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Mehdiyev, R.; Vos, P.

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Exploring students' learning experiences when using a Dynamic Geometry Software (DGS) tool in a geometry class at a secondary school in Azerbaijan

Rafiq Mehdiyev

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Supervised by Dr. Pauline Vos

AMSTEL Institute
Universiteit van Amsterdam
Postbus 94224
1090 GE Amsterdam, The Netherlands

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Abstract

This thesis describes a research project that studies the use of a Dynamic Geometry Software (DGS) tool in geometry learning and its implications for students' learning experiences. The research is based on an assumption that DGS facilitates the externalization of the internal representations of geometrical concepts, theorems and proofs. The dynamic and interactive medium provides students with the opportunity to share and discuss the emergent visual phenomena. Apart from its implication for students' cognitive development, a DGS tool may have an effect on students' learning experiences in geometry classes from different aspects.

I conducted this research study in a geometry class at a secondary school in Azerbaijan to explore the use of a DGS tool in a cooperative learning arrangement. I used GeoGebra as a DGS tool with research questions relating to the aspects of (1) motivation, (2) interactions and discussions, (3) student-centered learning, (4) conceptual understanding, and (5) problem solving strategies. The questions were embedded in an instructional research intervention. The intervention comprised the use of worksheets and applets I developed through GeoGebra. The research data were drawn from the used worksheets, classroom observations, results of pre- and post-test, a questionnaire and interview responses.

The collected data were analyzed separately and used to answer the research questions. The overall findings showed that the use of a DGS tool with the presence of other factors, such as group work and the use of worksheets, brought about certain changes in students' learning experiences of the geometrical concepts. Students were well motivated, but discussion and interaction were limited (due to time limitations) and results on students' conceptual understanding and problem-solving strategies were only partly satisfactory, but improved during the intervention.

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My special thanks also go to my cooperative teacher, Ms. Feride Guliyeva, who gave me the opportunity to give my research lessons in her own geometry class. By all means I am grateful to the students of her class who were willing to participate with great enthusiasm in my research lessons.

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1. Introduction

In Azerbaijan the mathematics curriculum includes the teaching of geometrical concepts. These are presented in textbooks through axioms, definitions, theorems and proofs. Often, the learning practices consist of the recitation of definitions, formulas and additionally, of solving routine exercises. Students are rarely encouraged to study the processes in which concepts and formulas are derived. Instead, the formulas are memorized with the aim of applying them directly, to solve typical exercises. This approach to school geometry is also driven by the available national assessment. Within the existing curriculum practice, one cannot expect a good performance from the students on exercises in which they have to vindicate the validity of given formulas or to solve conceptual problems. It is anticipated that this environment does not offer students the opportunity to deeply understand the introduced geometry concepts and formulas and thus, to solve conceptual problems differing from routine exercises.

A conceptual understanding of geometry, that is an understanding based on insight, calls for mental imagination, since the proofs and derivations of formulas are based on the flexibility and generalizability of figures or shapes. Textbooks cannot visualize the dynamic nature of geometrical figures on paper. As a consequence, students are forced to mentally investigate the possible properties of geometrical objects without an external way to increase understanding of the related concepts, and therefore students often fail to develop insights into the taught concepts. Thus, internalizing the geometrical representations is a psychological challenge to students in the pencil and paper medium, which makes learning geometry difficult to many students. This problem remains persistent in a learning environment which lacks dynamic features that may facilitate the justification and validation of definitions, axioms and theorems in a perceptive manner.

A possible solution to the above described problem was proposed by introducing a Dynamic Geometry Environment (DGE) to the teaching and learning of geometry. The Dynamic Geometry Software (DGS) could be incorporated into a regular geometry classroom in order to provide a dynamic as well as visual representation of geometry concepts in a physical sense. In the hands of the students, the DGS played a role of an intellectual tool for studying geometry objects empirically. The general objective of this research was to explore students' learning experiences when using DGS in order to provide support for learning processes based on the geometry textbook and regular geometry classes. For this, I designed a short intervention, consisting of a series of DGS-based geometry lessons in order to explore the use of a DGS tool in the learning of geometry and its affect on students' learning experiences.

1.1. Identified problems

I would describe the teaching and learning of geometry at secondary schools in Azerbaijan as teacher-centered in one word. The teacher dominates the classroom and turns students to mere listeners. They are not encouraged to discuss and interact with each other and to explore the presented content collaboratively. The teaching of geometrical objects is pedagogically authoritative in nature and therefore students are not encouraged to question the validation or construction of geometrical entities. By and large they are implicitly coerced to accept the delivery of geometrical content with absolute certainty. Consequently, they become demotivated to study geometry since it is not felt necessary in their life outside school. The teaching and learning of geometrical entities is primarily procedural-based and does not target the development of conceptual understanding. The employed strategy in problem solving is limited to performing routine exercises. Students are not involved in tackling problems with a number of possible alternative solutions.

In sum, the curriculum of school geometry in Azerbaijan on the whole lacks collaborative learning, discussions, the use of technology and the development of conceptual understanding and of non-routine problem solving strategies. By introducing a DGS tool in a geometry class, I attempted to implement these aspects and study them in this research project.

1.2. Motivation for the research

Personal motivation

The first motive to study the use of a DGS tool in a geometry class at a secondary school in Azerbaijan was inspired from my previous experience in the *Winter Project* of the *Theory of Teaching, Learning and Communication* course[1]. Within that project I had the opportunity to teach shortly at *Fons Vitae Lyceum*[2] on the concept of trigonometric functions. We developed electronic applets on trigonometric functions, representing these quite differently than in the textbook. The concept was presented in a dynamic, interactive and visual form. The students had the opportunity to empirically explore the concept through the applets. The students were enthusiastically involved in the organized learning activities. Working with the applets provided them with plenty of data on the basis of which they shared and discussed their views and ideas.

[1] http://studiegids.uva.nl/web/uva/2007_2008/en/c/255.html
[2] http://www.fonsvitae.nl/

Motivation based on other reasons

The second motive for this research was based on a Vygotskian view which aims at more effective education. A more effective education may be induced by the use of technology, as studied by a number of researchers (e.g. Falcade, Laborde and Mariotti 2007; Laborde 2003). They conducted research concerning cognitive and psychological effects arising from the use of technology in education. Yet, the consequences of the use of technology in the classrooms may differ depending on the social settings, because of interactive factors relating to cultural, social and motivational features. The aim of the proposed research was to elicit possible practical effects of the use of a DGS tool on geometry learning in terms of improved motivation, active participation, conceptual understanding and problem solving strategies. The incorporation of a DGS tool in a regular geometry class in Azerbaijan was an initiative towards the development and extension of a student-centered pedagogy. In this research technology was used to support the construction of theoretical knowledge. In a spectrum of using the supposed potentials of technology, the construction of theoretical knowledge is a new policy in Azerbaijan and leaves much space for research. It was assumed within the research that a technology intervention in geometry education would increase students' motivation and support them in constructing concepts and ideas through empirical experience.

2 Theoretical Framework

Within the realm of mathematics, geometry is the study of shapes and space (Senechal, 1990). To bring these objects into classroom, geometry is taught through a variety of representations, such as diagrams, schemes, drawings, and graphs. These representations are a contextual description of geometrical concepts or ideas and may support the process of conceptualization (Stephen & Tchoshanov, 2001). That is, the use of multiple representations facilitates students' development of geometrical concepts. Traditionally, geometry is taught and learned in a pencil and paper environment. Geometry textbooks at schools provide the above described illustrations. However, sometimes textbook-based illustrations may not be comprehensive, because they lack the visual description of a complete dynamic process needed for the construction of geometrical concepts. This incompleteness arises from the static feature of a textbook medium. In a textbook environment, a dynamic visualization of geometrical figures or shapes is left for an internal (mental) process. To be more explicit, students have to create dynamic geometrical constructions in their minds since a textbook depicts only ideal states of the figures or shapes. Such textbook descriptions do not explicitly picture the construction processes, through which the figures or shapes are idealized. The pencil and paper work only shows the result of the whole construction process, although this process is a product of mental performance. Hence, creating correct geometrical constructions calls for a spatial imagination, which, based on my own observation, turns out to be a hard experience for low-achieving students in Azerbaijan. Hence, because it is difficult for low-achieving students to create the required geometrical constructions, they may become demotivated in studying geometry. To supplement the pencil and paper medium in the teaching of geometry, in particular for the lesser motivated students, a new learning environment is proposed by Falcade, Laborde, and Mariotti (2007), Gawlick (2002), Hollebrands (2003), Laborde (2001), and Ruthven, Hennessy, and Deaney (2008).

They suggest that the use of technology promotes students' understanding of geometry and therefore, recommend a DGE for geometry teaching and learning. However, it is noted that very little is known about students' use of technology and their concept formation through the use of technology (Hollebrands, 2003). The central focus in most GDE-related research is on students' investigation of properties of geometrical concepts through the different types of dragging and students' explanations and justifications for geometric conjectures (Arzarello et al., 2002; Falcade et al., 2007; Gawlick, 2002, 2005). In line with these studies, this research project incorporated the use of DGS (GeoGebra) within a geometry class with the purpose of supporting students to develop deeper understanding of geometrical concepts. For this, the applets were developed under the DGS (GeoGebra), which represented the construction of the intended geometrical concepts.

2.1 Dynamic Geometry

Dynamic is understood as the opposite of static. Dynamic connotes motion, action and energy. Dynamic geometry is active, explorative geometry carried out with interactive computer software. It enables to visualize abstract geometrical concepts. Hershkowitz et al. (2002) stress that dynamic geometry tools like the Geometer's Sketchpad, the Geometric Inventor, and Cabri offer more opportunities to construct and justify geometrical concepts than the pencil and paper settings. According to them, a pencil and paper environment has a limited capacity in introducing a geometrical concept with an emphasis on its intrinsic properties. This insufficient feature of a pencil and paper medium causes the tendency in students to construct a limited concept image. Dynamic geometry thus patches up this insufficiency by providing students with the option of generating empirical evidence to progress from particular cases to the general case. In addition, a dynamic geometry medium plays an essential role in developing the proofs of geometrical conjectures (Hershkowitz et al., 2002). In the designed activities, usually, proving the validity of geometrical concepts by means of a dynamic geometry tool is realized through dragging the relevant points of the constructed objects towards a situation in which they satisfy predefined conditions. DGS enables the design of such activities in which students explore the relevant properties of the geometrical objects in order to construct a more appropriate concept image (Hershkowitz et al, 2002). Hence, learning geometry in a DGE can offer students opportunities to construct and manipulate geometrical figures and carry out empirical investigations. These activities are almost impossible in a static geometry environment (Laborde, 1999).

In Laborde's view, drawing refers to the material entity, while figuring refers to a theoretical object (Hershkowitz et al, 2002). She made a clear distinction between drawing and figuring for the following reasons:

1. Some properties of a drawing can be irrelevant. For example, if a rhombus has been drawn as an instance of a parallelogram, then the equality of the sides is irrelevant.
2. The elements of the figure have a variability that is absent in the drawing. For example, a parallelogram has many drawings; some of them are squares, some of them are rhombuses, and some of them are rectangles.
3. A single drawing may represent different figures. For example, a drawing of a square might represent a square, a rectangle, a rhombus, a parallelogram, or a quadrilateral.

Hence, it is not possible to provide an adequate representation for all properties simultaneously in a pencil and paper environment. However, this is an easier task in a DGE.

A DGE has a variety of tools that enable students to construct geometrical objects and visualize geometrical conjectures or ideas at a perceptive level. Also, the tools offer flexibility of the objects (the dragging tool). The flexibility of the geometrical construction grants students the opportunity to justify, validate or refute conjectures or ideas, as well as to build conjectures based on empirical evidence. Thus, DGS is a learning medium which ensures a new learning setting and new interactions, because it includes unique features that support the learning of geometry. A DGS offers tools to manipulate objects in a physical sense, and subsequently, these tools turn into psychological artifacts. A number of researchers (Arzarello et al., 2002; Gawlick et al., 2002) focused on the dragging modalities (different ways of dragging) provided by the tool. Clearly, this new learning medium provides tangible experiences to learners through physical interactions. This physical support to the emergence of mental processes are in line with a Vygotskian view (Vygotsky, 1978), stating that the use of technology in education has a promising potential in the internalization process (Falcade et al., 2007).

Based on the views discussed at this point, a series of activities was designed for a geometry class in this research project through the dynamic geometry tool GeoGebra (see Appendix IV Lesson plans and Appendix V Worksheets) in order to explore and construct the geometrical concepts. These activities were based on playing with the appropriate applets designed by me using GeoGebra (see Appendix IV Lesson plans). The applets were aimed to allow students to investigate the relevant properties of the geometrical objects in order to construct appropriate concept image and procedures. The underlying point to teach the intended geometrical concepts in a DGE was based on facilitating externalization of the representations of the concepts. Usually, such representations being implicitly described in the geometry textbook call for students to use mental performances. However, the dragging on the computer screen can facilitate the externalization of implicit ideas which become visible phenomena that can be shared and discussed (Zbiek et al., 2007).

Figure 1 below describes the interactions between externalization and internalization as a cyclic process. This interaction only becomes possible within a social interaction. As is seen from the picture, the externalization of the representations of geometrical concepts provides a medium for socialization which in turn ensures the internalization of the geometrical concepts. A DGS tool was assumed to support this cyclic process.

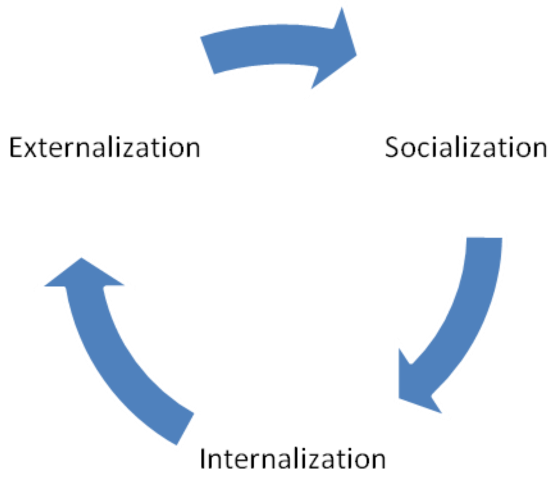

Figure 1.

In addition, dynamic geometry is a medium with which students are assumed to alleviate hard psychological experiences that are required for the geometrical constructions and manipulations with pencil and paper. This process is thus facilitated by the dynamic features of the geometry software. Further, the external experience supports the required internal processes needed for theoretical knowledge construction. Therefore, it is assumed that the successful integration of technology into mathematics education has the potential to bring about positive changes in the teaching and learning processes, in particular if combined with student-centered learning activities. In a DGE, students can be invited to develop deeper understanding on geometrical concepts and problem solving strategies informally (Hennessy & McCormick, 1994; Hershkowitz et al., 2002). Also, in a DGE, students can be invited to have mutual communications and interactions (Gilmore & Halcomb, 2004).

Nevertheless, the successful incorporation of DGS in the learning and teaching of geometry may differ, depending on the social and cultural domain. Therefore, I wanted to research the changes in a geometry class caused by the introduction of DGS under a social and cultural background that differed from earlier research. The general objective of this master research project was to investigate the practical changes that DGS could bring to students' learning experiences in geometry lessons. My research interest focused on the changes that might occur relating to five aspects; (1) motivation, (2) discussions and interactions, (3) student-centered learning, (4) procedural versus conceptual understanding and (5) problem-solving strategies. Herewith, I am convinced that these aspects are not independent. Rather, they are believed to be interrelated and the consequences of change made in one aspect may affect the other. Hence, it was assumed that the DGS-based learning medium should provide support to each of these aspects.

2.2 Motivation

Motivation was one of the aspects, which is important to students and teachers because of its affects on learning outcomes. Motivation is linked with the emotion which is manifested either in positive (interest, joy) or negative (frustration, anger) emotions depending whether the situation is in line with motivation or not (Hannula, 2006). It was supposed in the research that students would express positive emotions when working with computers in the classes. The value of these positive emotions was also added by employing student-centered group workings. Based on the computer supported student-centered instructions, it was assumed that students would be stimulated to interact with each other for discussing and sharing their ideas.

2.3 Discussions and Interactions

This aspect was based on the small-group workings that students were invited to employ during the classroom activities within the research. Students in small groups (two and three) were supposed to demonstrate cooperative learning by sharing and discussing their ideas and views. According to the relevant literature review (Good, Mulryan & McCaslin, 2005), working in small groups (pairs) plays an essential role in promoting students' achievements within heterogeneous classes. Accordingly, in the course of the intervention, it was desired to organize small groups of high and low achievers in order to provide the opportunity for them to learn from each other. By working collaboratively, low-achieving students may benefit from communicating with high-achievers. Further, in the course of social interactions based on sharing and discussing the visual phenomena appeared on computer screens, they come to better explore and understand the phenomena. Therefore, it was assumed in the research that computer-based instruction combined with working in pairs would increase both students' interactions with each other and their participation in class discussions. Also, according to a Vygotskian view (Vygotsky, 1978), a social environment plays an essential role in the development of individual thought. Therefore, through mediating the emergent phenomena during the classroom activities, students were supposed to promote a level of interactions and communications with each other. These interactions and communications in turn provide the internalization of a social/instructional environment (Good et al., 2005).

2.4 Student-centered Learning

This aspect manifested itself in the computer-based cooperative learning activities, in which the teacher interventions were reduced to a minimum level. It was assumed that students would feel more responsible for their learning. In a DGE it is believed that the use of computer technology provides the basis for the accomplishment of student-centered learning. According to Laborde (2001), the incorporation of technology into mathematics education changes the teaching system. All aspects in the classroom, such as the structure of activities and the content to be taught receive new shapes. This applies also to the DGS, acting as a mediator between students and content, and this mediation affects students' learning experience, in particular the interactions and the communication. Furthermore, students interact with the tools of the DGS and their activities result in representations to which they have to react. That is, interacting with DGS students receive feedback on the basis of which they make new interactions. Hence, this interaction-feedback cycle of working was assumed to provide support to student-centered learning activities. Furthermore, according to Gilmore and Halcomb (2004), it is unlikely to think that the use of technology based student-centered activities alone will enhance performance and collaboration among students. Rather, in order for technological integration in the classroom to be effective, the emphasis on instructional design must be increased. For this, the design of worksheets and applets as instructional materials was applied in the research.

2.5 Conceptual versus Procedural Understanding

Conceptual understanding was one of the research focuses in this project. According to Kilpatrick, Swafford, and Findell (2001), conceptual understanding is regarded as a key to grasping mathematical concepts and ideas. Conceptual understanding is an important strand in mathematical proficiency development. Students with conceptual understanding come to realize the interconnection between mathematical concepts and representations. Conceptual understanding thereby provides support for students to develop insights into mathematical procedures and ideas and to competently apply them in solving non-routine mathematical problems. Conceptual understanding assists students to acquire better competencies in formulating alternative solution methods and in connecting these methods with each other. Based on the elaborated definition of conceptual understanding as a mathematical proficiency strand, I assumed that a DGS-based learning medium provides insightful experience for students in learning geometry concepts. Because students develop conceptual understanding of geometry ideas and procedures, they are expected to know in what ways these geometry procedures are deduced and how to apply them in solving geometry problems. Additionally, conceptual understanding, according to Kilpatrick et al. (2001), provides support to develop procedural fluency which refers to knowledge of procedures, knowledge of when and how to use them appropriately and competence in performing them accurately and flexibly. Procedural fluency alone, to my own thinking, is not desirable, nor does it precede the former strand. That is, students without conceptual understanding may get better at performing procedures based on rote memorization in solving routine problems. However, when facing none-routine problems involving strategic skill, such students become at a loss to develop a new appropriate technique in solving them. Hence, conceptual understanding was also supposed to pave a way for developing problem solving strategies or strategic competencies as mentioned in the literature. Therefore, problem solving strategies was the next focus of the research.

2.6 Problem-solving Strategies

According to Kilpatrick et al. (2001), strategic competencies refers to the ability to formulate mathematical problems, represent them, and solve them. This strand is also known as problem solving. Thus, students should know how to develop a variety of solution strategies as well as which strategies are useful for solving non-routine problems. Consequently, this strand is interconnected with conceptual understanding. In this sense, it was assumed that DGS-based learning might also support students to develop geometry problem solving strategies. Because students learn how geometry concepts and procedures are developed, they are expected to know where geometry procedures may appropriately be used or applied. As well, developing a variety of solution strategies requires that students be able to generalize the representations generated for equally structured problems. This generalizability then becomes a resource for students to develop appropriate strategies in solving specific geometry problems. In sum, I assumed that by learning geometry concepts in a DGE, students develop conceptual understanding of geometry concepts and procedures which in turn become a ground for students to develop appropriate problem solving strategies. This view is also in line with the Vygotskian approach (Vygotsky, 1978), suggesting that students may be cognitively affected by the new learning resource. Because geometry was presented in visual and dynamic contexts, it was assumed that students were facilitated to develop conceptual understanding underlying the geometry procedures being taught, and accordingly, improve their problem-solving strategies (Hennessy & McCormick, 1994).

3 Research questions

Research goal

In this research project the traditional way of the teaching and learning of geometry was supplemented by a new approach, consisting in the use of a DGS tool along with working In small groups. The research goal then was to explore the extent to which the use of a DGS tool combined with small groups, and how this implicated students' learning experiences. It was assumed that a new approach could bring about positive changes to the learning of geometry. Unlike the traditional way of the teaching and learning of geometry, a new approach was believed to provide more offerings in increasing students' participation in whole class discussions, interactions and argumentations on theory construction, and in developing conceptual understanding, and problem solving strategies.

The effective role of the DGS-based lessons on students' learning experiences was assumed to be associated with the aspects of motivation, interactions and discussions, student-centered learning, conceptual understanding and problem solving strategies. The research questions were developed based on these aspects. Therefore, each of the research questions below represents one of the aspects.

The research goal is generally described in the overall question below;

- ✓ How far can the cyclic process described in Figure 1. be supported by a DGS tool in a geometry class?

The sub-questions are:

1) To what extent are students motivated to learn geometry with the support of DGS?
2) To what extent does the use of DGS increase students' participation in overall class discussions and interactions with each other and with a teacher?
3) In what ways does DGS provide support for student-centered learning activity in a geometry class?
4) To what extent does DGS amplify the shift from procedural to conceptual oriented understanding of geometry concepts?
5) To what extent does DGS have an effect on strategies developed by students in problems solving?

4 Research Design and Method

The research was designed as a classroom intervention, which supplemented the regular geometry lessons taught by the collaborative teacher. The research aligned with the curriculum topic (area and circumference of circles, radians) and was carried out in the same weeks in which the regular, traditionally taught lessons took place. Below, aspects of the design of the intervention will be explained.

4.1 Selection of Software tools

There are a great number of dynamic software products available which serve the purpose of increasing dynamism of geometry education. For instance, Cabri, WinGeom, Euclide, Cindrella, and GeoGebra all have advantages for geometry education. Some like WinGeom and GeoGebra are free and can easily be downloaded from their official web sites. Despite some common commands like drawing and dragging, the tools have differences. For instance, unlike other tools, GeoGebra provides the option of making interactive applets. Further, because of its controllable dynamic features and free and technical accessibility, GeoGebra 3.0.0.0 was proposed for use in a few regular geometry classes.

4.1.1 GeoGebra

For the research GeoGebra 3.0.0.0[3] was used. The software is technically available through the internet and can be installed independent of user platform. GeoGebra requires Java 1.4 which can also be downloaded from the web[4]. GeoGebra has a good option of dynamic manipulations with availability of a slider motion tool. This tool enables users to manually manipulate the drawn geometric objects and to monitor interactive changes. The software unites algebra, calculus and geometry subjects. Interchangeably, making algebraic and geometric representations in the same medium is the advantage of this tool. In addition, this software package has an option of applet construction which allows the author to determine the extent of interactivity for users in design time. Constructed applets with dynamic and visual representations thus become explorative sources to students.

4.2 Design of instructional materials

4.2.1 Electronic Applets

GeoGebra was used to develop applets representing the geometrical concepts taught in this research intervention. Five applets were designed to represent the intended geometrical concepts (see **Error! Not a valid bookmark self-reference.** in pictures). The first two applets were concerned with the circumference of a circle. The third and fourth applets represented the radian of an angle. The final applet was designed to illustrate the area of a regular polygon and circle. The design of all applets provided the students with an option to transform the geometrical constructions. In the applets dragging a given point or variable on a slidebar had certain consequences for the shapes of the geometrical constructions.

By dragging the points the students generated their own data and they were supposed to observe and record the consequences of the different values of the variable for the geometrical constructions. Based on the recorded values or numbers characterizing the different states of the constructions they were expected to develop answers to the questions given in the applets.

4.2.2 Worksheets

In order to support students' work with the applets, five worksheets were developed (see Appendix V Worksheets). Each of the worksheets contained tables, the tasks and the questions given in the applets. The tables were supposed to be filled with the appropriate

[3] http://www.geogebra.org/webstart
[4] http://java.com/en/download/index.jsp

values or numbers describing the different states of the geometrical constructions in the applets. The students were supposed to acquire these numbers by transforming the constructions through the slidebars. The tasks and questions in turn asked the students to develop a line of reasoning based on the recorded numbers in the tables.

4.3 Data collection

Along with the designed instructional materials some research instruments were developed to evaluate the intervention. There were classroom observations through field notes and videotape recordings, interviews with the cooperative teacher and with the students, a questionnaire and pre- and post-tests. Unfortunately, the videotape materials were lost due to an unexpected crash in the hard disk of my laptop. The rest of the collected data were used in developing the answers to the appropriate research questions.

- Observation of classroom activities

With permission of the school administration, classroom activities were videotaped. In addition, the field notes on the essential points observed during the classroom activities were collected. The field notes were used to analyze the extent to which students were motivated to learn geometry concepts when using DGS. The data of this source also helped to evaluate the point of students' participation in the overall class discussions and of their interactions with each other and with a teacher. In addition, the data helped in analyzing the role of DGS in student-centered learning activities. Thus, this type of data helped in answering research questions 1, 2, and 3.

- Interview with the cooperative teacher

After the intervention, an interview was arranged with a cooperative teacher to obtain her reflections (see Appendix III Interview responses). These data were used to evaluate, from a teacher's point of view, the role of DGS in supporting students' motivation, interactions and participations in class discussions, student-centered learning activities, conceptual understanding, and problem solving strategies. Therefore, these data were expected to help in answering research questions 1, 2, 3, 4, and 5.

- Interview with students

After the intervention, and with support of the cooperative teacher some students were interviewed (see Appendix III Interview responses). This source of data helped to analyze students' reflections and views on the role of DGS in supporting their motivation, interactions and participations in class discussions, student-centered learning activities, conceptual understanding and problem solving strategies. Their answers then were compared with the other data sources. Therefore, these data helped in answering the research questions 1, 2, 3, 4, and 5.

- Questionnaires

Also, after the intervention, students were asked to fill in a questionnaire (see Appendix VII Questionnaire and interview questions). In order to get extensive and reliable answers, each of the questions was asked in traversal ways. The data from the questionnaire were used to evaluate students' views on the role of DGS in supporting their motivation, interactions and participation in class discussions and student-centered activities. Thus, this type of data helped in answering the research questions 1, 2, and 3.

- Pre and post tests

Before and after the intervention a pre- and a post-test was administered. The difference between the results of the pre- and post-test were supposed to enable an analysis of changes made in students' conceptual understanding and problem solving strategies. The analysis of the comparison then helped in answering the questions relating to the roles of DGS in supporting the development of students' conceptual understanding and problems solving strategy. Therefore, the pre- and posttest touched on the research questions 4 and 5.

5 Setting of the research

5.1 Educational setting

After the collapse of the former Soviet Union, many fields including education have undergone reforms in Azerbaijan. Establishment of a free market economy, political, economical and cultural integration into the world community induced the country to renovate the old system in conformity with international experience. Due to financial and economic problems the country faced right after the decline of the Soviet system, the transition from the old Soviet system to new one engendered many constraints and difficulties. To reduce expenses for the renovation, the government decided to preserve some parts of the old system which could fit in the newly established structure.

In Azerbaijan education at school level consists of three parts[5]; primary, basic and secondary. The primary education covers four years. After primary education, the basic education starts and lasts five years. Up to this stage, education is compulsory for all citizens. Upon completion the basic education, students may leave school for continuing to study at technical or vocational schools. Those who aim for higher education (for universities) should complete the secondary education that covers two years.

Innovations in education, especially at school level, were primarily concerned with the curriculum content, teacher education content, and teaching and learning materials and textbooks. For this, the Ministry of Education of the Republic of Azerbaijan (MoE) has started to cooperate with the World Bank[6] (WB). A report from the WB advised the MoE to introduce more student-centered activities and to shift the role of teachers from leader to facilitator.

Integration of Information and Communication Technologies (ICT) in education became the prime focus of the educational authority. All schools were equipped with computers and a majority of schools have gained access to the World Wide Web. Yet, there remains a disparity in equipment between urban and rural areas. That is, schools in the rural areas have poorer equipment than those in the urban areas due to remoteness and lack of sufficient resources. Within the WB project the MoE plans to establish computer networking between the schools. The incorporation of technology into teaching and learning activities is also considered in the project. Furthermore, in-service training with regard to teaching with computers has been deployed. Teachers and educators are also trained and supported in applying new teaching methods and norms in their classroom activities. New lesson materials, in accordance with the new curriculum, have been published and distributed to schools.

[5] http://www.kurikulum.az/
[6] http://www.worldbank.org.az

Although the reforms and innovations are well-reflected in theory and in documents at the authority level, teachers at schools are still stuck to the conventional way of teaching[7]. In general they are presently not good at the use of technology in their classroom activities. Group work and classroom discussions are not considered relevant for mathematics education. Teachers deliver content through lectures and expect similar demonstrations from their students. During lessons the students are not encouraged to interact with each other. They are not motivated to discuss and construct mathematical knowledge in their own ways. The similar situation is met in geometry education. Teachers and students strictly follow the textbook. Students are not encouraged to construct geometrical concepts through experimental methods. The teaching and learning of geometry does not incorporate a DGS tool at all. The proofs of theorems, conjectures are performed in a pencil and paper setting as precisely dictated in the textbook.

5.2 Design of the intervention

The intervention took place in April 2009. As a target group for this research intervention, 9th grade students (14-15 years old) from a secondary school in Azerbaijan were selected through the cooperative communication with a mathematics teacher from that school. She would teach simultaneously the regular geometry lessons on the same topic.
The intervention was preceded by a pre-test, as a baseline (see Appendix VI Pre- and post-tests). After the pre-test the intended classroom activities were implemented. At the beginning of the intervention, for two lessons, the students were introduced to the Dynamic Geometry Software in order to obtain knowledge and skills in construction and manipulation of geometric objects (see Appendix IV Lesson plans).

Thereafter, in three sessions the students were introduced to the intended geometry concepts (see Appendix IV Lesson plans). The selection of the concepts was consistent with the schedule and content of the school curriculum. The concepts to be taught were developed into a series of activities based on the worksheets and applets that I designed as instructional materials for a classroom intervention. The worksheets were supposed to help the students in investigating the applets developed through GeoGebra.

In the course of each session the students were grouped (with support of the cooperative teacher) and guided to work with the electronic applets under the directions of the worksheets (see Appendix V Worksheets). At the end of the intervention, a post-test, questionnaire and interviews with both the students and the cooperative teacher were administered in order to evaluate the intervention (see Appendix VI Pre- and post-tests and Appendix VII Questionnaire and interview questions).

[7] These are my own observations.

School setting

The research was planned to be conducted at a secondary school which is situated in Baku city, the capital of Azerbaijan. This secondary school contains around 700 students and is considered one of the medium sized schools in the city. Also, it is a state school that provides free and equal education in the native language to all Azeri citizens regardless of ethnic, culture, gender, religious, and political affiliations. For the research intervention at the selected school the Ministry of Education of the Republic of Azerbaijan was officially requested and the school administration was asked to allow the use of the school laboratory for teaching the intended geometry lessons. The school laboratory was considered to have a basic facility to meet the need of the intervention. For instance, in the laboratory there were about 9 working computers.

Accordingly, a cooperative mathematics teacher working at this school agreed to allow her 9th grade students (14-15 years old) to be my research group. She had 20 students in her class. A majority of the students had basic skills to work with computers. The classroom teaching, which was the main part of the intervention, covered five lessons each of which was 45 minute period.

6 Data Analysis

The strategy to answer the research questions is based on making analysis of qualitative data collections from the appropriate data sources. In order to make the process of qualitative analysis easier the findings from the different data sources are first tabulated in the appropriate tables. In the analysis of the findings the relevant data from the different sources are then cross-referenced and appropriately combined in developing the answers to the research questions.

The analysis of the data from the worksheets, questionnaire, interviews and the classroom observations serves to answer research questions 1, 2, and 3 about the motivation, interactions and discussions, and student-centered learning. In answering research questions 4 and 5 about the conceptual understanding and problem solving strategies, the analysis of the data from the pre- and post-tests, worksheets, and interviews is used.

7 Results

7.1 Findings from the lessons

7.1.1 The class setting

All the intended geometry lessons were held in the computer laboratory at my testing school in Baku city, the capital of the Republic of Azerbaijan. The condition of the laboratory was satisfactory. Present in the laboratory were a beamer and eleven computers, two of which were not working properly. The computers were neither connected with a local network nor to the world-wide-web. The installation of the GeoGebra software as well as the distributions of the applets was enabled through an external flash drive. The computers had been placed along the wall side by side. The space and the number of computers were barely enough to work with my experimental group, especially when all students were present. The students were distributed into groups of two or three, and the groups sat very tightly to each other. Nevertheless, this setup was conducive for group discussions and interactions.

The participants in the study were grade 9 students (15-16 years old) numbered twenty, most of whom were girls. From the day of the pre-test until the day of the interview all students participated in the intended sessions, except for two or three students with certain excusable reasons. The planned research activities did not occur according to the predetermined schedule due to the organizational issues. Despite a shift of the timeline, the planned activities did happen in the planned sequence. The sequence of the data collection activities is tabulated in Table 1.

Table 1: Sequence of data collection

Date	Type of activity	Subject matter	Periods
13/04/2009	Pre-test		45 min
15/04/2009	Lesson	Introduction to the GeoGebra software	2 x 45 min
17/04/2009	Lesson	Circumference of a circle	45 min
20/04/2009	Lesson	Radian measurement of an angle	45 min
22/04/2009	Lesson	Area of a regular polygon and area of a circle	45 min
23/04/2009	Post-test, Questionnaire		45 min
24/04/2009	Interview with students and cooperative teacher		45 min

7.1.2 The in-class events and discussions

Lessons on 15th of April 2009

From the beginning of the lesson on this day, I observed that the students were sufficiently able to work with the software, having sufficient computer skills. I gave an introduction by means of the beamer connected to my laptop and they all followed the steps at the desktop computers[8]. The students were familiarized with some tools from the toolbar in order to draw triangles, rectangles, circles, regular polygons as well as to select and manipulate them. In addition, they were taught to use a slider motion for the manipulation of coherent geometric parts and objects. Some of the students were quick in performing the steps and in those cases I asked those students to help others. During the session, I walked along the groups in order to observe and offer my help whenever asked for. Based on my observations, I can say that the students appeared to be working enthusiastically with the software. This was obvious from their behavior, while they were interacting with one another and asking or showing their work. Especially, during the second lesson of this day I asked the students to control the radius of the circle through the motion slider. At first, it seemed difficult to them to do. At this point, I gave them time to work on it collaboratively. They were already acquainted with the tools of a circle with a radius, and the motion slider. After a while some of the groups managed to construct a circle whose radius was controllable with the motion slider. In the end, I asked the students to do a similar construction with regular polygons as homework[9]. They had to connect the side of a regular polygon with a motion slider in order to change it into the different types of a regular polygon.

Towards the end of the two lessons of the first day, I became impressed by the positive change in students' attitude towards the software. They were vividly expressing that learning geometry through the GeoGebra software started to make sense to them. One of the students was able to do the task and during the next lesson he showed it to me and his mates.

Lesson on 17th of April 2009

The lesson started in the computer laboratory with an arrangement of groups made by the cooperative teacher. All the groups took up their places at their computer desks. After the groups were seated, I asked questions to the students regarding what they remembered about the definition of a circle, a diameter, a radius, "π" and a chord. A few students could remember them from their previous lessons, though not in detail. For example, the definition of a circle was more difficult for them to remember than any of the other mentioned concepts.

[8] Because the computer: student ratio was 9:20, I had asked the students to alternately use the mouse.
[9] They would work until the next meeting and show their result to me in the following lesson.

After that, I presented a new topic: circumference of a circle. Asking for their prior knowledge regarding the concept, I found that the students did not have a conceptual understanding of it, although they recognized π, diameter, radius, and chord from the lower grade. For example, when the students were asked what they knew about the construction of π, they could not connect it with the diameter and circumference of a circle. They did not know how a formula for the circumference of a circle is constructed, either. Then, each group was given two paper worksheets in order to work with the appropriate electronic applets prepared and installed beforehand in their computers. In the meantime, I had all the groups open the appropriate directory to launch Applet 1 on their computers (see Appendix A). Next, I gave them a brief explanation what to do with the first paper and the applet. They were supposed to explore Applet 1 and to find out the numbers needed to fill in the table given in Worksheet 1 (see Appendix W). During the lesson the students were allowed to communicate with each other and with the teacher when needed.

The first question in Worksheet 1 asked the students to find out the relationship between the perimeter of a polygon inscribed inside a circle and the circumference of the circle, based on the numbers they accumulated from interacting with Applet 1. The students first gathered these numbers to fill in the table of Worksheet 1. After filling in the table, there was an open task to describe their observations. The responses developed by the groups are described in Table 2.

Table 2: Responses based on Worksheet 1

Groups	Responses
Group Et.Gy.Z.[10]	As the number of the side increases, the perimeter is proportionally increases, too. But, the circumference of the circle remains stable (constant).
Group Mu.Ln.S.	As n increases, the circumference of the circle remains unchanged. But, the perimeter of the polygon in the greatest numbers of n increases very little and overlaps with the circle.
Group Sh.La.	As n increases, the circumference does not change. But, the perimeter increases.
Group P.Ga.	As n increases, the perimeter increases. But, the circumference of the circle does not change.
Group Aj.H.	As n increases, the perimeter changes, increases. But, the circumference of the circle does not change.
Group K.Mo.	As n increases, the perimeter of the circle increases[11]. But, its circumference does not change.
Group U.F.	As n increases, the perimeter of the circle increases. But, the circumference of the circle does not change.
Group Me.Al.	As n increases, the perimeter of the circle increases. But, the circumference does not change.
Group N.El.	No answer

[10] The capital letters show the first letters of the names of the students. In the case of the names starting with the same letters, two lettered pseudonyms are used in order to avoid confusion. A period is used between the names (letters).

[11] Three groups have written "the perimeter of the circle" in their worksheets, which meant the perimeter of the polygon.

From Table 2 it is obvious that most of the groups have the same experience as a result of the work with Applet 1. All group express what they have seen from the applet. The responses show that the students did not compare the two different rows of numbers in Worksheet 1. Only one group named Group Mu.Ln.S. went a bit further to relate the two figures (the polygon and the circle) in the applet. Despite the question asking to look into a relational aspect of the two rows of numbers, the rest of the groups were unable with the task and did not look into the insights of the numbers. Furthermore, one group (Group N.El.) has not written any response to the question in Worksheet 1. They had only filled out the table with the appropriate numbers.

Following this task, the groups went to work with Worksheet 2 (see Appendix W). This time they had to study Applet 2 in order to answer the questions asked in Worksheet 2. The first question in Worksheet 2 asked the students to reason on the ratio of the circumference of a circle to its diameter. In addition to this, they had to find out the number that ratio approaches when n becomes larger. Their responses from the Worksheet 2's are illustrated in Table 3.

Table 3: Responses based on the first task of Worksheet 2

Groups	Responses
Group Mu.Ln.S.	The ratio approaches to π and this number helps to build the formula of the circumference of the circle ($l = 2\pi R$).
Group Et.Gy.Z.	The ratio gets closer to π.
Group P.Ga.	The ratio approaches to π.
Group Me.Al.	When n increases and R remains constant, the ratio does not change.
Group K.Mo.	No answer
Group Sh.La.	No answer
Group U.F.	No answer
Group Aj.H.	No answer
Group N.El.	No answer

As is shown in Table 3, five groups have not reasoned on the numbers they filled in the two tables of Worksheet 2 as a result of the interactions with Applet 2. They have only collected the numbers through playing with the applet. However, four groups have developed answers to the first question. The answers of three groups (Group Mu.Ln.S., Group Et.Gy.Z., and Group P.Ga.) are the same and only record what they saw. They recalled the number π from their previous lessons. The response of Group Me.Al. shows that those students did not focus on the number the ratio approaches. Apparently, the students did not compare the numbers they have filled in the two separate tables. They were expected to compare the two tables to find out that the number the ratio approaches is independent of the diameter and the perimeter of a polygon when n approaches the largest value.

The second question asked by Worksheet 2 wanted the students to construct a formula computing the circumference of a circle. The respective responses are shown in Table 4.

Table 4: Responses based on the second task of Worksheet 2

Groups	Responses
Group Mu.Ln.S.	L=2πR, $\frac{l}{d}=\frac{p}{2r}$ => $l=\frac{dp}{2r}=\frac{2rp}{2r}$=p π = $\frac{l}{2r}=\frac{l}{d}$ => $l = dp = 2r\pi$ because d = 2r
Group K.Mo.	L=2πR, the circumference = D*π=D*3.14, π = $\frac{l}{d}$ = 2πR
Group U.F.	L=2πR, π = $\frac{l}{d}$ = 2πR, the circumference, l = D*π=D*3.14
Group Me.Al.	L=2πR, π = $\frac{l}{d}$ = 2πR, l = d*π=d*3.14
Group Et.Gy.Z.	The circumference = D*π=D*3.14
Group P.Ga.	the circumference = D*π=D*3.14=2πR
Group Aj.H.	the circumference = D*π=D*3.14=2πR
Group N.El.	the circumference = D*π=D*3.14
Group Sh.La.	L = π*d

From Table 4 it is seen that all the groups have written a formula for computing the circumference of a circle. Evidently, most of the groups have used the ratio to build a formula. Four groups (Group Et.Gy.Z., Group P.Ga., Group Aj.H., and Group N.El.) have the same type of answer and they reproduce the formula from the textbook. These groups did not connect the construction of the formula with their prior experience of the first task of Worksheet 2. That is, the students apparently did not evolve the formula from the ratio. However, Group Mu.Ln.S. have tried to evolve a formula from the ratio described in Applet 2. They seem to have examined the applet and developed a formula. Their answer stands out by having considered a perimeter in the ratio. Their answer also indicates that the perimeter in the greatest values of n is interpreted by them as a circumference. The Group K.Mo., Group U.F., and Group Me.Al. also seem to have derived a formula from the ratio.

Lesson on 20th of April 2009

The teaching topic on that day was about a radian measurement of an angle. The lesson started as before with a brief revision of the prior learning. Afterwards, the students were distributed in the groups of two or three and each group was given out Worksheet 3. Again, I explained briefly what they had to do with it and had them open Applet 3. In their groups the students were exploring Applet 3 by manipulating its dynamic parts and writing the observed results in their worksheets. The first task of Worksheet 3 concerned the filling in the appropriate tables. The second task in the same worksheet asked the students to make a judgment on the relationship between the numbers characterizing arc-lengths and central angles subtending them. The responses from each group are provided in Table 5.

Table 5: Responses based on the second task of Worksheet 3

Groups	Responses
Group Ln.S.	As the central angle increases, so does the arc-length.
Group K.	As the central angle increases, so does the arc-length.
Group U.F.	As the central angle increases, so does the arc-length.
Group Al.Me.	As the central angle increases, so does the arc-length.
Group H.Et.	As the central angle increases, so does the arc-length.
Group Ga.Aj.	No answer
Group N.El.	No answer
Group P.	No answer
Group Mu.Gy.Z.	No answer

In this lesson, the groups were not the same as in the previous lesson. Because some students were absent, groups were rearranged. Moreover, two students (Group K. and Group P.) were reluctant to work together and because their group mates were not present they preferred to work alone. Nevertheless, these students were interacting with the other groups sitting closer to them.

All the groups did the first task of Worksheet 3 by filling in the tables through playing with Applet 3. For the second task, as is seen from Table 5, five groups have a conclusion based on the applet while four groups have not responded except for the filling in the tables of the worksheet. The groups with a response have not related the numbers with π. They were expected to relate the given angles in degrees with the number π. For example, when the angle is 180 degree, the arc-length is equal to 3.14 or π, and they overlooked such cases. It is evident from the responses that the students did not make a connection between the central angles in degrees and the relevant arc-lengths. Although they were supposed to focus on the correlative change between the values of the central angle and the arc-length, they have retrieved from the applet the numbers characterizing the arc-lengths.

The third task of Worksheet 3 asked students on their perception of the definition of radian measurement and of the difference between the degree and radian. The answers developed by the groups are tabulated in Table 6.

Table 6: Responses based on the third task of Worksheet 3

Groups	Responses
Group Al.Me.	1 radian is equal to both the arc-length subtended by it and the radius of the circle.
Group U.F.	1 radian is equal to both the arc-length subtended by it and the radius of the circle.
Group H.Et.	As the degree of the angle increases, so does the radian of the angle.
Group K.	As the degree of the angle increases, so does the radian of the angle.
Group Ln.S.	No answer
Group Ga.Aj.	No answer
Group N.El.	No answer
Group Mu.Gy.Z.	No answer
Group P.	No answer

Apparently, five groups again have no responses to this question. Group Al.Me. and Group U.F. have somehow struggled with the question. Their answers prove that the students have misconceptions about a radian since they have not properly indicated how the radius of a circle matters here. Applet 3 was supposed to enable the students to observe the inputs of a radius here as they could change its size. But, the responses show that the students have not had enough exploration of the opportunities of Applet 3. By keeping the radius at 1 the students have made their judgments based on a unit circle. In their responses, Group H.Et. and Group K. have merely expressed their observations on Applets 3 and 4. Hence, the overall result of working with Worksheet 3 points out that the students have developed their responses based on what they saw in the applets, and thereby did not come up with an expected outcome. That is, they have not been, up to this point, able to give and justify the answers that I expected to the questions regarding a radian measurement of an angle.

Following Worksheet 3, the groups continued to work with Worksheet 4 (see Appendix W) and with the associated Applet 4 (see Appendix A). In Applet 4 the definition of a radian has been defined through a degree and an arc-length. The groups were supposed to take it into account when filling in the appropriate tables in Worksheet 4. They were also asked to consider two different values of the radius in order to make sense of its relationship with the radian and the arc-length. The first task of Worksheet 4 was to fill in the relevant tables with consideration of those two different values of the radius. Although they were asked to write their answers as a fraction of π, the students indicated their answers in decimals. Obviously, they merely collected the values by manipulating Applet 4. Again, as was in the case of Worksheet 3, the students were not able to build up a connection between the degree and the radian of an angle except for having gathered the explicit answers of the formula illustrated in Applet 4.

The second task of Worksheet 4 asked the groups to reason on the conversion process between the degree and the radian of an angle. The groups gave the responses tabulated in Table 7.

Table 7: Responses based on the second task of Worksheet 4

Groups	Responses
Group K.	Radian is equal to α.
Group N.El.	Radian is equal to α.
Group H.Et.	Radian is equal to α.
Group P.	No answer
Group Mu.Gy.Z.	No answer
Group U.F.	No answer
Group Ga.Aj.	No answer
Group Ln.S.	No answer
Group Al.Me.	No answer

The groups initially did the first task of Worksheet 4, in which they filled in the three tables with the appropriate numbers they retrieved through working with Applet 4. After that they were expected to work on the second task based on making a sense of those numbers. Table 7 shows that the tasks of Worksheet 4 did not assist the students in developing a conceptual understanding of the concept of a radian. In the course of developing a conceptual understanding, the students were supposed to examine a relationship formulated between a degree and a radian and an arc-length through Applet 4. Also, the two groups (Group Mu.Gy.Z. and Group Ln.S.) did not fill in the third table given in Worksheet 4, which concerned the conversion of radians into degrees.

During the discussion with the students, it turned out that the students were unable to understand radian and its relationship with a degree and an arc-length. Some of them memorized the definition from the textbook. However, in the case of being asked to explain its connection with a degree and an appropriate arc-length, they became at a loss to reply.

By working through Applet 3 and 4, I was expecting them to realize the definition of a radian. It is true that the students came to convert a radian into a degree and vice versa. Seeing that the students were not proceeding as expected, I intervened with asking the groups to work with the applets more carefully. They had to change the degree and the radius and then, stop to think of the values of the radian and the appropriate arch-length. After that some students discussed that the radian is not, in fact, equal to the radius or the arc length, though it is inevitable in the case of a unit circle. Nonetheless, their perceptions are not reflected in their written responses.

Lesson on 22nd of April 2009

The lesson started as usual with an arrangement of groups as was ordered by the cooperative teacher. However, the students sometimes did not seem pleased with their group partners. That is why, the girls preferred to work with each other, while the boys wanted to work with themselves. The concept to be taught concerned the area of a regular polygon and the area of a circle. Again after a short array of questions regarding the previous lesson, the groups were given Worksheet 5 in order to work with Applet 5.

The first task of Worksheet 5 wanted the students to fill in the relevant table on the paper by investigating Applet 5. All the groups performed this task through the manipulations of a dynamic part in Applet 5. The second task asked the groups to make a judgment based on the numbers they had gathered about the area of a circle and the area of a regular polygon inscribed inside the circle. The groups developed the responses which are illustrated in Table 8.

Table 8: Responses based on the second task of Worksheet 5

Groups	Responses
Group Gy.Z.	As n increases, the area of the polygon changes – increasingly gets closer to the area of the circle.
Group Al.Me.	As n increases, the polygon is approaching the circle and the area of the triangle is getting smaller.
Group Mu.Aj.	The area of the circle does not depend on n. But, the area of the polygon is approaching π and overlapping with the circle as n increases.
Group P.Ga.	As n gets greater values, the area of the polygon inscribed inside the circle increases up to π. But, the area of the circle does not change.
Group H.Et.	As n gets greater values, the area of the polygon inscribed inside the circle increases up to π. But, the area of the circle does not change.
Group Ln.S.	The area of the polygon gets a value of π. The area of the circle does not change.
Group U.F.	The area of the circle does not change. But, the area of the polygon is approaching π as n increases.
Group Mo.La.	As n increases, the area of the polygon is approaching π. The area of the circle does not change. The area of the circle does not depend on the values of n.
Group N.El.	The area of the polygon increases. But, the area of the circle remains constant.

Based on Table 8, it is obvious that the students have expressed what they found out as a result of the interactions with Applet 5. The answers they developed are based on the first task of Worksheet 5, in which they filled in a table with the appropriate numbers they retrieved from the applet. After that, the students were expected to make a small investigation in order to find out the relationship between the area of a regular polygon and the area of a circle. Studying this relationship was supposed to give them a ground in order to construct an expected formula of an area of a circle.

Obviously, the students expressed what happened with the manipulation of the applet. Six groups (Group Mu.Aj., Group P.Ga., Group H.Et., Group Ln.S., Group U.F., and Group Mo.La.) recognized that the area approached π. Also, the responses show that the students recognized the distinction between the dynamic and motionless parts of the figures. Only Group Gy.Z., Group Al.Me., and Group Mu.Aj. have met the expectations of the task by indicating that the regular polygon is getting closer to a circle. These groups seemed to have developed an understanding that the area of a circle could be gained from the area of the regular polygon inscribed inside of it.

The third task of Worksheet 5 asked the students to construct a formula in order to compute the area of a circle. The responses of the groups are described in Table 9.

Table 9: Responses based on the third task of Worksheet 5

Groups	Responses
Group Mu.Aj.	$S_\Delta = \frac{1}{2}EO * OD * sin\alpha = \frac{1}{2}R * R * sin\alpha = \frac{1}{2}R^2 sin\alpha,$ EO=OD=R $\frac{360}{n} = \alpha$, $\frac{360}{n} = \frac{2\pi}{\alpha}$, $S_{circle} = n * \frac{1}{2} * R^2 * sin\alpha, S_{circle} = \frac{360}{\alpha} * \frac{1}{2} * R^2 * sin\alpha, \lim_{\alpha\to\infty} \frac{sin\alpha}{\alpha} = 1$ $S_{circle} = \frac{\pi}{\alpha} * R^2 * sin\alpha = \pi * R^2 * \frac{sin\alpha}{\alpha}, => S = \pi R^2$
Group U.F.	$S_\Delta = \frac{1}{2}EO * OD * sin\alpha = \frac{1}{2}R * R * sin\alpha = \frac{1}{2}R^2 sin\alpha,$ $\frac{360}{n} = \alpha$ $\frac{360}{n} = \frac{2\pi}{\alpha}$ $S_{circle} = n * \frac{1}{2} * R^2 * sin\alpha, S_{circle} = \frac{360}{\alpha} * \frac{1}{2} * R^2 * sin\alpha, \lim_{\alpha\to\infty} \frac{sin\alpha}{\alpha} = 1$ $S_{circle} = \frac{\pi}{\alpha} * R^2 * sin\alpha = \pi * R^2 * \frac{sin\alpha}{\alpha}, => S = \pi R^2$
Group Mo.La.	$S_\Delta = \frac{1}{2}EO * OD * sin\alpha = \frac{1}{2}R * R * sin\alpha = \frac{1}{2}R^2 sin\alpha,$ $\frac{360}{n} = \alpha$ $\frac{360}{n} = \frac{2\pi}{\alpha}$ $S = n * \frac{1}{2} * R^2 * sin\alpha, S = \frac{360}{\alpha} * \frac{1}{2} * R^2 * sin\alpha, \lim_{\alpha\to\infty} \frac{sin\alpha}{\alpha} = \frac{2\pi}{\alpha}$ $S = \frac{\pi}{\alpha} * R^2 * sin\alpha = \pi * R^2 * \frac{sin\alpha}{\alpha}, => S = \pi R^2$
Group H.Et.	$S_n = \frac{1}{2}P_n * h_n, S = \frac{1}{2}C * R, C = 2\pi R, => S = \pi R^2$
Group P.Ga.	$S = \frac{1}{2}CR, C = 2\pi R, => S = \pi R^2$
Group Al.Me.	$S = \frac{1}{2}EO * OD * sin\alpha = \frac{1}{2}R * R * sin\alpha = \frac{1}{2}R^2 sin\alpha$ $\frac{360}{n} = \alpha, S_n = \frac{1}{2} * R^2 * sin\frac{360}{n}, S_n = 2\pi R^2$
Group Gy.Z.	$n = 3, S_\Delta = EO * OD * \frac{1}{2} * sin\alpha,$ $\frac{360}{n} = \alpha$, n= $\frac{360}{\alpha}$ $S_\Delta = \frac{1}{2} * R^2 * sin\alpha, S_{circle} = n * \frac{1}{2} * sin\alpha, \frac{360}{\alpha} = 2\pi$
Group N.El.	$S=\pi R^2$
Group Ln.S.	No answer

In response to the third task of Worksheet 3, the students nearly could construct a formula based on the work with Applet 5. The Group Mu.Aj. was the first who developed a formula with a slight support received from the teacher (also the researcher). The way they developed the formula is grounded in a key idea that the total area of the triangles composed of an inscribed regular polygon is found through a limit when the number of the triangles infinitely increases. In the course of applying the idea, I offered them my help with the finding a limit that required L'Hopital's rule, since they did not know about it yet. Indeed, a lack of this input was the only hindrance for them to arrive at the expected answer. By taking the input into consideration, they discovered a formula to compute the area of a circle.

Nevertheless, not all of the students have developed the same line of reasoning during the completion of the task. More or less, all students have indicated the answers by showing a similar way of progression. However, it is important to note that the allowed time needed to accomplish the required task was not sufficient within a period of this lesson because the question was set open to the students. This is strongly felt when the students are supposed to discover an expected outcome on their own or through collaboration and therefore, to work with receiving limited guidance from the teacher's side.

Summary of the lesson report

As is seen from the classroom events described above, the lessons did not meet the expectations towards the effective use of a DGS tool. In the first lesson, the used learning materials (applets, worksheets) did not seem as helpful as they did in the latest. The lesson halfway the intervention concerning the radian was not successful at all. Also, the time allocated for whole class discussions was not sufficient. In all, not many students appeared to benefit from the materials in the lessons, although there was by and large increasing benefit toward the end of the intervention. I assume two reasons for this. The first reason may be that learning geometric concepts in a DGS-supported student-centered medium was a new experience for the students. That is, they might need a certain period of time to get accustomed to working in such environments. The second reason may be associated with the development of the instructional materials and the activities. That is, the used worksheets and applets could have been designed and developed more coherently had the students' prior knowledge and experience been known in advance. In addition, the teaching intervention was intentionally limited in order to allow the students to take charge of their own learning. As a teacher, I tried not to intervene much while the students were working with the materials. Whenever the class discussed, I attempted to intervene in order to stop the students from wasting time on getting nowhere. My interventions were not to reveal the correct answers, but to provide them with some clues to improve their investigations. This was obvious from the discussions during the fourth lesson on the radian. However, I also had to reveal a direct answer in the last lesson in which the students were not able to progress because of a limit which they did not know yet. In this case, I had to show them how to solve it so that they could continue with their investigation.

During the lesson activities, the students were encouraged to discuss their points with each other. At the end of each lesson, whole class discussions were initiated by me. Indeed, not always was there enough time to extend the whole class discussions because of short lesson periods. For example, the lesson on the radian (the fourth lesson) was not clear to students due to the insufficient time for the discussion. During this discussion, I asked questions to see if the students had a conceptual understanding of it (if they could relate the radian to the degree and arc-length). They stuck with the definition of the radian as they memorized from the textbook. I wanted to know if they could relate the central angle with the arc-length subtended by it. The common mistake was that they all perceived that the radian of an angle is equal to the subtended arc-length. This misconception emerged from the definition based on the unit circle. During the class discussions, based on my question they explored the relevant applet and found out that their perception was not correct. Furthermore, a whole class discussion initiated at the end of the last lesson on the area of a circle enabled some students to develop conceptual understanding to some extent.

In sum, the effective role of a DGS tool in teaching and learning geometry seemed to be related to the effective role of class discussions and the support of the teacher. Whenever the class discussions were not sufficient, the students did not develop the expected understanding. On the other hand, they seemed to have a better grasp of the concepts when they had enough discussions and arguments.

7.2 Findings from the pre-and post-tests

Before the intervention, the students were asked to take a pre-test to measure their conceptual ability to solve geometric problems. The presented problems were different than the textbook exercises which usually involve a direct application of procedures. However, the problems developed for the pre-test required the students to reason before applying a procedure. Intentionally, the pre-test was supposed to provide a support to the assessment of the difference between before and after the intended lessons. Accordingly, a post-test was applied right after the conducted lessons. The problems in the pre-test and post-test were equal, except for the first two problems which only differed in their numbers, but they had an identical structure.

The students were all present on the day of the pre-test. All but one student participated in the post-test. The responses of each student on both tests were tabulated in one table in order to facilitate the comparison. This enabled to assess the changes in the problem-solving strategies of the students. The tables below give the results of some students who have relatively distinctive responses of the pre- and post-tests.

For the results of pre- and post-tests, now let's look at student Aytaj's responses:

Table 10: Responses of student Aytaj

# Problem	Responses of Pre-test	Responses of Post-test
1.	D = 50 cm =0.5 m => 450:0.5=900 Answer is 900 times	D=40 cm =0.4 m => 650:0.4=1625 Answer is 1625 times
2.	L =85 cm and d = 5 cm 85:5 =17 Answer is 17 times	L =70 cm and d=3 cm 70:3=23.3 Answer is 23.3 times
3.	No response	No response
4.	$S_1 = \pi r^2 = 3.14 * 100 = 314$ sm $S_2 = \pi r^2 = 3.14 * 400 = 1256$ sm $S = S_2 - S_1 = 1256 - 314 = 942$ sm	No response
5.	$S_6 = \frac{1}{2}$ *6*R2 *sin$\frac{360}{6}$ = R2$\frac{\sqrt{3}}{2}$	n=6, $S_n = \frac{1}{2} nR^2 sin\frac{360}{n}$, $S_6 = \frac{1}{2} * 6 * R^2sin\frac{360}{6}$ =3R2sin60=3R2$\frac{\sqrt{3}}{2}$ $S_6 = 3R^2\frac{\sqrt{3}}{2}$

Table 10 shows student Aytaj's responses, which are similar to nine other students' responses. The strategy she developed to solve the first problem remains the same in both tests. Incorrectly, the given distance was divided by the given diameter of the wheels. The length of the wheel within the given distance was not considered. A similar solution is seen in the second problem, which had a similar structure. Again, the length of a tube for the length of a thread was not compared in both tests. Instead, the student divided the length of the thread into the diameter of the tube. Both in the pre- and post-tests the solutions were procedural, applying arithmetics to the given numbers. Here we can see, that neither the regular lessons from the collaborative teacher, nor the DGS-based lessons assisted these students in solving problems that did not explicitly ask for the application of the perimeter-formula.

The third problem was not tackled in both tests. The fourth problem was attempted in the pre-test. The solution was wrong in that the student was supposed to find out the arc-lengths of the described concentric circles and consequently, their difference. Instead, the student found the area of the circles and their difference. The same problem was not tried in the post-test. The methods taken to solve these problems, show that the student did not develop a conceptual understanding concerning the characteristics of a circle. The solution of the fifth problem in the pre-test indicates the area has been found for the case of a hexagon. The student did not try to find out the area of a circle through the inscribed regular polygon with infinitely many sides. However, the response of the post-test to the fifth question shows that the student managed to generalize the procedure to some extent. But, instead of writing a general procedure, it was contextualized to a hexagon. The results of the last problem show that the student started to develop a conceptual understanding regarding the area of a circle.

Now let's look at student Murad's responses, which are only similar to one student's responses:

Table 11: Responses of student Murad

# Problem	Responses of Pre-test	Responses of Post-test
1.	D = 50 cm =0.5 m => 450:0.5=900 Answer is 900 times	D=40 cm =0.4 m => 650:0.4=1625 Answer is 1625 times
2.	85:5 =17 Answer is 17 times	70:3=23.3 Answer is 23.3 times
3.	No response	$P_s = 4a, \; d_c = \frac{l}{\pi}, \frac{P_s}{d_c} = \frac{4a}{\frac{l}{\pi}} = \frac{4a\pi}{l}$
4.	$d_1 = 10\, sm\, r_1 = 5sm$ $d_2 = 20\, sm\, r_2 = 10sm$ $S_1 = \pi r^2 = 3.14 * 100 = 314$ sm $S_2 = \pi r^2 = 3.14 * 25 = 78.5$ sm $S = S_1 - S_2 = 314 - 78.5 = 235.5$ sm	$d_1 = 10\, sm\, r_1 = 5sm$ $d_2 = 20\, sm\, r_2 = 10sm$ $S_1 = 2\pi r_1 = 2 * 3.14 * 5 = 31.4$ sm $S_2 = 2\pi r_2 = 2 * 3.14 * 10 = 62.8$ sm $S_2 + x = S_1 => \; x = 62.8 - 31.4 = 31.4$ sm
5.	$S_6 = \frac{1}{2} 6sin\frac{360}{6},$ $S_6 = \frac{1}{2}$*6R2sin60 = 3R2$\frac{\sqrt{3}}{2}$	n=6 , S - ? $S_{\Delta EOD} = \frac{1}{2} EO * OD * sin\alpha,$ $S_c = n * \frac{1}{2} * R * R *\mathrm{sin}\alpha = n * \frac{1}{2} * R^2 *\mathrm{sin}\alpha$ $\frac{360}{n} = \alpha, n = \frac{360}{\alpha} = \frac{2\pi}{\alpha}, \lim_{n\to\infty} \frac{sin\alpha}{\alpha} = 1$ $S_c = \frac{2\pi}{\alpha}\frac{1}{2} * R^2\mathrm{sin}\alpha = \pi R^2 \frac{sin\alpha}{\alpha}$ $S_c = \pi R^2$

The responses by student Murad to the first and second problem are the same as those described before with student Aytaj. The third problem was tackled in the post-test only. In his answer the student used the definition of the ratio of a circumference to a diameter in a different way than most of the students. However, a solution to the problem was expected to produce the number π. This number was expected to come out as the description of the ratio of the perimeter of a regular polygon inscribed inside a circle to the diameter. Increasing the sides of the regular polygon should approximate the ratio to the expected number. Apparently, the student did not focus on the evolvement of the ratio through the different instances of the regular polygon. Not being able to perceive the number π in this way shows that the student could not de-contextualize the concept of the ratio of a perimeter to a diameter. Therefore, the student could only develop a limited conceptual understanding of the number π.

The solution of the fourth problem in the pre-test was incorrectly based on the areas of the concentric circles, and thereby, their difference. The strategy to solve this problem was completely different in the post-test, though still incorrect. This time the computation of areas was replaced with the computation of the circumferences and thereby, their difference. The incorrectness is that a completion of the circumferences was taken into account, whereas the points A and B are stated to move by 60 degree of a rotational angle. This student was unable to use the concept of a radian which was a key concept to solving the problem.

The solution to the fifth problem in the pre-test is limited to the case of a hexagon. In the post-test, the strategy to solve the same problem was different. The student correctly developed a formula to compute the area of a circle. The process of the development indicates that a formalization of the concept of the area of a circle was reached by the student. He managed to generalize the process from the case of the hexagon to the case of a circle. Hence, the applet and the worksheet used in the fourth lesson on the area of a circle helped the student to develop an expected conceptual understanding.

And now let's look at student Elnur's responses, which are similar to five other students' responses:

Table 12: Responses of student Elnur

# Problem	Responses of Pre-test	Responses of Post-test
1.	D = 50 cm =0.5 m => 450:0.5=900 Answer is 900 times	D=40 cm =0.4 m => 650:0.4=1625 Answer is 1625 times
2.	85:5 =17 Answer is 17 times	70:3=23.3 Answer is 23.3 times
3.	No response	No response
4.	$S_1 = \pi r^2 = 3.14 * 100 = 314$ sm $S_2 = \pi r^2 = 3.14 * 400 = 1256$ sm $S = S_2 - S_1 = 1256 - 314 = 942$ sm	$d_1 = 10\ sm\ r_1 = 5sm$ $d_2 = 20\ sm\ r_2 = 10sm$ $S_1 = 2\pi r_1 = 2 * 3.14 * 5 = 31.4$ sm $S_2 = 2\pi r_2 = 2 * 3.14 * 10 = 62.8$ sm $S_2 + y = S_1 \Rightarrow y = 62.8 - 31.4 = 31.4\ sm$
5.	$S_6 = \frac{1}{2} * 6 * R^2 * sin60 = 3R^2 \frac{\sqrt{3}}{2}$	$S = \pi r^2$

From the table we see that the first and second problem was treated in the same way as the previous student did. The third problem was not attempted in both tests. However, the fourth problem was tried in both the pre-test and post-test, though incorrectly. In the pre-test, the solution was based on the areas of the given concentric circles, and therefore, finding out their difference. In the post-test the computations of the areas of the circles were replaced with computations of circumferences, and therefore, their difference. This student did not consider the arc-lengths, covered by the points A and B on the concentric circles, either.

The fifth problem in the pre-test was treated in the same way as the previous student did and the solution lacks the generalizing of the process to the case of a circle. The same problem in the post-test was responded with a ready-made formula. Here, a way to deduct the formula is not shown. Since the student has not described the way how he arrived at this result, the response does not indicate the extent to which a conceptual understanding was developed.

Summary

Generally, all the students gave similar responses to the first and second problem of both the pre- and post-test (see Appendix I). There is no difference between the methods of solution used in both tests. The students were supposed to use the circumference of a circle as an input for solving the problems. Instead, they all performed straightforward arithmetic operations on the given numbers. This points out that the students did not develop a conceptual understanding regarding the circumference of a circle, neither from the regular lessons by the collaborative teacher, nor by the DGS-based lessons. As a consequence, the students did not manage to solve the problems with respect to the circumference in the post-test. The third problem remains unanswered in both tests. Apparently, the students did not relate this problem to their classroom activities. This also indicates that they have not well understood the basis of π. However, in the class discussions it turned out that they knew about the ratio of a circumference of a circle to its diameter, that produces π from the textbook. Yet, the tests show that they still have difficulty in constructing the number themselves. The students' responses to the fourth problem show that they did not conceptually understand the radian, either. This is obvious from the solutions they have developed in both pre- and post-test. In the pre-test most of the students confused it with the area. In the post-test they considered the circumference of a circle. Not one student attempted to apply the concept of radian to the correct measurement of the appropriate arc-lengths. In the final problem, however, two students achieved correct results thanks to the appropriate classroom activities. The distinctive results of pre- and post-tests point out that the students have managed to generalize their ideas in order to find out the area of a circle.

7.3 Findings from the questionnaire

Following the post-test, a questionnaire was administered in order to identify the students' impressions and attitudes towards the implemented lessons. The questionnaire was administered to nineteen students whose responses are tabulated. The quantitative data are provided in Table 13 (in Likert scale), while the qualitative responses are given in Table 14. The numbers in Table 13 show the number of responses. Zero indicates a non-response.

Table 13: Responses based on the questionnaire

RQ	Questions	Strongly agree	Agree	Nautral	Disagree	Strongly disagree	Mean (μ)	Standard Deviation (σ)	Sum
Motivation	I liked studying geometry lessons with the GeoGebra software.	10	8	0	0	0	**4.6**	**0.5**	**19**
	The DGS helped me a lot to learn the geometry concepts taught.	5	10	2	0	0	**4.2**	**0.6**	**19**
	I prefer lessons with the textbook, not with computers.	0	0	1	10	4	**1.8**	**0.5**	**19**
	From now on, I want to learn all geometry lessons with computers.	4	10	2	0	3	**3.6**	**1.3**	**19**
	Lessons with computers are messy.	2	0	2	12	3	**2.3**	**1.1**	**19**
Discussions and interactions	I interacted with my group mates or the teacher during the lessons.	2	15	0	0	2	**3.8**	**1.0**	**19**
	I discussed the result of our group work with the other group members.	2	16	1	0	0	**4.1**	**0.4**	**19**
	I asked questions to the teacher when I did not understand something.	4	12	3	0	0	**4.1**	**0.6**	**19**
Student-centered learning activity	The lessons with the DGS did not help me to understand what ‘radians’ are.	0	1	10	6	0	**2.7**	**0.6**	**19**
	The lessons with the DGS did not help me to understand the concept of the circumference of a circle.	0	2	1	13	1	**2.2**	**0.7**	**19**
	The lessons with the DGS did not help me to understand the concepts of the area of a regular polygon and the area of a circle.	1	2	2	11	0	**2.6**	**0.9**	**19**
	The textbook helps me a lot to learn the geometry concepts.	0	2	5	6	2	**2.5**	**0.9**	**19**
	Worksheets helped us a lot to work.	3	13	0	2	0	**3.9**	**0.8**	**19**
	Applets did not help to learn the topics taught.	4	1	2	10	1	**2.8**	**1.3**	**19**
	I felt helpless when asked to explore and study the learning materials presented in the lessons.	2	8	2	6	1	**3.2**	**1.2**	**19**

The results show that a majority of the students liked studying geometry lessons with the aid of the computer (μ=4.6, σ=0.5). Only one student is antagonistic. Most students expressed that they were helped by the computer to learn the taught concepts (μ=4.2, σ=0.6). A majority of the students think that the textbook does not help a lot to learn geometry concepts (μ=1.8, σ=0.5). A great number of students were not sure about the effective role of the computer in learning the radian (μ=2.7, σ=0.6). This position is also confirmed by the conducted test results which have revealed that the students did not develop an expected understanding regarding the concept of a radian. But, regarding the effective role of the computer in learning the circumference and the area of a circle, the majority of the students have positive response (μ=2.2, σ=0.7; μ=2.6, σ=0.9). Nevertheless, the test results do not support this fact. In fact, although the students think they have understood the concept relating to the circumference of a circle, the test results as well as the relevant classroom activities contradict this. The results of the test and classroom activities also show that not all of the students have developed the same degree of understanding regarding the concept of the area of a circle.

Also, a great number of students want to learn geometry with the aid of computers in future. A minority thinks that the ICT-based lessons cause a mess (μ=2.3, σ=1.1). The majority also thinks that the worksheets were helpful in the class (μ=3.9, σ=0.8). Similarly, applets were thought to be helpful in the class. Nearly, all students confirm that they have interacted with each other or with the teacher during the classes. Correspondingly, most students say they have discussed their group work with other group members. Also, the students show that they have asked enough questions to the teacher in the classes (μ=4.1, σ=0.6). A bit more respondents agree that they were helpless during the lessons.

The qualitative responses are described in Table 14. The numbers of responses to the sixteenth and seventeenth questions have been grouped into categories of equivalent responses.

Table 14: Responses based on the questionnaire

Open questions	Responses		Number of responses
Which lesson(s) did you like most? Why?	Circumference of circle		**5**
	All of the lessons	Because they were interesting.	**5**
	Radian		**1**
	Appelt 4 and Applet 5		**1**
	GeoGebra lesson	Because it was interesting.	**3**
	None of the lessons		**1**
	Area of a regular polygon and area of a circle	Because I undesrtood the concept.	**2**
Which lesson(s) did you dislike? Why?	Radian		**5**
	Applets	Because it seemed difficult to me	**3**
	None of the lessons	I was enthusiastic in all of the lessons	**9**
	All of the lessons		**1**
	Area of regular polygon and area of circle	Because I could not understand	**1**

Summary

The findings from the questionnaire show that with the exception of one student, all students liked the geometry lessons with the computer. They preferred computer-based learning to textbook-based learning. The responses point out that the students are not sure about whether they have developed an expected understanding of the radian. However, they are positive about the concepts of a circumference and area of a circle. Also, the learning materials appear to have been helpful to them during the lessons. The computer supported learning has also had a positive effect on the students' interactions with each other as well as with the teacher. They admit that they have participated in the group and class discussions. Nevertheless, three students did not like the applets because they found them difficult to work with. And one student indicates that s/he did not understand the area of a regular polygon and area of circle. Based on their answers to open questions, the rest of the students expressed positive remarks on the computer supported geometry lessons.

7.4 Findings from the interviews

Interview with the students

At the end of the intervention an interview was conducted with the students (see Appendix II). Three groups were present in the interview. One boy and a girl represented a group. Generally, the groups have developed similar views based on the interview questions. The result of the interview indicates that the students liked the geometry lessons with the computer. They think that it helped them to interact with each other in order to discuss and share their ideas. Generally, the applets helped them to learn the taught concepts. However, the lesson on the radian was not helpful, according to the students' view. Also, the students think that they were helpless without the guidance of a teacher. Hence, they suggest that the teacher should intervene more.

Interview with the cooperative teacher

After the interview with the students, an interview was also held with the cooperative teacher (see Appendix II). The cooperative teacher did not fully participate in the lessons. For this reason, the interview with her had a limited perspective on the intervention. Her answers are only based on the short visits she paid during the lessons. Also, because she did not know much more about the computer technology, she was not confident in giving justified answers with regard to the role of the computers in students' learning. However, her answers indicate that the students appeared motivated when working with the computers. They interacted with each other more in order to discuss their views and ideas based on the visual geometrical illustrations. According to her, the applets helped the students to develop insights into the concepts.

8 Conclusions and Discussion

8.1 Conclusions

In this section I will combine the results of the different data sources (the pre- and post-tests, worksheets, questionnaires, interviews, and classroom observations) to answer the research questions. In particular, each research question is looked at from all relevant data sources. In the case of contradiction between the data sources, I will give more weight to the most objective data sources[12]. In the end, I will attempt to answer the overall research question in view of the sub-questions.

Research Question 1: *To what extent are students motivated to learn geometry with the support of DGS?*

The findings of the questionnaire show that a majority of the students liked the lessons with the DGS. Very few students expressed that they would prefer the lessons with the textbook, but not with the DGS. Hence, the overall result of the questionnaire reflects that the students had positive expressions towards the use of DGS in learning geometry.

Furthermore, the students' responses to the interview questions with regard to the motivational aspect of using the DGS generally are positive. According to the interviews, the students liked the geometry lessons with the DGS.

Also, my observations during the classroom intervention reveal that the students became motivated while learning the geometrical concepts in a DGS-based medium. This was obvious from their positive attitudes while working with the learning materials. The students' behavior throughout the intervention reflected that they liked studying geometry with computers. From the students' discussions and interactions during the lessons it was noticeable that the DGS-based learning raised the students' interest and enthusiasm toward geometry as they interacted more and more with the learning materials.

Also in the interview, the cooperative teacher confirmed that the students were highly motivated when working with the DGS.

In summary, the findings from the relevant data sources reveal that a majority of the students liked the geometry lessons with the DGS and thus, they were motivated to participate in the lessons. Nevertheless, the DGS per se may not be the only input in the

[12] The objectivity of data sources may vary depending on the research questions. For example, the pre- and post-tests and worksheets are the most objective data sources for directly evaluating research questions 4 and 5, but not for directly evaluating research questions 1, 2, and 3.

emergence of students' motivation. The emerged motivation might also be related to the other factors such as independent group work, working with worksheets, and the change of teacher.

Research Question 2: *To what extent does the use of DGS increase students' participation in overall class discussions and interactions with each other and with a teacher?*

First, the results of the questionnaire reveal that a majority of the students participated in the whole class discussions and interactions with each other. They interacted with each other during the group works and asked the questions to the teacher whenever needed. Only a few students expressed that they were not involved in the interactions with their group mates to discuss the results of their group work. As far as the interactions with the teacher are concerned, the students stressed that they asked the questions to the teacher, but did not receive his support in return.

According to the interviews with the students, the DGS helped them interact with each other in order to share and discuss their findings and ideas regarding the geometrical concepts. However, they argue that the teacher did not intervene much and therefore, they felt helpless without the guidance of a teacher.

Based on my classroom observations, the students indeed were involved in the interactions with each other during the group work. Apparently, working in the DGS-based learning medium provided the students with the visual representations of the geometrical concepts, on the basis of which they discussed and shared their ideas with each other. By interacting with the dynamic illustrations, the students, in small groups, discussed and developed their ideas in response to the worksheet tasks. As planned from the onset, such groups were expected to discuss their findings with the rest of the class. Despite the group discussions, it was not possible to provide broad class discussions at the end of each lesson due to the insufficient lesson period for this kind of lesson format.

The above findings are confirmed in the interview with the cooperative teacher, who stated that the DGS ensured the students' interactions and discussions with each other.

In summary, the overall findings of the relevant data sources show that working with the DGS provided the students with the opportunity to interact and discuss their ideas with each other within the small groups. Nevertheless, the class wide discussions were not as much achieved as the in-group interactions and discussions because of the time insufficiency. In general, the DGS may not be the only factor in increasing students' participation in overall class discussions and interactions with each other. In this sense, the worksheets as well as the group setting in which the students sat tightly to each other may be the other key factors.

Research question 3: *In what ways does DGS provide support for student-centered learning activity in a geometry class?*

According to the findings of the questionnaire, most of the students emphasized that they were independent when studying the presented learning materials during the lessons. There was not much teacher intervention during the classroom activities.

The results of the interviews with the students confirm that the students felt helpless when working with the learning materials, though they expected the guidance from the teacher.

On the other hand, my classroom observations and field notes show that the designed instructional materials (the worksheets and applets) kept the students busy with studying the intended topics in all lessons. The DGS indeed played the role of a data provider for the students in the essential course of the lessons. Based on the immediate feedbacks in response to their actions, the students explored the properties of the geometrical concepts and procedures. Throughout the lessons, they collaboratively worked with the presented learning materials without much interference by the teacher.

Also in the interview, the cooperative teacher confirmed that the students used the worksheets and applets to investigate the concepts independently of intervention by the teacher.

In summary, the students collaborated with each other when involved in the learning investigations of the designed materials in the DGS-supported learning medium. The used worksheets and applets led them to explore the geometrical concepts without much need for intervention by the teacher. Apparently, the DGS along with the worksheets and group work supported for student-centered learning activities. Yet, the students expected the teaching guidance in the course of the activities. The reason for this might have to do with the lack of enough considerations regarding the students' prior knowledge, skills and needs in the design of instructional activities. The lack of time for the class wide discussions as an essential factor for the student-centered activities should also be taken into consideration.

Research question 4: *To what extent does DGS amplify the shift from procedural to conceptual oriented understanding of geometry concepts?*

First, the findings of the pre- and post-tests show that the students on the whole did not make a significant progress in developing a conceptual understanding as a result of the classroom intervention. There was not a significant difference between the results of the pre- and post-tests in terms of the developed solution methods for the given problems relating to the circumference and the radian. The test results show that only few students achieved a conceptual understanding with respect to the area of a circle.

Nevertheless, according to the results of the interviews with the students, the students think that they developed a better understanding of the taught geometrical concepts through the support of applets.

Furthermore, in her interview the cooperative teacher confirmed that the DGS indeed provided students with visual representations to develop insights into the geometrical concepts.

The above is not confirmed by the analysis of the classroom activities based on the worksheets. These reflect that the expected outcome related to a conceptual understanding was not fully achieved by all students. Indeed the students sought and found out the discrete numbers representing the certain states of the applet illustrations. The students were expected to eventually bring together these separate numbers in such a way that a reasonable interconnection between them could be constructed. But the worksheets show that most of the students stuck to the habit of memorizing the formulas from either the textbook or their fellow students. Although they gathered the expected numbers, they did not continue their investigation to make sense of those numbers and thus, they did not perceive possible interconnections between them. This behavior was more evident in the fourth lesson on the radian in that they had developed a misconception with regard to the definition of the radian. The students who memorized the definition from the textbook thought that the radian of an angle is equal to the arc-length subtended by it. Although they found out that this case was related to the unit circle through the relevant applet illustration, they did not develop a conceptual understanding concerning the relationship between the degree and radian and the arc-length. Neither did the students achieve a conceptual understanding in the third lesson on the circumference of a circle. Unlike in the third and fourth lessons, some students developed a conceptual understanding in the final lesson on the area of a circle. These students managed to construct a formula to compute the area of a circle.

In summary, the realized DGS-based learning activities on the whole did not provide an equal support to every student to eventually achieve the same degree of understanding of the taught concepts. The findings of the available different data sources explain that the students largely failed to develop a conceptual understanding regarding the intended geometrical concepts of the circumference and radian. Only very few students appeared to develop insights into the concept of the area of a circle. Generally, there was an increasing progress in the DGS-based lessons as the students got more and more experience with this type of lesson activities.

Research question 5: *To what extent does DGS have an effect on strategies developed by students in problems solving?*

The results of the pre- and post-tests show that the students did not develop new strategies to correctly solve the problems concerning the circumference and radian. They applied more or less the same solution methods in both tests for the problems with respect to the circumference of a circle and the radian. In particular, only few students in the post-test applied a new solution strategy for the problem with respect to the area of a circle.

Nevertheless, the students in the interview expressed that the DGS-based learning activities provided them with a support in developing a conceptual understanding and problem solving strategies.

In her interview the cooperative teacher stated that the students with better conceptual understanding managed to develop different problem solving strategies.

In summary, the findings of the relevant data sources demonstrate that the students did not overcome the problems given in the tests. The reason for this may be the lack of development of conceptual understanding regarding the taught geometrical concepts. Due to the insufficient conceptual understanding, the students failed to develop new problem-solving strategies in order to solve the problems relating to the circumference and the radian. With regard to the problem relating to the area of a circle, only few of them appeared to have developed new solution strategies. On the whole, the DGS-based learning did not support all students in developing new problem-solving strategies due to the lack of conceptual understanding regarding the taught geometrical concepts. Yet, because a few students developed conceptual understanding with regard to the area of a circle, they managed to solve the problem in the post-test concerning the area of a circle.

Overall question: *How far can the cyclic process described in Figure 1. be supported by a DGS tool in a geometry class?*

Generally, the use of a DGS tool in the learning of geometry supported the students' interactions with the applets and with each other. It supported a social medium in which the students developed and shared their views. Also, each of the assumed aspects received an effective role to some extent within this cyclic process.

For the motivational aspect of the students' learning, the use of the DGS combined with the use of the worksheets as well as the organization of small-groups work raised the students' motivation to learn the geometry. Also, there was a change in the students' learning behavior during the classroom activities. The students were involved in the small-groups work in order to carry out the learning investigations based on the presented applets and worksheets.

During the group work, the students interacted with each other in order to share and discuss their findings. Nevertheless, the students were not much involved in the class wide discussions due to the time constraints.

Without much interference of the teacher the students worked on the presented learning materials through collaborating with each other. The worksheet materials helped them in working with the applets. However, the students' performance as the whole did not benefit much from the learning materials.

Not all students managed to develop a conceptual understanding of the taught geometrical concepts. Yet, there was an increasing progress noticed towards the end of the classroom intervention. In the final lesson on the area of a circle some students developed the expected conceptual understanding. This progress was also reflected in the development of the problem solving strategy during the post-test.

8.2 Discussion

In this research my aim was to study the students' new learning experiences of the geometrical concepts in the DGS-based learning environment. For this, I focused my attention on the five interrelated aspects; motivation, discussions and interactions, student-centered learning, conceptual understanding and problem-solving strategy. There was an interplay among these aspects, which was presumably triggered by the use of the DGS.

The DGS as the central tool of the learning medium was assumed to have provided new learning experiences in the geometry lessons. Unlike earlier research (Arzarello et al., 2002; Falchade et al., 2007; Gawlick, 2002; Laborde, 2001) on the different types of dragging potentials of the DGS tool in geometry learning, the use of the DGS in this research was primarily based on the use of the pre-designed applets. The applets representing the geometrical concepts restricted the possible dragging interactions. These limited interactions reduced the arbitrary dragging and helped the students to pay attention to the externalization of the representations of the geometrical concepts. The limited interactions did not reduce the supposed efforts to externalize the implicit development of the geometrical concepts.

The research findings

As the available different data sources illustrated, the expectations with respect to the effective role of the DGS on the assumed aspects were not achieved in a broad sense. The students' motivation, their interactions and discussions with each other as well as student-centered learning activities were evidently present in the intervention, but their implication for geometrical understanding was not reflected in the evaluation of the results of the pre- and post-tests as well as in the worksheets. More explicitly, although the students demonstrated positive attitudes during the fourth lesson on the radian, the findings from the worksheets, interviews, and questionnaire showed that they did not have high learning achievements from the lesson. Apparently, the motivation they seemed to gain in the lessons did not have a significant effect on the learning outcomes. This is confirmed by the results of the pre- and post-tests and the classroom activities in the sense that the students did not develop the expected conceptual understanding and relevant problem solving strategies. In particular, due to time constraints, they did not manage to explore the radian. Although they managed to perform the preliminary task of collecting the required numbers from the applets in small groups, the remaining time given for the whole class discussions was not sufficient for the groups to discuss and correct their misconceptions regarding the radian.

Unlike in the fourth lesson on the radian, the students had a little bit more time to deliver their results and findings at the class level during the final lesson on the area of a circle. By my invitation two students presented their results to the whole class in that lesson. I went to ask them to closely explain to their fellow students how they had arrived at their findings. When these students ran into a difficulty, most students gave up the struggling and turned to ask for the revealing answers from me (the teacher). Apparently, the groups did not get the expected benefit from the class wide discussions. In spite of working independently, the students expected strong guidance from the teacher during the lessons. They indeed focused on the worksheets to explore the applets, but still they expected the teacher to correct and guide their work.

From a cognitive perspective, the DGS-based learning offered the opportunity for the students to investigate the ways the geometrical concepts were conceived. This happened in the way that the DGS made it possible to realize the externalization of the hidden ideas embedded in the formation of those concepts, and thereby made them commonly sensible for all students. At this opportunity, the ideas based on visual representations were shared and discussed by them. Nevertheless, the opposing process to internalize visual experience into conceptual understanding was not much achieved due to the insufficient time for the whole class discussions.

Furthermore, the evaluation of students' answers in the worksheets showed that they carried out the empirical investigations with the applets. They gathered the required numbers that characterized the particular states of the geometrical objects. However, the students did not synthesize the gathered numbers and did not make a connection between the objects. The reason for this may be that the students were not sufficiently involved in the class discussions which could have assisted in the internalization of the external representations of the geometrical concepts (Zbiek et al., 2007). That is to say, there was not sufficient time for class wide discussions on the applet illustrations. As a whole, the external representations of the geometrical concepts through the applets were expected to provide a setting for the social interactions and discussions. This in turn was expected to ensure the internalization of the social setting. The applets in themselves would not generate the conceptual understanding. Therefore, as the results of the intervention showed, the students on the whole could not benefit from the applet illustrations due to the insufficient class wide discussions.

Nevertheless, an increasing progress of geometrical understanding was observed in the course of using the DGS. The findings from the lesson activities revealed that the students benefited more in the final lesson than in the previous lessons. However, in general not all students benefited to the same extent in this intervention. This could be due to the short intervention period which might not be enough for the students in getting used to the DGS-based learning medium. As Laborde (2001) mentioned, the integration of technology into teaching is a long process. This increasing benefit manifested itself in a few students' work in the final lesson on the area of a circle. These students managed to develop a line of reasoning on the numbers they collected from the applet on the formula of the area of a circle. While constructing the formula, the students received the needed support from me regarding the area that they had not been taught yet. With regard to the whole class discussion, these students needed sufficient time to negotiate their findings with the others. All in all, the results and findings revealed that the insufficient whole class discussion was one of limitations of the research. Along with this, there are some other underlying reasons that are related to the research setup.

The setup and limitations of the research

The setup of this research was focused on the classroom activities built on the design and development of the DGS applets, the worksheets, small-group workings as well as the class discussions. Each of these design components bears a significant implication for the interrelated aspects of the research. Furthermore, the DGS applets together with the worksheets supported the externalization process of the implicit development of the geometrical concepts. At the same time, the work in small groups and class discussions supported the internalization process of the geometrical phenomena on the basis of sharing and discussing the external representations.

The limitation of this research is apparently related to both the design of the applets and worksheets representing the geometrical concepts and the organization of the class discussions. They are related to each other in the sense that the working with the designed applets and worksheets affected the discussion process. Therefore, the designed applet illustrations may externalize the representations, but without its discussions this may not lead to geometrical understanding. At the same time, shallow or insufficient discussions may not help to take advantage of the externalized representations. Henceforward, future research should put more emphasis on both the development of external representations and the organization of sufficient discussions based on them.

Another limitation was that the intervention took place parallel to regular lessons of the collaborative teacher on the same topic. It remains unclear, in how far the parallel courses interfered with each other: whether they hindered each other of strengthened each other. If there had been improvements between the pre- and posttest, it would have been hard to ascribe these to the intervention only. In the present research, there was hardly a difference. Hence both courses did hardly assist students in the transfer of the taught content toward applying the formula of the perimeter of a circle, when not asked for explicitly.

Possible extension of the research

Future research studies with respect to DGS-based learning, in my opinion, could focus on the development of external representations and the organization of sufficient discussions based on them. Initially, the question could be formulated regarding which mental processes of understanding could be externally represented (e.g. through applets) so that the social mediations of which can increase students' geometrical understanding. At a subsequent opportunity, in what way the social mediations could be organized so that those external representations can have more effect on student's learning.

For future research, both external representations and social mediations could be developed in coherence with each other. Also, the developed external representations should correspond to the students' prior knowledge and skills. Furthermore, time should be taken into account in the organization of class discussions. Thus the students should be provided with enough time so that they can mediate and reconcile their understandings through the offered external representations.

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Appendix I Student pseudonyms

1) Aysel - Al.
2) Aytaj - Aj.
3) Elnur - El.
4) Etibar - Et.
5) Feride - F.
6) Gulnara - Ga.
7) Gunay - Gy.
8) Hasan - H.
9) Kanan - K.
10) Leman - Ln.
11) Leyla - La.
12) Mehriban - Me.
13) Murad - Mu.
14) Movlana - Mo.
15) Nesib - N.
16) Parvana - P.
17) Sevinj - S.
18) Shafiga - Sh.
19) Ulviyya- - U.
20) Zeyneb - Z.

Appendix II Pre- and post-test responses

Table 15: Responses of student Parvana

# Problem	Responses of Pre-test	Responses of Post-test
1.	D = 50 cm =0.5 m => 450:0.5=900 Answer is 900 times	650*100=65000 cm 65000:40=1625 Answer is 1625 times
2.	85:5 =17 Answer is 17 times	70:3=23.3 Answer is 23.3 times
3.	No response	No response
4.	$S_1 = \pi r^2 = 3.14 * 100 = 314$ sm $S_2 = \pi r^2 = 3.14 * 400 = 1256$ sm $S = S_2 - S_1 = 1256 - 314 = 942$ sm	No response
5.	$S_6 = \frac{1}{2}$*6R2sin$\frac{360}{6}$ = 3R2$\frac{\sqrt{3}}{2}$	n=6 , S - ? $S_n = \frac{1}{2} nR^2 sin \frac{360}{n}$, $S_6 = \frac{1}{2} * 6 * R^2sin\frac{360}{6}$ =3R2sin60=3R2$\frac{\sqrt{3}}{2}$ $S_6 = 3R^2 \frac{\sqrt{3}}{2}$

Table 16: Responses of student Gulnara

# Problem	Responses of Pre-test	Responses of Post-test
1.	D = 50 cm =0.5 m => 450:0.5=900 Answer is 900 times	650*100=65000 cm 65000:40=1625 Answer is 1625 times
2.	85:5 =17 Answer is 17 times	70:3=23.3 Answer is 23.3 times
3.	No response	No response
4.	$S_1 = \pi r^2 = 3.14 * 100 = 314$ sm $S_2 = \pi r^2 = 3.14 * 400 = 1256$ sm $S = S_2 - S_1 = 1256 - 314 = 942$ sm	No response
5.	$S_6 = \frac{1}{2}$*6R2sin$\frac{360}{6}$ = 3R2$\frac{\sqrt{3}}{2}$	n=6 , S - ? $S_n = \frac{1}{2} nR^2 sin \frac{360}{n}$, $S_6 = \frac{1}{2} * 6 * R^2sin\frac{360}{6}$ =3R2sin60=3R2$\frac{\sqrt{3}}{2}$ $S_6 = 3R^2 \frac{\sqrt{3}}{2}$

Table 17: Responses of student Gunay

# Problem	Responses of Pre-test	Responses of Post-test
1.	D = 50 cm =0.5 m => 450:0.5=900 Answer is 900 times	D=40 cm =0.4 m => 650:0.4=1625 Answer is 1625 times
2.	85:5 =17 Answer is 17 times	70:3=23.33 Answer is 23.33 times
3.	No response	$P_s = 4a, \ d_c = \frac{l}{\pi} \quad \frac{P_s}{d_c} = \frac{4a}{\frac{l}{\pi}} = \frac{4a\pi}{l}$
4.	$r_1 = 5sm \quad r_2 = 10sm$ $S_1 = \pi r^2 = 3.14 * 100 = 314$ sm $S_2 = \pi r^2 = 3.14 * 25 = 78.5$ sm $S = S_1 - S_2 = 314 - 78.5 = 235.5$ sm	$d_1 = 10\ sm\ r_1 = 5sm$ $d_2 = 20\ sm\ r_2 = 10sm$ $S_1 = 2\pi r_1 = 2 * 3.14 * 5 = 31.4$ sm $S_2 = 2\pi r_2 = 2 * 3.14 * 10 = 62.8$ sm $S_2 + y = S_1 => \ y = 62.8 - 31.4 = 31.4$ sm
5.	$S_n = \frac{1}{2}$*6R2sin$\frac{360}{6}$ $S_6 = \frac{1}{2} * 6 * R^2 * sin60 = 3R^2\frac{\sqrt{3}}{2}$	n=6, S - ? if n is increased infinitely, the area of the regular polygon gets closer to the area of the circle. $S_{\Delta EOD} = \frac{1}{2}EO * OD * sin\alpha,$ $S_c = n * \frac{1}{2} * R * R *\sin\alpha = n * \frac{1}{2} * R^2 *\sin\alpha$ $\frac{360}{n} = \alpha, n = \frac{360}{\alpha} = \frac{2\pi}{\alpha}, \lim_{n\to\infty} \frac{sin\alpha}{\alpha} = 1$ $S_c = \frac{2\pi}{\alpha}\frac{1}{2} * R^2\sin\alpha = \pi R^2\frac{sin\alpha}{\alpha}$ $S_c = \pi R^2$

Table 18: Responses of student Zeyneb

# Problem	Responses of Pre-test	Responses of Post-test
1.	D = 50 cm =0.5 m => 450:0.5=900 Answer is 900 times	D=40 cm =0.4 m => 650:0.4=1625 Answer is 1625 times
2.	85:5 =17 Answer is 17 times	70:3=23.3 Answer is 23.3 times
3.	No response	No response
4.	No response	No response
5.	$S_6 = \frac{1}{2} * 6 * R^2 * sin60 = 3R^2\frac{\sqrt{3}}{2}$	No response

Table 19: Responses of student Movlana

# Problem	Responses of Pre-test	Responses of Post-test
1.	D = 50 cm =0.5 m => 450:0.5=900 Answer is 900 times	D=40 cm =0.4 m => 650:0.4=1625 Answer is 1625 times
2.	85:5 =17 Answer is 17 times	70:3=23.3 Answer is 23.3 times
3.	No response	No response
4.	No response	No response
5.	$S_6 = \frac{1}{2} * 6 * R^2 * sin60 = 3R^2 \frac{\sqrt{3}}{2}$	No response

Table 20: Responses of student Shafiga

# Problem	Responses of Pre-test	Responses of Post-test
1.	D = 50 cm =0.5 m => 450:0.5=900 Answer is 900 times	Absent
2.	85:5 =17 Answer is 17 times	Absent
3.	No response	Absent
4.	No response	Absent
5.	$S_6 = \frac{1}{2} * 6 * R^2 * sin60 = 3R^2 \frac{\sqrt{3}}{2}$	Absent

Table 21: Responses of student Sevinj

# Problem	Responses of Pre-test	Responses of Post-test
1.	D = 50 cm =0.5 m => 450:0.5=900 Answer is 900 times	D=40 cm =0.4 m => 650:0.4=1625 Answer is 1625 times
2.	85:5 =17 Answer is 17 times	70:3=23.3 Answer is 23.3 times
3.	No response	No response
4.	No response	No response
5.	$S_6 = \frac{1}{2} * 6 * R^2 * sin60 = 3R^2 \frac{\sqrt{3}}{2}$	No response

Table 22: Responses of student Ulviyya

# Problem	Responses of Pre-test	Responses of Post-test
1.	D = 50 cm =0.5 m => 450:0.5=900 Answer is 900 times	D=40 cm =0.4 m => 650:0.4=1625 Answer is 1625 times
2.	85:5 =17 Answer is 17 times	70:3=23.3 Answer is 23.3 times
3.	No response	No response
4.	No response	No response
5.	$S_6 = \frac{1}{2} * 6 * R^2 * sin60 = 3R^2 \frac{\sqrt{3}}{2}$	No response

Table 23: Responses of student Leman

# Problem	Responses of Pre-test	Responses of Post-test
1.	D = 50 cm =0.5 m => 450:0.5=900 Answer is 900 times	D=40 cm =0.4 m => 650:0.4=1625 Answer is 1625 times
2.	85:5 =17 Answer is 17 times	70:3=23.3 Answer is 23.3 times
3.	No response	No response
4.	$S_1 = \pi r^2 = 3.14 * 100 = 314$ sm $S_2 = \pi r^2 = 3.14 * 400 = 1256$ sm $S = S_2 - S_1 = 1256 - 314 = 942$ sm	$S_1 = 2\pi r_1 = 2 * 3.14 * 5 = 31.4$ sm $S_2 = 2\pi r_2 = 2 * 3.14 * 10 = 62.8$ sm $62.8 - 31.4 = 31.4$ sm
5.	$S_6 = \frac{1}{2} * 6 * R^2 * sin60 = 3R^2 \frac{\sqrt{3}}{2}$	No response

Table 24: Responses of student Feride

# Problem	Responses of Pre-test	Responses of Post-test
1.	D = 50 cm =0.5 m => 450:0.5=900 Answer is 900 times	D=40 cm =0.4 m => 650:0.4=1625 Answer is 1625 times
2.	85:5 =17 Answer is 17 times	70:3=23.3 Answer is 23.3 times
3.	No response	No response
4.	$S_1 = \pi r^2 = 3.14 * 100 = 314$ sm $S_2 = \pi r^2 = 3.14 * 400 = 1256$ sm $S = S_2 - S_1 = 1256 - 314 = 942$ sm	No response
5.	$S_6 = \frac{1}{2} * 6 * R^2 * sin60 = 3R^2 \frac{\sqrt{3}}{2}$	No response

Table 25: Responses of student Hasan

# Problem	Responses of Pre-test	Responses of Post-test
1.	D = 50 cm =0.5 m => 450:0.5=900 Answer is 900 times	D=40 cm =0.4 m => 650:0.4=1625 Answer is 1625 times
2.	85:5 =17 Answer is 17 times	70:3=23.3 Answer is 23.3 times
3.	No response	No response
4.	$S_1 = \pi r^2 = 3.14 * 100 = 314$ sm $S_2 = \pi r^2 = 3.14 * 400 = 1256$ sm $S = S_2 - S_1 = 1256 - 314 = 942$ sm	$S_1 = 2\pi r_1 = 2 * 3.14 * 5 = 31.4$ sm $S_2 = 2\pi r_2 = 2 * 3.14 * 10 = 62.8$ sm $62.8 - 31.4 = 31.4$ sm
5.	$S_6 = \frac{1}{2} * 6 * R^2 * sin60 = 3R^2 \frac{\sqrt{3}}{2}$	$S = \pi r^2$

Table 26: Responses of student Nesib

# Problem	Responses of Pre-test	Responses of Post-test
1.	D = 50 cm =0.5 m => 450:0.5=900 Answer is 900 times	D=40 cm =0.4 m => 650:0.4=1625 Answer is 1625 times
2.	85:5 =17 Answer is 17 times	70:3=23.3 Answer is 23.3 times
3.	No response	No response
4.	$S_1 = \pi r^2 = 3.14 * 100 = 314$ sm $S_2 = \pi r^2 = 3.14 * 400 = 1256$ sm $S = S_2 - S_1 = 1256 - 314 = 942$ sm	$S_1 = 2\pi r_1 = 2 * 3.14 * 5 = 31.4$ sm $S_2 = 2\pi r_2 = 2 * 3.14 * 10 = 62.8$ sm $62.8 - 31.4 = 31.4$ sm
5.	$S_6 = \frac{1}{2} * 6 * R^2 * sin60 = 3R^2 \frac{\sqrt{3}}{2}$	No response

Table 27: Responses of student Aysel

# Problem	Responses of Pre-test	Responses of Post-test
1.	D = 50 cm =0.5 m => 450:0.5=900 Answer is 900 times	D=40 cm =0.4 m => 650:0.4=1625 Answer is 1625 times
2.	85:5 =17 Answer is 17 times	70:3=23.3 Answer is 23.3 times
3.	No response	No response
4.	$S_1 = \pi r^2 = 3.14 * 100 = 314$ sm $S_2 = \pi r^2 = 3.14 * 400 = 1256$ sm $S = S_2 - S_1 = 1256 - 314 = 942$ sm	$S_1 = 2\pi r_1 = 2 * 3.14 * 5 = 31.4$ sm $S_2 = 2\pi r_2 = 2 * 3.14 * 10 = 62.8$ sm $62.8 - 31.4 = 31.4$ sm
5.	$S_6 = \frac{1}{2} * 6 * R^2 * sin60 = 3R^2 \frac{\sqrt{3}}{2}$	$S = \pi r^2$

Table 28: Responses of student Mehriban

# Problem	Responses of Pre-test	Responses of Post-test
1.	D = 50 cm =0.5 m => 450:0.5=900 Answer is 900 times	D=40 cm =0.4 m => 650:0.4=1625 Answer is 1625 times
2.	85:5 =17 Answer is 17 times	70:3=23.3 Answer is 23.3 times
3.	No response	No response
4.	$S_1 = \pi r^2 = 3.14 * 100 = 314$ sm $S_2 = \pi r^2 = 3.14 * 400 = 1256$ sm $S = S_2 - S_1 = 1256 - 314 = 942$ sm	$S_1 = 2\pi r_1 = 2 * 3.14 * 5 = 31.4$ sm $S_2 = 2\pi r_2 = 2 * 3.14 * 10 = 62.8$ sm $62.8 - 31.4 = 31.4$ sm
5.	$S_6 = \frac{1}{2} * 6 * R^2 * sin60 = 3R^2 \frac{\sqrt{3}}{2}$	No response

Table 29: Responses of student Etibar

# Problem	Responses of Pre-test	Responses of Post-test
1.	D = 50 cm =0.5 m => 450:0.5=900 Answer is 900 times	D=40 cm =0.4 m => 650:0.4=1625 Answer is 1625 times
2.	L =85 cm and d = 5 cm 85:5 =17 Answer is 17 times	L =70 cm and d=3 cm 70:3=23.3 Answer is 23.3 times
3.	No response	No response
4.	No response	No response
5.	$S_6 = \frac{1}{2}$*6R2sin$\frac{360}{6} = 3R^2 sin60 = 3R^2\frac{\sqrt{3}}{2}$ 6R2sin$\frac{360}{6}$ Answer is S_6 = 3R2$\frac{\sqrt{3}}{2}$	n=6 , S - ? $S_n = \frac{1}{2}nR^2 sin\frac{360}{n}$, $S_6 = \frac{1}{2} * 6 * R^2sin\frac{360}{6}$ =3R2sin60=3R2$\frac{\sqrt{3}}{2}$ $S_6 = 3R^2\frac{\sqrt{3}}{2}$

Table 30: Responses of student Kanan

# Problem	Responses of Pre-test	Responses of Post-test
1.	D = 50 cm =0.5 m => 450:0.5=900 Answer is 900 times	D=40 cm =0.4 m => 650:0.4=1625 Answer is 1625 times
2.	85:5 =17 Answer is 17 times	70:3=23.3 Answer is 23.3 times
3.	No response	No response
4.	$S_1 = \pi r^2 = 3.14 * 100 = 314$ sm $S_2 = \pi r^2 = 3.14 * 400 = 1256$ sm $S = S_2 - S_1 = 1256 - 314 = 942$ sm	No response
5.	$S_6 = \frac{1}{2} * 6 * R^2 * sin60 = 3R^2\frac{\sqrt{3}}{2}$	No response

Table 31: Responses of student Leyla

# Problem	Responses of Pre-test	Responses of Post-test
1.	D = 50 cm =0.5 m => 450:0.5=900 Answer is 900 times	D=40 cm =0.4 m => 650:0.4=1625 Answer is 1625 times
2.	85:5 =17 Answer is 17 times	70:3=23.3 Answer is 23.3 times
3.	No response	No response
4.	No response	No response
5.	$S_6 = \frac{1}{2} * 6 * R^2 * sin60 = 3R^2 \frac{\sqrt{3}}{2}$	No response

Appendix III Interview responses

<u>Interview with the students</u>

1) *Did you like the lessons taught with the support of DGS?*
- Yes we did. We liked learning geometry with the computer. The software helped us.
2) *In what way do you think the DGS helped you in your learning process? Be specific.*
- GeoGebra software helped me to better understand the concepts. It visualized the geometric objects. Especially, it helped us learn the area of a circle. Because we deduced the formula ourselves.
3) *Which lesson was the best in your opinion? Why?*
- All the lessons were good. But, the last lesson was the best because we understood it better.
4) *Which lesson was not good? Why?*
- The lesson on radians was not good because it was a difficult lesson.
5) *Did you interact with your class mates or with the teacher during the lessons?*
- Yes, we did. We discussed our findings with each other
6) *Did you join the group as well as whole class discussions?*
- Yes, we discussed our findings with the rest of the class.
7) *Did you receive a support from the teacher?*
- When we did not understand something we asked the teacher. But, he did not help us sufficiently.
8) *Did the applets help you to better understand the concepts taught? How?*
- Yes, we think the applets were good to play with the geomteric objects. And this playing helped us to better understand the concept.
9) *How well did the applets affect your conceptual (deep) thinking rather than procedural (ritual)?*
- The concepts helped us understand the concepts.
10) *What suggestions would you give in order to improve lessons like these?*
- It would be better if the teacher helped us more.

Interview with the cooperative teacher

1) *In your opinion what degree of importance does the use of DGS attach to the motivation of students in studying geometry?*
- The students evidently appeared to be enthusiastic with learning under the support of computers.

2) *In comparison with the traditional way of teaching, what are the differences with the DGS supported way in terms of increased interactions and participation in class discussions?*
- The computer based learning provides more data for the students to discuss with each other.

3) *Do you think that DGS can be supportive for student centered learning activities for geometry class? Why?*
- I think so. The students alone can investigate the topics based on the prepared instructions.

4) *In your opinion, how much can DGS be helpful in increasing conceptual (deep) understanding of geometry concepts? Do you think it is a good tool to help students develop a conceptual understanding in learning geometry rather than pure procedural? Why?*
- The DGS-based learning provides visual and dynamic illustrations which brings inner ideas to the surface. So it helps them to understand how procedures are developed from the concepts.

5) *How relevant do you think DGS is in improving students' problem solving strategies? Why?*
- Problem solving strategies are connected to the conceptual understanding. Once the students better understand the taught concepts they eventually demonstrate better performance in solving problems.

Appendix IV Lesson plans

I. LESSON PLAN

Subject: *Introduction to GeoGebra software*
Duration of lesson: *45 minutes*
Target group: *9th graders (14-15 years old)*

Teachers: *R. Mehdiyev*
Cooperating teacher: *F. Quliyeva*
Date: *13th of April 2009*

****

I. Prerequisites:

Students are familiar with basic computer operations. They are supposed to be able to use mouse and keyboard as inputs and to monitor corresponding outputs on the screen.

II. Required Materials:

GeoGebra 3.0.0.0 is required to be installed in computers. To run the software independently of the multiple platforms needs Java installation.

III. Learning Objectives:

At the end of the first lesson the students should be able to:

- recognize the working environment and some menu and toolbar
- recognize some basic tools in order to be able to draw basic geometric objects

IV. Activities:

<u>Lesson</u>

1) GeoGebra 3.0.0.0 and Java (if necessary) are installed in the computers in the presence of students. After a brief explanation about the philosophy of dynamic geometry software, the students are arranged to sit whether in pairs or individually depending on the number of computers available. Subsequently, the students follow the instructions of the teacher and apply them in the computers. The instructions are based on the GeoGebra introductory book taken from the official website of GeoGebra[13]. For introducing students to GeoGebra the first and second chapters are primarily covered.
2) After getting to know about GeoGebra, the students under support of the teacher are encouraged to draw basic geometric objects.

[13] http://www.geogebra.org/book/intro-en.pdf

II. LESSON PLAN

Subject: *Introduction to GeoGebra software*
Duration of lesson: *45 minutes*
Target group: *9th graders (14-15 years old)*

Teachers: *R. Mehdiyev*
Cooperating teacher: *F. Quliyeva*
Date: *15th of April 2009*

****

I. Prerequisites:

Students are familiar with some menu and toolbar functions and can draw some basic geometric objects in GeoGebra.

II. Required Materials:

GeoGebra 3.0.0.0 is required to be installed in computers. To run the software independently of the multiple platforms needs Java installation.

III. Learning Objectives:

At the end of the lesson the students should be able to:

- construct basic geometric objects (triangle, circle, polygon)
- construct and use motion slider

IV. Activities:

Lesson

1) The students are first asked questions about their prior learning and activities. Then they are involved in drawing basic geometric objects such as triangle, circle, and polygon.
2) They continue working with GeoGebra under instructions and support of the teacher.

III. LESSON PLAN

Subject: *Circumference of circle*
Duration of lesson: *45 minutes*
Target group: *9th graders (14-15 years old)*

Teachers: *R. Mehdiyev*
Cooperating teacher: *F. Quliyeva*
Date: *16th of April 2009*

****

I. Prerequisites:

Students are familiar with concepts of circle, radius, and degree of an angle, and diameter, perimeter, regular polygon that is both inscribed and circumscribed of the circle and rational as well as irrational numbers.

II. Required Materials:

The students work both on the computer and use a paper worksheet to write down their result and findings. The lessons are based on working with applets designed by me in GeoGebra.

III. Learning Objectives:

At the end of the lesson the students should be able to:

- compute the circumference of a circle,
- recognize the number π and understand where its constancy comes from

IV. Activities:

Lesson

1) At first the students are asked some questions in order to revise their prior knowledge about the circle, its radius and diameter as well as the computations of the radiuses of the circles both inscribed inside and circumscribed around the given regular polygon.
2) Later the students work in pairs with the first applet (I. Circumference of circle) with the help of the teacher whenever necessary. Each group is given a worksheet to write down their findings as developed through the interactions with computer applets.
3) Each group continues working with the second applet (II. Circumference of circle), again under guidance of the teacher. The pairs are encouraged to discuss their views and reflections and to prepare the verbalization for the whole class once they are finished. The earlier groups are given extra exercises to do to strengthen their findings. For extension work the groups finished earlier are asked to attempt to reconstruct the used applets on their own. When all the groups are finished with working on the applets they are asked to present their results in a class discussion.
4) At the end of the class within whole class discussions students are supposed to arrive at commonly shared knowledge with only slight guidance of the teacher.

IV. LESSON PLAN

Subject: *Radian measurement of angle*
Duration of lesson: *45 minutes*
Target group: *9^{th} graders (14-15 years old)*

Teachers: *R. Mehdiyev*
Cooperating teacher: *F. Quliyeva*
Date: *20^{th} of April 2009*

****

I. Prerequisites:

Students are familiar with concepts of the circumference of circle and the degree of an angle, and know about π number.

II. Required Materials:

The students work both on computer and paper worksheet to write down their result and findings. The lessons are based on working with applets designed in GeoGebra.

III. Learning Objectives:

At the end of the lesson the students should be able to:

- compute the radian of an angle
- convert degree into radian and vice versa

IV. Activities:

<u>Lesson</u>

1) In the beginning of the lesson, the students are given various questions about the previous lesson and learning with the emphasis on number π.
2) Then the students again work in pairs on the third applet (I. Radian of an angle) with the help of the teacher whenever necessary. The students are encouraged to write down their result and findings. They continue working on the fourth applet (II. Radian of an angle). Later each group is encouraged to discuss their views and reflections and to prepare the verbalization for the whole class once they are finished. Groups finished early are given extra exercises to do to strengthen their findings. In this extension work, students are asked to attempt to reconstruct the used applets on their own. When all the groups are finished with working on the applets they are asked to present their results in a class discussion.
3) At the end of the class within the whole class discussions the students are supposed to arrive at commonly shared knowledge under a slight guidance of the teacher.

V. LESSON PLAN

Subject: *Area of regular polygon, area of circle*
Duration of lesson: *45 minutes*
Target group: *9th graders (14-15 years old)*

Teachers: *R. Mehdiyev*
Cooperating teacher: *F. Quliyeva*
Date: *22nd of April 2009*

****

I. Prerequisites:

Students are familiar with concepts of the circumference of circle, and the radiuses of the circles both inscribed inside and circumscribed around the given regular polygon.

II. Required Materials:

The students work both on computer and paper worksheet to write down their result and findings. The lessons are based on working with applets designed in GeoGebra.

III. Learning Objectives:

At the end of the first lesson the students should be able to:

- compute the area of regular polygon
- compute the area of circle

IV. Activities:

<u>Lesson</u>

1) In the beginning, the students are involved in a revision of their previous learning and activities as well as thinking over the relevant questions concerning the area of basic geometric shapes such as triangle, rectangle etc.
2) Subsequently, the students are set to work in pairs on the fifth applet (Area of circle) with the help of the teacher whenever necessary. The students are encouraged to write down their results and findings. Each group is encouraged to discuss their views and reflections and to prepare the verbalization for the whole class once they are finished. The earlier groups are given extra exercises to do to strengthen their findings. For extension work the groups finishing earlier are asked to attempt to reconstruct the used applets on their own. When all the groups are finished with working on the applets they are asked to present their results in a class discussion.
3) At the end of the class within the whole class discussions the students are supposed to arrive at commonly shared knowledge under a slight guidance of the teacher.

Appendix V Worksheets

Names: ____

Grade level: _________

Date: _____/2009

Worksheet 1

Task: Working with the applet 1, fill in the table below with the appropriate data you make from the applet. After you have filled in the table with the appropriate data, based on those data what can you infer from the values the perimeter and circumference receive while *n* gets greater?

n	5	10	15	20	25	30
Perimeter						
Circumference						

Names: ____

Grade level: _________

Date: _____/2009

Worksheet 2

Task 1: Working with the applet 2, fill in the table below with the appropriate data you make from the applet. After you have filled in the table with the appropriate data, based on those data what sense can you make of the values of the ratio as *n* becomes greater and greater? What can you say about the characteristics of the ratio as *n* approaches its largest value?

Fill in when *R = 1*.

n	**5**	**10**	**20**	**30**	**35**	**40**
R	1	1	1	1	1	1
Perimeter						
Diameter						
Ratio						

Do the same operation when *R* is equal to any other possible value.

n	**5**	**10**	**20**	**30**	**35**	**40**
R						
Perimeter						
Diameter						
Ratio						

Task 2: Can you construct the formula to determine the circumference of circle?

Names: ____

Grade level: _________

Date: _____/2009

Worksheet 3

Task 1: Working with the applet 3, fill in the table below with the appropriate data you make from the applet.

Fill in when *R = 1*.

α	**60^0**	**120^0**	**180^0**	**240^0**	**300^0**	**350^0**
R	1	1	1	1	1	1
Arc-length						

Do the same operation when *R* is equal to any other possible value.

α	**60^0**	**120^0**	**180^0**	**240^0**	**300^0**	**350^0**
R						
Arc-length						

Task 2: Based on the data you have found for the arc length in both tables above in different values of *R*, what relations can you make between the arc-length and the angle subtending it?

Task 3: What in your opinion is radian? What is the difference between two types of angle measurement, degree and radian?

Names: ____

Grade level: _________

Date: _____/2009

Worksheet 4

Task 1: Working with the applet 4, fill in the table below with the appropriate data you make from the applet.

Fill in when *R* = *1.* Keep values of radian to π.

R	1	1	1	1	1	1
α	**60^0**	**120^0**	**180^0**	**240^0**	**300^0**	**350^0**
Arc-length						
Radian						

Do the same operation when *R* is equal to any other possible value. Keep values of radian to π.

R						
α	**60^0**	**120^0**	**180^0**	**240^0**	**300^0**	**350^0**
Arc-length						
Radian						

Find in the values of α in degree.

Radian	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$2\frac{\pi}{3}$	$\frac{\pi}{6}$	$3\frac{\pi}{4}$	π
α						

Task 2: Based on the data you have filled in the tables, can you explain how the conversion between two types of measurement, the degree and radian works?

Names: ____

Grade level: _________

Date: _____/2009

Worksheet 5

<u>Task 1:</u> Working with the applet 5, fill in the table below with the appropriate data you make from the applet.

n	**10**	**20**	**40**	**60**	**70**	**80**
Area of the polygon						
Area of the circle						

<u>Task 2:</u> Based on the data you have filled in the table, what can you judge regarding the areas of the polygon inscribed inside the circle and of the circle itself as *n* gets greater values?

<u>Task 3:</u> Can you construct the formula to compute the area of a circle? (Consider that α gets closer to *0* as the number of sides of the polygon becomes greater and greater.)

Appendix VI Pre- and post-tests

Name: __________________

Grade level: _________

Date: _____/2009

Questions for pre-and post-tests

<u>Solve the following problems:</u>

1) The diameter of the wheels of a bicycle is 50 cm. In order to get a distance 450 m, how many revolutions at least must the wheels make?
2) If you have an 85 cm. long thread, how many times can you wrap it around the tube with a diameter 5 cm?
3) Find out ratio of the perimeter of the square inscribed inside the circle to the diameter. Do the same thing with regular 8-gons, 36-gons. What is the approximating value?
4) Two points *A*, *B* are situated on the different circles (Fig. 1). The diameters of the circles are 10 and 20 cm, respectively. The points start to move at the same time counter clockwise by 60^0 of rotational angle. What extra way should the point *A* go in order to cover the same arc length like point *B* (π = 3.14)?

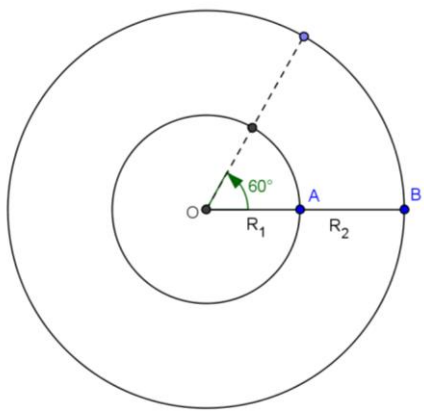


Figure 1.

5) In the picture below (Fig. 2) a regular polygon (hexagon) is inscribed inside a circle. Knowing R radius of the circle, can you find the area of the circle? (Hint: if you increase the number of sides of the polygon to the greater value, the area of that polygon can entirely fill inside the circle).

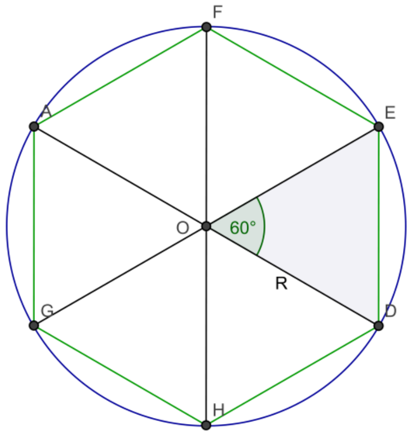


Figure 2.

Appendix VII Questionnaire and interview questions

Grade level: ___________

Date: ________/2009

Gender: ☐ Male ☐ Female

Questionnaire

The purpose of this questionnaire along with keeping anonymity of the respondents is only serving to deeper understanding of results of the planned research project intervention within the secondary school context in Azerbaijan. Your truly responses are highly appreciated and of high importance to the accuracy of the evaluation process of the research data.

RQ	Questions	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
Motivation	I liked studying geometry lessons with GeoGebra software.					
	The DGS helped me a lot to learn the geometry concepts taught.					
	I prefer lessons with the textbook, not with computers.					
	From now on, I want to learn all geometry lessons with computers.					
	Lessons with computers are messy.					
Discussions and interactions	I interacted with my group mates or the teacher during the lessons.					
	I discussed the result of our group work with the other group members.					
	I asked questions of the teacher when I did not understand something.					
Student-centered learning activity	The lessons with the DGS did not help me to understand the concepts of the area of a regular polygon and the area of a circle.					
	The lessons with the DGS did not help me to understand the concept of the circumference of a circle.					
	The lessons with the DGS did not help me to understand what 'radians' are.					
	The textbook helps me a lot to learn the geometry concepts.					
	Worksheets helped us a lot to work.					
	Applets did not help to learn the topics taught.					
	I felt helpless when asked to explore and study the learning materials presented in the lessons.					

Which lesson(s) did you like most? Why?

Which lesson(s) did you dislike? Why?

Interviewee: ___________________

Date: ________/2009

Interview with cooperative teacher

1) In your opinion what degree of importance does the use of DGS attach to the **motivation** of students in studying geometry?

2) In comparison with the traditional way of approach, what are the differences with the DGS supported study in terms of increased **interactions** and **participation** in class discussions?

3) Do you think that DGS can be a supportive means of **student centered learning activities** for geometry class? Why?

4) In your opinion, how much can DGS be helpful in increasing **conceptual (deep)** understanding of geometry concepts? Do you think it is a good tool to help students develop **conceptual** approach in learning geometry rather than pure **procedural**? Why?

5) How relevant do you think DGS is in improving students' **problem solving strategies**? Why?

Interviewee: ___________________

Date: ________/2009

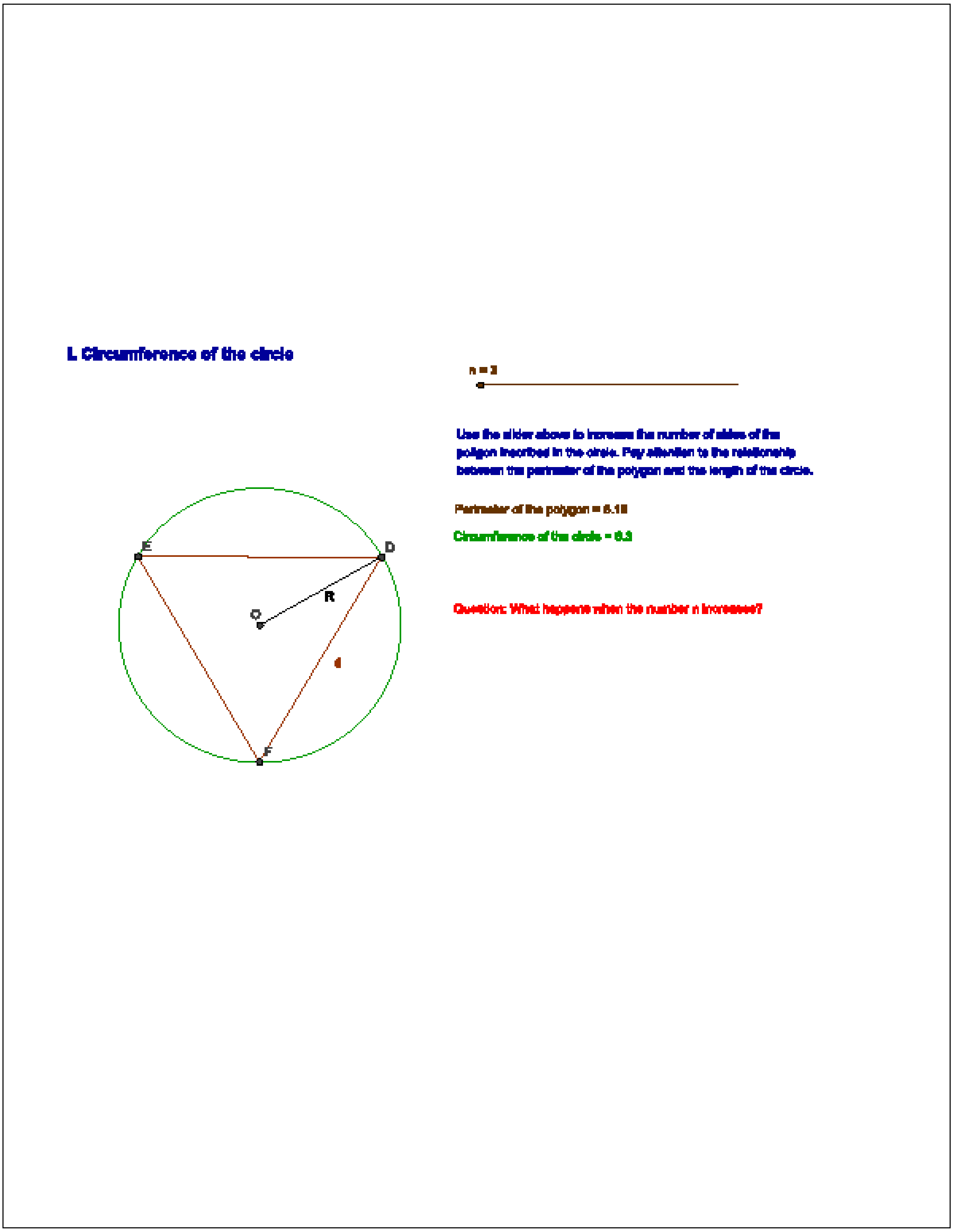
Interview questions with students

The purpose of this interview along with ensuring the anonymity of the respondents also serves acquire a deeper understanding the intervention within the secondary school context in Azerbaijan. Your honest responses are highly appreciated.

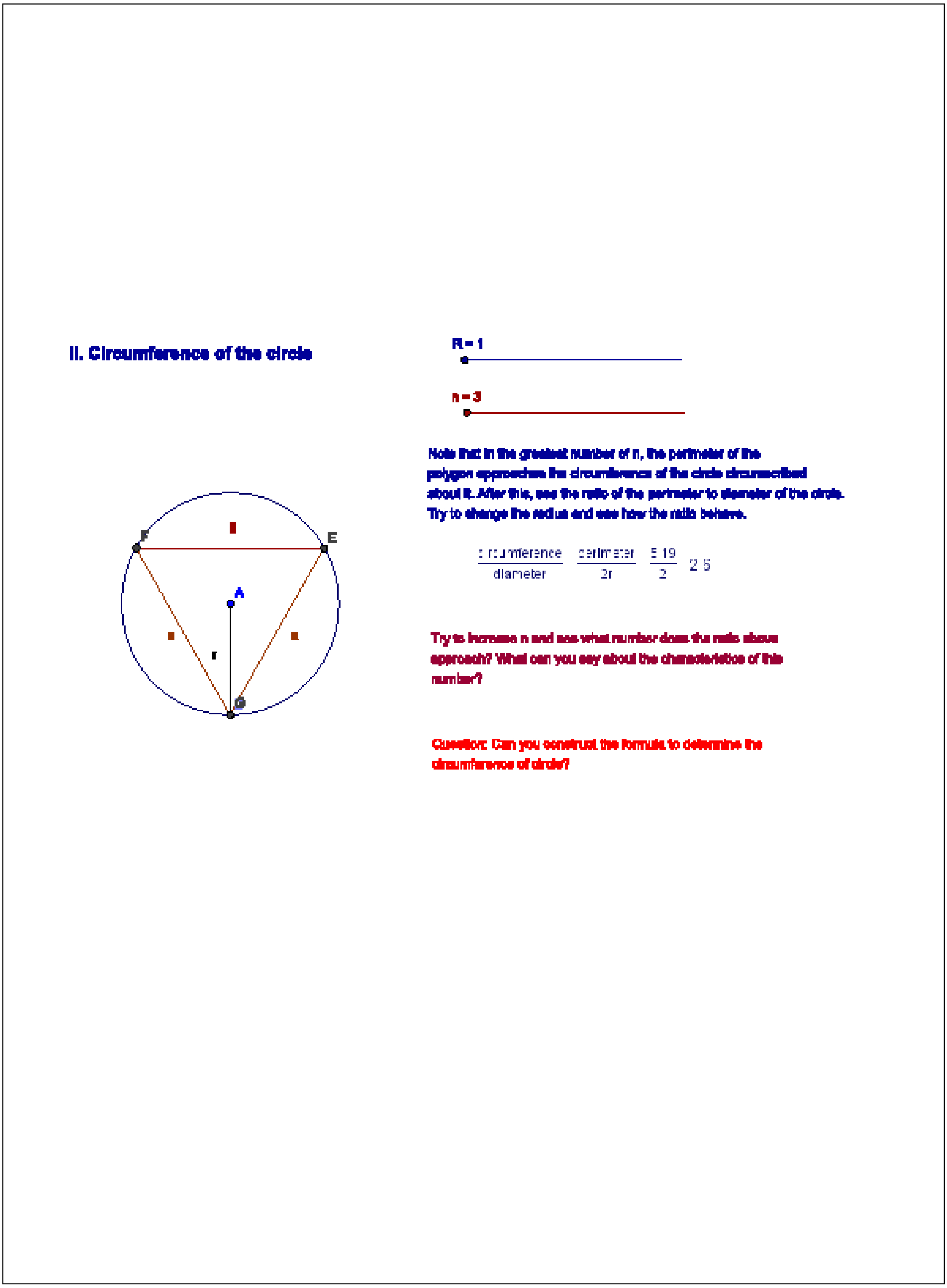
1) Did you like the lessons taught with the support of DGS?
2) In what way do you think the DGS helped you in your learning process? Be specific.
3) Which lesson was the best in your opinion? Why?
4) Which lesson was not good? Why?
5) Did you interact with your class mates or with the teacher during the lessons?
6) Did you join the group as well as whole class discussions?
7) Did you receive a support from the teacher?
8) Did the applets help you to better understand the concepts taught? How?
9) How well did the applets influence on your conceptual (deep) thinking rather than procedural (ritual)?
10) What suggestions would you give in order to improve lessons like these?

Appendix VIII Applets

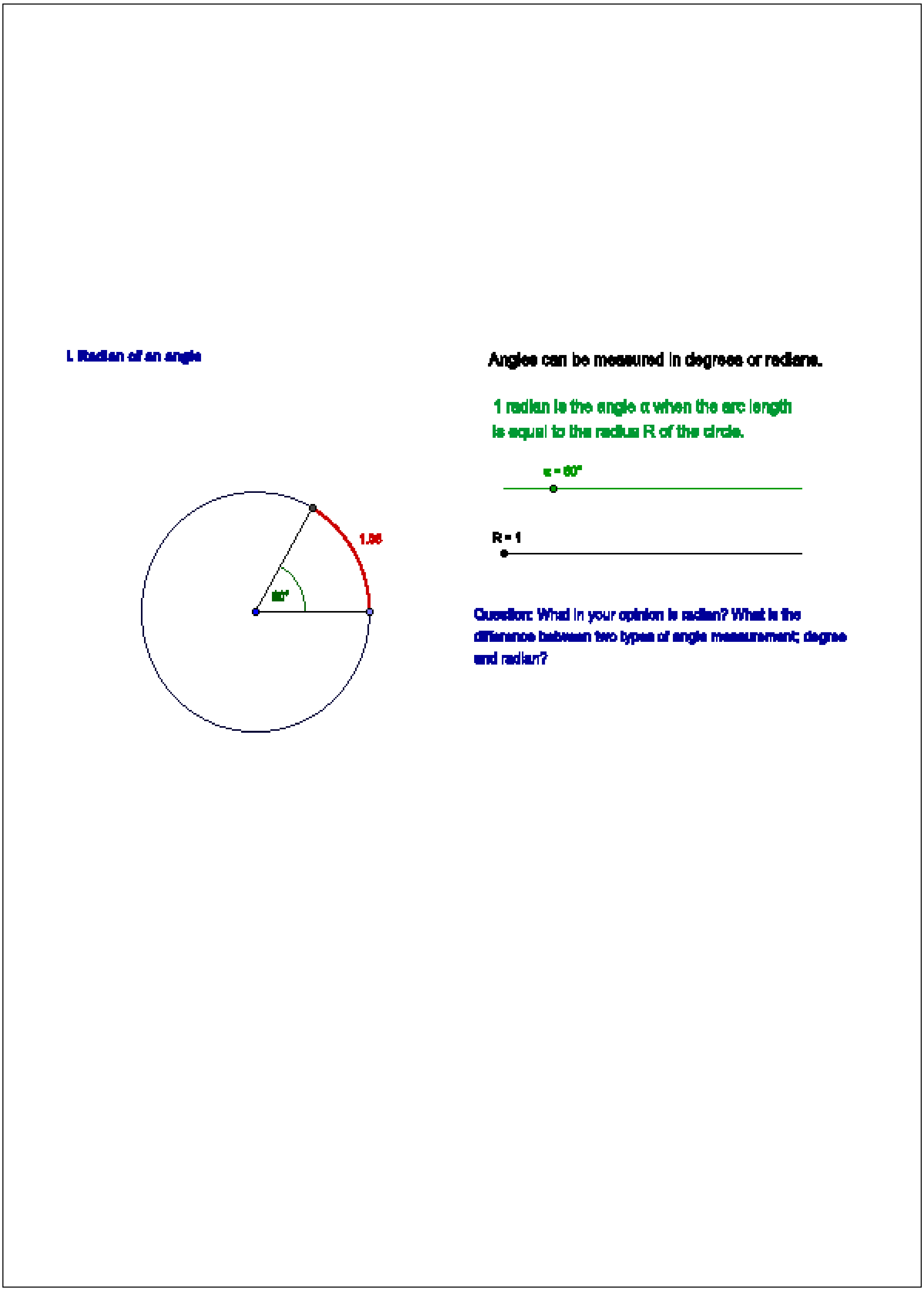
<u>Applet 1.</u>



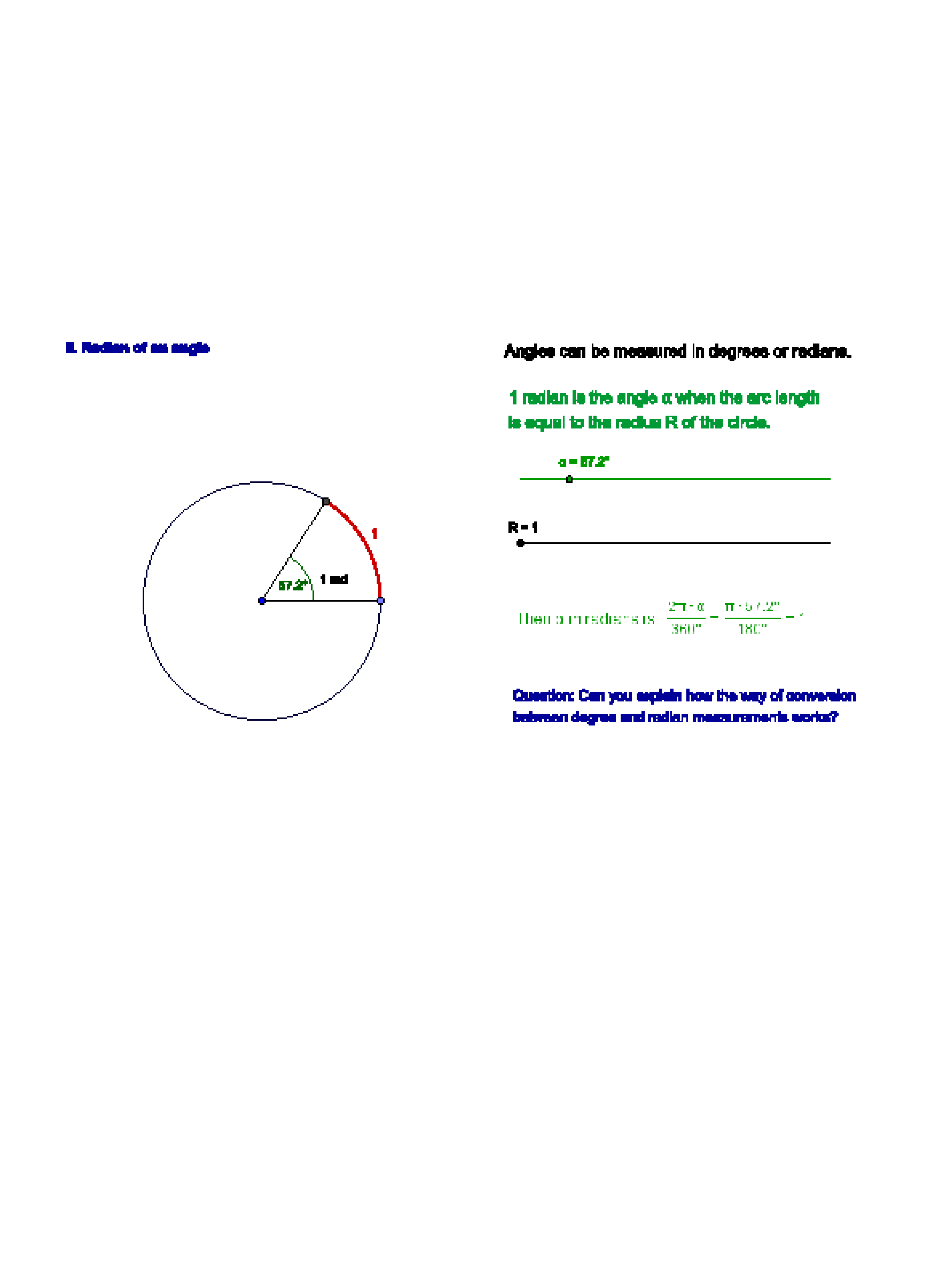
Applet 2.


II. Circumference of the circle
R = 1
n = 3
F
E
A
r
G
Note that in the greatest number of n, the perimeter of the polygon approaches the circumference of the circle circumscribed about it. After this, see the ratio of the perimeter to diameter of the circle.
Try to change the radius and see how the ratio behave.
circumference
diameter
perimeter
2r
5 19
2
2 5
Try to increase n and see what number does the ratio above approach? What can you say about the characteristics of this number?
Question: Can you construct the formula to determine the circumference of circle?

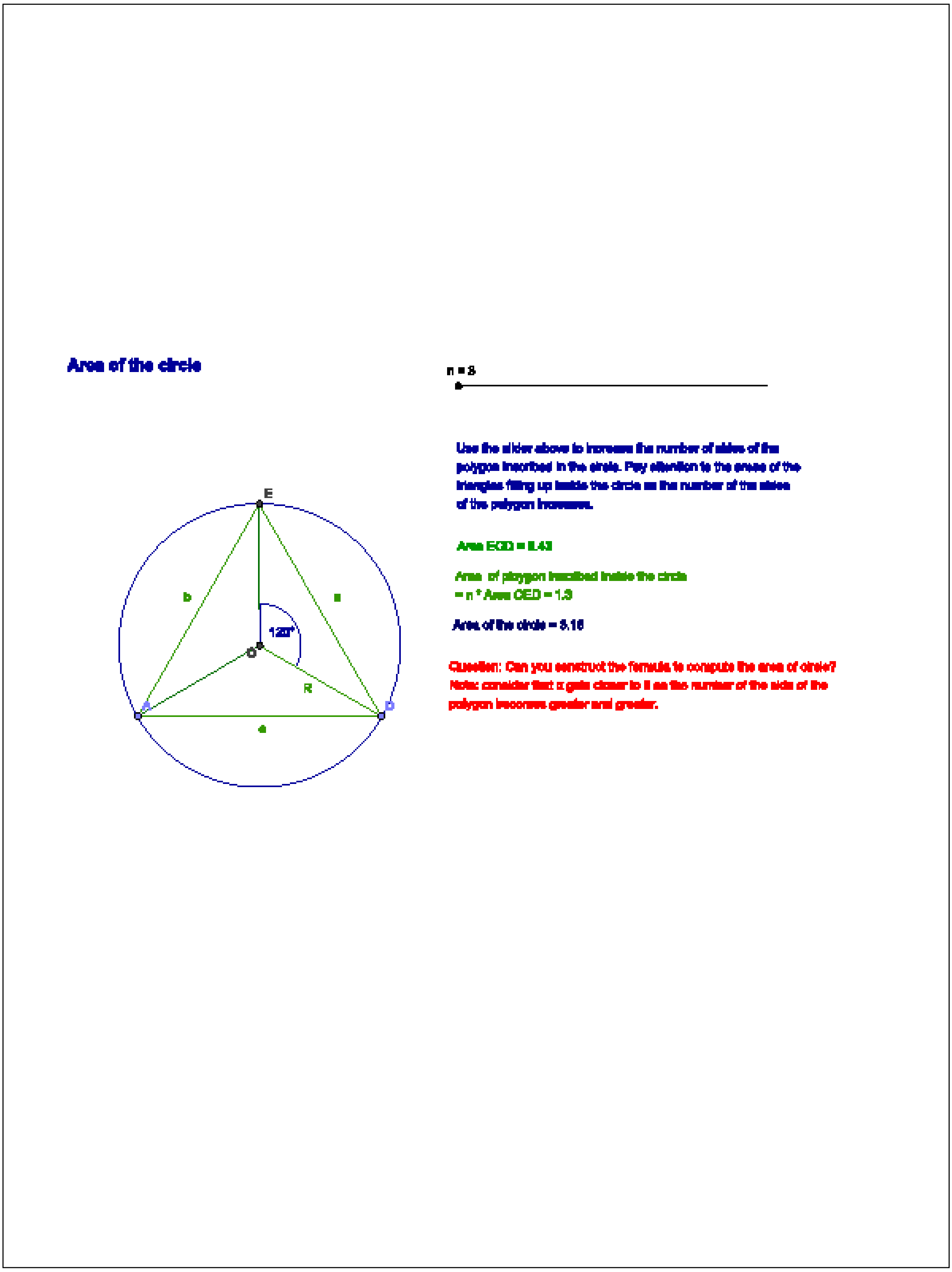
Applet 3.


I. Radian of an angle
Angles can be measured in degrees or radians.
1 radian is the angle α when the arc length
is equal to the radius R of the circle.
α = 60°
R = 1
1.05
60°
Question: What in your opinion is radian? What is the
difference between two types of angle measurement; degree
and radian?

Applet 4.


Radian of an angle
Angles can be measured in degrees or radians.
1 radian is the angle α when the arc length
is equal to the radius R of the circle.
α = 57.2°
R = 1
1
57.2°
1 rad
Then α in radians is $\frac{2\pi\cdot\alpha}{360°} = \frac{\pi\cdot 57.2°}{180°}$
Question: Can you explain how the way of conversion
between degree and radian measurements works?

Applet 5.


Area of the circle
n = 3
Use the slider above to increase the number of sides of the polygon inscribed in the circle. Pay attention to the areas of the triangles filling up inside the circle as the number of the sides of the polygon increases.
Area EOD = 0.43
Area of ploygon inscribed inside the circle = n * Area OED = 1.3
Area of the circle = 3.16
Question: Can you construct the formula to compute the area of circle?
Note: consider that α gets closer to 0 as the number of the side of the polygon becomes greater and greater.
E
b
a
120°
O
R
A
D
c