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Complementary Platforms

Patrick Van Cayseele and Jo Reynaerts

Abstract

This article investigates the pricing decisions in two-sided markets when several platforms are needed simultaneously for the successful completion of a transaction. The results indicate that the anticommons problem generalizes to two-sided markets. On the other hand, the limit of an atomistic allocation of property rights is not monopoly pricing.

KEYWORDS: two-sided markets, complements, the anticommons problem, advertising

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1 Introduction

On December 20, 2005 the Belgian Antitrust Authority approved of a merger between the only two remaining financial newspapers in the country, the Dutch language "De Tijd" and the French language "L'Echo." In this particular two-sided market, i.e. the Belgian market for financial and legal advertising, both papers act as platforms respectively connecting Flemish (Dutch speaking) and Walloon (French speaking) readers with companies that want to convey information, e.g. an announcement for the general assembly to be held in the near future. To protect investors' interests, companies located in Belgium are required by law to disseminate their information in a non-discriminatory manner.[1] Accordingly, companies simultaneously buy advertisement space in "De Tijd" and "L'Echo" to interact with respectively Dutch- and French-speaking investors. Both newspapers thus are necessary (and hence complementary) inputs into the provision of corporate information, forcing companies to simultaneously interact over both platforms (*multi*home). Investors from either linguistic regime on the other hand *single*home; they buy a single financial newspaper and through its complementary nature, consequently stay informed on all companies' activities. Figure 1 provides a schematic overview of this particular industry.

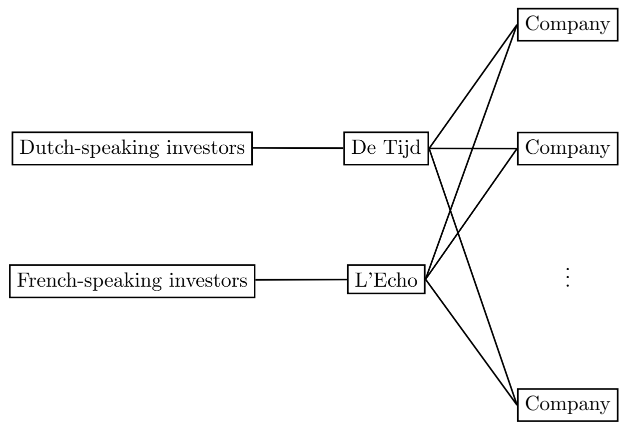
This example of newspapers serving as exclusive information gateways to two separate groups of consumers is a two-sided analogue to the classic (one-sided) complementary monopoly theory developed by Cournot (1838) and Ellet (1839) where two producers have a monopoly on goods that are complements in the production of a third composite good.[2] The striking conclusion of this theory is that welfare in such an industry *decreases* with the number of individual producers, a result also known as the *anti*commons problem.[3] The question therefore arises

[1] Art. 35, §1 of the Royal Decree of 14 November 2007 concerning the obligations for emittents of financial instruments allowed to trade on a regulated market, stipulates that (translated from Dutch) "Information must be made public (*i*) in a fast and non-discriminatory manner, (*ii*) so as to reach an audience as large as possible, (*iii*) in such a way that it becomes simultaneously available in Belgium and other member states. [...] In doing so, issuers rely on media for which it is reasonable to assume that they deliver an effective dissemination of the information within the entire European Economic Space."

[2] Cournot's example details the production of brass (the composite good), requiring zinc and copper (the inputs monopolized by separate producers). Ellet describes navigation on a canal (the composite good) the consecutive segments of which (the inputs) are owned by separate individuals.

[3] Stated otherwise, under non-cooperative complementary monopoly the equilibrium price level of the composite good exceeds its corresponding monopoly, or perfectly collusive complementary monopoly, level. Whereas the more generally known problem of the commons stems from inadequately defined property rights leading to *over*usage of the common resource (see e.g. Hardin, 1968), the anticommons problem is exactly opposite: the negative externality—the *under*usage of

Figure 1: Belgian financial newspapers



whether (and to what extent) the anticommons problem extends to two-sided markets when platforms are perfect complements in the mediation of a transaction between cross-market agents, or, coming back to the financial newspapers example, should the proposed merger indeed have been cleared by the competent authorities?

The literature on two-sided markets (see e.g Parker and Van Alstyne, 2005, Rochet and Tirole, 2003, 2006, Armstrong, 2006, Caillaud and Jullien, 2001, 2003) conceives of information and communication channels (or more succinctly, magazines) as platforms connecting two distinct sides of the same market. On one side, there is a group of consumers with informational needs. They buy a magazine to find out about their field of interest. This is the magazine's readership and constitutes the "target group" to producers located on the other side of the market. This side of the market wants to "get information across," and does so by buying advertising space in magazines. Throughout this paper we will refer to the reader and advertiser sides of the market as "receivers" (consumers) and "senders" (producers) respectively.

Prominent research efforts in the two-sided markets literature have addressed the issue of single- versus multihoming, i.e. the extent to which agents choose to affiliate with one (*single*homing) or more (*multi*homing) platforms. Two opposing arguments can be raised with respect to the localization (the side of the market) of these patterns. The first states that one side of the market singlehomes because of preferences or tastes, and hence that the other side has to consider multihoming, thus explaining why competing platforms are sometimes used simultaneously (see

the common resource—results from too many individual owners, who in their pricing decision do not take into account their impact on total demand, see e.g. Buchanan and Yoon (2000).

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e.g. Rochet and Tirole, 2003). Conversely, our paper deals with examples where the platforms are complements by necessity due to cultural or legal reasons, forcing one side to multihome while the other (rationally) singlehomes. This kind of complementarity therefore constitutes a sufficient condition for explaining distinct single-and multihoming patterns across the market, i.e., why one side of the market singlehomes, together with complete multihoming on the other side.

This paper's main contribution aims to illustrate the *implications* of platform complementarity on platform pricing structures: is complementarity beneficial to the sender (multihoming) or receiver (singlehoming) side? What about mergers between complementary platforms? (Can Cournot's results be extended to two-sided markets?) Does extreme fragmentation of property rights induce monopoly outcomes in the presence of complementary platforms? As such, the model developed in this paper lies at the crossroads of two important strands in the economic literature, borrowing elements and combining insights from (*i*) the two-sided markets literature (see above), and (*ii*) the theory on complementary goods (see e.g. Cournot, 1838, Ellet, 1839, Sonnenschein, 1968, Economides and Salop, 1992, Gaudet and Salant, 1992, Nalebuff, 2000, Feinberg and Kamien, 2001).

Our results indicate that the so-called anticommons problem indeed generalizes to two-sided markets. Compared to a situation in which complementary platforms individually set prices to maximize profits, we show that under joint ownership, the total fee per transaction (price *level*) is lower, and platform and industry profits are higher. Characterizing two-sided markets, the price *structure* also differs under both industry configurations, and joint ownership is shown to be detrimental to receivers and beneficial to senders (as reflected by respectively higher receiver and lower sender prices).

Allocative inefficiencies thus arise as individual platforms fail to internalize the negative pricing externality they exert on the other platforms. Mergers between such platforms then may be welfare enhancing, but involve a redistribution of surplus from one side of the market to the other. On the other hand, the limit of an atomistic allocation of property rights is not monopoly pricing. Unlike the Cournot-Ellet complementary monopoly theory the bundle price as set by independent complementary platforms does not approach the senders' "choke" level in the limit as the number of components (platforms) approaches infinity. The presence of the receiver side thus acts as a counterweight that limits the upward pressure on the bundle price exerted by an increasing number of components.

1.1 Literature Overview

While a number of papers have studied competition between substitute platforms, this paper adds to the two-sided market literature by explicitly examining competition between *complementary* platforms. This resembles the analysis of so-called *competitive bottlenecks* where platforms also have a local monopoly over the singlehoming agents they serve, and where cross-market agents have to multihome to realize network externalities, see Armstrong (2006) and Armstrong and Wright (2007). A crucial difference with both papers is that in our model multihoming agents, in order to complete the transaction, need to reach *all* cross-market agents, and to do so, need the services of *all* platforms in the market. This is opposed to the bottlenecks case where multihoming agents need to reach a specific (or single) agent on the other side of the market and accordingly only address the platform that agent is tied to.

Furthermore, while the competitive bottlenecks and complementary platforms cases share the localized monopoly over singlehoming agents,[4] the Armstrong and Wright setting embeds a dynamic element in that the number of singlehoming agents belonging to each platform varies as the result of competition. In our model the localized monopoly arises because of cultural and legal reasons. The nature of platform competition in both settings therefore is different: in the Armstrong-Wright setting, platforms compete for subscribers (singlehomers, receivers) whereas in our model, platforms compete for the budget share of the multihoming agents (senders).

Carrillo and Tan (2006) study consumers' single- or multihoming decisions in a setting where third parties offer goods and services that are complementary to the ones provided by two competing, horizontally differentiated, platforms. Whereas their focus lies on the impact of platform differentiation and the number of complementors on platform pricing structures, our paper—while simultaneously providing an explanation for asymmetric "homing" patterns—stresses the impact of *platform complementarity* on the pricing structure. We do so by comparing ensuing prices and profits under different platform ownership structures, taking our cue from the theory on complementary goods.

Related to the previous study, Economides and Katsamakas (2006) tackle the same issue of the optimal two-sided pricing strategy but from the point of view of proprietary versus open source platforms. Our paper shares their framework of analysis in the presence of different industry structures. However, while these authors consider vertical integration between platforms and complements, our model emphasizes horizontal integration between complementary platforms.

[4] In this sense, one could also label complementary platforms as *complementary* bottlenecks.

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Still another perspective is taken by Doganoglu and Wright (2006), studying the influence of consumer multihoming on compatibility decisions by firms. At the heart of their analysis lies the observation that although compatibility between firms increases consumers' network benefits, these can also be obtained when consumers choose to multihome should firms decide to remain incompatible. In our model, platform complementarity assures that singlehoming consumers (receivers) fully realize cross-market network benefits.

The remainder of this paper is structured as follows: in Section 2 we further elaborate on the merger between "De Tijd" and "L'Echo" and also give examples of complementarity between one- and two-sided firms providing services to a specific side of the market. Section 3 then introduces the model to analyze pricing behavior by bundles composed of one- and two-sided components and presents the basic results. This encompasses as special cases both Cournot's initial approach and the present model. Section 4 concludes.

2 Motivating Examples

2.1 The Merger between "De Tijd" and "L'Echo"

To give an idea about the quantities involved, Table 1 presents reader prices, advertising rates and circulation numbers for "De Tijd" and "L'Echo" for the years 2005 and 2010.[5] As mentioned above, the Belgian Antitrust Authority cleared the merger imposing a certain number of restrictions, see Raad voor de Mededinging (2005). Following up on European Commission practice (see Recoletos/Unidesa and Gruner and Jahr/Financial Times/JV),[6] the Authority partitioned the market for advertising in three distinct submarkets: (1) the market for thematic advertising, (2) the market for legal and financial advertising, and (3) the market for job advertisements, see Van Cayseele (2006). Especially the second market is important for the particular merger that was proposed since it involved the Dutch language financial newspaper "De Tijd" and the French language financial newspaper "L'Echo."

The results presented in Section 3 indicate that the merger between "De Tijd" and "L'Echo" induces lower total prices and higher industry profits. Hence, the condition imposed by the Authority so as to remedy the alleged negative consequences of the proposed merger hardly made sense, especially given the occurrence of lower *sender* prices: ignoring the complementary nature of advertising in this

[5]The declining circulation numbers for both financial newspapers are part of a general downward tendency that started in the beginning of the new millennium; for example, circulation for "De Tijd" in 2000 topped 67.209 copies (CIM, 2010).

[6]See respectively European Commission (1999a) and European Commission (1999b).

Table 1: Prices, rates and circulation 2005–2010[a]

	De Tijd		L'Echo	
	2005	2010	2005	2010
reader price	1,35	1,70	1,30	1,70
advertising rate	13.176	18.250	8.640	12.930
circulation	49.145[b]	43.879	24.986[b]	21.611

[a] Source: Raad voor de Mededinging (2005), CIM (2010) and Trustmedia (2010). Prices and rates in current euros; the reader price is the price for a single copy of the paper, the advertising rate is that for a single-page black-and-white financial or legal advertisement.

[b] Computed as the average circulation between January and June.

market, it prohibited discounts for joint advertising in "De Tijd" and other newspapers belonging to the merged group, such as "L'Echo." This is particularly the case because financial and legal advertising was explicitly mentioned to be precluded from discounts for a combined advertisement.[7] Moreover, we show that prices on the receiver side are likely to increase post-merger, but often what readers pay is subject to a price cap.

2.2 Other Examples

In order to generalize the setting and to show that both the Cournot-Ellet complementary monopoly problem and its two-sided equivalent arise as special cases from a particular industry configuration, we highlight two specific examples where platforms team up with traditional (one-sided) firms so as to provide a good or service to a specific side of the market.[8] First, consider shopping malls that can only be accessed through the use of transportation services (airline, train, ...) provided by monopolists. In its most simple form, consider the case where consumers that

[7] The merging parties agreed to satisfy this and other requirements for a five-year period starting at the date of the Authority's decision. Trustmedia (2010) presents the 2010 rates for publicity in "De Tijd" and "L'Echo," with page 1 of the document listing the rates for various types of financial and legal advertisements ("module financiële en wettelijke berichten"). Note that the rates for joint advertisements listed in column "De Tijd + L'Echo" are the exact sums of those listed in the separate columns of "De Tijd" and "L'Echo."

[8] We stress the fact that the paper's subject is the analysis of *horizontal* mergers between complementary components. For a vertical interpretation of the model, see Weyl (2008).

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want to visit an isolated shopping mall can only get there by train, and that the railroad company is a monopolist. Shopping malls have been identified as platforms that connect retailers with consumers. The provision of transport services on the other hand is typically a one-sided good. Thus, the bundle in this case consists of a platform (the mall) and a complementary one-sided component (the railroad company).[9]

A second example stems from the leisure industry where integrated tour operators combine travel agencies (platforms connecting tourists with holiday destinations) and airline companies (one-sided component) in providing travel arrangements. The complementarity between platform and component in this example arises as soon as such an operator were to exclusively offer ("monopolize") any specific destination.

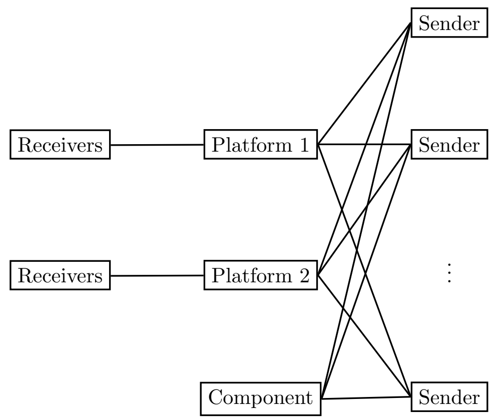
As in the example of "De Tijd" and "L'Echo," some factors explain for the segmentation of one side of the market while others for complementarity at the other. Here it is again specialization in production together with complementarity in consumption that entails the industry configuration shown in Figure 2. Regardless, in both cases one side of the market has to rely on all components present in the bundle in order to complete the transaction. This raises the question how the price structure of the two-sided platform is affected by the inclusion of a one-sided component.

3 The Model

As mentioned in Section 2.2, in reality bundles exist that simultaneously combine two-sided (platforms) with one-sided components (traditional firms), see Figure 2 for a schematic representation. We refer to these as *mixed* bundles, where "mixed" points to the simultaneous presence of one- and two-sided components. Let n be the total number of components present in the bundle, and respectively denote by n_1 and n_2 the number of one- and two-sided components. Given $n := n_1 + n_2$, this definition allows for a variety of bundle types, with extreme cases being compositions of either one-sided ($n_1 = n, n_2 = 0$), or two-sided components ($n_1 = 0, n_2 = n$). Any combination in between is a "mixed" bundle ($n_1, n_2 < n$ and $n_1 + n_2 = n$).

[9] McHardy (2006) in this context cites the required sequential use of different transport services by passengers in public transport as another well-known example of complementary monopoly. Barbot (2006) details the Ryanair-Charleroi Airport agreement as an example where a low-cost carrier (LCC) is subsidized by means of reduced landing fees and extremely low ground handling charges. The induced reduction in airport revenue is hoped to be compensated by increased spending of LCC passengers at the airport's bars, shops and restaurants.

Figure 2: Bundle with complementary one- and two-sided components



Throughout the paper, it is implicitly assumed that—from the point of view of the senders—the successful completion of a transaction requires the components to be combined in the bundle in a $1/1$ ratio. Hence the bundle price A in this most general a case is the sum of prices of one- and two-sided components:

$$A := \sum_{h=1}^{n_1} p_h^C + \sum_{i=1}^{n_2} p_i^S, \tag{1}$$

where p_h^C is the sender fee charged by one-sided components, and p_i^S the platforms' sender fees. We defer statement of further technical assumptions with respect to sender and receiver preferences until Section 3.2.

With demand for the bundle a function of the bundle price, three cases can be analyzed: (*i*) the pricing of mixed bundles, (*ii*) the pricing of one-sided bundles (complementary monopoly or Cournot), and (*iii*) the pricing of two-sided bundles (complementary platforms). For purposes of exposition, we first restate Cournot's complementary monopoly problem as one extreme case of mixed bundle pricing. This is subsequently compared with complementary platform pricing as the other logical extreme. We finally document the effects of the presence of both one- and two-sided components on bundle pricing.

3.1 One-Sided Bundles: Complementary Monopoly

With $n_1 = n, n_2 = 0$ and following Economides and Salop (1992), let D^S denote demand for the bundle composed of n_1 *one*-sided complementary goods produced by firms 1 to n_1, each having a monopoly on the production of their respective component. For ease of exposition, assume $n_1 = 2$. The defining feature of the complementary monopoly setting is that both monopolists face the same demand, i.e. the demand for the bundle as a whole, which is a function of the bundle price $A := \sum_h p_h^C$, where p_h^C is the price charged for complement $h = 1,2$.

Assuming for simplicity that each good is produced at constant marginal cost $c_h = 0$, each firm will independently set its price so as to maximize profits $\pi_h^C = p_h^C D^S(A)$. First-order conditions (FOCs) are[10]

$$\frac{\partial \pi_1^C}{\partial p_1^S} = D^S(A) + p_1^C \left[D^S(A)\right]' = 0$$
$$\frac{\partial \pi_2^C}{\partial p_2^S} = D^S(A) + p_2^C \left[D^S(A)\right]' = 0.$$

Summing across both FOCs yields

$$2D^S(A) + \left(p_1^C + p_2^C\right)\left[D^S(A)\right]' = 0,$$

and hence the price for the bundle is given by

$$A := p_1^C + p_2^C = -\frac{2D^S(A)}{[D^S(A)]'}. \tag{2}$$

For a bundle consisting of n_1 components, equation (2) naturally extends to

$$A := \sum_{h=1}^{n_1} p_h^C = -\frac{n_1 D^S(A)}{[D^S(A)]'}.$$

Now, suppose that both complements are produced by a single entity which sets the price of the bundle A so as to maximize joint profits

$$\Pi := \pi_1^C + \pi_2^C = \left(p_1^C + p_2^C\right) D^S(A) = AD^S(A).$$

Following the lead taken in Section 3, the FOCs with respect to p_h^C are[11]

$$\frac{\partial \Pi}{\partial p_1^C} = D^S(A) + \left(p_1^C + p_2^C\right)\left[D^S(A)\right]' = 0$$
$$\frac{\partial \Pi}{\partial p_2^C} = D^S(A) + \left(p_1^C + p_2^C\right)\left[D^S(A)\right]' = 0.$$

[10] We assume for simplicity that the second-order conditions for a maximum are also satisfied.
[11] Alternatively, taking the FOC with respect to the bundle price A yields the same result.

Summing across gives

$$2D^S(A) + 2\left(p_1^C + p_2^C\right)\left[D^S(A)\right]' = 0,$$

which yields the bundle price under joint ownership:

$$A^* := p_1^C + p_2^C = -\frac{D^S(A^*)}{[D^S(A^*)]'}. \tag{3}$$

Note that this result holds regardless the number of one-sided components. Therefore we can state the following:

Corollary 1 (Complementary Monopoly)**.** *For a bundle exclusively consisting of one-sided components ($n_1 = n, n_2 = 0$), the complementary mixed-bundle pricing problem leads to the complementary monopoly result for one-sided markets (Cournot, 1838, Ellet, 1839).*

Proof. Comparing (2) and (3), we have $A > A^*$. It follows that the price for the bundle under complementary monopoly is twice (n_1 times) as large as under integrated complementary monopoly, thus replicating the anticommons result for a *one*-sided market. ■

The economic rationale behind this result is that an integrated complementary monopolist, bearing the full cost of component price increases (i.e. the corresponding decrease in demand for the composite good D^S), will internalize the negative pricing externality exerted by individual producers and therefore set a lower, profit-maximizing bundle price (see e.g. Dari-Mattiacci and Parisi, 2006). As such, this is the horizontal equivalent of vertical integration to avoid the problem of double marginalization (Spengler, 1950). Relying on the definition of receiver and sender benefits in Section 3.2 and using subscripts I and J for prices set by respectively independent components and components under joint ownership, the following characteristics of the classic Cournot complementary monopoly result unfold:

Corollary 2. *(i) In a symmetric equilibrium with uniformly distributed sender benefits, independent one-sided components charge*

$$p_I^C = \frac{\bar{b}^S}{n_1 + 1}, \tag{4}$$

while one-sided components under joint ownership charge

$$p_J^C = \frac{\bar{b}^S}{2n_1}. \tag{5}$$

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The respective bundle prices are

$$A_I = \frac{n_1 \bar{b}^S}{n_1 + 1}, \quad and \quad A_J = \frac{n_1 \bar{b}^S}{2n_1}. \tag{6}$$

(ii) In the limit the bundle price as set by independent components approaches the choke level, while under joint ownership it attains half the choke level.

Proof. (i) Under symmetry, equations (4) and (5) follow directly from (2) and (3); (ii) As n_1 approaches infinity, by de l'Hôpital's rule we have respectively $\lim_{n_1\to\infty} A_I = \lim_{n_1\to\infty} \frac{n_1 \bar{b}^S}{n_1+1} = \bar{b}^S$ and $\lim_{n_1\to\infty} A_J = \lim_{n_1\to\infty} \frac{n_1 \bar{b}^S}{2n_1} = \frac{1}{2}\bar{b}^S$. ■

3.2 Two-Sided Bundles

Assume a market where platforms $i \in \{1, \ldots, n_2 = n\}$ provide their services to two distinct sides of the market, referred to as *senders* and *receivers*. Given the discussion in Sections 1 and 2, the characteristics of this market entail that the receiver side of the market singlehomes, whereas the sender side multihomes: senders rely on the services of all n platforms;[12] receivers on the other hand only need a single platform to successfully complete a transaction. As a result, for a single transaction senders pay the sum of prices of all platforms present in the market, as opposed to the receivers who pay a single fee to the platform they exclusively patronize.

3.2.1 Independent Complementary Platforms

Following Rochet and Tirole (2003, 2006), let receivers be heterogeneous in the gross benefit b_i^R they receive from a transaction mediated by a specific platform i. Benefits are uniformly distributed over the interval $\left[0, \bar{b}_i^R\right]$ with $0 < \bar{b}_i^R \leq \infty$. We further assume that the singlehoming receiver segments served by each platform are of equal size. Also note that receivers are perfectly segmented and are unable to migrate to and from other segments. Similarly, senders are heterogeneous in benefits b^S which are uniformly distributed over the interval $\left[0, \bar{b}^S\right]$ with $0 < \bar{b}^S \leq \infty$. In what follows we will refer to the combination of platform complementarity, perfect receiver segmentation and lack of intersegment mobility as the "Maintained Assumption."

The Maintained Assumption implies full multihoming on the sender side of the market: senders require the provision of services by all n platforms, entailing

[12] Similar to Cournot's perfect complements case, this is as if each platform's services are worthless to senders unless they are used together.

a total fee $A := \sum_{i=1}^{n} p_i^S$, where p_i^S denotes the sender fee charged by platform i. Therefore the per transaction net benefit senders derive from choosing the services delivered by the bundle of platforms is equal to $b^S - A$, and 0 otherwise. Note that this specification resembles the Cournot-Ellet "perfect complements" case where there is no consumer demand for the separate inputs and utility can only be derived from consumption of the composite good. Translated to the current setting, senders obtain zero benefits from using separate platforms for a transaction.

It follows that sender quasi-demand for the bundle of platform services is a function of the total fee charged and is defined as

$$D^S : \mathbb{R}_+ \to [0,1] : A \mapsto D^S(A) = D^S\left(p_1^S + p_2^S + \cdots + p_n^S\right), \tag{7}$$

with $D^S(A) = \text{Prob}\left\{b^S \geq A\right\} = 1 - G_S(A)$, where G_S is the uniform cumulative distribution function. From the definition of g_S, the uniform probability density function, we can see that sender quasi-demand is a decreasing function of the bundle price A:

$$\frac{\mathrm{d}}{\mathrm{d}A}\left[D^S(A)\right] = \frac{\mathrm{d}}{\mathrm{d}A}\left[1 - G_S(A)\right] = -g_S(A) < 0.$$

Zero conjectural variations (Bowley, 1924), i.e.

$$\frac{\partial A}{\partial p_i^S} = \frac{\partial}{\partial p_i^S}\left(\sum_{k=1}^{n} p_k^S\right) = \begin{cases} 0 & \forall k \neq i \\ 1 & \text{if } k = i, \end{cases} \tag{8}$$

also imply that sender quasi-demand is decreasing in the individual platform sender prices:

$$\frac{\partial}{\partial p_i^S}\left[D^S(A)\right] = -\frac{\partial}{\partial p_i^S}\left[G_S(A)\right] = -\frac{\partial G_S(A)}{\partial A}\frac{\partial A}{\partial p_i^S} = -g_S(A) < 0.$$

To guarantee the existence and uniqueness of an optimum we additionally impose log-concavity on sender quasi-demand, or $\partial^2\left[\ln D^S(A)\right]/\partial\left(p_i^S\right)^2 < 0$.

Equation (7) reveals that sender quasi-demand D^S is the same for all platforms. Note that we maintain the implicit assumption that platforms are used in a 1/1 ratio for a successful completion of a transaction, i.e., each platform is only needed once in the interaction with the receivers on the other side of the market.[13]

At first sight the multihoming characteristic of the sender side of the market seems to have important consequences for the quasi-demand structure on the

[13]This assumption can be relaxed by introducing variable production ratio's, see the working paper version of this article. The interpretation of these ratio's is then twofold, either (*i*) that one platform is used more (less) than the other in producing the composite good, or (*ii*) that platforms no longer are of equal size.

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receiver side: as receivers only need access to a single platform to complete a transaction with any of the senders on the other side, it is plausible to assume that they will singlehome. Receiver quasi-demand for platform i's services then becomes a function of all the platforms' prices charged to receivers (see Rochet and Tirole, 2003). In this case, let $\vec{p}^R$ be the vector of receiver prices. The fraction of receivers choosing platform i when all senders are affiliated with platform i is given by

$$d_i^R : \mathbb{R}_+^n \to [0,1] : \vec{p}^R \mapsto d_i^R\left(\vec{p}^R\right),$$

where $d_i^R\left(\vec{p}^R\right) = \text{Prob}\left\{b_i^R - p_i^R > \max\left(0, b_k^R - p_k^R\right) \forall k \neq i\right\}$.[14]

The Maintained Assumption however implies perfect segmentation on the receiver side of the market to the extent that each platform exclusively serves its own segment. In fact, complementarity and perfect segmentation are two sides of the same coin, as illustrated in Sections 1 and 2. Analogous to the sender case, if p_i^R denotes the receiver fee charged by platform i, the per transaction net benefit for a receiver in this case equals $b_i^R - p_i^R$, and 0 otherwise. As a consequence, receiver quasi-demand is a function of the own price only and is defined as

$$D_i^R : \mathbb{R}_+ \to [0,1] : p_i^R \mapsto D_i^R\left(p_i^R\right), \tag{9}$$

with $D_i^R\left(p_i^R\right) = \text{Prob}\left\{b_i^R \geq p_i^R\right\} = 1 - G_i^R\left(p_i^R\right)$, where G_i^R is the uniform cumulative distribution function. Similar to sender quasi-demand, we require D_i^R to be decreasing, $\mathrm{d}D_i^R\left(p_i^R\right)/\mathrm{d}p_i^R < 0$, and log-concave in prices, $\mathrm{d}^2\left[\ln D_i^R\left(p_i^R\right)\right]/\mathrm{d}\left(p_i^R\right)^2 < 0$.

Given (7) and (9), and assuming independence between sender and receiver benefits, platform i's expected transaction demand D is the product of receiver and sender quasi-demand:[15]

$$D : \mathbb{R}_+^2 \to [0,1] : \left(p_i^R, A\right) \mapsto D\left(p_i^R, A\right) = D_i^R\left(p_i^R\right) \cdot D^S(A).$$

Assuming for simplicity that platforms incur a constant marginal cost $c = 0$ per transaction, each platform i's optimization problem then becomes

$$\max_{p_i^R, p_i^S} \pi_i = \left(p_i^R + p_i^S - c\right) D_i^R\left(p_i^R\right) D^S(A). \tag{10}$$

In stating the platform's profit function we appeal to the technical assumptions of the Rochet and Tirole (2003) model underlying the multiplicative form

[14] Note that this is equivalent to a discrete choice model where a receiver chooses the platform that maximizes utility, see e.g. Anderson and Gabszewicz (2006).

[15] Exogenously fixing the number of potential transactions in the market at N, platform i's total expected transaction demand is $ND_i^R\left(p_i^R\right) D^S(A)$. For simplicity we normalize N to 1.

in equation (10). These assumptions fit this particular market well given the constraints on available space in daily issues of newspapers implying that not all financial adds appear in a single issue of the paper. From the point of view of the reader, when buying a financial newspaper there then is no guarantee that the firms in his portfolio of stocks will post their financial or legal advertisements on any particular day. Equally on the other side, advertising does not guarantee that any particular stockholder will have read the firm's announcement. This inherent uncertainty with respect to the cross-market participant's identity is well captured by the concept of receiver and sender quasi-demand, which in essence is the probability that a heterogeneous agent (receiver or sender) will participate. This is amplified by multiplying the resulting quasi-demands. Related arguments, notably on insulating tariffs, lead Weyl (2010) to the conclusion that the newspaper market can adequately be cast in the generalized Rochet and Tirole (2003) model.

Additionally, assume that it is costless to produce the composite good, i.e., the bundle of platform services. Imposing log-concavity on receiver and sender quasi-demand ensures that the first-order conditions for program (10) are both necessary and sufficient for a maximum. For ease of notation, let $\phi := \bar{b}^R \cdot \bar{b}^S$. We can now state the following:

Lemma 1 (Independent Complementary Platforms)**.** *In a symmetric equilibrium with uniformly distributed sender and receiver benefits, independent complementary platforms charge*

$$p_I^R = \frac{(n+1)\bar{b}^R - \bar{b}^S}{2n+1} \tag{11}$$

$$p_I^S = \frac{2\bar{b}^S - \bar{b}^R}{2n+1} \tag{12}$$

and make profits equal to

$$\pi_I = \frac{1}{\phi}\left(\frac{n\bar{b}^R + \bar{b}^S}{2n+1}\right)^3. \tag{13}$$

The price level, the bundle price and industry profits respectively equal

$$p_I := p_I^R + p_I^S = \frac{n\bar{b}^R + \bar{b}^S}{2n+1} \tag{14}$$

$$A_I := np_I^S = n\left(\frac{2\bar{b}^S - \bar{b}^R}{2n+1}\right) \tag{15}$$

$$\Pi_I := n\pi_I = \frac{n}{\phi}\left(\frac{n\bar{b}^R + \bar{b}^S}{2n+1}\right)^3. \tag{16}$$

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Proof. See Appendix A.1. ■

A first important result—from a welfare point of view—is that, unlike the Cournot-Ellet complementary monopoly theory (see Corollary 2), the bundle price as set by independent complementary platforms does not approach the senders' "choke" level in the limit as the number of components (platforms) grows large:

Proposition 1 (Bundle Limit Price in Two-Sided Markets)**.** *As the number of platforms grows to infinity, the bundle price does not attain the upper bound imposed by the sender choke level $\bar{b}^S$.*

Proof. Taking the limit of the bundle price (15) as $n \to \infty$, we have $\lim_{n\to\infty} A_I = \lim_{n\to\infty} np_I^S = \lim_{n\to\infty} \frac{n(2\bar{b}^S - \bar{b}^R)}{2n+1} = \frac{2\bar{b}^S - \bar{b}^R}{2}$, where the last equality follows from de l'Hôpital's rule. Because $\bar{b}^S - \frac{1}{2}\bar{b}^R \leq \bar{b}^S$, this limit value is smaller than the sender choke level. ■

The presence of the receiver side thus acts as a counterweight that limits the upward pressure on the bundle price exerted by an increasing number of components (platforms). This result arises from the interaction between two opposing forces: (*i*) the negative externality exerted by independent complementary platforms on each other, causing too high a bundle price (see Proposition 2), and (*ii*) the positive cross-market externality between senders and receivers, mitigating the former.

3.2.2 Complementary Platforms: Joint Ownership

Suppose now that the platforms are owned by a single entity which sets receiver prices p_i^R and sender prices p_i^S, where $A := \sum_i p_i^S$ is the price of the *bundle* of platforms, so as to maximize joint profits:

$$\Pi := \sum_{i=1}^{n} \pi_i = \sum_{i=1}^{n} \left(p_i^R + p_i^S\right) D_i^R \left(p_i^R\right) D^S(A).$$

With subscript J referring to the actions taken by the joint entity, we can now state the following:

Lemma 2 (Complementary Platforms: Joint Ownership)**.** *In a symmetric equilibrium with uniformly distributed sender and receiver benefits, complementary platforms under joint ownership charge*

$$p_J^R = \frac{2n\bar{b}^R - \bar{b}^S}{3n} \tag{17}$$

$$p_J^S = \frac{2\bar{b}^S - n\bar{b}^R}{3n} \tag{18}$$

and make profits equal to

$$\pi_J = \frac{1}{n^2\phi}\left(\frac{n\bar{b}^R + \bar{b}^S}{3}\right)^3. \tag{19}$$

The price level, the bundle price and industry profits respectively equal

$$p_J := p_J^R + p_J^S = \frac{n\bar{b}^R + \bar{b}^S}{3n} \tag{20}$$

$$A_J := np_J^S = \frac{2\bar{b}^S - n\bar{b}^R}{3} \tag{21}$$

$$\Pi_J := n\pi_J = \frac{1}{n\phi}\left(\frac{n\bar{b}^R + \bar{b}^S}{3}\right)^3. \tag{22}$$

Proof. See Appendix A.2. ■

The major difference with the results under independent platforms is the presence of the term $\left[D^S(A)\right]' \sum_{k \neq i}\left(p_k^R + p_k^S\right) D_k^R\left(p_k^R\right)$ in the first-order condition with respect to the individual sender prices, see equation (44). This second-order effect, which is absent in the case of independent platforms, embodies the negative externality–the corresponding decrease in sender quasi-demand–imposed by an increase in platform i's sender price on the remaining platforms' revenues, which is now borne in its entirety by the joint entity. It is exactly this which allows us to state the following proposition, extending Cournot's complementary monopoly result to two-sided markets:[16]

Proposition 2 (The Anticommons Problem in Two-Sided Markets)**.** *Compared with independent complementary platforms, in a symmetric equilibrium with uniformly distributed sender and receiver benefits, platforms under joint ownership set receiver and sender prices such that*

[16]For $n = 1$, prices and profits are the same under both ownership structures. We therefore consider the case for $n \geq 2$.

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(i) the price level is lower:

$$p_J < p_I, \tag{23}$$

(ii) the price structure is beneficial to senders and detrimental to receivers:

$$p_J^S < p_I^S \tag{24}$$

$$A_J < A_I \tag{25}$$

$$p_J^R > p_I^R, \tag{26}$$

(iii) platform and industry profits are higher:

$$\pi_J > \pi_I \tag{27}$$

$$\Pi_J > \Pi_I. \tag{28}$$

Proof. By comparison of the results in Lemmas 1 and 2. ■

As in the classic anticommons result, independent platforms charge too high a sender price, exerting a negative pricing externality on the other platforms. As a result, sender quasi-demand for the bundle of platform services decreases. Being a two-sided market, this increase in sender prices is offset by a decrease in receiver prices. This decrease however fails to compensate for the losses incurred on the sender side, causing individual and industry profits to decrease.

Similar to a multi-product monopoly (see e.g. Tirole, 1988, pp. 69–72), complementary platforms under joint ownership internalize negative pricing externalities, charging lower sender prices so as to decrease the bundle price [see equation (25)], thereby increasing sender quasi-demand. The two-sidedness of the market is mirrored however by higher receiver prices, as indicated by the price structure, see equations (24) and (26). Figure 3 gives a representation of the basic results of Propositions 1 and 2.

3.2.3 Welfare Analysis

Proposition 2 indicates that merger is beneficial to senders and platforms, and detrimental to receivers. Lemmas (1) and (2) allow for an explicit determination of *total welfare* under independent and joint ownership. In order to do so, we define total welfare as the unweighted (utilitarian) sum of receiver surplus, sender surplus and industry profits for $k \in \{I, J\}$:

$$W_k\left(n, \bar{b}^R, \bar{b}^S\right) = CS_k^R\left(n, \bar{b}^R, \bar{b}^S\right) + CS_k^S\left(n, \bar{b}^R, \bar{b}^S\right) + \Pi_k\left(n, \bar{b}^R, \bar{b}^S\right). \tag{29}$$

Note that in (29) we explicitly define the components of welfare as functions of the triple $\left(n, \bar{b}^R, \bar{b}^S\right)$ for reasons that will become clear below. The extent to which total

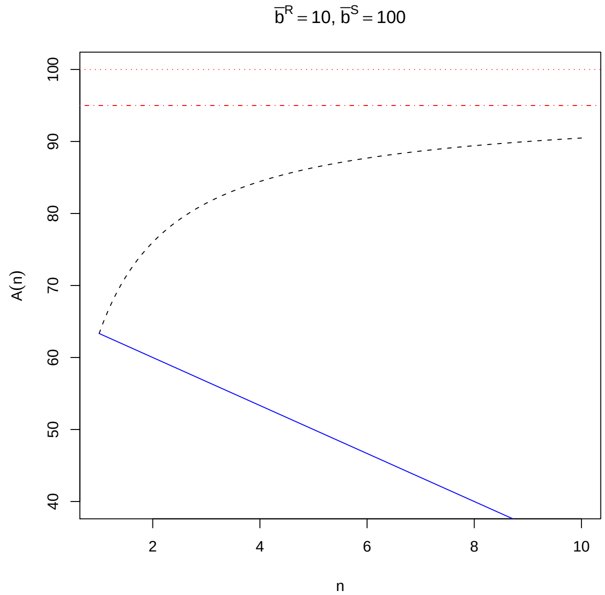

Figure 3: Equilibrium bundle price A_k as a function of the number of components n for fixed combination of upper bounds $(\bar{b}^R, \bar{b}^S) = (10, 100)$ under independent (dashed line, $--$) and joint ownership (solid line, $-$), highlighting Proposition 2. As shown by Proposition 1, the bundle price when platforms are independent converges to its limit value $\bar{b}^S - \frac{1}{2}\bar{b}^R$ (dashed-dotted line, $- \cdot -$), which is below the Cournot limit value $\bar{b}^S$ (dotted line, $\cdots$).

welfare is affected under joint ownership then is the difference $\Delta W(\cdot) = W_J(\cdot) - W_I(\cdot)$, where $(\cdot)$ is shorthand notation to indicate that the change in social welfare is also a function of the triple $\left(n, \bar{b}^R, \bar{b}^S\right)$. As both sender surplus and industry profits increase ($\Delta CS^S = CS_J^S - CS_I^S > 0, \Delta\Pi = \Pi_J - \Pi_I > 0$), this will ultimately depend on the redistribution of surplus from receivers ($\Delta CS^R = CS_J^R - CS_I^R < 0$) to the latter, or $\Delta W > 0$ if $\Delta CS^S + \Delta\Pi > -\Delta CS^R$. We therefore first quantify total welfare as follows:

Lemma 3 (Total Welfare with Complementary Platforms)**.** *In a symmetric equilibrium with uniformly distributed sender and receiver benefits, total welfare defined under* (29) *can be computed as*

$$W_I\left(n, \bar{b}^R, \bar{b}^S\right) = \frac{n}{2\bar{b}^R}\left(\frac{n\bar{b}^R + \bar{b}^S}{2n+1}\right)^2 + \frac{1}{2\bar{b}^S}\left(\frac{n\bar{b}^R + \bar{b}^S}{2n+1}\right)^2 + \frac{n}{\phi}\left(\frac{n\bar{b}^R + \bar{b}^S}{2n+1}\right)^3 \quad (30)$$

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when platforms are independent, and

$$W_J\left(n,\bar{b}^R,\bar{b}^S\right)=\frac{n}{2\bar{b}^R}\left(\frac{n\bar{b}^R+\bar{b}^S}{3n}\right)^2+\frac{1}{2\bar{b}^S}\left(\frac{n\bar{b}^R+\bar{b}^S}{3}\right)^2+\frac{1}{n\phi}\left(\frac{n\bar{b}^R+\bar{b}^S}{3}\right)^3 \tag{31}$$

under joint ownership.

Proof. See Appendix A.3. ■

As equations (30) and (31) show, total welfare not only depends on the number of platforms present in the market, but also on the upper bounds of the support of the distributions of receiver and sender benefits. In addition, these expressions are highly nonlinear, complicating the analysis. We therefore assess the merger effects numerically using (30) and (31). Figure 4 shows total welfare as a function of the number platforms n under independent (dashed line, $--$) and joint ownership (solid line, $-$) for various combinations $(\bar{b}^R,\bar{b}^S)$ of upper bounds on the support of receiver and sender benefits. The picture reveals two basic findings: (i) for a wide class of combinations, joint ownership dominates independent pricing in the sense of equations (30) and (31), and (ii) for certain combinations $(\bar{b}^R,\bar{b}^S)$ there exists a critical number of components n^* for which an independent industry configuration is preferable to joint ownership.

Furthermore, Figure 4 also displays a *positive* relationship between n^* and $\bar{b}^S$, or, the higher the sender willingness-to-pay (WTP), the more components are needed for a merger to be welfare enhancing. This result is easily explained by noting that receiver surplus is a nonlinear function of the triple $\left(n,\bar{b}^R,\bar{b}^S\right)$ and by relying on a *vertical* and an *horizontal* argument. Using column 1 from Figure 5 which decomposes the top row from Figure 4 in receiver surplus, sender surplus and industry profits, the vertical argument is that, for fixed n and $\bar{b}^R$, receiver surplus is increasing in $\bar{b}^S$ irrespective of the platform ownership structure. This is purely due to the two-sidedness of the market because receiver prices decrease with $\bar{b}^S$, see equations (11) and (17). The horizontal argument is that, for fixed $\bar{b}^S$, receiver surplus is not only a nonlinear function of the number of components, but is also increasing in n for low values of $\bar{b}^S$, and decreasing in high values of $\bar{b}^S$. Combining both arguments, these imply that the losses incurred by receivers in passing from independent to joint pricing are higher at higher values of $\bar{b}^S$, requiring more components to be included in the merger before these losses are compensated by increased sender surplus and industry profits.[17]

[17] For fixed n and $\bar{b}^S$, a similar reasoning holds for lower values of $\bar{b}^R$, see Column 3 in Figure 4.

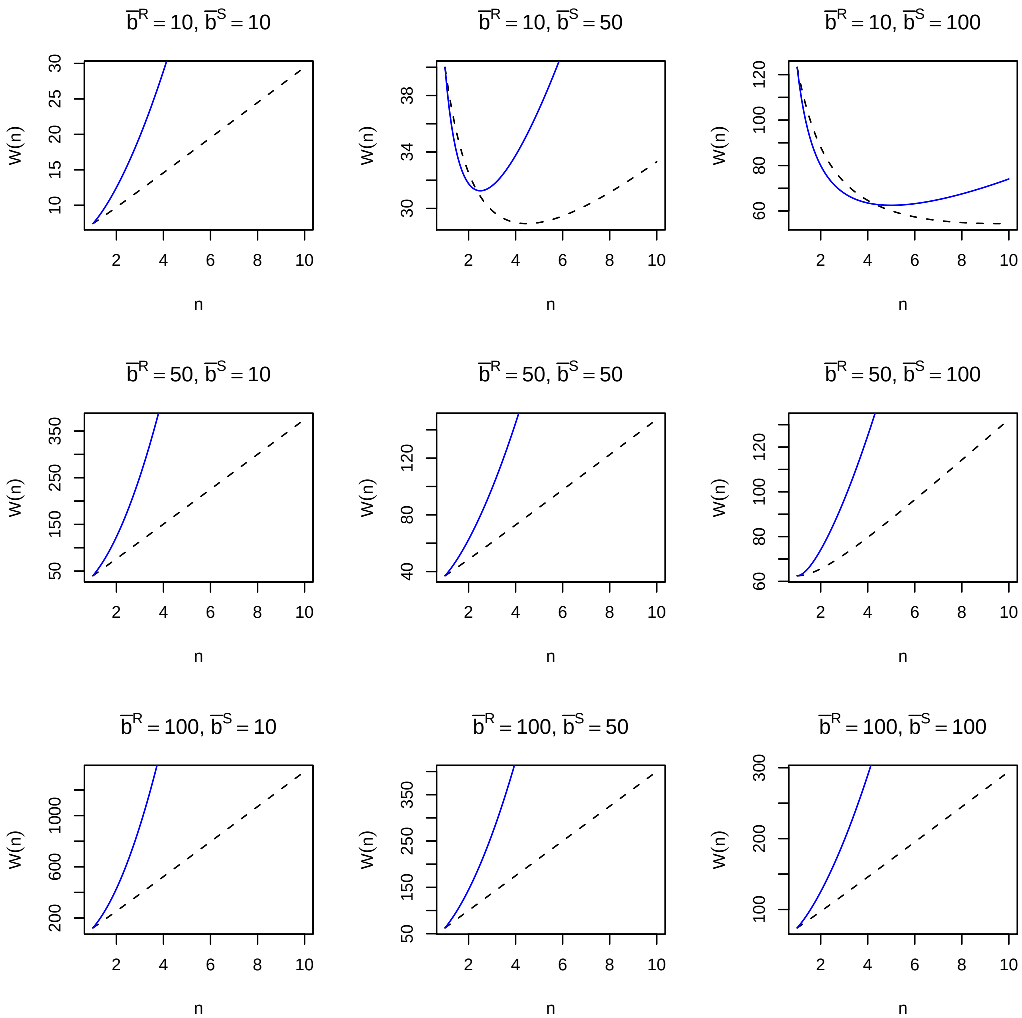

Figure 4: Welfare as a function of the number of platforms n under independent (dashed line, $--$) and joint ownership (solid line, $-$) for various combinations $(\bar{b}^R, \bar{b}^S)$ of upper bounds on the support of receiver and sender benefits.

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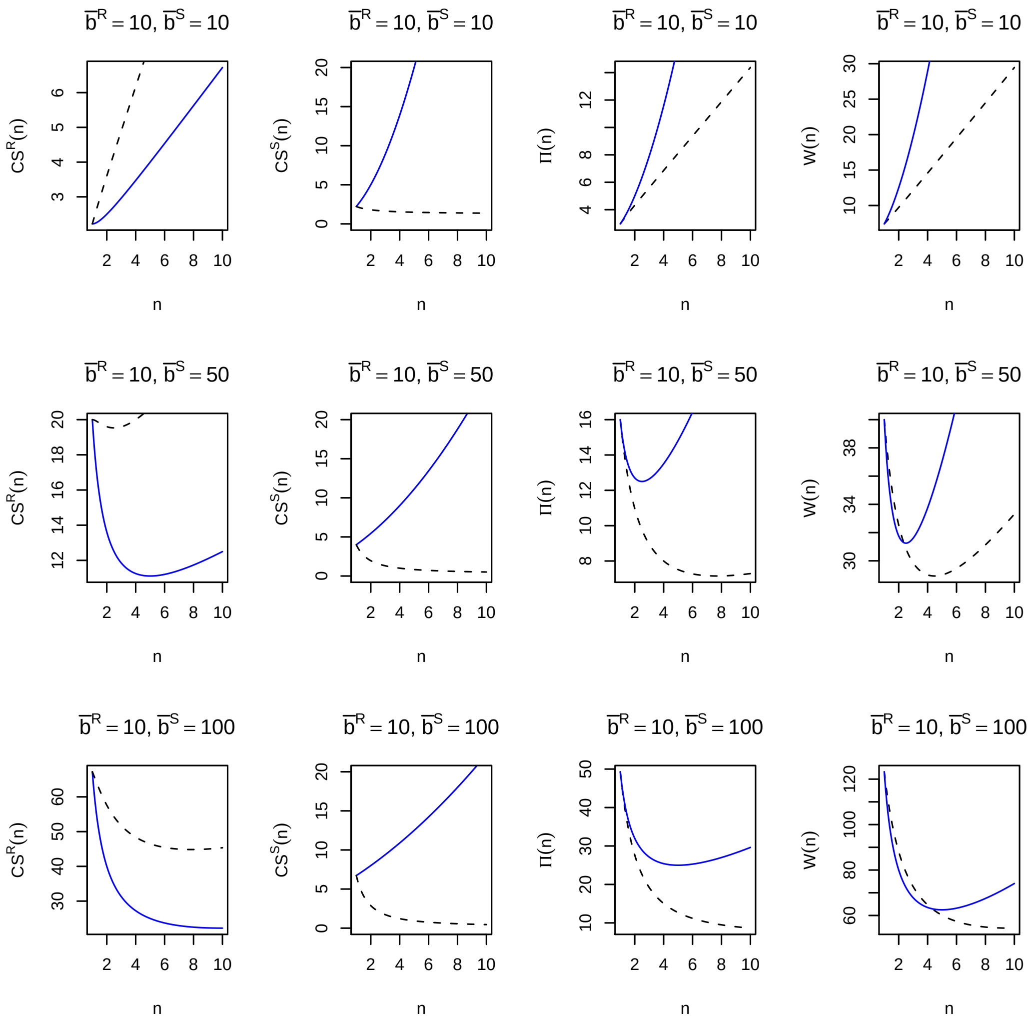

Figure 5: Decomposition of welfare (column 4) in receiver surplus (column 1), sender surplus (column 2), and industry profits (column 3) for fixed upper bound of receiver benefits $\bar{b}^R = 10$ and increasing levels of upper bounds of sender benefits $\bar{b}^S \in \{10, 50, 100\}$ revealing the trade-off between decreased receiver surplus ("losers") and increased sender surplus and industry profits ("winners") in passing from independent (dashed line, $--$) to joint ownership (solid line, $-$).

As the value of n^* critically depends on the values of $(\bar{b}^R, \bar{b}^S)$, this highlights the need to appropriately determine the WTP on both sides of the market. Summarizing, as long as $\bar{b}^R > \bar{b}^S$, mergers are welfare enhancing. For the opposite case where $\bar{b}^R < \bar{b}^S$, a merger can still be welfare enhancing provided that either of the following two conditions are satisfied, individually or jointly: (*i*) for a given number of components n, $\bar{b}^S$ is not too big vis-à-vis $\bar{b}^R$, or (*ii*) n is not too big given $\bar{b}^S/\bar{b}^R > 1$. As for the example of "De Tijd" and "L'Echo," given that n is small in this case, the welfare analysis corroborates the decision taken by the Belgian Antitrust Authority to clear the merger for the latter case.

3.3 Mixed Bundles

Surpassing both extreme cases, this subsection details the analysis of price-setting behavior in markets where platforms team up with one-sided firms to create a bundle which senders need to consume as a whole to successfully interact with cross-market agents. From a methodological point of view, we apply the blueprint developed in Subsections 3.1 and 3.2 to derive equilibrium prices and profits, and emphasize the role played by the number of one- and two-sided components in this particular industry setup, i.e. we investigate the effect of the number of one- and two-sided components present in the bundle (the fraction $\frac{n_1}{n_2}$) on the limit price of the bundle itself.[18]

Sender demand D^S is now governed by the total fee (1). Just as in the standard one-sided complementary monopoly setting, transaction volume for the components is equal to the demand for the entire bundle, $D^S(A)$. With c_h^C and c_i respectively denoting the components' and the platforms' marginal costs per transaction, the profit function for the components can then be written as

$$\pi_h^C = \left(p_h^C - c_h^C\right) D^S(A),$$

and for the platforms

$$\pi_i = \left(p_i^R + p_i^S - c_i\right) D_i^R\left(p_i^R\right) D^S(A).$$

[18] We do not focus on prices under different ownership structures here as one can see that in this setting the analysis of (*i*) mergers between one-sided components only hinges on the classic Cournot (one-sided) complementary monopoly result, see Subsection 3.1, (*ii*) mergers between two-sided components only where a single entity sets platform prices on both sides of the market is a simple extension of the results found in Subsections 3.2.1 and 3.2.2, and (*iii*) mergers between one- and two-sided components where a single entity sets all prices, in particular the bundle price, combines elements from both (*i*) and (*ii*).

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Next, assume $c_i = c \geq 0$ (i.e., platforms are symmetric), and $c_h^C = \theta c$ with $\theta \in [0,1]$. For example, with $\theta = 1$, components incur the same marginal cost per transaction as do the platforms. For simplicity we again assume $c = 0$.

Lemma 4 (Mixed Bundles)**.** *In a symmetric equilibrium with uniformly distributed sender and receiver benefits, platforms in bundles composed of* n_1 *one-sided and* n_2 *two-sided complementary goods, charge*

$$p_I^R = \frac{(n_1+n_2+1)\bar{b}^R - \bar{b}^S}{n_1+2n_2+1} \tag{32}$$

$$p_I^S = \frac{2\bar{b}^S - (n_1+1)\bar{b}^R}{n_1+2n_2+1} \tag{33}$$

and make profits

$$\pi_I = \frac{1}{\phi}\left(\frac{n_2\bar{b}^R + \bar{b}^S}{n_1+2n_2+1}\right)^3, \tag{34}$$

while one-sided components charge

$$p_I^C = \frac{n_2\bar{b}^R + \bar{b}^S}{n_1+2n_2+1} \tag{35}$$

and make profits

$$\pi^C = \frac{1}{\bar{b}^S}\left(\frac{n_2\bar{b}^R + \bar{b}^S}{n_1+2n_2+1}\right)^2. \tag{36}$$

The bundle price equals

$$A_I = \frac{(n_1+2n_2)\bar{b}^S - n_2\bar{b}^R}{n_1+2n_2+1}. \tag{37}$$

Proof. See Appendix A.4. ■

The first thing to note is that, contrary to one-sided complementary monopoly, the equilibrium fee charged by one-sided components now positively depends on both the senders' and the receivers' choke level, see equation (35). Second, a closer look reveals that this general result encompasses both symmetric cases, the Cournot-Ellet complementary monopoly result and the complementary platform result: for $n_1 = n$ and $n_2 = 0$, the mixed bundle model yields (4), the equilibrium price for one-sided components. For $n_1 = 0$ and $n_2 = n$, the model replicates results (11) and (12) for equilibrium receiver and sender prices set by independent complementary platforms.

A final result following from equation (37) is that the two-sided characteristic of the market tends to disappear as the number of one-sided components grows large. Despite the presence of platforms, the market behaves as if it were one-sided when the number of one-sided components approaches infinity. On the other hand, if the number of platforms approaches infinity, the one-sided components become relatively unimportant and the market tends to a two-sided market with complementary platforms.

Proposition 3 (Mixed Bundle Limit Prices). *(i) If the number of one-sided components approaches infinity the bundle price approaches the senders' choke level, replicating the Cournot-Ellet complementary monopoly result of Corollary 2; (ii) if the number of two-sided components grows large, the bundle price does not approach the senders' choke level and replicates the complementary platform result from Proposition 1.*

Proof. (i) In this case we have $\lim_{n_1\to\infty} A_I = \lim_{n_1\to\infty} \frac{(n_1+2n_2)\bar{b}^S - n_2\bar{b}^R}{n_1+2n_2+1} = \frac{n_1\bar{b}^S}{n_1} = \bar{b}^S$; ($ii$) Here we have $\lim_{n_2\to\infty} A_I = \lim_{n_2\to\infty} \frac{(n_1+2n_2)\bar{b}^S - n_2\bar{b}^R}{n_1+2n_2+1} = \frac{2n_2\bar{b}^S - n_2\bar{b}^R}{2n_2} = \bar{b}^S - \frac{1}{2}\bar{b}^R$. ∎

4 Conclusion

In this paper we introduced a model that allows for the investigation of pricing decisions by complementary platforms, extending Cournot's anticommons problem to two-sided markets. At the same time, this complementary setting offers a natural explanation for the presence of distinct single- and multihoming patterns across the market.

We show that in markets characterized by the presence of independent complementary platforms, the total fee per transaction is higher and platform and industry profits are lower than with complementary platforms under joint ownership. Similar to the anticommons problem in traditional one-sided markets, this result arises because independent platforms fail to internalize the negative pricing externality they exert on others. Under joint ownership, platforms charge lower sender prices (and therefore a lower bundle price) and correspondingly make higher profits. The two-sidedness of the market however also induces higher receiver prices, thus creating both "winners" and "losers" of mergers.

Unlike the Cournot-Ellet complementary monopoly theory, the bundle price as set by independent complementary platforms does not approach the senders' choke level in the limit as the number of components (platforms) grows large. The adverse effects of the dilution of property rights on equilibrium pricing are mitigated by the presence of the receiver side that acts as a counterweight limiting the upward pressure on the bundle price exerted by an increasing number of components (platforms).

A Appendix

A.1 Proof of Lemma 1

Proof. Following the lead taken in Subsection 3.1, the first-order conditions for profit maximization are

$$\frac{\partial \pi_i}{\partial p_i^R} = D^S(A)\left\{D_i^R\left(p_i^R\right)+\left(p_i^R+p_i^S\right)\left[D_i^R\left(p_i^R\right)\right]'\right\}=0 \tag{38}$$

$$\frac{\partial \pi_i}{\partial p_i^S} = D_i^R\left(p_i^R\right)\left\{D^S(A)+\left(p_i^R+p_i^S\right)\left[D^S(A)\right]'\frac{\partial A}{\partial p_i^S}\right\}=0. \tag{39}$$

Zero conjectural variations (8) imply that we obtain a system of $2n$ equations in $2n$ unknowns which implicitly define the optimal sender and receiver prices:

$$p_i^R+p_i^S=-\frac{D_i^R\left(p_i^R\right)}{\left[D_i^R\left(p_i^R\right)\right]'} \tag{40}$$

$$p_i^R+p_i^S=-\frac{D^S(A)}{[D^S(A)]'}. \tag{41}$$

Combining (40) and (41) we obtain

$$-\frac{D_i^R(p_i^R)}{[D_i^R(p_i^R)]'}=-\frac{D^S(A)}{[D^S(A)]'}, \tag{42}$$

which replicates the result of Rochet and Tirole (2003): when platforms set prices p_i^R and p_i^S to maximize volume for a given total price $p_i := p_i^R+p_i^S$, the volume impact of a small variation in prices has to be the same on both sides, keeping in mind that here the volume impact on the sender side is triggered by a change in the total sender fee $A := \sum_{i=1}^n p_i^S$.

Summing equations (40) and (41) over i we obtain

$$\sum_{i=1}^n\left(p_i^R+p_i^S\right)=-\sum_{i=1}^n\frac{D_i^R\left(p_i^R\right)}{\left[D_i^R\left(p_i^R\right)\right]'}$$

$$\sum_{i=1}^n\left(p_i^R+p_i^S\right)=-\frac{nD^S(A)}{[D^S(A)]'}.$$

Then, under perfect segmentation and assuming equal supports of the distribution of receiver benefits, meaning $\underline{b}_i^R = \underline{b}^R$ and $\bar{b}_i^R = \bar{b}^R$ for all i, we obtain in a symmetric equilibrium where $p_i^S = p_I^S$ and $p_i^R = p_I^R$ for all i (such that receiver quasi-demand $D_i^R\left(p_I^R\right) := D^R\left(p_I^R\right)$ is symmetric), the system of best-response functions above can be simplified and written in matrix notation as

$$\begin{bmatrix} 2 & 1 \\ 1 & n+1 \end{bmatrix} \begin{bmatrix} p_I^R \\ p_I^S \end{bmatrix} = \begin{bmatrix} \bar{b}^R \\ \bar{b}^S \end{bmatrix},$$

where we have used a uniform distribution of receiver and sender benefits $G_k(x) := x/\bar{b}^k$ for $k = R, S$ to obtain a closed form for equations (40) and (41).[19] Applying Cramer's rule, $p_I^k = \frac{|A_k|}{|A|}$ for $k = R, S$, yields the desired results. ■

A.2 Proof of Lemma 2

Proof. The first-order conditions with respect to receiver prices are identical to the ones obtained under independent platforms, embodied by equation (40):

$$\frac{\partial \Pi}{\partial p_i^R} = D^S(A)\left\{D_i^R\left(p_i^R\right) + \left(p_i^R + p_i^S\right)\left[D_i^R\left(p_i^R\right)\right]'\right\} = 0.$$

This yields the familiar expression

$$p_i^R + p_i^S = -\frac{D_i^R\left(p_i^R\right)}{\left[D_i^R\left(p_i^R\right)\right]'}.$$

Summing over i we obtain

$$\sum_{i=1}^{n}\left(p_i^R + p_i^S\right) = -\sum_{i=1}^{n}\frac{D_i^R\left(p_i^R\right)}{\left[D_i^R\left(p_i^R\right)\right]'}. \tag{43}$$

Taking the first derivative with respect to sender prices we obtain

$$\begin{aligned} \frac{\partial \Pi}{\partial p_i^S} =& D_i^R\left(p_i^R\right)\left\{D^S(A) + \left(p_i^R + p_i^S\right)\left[D^S(A)\right]'\right\} \\ &+ \left[D^S(A)\right]'\sum_{k\neq i}\left(p_k^R + p_k^S\right)D_k^R\left(p_k^R\right) = 0. \end{aligned} \tag{44}$$

Summing over i and grouping common elements yields

$$D^S(A)\sum_{i=1}^{n}D_i^R\left(p_i^R\right) + n\left[D^S(A)\right]'\sum_{i=1}^{n}\left(p_i^R + p_i^S\right)D_i^R\left(p_i^R\right) = 0,$$

[19] Additionally, the use of uniform distributions avoids corner solutions arising from skewed pricing distributions, see e.g. Bolt and Tieman (2006, 2008).

or

$$\sum_{i=1}^{n}\left(p_i^R+p_i^S\right)D_i^R\left(p_i^R\right)=-\frac{D^S(A)}{n\left[D^S(A)\right]'}\sum_{i=1}^{n}D_i^R\left(p_i^R\right). \tag{45}$$

In a symmetric equilibrium equations (43) and (45) can again be simplified and written as

$$\begin{bmatrix}2 & 1\\ n & 2n\end{bmatrix}\begin{bmatrix}p_J^R\\ p_J^S\end{bmatrix}=\begin{bmatrix}\bar{b}^R\\ \bar{b}^S\end{bmatrix}.$$

Applying Cramer's rule, $p_J^k=\frac{|A_k|}{|A|}$ for $k=R,S$, yields the desired results. ■

A.3 Proof of Lemma 3

Proof. Following conventional economic practice we compute receiver surplus on an individual platform as the surface of the area below the receiver quasi-demand curve and above the equilibrium receiver price p_k^R for $k\in\{I,J\}$. Given $b^R\sim\mathscr{U}\left[0,\bar{b}^R\right]$, we exploit the resulting linearity of receiver quasi-demand and compute this area as the surface of a triangle, in this case

$$\widetilde{CS}_k^R\left(n,\bar{b}^R,\bar{b}^S\right)=\frac{1}{2}\left(\bar{b}^R-p_k^R\right)D^R\left(p_k^R\right),$$

yielding

$$\widetilde{CS}_k^R\left(n,\bar{b}^R,\bar{b}^S\right)=\begin{cases}\dfrac{1}{2\bar{b}^R}\left(\dfrac{n\bar{b}^R+\bar{b}^S}{2n+1}\right)^2 & \text{if } k=I\\[2ex] \dfrac{1}{2\bar{b}^R}\left(\dfrac{n\bar{b}^R+\bar{b}^S}{3n}\right)^2 & \text{if } k=J.\end{cases}$$

With n platforms, total receiver surplus $CS_k^R(\cdot)$ then equals $n\cdot\widetilde{CS}_k^R(\cdot)$. Likewise, sender surplus is computed as the surface of the area below the sender quasi-demand curve and above the equilibrium bundle price A_k:

$$CS_k^S\left(n,\bar{b}^R,\bar{b}^S\right)=\frac{1}{2}\left(\bar{b}^S-A_k\right)D^S\left(A_k\right).$$

This yields

$$CS_k^S\left(n,\bar{b}^R,\bar{b}^S\right)=\begin{cases}\dfrac{1}{2\bar{b}^S}\left(\dfrac{n\bar{b}^R+\bar{b}^S}{2n+1}\right)^2 & \text{if } k=I\\[2ex] \dfrac{1}{2\bar{b}^S}\left(\dfrac{n\bar{b}^R+\bar{b}^S}{3}\right)^2 & \text{if } k=J.\end{cases}$$

Using expressions (16) and (22) for total industry profits and summing all components yields the expressions for total welfare under (30) and (31). ■

A.4 Proof of Lemma 4

Proof. As platforms individually set prices to maximize profits, we again obtain the system of $2n_2$ FOCs (38) and (39). Summing over i, we have

$$\begin{aligned} D^S(A)\sum_{i=1}^{n_2} D_i^R\left(p_i^R\right) + D^S(A)\sum_{i=1}^{n_2}\left(p_i^R+p_i^S\right)\left[D_i^R\left(p_i^R\right)\right]' &= 0 \\ D^S(A)\sum_{i=1}^{n_2} D_i^R\left(p_i^R\right) + \left[D^S(A)\right]'\sum_{i=1}^{n_2}\left(p_i^R+p_i^S\right)D_i^R\left(p_i^R\right) &= 0. \end{aligned}$$

Invoking symmetry, the latter respectively simplify to

$$\begin{aligned} D^S(A)n_2D^R\left(p^R\right) + D^S(A)n_2\left(p^R+p^S\right)\left[D^R\left(p^R\right)\right]' &= 0 \\ D^S(A)n_2D^R\left(p^R\right) + \left[D^S(A)\right]'n_2\left(p^R+p^S\right)D^R\left(p^R\right) &= 0, \end{aligned}$$

thus yielding

$$p^R+p^S = -\frac{D^R\left(p^R\right)}{\left[D^R\left(p^R\right)\right]'} \tag{46}$$

$$p^R+p^S = -\frac{D^S(A)}{\left[D^S(A)\right]'}. \tag{47}$$

Taking the first derivative with respect to component prices we find the following n_1 FOCs:

$$\frac{\partial \pi_h^C}{\partial p_h^C} = D^S(A) + p_h^C\left[D^S(A)\right]' = 0.$$

Summing over h we have

$$n_1D^S(A) + \sum_{h=1}^{n_1} p_h^C\left[D^S(A)\right]' = 0,$$

which, when invoking symmetry ($p_h^C = p^C\,\forall h$), simplifies to

$$n_1D^S(A) + n_1p^C\left[D^S(A)\right]' = 0,$$

or

$$p^C = -\frac{D^S(A)}{\left[D^S(A)\right]'}. \tag{48}$$

Noting that the bundle price under symmetry amounts to

$$A = n_1p^C + n_2p^S,$$

and combining (46), (47) and (48), we obtain the following system of three equations in three unknowns:

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & n_2+1 & n_1 \\ 0 & n_2 & n_1+1 \end{bmatrix}\begin{bmatrix} p^R \\ p^S \\ p^C \end{bmatrix} = \begin{bmatrix} \bar{b}^R \\ \bar{b}^S \\ \bar{b}^S \end{bmatrix}.$$

The application of Cramer's Rule, $p_I^k = \frac{|A_k|}{|A|}$ for $k \in \{R,S,C\}$, gives the desired results. ■

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