Studies of accreting and non-accreting neutron stars

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CHAPTER IV

PULSAR STATISTICS

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Summary

It is shown that for radio pulsars the observed probability distributions of periods, magnetic field strengths, characteristic ages and heigths above the galactic plane can be well understood if:
(1) pulsars are born close to the galactic plane, within a band of scale heigth 175 pc, and with a maxwellian velocity distribution, having a standard deviation of 107 km/s,
(2) their initial magnetic field strengths, B_0, have a gaussian distribution in $\ln B_0$, with standard deviation 0.69 and centered around $B_0 = 3.2\ 10^{12}$ G,
(3) their initial periods are short, typically between 1 and 50 msec,
(4) their magnetic field strengths decay on a timescale of $5.3\ 10^6$ yr, and
(5) their radio luminosities are proportional to B/P^2 for $B/P^2 \leq 10^{13}\ Gs^{-2}$ and constant above this value.

Important differences with previous studies of pulsar statistics are that the adopted luminosity law is consistent with the observations and that pulsars are not born with long periods (i.e. no "late injection").

It is, furthermore, shown that there may be evidence for the existence of a small population of recycled pulsars. These, probably, comprise 10 to 15% of the total pulsar population.

Key words: pulsars

1. Introduction

Recently it has become clear that our understanding of the origin and evolution of radio pulsars is greatly affected by the precise way in which their radio luminosity, L, depends on the more basic properties of the underlying neutron star, such as its rotation period P, its period derivative $\dot{P}$, and its magnetic field strength B. In earlier studies (e.g. Gunn and Ostriker,1970;

Lyne, Manchester and Taylor,1985) the assumption was made that the luminosity is proportional to the square of the magnetic field strength. Using this law, Lyne, Manchester and Taylor (1985) showed that the observed distributions of P, $\dot{P}$, T (= $P/2\dot{P}$, the so-called characteristic age of the pulsar) and $|z|$ (the height above the galactic plane) could be well understood if it is assumed that pulsars are born:

(1) close to the galactic plane with a Maxwellian velocity distribution, having a standard deviation of 107 km/s,

(2) with a gaussian distribution in ln B with a standard deviation of $\sigma = 0.69$, and centered around $B = 0.75\ 10^{12}$ G,

(3) with short periods,

and further by:

(4) assuming that their magnetic field strengths decay on a timescale of 9.1 million years.

However, it was shown by Vivekanand and Narayan (1981) and Proszynski and Przybycien (1985) that the above-mentioned luminosity law is not consistent with the observed luminosities of pulsars. These authors found that a law of the form $L \propto P^{-1} \dot{P}^{1/3}$ is in better agreement with the observations. Based upon this law Chevalier and Emmering (1986) made a new study of the galactic pulsar population. They found that this luminosity law leads to the conclusion that pulsars are born with magnetic field strengths equally distributed between $9.4\ 10^{11}(4\ 10^{6}\text{yr}/\tau_D)^{1/2}$ G and $6.3\ 10^{12}(4\ 10^{6}\text{yr}/\tau_D)^{1/2}$ G, where τ_D is the decay time of the field in years and, furthermore, that both $\tau_D = 4\ 10^6$ yr and $9\ 10^6$ yr give a good fit to the observed distributions in T and $\dot{P}$. However, their main conclusion was that if one adopts this luminosity law, pulsars are born with much longer periods (i.e., between 0.09 and 0.25 sec) than was assumed by Lyne, Manchester and Taylor (1985). This so-called late injection of pulsars in the observable population seems to be a natural consequence of the assumed luminosity law and was also proposed by Vivekanand and Narayan (1981).

In order to examine which of the two luminosity laws mentioned sofar (i.e. $L_1 \propto B^2 \propto P\dot{P}$, versus $L_2 \propto P^{-1} \dot{P}^{1/3}$) gives the best fit to the statistical characteristics of pulsars, I have made a comparison of the B vs. P diagrams predicted by both laws (Stollman, 1986a), and I have shown that the law L_1 leads to a B vs. P diagram that more closely seems to resemble the observed diagram than does L_2. The latter law predicts too many pulsars that have both a short period and a low field strength. I assumed in my study that pulsars are born with the same short periods as adopted by Lyne et al. (1985). When this last constraint was dropped and pulsars were assumed to be born with periods between 0.09 and 0.25 seconds, as suggested by Chevalier and Emmering (1986), the

theoretically predicted B vs. P diagram improved somewhat but still showed an excess of pulsars with weak fields. A way to solve this problem was suggested in a recent paper (Stollman, 1986b), where it was shown that the luminosity of pulsars may be proportional to the potential drop across the polar gap in the pulsar model of Ruderman and Sutherland (1975). The potential drop in this model is proportional to B/P^2 below a certain value of this quantity, and more or less constant above that value. Plotting the observed radio luminosities versus B/P^2 it was found that below $B/P^2 = 10^{13}$ Gs^{-2} the luminosity is proportional to B/P^2 and above 10^{13} Gs^{-2} the luminosity is approximately constant. Using this new luminosity law to generate the B vs. P diagram it was found (Stollman, 1986b) that this law can explain the observed distribution of pulsars in this diagram quite well, without the constraint that pulsars should be born with long periods. It was also found that this luminosity law does not generate too many pulsars with weak fields.

In this paper a more detailed statistical study is made of the pulsar population, using the luminosity law found by Stollman (1986b). In section 2 the method used (i.e. a Monte Carlo method) is described and justified. In section 3 the theoretically predicted distributions of P, B, T and $|z|$ are fitted to the observed ones and the appropriate fitting parameters determined. A critical comparison is made with the results of Lyne, Manchester and Taylor (1985) and Chevalier and Emmering (1986). In section 4 the Monte Carlo method is used to examine whether the observed P- and B-distribution of pulsars are influenced by a possible existence of a population of recycled pulsars. In section 5 our conclusions are summarized.

2. The computer model

In this paper the expected radio pulsar population is generated by using a Monte Carlo method. Choosing cylindrical coordinates with the center of the galaxy at the origin and the z-axis perpendicular to the galactic plane, the pulsars are generated by selecting them from a probability distribution $\Pi(\rho_0,\theta_0,z_0,\vec{v},\ln B_0,P_0)$ that can be written as

$$\Pi(\rho_0,\theta_0,z_0,\vec{v},\ln B_0,P_0)d\rho_0 d\theta_0 dz_0 d\vec{v} d\ln B_0 dP_0$$

$$= R(\rho_0,\theta_0)Z(z_0)V(\vec{v})B(\ln B_0)P(P_0)d\rho_0 d\theta_0 dz_0 d\vec{v} d\ln B_0 dP_0 \qquad (1)$$

Here $R(\rho_0,\theta_0)d\rho_0 d\theta_0$ is the probability that a pulsar is born with coordinates ρ_0 and θ_0 in the range (ρ_0,θ_0) and $(\rho_0 + d\rho_0,\ \theta_0 + d\theta_0)$. It will be assumed that R is independent of ρ_0 and θ_0 for $\rho_0 < 15$ kpc and $0 < \theta_0 < 2\pi$ and that R is zero

for $\rho_0 > 15$ kpc. (Here it is assumed that the radius of the galactic disk is approximately 15 kpc). $Z(z_0)dz_0$ is the probability that a pulsar is born with coordinate z_0 in the range $(z_0, z_0 + dz_0)$. In accordance with the work of Gunn and Ostriker (1970), Lyne, Manchester and Taylor (1985) and Chevalier and Emmering (1986) this probability distribution is defined as

$$Z(z_0)dz_0 = \frac{1}{2H} \exp(-|z_0|/H)\ dz_0 \tag{2}$$

where H is the scaleheight of the pulsar progenitors.

In equation (1) $V(\vec{v})d\vec{v}$ is the probability that a pulsar is born with velocity components v_x, v_y and v_z in the range (v_x, v_y, v_z) and $(v_x + dv_x, v_y + dv_y, v_z + dv_z)$ and is given by

$$V(\vec{v})dv_x dv_y dv_z = \frac{1}{\sigma_v^3 (2\pi)^{3/2}} \exp\left[-\frac{v_x^2 + v_y^2 + v_z^2}{2\ \sigma_v^2}\right] dv_x dv_y dv_z \tag{3}$$

The standard deviation σ_v will be set equal to 107 km/s, which is consistent with the distribution of transverse velocities as measured by Lyne et al. (1982) and which is equal to the value used by Lyne, Manchester and Taylor (1985) and also consistent with the velocity distribution applied by Chevalier and Emmering (1986).

$B(\ln B_0)d\ln B_0$ is the probability that a pulsar is born with a value $\ln B_0$ in the range $\ln B_0$ to $\ln B_0 + d\ln B_0$, where B_0 is the dipole magnetic field strength at the pole of the neutron star. Following Gunn and Ostriker (1970) the form of $B(\ln B_0)$ is chosen to be gaussian, i.e:

$$B(\ln B_0)d\ln B_0 = \frac{1}{\sigma_B (2\pi)^{1/2}} \exp\left[-\frac{(\ln B_0 - \langle \ln B_0 \rangle)^2}{2\ \sigma_B^2}\right] d\ \ln B_0 \tag{4}$$

where $\langle \ln B_0 \rangle$ is the average value of $\ln B_0$, with which pulsars are born and σ_B is the standard deviation.

In equation (1) $P(P_0)dP_0$ is the probability that a pulsar is born with rotation period P_0 in the range between P_0 and $P_0 + dP_0$. In this paper it is assumed that this probability is the same for all periods between 1 and 50 milliseconds and zero outside this range, except in those cases indicated.

It is then assumed that in our galaxy once every 100 yrs a pulsar is born (Taylor and Stinebring, 1986). After a certain time t a population of t/100 pulsars is created. For each i^{th} pulsar in this population the position $\vec{r}_i$ with respect to the galactic center is determined by

$$\vec{r}_i = \vec{r}_{0i} + \vec{v}_i t_i \tag{5}$$

where $\vec{r}_{0i}$ is the place of birth and t_i is the age of the pulsar. At this position all the characteristics of the pulsar that are of interest can be calculated. The value of its magnetic field strength $B_i(t_i)$ is given by

$$B_i(t_i) = B_{0i}\exp(-t_i/\tau_D) \tag{6}$$

where B_{0i} is the initial magnetic field strength and τ_D is the decay time of the field. The value of the rotation period, P, and period derivative, $\dot{P}$, are given by (see e.g. Stollman, 1986a)

$$P_i(t_i) = \beta_i\left[1 - \exp(-2t_i/\tau_D) + P_{0i}^2/\beta_i^2\right]^{1/2} \tag{7}$$

$$\dot{P}_i(t_i) = \frac{\beta_i}{\tau_D}\exp(-2t_i/\tau_D)/\left[1 - \exp(-2t_i/\tau_D) + P_{0i}^2/\beta_i^2\right]^{1/2} \tag{8}$$

where β_i is defined as $\beta_i^2 = B_{0i}^2\tau_D/\alpha$ and α by the equation $B_i^2(t_i) = \alpha\, P_i(t_i)\, \dot{P}_i(t_i)$ (cf. Manchester and Taylor, 1977), with the canonical value $\alpha = 1.0\ 10^{39}$ G^2s^{-1}.

The radio luminosity of pulsars is usually defined as $L_{400} = S_{400}\, d^2$, where S_{400} is the mean flux density at 400 MHz in mJy and d is the distance to the pulsar in kpc. It was shown by Stollman (1986b) that L_{400} depends on B/P^2 in the following way

$$L_{400} = 10^{-10.05 \pm 0.84}\left(B/P^2\right)^{0.98 \pm 0.03}\ \text{mJy kpc}^2 \tag{9a}$$

for $B/P^2 < 10^{13}\ Gs^{-2}$ and

$$L_{400} = 10^{2.71 \pm 0.60}\ \text{mJy kpc}^2 \tag{9b}$$

for $B/P^2 > 10^{13}\ Gs^{-2}$.

For the pulsars in the computer-generated population the value of B/P^2 is determined from equations (6) and (7) and then the value of L_{400} is calculated from either eq. (9a) or (9b) depending on the value of B/P^2. From its position $\vec{r}_i$ with respect to the center of the galaxy the distance of the pulsar to the earth can be determined from

$$d = \left|\vec{r}_i(t_i) - \vec{r}_0\right| \tag{10}$$

where $\vec{r}_0$ is the position vector of the sun. It is assumed that the sun is situated in the plane of the galaxy at a distance of 10 kpc from the center. Once the distance d to the pulsar is known, one may evaluate its mean flux

density S_{400} from

$$S_{400} = L_{400} / d^2 \text{ mJy} \qquad (11)$$

In order that the computer-generated population can be compared to the observed one, one has to exclude from both populations those pulsars for which S_{400} is smaller than a certain minimum value S_{min}. This should be done because the pulsar samples that are compared must be complete down to a certain limiting flux value. For the observed population this is not the case because it includes a number of surveys with different limiting fluxes. The value of S_{min} is uncertain. Chevalier and Emmering (1986) have assumed it to be equal to 1 mJy, which is the minimum detected flux listed in the pulsar catalog of Manchester and Taylor (1981). In the main section of this paper the value of S_{min} is set equal to 5 mJy in order to obtain a more complete sample.

It is generally assumed that pulsars do not pulse forever. In the theory of Ruderman and Sutherland (1975), on which the luminosity law used in this paper is based, it is found that the pulsar radiation mechanism stops when the potential difference across the polar gap drops below a certain critical value. This amounts to a minimum value for B/P^2 of $2\ 10^{11}$ Gs^{-2}, which defines the so-called death line in the B vs. P diagram. When the value of B/P^2 drops below this critical value for pulsars in the computer-generated population they are excluded from the sample.

For the thus generated computer-population of radio pulsars it is possible to find the distributions of P, B etc. and to compare these to the observed distributions. Since the age of the galaxy is much larger than the expected lifetime of pulsars it seems likely that the pulsar population has reached a steady state. This requires that the age, t, of the computer-generated population must be chosen larger than a certain value t_{max}, such that for all $t > t_{max}$ the distributions in P, B etc. are constant in time. It is found that for all the distributions calculated in this paper t_{max} is of the order of 5 to 6 times the decay time of the magnetic field.

In order to see whether the computer model, described sofar, does indeed generate the correct pulsar distribution functions (i.e. those predicted analytically), the distribution of expected periods is calculated using first the assumptions of Lyne, Manchester and Taylor (1985) and secondly those of Chevalier and Emmering (1986). The P-distributions thus calculated are then compared to the (semi-) analytical distribution functions obtained by these authors.

To calculate the P-distribution obtained by Lyne, Manchester and Taylor (1985) and Chevalier and Emmering (1986) it is first of all assumed that the sun

is at the center of the galaxy and that the plane in which pulsars are born extends to infinity. In the studies mentioned this assumption had to be made in order to find analytical expressions for the distribution functions. In the case of Lyne, Manchester and Taylor (1985) the following additional assumptions are made:

(1) the pulsars are selected from the same distributions as presented above in eq. (2) with a scaleheight H equal to 100 pc, eq. (3) with σ_v = 107 km/s and eq. (4) with $\langle \ln B_0 \rangle$ = 27.343 and σ_B = 0.69.

(2) the initial periods are selected from a flat distribution with 1 msec < P_0 < 50 msec (i.e. no late injection is assumed).

(3) the decay time of the magnetic field is $\tau_D = 9.1\ 10^6$ yr.

(4) the luminosity is proportional to the square of the magnetic field strength and is given by (see Stollman and van den Heuvel, 1986)

$$L_{400} = 23\ (B/10^{12})^2 \text{ mJy kpc}^2 \qquad (12)$$

(5) the minimum detectable flux, S_{min}, is set equal to 1 mJy.

(6) no death line is assumed.

For the case of Chevalier and Emmering (1986) in addition to the assumption of an infinitely extending galactic plane the following assumptions are made:

(1) the pulsars are selected from the same distribution functions as presented above in eq. (2) with H equal to 225 pc and eq. (3) with σ_v = 107 km/s. (Note that Chevalier and Emmering do not use equations (2) and (3) to calculate the P-distribution, but assume that the pulsars stay in the plane of the galaxy: the values for H and σ_v, used here, are obtained from their $|z|$-distribution.)

(2) the period P_0 is selected from a flat distribution in P_0, with 0.091 sec < P_0 < 0.250 sec.

(3) the magnetic field strength B_0 is selected from a flat distribution in B_0, with $9.4\ 10^{11}$ G < B_0 < $6.3\ 10^{12}$ G.

(4) the decay time of the field τ_D is $4\ 10^6$ yr.

(5) the luminosity of the pulsars is given by the relation found by Proszynski and Przybycien (1985), i.e.

$$L_{400} = 10^{6.94 \pm 0.61}\ (P^{-3}\dot{P})^{0.348 \pm 0.043} \text{ mJy kpc}^2 \qquad (13)$$

(6) the minimum flux density is set equal to 1 mJy,

(7) and also the assumption of no death line is made.

In Fig. 1a the computer-generated log P - distribution obtained with the assumptions of Lyne et al. (1985) is plotted together with the P - distribution

obtained analytically by these authors.

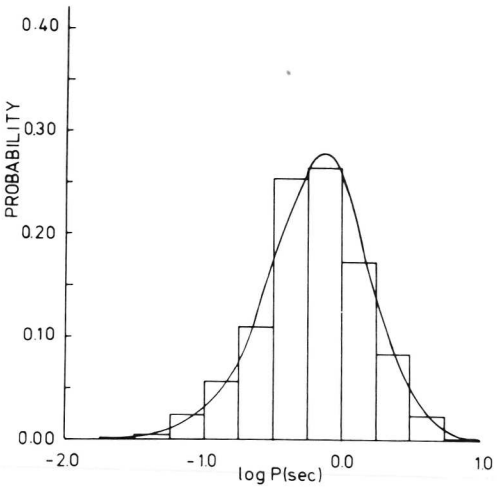

Fig. 1a. The computer generated log P - probability distribution (bars) and the one predicted analytically (curve). In both cases the model of Lyne, Manchester and Taylor (1985) is used.

The figure shows that the log P - distribution generated by the Monte Carlo method does fit the analytical distribution very well. In Fig. 1b the computer generated distribution and the one found by Chevalier and Emmering (1986) are plotted. Considering the fact that the latter one was also calculated numerically both distributions are remarkebly similar to one another.

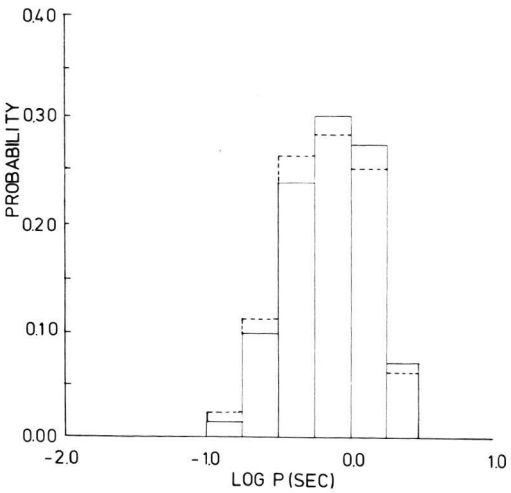

Fig. 1b. The computer generated log P - probability distribution (fully drawn) and the one predicted semi-analytical (dashed). In both cases the model of Chevalier and Emmering (1986) is used.

The above calculation shows that the Monte Carlo method described above does indeed generate the analytically predicted distribution functions. In the following section this method will be used to fit the theoretically predicted P -, B -, T- and $|z|$ - distributions as well as possible to the observed ones, and to thus obtain the best values for τ_D, $\langle \ln B_0 \rangle$, σ_B and H, assuming that the luminosity of pulsars is given by eq. (9) and that they are born with short periods, typically between 1 and 50 msec.

3. Fitting the observed distributions

The observed probability distributions for the period P, magnetic field strength B, characteristic age T and height $|z|$ are found from the catalog of Manchester and Taylor (1981), where the pulsars with $S_{400} < 5$ mJy are excluded. The period distribution is corrected for the selection against short periods as suggested by Lyne, Manchester and Taylor (1985), using formula A3 in the appendix A of that paper. Then the expected distributions are calculated using the method described in section 2 in combination with the luminosity law found by Stollman (1986b) as given by eq. (9). The variables that are to be determined are τ_D (i.e. the decay time of the field), $\langle \ln B_0 \rangle$ (i.e the average magnetic field strength with which pulsar are born), σ_B and the scaleheigth H.

In order to determine the quality of the fits the following quantity is used

$$\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}} = \sum_{i=1}^{k} \frac{(x_i - N f_i)^2}{N f_i} \qquad (14)$$

where x_i is the observed number of pulsars in the i^{th} log P-, log B-, log T- or log $|z|$ - bin, f_i is the expected probability of finding pulsars in the i^{th} bin and N is the total number of observed pulsars. The quality of the fits is determined by the probability $\text{Prob}(\chi^2, \nu)$ of finding a $\chi^2 > \chi^2_{fit}$, where ν is the number of degrees of freedom. This probability is given by

$$\text{Prob}(\chi^2_{fit}, \nu) = \frac{1}{2^{\nu/2}\Gamma(\nu/2)} \int_{\chi^2_{fit}}^{\infty} (\chi^2)^{\nu/2} \exp(-\chi^2/2)\, d\chi^2 \qquad (15)$$

In general the difference between the expected distribution and the observed distribution is called "probably significant" for Prob = 5 %, "significant" for Prob = 1 % and "very significant" for Prob = 0.1 % (see e.g. Carnahan et al., 1969).

In the fitting procedure not all of the parameters τ_D, $\langle \ln B_0 \rangle$, σ_B and H are used as independent fitting parameters. Since pulsars are observed to be significantly confined to the galactic plane the distributions in P, B and T are not much influenced by the value of H, and this value will initially be set

equal to 175 pc. Also the value of σ_B will be fixed to 0.69 since this is approximately the width of the observed distribution of lnB - (this amounts to $\sigma'_B = 0.3$ for the distribution in $^{10}\log B$, which will be plotted in this paper) - and the value found by Lyne, Manchester and Taylor (1985). With these parameters fixed, τ_D and $\langle \ln B_0 \rangle$ will be determined by fitting the expected distribution functions of log P and log B to the observed ones and maximising $\mathrm{Prob}(\chi^2_{fit}, \nu)$. With the value of τ_D and $\langle \ln B_0 \rangle$ thus found the distribution functions of log T and $|z|$ are determined and compared to the observed ones.

In Figs. 2 and 3 the expected and observed probability distributions are plotted for log P and log B. The best values found for the fitting parameters are: $\tau_D = 5.3\ 10^6$ yr and $\langle \ln B_0 \rangle = 28.8$ (or $\langle \log B_0 \rangle = 12.5$). In the case of the log P - distribution the value for χ^2 was calculated using eq. (14). The bins were chosen as plotted in Fig. 2 except that the ranges log P < -0.75 and log P > 0.25 were considered as one bin each, to improve the statistics (i.e. the χ^2 determined in eq. (14) is distributed as given by the probability distribution under the integral in eq. (15), when for each i^{th} bin, Nf_i in eq. (14) is larger than approximately 10). The number of degrees of freedom was therefore 4 and the fit of Fig. 2 has the value $\chi^2 = 4.8$ leading to Prob(4.8,4) = 32 %, which implies a very good fit.

In the case of the log B - distribution the bins were also chosen as plotted in Fig. 3 except that also here the edges of the distribution are rebinned such that the ranges log B < 11.25 and log B > 12.75 were considered 1 bin each. The fit presented in Fig. 3 has the value $\chi^2 = 9.5$ which implies a probability of 15%, which is also very acceptable.

For all other values of τ_D and $\langle \ln B_0 \rangle$ than presented above the fits to the observed distributions of log P and log B were worse in the sense that the average value of $\mathrm{Prob}(\chi^2_{fit}, \nu) = (\mathrm{Prob}_{\log P} + \mathrm{Prob}_{\log B})/2$ was a maximum for these values. It was for example noticed that for $\tau_D = 5.5\ 10^6$ yr the fit to the log P distribution was improved but that it made the fit to the log B distribution worse.

To obtain the ranges of acceptable values of τ_D and $\langle \ln B_0 \rangle$ the minimum value of $\mathrm{Prob}(\chi^2_{fit}, \nu)$ was set equal to 10% leading to

$$4.5\ 10^6 \text{ yr} < \tau_D < 6.0\ 10^6 \text{ yr} \qquad (16)$$

and

$$28.65 < \langle \ln B_0 \rangle < 28.90$$

or

$$12.44 < \langle \log B_0 \rangle < 12.55 \qquad (17)$$

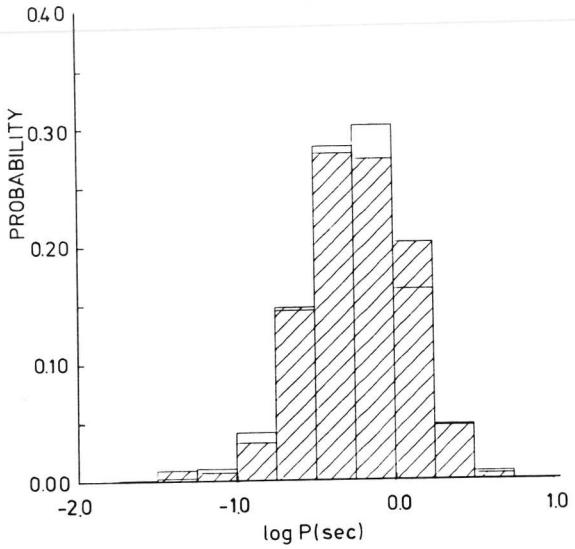

Fig. 2. The probability distribution of log P, generated with the model described in section 2, and the observed one (hatched). χ^2 = 4.8 and Prob(χ^2_{fit},ν) = 32%.

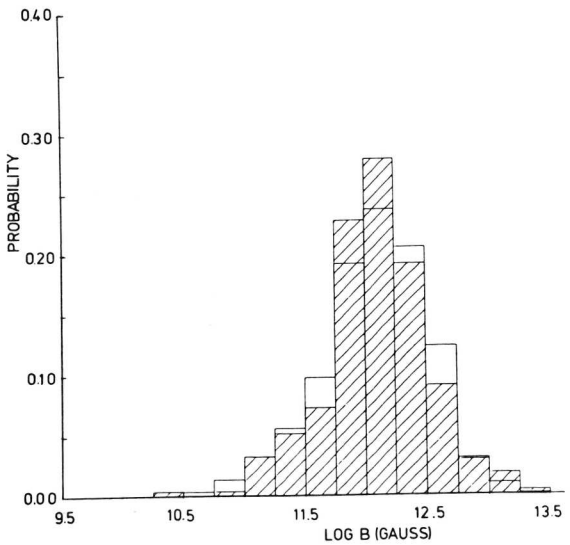

Fig. 3. The probability distribution of log B, generated with the model described in section 2, and the observed one (hatched). χ^2 = 9.5 and Prob(χ^2_{fit},ν) = 15%.

Using the values τ_D = 5.3 10^6 yr and $\langle \ln B_0 \rangle$ = 28.8 the values of σ_B and H were changed and again the ranges determined by setting Prob = 10 %. The ranges found are

$$0.6 < \sigma_B < 0.8 \tag{18}$$

and

$$100 \text{ pc} < H < 225 \text{ pc} \tag{19}$$

In Fig. 4 the observed and expected distributions of log T, where T is the characteristic age $P/2\dot{P}$, are plotted using τ_D = 5.3 10^6 yr, $\langle \ln B_0 \rangle$ = 28.8, σ_B = 0.69 and H = 175 pc. The value of χ^2 is calculated using the binning presented in Fig. 4, where the ranges log T < 5 and log T > 8 are considered as one bin each. The value of χ^2 found is 20, leading to Prob = 1 %. Therefore, the quality of this fit is not very good. However, exploring the parameter space defined by equations (16) to (19) did not lead to a better fit. Since both B and T are simple functions of P and $\dot{P}$, it is not clear why the fits for log B and for log P are so much better than the one for log T. It should also be noticed that the fits for log T presented by Chevalier and Emmering (1986) were considered as acceptable by the authors, while the value for the probability is less than 0.1 % and the fit is therefore even worse than the one presented here.

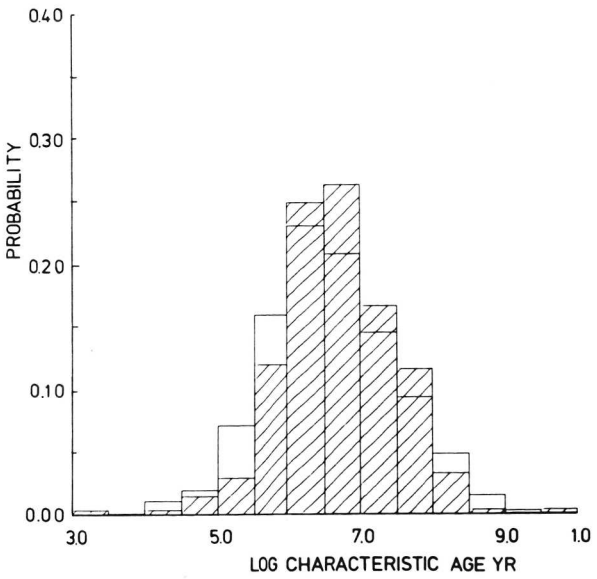

Fig. 4. The probability distribution of log T, generated with the model described in section 2, and the observed one (hatched). χ^2 = 20 and Prob(χ^2_{fit},ν) = 1%.

The distribution in $|z|$ generated by the Monte Carlo method, using τ_D = 5.3 10^6 yr, $\langle \ln B_0 \rangle$ = 28.8, σ_B = 0.69 and H = 175 pc, is plotted in Fig. 5 and is compared to the observed one. The latter one is affected by the uncertainties in the distances and in Fig. 5 for each $|z|$-bin the uncertainties are indicated are given by Lyne, Manchester and Taylor (1985). The last bin in Fig. 5 contains all the pulsars with $|z| \geq 1.1$ kpc. The value of χ^2 is determined using the bins in Fig. 5 , where the range $|z| \geq 0.9$ is considered as one bin. The value found is 29 leading to Prob = 0.1 %. The fit therefore is not very good. However, considering the uncertainties in the observed $|z|$-distribution and comparing the fit to those found by Lyne, Manchester and Taylor (1985) and Chevalier and Emmering (1986) - in both cases Prob is much less than 0.1 % - the fit seems quite acceptable. Changing the value of H within the range given by (19) did not improve the fit.

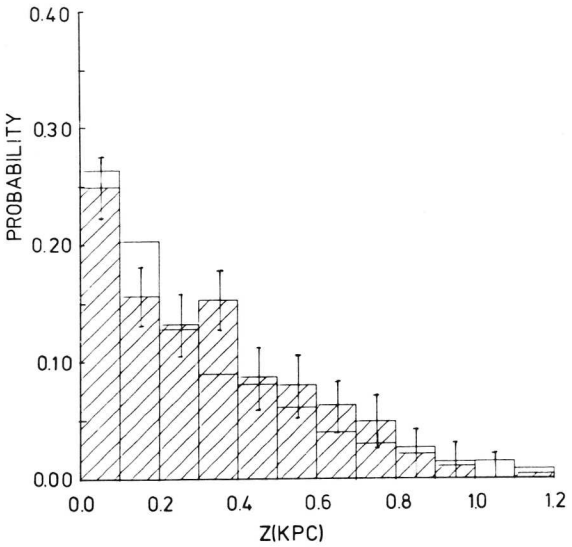

Fig. 5. The probability distribution of $|z|$, generated with the model described in section 2, and the observed one (hatched). χ^2 = 29 and Prob(χ^2_{fit}, ν) = 0.1%.

It is interesting to see whether the assumptions made by Lyne, Manchester and Taylor (1985) and Chevalier and Emmering (1986) and the values which these authors use for the different parameters in their fits (see section 2) lead to better fits to the observed distributions of P and B than those presented above. The calculations will be the same as the ones used to generate the distributions in Figs. 1a and 1b, except that now the minimum value for S_{400} is set equal to 5

mJy, as was done for the observed pulsars and in the calculations above. Furthermore, the finite dimensions of our galaxy and the position of the sun away from the center are taken into account. In order to calculate the values of χ^2 for the various fits the same binning is assumed as in the above calculations.

In Figs. 6 and 7 the computer generated distributions are plotted and compared to the observed ones, using the assumptions of Lyne, Manchester and Taylor (1985) as well as the values for τ_D, σ_B, $\langle \ln B_0 \rangle$ and H derived by these authors. The fits look quite acceptable. However, the values of χ^2 are much larger than for the fits in Figs. 2 and 3. For the log P - distribution this value is 14.5 and for the log B - distribution 66.5. In both cases the value of Prob(χ^2,ν) is therefore unacceptable, even considering the fact that the number of the degrees of freedom is larger than in the case of Figs. 2 and 3. Maybe the fits could be improved by changing the appropriate parameters. However, this was not the object of the calculation, for the idea was to see whether the model plus its parameters, as derived by Lyne, Manchester and Taylor (1985) does give better fits to the observations, than the model presented in this paper. The conclusion is that it does not.

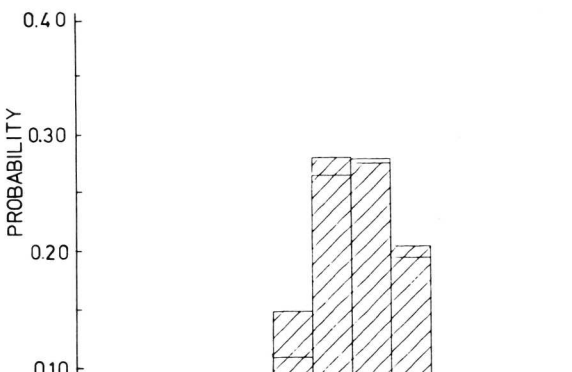

Fig. 6. The probability distribution of log P, generated by using the model of Lyne, Manchester and Taylor (1985) where S_{min} is set to 5 mJy and where the finite dimension of the galaxy is taken into account, together with the observed distribution (hatched). χ^2 = 14.5

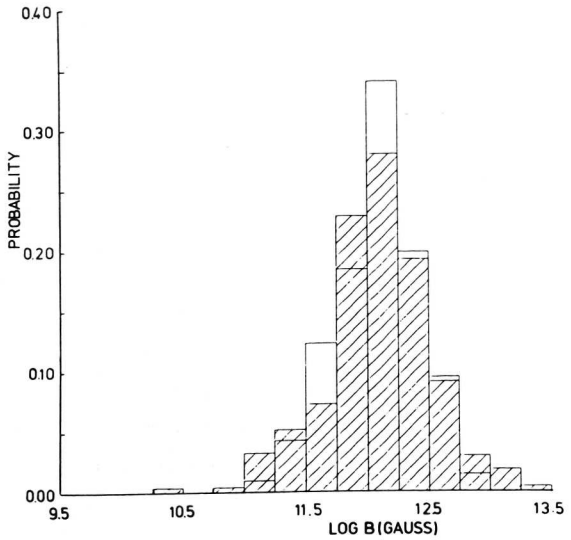

Fig. 7. The probability distribution of log B, generated, using the model of Lyne, Manchester and Taylor (1985) with the same assumptions as in Fig. 6, and the observed distribution (hatched). $\chi^2 = 66.5$.

In Figs. 8 and 9 the computer-generated distributions of log P and log B are plotted together with the observed ones, using the model of Chevalier and Emmering (1986), for which the most important ingredient is the late injection of pulsars. The first thing that is evident is that with respect to Fig. 1b the period distribution has shifted to somewhat longer periods. In the luminosity model used by Chevalier and Emmering (1986) the pulsars with the highest luminosity have on average short periods. The highest luminosities expected are of the order of 7000 mJy kpc^2. With a flux limit of 5 mJy this implies a maximum distance of about 40 kpc to the pulsar, which is larger than the dimensions of the galaxy. In the calculation of Fig. 8 the finite dimension of our galaxy are taken into account, while in Fig. 1b this was not the case. This implies that the relative number of high luminosity pulsars decreases somewhat, which, therefore, leads to a small shift in the period distribution, as observed. This effect is not seen in Fig. 6, since in the $L \propto B^2$ model the highest expected luminosities are of the order of 750 mJy kpc^2, leading to a maximum distance of about 12 kpc, which falls within the dimensions of our galaxy.

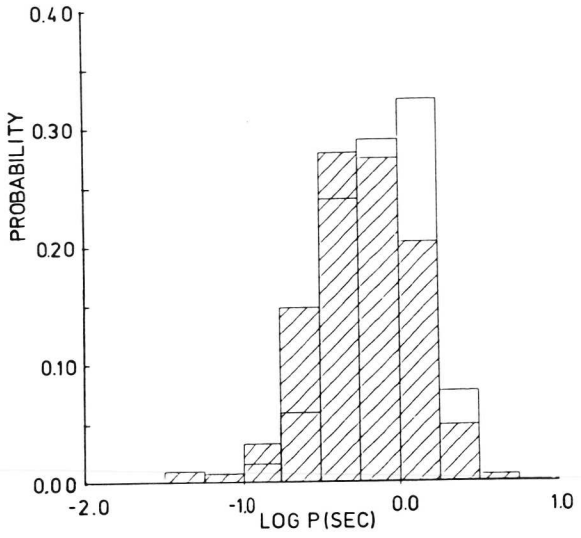

Fig. 8. The same as Fig. 6 but now using the model of Chevalier and Emmering (1986). χ^2 = 98.1.

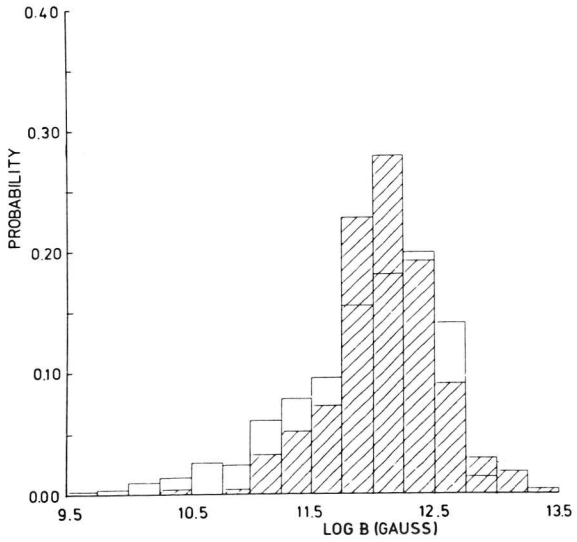

Fig. 9. The same as Fig. 7 but now using the model of Chevalier and Emmering (1986). χ^2 = 80.6.

Another important thing to be noticed is the distribution of log B, expected from the model of Chevalier and Emmering (1986). Here it is clearly seen that the model predicts too many pulsars with weak fields, as pointed out by Stollman (1986a,b).

It is clear from Figs. 8 and 9 that the model of Chevalier and Emmering (1986) gives very poor fits to the observations especially if the finite dimensions of the galaxy are taken into account. The values found for χ^2 are 98.1 and 80.6 for the fits to log P and log B respectively. Again the conclusion is that the model of Chevalier and Emmering (1986) does not lead to better fits to the observed distributions in log P and log B, than the model presented in this paper. In fact the conclusions drawn by Stollman (1986a,b) from the expected B vs. P diagram are confirmed by the present calculations.

The conclusion from the calculations presented sofar is that the evolutionary model, described in section 2, leads to period and magnetic field strength distributions that seem to fit the observations quite well. Only the fits to the observed distributions in characteristic age and height above the galactic plane are not excellent, but these fits are still acceptable when compared to fits found in the earlier studies by Lyne, Manchester and Taylor (1985) and Chevalier and Emmering (1986). The important differences with the former analysis are that the luminosity law used (see eq. (9)) is not an assumption but is consistent with the observed radio luminosities of pulsars and, furthermore, that pulsars are born with considerably stronger magnetic fields (Lyne, Manchester and Taylor find $\langle B_0 \rangle = 7.5\ 10^{11}$ G and the model presented here gives $\langle B_0 \rangle = 3.2\ 10^{12}$ G), and that the magnetic field strength decays on a somewhat shorter timescale ($5.3\ 10^6$ yr versus $9.1\ 10^6$ yr).

The differences with the model of Chevalier and Emmering (1986) are the inclusion of the finite dimensions of the galaxy and most importantly the fact that pulsars are not injected into the observable population with longer periods than generally accepted. It is found that pulsars are born with short periods, typically between 1 and 50 msec.

4. Recycled pulsars?

After the discovery of several radio pulsars in binary systems and the single 1.6 msec. pulsar, it is now generally believed that part of the observed population consists of so-called recycled pulsars. These are the product of binary evolution in which mass was transferred from a companion star to the neutron star, thereby spinning it up. At the end of this mass-transfer phase the neutron star may be observable as a radio pulsar in a binary or as a single pulsar (see for a review van den Heuvel, 1984, 1985). The exact value of the

spin period and the magnetic field strength of the neutron star, after this mass-transfer phase, depend on the parameters of the binary system at the onset of the accretion (see for details de Kool and van Paradijs, 1986). But it is generally assumed that, depending on their magnetic field strength and the mass-accretion rate, the neutron stars are spun up to a minimum period P_{min}, given by (Ghosh and Lamb, 1979; Henrichs, 1983)

$$P_{min} = (2.4 \text{ msec.}) (B_9)^{6/7} M^{-5/7} (\frac{\dot{M}}{\dot{M}_{Edd}})^{-3/7} R_6^{15/7} \qquad (20)$$

Here B_9, M and R_6 are the surface dipole magnetic field strenth of the neutron star in units of of 10^9 G, its mass in solar masses, and its radius in units of 10^6 cm, respectively. $\dot{M}_{Edd}$ is the maximum possible "Eddington-limit" accretion rate. Equation (20) shows that for a "standard" neutron star with M = 1, R_6 = 1, the shortest possible spin-period P_{min} that can be reached - for $\dot{M} = \dot{M}_{Edd}$ - depends only on the value of B_9, as $P_{min} \propto B_9^{6/7}$. This relation defines a line in the B vs. P diagram above which no recycled pulsars are expected. This line is the so-called spin-up line. Furthermore, as explained in section 2, no pulsars are expected under the so-called death line (i.e. $B/P^2 = 2.0\ 10^{11}$ Gs^{-2}). Therefore, one expects the recycled pulsars to be situated in the wedge-shaped region between these two lines. Indeed, all the known recycled pulsars lie in this region of the B vs. P diagram (cf. Taylor and Stinebring, 1986). Above the spin-up line one expects only so-called "normal" pulsars (that is: non-recycled pulsars). The model presented in section 2 is only valid for these "normal" pulsars. However, the results in section 3 were obtained from fits of this model to the observed pulsar population, which consists of both recycled and "normal" pulsars.

To see whether the expected population of recycled pulsars, situated below the spin-up line, might affect the observed distributions in period and magnetic field strength, the results of section 3 are used to create a sample of pulsars for which the period, P, and field strength ,B, lie above the spin-up line. This sample is then compared to the observed sample of "normal" pulsars above the spin-up line. Using $\tau_D = 5.3\ 10^6$ yr, $\langle \ln B_0 \rangle = 28.8$, $\sigma_B = 0.69$ and H = 175 pc, the model distributions of log P and log B and the observed ones are plotted in Figs. 10 and 11. The χ^2 values are respectively 1.0 and 8.4, leading to Prob = 98 % for the log P - fit and Prob = 40 % for the log B -fit. Therefore these fits are better than those for the complete population, as presented in Figs. 2 and 3.

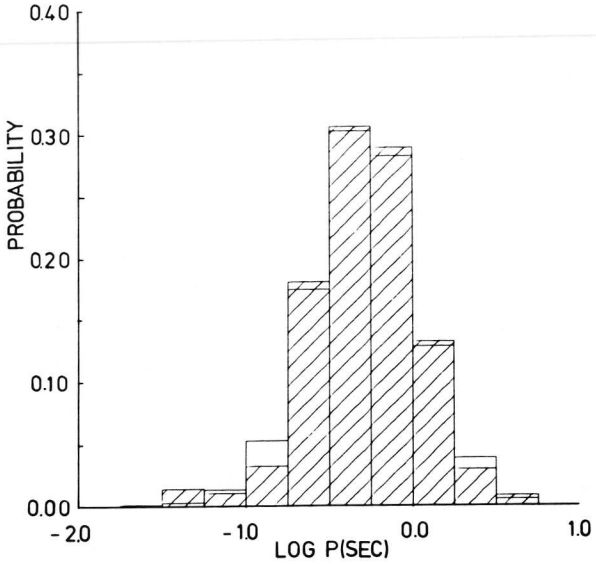

Fig. 10. The observed (hatched) and expected probability distribution of log P, for pulsars above the spin-up line. For the expected distribution the model of section 2 is used with τ_D = 5.3 10^6 yr, $\langle \ln B_0 \rangle$ = 28.8, σ_B = 0.69 and H = 175 pc. χ^2 = 1.0 and Prob(χ^2_{fit},ν) = 98%.

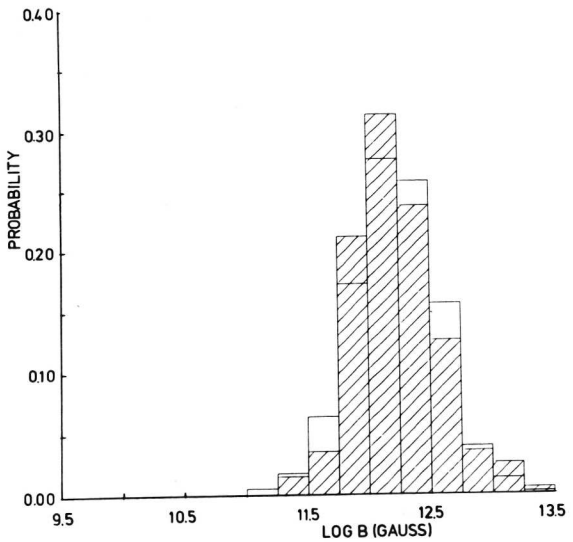

Fig. 11. The same as Fig. 10, but now for the log B distribution. χ^2 = 8.4 and Prob(χ^2_{fit},ν) = 40%.

However, it has to be remembered that the total number of pulsars considered in this case is somewhat smaller, which may lead to somewhat smaller values of χ^2 and it is therefore not clear whether this result is significant enough to conclude that the model fits to the complete population are influenced by the presence of a class of recycled pulsars. One may, however, conclude that if there is such a population of recycled pulsars it does not comprise a large percentage of the total population. And it may also be concluded that the parameters τ_D, $\langle \ln B_0 \rangle$, σ_B and H, derived in section 3, do describe the origin and evolution of radio pulsars quite well.

With the above in mind one may now generate the expected population of "normal" pulsars below the spin-up line and compare it to the observed one. The expected and observed distributions of log P and log B for pulsars below the spin-up line are plotted in Figs. 12 and 13. The χ^2 values are respectively 17.2 and 7.0.

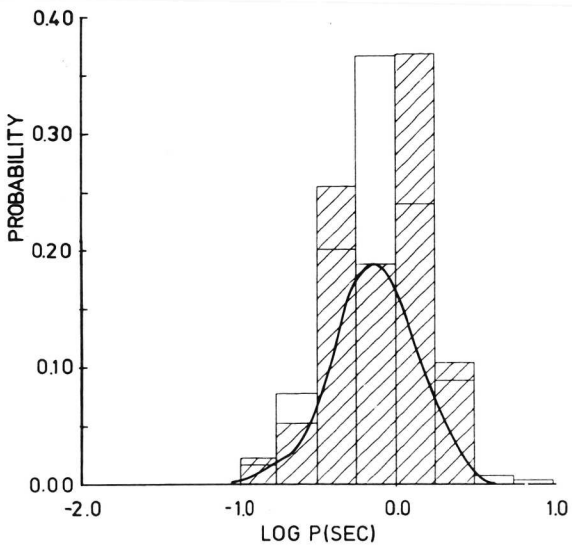

Fig. 12. The same as Fig. 10 but now for pulsars below the spin-up line. χ^2 = 17.2 and Prob(χ^2_{fit},ν) = 0.9%. The drawn curve represents the expected distribution rescaled such that the ratio of the areas of the observed and expected distributions correspond to the ratio of the number of observed and expected pulsars (see text).

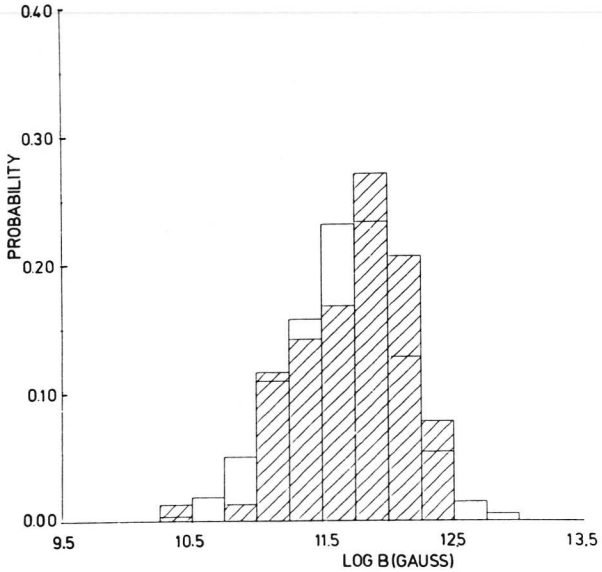

Fig. 13. The same as Fig. 11 but now for pulsars below the spin-up line. χ^2 = 7.0 and Prob(χ^2_{fit},ν) = 32%.

For Fig. 13 the value of χ^2 was calculated with a somewhat different binning than used previously; i.e., the ranges with log B < 11.25 and log B > 12.25 were considered as one bin each. Since the parameters τ_D, $\langle \ln B_0 \rangle$, σ_B and H were fixed and not used as free parameters the number of degrees of freedom for each plot is 6. Together with the calculated values of χ^2 this leads to Prob(17.2,6) = 0.9% for the log P - fit and Prob(7.0,6) = 32 % for the log B - fit. Therefore, the model fit to the observed distribution of log B is acceptable while the one for log P is clearly not. The latter is mainly due to the double peaked character of the observed log P distribution for pulsars below the spin-up line. The small value of the probability Prob(χ^2,ν) (i.e. 0.9%) also implies that it is very unlikely to find this double-peaked distribution if the real underlying distribution is given by the non-hatched one in Fig. 12.

From the above one may, therefore, conclude that the observed double peaked distribution in periods below the spin-up line is probably real and likely to be due to the presence of recycled pulsars in the observed sample. One way to make a rough estimate of the expected percentage of these recycled pulsars is the following. If one makes the assumption that the left "hump" in Fig. 12 is due to recycled pulsars then this amounts to approximately 35 % (allowing for some

overlap between the two "humps"). Since the number of pulsars in the sample below the spin-up line is 78 and in the total sample is 277, 10% of all observed pulsars could be recycled. However, it is clear from Fig. 12 that the maximum of the expected distribution is somewhat shifted to the left in comparison with the right-hand maximum in the observed distribution, which one would not expect if the left-hand peak is only due to recycled pulsars and the right-hand one only to normal pulsars. Therefore, it is probably better to compare the expected number of pulsars below the spin-up line with the number observed (i.e., 78). One may find the number of expected pulsars by using a scaling factor that is determined by the ratio of observed over expected pulsars above the spin-up line. The number of expected pulsars below the spin-up line is then $\sim$ 40 and; therefore, 50% of all pulsars below the spin-up line may be recycled, leading to $\sim$ 15% of all single pulsars. (Notice that all binary pulsars were left out in the analysis in this paper). In Fig. 12 the expected distribution, rescaled to $\sim$ 50% of the observed one, is also drawn.

These rough estimates seem to be consistent with the findings of Stollman and van den Heuvel (1986), where it was found that the observed correlation between the transverse velocities and the magnetic field strengths of pulsars, found by Anderson and Lyne (1983) and Cordes (1986), could be understood if 10 to 20 percent of the pulsars is recycled.

5. **Conclusions**

In this paper it is shown that the observed distributions in P, B, T and $|z|$ of radio pulsars can be understood if they are born:

(1) close to the galactic plane, having a typical scaleheight of 175 pc, and with a maxwellian velocity distribution, having a standard deviation of 107 km/s,
(2) with a gaussian distribution in ln B_0, centered around $B_0 = 3.2\ 10^{12}$ G and with a standard deviation of 0.69,
(3) with short periods, typically in the range of 1 to 50 milliseconds,

and if it is, furthermore, assumed that

(4) the magnetic field decays on a timescale of $5.3\ 10^6$ yr,
(5) the radio luminosity is proportional to B/P^2 for $B/P^2 \leq 10^{13}$ Gs^{-2}, and constant above that value.

The most important difference with the work of Lyne, Manchster and Taylor (1985) is that the luminosity law used in the present analysis is consistent

with the observations. Furthermore it is found that the average initial magnetic field is a factor 4 stronger and the decay time a factor 1.7 smaller.

. With regards to the analysis of Chevalier and Emmering (1986) it is shown that the distributions in initial periods and magnetic field strengths used by these authors, combined with the luminosity law of Proszynski and Przybycien (1985), lead to expected distributions in log P and log B that are different from the observed ones. For the period distribution this is probably due to the fact that in their (semi-) analytical calculations these authors assumed the galactic disk to be infinitely extended. For the log B-distribution the difference is probably due to the luminosity law used, which confirms the conclusions of Stollman (1986b), based on the comparison of the expected and observed B vs. P diagrams. Therefore, the most important differences between the results of the analysis presented in this paper and the work of Chevalier and Emmering (1986) are that firstly it is found that pulsars are not born with long periods but typically have initial periods in the range 1 to 50 msec., and secondly that the luminosity law which is used in this paper is consistent with the observations but does not predict an excess of pulsars with short periods or weak fields, as is predicted by the luminosity law used by Chevalier and Emmering (1986).

In general, an important difference between the model presented in this paper and the work done sofar is that the Monte Carlo method, used here, allows for a better modelling of the luminosity law, since the large statistical deviations from the general laws can be taken into account. The latter is not possible in an analytical calculation. The same holds true for the finite dimensions of the galaxy. These are probably not so important in the analysis of Lyne, Manchester and Taylor (1985) but probably do affect the results found by Chevalier and Emmering (1986).

It is, furthermore, shown that there is some evidence for a small population of recycled pulsars of order 10 to 20% of the total (single) pulsar population. These are believed to have had a different evolutionary history and cannot be treated by the model presented in this paper.

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