing a lower bound on inflation. Arifovic and Petersen (2015) and Hommes, Massaro, and Salle (2016) recently tested these policies in a ZLB laboratory environment and found that fiscal policy is more effective than aggressive monetary policy in escaping liquidity traps.

2. Simsek (2013) and Scheinkman and Xiong (2003) have stressed the role of overly optimistic (over confident) believers in driving bubble like phenomena in a framework where rational agents take into account the presence of overly optimistic believers, but without endogenous strategy selection. Our model contains rational as well as boundedly rational agents; see Hommes (2006) for an overview and extensive discussion of heterogeneous expectations and bounded rationality.

1. More recently, Evans, Guse, and Honkapohja (2008) and Benhabib, Evans, and Honkapohja (2008) study the New Keynesian model under learning with an interest-rate rule subject to a zero lower bound (ZLB). Large pessimistic shocks

interest rate for the loan, but more generally of “qualification adjusted contract terms” describing today’s difficulties of getting a loan, for example, by raising credit score qualifications, increasing down payment requirements for the loan, and so forth. A very high contract rate represents a liquidity crisis, which may lead to an economic crisis with persistently low output.

We borrow from recent work by Brock and Manski (2008, 2011) (B&M hereafter) to describe and conceptualize ambiguity and pessimism in a credit market economy. In particular, B&M take into account the existence in credit markets of an informational problem because of partial knowledge of loan repayments, that is, lenders do not know a priori whether a borrower will totally repay his debt or only part of it, or, in the worst case scenario, he will not repay at all. In B&M, lenders must build a model of borrower behavior, which they are unable to completely specify because of lack of knowledge. We assume that most lenders lack fully rational expectations in forming expectations about the future share of loans that will be paid back. Although B&M use a static model, we study the role of expectations in a dynamic equilibrium model for loanable funds driven by an exogenous stochastic process for the probability that loans will be paid back.

There is a large behavioral macro-finance literature stressing the importance of pessimistic expectations and extrapolation bias. Most notably, a strand of the behavioral finance literature emphasizes investors’ over-extrapolation from recent good/bad performance as an important factor in understanding stock prices. In fact, the extrapolation bias is the most prominent nonrisk-based explanation for the so-called value anomaly in stock returns: why stocks with high book-to-market value earn higher returns than stocks with low book-to-market value (see Lakonishok, Shleifer, and Vishny 1994 and La Porta et al. 1997). In addition, over-extrapolation can also explain various puzzles about aggregate stock returns, for example, excess volatility and predictability puzzles (see Barberis and Thaler 2003). Furthermore, there are papers that emphasize Bayesian versions of the extrapolation bias. When agents do not have many observations (i.e., in the short run), Bayesian learning can also generate effects that are similar to the extrapolation bias. Using this approach, Cogley and Sargent (2008) attempt to explain the high historical equity premium as being driven by the influence of a negative realization, namely the Great Depression, on agents’ beliefs about

stock returns. More recently, Boz and Mendoza (2010) emphasize learning-driven optimism as a cause of the recent subprime financial crisis. In their model, periods with early success induce Bayesian agents to become optimistic about asset prices. Subsequently, a bad realization induces them to become pessimistic. This reduces prices and causes a crisis. The crisis is exacerbated because the learning effects interact with fire sale externalities. Bianchi, Boz, and Mendoza (2012) quantify the welfare implications of this analysis.

In relation to the importance of pessimistic expectations, there is a large literature that emphasizes ambiguity aversion or Knightian uncertainty for understanding financial crises and recessions. This literature is related because Knightian uncertainty also naturally generates pessimistic expectations. Recent contributions include Caballero and Krishnamurthy (2008) in the context of financial crises, and Ilut and Schneider (2014) in the context of business cycle fluctuations. In addition, the robustness literature initiated by Hansen and Sargent (2001, 2008, 2010) is also closely related.

The key distinguishing feature from this behavioral macro-finance literature is the endogenous switching between *heterogeneous expectations* rules based upon their relative forecasting performance, as by Brock and Hommes (1997, 1998). Beliefs are symmetric and include rational, naive, average, trend-extrapolation and pessimistic as well as optimistic expectations. Due to strong nonlinearities of the model, however, the aggregate effect may be asymmetric. A relatively small fraction of pessimistic agents may have a strong aggregate effect and amplify a crisis. More precisely, after an unexpected negative shock endogenous selection among heterogeneous expectation rules may enforce coordination on pessimistic expectations, causing a substantial increase of the contract rate for loans, a subsequent decline of lending, output and wages, and a slowdown of the recovery from such an economic crisis.³

This article is organized as follows. Section II introduces the modeling framework describing households and firms and the dynamic equilibrium. In Section III, we consider a number of homogeneous expectations benchmarks,

3. An interesting related article by Mamatzakis (2013) reports empirical evidence that market level pessimism has important effects on the Euro area sovereign debt crisis and that substantial elements of the market may not share common preferences/beliefs.

including rational, naive, average, trend following, optimistic (maximum), and pessimistic (minimum) expectations. Section IV presents the model with heterogeneous expectations. Section V investigates a 6-type example with all previous homogeneous rules. We first study the dynamical behavior for a typical realization of the exogenous stochastic AR(1) probability series. We then provide intuition why nonrational strategies survive in the market. This intuition is partly based on studying analytically the case where at each point in time all agents switch to the best (possibly nonrational) forecasting rule. We then perform a Monte Carlo analysis based on 1,000 runs of the stochastic probability series and show that the pessimism bias, the amplification of the crisis, and the persistently slow recovery of the economy after a crisis are statistically significant (see especially Figure 8). Finally, Section VI concludes.

II. THE MODEL

This section describes the basic ingredients of our framework. We consider a market for loanable funds that is populated by households/lenders and firms/borrowers. The households' sector, which also represents the supply side of the market for loanable funds, is built by means of an overlapping generations framework in which each agent when young consumes ($c_{t,t}$) and saves earnings (s_t) from work, with wages w_t . Savings are invested either in a safe asset⁴ or in a "fund" (productive investment)⁵ with an uncertain return (λ_{t+1}) which must be forecasted. When old, the agent consumes ($c_{t,t+1}$) an endowment (ω_o) and the average return on investments.

The demand side of the market for loanable funds in our economy is represented by firms that borrow a certain amount of capital (x_t) for production and remunerate work after paying back their debt. The remuneration for work is used by households to consume and to save. Savings are used to extend loans to the firms' sector.

A. Households

The supply side of our economy is described by means of a two-period overlapping generations structure. We assume that the

young agent at date t has preferences defined over consumption when young $c_{t,t}$ and when old $c_{t,t+1}$. For the sake of convenience, we assume a logarithmic utility function. The objective function therefore is

$$(1) \quad u_t = \ln c_{t,t} + \ln c_{t,t+1}^e,$$

where $c_{t,t+1}^e$ is expected consumption when old. When young, the agent works and earns a real wage w_t (i.e., wages from the productive sector). He invests his savings s_t partly in a safe asset, which yields a known fixed return $\rho > 1$ at $t + 1$, and partly in a fund whose (uncertain) rate of return λ_{t+1} in period $t + 1$ has to be forecasted. Investment in the fund can be conceived of as employment of resources (capital) in the productive sector, whose output is uncertain. The expectations by the young formed at date t on the return of the fund at date $t + 1$ are denoted by λ_{t+1}^e . When old, the agent retires and receives an (exogenous) endowment ω_o (at the beginning of old age) and the return on asset investments. The budget constraint of the agent when young and when old respectively, therefore, are

$$(2) \quad c_{t,t} \leq w_t - s_t,$$

$$(3) \quad c_{t,t+1}^e \leq \omega_o + s_t \left[(1 - \delta_t) \rho + \delta_t \lambda_{t+1}^e \right],$$

where w_t is labor income. The decision problem of the young is to optimize Equation (1) subject to Equations (2) and (3). At date t and allocates a fraction δ_t to the fund which he anticipates to produce a real amount $s_t \delta_t \lambda_{t+1}^e$ available for consumption in $t + 1$. Therefore, $s_t \delta_t \lambda_{t+1}^e$ can be interpreted as expected production obtained employing $s_t \delta_t$ in the productive sector. It follows that λ_{t+1}^e can be interpreted as the expected *average productivity of capital* in this context. The amount $s_t(1 - \delta_t)$ allocated at date t to the safe asset is known by the young at date t to produce $s_t(1 - \delta_t)\rho$ available for consumption in period $t + 1$. The expression in brackets in Equation (3), that is,

$$(4) \quad \mu_{t+1}^e =: (1 - \delta_t) \rho + \delta_t \lambda_{t+1}^e,$$

will be denoted as the expected *average return on investment*. Substituting the constraints into the objective function one ends up with the following maximization problem

$$(5) \quad \max_{s_t} \ln(w_t - s_t) + \ln(\omega_o + s_t \mu_{t+1}^e).$$

4. The risk free asset is a constant returns deterministic storage technology that returns $\rho > 1$ per unit stored.

5. In the article, we will refer to it as fund for short.

The first order conditions (FOCs) give the following expression for savings and the fraction δ_t allocated to the fund:

$$(6) \quad \begin{cases} s_t = \frac{1}{2} \left(w_t - \frac{\omega_o}{\mu_{t,t+1}^e} \right), \\ \delta_t = \max_{\delta_t \in [0,1]} \{ [(1 - \delta_t) \rho + \delta_t \lambda_{t+1}^e] \} \end{cases}$$

Assuming, for the sake of simplicity, zero endowment when old, that is, $\omega_o = 0$, the FOC for s_t simplifies to

$$(7) \quad s_t = \frac{w_t}{2}.$$

Note that Equation (7) says that, conditional on w_t , the supply for investment, that is, the total amount of savings, s_t , is perfectly inelastic with respect to known and unknown returns on assets next period. How these savings will be distributed over the risk free and the fund will be discussed in Section II.C.

B. Firms' Demand for Loanable Funds

Following Brock and Manski (2008, 2011), we assume that borrowers get into debt in order to finance productive investments. Moreover, if returns on investments turn out to be too low, they may not be able to pay back. Therefore, we introduce a (time varying) *probability of success*, p_t or, equivalently, a *probability of bankruptcy* $1 - p_t$. The probability of success represents the share of firms that will be able to pay back their loans. Firms choose the amount of capital x_t at time t borrowed from the lending side of the economy, and labor l_{t+1} at time $t + 1$ solving the maximization problem:

$$(8) \quad \max_{x_t, l_{t+1}} E_t I_t \{ p_t [g(x_t, l_{t+1}) - r_t x_t - w_{t+1} l_{t+1}] \},$$

where $r_t > 1$ is the gross interest rate, w_{t+1} are wages from the productive sector, $g(x_t, l_{t+1})$ is the production function, assumed to be strictly concave with decreasing returns to scale,⁶ and I_t is an indicator variable that equals one if the investment is successful in $t + 1$ and zero if not.⁷ When the labor market equilibrium condition,

6. More precisely, we assume $g'(x_t) > 0$, $g''(x_t) < 0$ with right-hand and left-hand Inada conditions, that is, $g(0) = 0$, $g'(\infty) = \infty$, $g'(\infty) = 0$.

7. The timing of the model works as follows: young workers at time t lend to firms capital x_t that leads to time $t + 1$ production $g(x_t)$ by the successful borrowing firms when capital x_t is combined with labor $l_{t+1} = 1$ hired in $t + 1$. Output gives returns on saving to the old worker in $t + 1$.

$l_{t+1} = 1$ is imposed at each date, and we restrict ourselves to the Cobb–Douglas case, that is, we define $g(x_t) = x_t^\alpha l_{t+1}^{1-\alpha}$, the maximization problem yields the following FOCs:⁸

$$(9) \quad x_t = g'^{-1}(r_t),$$

$$(10) \quad w_{t+1} = (1 - \alpha) g(x_t).$$

Given the features of the production function $g(x_t)$, Equation (9) represents a decreasing relation between the amount of capital at period t and the rental rate on capital in the same period, therefore it defines the *demand* for capital in this setting. In our economy at each date t , a fraction of firms $1 - p_t$ fail and hence pay zero to their workers. We assume that a lump sum tax (τ_t)⁹ is levied on the successful firms' workers who receive wages $(1 - \alpha)g(x_{t-1})$ to pay insurance to workers of the fraction $1 - p_t$ firms who failed at date t . In this way, each worker at a firm whether it fails or not receives actual net of tax wages at date t equal to:¹⁰

$$(11) \quad w_t = (1 - \alpha) g(x_{t-1}) p_t.$$

In the case of a Cobb–Douglas production function $g(x_t) = x_t^\alpha l_{t+1}^{1-\alpha}$ and imposing the labor market equilibrium condition ($l_{t+1} = 1$) at each date, Equations (9) and (11) specialize to the *demand function* and wages given by:

$$(12) \quad x_t = \left(\frac{r_t}{\alpha} \right)^{\frac{1}{\alpha-1}},$$

$$(13) \quad w_t = p_t (1 - \alpha) x_{t-1}^\alpha.$$

Substituting the demand for capital x_t from Equation (12) into Equation (13), we get the labor income in the case of Cobb–Douglas production function:

$$(14) \quad w_t = \eta p_t r_{t-1}^{\frac{\alpha}{\alpha-1}},$$

where $\eta = \alpha^{\frac{\alpha}{1-\alpha}} (1 - \alpha)$. For later use it will also be useful to define the inverse demand function as:

$$(15) \quad r_t = \alpha x_t^{\alpha-1}.$$

8. $0 < \alpha < 1$ represents the capital's share and $(1 - \alpha)$ the labor share.

9. The lump sum tax τ_t is subtracted off the wages of the successful firms so that $(1 - \alpha)g(x_{t-1})p_t = (1 - \alpha)g(x_{t-1}) - \tau_t$ and every worker gets a net wage $(1 - \alpha)g(x_{t-1})$.

10. We thank an anonymous referee for this clarifying suggestion.

C. Equilibrium

In this section, we will compute the equilibrium of our economy. Following Brock and Manski (2008, 2011), we indicate with $x_j(r_t)$ the j -th borrower's loan demand at a contract rate r_t . Hence for a "sample" of J firms the lender's expected loan return is given by:

$$(16) \quad \lambda_{t+1}^e(r_t) = \frac{\frac{1}{J} \sum_{j=1}^J \min \{ i(j \in S_t) g(x_{j,t}), r_t x_{j,t} \}}{\frac{1}{J} \sum_{j=1}^J x_{j,t}},$$

where $i(j \in S_t)$ is the indicator function which is unity if firm j is successful at date t and is zero otherwise. Moreover, the numerator represents aggregate repayment and the denominator represents aggregate loan demand. We assume success is independently distributed across firms at each date t . Therefore, firm j chooses $x_{j,t}$ to satisfy:

$$(17) \quad x_{j,t} = \max_{x_{j,t}} p_{j,t} [g(x_{j,t}) - r_t x_{j,t}],$$

provided that the maximized quantity is non-negative, otherwise firm j shuts down and does not operate in period t , that is, it chooses $x_{j,t} = 0$.

Assume that the probability of success is the same for all firms at date t , that is, $p_{j,t} \equiv p_t$, for all j . Then each firm solves the same maximization problem and the optimal solution is the same for all firms. Apply the law of large numbers to Equation (16) to obtain the "population" loan return function:

$$(18) \quad \lambda_{t+1}^e(r_t) = p_{t+1}^e r_t.$$

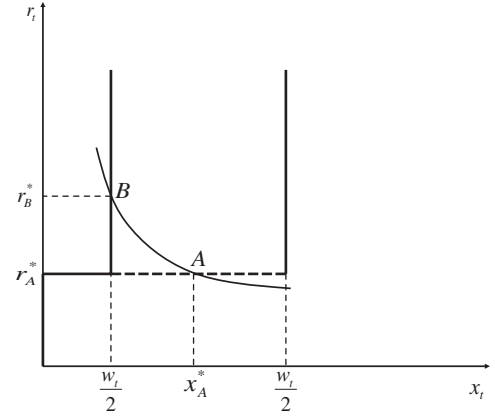
where p_{t+1}^e is the expected probability of success, that is, the share of firms that is expected to be able to pay back the loan. The expected probability of success may be seen as a measure of "confidence" in our economy. Assuming risk neutrality, the *no arbitrage condition* is such that the return on the fund equals the return on the risk free investment, that is, $\lambda_t = \rho$. It follows that the no arbitrage value of the contract rate (r_t^*) is given by the following relation:

$$(19) \quad r_t^* = \frac{\rho}{p_{t+1}^e}.$$

At this stage, we have all the necessary ingredients to compute the equilibrium of our model.

FIGURE 1

The loan supply correspondence (21) and the demand curve (9)



Note: Points A and B represents the two possible configurations of the temporary equilibrium allocations (x^*, r^*) depending on the time-varying features of the supply curve.

Let us define

$$(20) \quad \Delta_t^*(r_t) := \bar{i} \left[r_t > \frac{\rho}{p_{t+1}^e} \right] := \bar{i} [r_t > r_t^*],$$

where the upper bar over the indicator function means that it is the set $[0, 1]$ when $=$ holds instead of $>$. Hence we can define the *loan supply correspondence*, when old age endowment ω_o is zero, by

$$(21) \quad S_t(r_t) := \frac{w_t}{2} \bar{i} \left[r_t > r_t^* = \frac{\rho}{p_{t+1}^e} \right],$$

that is, when $r_t > r_t^*$ ($r_t < r_t^*$) all savings are invested into loans (the risk free asset). Note that it is the belief p_{t+1}^e formed at date t about the probability of success in $t+1$ that determines the loan supply at time t .

The demand for capital and the equilibrium value for the contract rate are determined by market clearing, that is,

$$(22) \quad x(r_t) = S_t(r_t).$$

As the supply correspondence is a (time varying) step function, there are two possibilities for the equilibrium, points A and B, as illustrated in Figure 1.

The first possibility for equilibrium (point A) is given by:

$$(23) \quad r_A^* = \frac{\rho}{p_{t+1}^e}$$

$$(24) \quad x_A^* = x(r_A^*) = \left(\frac{\rho}{\alpha p_{t+1}^e} \right)^{\frac{1}{\alpha-1}},$$

arising when $x(r_A^*) < w_t/2$, where $x(\cdot)$ is the demand function Equation (12).

The other possibility (point B) is given by

$$(25) \quad r_B^* = r(x_B^*) = \alpha \left[\frac{\eta}{2} p_{t-1} r_{t-1}^{\frac{\alpha}{\alpha-1}} \right]^{\alpha-1}$$

$$(26) \quad x_B^* = \frac{w_t}{2} = \frac{\eta}{2} p_{t-1} r_{t-1}^{\frac{\alpha}{\alpha-1}}$$

and it arises when $x(r_A^*) > w_t/2$.

Note the crucial role played by expectations about the firms' probability of success (p_{t+1}^e), the confidence measure in our economy. In fact, given the return ρ on the risk free asset, the lower the expected probability of success the higher will be the non arbitrage contract rate (r_A^*) and, consequently, the lower will be the demand for capital (x_A^*). On the other hand, a high expected probability of success p_{t+1}^e causes the contract equilibrium rate r_A^* to drop. Hence pessimistic expectations amplify a bust or crisis of the economy (high contract rate and low output).

III. HOMOGENEOUS BELIEFS

We have not yet specified the probability of success p_t and how lenders form expectations about this probability to repay the loan. We are particularly interested in the situation where there is a series of "bad" exogenous shocks to the economy and the probability of success suddenly drops. Instead of focusing on a single stochastic negative shock and an impulse response analysis, we assume an exogenous dynamic stochastic process with some persistence for the probability of success and then study the corresponding equilibrium dynamics. We focus on the simplest case of an exogenous AR(1) process for the probability of success, given by:

$$(27) \quad p_{t+1} = \mu + a(p_t - \mu) + \varepsilon_t,$$

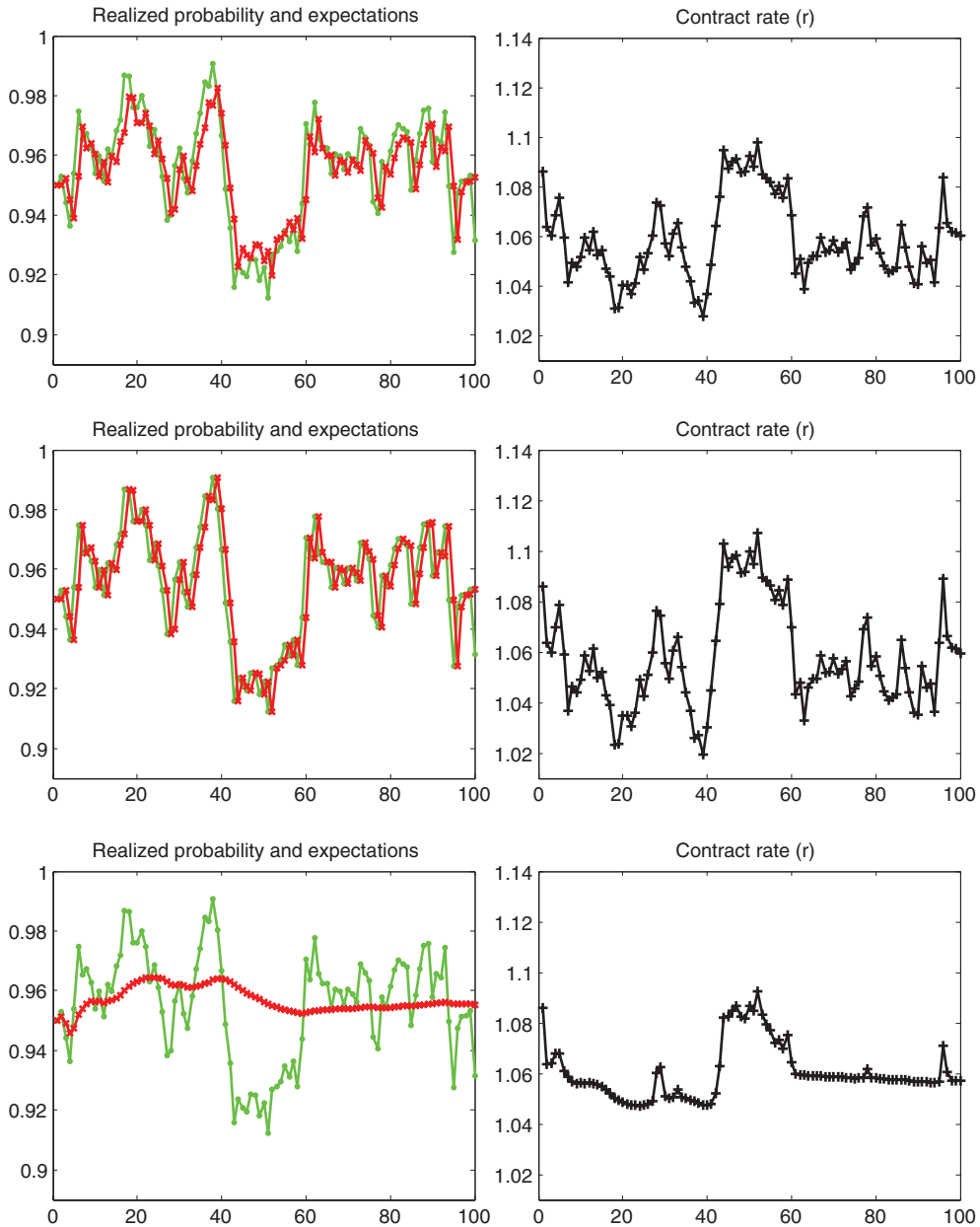
where μ is the long run average, a is the first order autocorrelation coefficient, and ε_t is an identically and independently distributed random variable drawn from a normal distribution. Throughout the article, we fix $\mu = 0.95$, $a = 0.8$, and $\sigma = 0.01$, so that the (long run) average is 0.95 and there is some persistence in the probability of success. One typical realization of the exogenous

probability time series ε_t is illustrated in Figure 2 (left panels in green). The success probability fluctuates between 0.9123 and 0.9909 over 100 periods. Between periods 39 and 43, the probability gradually declines, it stays persistently low between periods 44 and 51 and hits its lowest value 0.9123 in period 51. From period 52 to period 60, it slowly recovers. We will refer to this lowest value as the "crisis" because of the exogenous shocks. This example serves to illustrate the typical behavior of the model. In Section V.C we will perform Monte Carlo simulations over 1,000 runs of the stochastic probability series to check the robustness and statistical significance of our main findings.

Our main interest here is how *confidence* of the lenders, that is, their expectations about the probability of success of the firms, affects temporary equilibrium dynamics of contract rates, wages, and output. In particular, we study what happens after the exogenously generated crisis under endogenous selection of heterogeneous expectations by their relative performance. Agents are forecasting an exogenous probability process and in our model these forecasts have no effect upon the realizations of the success or default probabilities. Agents' forecasts however feed back into the real economy through their loan supply functions and thus affect aggregate equilibrium outcomes. As the probability process to be forecasted is exogenous (a simple AR(1) process), it should be relatively easy to learn the true data generating process and converge to rational expectations. But this is not what happens. As we will see pessimistic expectations have a large and persistent effect upon market equilibrium outcomes. Under endogenous selection of heterogeneous expectations, at times of high default probabilities pessimistic agents temporarily dominate the economy amplifying busts and crises and slowing down recovery.

Before investigating the role of heterogeneous expectations, however, by way of comparison it is useful to consider a number of benchmark specifications of the lender's expectations in the simple case of a representative agent, that is, we will consider some homogeneous expectations benchmarks. In addition to standard rational expectations, we allow for bounded rationality and consider a number of benchmark cases with a simple forecasting rule, motivated by empirical evidence from laboratory experiments. Hey (1994), for example, showed that in laboratory experiments where individuals forecast an exogenous stochastic AR(1) time series, rational

FIGURE 2
Homogeneous expectations benchmarks



Note: Upper panels: rational (AR1); middle panels: naive; bottom panels: average. Left panels: realized (green) and expected (red) probability of success. Right panels: corresponding equilibrium contract rates r_t .

expectations is rejected in most cases and simple forecasting rules such as adaptive expectations provide a better description of individual forecasting behavior; see also Dwyer et al. (1993). In more recent learning to forecast laboratory

experiments simple forecasting rules, such as naive expectations or a trend following rule, as described below, fit individual forecasting behavior quite nicely, see for example the surveys by Assenza et al. (2014) and Hommes (2011).

A. Rational Expectations

In the case of rational expectations, lenders are assumed to have perfect knowledge about the true stochastic probability process. Agents know that the probability of success follows the AR(1) process (Equation (27)) with perfect knowledge about its parameters. The rational forecast of the probability of success at period $t + 1$ is given by:

$$(28) \quad p_{t+1}^e = \mu + a(p_t - \mu).$$

Figure 2 (upper panels) illustrates time series of the realized probability p_t , the rational AR(1) forecast, and the equilibrium contract rate r_t . The rational forecast closely tracks the realized probability and the contract rate spikes exactly in the crisis period 51 when the probability of success hits its lowest value or, equivalently, when the probability of default hits its highest value. Under rational expectations, the dynamics of the contract rates is characterized by mean reversion to its long run equilibrium value $\bar{r} = \rho/\mu = (1.01/0.95) \approx 1.063$,¹¹ with exactly the same speed as the exogenous true probability process.

B. Naive Expectations

Under naive expectations, the forecast of the probability of success at period $t + 1$ is given by last period's observation, that is,

$$(29) \quad p_{t+1}^e = p_t.$$

Figure 2 (middle panels) illustrates time series of the realized probability p_t , the naive forecast, and the equilibrium contract rate r_t . Clearly the naive forecast lags realized probability by one period and the contract rate spikes in period 52, immediately after the probability of success hits its lowest value in the "crisis-period" 51 (or equivalently the probability of default hits its highest value). The dynamics of the contract rate under naive expectations is characterized by mean reversion to its long run equilibrium value $\bar{r} = \rho/\mu = (1.01/0.95) \approx 1.063$. Under naive expectations, the dynamics of the contract rate r_t is thus completely driven by the exogenous probability of success, just lagging one period behind. The speed of recovery of the economy after the exogenous crisis in period 31 is the same as the speed of mean reversion of the realized probability of success, and lags only one period behind the true probability. Notice that under

naive expectations the peaks of the contract rate are more extreme, because the rational AR(1) rule correctly predicts mean reversion (on average) after an extreme observation, whereas naive expectations uses the last observation and does not predict mean reversion.

C. Average Beliefs

Another interesting case is when agents use long run averages in forecasting. In the case of average expectations, the forecast of the probability of success is given by the sample average of past observation, that is,

$$(30) \quad p_{t+1}^e = \frac{1}{t+1} \sum_{i=0}^t p_i.$$

Figure 2 (lower panels) illustrates time series of the realized probability p_t , the average forecast, and the equilibrium contract rate r_t . The average forecast adjusts slowly following realized probability and decreases gradually between periods 40 and 60. As a result, the contract rate gradually increases within the same time span and afterwards (when the economy has recovered and the probability of success has gradually increased), it slowly converges back to its long run equilibrium level $\bar{r} = \rho/\mu \approx 1.063$. Hence, when all agents in the economy give *equal weight to all past observations*, the economy in principle should hardly fluctuate, but converge slowly to its long run equilibrium steady state. However, because of the persistent low probability of success between periods 44 and 51 we observe a persistent decline in average expectations that stays until about period 60 when the economy (between periods 54 and 60) has slowly recovered. In turn, we observe within the same period an analogous behavior of the contract rate.

D. Trend Following Expectations

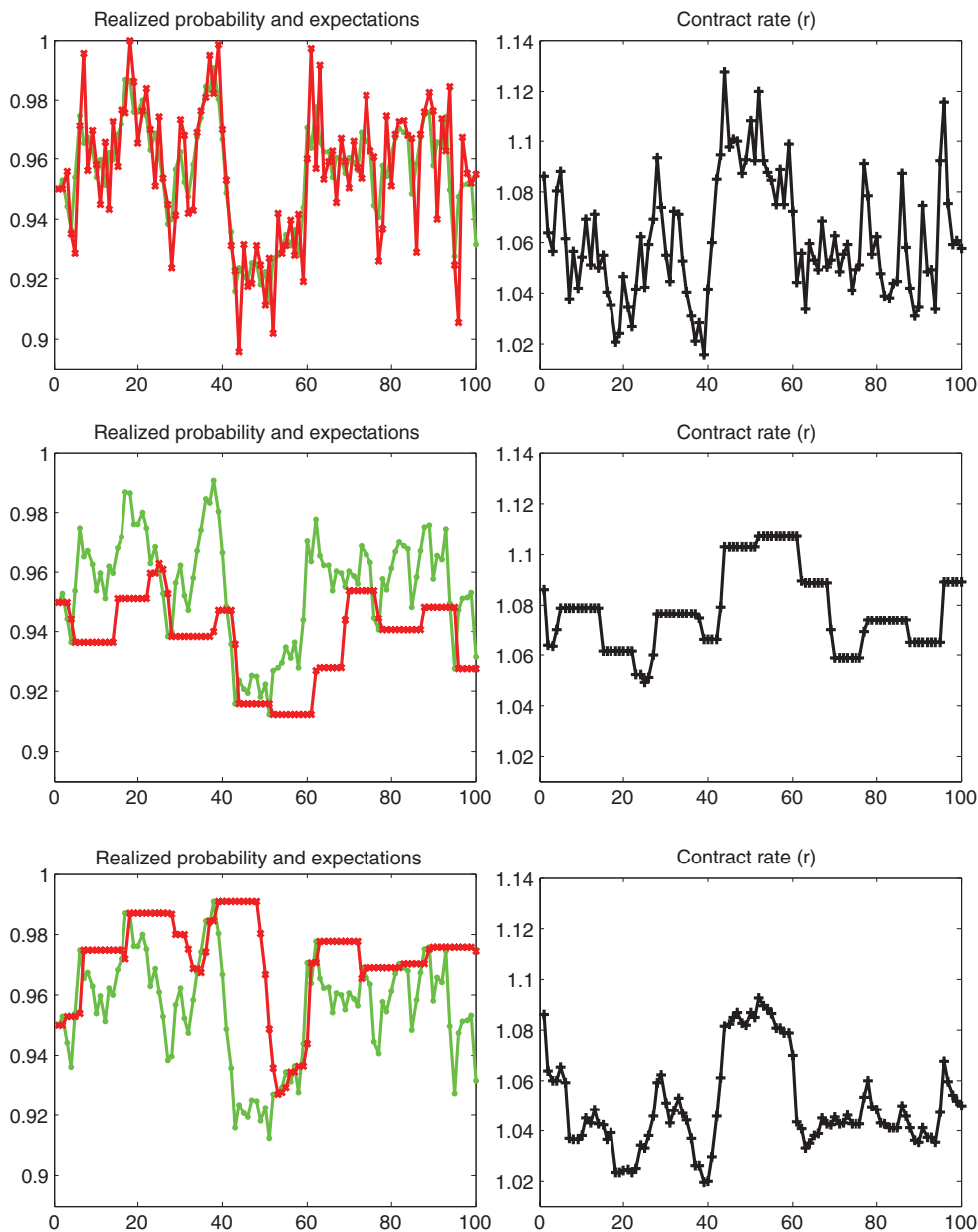
In the case of trend following expectations the forecast of the probability of success is given by a simple linear extrapolation rule

$$(31) \quad p_{t+1}^e = p_{t-1} + g(p_{t-1} - p_{t-2}).$$

Simple trend following rules belong to the most popular rules used in learning to forecast laboratory experiments with human subjects (e.g., Hommes 2011) and are also popular among chartists' trading rules in financial markets as has been documented in survey data analysis (e.g., Allen and Taylor 1990; Frankel and Froot

11. Where ρ is the risk free rate of return and μ is the long run mean of the AR(1) stochastic probability process.

FIGURE 3
Homogeneous expectations benchmarks



Note: Upper panels: trend followers; middle panels: minimum; bottom panels: maximum. Left panels: realized (green) and expected (red) probability of success. Right panels: corresponding equilibrium contract rates r_t .

1990). Figure 3 (upper panels) illustrates time series of the realized probability p_t , the trend follower forecast (31), and the equilibrium contract rate r_t . Trend followers may lead to overly pessimistic expectations, when the trend following

forecast undershoots its minimum realized value. As a consequence, this leads to more extreme spikes in the contract rate, for example, higher maximum values of the contract rate in periods 52 – 53, immediately following the exogenously

generated crisis period 51. Hence, the presence of trend followers *amplifies* booms and busts.

E. Pessimistic Expectations

Now consider the homogeneous benchmark case of pessimistic expectations. We model pessimistic expectations by a forecast that predicts that the probability of success remains at its lowest observed value in the last T periods, that is,

$$(32) \quad p_{t+1}^e = \min \{p_{t+1-T}, p_{t+2-T}, \dots, p_{t-1}, p_t\}.$$

As a typical example in the simulations below we choose $T = 10$. Figure 3 (middle panels) illustrates time series of the realized probability p_t , the minimum forecast, together with the corresponding equilibrium contract rate r_t . The minimum forecast adjust according to the local minima of the observed probability and decreases until its lowest value in period 52 to stay there for 10 periods, after the probability of success hits its lowest value, in period 51. As a result, the contract rate increases gradually and hits its highest value in period 52 to stay there for 10 periods. Under pessimistic beliefs after each local minimum of the probability of success, the contract rate spikes at a local maximum and stays there for at least $T = 10$ periods or jumps to a new (local) maximum.¹² Pessimistic expectations thus considerably slow down the mean reversion of the dynamic equilibrium process and therefore *amplify the duration* of an economic crisis and slow down economic recovery.

F. Optimistic Expectations

Finally consider the symmetrically opposite homogeneous benchmark case of what we call optimistic expectations, forecasting that the probability of success remains at its highest observed value in the last T periods, that is,

$$(33) \quad p_{t+1}^e = \max \{p_{t+1-T}, p_{t+2-T}, \dots, p_{t-1}, p_t\}.$$

As for the pessimistic rule, in the simulations below we take $T = 10$. Figure 3 (lower panels)

illustrates time series of the realized probability p_t , the maximum forecast, together with the corresponding equilibrium contract rate r_t . When the probability of success hits high values, the maximum forecast adjusts according to the local maxima of the observed probability.

Figure 3 suggests an asymmetry of how optimists and pessimists affect aggregate outcomes. Pessimistic expectations lead to higher contract rates and slow mean reversion, as discussed above. Optimistic expectations lead to lower contract rates, but do not affect mean reversion of the contract rate as much as do pessimists. This asymmetry is because of nonlinearities in the model and may be explained by looking at the loan market equilibrium points in Figure 1. When expectations become more pessimistic, the horizontal part in the loan supply correspondence shifts upwards, and the equilibrium point B (or A) shifts upwards. This may lead to very high contract rates and amplification of a crisis. When expectations become more optimistic, the horizontal part in the loan supply correspondence shifts downwards, so the equilibrium point A (or B) shifts downwards to the right, until it reaches the endpoint of the supply curve at $w_t/2$. All savings by households are then invested into loans and this is the maximum amount firms can borrow. This constraint limits the effects of optimistic expectations on booms of the economy. In our model, because of this nonlinearity, the amplification effect of pessimistic expectations on busts is larger than the amplification effect of optimistic expectations on booms.¹³

IV. HETEROGENEOUS BELIEFS

It is intuitively clear that exogenously assumed pessimistic expectations may become self-fulfilling and amplify crises. Our endogenous selection among heterogeneous expectations based upon relative forecasting performance is an endogenous amplification mechanism reinforcing pessimistic expectations in bad times thus amplifying busts. We follow Brock and Hommes (1997) to model heterogeneous expectations by a discrete choice model and evolutionary strategy selection based on their

12. As an example of a pessimistic agent, a referee referred to Nassim Taleb and his idea of a Black Swan. Google Trends show that Taleb's popularity (measured by Google searches) increased during the financial crisis and remains higher than before 2007 to this day. Just like Taleb, the pessimistic forecasting rule in the model enjoys increased popularity for many periods after a severe negative realization.

13. In a model where one would allow optimistic households to borrow more than their wage income to firms, the amplification effect of optimistic expectations upon booms would increase. Here we mainly focus on amplification of crises by pessimistic expectations.

relative past performance. There is quite some empirical evidence for heterogeneity of expectations and performance-based strategy switching in various economic settings. For example, Branch (2004, 2007) estimates a simple switching model with heterogeneous expectations using exchange rate survey data, Vissing-Jorgensen (2003) presents evidence of heterogeneous beliefs of individual investors about the prospect of the stock market, and Shiller (2000) finds evidence that investor's sentiment changes over time, with both institutions and individual investors becoming more optimistic in response to recent significant increases of the stock market. Heterogeneous expectations switching models have been successfully estimated/calibrated in various empirical applications, for example, to explain bubbles and crashes in stock prices (e.g., Amilon 2008; Boswijk, Hommes, and Manzan 2007; de Jong, Verschoor, and Zwinkels 2009; Lof 2012, 2015), large movements in exchange rates (e.g., Gilli and Winker 2003; Westerhoff and Reitz 2003), persistent high and low inflation (Cornea, Hommes, and Massaro 2013), and bubbles and crashes in commodities (e.g., gold prices Alfarano, Lux, and Wagner 2005, and oil prices Ellen and Zwinkels 2010). Anufriev and Hommes (2012) and Assenza et al. (2013) fitted a heuristics switching model to laboratory data of asset pricing and inflation/output forecasting experiments.

A. Heterogeneous Expectations

Assume there are J types of lenders in our economy. At date t , type j 's forecast for period $t+1$ of the return of the fund is given by:

$$(34) \quad \lambda_{j,t+1}^e = p_{j,t+1}^e r_t.$$

Hence, each forecasting rule is determined by its forecast $p_{j,t+1}^e$ of the probability of success, that is, the probability that the firm will pay back the loan. Agents can choose between J different forecasting rules. The main idea underlying the switching model is that agents are boundedly rational and choose a forecasting strategy based upon its relative past performance. Let $U_{j,t}$ be a weighted average of past squared forecasting errors of type j of the returns $\lambda_{j,t}$, which, using Equation (34), becomes

$$(35) \quad U_{j,t} = r_{t-1}^2 \left(p_t - p_{j,t}^e \right)^2 + \gamma U_{j,t-1},$$

where γ is the weight given to past fitness. Let $u_{j,t}$ be the relative past squared forecasting errors of the returns of the fund, that is,

$$(36) \quad u_{j,t} = U_{j,t}/U_t^{\text{tot}}, \quad U_t^{\text{tot}} = \sum_{j=1}^J U_{j,t}.$$

The fraction of the expectations rule j is updated according to a *discrete choice model with asynchronous updating* (Diks and van der Weide 2005; Hommes et al. 2005a; Hommes, Huang, and Wang 2005b)

$$(37) \quad n_{j,t} = \delta n_{j,t-1} + (1 - \delta) \frac{e^{-\beta u_{j,t}}}{z_t},$$

where $z_t = \sum_{j=1}^J \exp(-\beta u_{j,t})$ is a normalization factor. The asynchronous updating parameter $0 \leq \delta \leq 1$ reflects inertia in the choice of the heuristics.¹⁴ In the extreme case $\delta = 1$, the initial impacts of the rules never change, no matter what their past performance was. At the other extreme, $\delta = 0$, we have the special case of synchronous updating, as by Brock and Hommes (1997), where all agents switch to better strategies in each period. In general, in each period only a fraction $1 - \delta$ of the heuristic's weight is updated according to the discrete choice model with asynchronous updating. The parameter $\beta \geq 0$ represents the intensity of choice measuring how sensitive predictor choice is to differences in heuristics' performance. In the extreme case $\beta = 0$, the relative weights of heuristics are not updated; at the other extreme $\beta = +\infty$, a fraction $1 - \delta$ of agents switch immediately to the best predictor. In the special case, where $\delta = 0$ and $\beta = \infty$ coined the *neoclassical limit* by Brock and Hommes (1997), all agents switch immediately to the best forecasting strategy. In the simulations of heterogeneous market equilibrium dynamics below, the parameters will be fixed at $\beta = 5$, $\delta = 0.5$, and $\gamma = 0$, but the results are fairly robust with respect to changes of these parameters. We will also check the robustness by studying analytically the neoclassical limit case.

14. In recent laboratory experiments in various settings, for example in asset pricing forecasting (Anufriev and Hommes 2012), in positive feedback (asset) and negative feedback (cobweb) markets (Anufriev, Hommes, and Philipse 2013b) and in a New Keynesian macro framework (Assenza et al. 2013), it has been found that the value of the inertia parameter $\delta = 0.8$ or 0.9 is high so that there is a strong tendency to stick to some rule before switching to another rule.

B. Heterogeneous Market Equilibrium

Under heterogeneous expectations, we define total supply of loans at date t as

$$\begin{aligned} S_t(r_t) &= \frac{w_t}{2} \sum_{j=1}^J n_{j,t} \bar{i} \left[\lambda_{j,t+1}^e(r_t) > \rho \right] \\ (38) \quad &= \frac{w_t}{2} \sum_{j=1}^J n_{j,t} \bar{i} \left[p_{j,t+1}^e r_t > \rho \right], \end{aligned}$$

where $p_{j,t+1}^e$ represents expectations of type j about the probability of success and $n_{j,t}$ represents the fraction of agents of type j at time t . Here the last equality follows from Equation (18). Temporary equilibrium in the loan market is given by:

$$(39) \quad x(r_t) = S_t(r_t) = \frac{w_t}{2} \sum_{j=1}^J n_{j,t} \bar{i} \left[p_{j,t+1}^e r_t > \rho \right].$$

Figure 4 illustrates market equilibrium in the case of heterogeneous expectations with two types of agents ($J=2$). Recall that, in the homogeneous case, the loan supply correspondence (Equation (21)) is a step function (see Figure 1), with the loan supply switching from 0 to $w_t/2$ at the threshold $r^* = \rho/p_{t+1}^e$, whose value is determined by the expected probability of success. In the heterogeneous case with two types of expectations, $p_{1,t+1}^e$ and $p_{2,t+1}^e$, the loan supply correspondence is a 2-step function. If, for example, $p_{1,t+1}^e > p_{2,t+1}^e$, that is, type 2 are more pessimistic, then the threshold levels are $r_1^* = \rho/p_{1,t+1}^e$, where the loan supply switches from 0 to $n_{1,t}w_t/2$, and $r_2^* = \rho/p_{2,t+1}^e$, where the loan supply switches from $n_{1,t}w_t/2$ to $w_t/2$.

Note that Equation (39) is a *temporary equilibrium* relation, with time-varying supply correspondence in Equation (38), depending on time-varying aggregate savings $w_t/2$, time-varying fractions $n_{j,t}$ and individual expectations $p_{j,t+1}^e$. Figure 4 illustrates four possible temporary equilibrium points A, B, C, and D, depending on the temporary supply curve, in a simple 2-type case. It is important to note that as the fraction of the most pessimistic forecasting rule 2 increases (given $w_t/2$), the vertical threshold at $n_{1,t}w_t/2$ shifts to the left and the equilibrium point shifts from A to B or C (Figure 4, respectively, panels (a), (b), and (c)). Consequently, an increase of the fraction of the most pessimistic forecasting rule leads to a high contract

rate r^* and a lower equilibrium loan x^* and a correspondingly lower output. Similarly, an increase in the most optimistic forecasting rule leads to a lower equilibrium contract rate and a higher equilibrium output. Stated differently, an increase in the fraction of the most pessimistic forecasting rule amplifies an economic crisis or bust. In Figure 4D, we show the effect of a decrease of the time-varying aggregate savings $w_t/2$ (given $n_{1,t}$), due for example to a decrease of the probability of success p_t (see Equation (13)). As the probability of success decreases aggregate savings decreases and the temporary equilibrium moves from A to D with higher interest rate and lower equilibrium loan.

V. A STYLIZED EXAMPLE WITH SIX BELIEF TYPES

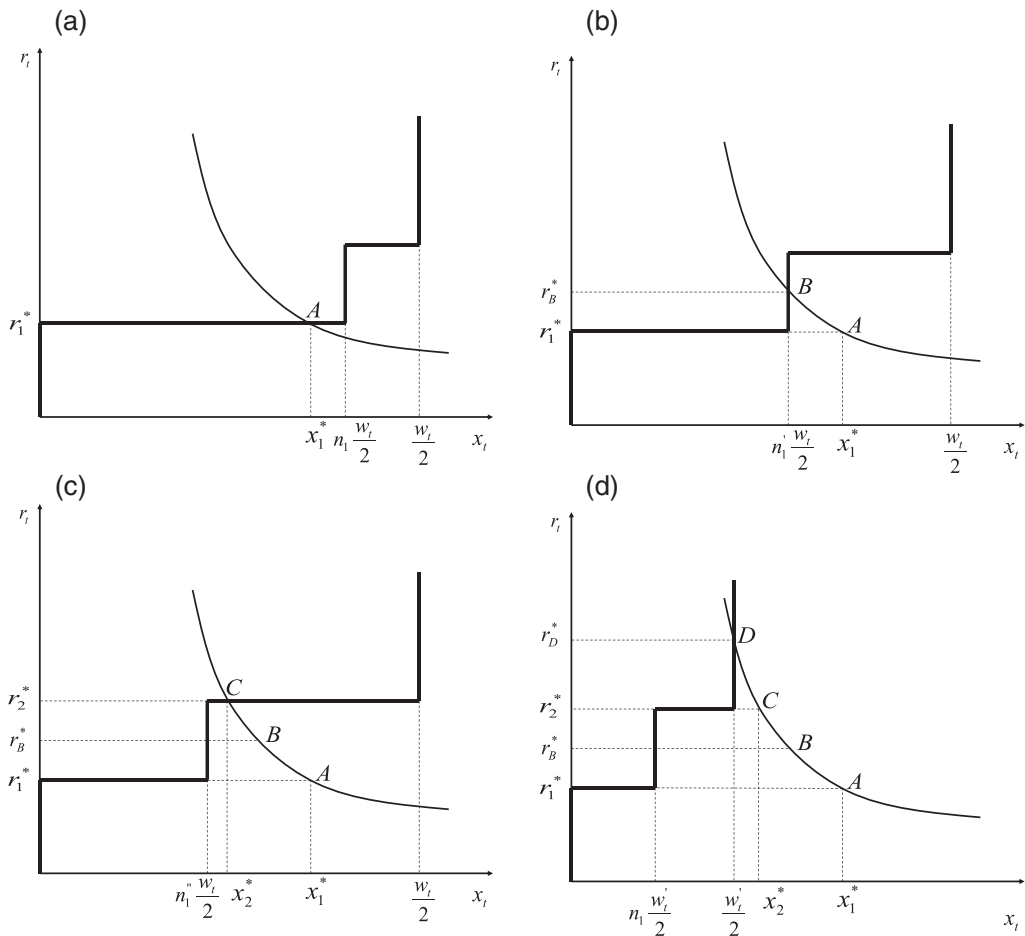
In this section, we consider the heterogeneous beliefs case with six expectations rules taken from the homogenous benchmarks: rational, naive, average, trend-following, pessimistic, and optimistic expectations. Rational agents know the true exogenous probability generating process (27) and therefore use the optimal, model consistent AR(1) forecasting rule to predict the firms' probability of success. Notice that AR(1) forecasters are not only rational forecasters, but also rational optimizers maximizing utility Equation (1) under the budget constraint Equations (2 and 3), given their forecast of the expected loan return $\lambda_{t+1}^e = p_{t+1}^e r_t$ in Equation (18). As the equilibrium contract rate r_t is known before making the forecasts, agents correctly take the behavior of other non-rational agents, who affect this equilibrium contract rate r_t , into account.¹⁵

We divide in three subsections. In the benchmark simulations in Section V.A the exogenous AR(1) stochastic time series of the probability of success is the same as for the homogeneous benchmarks before, with its minimum realization in the "crisis-period" 51. In Section V.B, we provide the main intuition why nonrational strategies survive and how they explain our results. In Section V.C, we perform Monte Carlo simulations to show the robustness and statistical significance of our results.

15. The same is true for other subjective forecasting rules, but AR(1) forecasters are the only agents who are both rational optimizers and rational forecasters, while other forecasting rules are not rational forecasters as they are not model consistent with the exogenous stochastic probability process. See Sargent (1993) for a discussion of optimization and forecasting as two different aspects of rationality.

FIGURE 4

Temporary equilibria in the 2-type case, with four possible loan market equilibrium points A, B, C, and D, depending on the time-varying supply curve



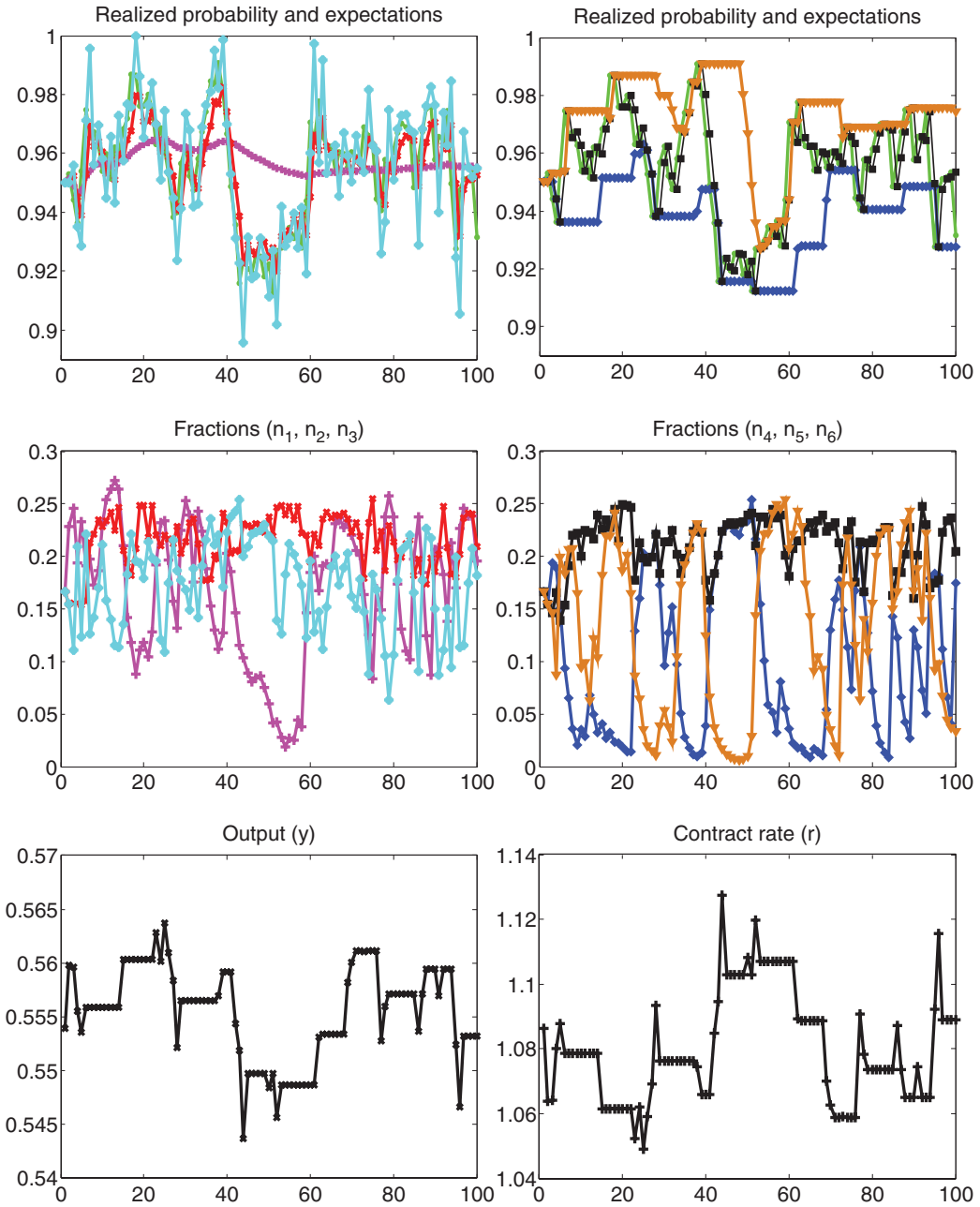
Note: The figure illustrates the case $p_{1,t+1}^e > p_{2,t+1}^e$, that is, type 2 are more pessimistic. The loan supply correspondence is a 2-step function with critical threshold levels at $r_1^* = \rho/p_{1,t+1}^e$, where the loan supply switches from 0 to $n_1 w_t/2$, and at $r_2^* = \rho/p_{2,t+1}^e$, where the loan supply switches from $n_1 w_t/2$ to $w_t/2$. As the fraction of the pessimistic type 2 increases, the equilibrium shifts from A to B, C or D.

A. Benchmark Simulations

Figure 5 illustrates the dynamics of the 6-type case. The fractions of all types (middle panels) show considerable fluctuations, fluctuating between 0 and 0.28, with rational and naive expectations dominating (ranging from 0.15–0.25), trend followers somewhere in between (ranging from 0.05–0.25), and average, pessimistic and optimistic expectations wildly fluctuating (between 0 and 0.27) at times being the minority types, but never completely driven out of the market.

The contract rate (bottom right panel) gradually increases and remains persistently high between periods 40–60. Overall, the contract rate is persistently higher than the long run equilibrium rate $\bar{r} = \rho/\mu = 1.063$, because of the presence of pessimistic forecasters, even when their fractions are relatively small. Around the exogenously generated crisis of period 51, the fraction of pessimistic expectations is at a peak, around 0.25, and only decrease gradually thereafter. A relatively small fraction of pessimistic traders thus has a significant impact on aggregate

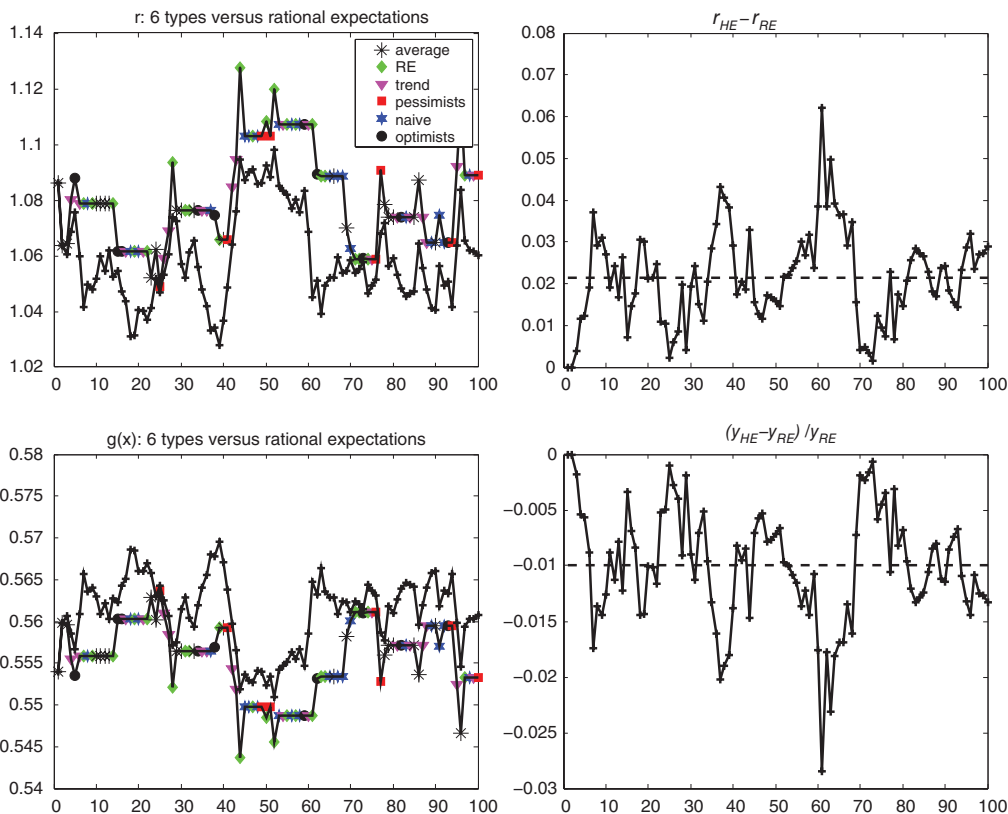
FIGURE 5
Heterogeneous expectations with 6 types ($\beta = 5$, $\delta = 0.5$, and $\gamma = 0$)



Note: Upper left panel: realized probability of success (green), rational expectations (AR(1)) (red), average expectations (purple), and trend follower expectations (cyan). Upper right panel: pessimistic (minimum) expectations (blue), naive expectations (black), and optimistic (maximum) expectations (yellow). Mid-left panel: fractions of rational, average, and trend-following believers (red, purple, and cyan, respectively). Mid-right panel: fractions of pessimistic, naive, and optimistic (blue, black, and yellow, respectively). Bottom panel: output (left) and contract rate (right).

FIGURE 6

Differences between homogeneous rational expectations benchmark and heterogeneous expectations ($\beta = 5, \delta = 0.5,$ and $\gamma = 0$)



Note: Upper panel left: contract rate of 6 types benchmark (colored marks) versus rational (AR(1)) benchmark (black). Upper right: the heterogeneous expectations bias, that is, the differences in contract rates. Bottom panel left: output for 6 types benchmark (colored marks) versus rational (AR(1)) benchmark (black). Bottom right: the heterogeneous expectations bias, that is, the relative differences in output.

outcomes and contributes to a high equilibrium contract rates for more than 10 periods. The time series of output $y_t = g(x_t)$ is also shown (bottom left panel), it gradually decreases and remains persistently low between periods 40 and 60, only slowly recovering in subsequent periods.

Figure 6 compares the 6-type heterogeneous expectations simulations with the homogeneous rational expectations benchmark. In particular, Figure 6 (top right panel) illustrates that the difference of the contract rates under boundedly rational heterogeneous expectations and homogeneous rational expectations is always positive and highly persistent. The average *heterogeneous expectations bias* of the contract rate for loans are $\bar{r}^{HET} - \bar{r}^{RE} \approx 3.04\%$. Its peak is substantial, about 6.4 %, and occurs in period 61, much later

than the worst exogenous shock in the crisis period 51, at times when the rational forecast has already correctly predicted the mean reversion of the probability of success toward its mean. Apparently, under heterogeneous expectations the influence of a relatively small fraction of pessimistic agents on aggregate behavior is still substantial.

Similarly, the bottom panel of Figure 6 illustrates differences in output under heterogeneous versus homogeneous rational expectations. Under heterogeneous expectations, output is significantly lower than under rational expectations. On average, the relative output loss $(y^{RE} - y^{HET}) / y^{RE}$ because of boundedly rational heterogeneous expectations is about 1.4 %, with a peak of almost 3 %. As for the peak in the

differences in the contract rate, the biggest output loss because of heterogeneous expectations occurs in period 61, much later than the crisis period 51, and it occurs when in fact the exogenous probability of success already has recovered to normal levels. A drop of confidence, however, because of boundedly rational heterogeneous expectations still affects output at the macro level substantially, even when the fractions of pessimists at the micro level is relatively small.

B. Why Do Nonrational Strategies Survive?

Why then are the fully rational agents, using the AR(1) model consistent forecasting rule of the exogenous probability process, not driving out all other forecasting rules, as has been suggested by the traditional rational approach, advocated, for example, by Friedman (1953) and Fama (1970).¹⁶ In order to address this question we first study analytically the case $\delta = 0$ (i.e., synchronous updating), $\gamma = 0$ (i.e., no memory) and $\beta = \infty$ (immediate switching to the best performing rule). Brock and Hommes (1997) coined this case a “neoclassical limit,” with all agents immediately switching to the best rule. We will refer to this benchmark as *immediate optimal switching*.

A first observation is that, as a result of the noise, the RE rule does *not* always yield the best forecast. Figure 7 shows a histogram of how often each of the six rules yields the best forecast for the same exogenous probability series of 100 periods as before.¹⁷ Only in 24% of the cases RE is the best forecast. RE is closely followed by the average (20%), naive (19%), and the trend-following (19%) rules. The pessimistic (8%) and optimistic (10%) are less frequently used, but still perform best for a significant amount of time. The pessimistic rule performs particularly well during crisis phases (when the probability of success is low).

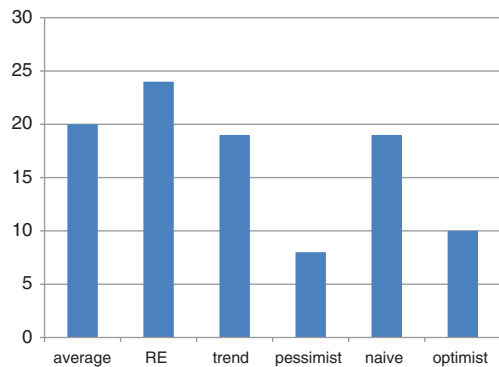
There are three key elements of why non-rational forecasting rules survive in our economy with performance based strategy selection: (1) bounded rationality, (2) finite memory and

16. We stress once more that AR(1) forecasters are fully rational in our framework, as they are both rational forecasters and utility maximizers taking the behavior of other non-rational agents into account through their knowledge of the equilibrium contract rate r_t ; see the discussion in Section 5.

17. We stress that the histogram in Figure 7 for the neoclassical limit $\beta = \infty$ should be viewed as an analytical result since, given the random realizations of the exogenous probability time series, a comparison of the forecasting errors of the six strategies *exactly* determines the best rule at each period in time.

FIGURE 7

Histogram of the best performing forecasting rule for the same exogenous probability series



Note: RE is the best rule, but closely followed by the average, naive, and trend-following rules. Each of the other rules, pessimists and optimists still dominate more than 8% of the time.

more weight to recent performance, and (3) inertia because of asynchronous strategy updating.

1. Agents choose between heterogeneous forecasting rules based upon recent forecasting performance. Their choice is *boundedly rational* in the sense that their intensity of choice to switch strategies is finite, that is, $\beta < \infty$, implying that some agents will *not* switch to the best strategy, but choose an alternative rule. Hence, for $\beta < \infty$, each rule attracts some followers. When $\beta \approx 0$, the distribution of the population over the forecasting rules is flat, with fractions approximately equal. For $\beta \approx \infty$, the distribution over rules is peaked, with most agents choosing the best strategy.

2. In general, the performance measure is a weighted average of past (relative) forecasting errors, as in Equation (35). In the (special) case when the contract rate r_t would be constant over time and memory would be infinite (i.e., $\gamma = 1$), the performance measure is, up to a scaling factor, equivalent to the mean squared error (MSE). In the special case of infinite memory ($\gamma = 1$) in the long run the RE AR(1) forecast would drive out all other forecasting rules. Hence, the rational benchmark is nested within our framework as a special case. However, in the more realistic case when *memory is finite*, that is, $0 \leq \gamma < 1$, agents give more weight to recent observations. When more weight is given to recent observations, RE can be suboptimal. Indeed, as we have seen for the immediate optimal switching case, at times

other rules perform better than RE. In particular, in “bad” times the pessimistic rule performs relatively well, while in “good” times the optimistic rule performs relatively well. There is empirical evidence that *recent* performance is important for strategy selection. For example, evidence from empirical finance suggests that the flow in and out of mutual funds is strongly driven by the recent past performance of these funds (e.g. Karceski 2002; Sirri and Tufano 1998). Similarly, using data, Vanguard, Benartzi and Thaler (2007) have shown for retirement savings decisions that equity allocation of new participants rose from 58% in 1992 to 74% in 2000, following a strong rise in stock prices in the late 1990s, but dropped, back to 54% in 2002, following a strong fall in stock prices. In recent laboratory experiments with human subjects, Anufriev, Bao, and Tuinstra (2015) show that individuals switch to alternative strategies which performed better in the recent past, even when such performance was driven by an exogenous random sequence and individuals had enough information about which strategy was optimal on average.

3. Our expectations selection framework (Equations (35)–(37)) is an extension of the model with synchronous updating of Brock and Hommes (1997), allowing for *asynchronous updating* (Diks and van der Weide 2005; Hommes, Huang, and Wang 2005b). Asynchronous updating introduces inertia in strategy switching, through the parameter $0 < \delta < 1$, representing the fraction of agents that will stick to their previous strategy, while in a given period only a fraction $1 - \delta$ switches strategy based on relative performance. This inertia in strategy switching because of asynchronous updating is in some sense similar to rational inattention (Sims 1998, 2003). Anufriev and Hommes (2012) fitted the heterogeneous expectations switching model with asynchronous updating to experimental data and found relative large values around $\delta = 0.8$. Consequently, once non-rational expectations rule gain some weight, for example in “bad” times, when a fraction of agents becomes pessimistic, asynchronous strategy updating implies that they only disappear gradually afterwards. As a consequence, a relatively small fraction of pessimistic agents may increase the persistence of crisis considerable leading to a very slow recovery of the economy.

These three plausible and empirically relevant elements of strategy switching cause non-rational rules to survive in a heterogeneous population. In particular, in bad times they cause (at least)

a small fraction of agents to have pessimistic expectations. But even a relatively small fraction of pessimistic believers has a significant effect upon aggregate behavior and causes crises to be deeper and more persistent.

C. Monte Carlo Simulations

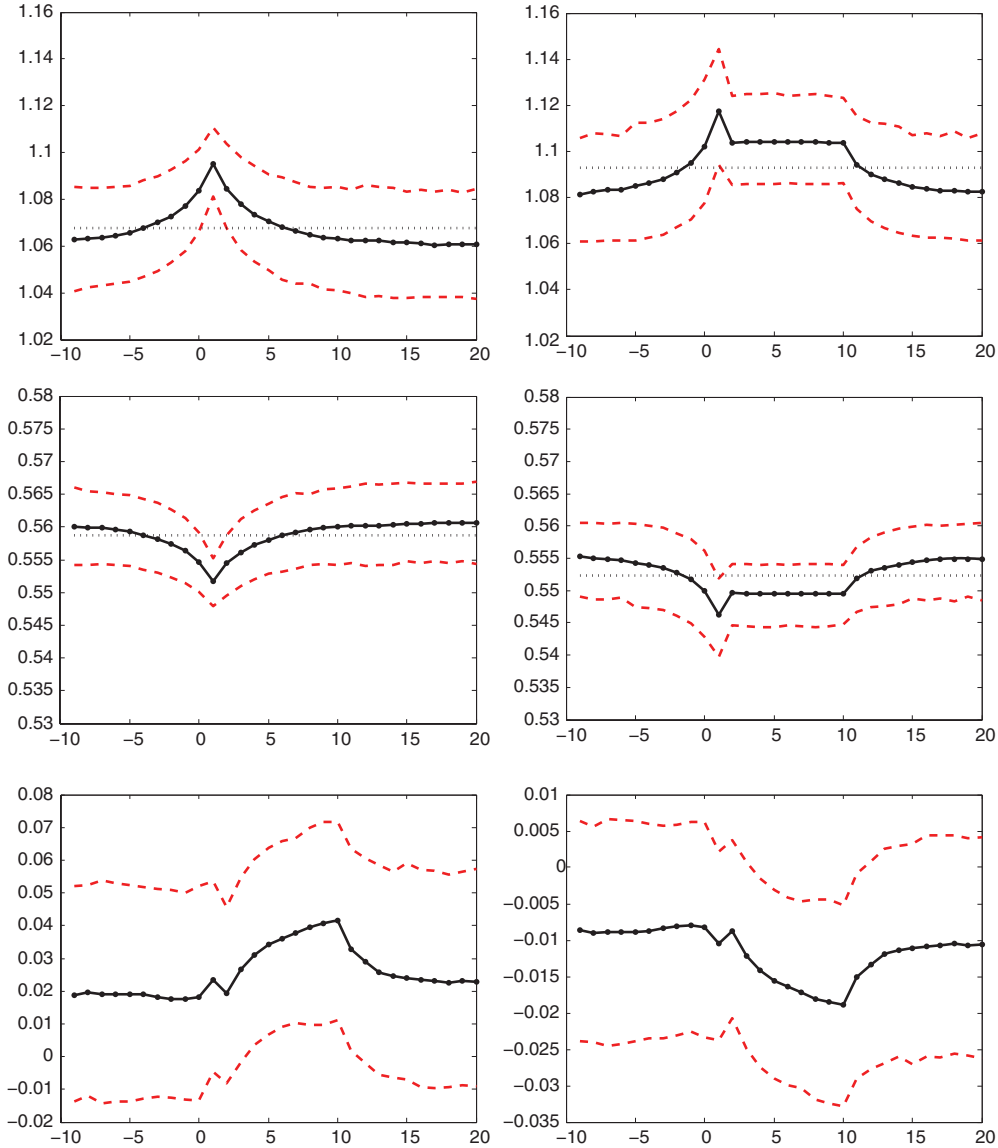
To check the robustness and statistical significance of these results, this section presents Monte Carlo (MC) simulations of the contract rate r_t and output y_t , averaged over $B = 1,000$ runs of the exogenous stochastic AR(1) time series of probabilities p_t of length 100. We compare the average behavior of the 6-type benchmark model (i.e., $\beta = 5$, $\gamma = 0$, and $\delta = 0.5$) to the RE benchmark and compute 95% confidence bounds over $B = 1,000$ runs.

Behavior Around Crises. Figure 8 shows MC simulations of the mean contract rate r_t and mean output y_t around a crisis. Here a crisis is defined as the *global minimum* of the corresponding AR(1) probability series p_t of length 100, and the plots show the mean contract rate r_t and mean output y_t , together with the 95% confidence bounds, from 10 periods *before* until 20 periods *after* the crisis, averaged over $B = 1,000$ MC simulations.¹⁸ Under RE (left panels), the behavior around the crisis (i.e., period 0) is symmetric, with for example, an exponential increase of the contract rate before and an exponential decrease after the crisis until the long run steady state levels (dotted horizontal lines) are reached. In the heterogeneous expectations 6-type benchmark model (right panels), the behavior is asymmetric, with again an exponential increase of the contract rate before the crisis, but a much slower decline of the contract rate afterwards. In the benchmark 6-type model, the decline of the contract rate after the crisis is very slow, almost flat (top right panel) because of the presence and persistence of (a small fraction of) pessimists, with agents only gradually updating their pessimistic expectations. The same is true for output (second row, right panel), which recovers only extremely slowly after the crisis. The slow recovery lasts 10 periods, exactly the memory length of the pessimistic agents. The bottom row panels of Figure 8 illustrate the differences between the contract rates (left) and the relative

18. For our single benchmark run of the AR(1) probabilities in Section V.A the crisis occurs at period 51, so the period around the crisis covers $41 \leq t \leq 71$.

FIGURE 8

The mean contract rate r_t and the output y_t , with the 95% confidence bounds, from 10 periods before the crisis (i.e., the global minimum of the corresponding probability series p_t of length 100) until 20 periods after the crisis, averaged over $B = 1,000$ Monte Carlo simulations



Note: Top panels: mean r_t under RE (left panel) and for benchmark 6-type model (right panel); Middle panels: mean y_t under RE (left panel) and for benchmark 6-type model (right panel); Bottom panels: mean difference r_t (left panel) and y_t (right panel) between 6-type benchmark model and RE.

differences between output (right) in the 6-type heterogeneous expectations (HE) model and the RE benchmark. On average, under HE the contract rate r_t is about 2% higher, while output is about 1% lower, consistent with the single

simulation run in Section V.A. The mean difference in contract rate r_t increases from 2% in normal times to 4% with a peak 10 periods after the crisis, while the mean relative difference in output increases from 1% in normal times to almost

2% 10 periods after the crisis. These numbers are consistent with the single stochastic simulation run in Section V.A, which is within the 95% confidence bounds. Thus pessimistic beliefs show a high persistence because of gradual strategy updating. The slow recovery of the economy lasts 10 periods, exactly the horizon of a relatively small fraction of pessimistic beliefs.

VI. CONCLUSION

This article is an attempt to build a model of “animal spirits” and “confidence,” as advocated by Akerlof and Shiller (2009). Our building block is the heterogeneous expectations switching model of Brock and Hommes (1997). We have studied an equilibrium model for loans and compared the case of expectations heterogeneity to the standard case of homogeneous rational expectations. Heterogeneous expectations are disciplined by evolutionary selection or reinforcement learning based upon recent forecasting performance. Survey data on expectations, laboratory forecasting experiments and time series data lend empirical support to such a heterogeneous expectations hypothesis. Costless rational expectations, whose forecast uses the correct model consistent specification of the stochastic probability of success, are unable to drive out simple forecasting heuristics such as naive expectations, trend following rules and pessimistic or optimistic expectations. In particular, a small fraction of pessimistic expectations survives, and even a small fraction of pessimistic believers has a large impact on aggregate macro behavior. Even in the presence of costless fully rational expectations, a small fraction of pessimistic agents at the microlevel may have a relatively large aggregate effect at the macrolevel and cause economic busts to be deeper and recovery from crisis to be much slower. These busts are amplified by almost self-fulfilling expectations because of the positive feedback in our macroeconomic system.¹⁹ The different timing bust episodes under heterogeneous expectations may have important consequences for the timing of monetary and fiscal policy.

In our stylized model only expectations about an exogenous process of default probabilities

are formed. The forecasts themselves have no effect upon the realizations of these default probabilities. Agents’ forecasts, however, feed back into the real economy through their loan supply functions and thus affect aggregate equilibrium outcomes. Our results show that in a heterogeneous forecasting competition of this exogenous process, nonrational rules survive and have a large impact upon aggregate outcomes. In a more extensive model, and in reality, forecasts of default probabilities are likely to exhibit endogenous feedback, that is, default probability forecasts have an impact on realized default probabilities. In the presence of direct endogenous expectations feedback, one would expect coordination on self-fulfilling pessimistic expectations to be even more likely and lead to even stronger amplification effects of crises and the duration of its recovery. This remains an important topic for future work.

In a recent survey, Brunnermeier, Eisenbach, and Sannikov (2012) focus on financial frictions as the key mechanism causing persistence, amplification, and instability at the macroeconomic level. Although financial frictions may play an important role, our results show that persistence, amplifications, and instability arise even without any financial frictions in a simple stylized equilibrium model of boundedly rational agents with heterogeneous expectations. If bounded rationality, animal spirits, and expectations heterogeneity are indeed important drivers of macroeconomic instability amplifying economic crises and slowing down recovery, policy should focus not only on financial frictions but also on managing heterogeneous expectations, trend following behavior, and overpessimistic beliefs about the economy. Moreover, the economics profession should pay more attention to animal spirits and expectational heterogeneity and their potentially destabilizing role and negative welfare effects in order to prevent economic losses.²⁰

19. This is consistent with laboratory experiments, for example, Assenza et al. (2013), Bao et al. (2012), and Heemeijer et al. (2009), where in positive feedback systems subjects coordinate expectations on almost self-fulfilling forecasting rules, such as trend-following or anchor and adjustment rules, thus amplifying market fluctuations.

20. For example, Brock, Hommes, and Wagener (2009) discuss the role of financial innovation in generating financial instability. In the traditional financial economics view, under full rationality financial innovation is usually considered to be stabilizing and welfare improving. In contrast, in a simple stylized model with boundedly rational heterogeneous investors, Brock, Hommes, and Wagener (2009) show that financial innovation may destabilize price fluctuations and decrease average welfare. The main reason is that, in the presence of more financial hedging instruments, investors take bigger positions (leverage) amplifying wins or losses of boundedly rational agents, thus destabilizing the market. Policy implications concerning regulating financial innovation may thus be completely opposite whether one adopts a homogeneous rational or a boundedly rational heterogeneous expectations market view.

Off course our model is very stylized, but the same heterogeneous expectations framework can be applied to richer and more advanced models with endogenous expectations feedback, for example New Keynesian macromodels (e.g. Anufriev et al. 2013a; De Grauwe 2011), including models with infinite horizon (Branch and McGough 2009; Massaro 2013). Future work should investigate theoretically and empirically the size and persistence of heterogeneity, differences in real variables, such as wages and output, and the implications for the timing of monetary and fiscal policy.

REFERENCES

- Adam, K. "Experimental Evidence on the Persistence of Output and Inflation." *The Economic Journal*, 117(520), 2007, 603–36.
- Akerlof, G. A., and R. J. Shiller. *How Human Psychology Drives the Economy, and Why It Matters for Global Capitalism*. Princeton, NJ: Princeton University Press, 2009.
- Alfarano, S., T. Lux, and W. Wagner. "Estimation of Agent-Based Models: The Case of an Asymmetric Herding Model." *Computational Economics*, 26, 2005, 19–49.
- Allen, H., and M. P. Taylor. "Charts, Noise and Fundamentals in the London Foreign Exchange Market." *The Economic Journal*, 100(400), 1990, 49–59.
- Amilon, H. "Estimation of an Adaptive Stock Market Model with Heterogeneous Agents." *Journal of Empirical Finance*, 15, 2008, 342–62.
- Anufriev, M., and C. H. Hommes. "Evolutionary Selection of Individual Expectations and Aggregate Outcomes in Asset Pricing Experiments." *American Economic Journal: Microeconomics*, 4(4), 2012, 35–64.
- Anufriev, M., T. Assenza, C. H. Hommes, and D. Massaro. "Interest Rate Rules and Macroeconomic Stability under Heterogeneous Expectations." *Macroeconomic Dynamics*, 17, 2013a, 1574–604.
- Anufriev, M., C. H. Hommes, and R. Philipse. "Evolutionary Selection of Expectations in Positive and Negative Feedback Markets." *Journal of Evolutionary Economics*, 23, 2013b, 663–88.
- Anufriev, M., T. Bao, and J. Tuinstra. "Microfoundations for Switching Behavior in Heterogeneous Agent Models: An Experiment." CeNDEF Working Paper 2015–09, University of Amsterdam, 2015.
- Arifovic, J., and L. Petersen. "Escaping Expectations Driven Liquidity Traps: Experimental Evidence." Working Paper No. 15-03, Simon Fraser University, 2015.
- Assenza, T., and M. Berardi. "Learning in a Credit Economy." *Journal of Economic Dynamics and Control*, 33(5), 2009, 1159–69.
- Assenza, T., P. Heemeijer, C. Hommes, and D. Massaro. "Individual Expectations and Aggregate Macro Behavior." Tinbergen Institute Discussion Paper No. TI 2013-016/II, 2013.
- Assenza, T., T. Bao, C. H. Hommes, and D. Massaro. "Experiments on Expectations in Macroeconomics and Finance," in *Experiments in Macroeconomics, Research in Experimental Economics*, Vol. 17, edited by J. Duffy. Cambridge, MA: Emerald Publishing, 2014.
- Bao, T., C. H. Hommes, J. Sonnemans, and J. Tuinstra. "Individual Expectations, Limited Rationality and Aggregate Outcomes." *Journal of Economic Dynamics and Control*, 36, 2012, 1101–20.
- Barberis, N., and R. H. Thaler. "A Survey of Behavioral Finance," in *Handbook of the Economics of Finance*, edited by G. M. Constantinides, M. Harris, and R. Stulza. Amsterdam: Elsevier, 2003, 1051–121.
- Benartzi, S., and R. H. Thaler. "Heuristics and Biases in Retirement Savings Behavior." *Journal of Economic Perspectives*, 21, 2007, 81–104.
- Benhabib J., G. W. Evans, and S. Honkapohja. "Liquidity Traps and Expectation Dynamics: Fiscal Stimulus or Fiscal Austerity?" NBER Working Paper No. 18114, 2008.
- Berardi, M. "Heterogeneity and Misspecifications in Learning." *Journal of Money, Credit and Banking*, 31, 2007, 3203–27.
- Bianchi, J., E. Boz, and E. Mendoza. "Macro-Prudential Policy in a Neo-Fisherian Model of Financial Innovation." *IMF Economic Review*, 60, 2012, 223–69.
- Boswijk, H. P., C. H. Hommes, and S. Manzan. "Behavioral Heterogeneity in Stock Prices." *Journal of Economic Dynamics and Control*, 31, 2007, 1938–70.
- Boz, E. and E. Mendoza. "Financial Innovation, the Discovery of Risk, and the U.S. Credit Crisis." NBER Working Paper No. 16020, 2010.
- Branch, W. "The Theory of Rationally Heterogeneous Expectations: Evidence from Survey Data on Inflation Expectations." *The Economic Journal*, 114(497), 2004, 592–621.
- Branch, W. A. "Sticky Information and Model Uncertainty in Survey Data on Inflation Expectations." *Journal of Economic Dynamics and Control*, 31, 2007, 245–76.
- Branch, W., and G. Evans. "Intrinsic Heterogeneity in Expectation Formation." *Journal of Economic Theory*, 127, 2006, 264–95.
- Branch, W., and B. McGough. "Monetary Policy in a New Keynesian Model with Heterogeneous Expectations." *Journal of Economic Dynamics and Control*, 33, 2009, 1036–51.
- Brazier, A., R. Harrison, M. King, and T. Yates. "The Danger of Inflating Expectations of Macroeconomic Stability: Heuristic Switching in an Overlapping Generations Monetary Model." *International Journal of Central Banking*, 4(2), 2008, 219–54.
- Brock, W., and P. de Fontnouvelle. "Expectational Diversity in Monetary Economies." *Journal of Economic Dynamics and Control*, 24(5–7), 2000, 725–59.
- Brock, W. A., and C. H. Hommes. "A Rational Route to Randomness." *Econometrica*, 65(5), 1997, 1059–95.
- . "Heterogeneous Beliefs and Routes to Chaos in a Simple Asset Pricing Model." *Journal of Economic Dynamics and Control*, 22, 1998, 1235–74.
- Brock, W. A., and C. Manski. "Competitive Lending with Partial Knowledge of Loan Repayment." NBER Working Paper No. 14378, 2008.
- . "Competitive Lending with Partial Knowledge of Loan Repayment." *Journal of Money, Credit and Banking*, 43(2–3), 2011, 441–59.
- Brock, W. A., C. H. Hommes, and F. O. O. Wagener. "More Hedging Instruments May Destabilize Markets." *Journal of Economic Dynamics and Control*, 33, 2009, 1912–28.
- Brunnermeier, M. K., T. M. Eisenbach, and Y. Sannikov. *Macroeconomics with Financial Frictions: A Survey*. Princeton, NJ: Princeton University Press, 2012.
- Caballero, R., and A. Krishnamurthy. "Collective Risk Management in a Flight to Quality Episode." *Journal of Finance*, 63, 2008, 2195–230.

- Cogley, T., and T. J. Sargent. "Anticipated Utility and Rational Expectations as Approximations of Bayesian Decision Making." *International Economic Review*, 49, 2008, 185–221.
- Colander, D., R. P. F. Holt, and B. J. Rosser, ed. *The Changing Face of Economics. Conversations with Cutting Edge Economists*. Ann Arbor: University of Michigan Press, 2004.
- Colander, D., P. Howitt, A. Kirman, A. Leijonhufvud, and P. Mehrling. "Beyond DSGE Models: Toward an Empirically Based Macroeconomics." *American Economic Review*, 98(2), 2008, 236–40.
- Cornea, A., C. H. Hommes, and D. Massaro. "Behavioral Heterogeneity in U.S. Inflation Dynamics." Tinbergen Institute Discussion Paper No. TI 2013-015/II, 2013.
- De Grauwe, P. "Animal Spirits and Monetary Policy." *Economic Theory*, 47(2–3), 2011, 423–57.
- Diks, C. G. H., and R. van derWeide. "Herding, a-Synchronous Updating and Heterogeneity in Memory in a CBS." *Journal of Economic Dynamics and Control*, 29, 2005, 741–63.
- Duffy, J. "Macroeconomics: A Survey of Laboratory Research." University of Pittsburgh Working Papers No. 334, 2008.
- Dwyer, G. P., A. W. Williams, R. C. Battalio, and T. I. Mason. "Tests of Rational Expectations in a Stark Setting." *The Economic Journal*, 103, 1993, 586–601.
- Ellen, T. S., and R. C. J. Zwickels. "Oil Price Dynamics: A Behavioral Finance Approach with Heterogeneous Agents." *Energy Economics*, 32(6), 2010, 1427–34.
- Evans, G. W., and S. Honkapohja. *Learning and Expectations in Macroeconomics*. Princeton, NJ: Princeton University Press, 2001.
- . "Expectations and the Stability Problem for Optimal Monetary Policies." *Review of Economic Studies*, 70, 2003, 807–24.
- Evans, G., and S. Honkapohja. "Monetary Policy, Expectations and Commitment." *Scandinavian Journal of Economics*, 108, 2006, 15–38.
- Evans, G. W., E. Guse, and S. Honkapohja. "Liquidity Traps, Learning and Stagnation." *European Economic Review*, 52(8), 2008, 1438–63.
- Fama, E. F. "Efficient Capital Markets: A Review of Theory and Empirical Work." *Journal of Finance*, 25, 1970, 383–423.
- Frankel, J. A., and K. A. Froot. "The Rationality of the Foreign Exchange Rate. Chartists, Fundamentalists and Trading in the Foreign Exchange Market." *American Economic Review: Papers and Proceedings*, 80(2), 1990, 181–85.
- Friedman, M. "The Case of Flexible Exchange Rates," in *Essays in Positive Economics*. Chicago: University of Chicago Press, 1953.
- Gilli, M., and P. Winker. "A Global Optimization Heuristic for Estimating Agent Based Models." *Computational Statistics & Data Analysis*, 42, 2003, 299–312.
- Hansen, L. P., and T. J. Sargent. "Robust Control and Model Uncertainty." *American Economic Review*, 91, 2001, 6066.
- . *Robustness*. Princeton, NJ: Princeton University Press, 2008.
- . "Fragile Beliefs and the Price of Uncertainty." *Quantitative Economics*, 1, 2010, 129–62.
- Heemeijer, P., C. H. Hommes, J. Sonnemans, and J. Tuinstra. "Price Stability and Volatility in Markets with Positive and Negative Expectations Feedback." *Journal of Economic Dynamics & Control*, 33, 2009, 1052–72.
- Hey, J. D. "Expectations Formation: Rational or Adaptive or ...?" *Journal of Economic Behavior and Organization*, 25, 1994, 329–49.
- Hommes, C. "Heterogeneous Agent Models in Economics and Finance," in *Handbook of Computational Economics. Agent-Based Computational Economics*. Handbooks in Economics Series, Vol. 2, edited by K. Judd and L. Tesfatsion. Amsterdam: Elsevier/North-Holland, 2006.
- Hommes, C. H. "The Heterogeneous Expectations Hypothesis: Some Evidence from the Lab." *Journal of Economic Dynamics and Control*, 35(1), 2011, 1–24.
- Hommes, C., J. Sonnemans, J. Tuinstra, and H. van de Velden. "Coordination of Expectations in Asset Pricing Experiments." *Review of Financial Studies*, 18(3), 2005a, 955–80.
- Hommes, C. H., H. Huang, and D. Wang. "A Robust Rational Route to Randomness in a Simple Asset Pricing Model." *Journal of Economic Dynamics and Control*, 29, 2005b, 1043–72.
- Hommes, C.H., D. Massaro, and I. Salle. "Monetary and Fiscal Policy Design at the Zero Lower Bound—Evidence from the Lab." CeNDEF Working Paper No. 15-11, 2016.
- Ilut, C. L., and M. Schneider. "Ambiguous Business Cycles." *American Economic Review*, 104, 2014, 2368–99.
- de Jong, E., W. F. C. Verschoor, and R. C. J. Zwickels. "Behavioural Heterogeneity and Shift-Contagion: Evidence from the Asian Crisis." *Journal of Economic Dynamics and Control*, 33, 2009, 1929–44.
- Karceski, J. "Returns-Chasing Behavior, Mutual Funds, and beta's Death." *Journal of Financial and Quantitative Analysis*, 37, 2002, 559–94.
- La Porta, R., J. Lakonishok, A. Shleifer, and R. Vishny. "Good News for Value Stocks: Further Evidence on Market Efficiency." *Journal of Finance*, 52, 1997, 859–74.
- Lakonishok, J., A. Shleifer, and R. Vishny. "Contrarian Investment, Extrapolation, and Risk." *Journal of Finance*, 49, 1994, 1541–78.
- Lines, M., and F. Westerhoff. "Inflation Expectations and Macroeconomic Dynamics: The Case of Rational versus Extrapolative Expectations." *Journal of Economic Dynamics and Control*, 34, 2010, 246–57.
- Lof, M. "Heterogeneity in Stock Prices: A STAR Model with Multivariate Transition Function." *Journal of Economic Dynamics and Control*, 36, 2012, 1845–54.
- . "Rational Speculators, Contrarians and Excess Volatility." *Management Science*, 61, 2015, 1889–901.
- Mamatzakis, E. "Are There Any Animal Spirits Behind the Scenes of the Euro Area Sovereign Debt Crisis?" Hellenic Observatory Papers on Greece and Southeast Europe, GreeSE Paper No. 72, London School of Economics and Political Science, London, UK, 2013.
- Mankiw, N., R. Reis, and J. Wolfers. "Disagreement about Inflation Expectations," in *NBER Macroeconomics Annual*, edited by M. Gertler and K. Rogoff. Cambridge, MA: National Bureau of Economic Research, 2003.
- Massaro, D. "Heterogeneous Expectations in Monetary DSGE Models." *Journal of Economic Dynamics and Control*, 37(3), 2013, 680–92.
- Pfajfar, D., and E. Santoro. "Heterogeneity, Learning and Information Stickiness in Inflation Expectations." *Journal of Economic Behavior and Organization*, 75(3), 2010, 426–44.
- Pfajfar, D., and B. Zakelj. "Experimental Evidence on Inflation Expectation Formation." *Journal of Economic Dynamics and Control*, 44, 2014, 147–68.
- . Forthcoming "Inflation Expectations and Monetary Policy Design: Evidence from the Laboratory." *Macroeconomic Dynamics*, 2016.
- Roos, M. W. M., and W. J. Luhan. "Information, Learning, and Expectations in an Experimental Model Economy." *Economica*, 80(319), 2012, 513–31.

- Sargent, T. J. *Bounded Rationality in Macroeconomics: The Arne Ryde Memorial Lectures*. Oxford: Oxford University Press, 1993.
- Scheinkman, J., and W. Xiong. "Overconfidence and Speculative Bubbles." *Journal of Political Economy*, 111, 2003, 1183–219.
- Shiller, R. J. "Measuring Bubble Expectations and Investor Confidence." *Journal of Psychology and Financial Markets*, 1, 2000, 49–60.
- Sims, C. A. "Stickiness." *Carnegie-Rochester Conference Series on Public Policy*, 49, 1998, 317–56.
- . "Implications of Rational Inattention." *Journal of Monetary Economics*, 50(3), 2003, 665–90.
- Simsek, A. "Belief Disagreements and Collateral Constraints." *Econometrica*, 81(1), 2013, 1–53.
- Sirri, E. R., and P. Tufano. "Costly Search and Mutual Fund Flows." *Journal of Finance*, 53, 1998, 1589–621.
- Vissing-Jorgensen, A. "Perspectives on Behavioral Finance: Does 'Irrationality' Disappear with Wealth? Evidence from Expectations and Actions," in *NBER Macroeconomics Annual*, edited by M. Gertler and K. Rogoff. Cambridge, MA: MIT Press, 2003.
- Westerhoff, F. H., and S. Reitz. "Nonlinearities and Cyclical Behavior: The Role of Chartists and Fundamentalists." *Studies in Nonlinear Dynamics & Econometrics*, 7(4), 2003, Article 3.