# Implicit versus explicit comparatives

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Link to publication

**Citation for published version (APA):**

# Implicit versus explicit comparatives

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**Abstract**

It is natural to assume that the explicit comparative – John is taller than Mary – can be true in cases the implicit comparative – John is tall compared to Mary – is not. This is sometimes seen as a threat to comparison-class based analyses of the comparative. In this paper it is claimed that the distinction between explicit and implicit comparatives corresponds to the difference between (strict) weak orders and semi-orders, and that both can be characterized naturally in terms of constraints on the behavior of predicates among different comparison classes.

## 1  Introduction

Consider the following figure, picturing the lengths of John and Mary.
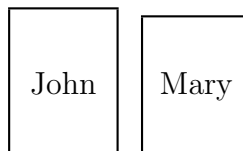


Chris Kennedy (this volume) observed that according to most people's intuition, this picture allows us to say (1-a). At the same time, most people would say that (1-b) is false.

(1)  a.  John is taller than Mary.
     b.  Compared to Mary, John is tall, but compared to John, Mary is not tall.

This observation is stated as a challenge to followers of Klein (1980). According to Klein (1980), (1-a) is true if and only if *there is* a comparison class according to which John is tall and Mary is not, i.e. (Klein). It is standardly assumed (e.g. Von Stechow, 1984) that Klein's actual analysis of (1-a) is *equivalent* with the simpler analysis of van Benthem, (Benthem), which will be discussed in section 2:

(Klein)     (1-a) is true iff *there is* a comparison class such that John is tall here and Mary is not.

(Benthem) (1-a) is true iff John is tall in comparison class {John, Mary}, but Mary is not.

But according to (Benthem), (1-a) and (1-b) have the same truth conditions, and it is impossible to account for the contrast between them in truth conditional terms. We will meet Kennedy's challenge by making a distinction between analyses (Klein) and (Benthem).

## 2   Comparatives and comparison classes

Although expressions of many lexical categories are vague, most research on vagueness concentrates on adjectives like 'tall' and 'bald'. In linguistics these adjectives are known as *gradable* adjectives and should be distinguished from non-gradable adjectives like 'pregnant' and 'even'. The latter adjectives do not give rise to (much) vagueness. There exist two major types of approaches to the analysis of gradable adjectives: degree-based approaches and delineation approaches. Degree-based approaches (e.g. Seuren, 1973; von Stechow, 1984; Kennedy, 1999), analyze gradable adjectives as relations between individuals and degrees, where these degrees are thought of as scales associated with the dimension referred to by the adjective. Individuals can possess a property to a certain measurable degree. The truth conditions of sentences involving these adjectives are stated in terms of degrees. According to the most straightforward degree-based approach, a sentence like *John is tall* is true iff the degree to which John is tall is (significantly) greater than a (contextually given) standard of height. The comparative *John is taller than Mary* is true iff the (maximal) degree to which John is tall is greater than the (maximal) degree to which Mary is tall.[1]

Delineation approaches (Lewis, 1970; Kamp, 1975; Klein 1980, 1991) analyze gradable adjectives like 'tall' as simple predicates, but assume that the

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[1]More complex sentences suggest that this simple picture is naive, and there has been a lot of discussion of how to improve on it. I will ignore this discussion in this paper.

extension of these terms are crucially context-dependent. If one accounts for comparatives in terms of supervaluation structures as Lewis and Kamp do, the most obvious way to account for this context-dependence is very similar to the one used in degree-based approaches: context just determines the cutoff-point. But, of course, context-dependence is more fine-grained than that. For Jumbo to be *a small elephant*, for instance, means that Jumbo is being small for an elephant, but that does not mean that Jumbo is small. For instance, Jumbo will be much bigger than any object that counts as *a big mouse*. One way to make this explicit is to assume with Klein (1980) that every adjective should be interpreted with respect to a *comparison class*, i.e. a set of individuals. The truth of a sentence like *John is tall* depends on the contextually given comparison class: it is true in context (or comparison class) $c$ iff John is counted as tall in this class. Klein (1980) further proposes that the meaning of the comparative *John is taller than Mary* is context-independent and the sentence is true iff there is a context (henceforth we will use 'context' instead of the more cumbersome 'comparison class') according to which John counts as tall, while Mary does not. If there is any context in which this is the case, it will also be the case in the context containing only John and Mary.

Klein (1980) favors the delineation approach towards comparatives for a number of reasons. First, a degree-based approach only makes sense in case the comparative gives rise to a *total* ordering. But for at least some cases (e.g. *more clever than*) this does not seem to be true, because *clever* is a multi-dimensional adjective. Second, the delineation account assumes that the meaning of the comparative 'taller than' is a function of the meaning of 'tall'. This is much in line with Frege's principle of compositionality,[2] and also accounts for the fact that in a wide variety of languages the positive is formally unmarked in relation to the comparative.

An analysis of comparatives in terms of comparison classes is sometimes stated as if it *presupposes* that the domain of all individuals of any gradable adjective has an inherent ordering imposed upon it, and that the ordering on a comparison class must preserve the initial ordering on the domain of the adjective in order to avoid undesirable entailments. Kennedy (1997) states this in terms of the following *consistency postulate*.

> For any context in which *a is $\varphi$* is true and $b \geq a$ with respect to the original ordering on the domain of $\varphi$, then *b is $\varphi$* is also true, and for any context in which *a is $\varphi$* is false, and $a \geq b$, then *b is $\varphi$* is also false.

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[2]But see Von Stechow (1984) for an argument saying that also the degree-based approach is in line with Frege's principle.

But if this were so, the delineation analysis of comparatives would be reduced to an initial comparison ordering, and thus the delineation approach would not take the positive use of the adjective as basic. Fortunately, Van Benthem (1982) has shown that what Kennedy calls the initial ordering can be *derived* from how we positively use the adjective in certain contexts, plus some additional constraints on how the meaning of the adjective can change from context to context. This is done in terms of the notion of a *context structure*, $M$, being a triple $\langle X, C, V \rangle$, where $X$ is a non-empty set of individuals, the set of contexts, $C$, consists of all finite subsets of $X$, and the valuation $V$ assigns to each context $c \in C$ and each property $T$ those individuals in $c$ which are to count as 'being $T$ in $c$'.

This definition leaves room for the most diverse behavior of individuals across contexts. Based on intuition (for instance by visualizing sticks of various lengths), however, the following plausible cross-contextual principles make sense, which constrain the possible variation. Take two individuals $x$ and $y$ in context $c$ such that $M, c \models T(x) \wedge \neg T(y)$. We now constrain the set of contexts $C$ by the following three principles: No Reversal (NR), which forbids $x$ and $y$ to change roles in other contexts:

(NR) $\neg \exists c' \in C : M, c' \models T(y) \wedge \neg T(x)$.

This constraint does not prevent $x$ and $y$ both to be tall in larger contexts than $c$. However, once we look at such larger contexts, the Upward Difference (UD) constraint demands that there should be at least one difference pair:

(UD) $\forall c' \in C[c \subseteq c' \rightarrow \exists z_1, z_2 : M, c' \models T(z_1) \wedge \neg T(z_2)]$

The final Downward Difference (DD) principle constrains in a very similar way what is allowed if we look at subsets of $c$: if $x$ and $y$ are elements of this subset, there still should be a difference pair:

(DD) $\forall c' \in C[(c' \subseteq c \ \& \ x, y \in c') \rightarrow \exists z_1, z_2 : M, c' \models T(z_1) \wedge \neg T(z_2)]$

If we say that 'John is taller than Mary' is true if and only if there is a context $c$ such that $M, c \models T(j) \wedge \neg T(m)$, Van Benthem shows that the comparative (given the above constraints on context strucures) as defined above has exactly those properties which we intuitively want for most comparatives (see below). Thus we have seen that on the basis of the initial idea of the delineation approach we can *derive* the ordering relation that Kennedy (1997) claims delineation approaches must already take for granted to begin with.

In the definition of a context structure we used above, context structures

give rise to orderings for any context-dependent adjective. For convenience, we will just limit ourselves to one adjective: $P$. If we do so, we can think of a context structure as a triple $\langle X, C, P \rangle$, where $P$ can be though of as a *choice function*, rather than a general valuation function.

**Definition 1.**
*A <u>context structure</u> $M$ is a triple $\langle X, C, P \rangle$, where $X$ is a non-empty set of individuals, the set of contexts, $C$, consists of all finite subsets of $X$, and the choice function $P$ assigns to each context $c \in C$ one of its subsets.*

Notice that $P(c)$ (with respect to context structure $M$) corresponds to the set $\{x \in X : M, c \models P(x)\}$ in our earlier formulation. To state the cross-contextual constraints somewhat more compactly than we did above, we define the notion of a difference pair: $\langle x, y \rangle \in D_P(c)$ iff$_{def}$ $x \in P(c)$ and $y \in (c - P(c))$. Now we can define the constraints as follows (where $c^2$ abbreviates $c \times c$, and $D_P^{-1}(c) =_{def} \{\langle y, x \rangle : \langle x, y \rangle \in D_P(c)\}$):

(NR) $\forall c, c' \in C : D_P(c) \cap D_P^{-1}(c') = \varnothing$.
(UD) $c \subseteq c'$ and $D_P(c) \neq \varnothing$, then $D_P(c') \neq \varnothing$.
(DD) $c \subseteq c'$ and $D_P(c') \cap c^2 \neq \varnothing$, then $D_P(c) \neq \varnothing$.

If we say that $x >_P y$, iff$_{def}$ $x \in P(\{x, y\})$ and $y \notin P(\{x, y\})$, van Benthem (1982) shows that the ordering as defined above gives rise to a *strict weak order*. A structure $\langle X, R \rangle$, with $R$ a binary relation on $X$, is a strict weak order just in case $R$ is irreflexive (IR), transitive (TR), and almost connected (AC):

**Definition 2.**
*A <u>(strict) weak order</u> is a structure $\langle X, R \rangle$, with $R$ a binary relation on $X$ that satisfies the following conditions:*
*(IR) $\forall x : \neg R(x, x)$.*
*(TR) $\forall x, y, z : (R(x, y) \land R(y, z)) \rightarrow R(x, z)$.*
*(AC) $\forall x, y, z : R(x, y) \rightarrow (R(x, z) \lor R(z, y))$.*

The constraint that $R$ should be almost connected is in some circles better known under its contrapositive guise as co-transitivity: $\forall x, y, z : (\neg R(x, z) \land \neg R(z, y)) \rightarrow \neg R(x, y)$. If we now define the indifference relation, '$I$', or in our case '$\approx_P$', as follows: $x \approx_P y$ iff$_{def}$ neither $x >_P y$ nor $y >_P x$, it is clear that '$\approx_P$' is an equivalence relation. It is well-known from measurement theory (e.g. Krantz et al, 1971) that in case '$>_P$' gives rise to a (strict) weak order, it can be represented numerically by a real valued function $f_P$ such that for all $x, y \in X$: $x >_P y$ iff $f_P(x) > f_P(y)$, and $x \approx_P y$ iff $f_P(x) = f_P(y)$.

# 3   Explicit versus implicit comparison

Consider again the following figure, picturing the lengths of John and Mary.
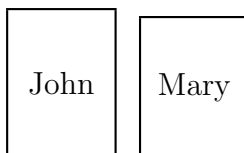


How can we account for the fact that the *explicit* comparative (1-a) is intuitively true, while the *implicit* comparative (1-b) is false?[3]

(1-a)    John is taller than Mary.

(1-b)    Compared to Mary, John is tall, but compared to John, Mary is not tall.

It is clear that according to (Benthem) (1-a) and (1-b) have the same truth conditions, and it is impossible to account for the contrast between them in truth conditional terms.

> (Benthem) (1-a) is true iff John is tall in comparison class {John, Mary}, but Mary is not.

Of course, Klein's (1980) original analysis (Klein) was a bit different, so it seems possible to account for the difference between (1-a) and (1-b) in terms of the difference between (Klein) and (Benthem).

> (Klein)    (1-a) is true iff *there is* a comparison class s.t. John is tall here and Mary is not.

It can be easily shown, however, that *in case* Klein would have adopted Van Benthem's (1982) cross-contextual constraints, (Klein) is equivalent to (Benthem). It is immediately clear that by Van Benthem's definition of a context structure and by adopting his constraints, analysis (Klein) *follows from* analysis (Benthem). But it is important to see why also the reverse holds. So suppose that (1-a) is true according to (Klein). That means that there exists a comparison class $c \in C$ containing John and Mary such that $j \in P(c) \wedge m \notin P(c)$. But this means that $\langle j, m \rangle$ is a difference pair in $c$, and by (DD) it follows that $\langle j, m \rangle$ will be a difference pair in all $c' \in C$ that are subsets of $c$ containing both John and Mary. By the assumption that $C$

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[3]According to Chris Kennedy (p.c.) the names 'explicit' versus 'implicit' comparatives goes back to Sapir (1944).

contains all finite subsets of $X$, it follows that $\langle j, m \rangle$ is also a difference pair for comparison class {John, Mary}, which means that (1-a) is also predicted to be true by (Benthem).

I would like to point out that the derivation of the truth of the comparative according to (Benthem) from its truth according to (Klein) is based on (at least) *two assumptions*. The first assumption is that we do not really make a distinction between it not being the case that Mary is tall, and Mary's being *not-tall*, or, perhaps equivalently, Mary's being *short*. A second crucial presupposition on which the above derivation is based is the assumption that $C$ contains *all* finite subsets of $X$, in particular that {John, Mary} is an element of $C$. The *first assumption*, however, was explicitly rejected by Klein (1980). Klein (1980) explicitly proposed that an adjective gives rise to a *three*-way partition of the comparison class $c$: some individuals in $c$ are (definitely) tall, some are (definitely) not-tall, and some are neither. Klein used a three-valued logic, but the same intuition can be captured by inducing the three-way partition by a set of contrary predicates: e.g., the adjective 'tall' and its antonym 'short'. Although no individual in $c$ is tall *and* short, it is possible that some individuals are neither. Klein (1980) assumed that an adjective gives rise to a three-way partition to account for *vagueness*. I will argue in the next section that doing so is indeed natural to account for the Sorites paradox. In the next section I will also argue that *for the very same reason* it is also natural to give up van Benthem's (1981) assumption that *all* finite subsets of $X$ are appropriate comparison classes. In section 5 I will then show that if one gives up either of these assumptions one can generate an ordering relation that properly represents vagueness, and, or so I will argue, in terms of which one can give a natural account of the distinction between (1-a) and (1-b).

# 4   The Sorites and semi-orders

## 4.1   Vagueness and Semi-orders

Consider a long series of people ordered in terms of their height. Of each of them you are asked whether they are tall or not. We assume that the variance between two subsequent persons is always *indistinguishable*. Now, if you decide that the first individual presented to you, the tallest, is tall, it seems only reasonable to judge the second individual to be tall as well, since you cannot distinguish their heights. But, then, by the same token, the third person must be tall as well, and so on indefinitely. In particular, this makes also the last person tall, which is a counterintuitive conclusion, given that it is in contradiction with our intuition that this last, and shortest individual, is short, and thus not tall.

This so-called Sorites reasoning is elementary, based only on our intuition that the first individual is tall, the last short, and the following inductive premise, which seems unobjectable:

> [**P**] If you call one individual tall, and this individual is not visibly taller than another individual, you have to call the other one tall too.

Our above Sorites reasoning involved the predicate 'tall', but that was obviously not essential. Take any predicate $P$ that gives rise to a complete ordering 'as $P$ than'. Let us assume that '$\sim_P$' is the indistinguishability, or indifference, relation between individuals with respect to predicate $P$. Now we can state the inductive premise somewhat more formally as follows:

> [**P**] For any $x, y \in X : (P(x) \land x \sim_P y) \to P(y)$.

If we assume that it is possible that $\exists x_1, \ldots, x_n : x_1 \sim_P x_2 \land \cdots \land x_{n-1} \sim_P x_n$, but $P(x_1)$ and $\neg P(x_n)$, the paradox will arise. Recall that if $P(x_1)$ and $\neg P(x_n)$, it is required that $x_1$ must be *visibly* or *significantly $P$-er than $x_n$*, denoted by $x_1 \succ_P x_n$. In section 2 of this paper we have defined a relation '$>_P$' in terms of the behavior of predicate $P$. The constraints discussed there, however, did not allow for the possibility that $\exists x_1, \ldots, x_n : x_1 \sim_P x_2 \land \cdots \land x_{n-1} \sim_P x_n$, but $P(x_1)$ and $\neg P(x_n)$, and the defined comparison relation could not really be interpreted as meaning 'being *visibly/significantly $P$-er than*'. Fortunately, there is a well-known ordering that should be interpreted this way: what Luce (1956) calls a *semi-order*. Following Scott & Suppes' (1958) (equivalent, but still) simpler definition, a structure $\langle X, R \rangle$, with $R$ a binary relation on $X$, is a semi-order just in case $R$ is irreflexive (IR), satisfies the interval-order (IO) condition, and is semi-transitive (STr).[4]

**Definition 3.**
*A semi-order is a structure $\langle X, R \rangle$, with $R$ a binary relation on $X$ that satisfies the following conditions:*
*(IR) $\forall x : \neg R(x, x)$.*
*(IO) $\forall x, y, v, w : (R(x, y) \land R(v, w)) \to (R(x, w) \lor R(v, y))$.*
*(STr) $\forall x, y, z, v : (R(x, y) \land R(y, z)) \to (R(x, v) \lor R(v, z))$.*

It is important to see that if we interpret the relation '$\succ_P$' as a semi-order, it is irreflexive and transitive, but need not be almost connected. Perhaps the

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[4]Any relation that is irreflexive and satisfies the interval-order condition is called an *interval order*. All interval orders are also transitive, meaning that they are stronger than strict partial orders.

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easiest way to grasp what it means to be a semi-order is to look at its (intended) measure-theoretical interpretation. On the intended interpretation, '$x \succ_P y$' is true iff the height of $x$ is higher than the height of $y$ plus some fixed (small) real number $\epsilon$, which can be thought of as a *margin of error*.[5] Indeed, as already suggested by Luce (1956) and rigorously proved by Scott & Suppes (1958), if $X$ is a finite set and $\epsilon$ a positive number, $\langle X, \succ_P \rangle$ is a semi-order if and only if there is a real-valued function $f_P$ such that for all $x, y \in X : x \succ_P y$ iff $f_P(x) > f_P(y) + \epsilon$. This fact helps to explain the constraints. That the order should be irreflexive is trivial, because there can be no $x$ such that $f_P(x) > f_P(x) + \epsilon$. As for (IO), consider two cases: $f_P(x) \geq f_P(v)$ or $f_P(v) \geq f(x)$. In the first case we have $f_P(x) \geq f_P(v) > f_P(w) + \epsilon$, and thus $x \succ w$. In the second case we have $f_P(v) \geq f_P(x) > f_P(y) + \epsilon$, and thus $v \succ y$. To see that (STr) holds, suppose that $x \succ_P y$ and $y \succ_P z$. Then $f_P(x) >_\epsilon f_P(y) >_\epsilon f_P(z)$ (with $a >_\epsilon b$ iff $a > b + \epsilon$). But then $f_P(v) \geq f_P(y)$ implies $v \succ z$, and $f_P(v) \leq f_P(y)$ implies $x \succ_P v$.

In terms of '$\succ_P$' we can define a similarity relation '$\sim_P$' as follows: $x \sim_P y$ iff neither $x \succ_P y$ nor $y \succ_P x$. The relation '$\sim_P$' is reflexive and symmetric, but need not be transitive. Thus, '$\sim_P$' does not give rise to an equivalence relation. One should think of this similarity relation as one of *indifference* or *indistinguishability*. Measure theoretically '$x \sim_P y$' is true iff the difference in height between $x$ and $y$ is less than $\epsilon$.[6] In case $\epsilon = 0$, the semi-order is a weak order.

## 4.2   Some proposed solutions to the Sorites

The standard reaction to the Sorites paradox taken by proponents of fuzzy logic and/or supervaluation theory is to say that the argument is valid, but that the inductive premise [**P**] (or one of its instantiations) is *false*. But why, then, does it at least *seem* to us that the inductive premise is true? According to the standard accounts of vagueness making use of fuzzy logic and supervaluation theory, this is so because the instantiations of the inductive premise are *almost* true (in fuzzy logic), or almost all instantiations are true in the complete valuations (in supervaluation theory).

Linguists (e.g. Kamp, 1975; Klein, 1980; Pinkal, 1995) typically do not like the fuzzy logic approach to vagueness, because that cannot account for

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[5]One can think of Williamson's (1990) epistemic analysis of vagueness based on semi-orders as well.

[6]The fact that '$\sim_P$' is intransitive has the consequence that semi-orders cannot be given a full measure-theoretic interpretation $f$ in the sense that there is no set of transformations such that $f$ is *unique* up to this set of transformations. This fate it shares with, among others, partial orders.

what Fine (1975) called 'penumbral' connections. The treatment of vagueness and the Sorites paradox in supervaluation theory is not unproblematic either, however. The use of complete refinements in supervaluation theory assumes that we *can* always make sharp cutoff-points: vagueness exists only because in daily life we are *too lazy* to make them. But this assumption seems to be wrong: vagueness exists, according to Dummett (1975), because we *cannot* make such sharp cutoff-points even if we wanted to. In terms of what we discussed above, this suggests that the relation '$\sim_P$' of indifference or indistinguishability should be intransitive, just as it is for a semi-order. But because this relation is still symmetric, it is very natural to claim that something like [**P**] is true.

For a while, the so-called 'contextualist' solution to the Sorites paradox was quite popular (e.g. Kamp, 1981; Pinkal, 1984; Raffman, 1994, 1996). Most proponents of the contextuallist solution follow Kamp (1981) in trying to preserve (most of) [**P**] by giving up some standard logical assumptions, and/or by making use of a mechanism of *context change*. But with Keefe (2007) we do not believe that context change is essential to save natural language from the Sorites paradox. We rather believe that any solution involves some notion of *partiality*. We will briefly discuss two such proposed 'solutions' in this section (without pretending to be complete or assuming that they are undoubtfully successful), and use the motivations behind those 'solutions' in the following sections to propose some new cross contextual constraints on the behavior of predicates which generate semi-orders.

A first solution is closely related with recent work of Raffman (2005) and Shapiro (2006) and makes use of partiality in a rather direct way:[7] in terms of three valued logic (Shapiro), or in terms of pairs of contrary antonymns (Raffman). The idea – just as what Klein (1980) proposed earlier – is that with respect to a comparison class $c$, predicate $P$ and its antonym $\overline{P}$ do not necessarily partition $c$, and there might be elements in $c$ that neither (clearly) have property $P$ nor property $\overline{P}$, but are somewhere 'in the middle'. Once one makes such a move it is very natural to assume that the inductive principle [**P**] is not valid, but a weakened version of it, [**P_w**], is:

> [**P_w**] If you call one individual tall in a particular context, and this individual is not visibly/relevantly taller than another individual, you will/should not call the other one short/not tall.
> Thus, for any $x, y \in I, c \in C : (P(x, c) \wedge x \sim_P y) \rightarrow \neg\overline{P}(y, c)$.

Of course, principle [**P_w**] can only be different from the original [**P**] if

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[7]Shapiro (2006) argues that his solution is closely related to Waisman's notion of 'Open Texture'. For what it is worth, I believe that Waisman's notion is more related to what I call the second 'solution' of the Sorites.

$\neg\overline{P}(y,c)$ does not come down to the same as $P(y,c)$.[8] Thus, a gap between the sets of $P$- and $\overline{P}$-individuals (with respect to $c$) is required. Notice that the Sorites paradox can now be 'solved' in a familiar way: $P(x_1,c)$ and $\overline{P}(x_n,c)$ are true in context $c$, and modus ponens is valid, but the inductive hypothesis, or (all) its instantiations, are not. However, because we adopt $[\mathbf{P_w}]$ as a valid principle of language use, we can explain why inductive hypothesis $[\mathbf{P}]$ *seems* so natural. To illustrate, if $c = \{x,y,z\}$, it might be that with respect to a particular context structure $P(c) = \{x\}, \overline{P}(c) = \{z\}$, and $x \sim_P y \sim_P z$. Notice that such a context structure satisfies $[\mathbf{P_w}]$ but not $[\mathbf{P}]$.

A second 'solution' is more radically *pragmatic* in nature and seems very much in line with Wittgenstein's *Philosofische Untersuchungen*.[9] In normal discourse, we talk about relatively few objects, all of which are easily discernible from the others. In those circumstances, $[\mathbf{P}]$ will not give rise to inconsistency, but serves its purpose quite well. Only in exceptional situations i.e., when we are confronted with long sequences of pairwise indistinguishable objects — do things go wrong. But in such situations, we should not be using vague predicates like 'tall' but precisely measurable predicates instead. A weak version of this reaction can be formalized naturally in terms of comparison classes. The idea is that it only makes sense to use a predicate $P$ in a context – i.e. with respect to a comparison class –, if it helps to clearly demarcate the set of individuals that have property $P$ from those that do not. Following Gaifman (1997),[10] we will implement this idea by assuming that any subset of $X$ can only be an element of the set of *pragmatically appropriate* comparison classes $C$ just in case the gap between the last individual(s) that have property $P$ and the first that do(es) not must be between individuals $x$ and $y$ such that $x$ is clearly, or significantly, $P$-er than $y$. This is not the case if the graph of the relation '$\sim_P$' is closed in $c \times c$.[11] Indeed, it is exactly in those cases that the Sorites paradox arises. Notice that also this analysis makes use of partiality, but this now consists in the idea that certain comparison classes are not appropriate for the use of a particular predicate $P$.

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[8]In this paper I don't really distinguish thinking of the comparison class as part of the context (as I did until now, or of thinking of it as an argument of an adjective. For present purposes, this distinction is irrelevant.

[9]See in particular section 85-87: 'A rule stands like a signpost ... The signpost in order in in normal circumstances it fulfils its purpose.' The observation that our pragmatic solution is very much in line with Wittgenstein's later philosophy, I owe to Frank Veltman.

[10]One might argue that Gaifman's solution was already anticipated – though in a rather different way – by Kamp (1981). A theory much more similar to Gaifman's was proposed by Pagin (this volume). The editors of this volume pointed out to me that Gomez-Torrente (2008) argues for much the same idea.

[11]Notice that also in discrete cases the relation '$\sim_P$' can be closed in $c \times c$. It just depends on how '$\sim_P$' is defined.

How does such a proposal deal with the Sorites paradox? Well, it claims that *in all contexts in which P can be used appropriately*, [**P**] is true. If we assume in addition that the first element $x_1$ of a Sorites series is the absolute most $P$-individual, and the last element $x_n$ the absolute least $P$-individual, it also claims that *in all contexts c in which it is appropriate to use predicate P in combination with $x_1$ and $x_n$, 'P($x_1$, c)'* is *true* and *'P($x_n$, c)'* is *false*. Thus, in all *appropriate* contexts, the premises of the Sorites argument are considered to be true. Still, no contradiction can be derived, because using predicate $P$ when *explicitly* confronted with a set of objects that form a Sorites series is *inappropriate*. Thus, in contrast to the original contextualist approaches of Kamp (1981), Pinkal (1984), and others, the Sorites paradox is not avoided by assuming that the meaning (or extension) of the predicate changes as the discourse proceeds. Rather, the Sorites paradox is avoided by claiming that the use of predicate $P$ is inappropriate when confronted with a Sorites series of objects.[12]

The above sketch of some proposed solutions was rather unsophisticated and I do not want to claim in this paper that they, or their more sophisticated variants, are completely successful. I also don't want to go into their relative pros and cons, or argue that they are (clearly) preferred to other proposals (though I sympathize with them). I only sketched them because the motivations behind those proposals clearly suggest some new cross contextual constraints on the behavior of predicates which can be shown to generate (or even characterize) semi-orders.

## 4.3 Semi-orders and semantic gaps

The first of the above 'solutions' to the Sorites paradox is in essence *three-valued*. Either because a three-valued logic was used, or because we made use of pairs of antonyms. Recall again that Klein (1980) already used a three-valued logic: not all individuals in a particular comparison class need to be either tall or not-tall (in fact, Klein used supervaluations to make up for this 'deficiency'). In this section we will indicate that once we follow this line of thought, it becomes easy to generate semi-orders, instead of weak orders. Our derivation makes use of *two choice functions*. Let us say that $P(c)$ selects the elements of $c$ that (clearly) have property $P$, while $\overline{P}(c)$ (e.g. 'tall') selects the

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[12]Williamson (p.c.) and a reviewer of this paper ask what are the *semantic* consequences of using a pragmatically inappropriate comparison class. The main answer is that if pushed one can still choose between, for instance, an epistemic approach or a three valued approach. Adopting this approach, the answer to this question should, I think, be of little theoretical importance: I do not think we have very strong semantic intuitions about things that go against what we ought to do and normally do.

elements that (clearly) have property $\overline{P}$) (e.g. 'short'). Now we can give the following four constraints:[13]

(P*) $\forall c \in C : P(c) \cap \overline{P}(c) = \varnothing$.
(NR*) $\forall c, c' : D_P(c) \cap D_P^{-1}(c') = \varnothing$ and $D_{\overline{P}}(c) \cap D_{\overline{P}}^{-1}(c') = \varnothing$.
(UD*) If $c \subseteq c'$ and $D_{P\overline{P}}(c) \neq \varnothing$, then $D_{P\overline{P}}(c') \neq \varnothing$.
(DD*) If $c \subseteq c'$ and $D_{P\overline{P}}(c') \cap c^2 \neq \varnothing$, then $D_{P\overline{P}}(c) \neq \varnothing$.

Constraint (P*) assures that $P$ and $\overline{P}$ behave as contraries, while (NR*) is the obvious generalization of van Benthem's (1982) No Reversal constraint. Constraints (UD*) and (DD*) are very similar to the earlier Upward and Downward Difference constraints of van Benthem (1982), but still crucially different. The difference is that in this case we look at *contrary* pairs, and not merely at *contradictory* pairs. We define the ordering relation as follows: $x \succ_P y$ iff$_{def}$ $x \in P(\{x,y\})$ and $y \in \overline{P}(\{x,y\})$. Then we can prove that this relation is irreflexive and transitive, but it need *not* satisfy *almost connectedness*: If $x \succ_P y$, it is possible that neither $x \succ_P z$ nor $z \succ_P y$, because (DD*) does not require either of them to hold if $P(\{x,y,z\}) = \{x\}$ and $\overline{P}(\{x,y,z\}) = \{y\}$. Now we can prove the following theorem (see van Rooij, 2009):

**Theorem 1.** *Any context structure $\langle X, C, P, \overline{P} \rangle$ with $X$ and $C$ as defined above such that $P$ and $\overline{P}$ obey axioms (P*), (NR*), (UD*), and (DD*), gives rise to a semi-order $\langle X, \succ_P \rangle$, if we define $x \succ_P y$ as $x \in P(\{x,y\})$ and $y \in \overline{P}(\{x,y\})$.*

## 4.4 Semi-orders and pragmatic gaps

Recall that according to the 'pragmatic solution' of the Sorites paradox not all subsets of $X$ are assumed to be *appropriate* comparison classes. Whether $c$ is an appropriate comparison class/context set was defined in terms of the relations '$\succ_p$' and '$\sim_P$': the relation '$\sim_P$' should not connect all elements in $c$. In this section we want to turn that idea around: find some principles to generate all and only all appropriate context sets and then derive the relations '$\succ_P$' and '$\sim_P$' from that. The idea is that we just start with subsets of $C$ that consist of two distinguishable elements and close this set of subsets of $C$ under some closure conditions such that they will generate *all and only all appropriate contexts*. That is, the idea is to find some closure conditions such that we will generate just those subsets of $X$ for which also vague predicates can clearly partition the context without giving rise to the Sorites paradox.

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[13]The formulation of the constraints is much simpler, though equivalent, to the formulation I used in van Rooij (2009). I thank Frank Veltman for pointing out that my earlier formulation was needlessly complex.

In conjunction with this, we will assume the same cross-contextual constraints on the behavior of $P$ as van Benthem (1982) did, and define also $x \succ_P y$ as he did: $x \succ_P y$ iff$_{def}$ $x \in P(\{x,y\})$ and $y \notin P(\{x,y\})$.

The closure conditions that jointly do this job are the following:

(P1) $\forall c \in C : \forall x \in \bigcup C : c \cup \{x\} \in GAP \rightarrow c \cup \{x\} \in C$.
(OR) $\forall c \in C, \{x,y\} \in C : c \cup \{x\} \in C$ or $c \cup \{y\} \in C$.
(P2) $\forall c \in C, x \in X : c \in GAP_2 \rightarrow c \cup \{x\} \in C$.

In terms of these constraints we want to generate all appropriate comparison classes starting with a set of appropriate comparison classes of just 2 elements. These constraints mention '$GAP$' and $GAP_2$', which intuitively stand for gaps. Still, it is important to realize that the formalization does not make use of any predefined notion of a gap. The two notions '$GAP$' and $GAP_2$' will be defined in terms of such sets of appropriate comparison classes.

Before we discuss these constraints, it is important to see that the closure conditions do not guarantee that $C$ necessarily contains *all* finite subsets of $X$ (or better, not all subsets of $X$ with cardinality 2 or 3). This is essential, because otherwise we could conclude with van Benthem (1982) that the resulting ordering relation would be a weak order and satisfies (AC) $\forall x, y, z : x \succ_P y \rightarrow (x \succ_P z \lor z \succ_P y)$. It suffices to observe that because neither $GAP$ nor $GAP_2$ (both notions are defined below) is always satisfied, no constraint formulated above forces us to assume that $\{x,y,z\} \in C$ if $x \succ_P y$, which is all that we need.

Now we will discuss these constraints. Constraint (P1) says that to any element $c$ of $C$ one can add any element $x \in X$ (thus, also $c \cup \{x\} \in C$) that is in an ordering relation with respect to at least one other element, on the condition that $c \cup \{x\}$ satisfies constraint $GAP$. To state this constraint, suppose that $c$ contains $n$ elements (written by $c^n$). Then the constraint says that there must be at least $n - 1$ subsets $c'$ of $c$ with cardinality $n - 1$ such that all these $c'$ are also elements of $C$.

$$c^n \in GAP \quad \text{iff}_{def} \quad \exists_{n-1} c' \subset c : card(c') = n - 1 \land c' \in C.$$

The *intuition* behind this condition is that only those subsets of $X$ satisfy $GAP$ if there is at least one gap in this subset with respect to the relevant property. It is easy to show that this closure condition guarantees that the resulting ordering relation will satisfy transitivity and will thus be a strict partial order.[14] Constraint (OR) guarantees that if $\{x,y\}$ and $\{v,w\}$ are in $C$, then

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[14] For a proof of this result, and the others mentioned below, see van Rooij (to appear).

either $\{x, y, v\}$ or $\{x, y, w\}$ belongs to $C$ as well. We will see below that by adopting this constraint the resulting ordering relation will satisfy the interval ordering condition. Constraint (P2) implements the intuition that if $c$ gives rise to *two* gaps (again, only intuitively speaking), one can always add at least one arbitrary element of the domain to it, without closing all gaps. Constraint (P2) is defined in terms of predicate $GAP_2$ which is defined as follows:

$$c^n \in GAP2 \quad \text{iff}_{def} \quad \exists_n c' \subset c : card(c') = n - 1 \wedge c' \in C.$$

Notice the subtle difference between $GAP_2$ and $GAP$: whereas the former requires that there are at least $n$ subsets of $c^n$ in $C$ with cardinality $n - 1$, the latter requires this only for $n - 1$ subsets. The intuition between this formal difference is that whereas $c$ satisfies $GAP$ already if it contains at least *one* gap, $c$ can only satisfy $GAP_2$ if it has at least *two* gaps. Consider subsets of the natural numbers and assume that such a subset has a gap if at least one number in the order is missing. Thus, $\{1, 2, 3, 4\}$ has no gap, $\{1, 2, 3, 5\}$ has 1, but $\{1, 3, 4, 6\}$ and $\{1, 3, 5, 7\}$ have two or more gaps. The set $\{1, 2, 3, 5\}$ has the following subsets of 3 numbers with a gap: $\{1, 2, 5\}, \{1, 3, 5\}$, and $\{2, 3, 5\}$. Thus it has 3 such subsets, which means that $\{1, 2, 3, 5\}$ satisfies $GAP$, but not $GAP_2$. The set $\{1, 3, 4, 6\}$, on the other hand, has 4 four subsets of 3 elements with a gap: $\{1, 3, 4\}, \{1, 3, 6\}, \{1, 4, 6\}$, and $\{3, 4, 6\}$ which means that it satisfies both $GAP$ and $GAP_2$. The same holds for the set $\{1, 3, 5, 7\}$. The idea behind constraint (P2) is that to the set $\{1, 3, 4, 6\}$ we can always add an arbitrary natural number and still have a gap (and thus be an appropriate context), but that this doesn't hold for $\{1, 2, 3, 5\}$: adding 4 would result in an inappropriate context. Now we can state the desired theorem (see van Rooij, 2009):

**Theorem 2.** *Any context structure $\langle X, C, P \rangle$ with $X$ a set of individuals, where $P$ obeys axioms (NR), (DD), (UD) of section 2, and where $C$ is closed under (P1), (OR), and (P2) gives rise to a semi-order $\langle X, \succ_P \rangle$, if we define $x \succ_P y$ as $x \in P(\{x, y\})$ and $y \notin P(\{x, y\})$.*

# 5  Comparisons revisited

Consider once more the following figure, picturing the lengths of John and Mary.
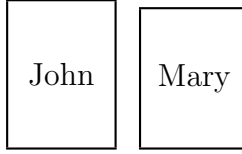
How can we account for the intuition that this picture allows us to say that the *explicit* comparative (1-a) is true while the *implicit* comparative (1-b) is false?

(1-a)    John is taller than Mary.

(1-b)    Compared to Mary, John is tall, but compared to John, Mary is not tall.

I would like to suggest that the difference between explicit and implicit comparatives is closely related with the difference between weak orders and semi-orders. As already suggested in section 1, weak orders are very natural representations of standard explicit comparatives like (1-a). I propose that the semi-order relation *significantly taller than*, i.e. '$\succ_T$', is what is relevant to evaluate the truth of implicit comparatives like (1-b). Thus, (1-b) is true just in case John is significantly taller than Mary. This immediately explains why (1-a) can be inferred from (1-b), but not the other way around.

A weak order '$>_P$' can be thought of as *at least as informative*, or *refined*, as a corresponding semi-order '$\succ_P$' in the sense that for all $\langle x, y \rangle \in X \times X$, if $x \succ_P y$, then $x >_P y$ as well. There is, however, another sense in which it is natural to think of the semi-order as the basic one, and *derive* a corresponding weak order. Note, though, that for an arbitrary semi-order there might always be several weak orders that are compatible with it. The following two weak orders can, for instance, be derived from the semi-order '$\succ_P$': '$>_P^1$' defined as $x >_P^1 y$ iff$_{def}$ $\exists z : (x \sim_P z \land z \succ_P y)$ and '$>_P^2$' defined as $x >_P^2 y$ iff$_{def}$ $\exists z : (x \succ_P z \land z \sim_P y)$. Fortunatetely, for each semi-order there is also a *unique most refined* weak order that can be derived from it. As already shown by Luce (1956), this *unique* strict weak order '$>_P$' can be defined as follows: $x >_P y$ iff$_{def}$ $\exists z : (x \sim_P z \land z \succ_P y) \lor (x \succ_P z \land z \sim_P y)$. The corresponding relation '$\approx_P$' defined as $x \approx y$ iff$_{def}$ $x \not>_P y$ and $y \not>_P x$ is an equivalence relation, which could also be defined directly as $x \approx y$ iff$_{def}$ $\forall z \in I : x \sim_P z$ iff $y \sim_P z$. What I would like to suggest is that if we start with the semi-order '$\succ_P$' in terms of which we interpret implicit comparatives, it is the strongest derived weak order '$>_P$' that is relevant to interpret explicit comparatives.

Recall from section 3.1 that if '$P$' is 'tall', measure theoretically '$x \sim_P y$' is true iff the difference in length between $x$ and $y$ is less than a fixed *margin of error* $\epsilon$. Suppose that John's length is $\delta < \epsilon$ higher than Mary's length, which

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is $\delta < \epsilon$ higher than Sue's length, but that the difference between John's and Sue's length is higher than $\epsilon$. This situation can be pictured as follows:



In this situation, $j \sim_P m$, $m \sim_P s$, but $j \succ_P s$. Given our claim that implicit comparatives should be interpreted as 'significantly taller than' using semi-orders, we correctly predict that (1-b) is false: $j \not\succ_P m$. But given that we interpret implicit comparatives in terms of the weak order '$>_P$' defined in terms of the semi-order, we also correctly predict that (1-a) comes out as being true. The reason is that $j >_P m$, because there is an $s$, i.e. Sue, such that $j \succ_P s$ and $s \sim_P m$. This is an encouraging result, and enough to 'explain' the difference between (1-a) and (1-b). But we wanted more: we wanted to explain the difference between explicit and implicit comparatives in terms of the behavior of adjectives in comparison classes. In the remainder of this section I want to discuss (i) how we can do that, and (ii) whether the way we suggested to interpret explicit comparatives is really strong enough.

An obvious way to account for the distinction between explicit and implicit comparisons in terms of comparison classes is just to look at the differences between the constraints on context structures in section 1 versus those in sections 3.3 and 3.4. If we adopt the semantic model of section 3.3 we might say that the distinction between explicit versus implicit comparatives corresponds to whether we assume the equivalence of 'It is not the case that Mary is tall in $c$' and 'Mary is small in $c$' or not. If we adopt the pragmatic model of section 3.4, on the other hand, the distinction between explicit versus implicit comparatives can be said to correspond with what we take to be appropriate comparison classes: *all* subsets of $X$, or only those in which there is a significant gap.

But what if this freedom is not allowed? What if we cannot play with what is an appropriate comparison class and assume the equivalence of 'It is not the case that Mary is tall in $c$' and 'Mary is small in $c$'? Even in that case, I claim, we can make a distinction between explicit and implicit comparatives, because adopting the analysis of section 3.3, we can make a difference between (Klein) and (Benthem), as introduced in the beginning of this paper. Thus, I propose the following interpretation rules:

(Klein)    (1-a) is true iff *there is* a comparison class such that John is

tall here and Mary is not: $\exists c : M, c \models T(j) \land \neg T(m)$.

(Benthem) (1-b) is true iff John is tall in comparison class $\{\text{John, Mary}\}$, and Mary is not: $M, \{j, m\} \models T(j) \land \neg T(m)$.

Let us first note that in the semantic model of section 3.3, it is now at least possible that (1-a) is true, but (1-b) is false. It is very natural to assume that in the comparison class $\{\text{John, Mary, Sue}\}$, we count John as tall, Sue as short, and Mary as neither tall nor short. But this is enough for the explicit comparative (1-a) to be true in the above situation. Moreover, it is natural to assume that in the comparison class $\{\text{John, Mary}\}$, we count John as tall if and only if Mary is counted as tall, which means that the implicit comparative (1-b) is correctly predicted to be false. Unfortunately, the constraints given in section 3.3 do not rule out the possibility that given the above situation, John is counted as tall in comparison class $\{\text{John, Mary}\}$ but Mary is not. But in that situation (1-b) is falsely predicted to be true, just as (1-a). A situation like this is ruled out if we adopt the natural constraint that for all comparison classes $c$, $P(c) = \varnothing$ iff $\overline{P}(c) = \varnothing$. Adopting this constraint, it can only be the case that John is counted as tall in comparison class $\{\text{John, Mary}\}$ but Mary not if Mary is counted as *short* in this comparison class, and thus that John is *significantly taller* than Mary. But this is in contradiction with what we assumed.

How does this proposal relate with our earlier suggestion to interpret explicit comparatives in terms of the unique strongest weak order derived from a semi-order? We can prove that it comes down to the same thing. Suppose an explicit comparative '$x$ is $P$-er than $y$' is true according to (Klein) because there is a comparison class $c$ such that $x \in P(c)$ and $y \in c - P(c)$. If $c = \{x, y\}$ then $x \succ_P y$ holds. Because $\succ_P \subseteq >_P$ it follows that $x >_P y$, so that is ok. But now suppose that it is not the case that $x \in P(\{x, y\})$ and $y \notin P(\{x, y\})$. Then there must be a superset $c$ of $\{x, y\}$ for which $x \in P(c)$ and $y \notin P(c)$ holds. Adopting constraint (DD*) we have already ruled out the possibility that $y \in \overline{P}(c)$. So it must be that $y \in c$ but $y \notin P(c)$ and $y \notin \overline{P}(c)$. Let $c$ for instance be $\{x, y, z\}$. By our constraint that for all comparison classes $c$, $P(c) = \varnothing$ iff $\overline{P}(c) = \varnothing$ it follows that $z \in \overline{P}(c)$. By (DD*) it follows that $x \in P(\{x, z\})$ and $z \in \overline{P}(\{x, z\})$, and thus that $x \succ_P z$. What we have to show is that $z \sim_P y$. Because $z \in \overline{P}(c)$, $y \notin P(c)$ and $y \notin \overline{P}(c)$, it follows by (NR*) that it is not the case that $z \in P(\{y, z\})$ and $y \in \overline{P}(\{y, z\})$, which means that $z \not\succ_P y$. But that means that $y \succeq_P z$. If $y \not\succ_P z$ we are done, so suppose $y \succ_P z$. In that case it *is natural to assume* that *there is another* (i.e. taller) $z'$ such that $x \in P(\{x, y, z'\}), z' \in \overline{P}(\{x, y, z'\}), y \notin P(\{x, y, z'\})$ and $y \notin \overline{P}(\{x, y, z'\})$ and for which $y \not\succ_P z'$. But this means that $y \sim_P z'$ which is what we wanted.

The one but last sentence indicated something important that, in fact, appeared already much earlier: if we want to guarantee that John is counted as taller than Mary if their relative lengths are pictured as below,

John | Mary
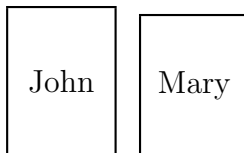
we have to assume that we have enough other individuals John and Mary can be compared with. There either has to be somebody like Sue who is significantly shorter than John but similar to Mary, or another individual who is significantly taller than Mary but similar to John. A very natural way to guarantee these kind of *witnesses* to exist is to adopt the following constraints on models: for all individuals $y$ with at most two exceptions (the tallest and shortest individuals, if they exist), $\exists x, z : x \succ_P z \wedge x \sim_P y \sim_P z$. For the other two individuals $v$, if they exist, it just hold that $\exists w : w \neq v \wedge v \sim_P w$. Thus, we demand that with at most two exceptions, any object is 'indistinguishable' from at least two others. If we take semi-orders to be primitive, this constraint has a direct effect. Otherwise, the constraint should be reformulated in terms of context structures. In whatever way we do this, it is clear that it has the desired effect: any small difference in length between John and Mary is enough to make the explicit comparative true.

Adopting the above type of witness constraint is costly, but how costly is it really? Degree-based theories of comparatives make use of witness constraints as well. If we look at the algebraic structures that are faithfully represented by means of the measure functions (see Krantz et al., 1971) we see that in case the numbers are really crucial (as is the case in so-called 'interval scales' and 'ratio scales') it has to be assumed that there *exists a witness for every possible degree* required for the homomorphic function. It is clear that our witness constraint is much less involved, i.e., we did not secretly presuppose degrees after all.

# 6    Conclusion

I claimed in this paper that the distinction between explicit and implicit comparatives corresponds to the difference between (strict) weak orders and semi-orders. Moreover, I showed that both can be characterized naturally in terms of constraints on the behavior of predicates among different comparison classes, and thereby meeting the challenge Kennedy and others have posed upon comparison class-based approaches of comparative statements. How can degree-

based approaches account for the difference between explicit and implicit comparatives? The most natural way for them to make a distinction between (1-a) and (1-b) would be to claim that for the latter there must be a specific number $\epsilon > 0$ such that the length of John minus $\epsilon$ is more than the length of Mary. But if we think of $\epsilon$ as the threshold, degree-based approaches must make a distinction between explicit and implicit comparatives very much like we did involving weak and semi-orders.

An interesting question that arises is whether we really want the threshold to be the same for any pair of individuals. This was assumed in this paper (by making use of semi-orders), but should perhaps be rejected in general. In De Jaegher and van Rooij (to appear) it is shown that Prospect Theory can be used to account for the intuition that the threshold depends on the individuals involved. It would take us too far to investigate this issue here.

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