

## UvA-DARE (Digital Academic Repository)

### Hurwitz numbers, moduli of curves, topological recursion, Givental's theory and their relations

Spitz, L.

**Publication date**  
2014

[Link to publication](#)

#### Citation for published version (APA):

Spitz, L. (2014). *Hurwitz numbers, moduli of curves, topological recursion, Givental's theory and their relations*. [Thesis, fully internal, Universiteit van Amsterdam].

#### General rights

It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

#### Disclaimer/Complaints regulations

If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: <https://uba.uva.nl/en/contact>, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.

## Bibliography

- [1] The On-Line Encyclopedia of Integer Sequences,  
<http://oeis.org/A008955> and <http://oeis.org/A008956>.
- [2] D. Abramovich, T. J. Jarvis. *Moduli of twisted spin curves* – Proc. Amer. Math. Soc. **131** (2003), no. 3, 685–699.
- [3] M. Aganagic, R. Dijkgraaf, A. Kleemann, M. Mariño, and C. Vafa, *Topological Strings and Integrable Hierarchies*, Commun. Math. Phys. **261**, 451–516 (2006).
- [4] A. Alexandrov, A. Mironov, A. Morozov, *Solving Virasoro Constraints in Matrix Models*, Fortsch.Phys. **53**, 512-521 (2005).
- [5] A. Alexandrov, A. Mironov, A. Morozov, *M-Theory of Matrix Models*, arXiv:hep-th/0605171.
- [6] A. Alexandrov, A. Mironov, A. Morozov, *Instantons and Merons in Matrix Models*, Physica D235:126-167,(2007).
- [7] A. Alexandrov, A. Mironov, A. Morozov, S. Natanzon, *Integrability of Hurwitz Partition Functions. I. Summary*, arXiv:1103.4100v1.
- [8] G. Borot, B. Eynard, M. Mulase and B. Safnuk, *Hurwitz numbers, matrix models and topological recursion*, Journal of Geometry and Physics **61**, 522–540 (2011).
- [9] V. Bouchard, D. Hernandez Serano, X. Liu, and M. Mulase, *Mirror symmetry for orbifold Hurwitz numbers*, arXiv:1301.4871 [math.AG].
- [10] V. Bouchard and M. Mariño, *Hurwitz numbers, matrix models and enumerative geometry*, in “From Hodge theory to integrability and TQFT  $tt^*$ -geometry,” Proc. Symposia Pure Math. **78**, 263–283 (2008).
- [11] S. Brassesco, M. A. Méndez, *The asymptotic expansion for the factorial and Lagrange inversion formula*, arXiv:1002.3894.
- [12] L. Caporaso, C. Casagrande, M. Cornalba. *Moduli of roots of line bundles on curves*. – Trans. Amer. Math. Soc. **359** (2007), no. 8, 3733–3768.
- [13] R. Cavalieri, P. Johnson, H. Markwig, *Tropical Hurwitz Numbers*, J. Algebr. Comb. **32** (2010), no. 2, 241-265.
- [14] R. Cavalieri, P. Johnson, H. Markwig, *Chamber Structure of Double Hurwitz numbers*, arXiv:1003.1805v1

## 8.4. BIBLIOGRAPHY

- [15] L. Chekhov, B. Eynard. *Matrix eigenvalue model: Feynman graph technique for all genera.* – J. High Energy Phys. **2006**, no. 12, 026, 29 pp.
- [16] A. Chiodo. *Stable twisted curves and their r-spin structures.* – Ann. Inst. Fourier (Grenoble) **58** (2008), no. 5, 1635–1689,
- [17] A. Chiodo, *Towards an enumerative geometry of the moduli space of twisted curves and r-th roots,* Compos. Math. **144** (2008), no. 6, 1461–1496.
- [18] A. Chiodo. *Witten’s top Chern class via K-theory.* – J. Algebraic Geom. **15** (2006), no. 4, 681–707.
- [19] A. Chiodo, Y. Ruan. *Landau-Ginzburg/Calabi-Yau correspondence for quintic three-folds via symplectic transformations* – Invent. Math. **182** (2010), no. 1, 117–165.
- [20] A. Chiodo, D. Zvonkine. *Twisted Gromov-Witten r-spin potential and Givental’s quantization.* – Adv. Theor. Math. Phys. **13** (2009), no. 5, 1335–1369.
- [21] R.M. Corless, G.H. Gonnet, D.E.G. Hare, D.J. Jeffrey and D.E. Knuth, *On the Lambert W-function,* Adv. Computational Math. **5**, 329–359 (1996).
- [22] R. Dijkgraaf, *Intersection Theory, Integrable Hierarchies and Topological Field Theory, New symmetry principles in quantum field theory* (Cargése, 1991), 95–158, NATO Adv. Sci. Inst. Ser. B Phys., 295, Plenum, New York, (1992).
- [23] R. Dijkgraaf, L. Hollands, and P. Sulkowski, *Quantum curves and  $\mathcal{D}$ -modules,* Journal of High Energy Physics **0810.4157**, 1–58 (2009).
- [24] R. Dijkgraaf, L. Hollands P. Sulkowski, and C. Vafa, *Supersymmetric gauge theories, intersecting branes and free Fermions,* Journal of High Energy Physics **0802.106**, (2008).
- [25] R. Dijkgraaf and C. Vafa, *Two Dimensional Kodaira-Spencer Theory and Three Dimensional Chern-Simons Gravity,* arXiv:0711.1932 [hep-th].
- [26] N. Do, O. Leigh, and P. Norbury, *Orbifold Hurwitz numbers and Eynard-Orantin invariants,* arXiv:1212.6850.
- [27] B. Dubrovin, *Geometry of 2D topological field theories, Integrable systems and quantum groups* (Montecatini Terme, 1993), 120348, Lecture Notes in Math., 1620, Springer, Berlin, (1996).
- [28] B. Dubrovin, *Painlevé’ transcedents and two-dimensional topological field theory,* arXiv:math/9803107
- [29] B. Dubrovin, Y. Zhang, *Normal forms of hierarchies of integrable PDEs, Frobenius manifolds and Gromov-Witten invariants,* a new 2005 version of arXiv:math/0108160v1.
- [30] O. Dumitrescu, M. Mulase, A. Sorkin and B. Safnuk, *The spectral curve of the Eynard-Orantin recursion via the Laplace transform,* arXiv:1202.1159 [math.AG].
- [31] Ekedahl, S. K. Lando, M. Shapiro, A. Vainshtein. *Hurwitz numbers and intersections on moduli spaces of curves.* – Invent. Math., **146** (2001), 297–327,
- [32] B. Eynard, *Recursion between Mumford volumes of moduli spaces,* arXiv:0706.4403.

- [33] B. Eynard, *Intersection numbers of spectral curves*, arXiv:1104.0176 [math-ph].
- [34] B. Eynard, *Invariants of spectral curves and intersection theory of moduli spaces of complex curves*, arXiv:1110.2949.
- [35] B. Eynard, M. Mulase and B. Safnuk, *The Laplace transform of the cut-and-join equation and the Bouchard-Mariño conjecture on Hurwitz numbers*, Publications of the Research Institute for Mathematical Sciences **47**, 629–670 (2011).
- [36] B. Eynard and N. Orantin, *Invariants of algebraic curves and topological expansion*, Communications in Number Theory and Physics **1**, 347–452 (2007).
- [37] B. Eynard, N. Orantin, *Weil-Petersson volume of moduli spaces, Mirzakhani’s recursion and matrix models*, arXiv:0705.3600.
- [38] B. Eynard, N. Orantin, *Algebraic methods in random matrices and enumerative geometry*, arXiv:0811.3531
- [39] B. Eynard, N. Orantin, *Computation of open Gromov-Witten invariants for toric Calabi-Yau 3-folds by topological recursion, a proof of the BKMP conjecture*, arXiv:1205.1103.
- [40] C. Faber, R. Pandharipande, *Hodge integrals and Gromov-Witten theory*, Invent. Math. **139**, 173–199 (2000).
- [41] B. Fantechi, R. Pandharipande, *Stable maps and branch divisors*, Compositio Mathematica, **130** (2002), p. 345–364.
- [42] C. Faber, R. Pandharipande, *Hodge integrals, partition matrices, and the  $\lambda_g$  conjecture*, Ann. of Math. **157**, 97–124 (2003).
- [43] C. Faber, R. Pandharipande, *Relative maps and tautological classes*. J. Eur. Math. Soc. (JEMS) **7** (2005), no. 1, 13–49.
- [44] C. Faber, S. Shadrin, D. Zvonkine, *Tautological relations and the r-spin Witten conjecture*, arXiv:math/0612510.
- [45] A. Givental, *Semisimple Frobenius structures at higher genus*, IMRN (2001), no. 23, 1265–1286.
- [46] A. Givental, *Gromov-Witten invariants and quantization of quadratic hamiltonians*, Mosc. Math. J. **1** (2001), no. 4, 551–568.
- [47] A. Givental, *Symplectic geometry of Frobenius structures*, In “Frobenius manifolds”, 91–112, Aspects Math., E36, Vieweg, Wiesbaden, 2004.
- [48] I.P. Goulden and D.M. Jackson, *The Number of Ramified Coverings of the Sphere by the Double Torus, and a General Form for Higher Genera*, J. Comb. Theory, Ser. A **88** (1999), no. 2, 259–275.
- [49] I. P. Goulden, D. M. Jackson, *Transitive factorisations into transpositions and holomorphic mappings on the sphere*, Proc. Amer. Math. Soc. **125** (1997), no. 1, 51–60.
- [50] I.P. Goulden, D.M. Jackson, R. Vakil, *The Gromov-Witten Potential of A Point, Hurwitz Numbers, and Hodge Integrals*, Proc. Lond. Math. Soc. **83** (2001), no. 3, 563–581.

## 8.4. BIBLIOGRAPHY

- [51] I. P. Goulden, D. M. Jackson, R. Vakil, *Towards the geometry of double Hurwitz numbers*, Adv. Math. **198** (2005), no. 1, 43–92.
- [52] T. Graber, R. Vakil, *Hodge integrals and Hurwitz numbers via virtual localization*, Compositio Math. 135 (2003), no. 1, 25–36.
- [53] S. Gukov and P. Sułkowski, *A-polynomial, B-model, and quantization*, arXiv:1108.0002v1 [hep-th].
- [54] R. Hain, *Normal Functions and the Geometry of Moduli Spaces of Curves*. arXiv:1102.4031.
- [55] E. N. Ionel, *Topological Recursive Relation in  $H^{2g}(\mathcal{M}_{g,n})$* . Invent. Math., **148** (2002), no. 3, pp. 627–658.
- [56] C. Itzykson, J.-B. Zuber. *The planar approximation. II.* – J. Math. Phys. **21** (1980), no. 3, 411–421.
- [57] G. D. James, *The representation theory of the symmetric groups*, Lecture Notes in Math., 682. Springer-Verlag, Berlin-Heidelberg-New York, 1978.
- [58] T. J. Jarvis. *Geometry of moduli of higher spin curves*. – Internat. J. Math., **11** (2000), 637–663, arXiv:math/9809138.
- [59] P. Johnson, *Double Hurwitz numbers via the infinite wedge*, arXiv:1008.3266.
- [60] P. Johnson, R. Pandharipande, H.H. Tseng, *Abelian Hurwitz-Hodge integrals*, Michigan Math. J. **60**, 171–198 (2011).
- [61] V. Kac, A. K. Raina, *Bombay lectures on highest weight representations of infinite-dimensional Lie algebras*, Adv. Ser. Math. Phys., 2. World Scientific, Teaneck, NJ, (1987).
- [62] M. Kazarian, *Deformations of cohomological field theories*, preprint 2007.
- [63] M. Kazarian, S. Lando. *An algebro-geometric proof of Witten’s conjecture*. – J. Amer. Math. Soc. **20** (2007), no. 4, 1079–1089.
- [64] M. Kazarian, S. Lando, D. Zvonkine, *Universal cohomological expressions for singularities in  $\overline{\mathcal{M}}_{0,n}(\mathbb{C}\mathbb{P}^1)$* . In preparation.
- [65] S. Kerov, G. Olshanski, *Polynomial functions on the set of Young diagrams*, C. R. Acad. Sci. Paris Sér. I Math. **319** (1994), no. 2, 121–126.
- [66] M. Kontsevich, *Intersection theory on the moduli space of curves and the matrix Airy function*, Commun. Math. Phys. 147, 1–23 (1992).
- [67] I. Kostov, N. Orantin, *CFT and topological recursion*, JHEP 1011:056 arXiv:1006.2028.
- [68] Y.-P. Lee, *Witten’s conjecture, Virasoro conjecture, and invariance of tautological equations*, arXiv:math/0311100.
- [69] Y.-P. Lee, *Invariance of tautological equations I: conjectures and applications*, J. Eur. Math. Soc. (JEMS) **10** (2008), no. 2, 399–413.

## 8. BIBLIOGRAPHY

- [70] Y.-P. Lee, *Invariance of tautological equations II: Gromov–Witten theory* (with Appendix A by Y. Iwao and Y.-P. Lee), J. Amer. Math. Soc. **22** (2009), no. 2, 331–352.
- [71] Y.-P. Lee, *Notes on axiomatic Gromov–Witten theory and applications*. Algebraic geometry –Seattle 2005. Part 1, 309–323, Proc. Sympos. Pure Math., 80, Part 1, Amer. Math. Soc., Providence, RI, 2009.
- [72] J. Li. *Stable morphisms to singular schemes and relative stable morphisms*. J. Differential Geom. **57** (2001), no. 3, 509–578.
- [73] J. Li. *A degeneration formula of GW invariants*. J. Differential Geom. **60** (2002), no. 2, 199–293.
- [74] C.-C. M. Liu, *Lectures on the ELSV formula*, In: Transformation groups and moduli spaces of curves, 195–216, Adv. Lect. Math. (ALM), 16, Int. Press, Somerville, MA, 2011.
- [75] A. Losev, Y. Manin, *New moduli spaces of pointed curves and pencils of flat connections*. Michigan Math. J. **48** (2000), 443–472.
- [76] Y. L. Luke. *The Special Functions and their Approximations*, vol. I, Academic Press, New York, 1969.
- [77] T. Milanov. *The Eynard–Orantin recursion for the total ancestor potential*.  
[arXiv:1211.5847](https://arxiv.org/abs/1211.5847).
- [78] T. Miwa, M. Jimbo, E. Date, *Solitons. Differential equations, symmetries, and infinite-dimensinal algebras*, Cambridge Tracts in Math., 135. Cambridge University Press, Cambridge, 2000.
- [79] A. Mironov, A. Morozov, S. Natanzon, *Complete Set of Cut-and-Join Operators in Hurwitz-Kontsevich Theory*, [arXiv:0904.4227](https://arxiv.org/abs/0904.4227)
- [80] A. Mironov, A. Morozov, S. Natanzon, *Algebra of differential operators associated with Young diagrams*, [arXiv:1012.0433](https://arxiv.org/abs/1012.0433)
- [81] M. Mulase and P. Sulkowski, *Spectral curves and the Schrödinger equations for the Eynard–Orantin recursion*, [arXiv:1210.3006 \[math-ph\]](https://arxiv.org/abs/1210.3006).
- [82] M. Mulase, N. Zhang, *Polynomial recursion formula for linear Hodge integrals*,  
[arXiv:0908.2267](https://arxiv.org/abs/0908.2267)
- [83] D. Mumford. *Towards enumerative geometry on the moduli space of curves*. – In: Arithmetics and Geometry (M. Artin, J. Tate eds.), v.2, Birkhäuser, 1983, 271–328.
- [84] P. Norbury, N. Scott, *Gromov–Witten invariants of  $\mathbf{P}^1$  and Eynard–Orantin invariants*,  
[arXiv:1106.1337](https://arxiv.org/abs/1106.1337).
- [85] A. Okounkov *Toda equations for Hurwitz numbers*, Math. Res. Lett. **7**, 447–453 (2000).
- [86] A. Okounkov, R. Pandharipande, *Gromov–Witten theory, Hurwitz numbers, and Matrix models, I*, [arXiv:math/0101147](https://arxiv.org/abs/math/0101147).
- [87] A. Okounkov and R. Pandharipande, *The equivariant Gromov–Witten theory of  $\mathbb{P}^1$* ,  
[arXiv:math/0207233 \[math.AG\]](https://arxiv.org/abs/math/0207233).

## 8.4. BIBLIOGRAPHY

- [88] A. Okounkov and R. Pandharipande, *Gromov-Witten theory, Hurwitz theory, and completed cycles*, Ann. Math. **163**, 517–560 (2006).
- [89] A. Okounkov, R. Pandharipande, *Virasoro constraints for target curves*, Invent. Math., **163** (2006), no. 1, 47–108.
- [90] N. Orantin, *From matrix models' topological expansion to topological string theories: counting surfaces with algebraic geometry*, Ph.D. thesis Université Paris 6 – Pierre et Marie Curie, arXiv:0709.2992.
- [91] N. Orantin, *Symplectic invariants, Virasoro constraints and Givental decomposition*, arXiv:0808.0635.
- [92] A. Pixton, R. Pandharipande, D. Zvonkine *Relations on  $\overline{\mathcal{M}}_{g,n}$  via 3-spin structures*. – arXiv:1303.1043.
- [93] A. Polishchuk, A. Vaintrob. *Algebraic construction of Witten's top Chern class*. – Advances in algebraic geometry motivated by physics (Lowell, MA, 2000), pp. 229–249, Contemp. Math., vol **276**, Amer. Math. Soc., Providence, RI, 2001, math.AG/0011032.
- [94] P. Rossi, *Gromov-Witten invariants of target curves via symplectic field theory*, J. Geom. Phys. **58** (2008), no. 8, 931–941.
- [95] S. Shadrin, *Some relations for double Hurwitz numbers*, Funct. Anal. Appl. **39** (2005), no. 2, 160–162.
- [96] S. Shadrin, *On the structure of Goulden-Jackson-Vakil formula*, Math. Res. Lett. **16** (2009), no. 4, 703–710.
- [97] S. Shadrin, *BCOV theory via Givental group action on cohomological fields theories*, Mosc. Math. J. **9** (2009), no. 2, 411–429.
- [98] S. Shadrin, M. Shapiro, A. Vainshtein, *Chamber behavior of double Hurwitz numbers in genus 0*, Adv. Math. **217** (2008), no. 1, 79–96.
- [99] S. Shadrin, D. Zvonkine, *Changes of variables in ELSV-type formulas*, Michigan Math. J. **55** (2007), no. 1, 209–228.
- [100] S. Shadrin, D. Zvonkine, *Intersection numbers with Witten's top Chern class*, Geom. Topol. **12** (2008), no. 2, 713–745.
- [101] Jun S. Song, Yun S. Song, *On a conjecture of Givental*, J. Math. Phys. **45** (2004), no. 12, 4539–4550.
- [102] Jun S. Song, *Descendant Gromov-Witten invariants, simple Hurwitz numbers, and the Virasoro conjecture for  $P^1$* , Adv. Theor. Math. Phys. **3** (1999), no. 6, 17211768.
- [103] C. Teleman, *The structure of 2D semi-simple field theories*, arXiv:0712.0160.
- [104] R. Vakil. *The moduli space of curves and Gromov-Witten theory*. – Enumerative invariants in algebraic geometry and string theory, 143–198, Lecture Notes in Math. **1947**, Springer, Berlin, 2008, math.AG/0602347.

## 8. BIBLIOGRAPHY

- [105] R. Vakil, *Problem session of the workshop “Diamant meet GQT”*, Lorentz Center, Leiden, October 2008, available at:  
<http://www.lorentzcenter.nl/lc/web/2008/313/extra.pdf>
- [106] A. Vershik, A. Okounkov, *A new approach to the representation theory of the symmetric groups*, Selecta Math. (N.S.) **2** (1996), no. 4, 581–605.
- [107] E. Witten, *Two dimensional gravity and intersection theory on moduli space*, Surveys in Differential Geometry 1, 243–310 (1991).
- [108] E. Witten. *Algebraic geometry associated with matrix models of two-dimensional gravity*. – Topological methods in modern mathematics (Stony Brook, NY, 1991), 235–269, Publish or Perish, Houston, TX, 1993.
- [109] J. Zhou, *Intersection numbers on Deligne-Mumford moduli spaces and the quantum Airy curve*, arXiv:1206.5896.
- [110] J. Zhou, *Quantum mirror curves for  $\mathbb{C}^3$  and the resolved conifold*, arXiv:1207.0598.
- [111] D. Zvonkine, *A preliminary text on the r-ELSV formula*, preprint 2006.

### PUBLICATIONS OF L. SPITZ RELATED TO THE TOPICS OF THE THESIS

- [112] S. Shadrin, L. Spitz, D. Zvonkine, *On double Hurwitz numbers with completed cycles*, J. Lond. Math. Soc. (2) **86**, no. 2, 407–432 (2012).
- [113] P. Dunin-Barkowski, N. Orantin, S. Shadrin, L. Spitz. *Identification of the Givental formula with the spectral curve topological recursion procedure*. – to appear in Comm. Math. Phys., arXiv:1211.4021.
- [114] M. Mulase, S. Shadrin, L. Spitz. *The spectral curve and the Schrödinger equation of double Hurwitz numbers and higher spin structures*. Communications in number theory and physics, Volume 7, Number 1, 125143, 2013.
- [115] A. Buryak, S. Shadrin, L. Spitz, D. Zvonkine. *Integrals of  $\psi$ -classes over double ramification cycles*. –arXiv:1211:5237
- [116] S. Shadrin, L. Spitz, D. Zvonkine, *Equivalence of ELSV and Bouchard-Mario conjectures for r-spin Hurwitz numbers*, –arXiv:1306:6226.
- [117] P. Dunin-Barkowski, M. Kazarian, N. Orantin, S. Shadrin, L. Spitz. *Polynomiality of Hurwitz numbers, Bouchard-Mariño conjecture, and a new proof of the ELSV formula* – arXiv:1307:4729

### OTHER PUBLICATIONS OF L. SPITZ

- [118] P. Dunin-Barkowski, S. Shadrin, L. Spitz. *Givental graphs and inversion symmetry*. Letters in Mathematical Physics May 2013, Volume 103, Issue 5, pp 533–557