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Math Garden: A new educational and scientific instrument

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CHAPTER

General introduction

The initial aim of this thesis project was to study the dynamics of cognitive development in children, specifically the development of mathematical knowledge and abilities. To study and understand the complex processes that play a role in children's mathematical development we chose a time-serial approach. High frequent measurements enable the detection of developmental characteristics such as transitions, accelerations, but also stagnation and relapse. Measuring children on a high frequent basis is, however, easier said than done as measuring on a daily or weekly basis is very invasive in the life of children. The solution seems simple as children are already being measured on a daily basis. Every weekday children solve math problems in their notebooks and workbooks. Why not use this information to study their mathematical development?

This is the first path we took. We extracted data from children's notebooks, workbooks, and the software of math methods used by schools. To achieve this we had to face severe practical problems such as deciphering children's handwriting, linking answers in notebooks to their corresponding problems and finding the relevant raw data in inaccessible data archives. In addition, more fundamental problems arose. These measurements were curriculum based and not aimed to assess children's general mathematical development. We concluded that a new approach was needed that combined both educational and scientific aims.

The focus of the project shifted, therefore, to the development of a new instrument that would meet both educational and scientific aims: Math Garden (EN: www.mathsgarden. com / NL: www.rekentuin.nl). On the one hand Math Garden enabled us to study children's mathematical development in detail. On the other hand it is an educational tool that enables children to work at their own level and at the same time gives teachers insight into the mathematical abilities (or more specifically the arithmetical abilities[1]) of their pupils.

Math Garden is a web application in which children can play math games. The garden metaphor is used to stimulate children to maintain their mathematical abilities as they can nurture their plants by improving their mathematical ability and prevent their plants from withering by playing on a regular basis. Math Garden uses a new method for computer adaptive testing by which the difficulty of the exercises is automatically adjusted to the skills of the child. Before introducing the instrument in more detail we will discuss some key ideas on which Math Garden is based.

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[1] We consider arithmetic as a branch of mathematics and both terms will be used interchangeable in this thesis.

Key ideas that have influenced Math Garden

Mathematical development is a complex dynamical system

The developing cognitive system can be understood as a complex dynamical system (Kan, Ploeger, Raijmakers, Dolan, & van der Maas, 2010; Van der Maas, Dolan, Grasman, Wicherts, Huizenga, & Raijmakers, 2006; Van der Maas & Molenaar, 1992). Complex dynamical systems are networks of many elements that interact iteratively with each other. They develop and adapt dynamically and are often characterized by nonlinearity and self-organization. The study of complex systems is a topic in many disciplines, such as physics, neuroscience, biology, and psychology.

Why do we consider math learning and development a complex dynamic system? Mathematical ability consists of a large number of sub abilities. According to Dowker (2005, p.1) "there is no such thing as arithmetical ability – only arithmetical abilities". These abilities can be grouped into three main categories. *Factual knowledge* consists of knowledge about arithmetic facts and the names for numbers and operations. *Procedural knowledge* concerns knowledge about arithmetical procedures and how to perform them. Finally, *conceptual knowledge* concerns knowledge about arithmetical principles, such as commutativity, but according to Dowker also the understanding of the meaning of word problems and approximate arithmetic. All these abilities interact recursively with each other. For example, better performance of calculation procedures (procedural knowledge) may lead to a better consolidation of answers to problems in an associative memory network (factual knowledge), which in turn facilitates the calculation of more complex arithmetic problems (procedural knowledge). The relation between conceptual and procedural knowledge has received much attention and the results seem to favor a causal bidirectional relation between these forms of knowledge instead of one preceding the other (Rittle-Johnson & Alibali, 1999). Moreover, several basic cognitive skills are related to these arithmetical abilities, such as, but not exlusively, working memory, long-term memory, intelligence, logical reasoning, spatial ability, and verbal ability (Dowker, 2005). These abilities are assumed to mutually influence each other and are each under the influence of maturation of the brain and of the environment.

This view of a complex system of bidirectional relations between cognitive abilities has been formally described as a mutualism model (Van der Maas et al., 2006). In this nonlinear dynamical system model basic cognitive abilities are initially uncorrelated. Each component, i.e. ability, follows a logistic growth curve determined by parameters sampled from independent normal distributions. The growth curve of each ability is, however, also positively influenced by the growth of other abilities. Over time different cognitive abilities

then become correlated. For example, improvement in basic memory processes does not only improve the functioning of short and long term memory but also stimulates the development of better cognitive strategies and scholastic performance in areas such as reading and arithmetic. On the other hand, better cognitive strategies make it possible to increase the efficiency of short-term memory (Siegler & Abali, 2005).

The mutualism model can explain correlational patterns often found in intelligence research. No general intelligence factor, such as g or mental power (Spearman, 1904, 1927) is needed to explain these correlational results. A main challenge in testing the mutualism model is fitting the model to time-serial data with bidirectional relations between abilities. There are several new developments in the statistical analysis of network data that have promise for educational data (Borsboom, Cramer, Schmittman, Epskamp, & Waldorp, 2011; Epskamp, Cramer, Waldorp, Schmittman, & Borsboom, 2012; Schmittman et al., 2013). Clearly, such techniques need high frequent reliable data and that is the focus of this dissertation.

We argue that an important way to study children's complex system of interacting abilities is the microgenetic approach (Siegler & Crowley, 1991). Researchers using the microgenetic method are specifically interested in the process of change, not just in the product of change (Granott & Parziale, 2002). The microgenetic method is characterized by high frequent measurements during a period of development. The density of these measurements should be high relative to the rate of change and the study should span the whole developmental process, that is, until a relatively stable state is reached. In addition, the collected data should be analyzed with intensive trial-by-trial analyses in order to detect the dynamics of the developmental processes (Flynn, Pine, & Lewis, 2006; Siegler & Crowley, 1991). With this method key features of developmental processes can be detected, such as transitions, sensitive periods, but also relapses and stagnations (van der Maas, Jansen, & Raijmakers, 2004; van der Maas & Molenaar, 1992; van der Maas & Raijmakers, 2009).

Although the microgenetic method has promising features, its application in developmental research is still limited. The reason for this is often practical. The high frequency measurements make the microgenetic method time-consuming and expensive. Moreover, because of its invasiveness in everyday life it is difficult to find and maintain subjects for these studies. But other factors complicate microgenetic studies as well. One might wonder if the study itself does not alter the developmental process. If, for example, children have to make an arithmetic test every day, this could promote learning itself and lead to other developmental trajectories than those that would have occurred without the intervention of the study. This is a difficult issue, as it is not possible to compare the results of a microgenetic study to an independent characterization of the natural change process (Kuhn, 1995).

We believe that the limitations of the microgenetic method are subordinate to its potentials. Microgenetic studies may give us much better insights into the dynamics of development both within and between domains. These findings could also have a large effect on education. If, for example, we are able to detect sensitive periods in which children are especially susceptible to specific instructions, more effective instructions can be given. In this thesis we present a new instrument for the microgenetic study of mathematical development: Math Garden. This instrument solves many of the problems of microgenetic studies and is described below.

Mathematical abilities can be considered as a form of cognitive expertise

Our second key idea is that children's arithmetical development and learning should be seen as a form of expertise development. With expertise we mean the outstanding performance within a certain domain, for example, sports, music, or science. Much research has been done to understand the principles that underlie the achievement of expert performance. Ericsson, one of the leading experts in the domain of expertise development, claims that motivation and practice are the key principles in expertise development (Ericsson, 2006; Ericsson & Ward, 2007). According to Ericsson the development of expert performance is gradual and expertise is only acquired after many years of special practice activities, which he calls deliberate practice. Deliberate practice is characterized by goal-directed training with repeated exercises just beyond the current ability level and with immediate feedback. Because of the intensity of these goal-directed training sessions regular sessions are preferred over a small number of long training sessions (Erisccon, 2006). In addition, Krampe and Charness (2006) argue that deliberate practice is also necessary for maintaining expert performance.

We all learn math in school and we are, therefore, not inclined to consider this as an expertise. However, mathematical learning also requires many hours of practice, rehearsal of exercises and motivation over a prolonged period of time. It is often debated whether math learning requires innate ability or talent (Pesenti, 2005). Yet, in any case, expertise requires extensive practice over long periods. For example, American students became calculating experts after 300 hours of training over a period of two to three years (Staszewski, 1988).

To optimize mathematics education one should strive to implement the principles of deliberate practice into everyday education. Dutch policymakers also argue that more time should be dedicated to practice in everyday education (Expert group "Doorlopende leerlijnen", 2008). The Dutch ministry of education acknowledges the importance of maintaining and extending one's knowledge with age by implementing basic math (i.e., arithmetic) exams at several time points in higher education. We believe that there is much to win if

practice in education is implemented according to the principles of deliberate practice. However, one of the main requirements of deliberate practice is intensive one-on-one guidance by a teacher or coach. Bloom (1984) claimed that one-to-one tutoring is very beneficial and refers to this effect as the two sigma problem. He found that students who received one-to-one tutoring performed on average two standard deviations better than students who received conventional instruction within a classroom setting.

Hence the challenge is to initiate deliberate practice in classrooms of 30 children who are on average less motivated than promising athletes or musicians. Providing each student with the most optimal learning settings according to the principles of deliberate practice is further complicated by the individual differences that exist within a classroom, which brings us to our third key idea.

Individual differences in math are huge

Our third idea or observation is that individual differences in mathematical ability are huge. These include individual differences in procedural, factual, and conceptual knowledge. Dowker's book (2005) reviews a large number of studies concerning individual differences in math ability. Cockcroft (1982) stated that it is very likely that there is a 7-year age range in arithmetical ability in a class of 11-year olds. In the Math Garden dataset we found huge differences in mathematical ability in all grades. Figure 1 shows the proportion of children per grade that score above or below the mean of children one or two grades higher and lower, averaged across addition and subtraction. In all grades a substantial number of children score above the mean of children two grades higher: between 6.5% and 14.9%. On the other hand there are also many children, between 7.0% and 26.5%, who score below the mean of children two grades lower. Results from the periodical educational assessments performed in the Netherlands by Cito also demonstrated considerable individual differences in math ability within Grade 3 (Hop, Janssen, Hemker, van Weerden, & Til, 2012) and Grade 6 (Scheltens, Hemker, & Vermeulen, 2013). Summarized, the results from Math Garden and previous studies illustrate the enormous challenges that teachers face when teaching 30 children with varying abilities.
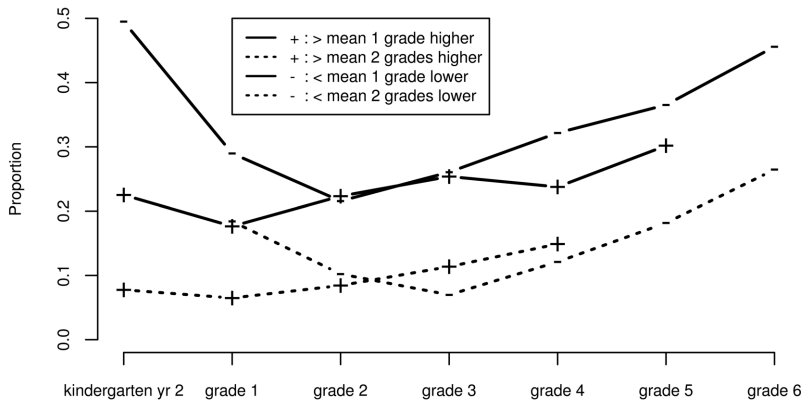
Figure 1 *Proportion of children scoring above the mean of children in one or two grades higher and lower, averaged across the domains addition and subtraction, in the months April till June 2013.*

These large individual differences make it especially difficult for teachers to provide optimal practice sessions for each child. At the same time the Dutch ministry of Education demands that all children, also children who are weak or excelling in math, receive education at their own level. This is referred to as adaptive education ("passend onderwijs") and should preferably take place in standard classes instead of by placing children in schools for special education. However, resources for adaptive education are limited, making it a seemingly impossible task.

One of the prerequisites of good adaptive education is knowledge about the ability levels of pupils. One can only provide suitable instruction if it is known how skilled the child is and what his or her strengths, flaws, and common errors are. Only then is it possible to provide instruction according to the principles of deliberate practice. The parliamentary research committee Dijsselbloem (2008) and the expert group "Doorlopende Leerlijnen" (2008) also argued that pupils' progress should be monitored in order to detect possible developmental delays. They state that information acquired with monitoring instruments should be fully exploited. Many schools in the Netherlands use progress-monitoring systems (Blok, Otter, & Roeleveld, 2002) but most method-independent progress-monitoring systems, such as the child monitoring system of Cito (Janssen, Verhelst, Engelen, & Scheltens, 2010), measure children's abilities only once or twice per year. In order to detect problems early and start remediation in time, measurements should take place more frequently. However, the more educational time is used for testing, the less time remains for practice and instruction.

ICT is sufficiently developed to enable optimal practice activities in classroom settings

To summarize, in order to develop an instrument that meets both educational and research goals, we need an instrument that measures children's math abilities on a high frequent basis, can cope with the individual differences in ability, and implements the principles of deliberate practice. The solution can be found in ICT. We note that the introduction of computers into the schools has been slow for different reasons. Last century computers were expensive, large, error prone, and required lots of maintenance. Teachers were not skilled in ICT. But in the last 10 years we have witnessed the rapid and large-scale introduction of computer technology in households and schools, due to the availability of small computers (mini-laptops, tablets) and fast WIFI access to the internet. According to Statistics Netherlands (2011), 92% of the Dutch households owned a computer in 2010 and 91% of the households was connected to the Internet. Moreover, the increasing use of smartphones and tablets has lead to a strong increase in Internet traffic. The Digital Agenda of the EU 2020 strategy recommends that broadband access be available to all by 2013, and fast Internet access (>30MB/s) by 2020.

Developing educational methods that exploit these new possibilities is a major challenge, which is being taken up by many research groups, companies, and schools. With Math Garden we used the possibilities of ICT in a new progress monitoring and research instrument. Math Garden is described in detail in the next section.

Math Garden

Math Garden is a web based training-tracking system in which children can train their mathematical skills while at the same time their development is being tracked. After logging in, children enter their personal garden in which every plant represents a math game. Several game principles are implemented to motivate children to practice their math abilities on a regular (weekly) basis. The state of the garden gives children an indication of their mathematical skills as the size of the plants represents their ability level. Plants grow when their ability increases. If children do not maintain their garden by playing regularly, the plants will wither, indicating that these math skills need to be practiced.

When children first enter their garden only one plant is visible. New plants (i.e., games) appear when children reach pre-set levels of ability in the games already present in the garden. This thesis focuses on the data collected with the four games that measure children's ability on the operations addition, subtraction, multiplication, and division. In September 2013 Math Garden contained sixteen games measuring math-related skills such as count-

ing, number series, telling time, and fractions. Each game administers items from a large item bank (> 400 items per game). These item banks consist of items, varying in difficulty, representing the curriculum of primary education and beyond.

Both children's answers (accuracy) and their responses are combined to estimate their mathematical ability with a new scoring rule: the High Speed High Stakes (HSHS) scoring rule (Maris & van der Maas, 2012). For each item children have limited time to give an answer. After an answer is given, the score on an item equals the remaining time and this score is either positive, in case of a correct answer, or negative, in case of an incorrect answer. This scoring rule is explained in more detail in Chapter 2. The scoring rule is implemented in the games in a playful manner. The deadline for an item is visualized by coins on the screen and each second a coin disappears. Children either win or lose the remaining coins when providing an answer. Children can use these coins to buy trophies for their virtual trophy cabinet.

In summary, game principles are used to motivate children to practice their mathematical skills on a regular basis, one of the key principles of deliberate practice. As was said above, it is not only essential to practice on a regular basis but also that these practice sessions are adjusted to the ability level of the child. This was achieved by using a new adaptive testing procedure, using state of the art psychometric modeling (see Chapter 2 and Maris and van der Maas, 2012). With this procedure the difficulty of the items in the math games are automatically adapted to children's abilities. In most schools different programs are used for gifted children and for special training for weak students. Within Math Garden both weak and strong students can practice at their own level, therefore being equally motivating and challenging for children of all abilities.

The educational application of Math Garden is based on the concept of Game, Train, Track, and Teach. Within Math Garden children play adaptive **games** in a stimulating on-line environment and thereby **train** their mathematical skills according to the principles of deliberate practice. The data of these training sessions are **tracked** as the answers and response times of every solved problem are being registered in an online database. This database enables comparisons of pupils and school classes to their reference groups, informing teachers about strengths and weaknesses of their pupils. In addition, information about children's specific errors and strategies can be given. Teachers can use this information to optimize **teaching** at both the individual and school class level. With Math Garden we aimed to take over the less appealing parts of education, that is, many hours of practice on the student level and correction of schoolwork on the teacher level, and making them more pleasant and better.

One of the key principles behind the set-up of Math Garden is self-organization. Firstly, this applies to the computer adaptive testing (CAT) system that is used to administer items in the math games. One of the requirements of an adaptive system is that the difficulties of the items are known on beforehand. Adjusting item difficulty to the ability level of a child, demands knowledge on the difficulty of items. This requires pre-testing of items, which is expensive and time-consuming. The use of CAT in education is, therefore, limited. CAT is mainly used by large companies (i.e., Cito) and is only applied in large-scale educational settings. The new CAT system in Math Garden makes the process of pretesting unnecessary. It is based on the Elo rating system that has been developed to compare chess players (Elo, 1978). Comparable rating systems are used in various sports and games to measure the ability of players and to match opponents of equal strength.

In Math Garden children and items are considered opponents and thus a child solving an item is seen as a match. Within this system both items and children have a rating, indicating their difficulty or ability level, respectively. After each solved item the rating of the child and the rating of the item are adjusted. For example, if a child answers an item incorrectly, the item wins. The item gains ratings points, that is, becomes more difficult, and the child loses rating points. The number of rating points won or lost depends on the difference in rating between the child and the item. This new CAT system and the online set-up of Math Garden enable the estimation of both item difficulties (item ratings) and children's abilities (person ratings) on the fly. Estimations of item difficulties are, therefore, based on the answers of all children playing online in Math Garden. This self-organizing set-up of the CAT system enables the fast and easy development of new adaptive games. After an item set is constructed with items of various difficulties, the CAT system will do the rest. A schematic overview of the Math Garden system can be seen in Figure 2.
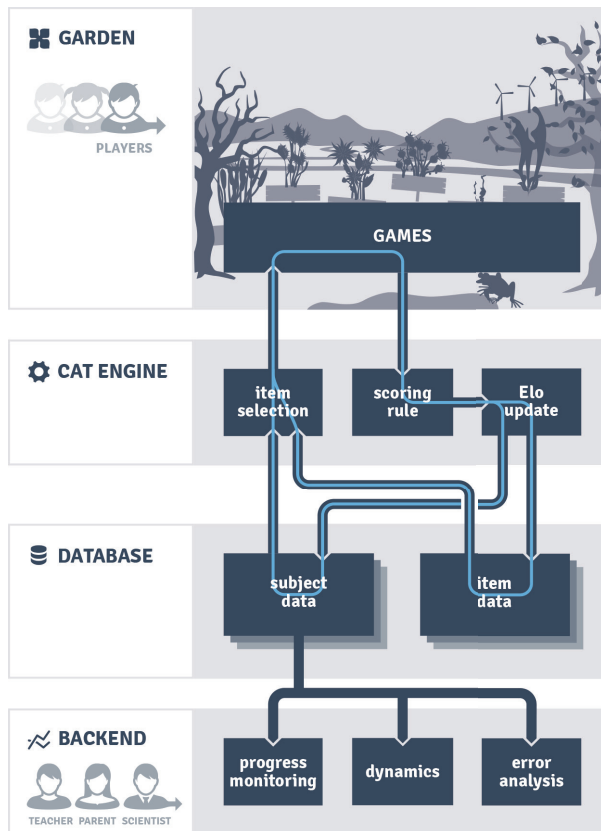
Figure 2 *A schematic overview of the web application Math Garden. Children maintain their garden by playing math games. The size of the plants depends on the ability level of the child. Item choice is adapted to the ability of the child by the CAT engine. Estimates of children's ability and item difficulty are updated after each answered item according to an extended Elo algorithm with the HSHS scoring rule. Teachers, parents, and scientist receive automatically generated reports on the data.*

Another self-organizational part of Math Garden concerns the reports on children's performance, discussed above. One of the important goals of testing in education is the comparison of children's performance to their norm group, which enables the detection of children that are ahead or behind and need extra attention. The construction of norm groups is, however, a very costly and time-consuming process. It requires a large representative sample of children in each age group. Moreover, norm groups may become outdated as the educational curriculum changes and they are often bound to certain time points of the

school year. The online set-up of Math Garden enables the comparison to other users at every time point, as all data is stored in a central database.

We use the term reference groups instead of norm groups because these groups have not been formed according to standard norm requirements. For example, there is no control over the type of schools that use Math Garden. Also, the conditions under which children use Math Garden are less controlled than the conditions under which norm-referenced tests are being administered. Children can use Math Garden anytime anywhere as long as there is an Internet connection. Some selection criteria are used to ensure the reliability of the reference groups: only children that have played sufficient items in a recent time frame are included.

The advantage of the reference groups in Math Garden lies, however, in the power of the data. The more users play the games in Math Garden, the more reliable the reference groups become. For example, the norm groups in the monitoring system for Math of Cito consist of 778 to 1516 children in each grade (Janssen et al., 2010). In September 2013 Math Garden has between 11,000 to 17,000 active users in each grade (grades 1 to 6[2]). The reference group comparison can give a fairly good indication of children's performance at every desired time point. Moreover, the online set-up allows for self-organization of the reference groups. Changes in performance across the school year, either developmental or due to changes in the curriculum, will be reflected in the performance of the reference group because the performance of all users is tracked over time.

Summarized, in contrast to off-line and online tests with static norm groups, Math Garden does not have the disadvantage of repeatedly needing to conduct new research to determine norms. The larger the group of Math Garden users, the more specific the reference groups can be. We could then, for example, compare 8-year old girls who attend a "Montessori"-school in the region Amsterdam with each other. International use of Math Garden would allow assessment of international differences in math performance, possibly as an alternative for large expensive research projects such as TIMMS and PISA. On the other hand, Math Garden has the typical disadvantages of low-stakes testing.

This brings us to the final characteristic of Math Garden we would like to discuss: Math Garden combines practice and testing in one program. Within the educational field there is a clear distinction between practice and testing. Programs for practice and testing are often developed by different publishers, each having their own expertise. Much time can be saved if these two goals are combined. This is what Math Garden does. Math Garden uses children's daily practice to track their performance, using psychometric modeling.

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[2] In this thesis we use the US grade system. In the Dutch grade system grade 1 to 6 are numbered grade 3 to 8.

Again the advantage lies in the power of data. The introduction of tablets and small computers in education enables children to practice in Math Garden on a weekly or daily basis, by which their performance is automatically tracked. The more frequently children use Math Garden, the better the insight in children's math development. In addition, a bad day or help from a sibling or parent can easily be detected when comparing a child's performance to his or her performance on other occasions. Thus, Math Garden enables high frequent monitoring of children's performance while at the same time children practice according to the principles of deliberate practice. Thereby, no valuable time is lost on the administration of tests.

Math Garden was developed to enable a large-scale microgenetic study of children's mathematical development. To ensure that schools would be willing to use Math Garden on a daily or weekly basis it was important to meet several educational aims. Normally, it takes a lot of effort to find schools that are prepared to participate in such an extensive research project. Because Math Garden filled some gaps in the educational system, schools were enthusiastic about participating in the project. In the first school year (2007-2008) 8 schools participated and in the second year even 21 schools participated. Not much acquisition was needed to persuade these schools to participate. The success of the Math Garden project has led to the start-up of Oefenweb.nl. Oefenweb.nl is a spin-off company of the University of Amsterdam that aims to improve Math Garden for educational use and to develop other online learning tools based on the same principles. Nowadays, users of Math Garden are still asked for permission for use of their data for scientific research, thereby maintaining the link between education and research. Besides the advantages, combining research and education may also pose problems for the research questions addressed. The data contain more noise and by integrating measurement with the learning process itself, the learning process may be altered. But this is an inevitable property of a new instrument that meets both educational and scientific goals.

Overview thesis

This thesis is only the starting point of research with Math Garden. So far, more than 200 million problems have been solved on the Dutch website of Math Garden. Math Garden is growing very rapidly. However, the important question is whether Math Garden is indeed a valuable tool for scientific research. This is the central question of the research presented in this thesis. Because the main chapters of this thesis were written as independent research papers, some overlap was inevitable.

Chapter 2 provides a detailed description of the Math Garden system, in particular the new model for computerized adaptive practice and monitoring. This chapter evaluates the computer adaptive method by investigating the reliability and validity of the ratings resulting from this method. In addition, this chapter addresses whether the educational aims of Math Garden are met, that is, whether children indeed practice on their own ability level and are motivated to practice regularly. Finally, examples concerning the diagnostic and developmental measurement possibilities of Math Garden are provided.

Chapter 3 and 4 address the validity of Math Garden from a different angle, namely by investigating problem characteristics that affect difficulty of the items used in Math Garden. In previous research problem difficulty is almost always operationalized by error rates or response times. The adaptive system of Math Garden produces item ratings in which speed and accuracy are integrated. If this adaptive system produces valid item and person ratings, then problem characteristics that were found to affect problem difficulty (either in accuracy or response times) in previous studies, should also affect the item ratings of Math Garden. Chapter 3 investigates the problem characteristics that affect the difficulty of simple multiplication problems. Chapter 4 addresses the difficulty of simple and complex addition and subtraction problems. Besides establishing the validity of the item ratings, both chapters also attempt to provide new insights into the problem characteristics affecting item difficulty. In both chapters the robustness and dependencies of the effects are investigated.

In Chapter 5 children's errors on multiplication items are addressed. With the Math garden dataset we have a very large database of children's arithmetical errors. Children's errors provide insight into arithmetic development. Errors indicate erroneous strategies and shed light on the organization of the memory network of multiplication facts. More insight into the order in which errors occur can also improve our knowledge of the developmental process. In their review of possible subtypes of mathematical difficulties Stock, DeSoete, and Roeyers (2007) argue that errors are indicative of these different subtypes. For example, reversal of numbers may be indicative of visuospatial dyscalculia. A starting point for all research concerning children's errors is the classification of different error types. This is, however, not a straightforward process, as different erroneous strategies can lead to similar erroneous answers. The issues faced when developing a classification method for errors are addressed in Chapter 5 and a new classification method for multiplication errors is presented.

Chapter 6 concerns a different research field, namely children's knowledge of the earth, and is therefore independent of the other chapters. This chapter gives, however, a good example of the type of analyses that we also aim to perform with Math Garden data. It illustrates how different theories concerning the developmental process in a specific research

field can be investigated by testing the assumptions underlying these theories with statistical techniques. In Chapter 6 two theories concerning children's knowledge of the earth are compared. One is that children form consistent mental models of the earth and the other that development is fragmented. These theories are tested with latent class analysis. This type of analysis can also be conducted on Math Garden data. In Chapter 6, data of children from different ages at the same time point are compared. However, the theories also differ in their assumptions regarding development of knowledge of the earth. Where one theory assumes a stepwise development, the other assumes gradual development. The next step is to track the same children over time with a microgenetic method, which is done in Math Garden, and to test these diverging assumptions concerning development.

Chapter 7 summarizes and discusses the research reported in the previous chapters but also provides an overview of other research projects and papers related to the Math Garden project.