UNIVERSITY OF AMSTERDAM

UvA-DARE (Digital Academic Repository)

How unfounded microfoundations pollute our thinking and possibility also the world

Vermeylen, K.

Publication date
2011

Document Version
Submitted manuscript

Citation for published version (APA):
Vermeylen, K. (2011). *How unfounded microfoundations pollute our thinking and possibility also the world*. Universiteit van Amsterdam.

How Unfounded Micro-Foundations Pollute our Thinking and Possibly also the World

Koen Vermeylen[*]

University of Amsterdam

January 31, 2011

4th draft

Abstract

The rate of time preference or the subjective discount rate in environmental cost-benefit analyses is often calibrated with the Ramsey rule to match market observations of the interest rate. However, this practice suffers from a fallacy of composition, and is illustrative for a methodological flaw in some of the modern macroeconomic literature. I illustrate this by showing how a fictitious economy would be analyzed by a modern (new-classical) economist. Beguiled by micro-foundations that lack a strong empirical foundation, he would derive a subjective discount rate that bears no clear relation with the true preferences of the population; in addition, his policy advice would be time-inconsistent. I argue that an old-style (neo-classical) economist would do a better job.

1   Introduction

A long-standing debate in environmental economics is how to discount the costs and benefits of environmental policy. Most economists agree that an appropriate discount rate can be found with the Ramsey rule, which is the sum of two components: a pure rate of time preference, also called the subjective discount rate (to capture the idea that most people seem to care less about future felicity than about current felicity); and (the absolute value of) the elasticity of the marginal social value of consumption *times* the growth rate of consumption (to capture the idea that the marginal social value of consumption decreases as societies grow richer). But economists disagree about how we should pin down appropriate values for these two components.

To fix ideas, let us assume that there is full agreement about appropriate values for the elasticity of the marginal social value and the future growth rate of consumption,[1] and let us focus on the subjective discount rate.

According to economists such as Ramsey (1928), Sen (1982) and Cline (1992), the choice of an appropriate subjective discount rate is not an economic question but an ethical issue, and should be set as low as possible - possibly even zero. This *prescriptive approach* is followed by Nick Stern in his *Stern Review* (2007), where he uses a subjective discount rate of a mere 0.1%.

That seems to be a minority view, however. Most modern macroeconomists take a *descriptive approach*, and argue that the subjective discount rate should in principle be derived from rates of return which we observe in economies with well-developed financial markets - see, for instance, Tol and Yohe (2006), Nordhaus (2007), Weitzman (2007) and Mendelsohn (2007) in their critique on the Stern Review. This is not easy, however: there exists a wide range of rates of return, and it is not *a priori* clear which one we should use; it is even not clear why there exists such a wide range in the first place, as is documented in a large literature on the equity premium puzzle (for an overview, see Mehra, 2006; and DeLong and Magin, 2009). But, nevertheless, there seems to be a consensus that a market-based subjective discount rate should be somewhere between 1 and 3%.

The difference between a subjective discount rate of almost 0% and one in the range of 1-3% turns out to be very important: Weitzman (2007), for instance, argues that this is the main reason why the Stern Review calls for sharp and immediate reductions in greenhouse gas emissions, while cost benefit analyses based on market-based subjective discount rates such as Tol (2002a, 2002b) and Nordhaus (2008) yield much more moderate policy implications.

---

[1]...which, of course, is not the case: Dasgupta (2008) gives an account of the disagreements.

In this paper, I argue that trying to distill a reasonable value for the subjective discount rate out of observations of rates of return in the financial markets is doomed to fail.

For the sake of argument, let us assume that there is only one rate of return in the financial markets, the (risk-free) interest rate. The interest rate, as any other macroeconomic variable, is the result of the interactions of millions of economic agents, all of them endowed with different preferences and facing different constraints. If we had sufficient micro-economic evidence about the preferences of all these agents, we could simply take some sort of average of their subjective discount rates and use it in an environmental cost-benefit analysis[2] - we would then not even need information about the interest rate. But we don't have this luxury. Devoid of sufficient micro-economic evidence, modern macro-economists therefore assume away much if not all of the heterogeneity at the micro level, and simply assume that all agents can be represented by some *representative* household, who presumably is endowed with some average subjective discount rate; they then derive how the interest rate in such a simplified economy would depend on this representative household's subjective discount rate, and compute his subjective discount rate in such a way that the implied value of the interest rate matches observations of the interest rate in the real world.

But this procedure suffers from a fallacy of composition. It is not at all obvious why the subjective discount rate of a non-existing representative household, calibrated to match a macroeconomic variable such as the interest rate, would be a good estimate of the average subjective discount rate in a population of agents that are widely heterogeneous.

Note that this critique is similar to some profound concerns about modern macroeconomics that have recently been raised by Solow (2007, 2008), Colander et. al. (2009), and many others following the lead of Buiter (2009) and Krugman (2009). Modern macroeconomics arose in the wake of the *great critiques* by Lucas and Sims on neo-classical macroeconomics, that reigned supreme in the 1950s and the 1960s. Neo-classical macro-economists simply posited some stylized relations between aggregate variables in their models. Unfortunately, relations between aggregate variables may not be stable: they may change if policy changes, especially if economic agents behave strategically in anticipation of or in response to policy changes (Lucas, 1976); and policy itself may be endogenous as well (Sims, 1980). So to make sure that their models are immune against the critiques by Lucas and Sims, modern

---

[2]Even this may be hard: people seem to have different discount rates for different types of trade-offs between the present and the future (see Frederick, Loewenstein and O'Donoghue, 2002).

macro-economists build their models from micro-foundations: they specify the objectives and the constraints of a number of forward-looking agents (who are often fully rational and self-interested), they then derive their optimal behavior, and then impose an equilibrium device to derive the aggregate behavior of the economy.[3]

However, these micro-foundations almost always lack a strong empirical foundation: the amount of behavioral complexity, heterogeneity and institutional detail in most modern macroeconomic models is way too small to establish clear links with the available micro-economic evidence, or is even conflicting with it. So instead, deep structural parameters such as those that describe the agents' preferences, are calibrated or estimated in such a way that the aggregate behavior of the model mimics macroeconomic data. Unfortunately, empirically unfounded micro-foundations make these models vulnerable for fallacies of composition. And indeed, many of these models impose constraints on the data that do not pass any serious econometric test (see, for instance, Johansen, 2006; and Juselius and Franchi, 2007), which casts serious doubts upon these models' empirical validity[4] - just as the Ramsey rule, calibrated with the interest rate and aggregate consumption data, forces the data in a straightjacket that is not derived from empirically sound micro-foundations, and does not reveal the average subjective discount rate of a heterogeneous population.

This paper illustrates this issue. In the next section, I present an economy where production pollutes the environment, and where environmental degradation leads to abatement costs for the government. I assume overlapping generations of households, which introduces one of the most elementary dimensions of heterogeneity among households (namely their age). This substantially obscures the link between the interest rate and the households' sub-

---

[3]This is what is usually understood with the term *micro-foundations*. It is important to realize, however, that micro-foundations were also implicit in much of the work by neo-classical economists (such as James Tobin and Franco Modigliani). Hoover (2009) traces back the role of micro-foundations in macroeconomics to the very origin of the terms *micro-economics* and *macroeconomics*, when they were coined by (presumably) Ragnar Frisch in the 1930s.

[4]I lump all modern macroeconomists together in this paper, and treat them unkindly. I do so for practical reasons, but I readily admit that this is unfair. First, there are several macroeconomic models of which the key microeconomic parameters are not simply calibrated or estimated, but are carefully derived from microeconomic evidence; examples are Storesletten, Telmer and Yaron (2004) and Bartelsman, Haltiwanger and Scarpetta (2009). Second, a model can always serve as a thought experiment, and in that case a parameterization may help to make the model more transparent. But things may go wrong if a model without a strong empirical foundation for its micro-foundations is presented as a good description of the real world simply because its dynamics mimic the behaviour of aggregate data.

jective discount rate; furthermore, it establishes a link between the interest rate and the environmental quality: as the environmental quality deteriorates and abatement costs for the government increase, the national saving rate goes down, and the interest rate increases.

I then consider a *social evaluator* who wants to determine the optimal path of emission allowances. This social evaluator faces a trade-off between high emission levels which allow firms to produce a lot today, or low emission levels to avoid the impact of environmental degradation in the future - a trade-off which he wants to settle based upon a subjective discount rate. As the social evaluator does not understand well how the economy works, he seeks advice from a neo-classical and a new-classical economist.

In section 3, I describe the advice of a *neo-classical economist.* I will assume that the neo-classical economist does not know much about the household sector and therefore simply assumes that aggregate saving is a constant fraction of aggregate disposable income. Nevertheless, it turns out that his model captures some essential features of the economy; and recognizing that he does not know the social evaluator's subjective discount rate, he computes the optimal emissions policy for a range of possible discount rates.[5]

In section 4, I consider the analysis of a *new-classical economist*, a modern economist who attempts to model the economy with micro-foundations. The new-classical economist, not having much empirical information about the household sector either, introduces an immortal representative household in his model, endows her with a utility function, and derives her optimal consumption and saving decisions. He then finds that the relation between the representative household's subjective discount rate, the interest rate and the growth rate of aggregate consumption is described by the Ramsey rule, which he exploits to calibrate the representative household's subjective discount rate. Claiming that his model has revealed the population's subjective discount rate (which is not correct because of a fallacy of aggregation), he then advises the social evaluator to adopt this subjective discount rate, and traces out the corresponding optimal emissions policy. Furthermore, as the new-classical economist's model misses the point that environmental degradation leads to higher interest rates, he will steadily revise his estimate of the subjective discount rate upwards as the environmental quality deteriorates - implying a steady state which might be much more polluted than is socially desirable.

In section 5, I give some concluding remarks.

---

[5]In reality, neo-classical economists typically did not do this - be it alone because it was much harder to make nice graphs in the pre-Matlab era than it is now. But they typically did acknowledge that pinning down an appropriate subjective discount rate is beyond the scope of positive economics.

2 An economy with pollution

Let us consider a simple economy where production causes environmental damage, and where environmental damage leads to production losses and abatement costs for the government; the extent to which production pollutes the environment depends on the level of emissions, which is determined by the government's environmental policy.

I first present the set-up of the model. I then show how the economy's steady state depends on the steady state emission level. I conclude this section by assuming a social welfare function and by deriving how the optimal environmental policy depends on the social evaluator's subjective discount rate; this will then serve as a benchmark to assess the policy advice of neo-classical and new-classical economists in sections 3 and 4.

2.1 The set-up

The model is set up in three steps. First, I explain the relation between production, pollution and environmental quality, and its consequences for the taxes that are needed to cover the government's abatement costs. I then describe the economy's population and their consumption and saving decisions. I complete the economy's set-up by characterizing its equilibrium.

2.1.1 Production, pollution, the environmental quality, abatement costs and taxes

The supply side of the economy is described by the production decisions of a representative firm, operating under perfect competition. The representative firm produces output Y according to a Cobb-Douglas production function with capital K, technology A and labor input L; in addition, the production function depends on the emissions E that are allowed by the government and on the environmental quality M:

$$Y_t \quad = \quad E_t^\varepsilon M_t^{\mu_Y} K_t^\alpha (A_t L_t)^{1-\alpha} \tag{1}$$

...where the subscript t denotes the time period, and $0 < \alpha < 1$. E and M are measured on a scale from 0 to 1 and ε and μ_Y are both positive: $\varepsilon > 0$ and $\mu_Y > 0$. So the higher the permitted emission level E and the better the environmental quality M, the more the representative firm can produce.

A period lasts very long (several decades). I therefore assume that the capital stock fully depreciates within a period, such that next period's capital stock is always equal to current period's investment. In addition, I assume

that the state of technology grows at an exogenous rate g, and that labor input remains constant over time and is normalized to 1:

$$K_{t+1} = I_t \tag{2}$$
$$A_{t+1} = A_t(1 + g) \tag{3}$$
$$L_t = 1 \tag{4}$$

The firm hires labor and invests in new capital taking as given the real wage w, the real interest rate r, the permitted emission level E and the environmental quality M. Profit maximization yields then the following first-order conditions:

$$(1 - \alpha)Y_t = w_t \tag{5}$$
$$\alpha \frac{Y_t}{K_t} = 1 + r_t \tag{6}$$

The environmental quality M not only affects aggregate production, it also determines the abatement costs G which the government has to incur. I assume that these abatement costs are proportional with aggregate output; and as the government balances its budget in every period, taxes T are proportional with aggregate output as well:

$$G_t = T_t = \tau_t Y_t \quad \text{where} \quad \tau_t = z - \zeta M_t^{\mu_Z} \tag{7}$$

...with $0 < \zeta < z < 1$ and $\mu_Z > 0$. So the lower the environmental quality, the higher are the abatement costs for a given production level and the higher is the share of taxes in aggregate income. Note that as M is measured on a scale from 0 to 1, τ_t is always between z (for $M_t = 0$) and $z - \zeta$ (for $M_t = 1$).

I assume that initially, in period 0, the economy is in a steady state where the environmental quality is optimal, and where emissions do not pollute the environment; consequently, the government sets E to its maximum level in period 0. From period 1 onwards, however, emission levels affect the dynamics of M according to the following law of motion:

$$M_{t+1} = 1 + \phi(M_t - 1) - \psi E_t \quad \text{for } t \geq 1 \tag{8}$$

...where $0 < \phi < 1$ and $0 < \psi < 1 - \phi$. So from period 1 onwards, the government faces a trade-off between setting a high emission level and allowing firms to produce a lot today, or setting a low emission level to protect the environment and avoid the impact of environmental degradation on production and government spending in the future.

2.1.2 Consumption and saving

Households live for two periods. In the beginning of every period, a new generation is born, and at the end of every period, the oldest generation dies. In the first period of life, households supply labor, earn labor income, pay a lump sum tax, and consume part of their disposable income; the rest of their disposable income is saved to finance their consumption in their second period of life, when they are retired.

I assume that all households have the same preferences. The consumption and saving decisions of the generation born in period t can then be derived by maximizing the utility function of a representative household,

$$U_t = \ln c_{1,t} + \frac{1}{1+\theta} \ln c_{2,t+1} \quad \text{with } \theta > 0 \tag{9}$$

subject to her lifetime budget constraint,

$$c_{1,t} + \frac{1}{1+r_{t+1}} c_{2,t+1} = w_t - T_t \tag{10}$$

...where $c_{1,t}$ and $c_{2,t+1}$ are her consumption in her first and second period of life, and T_t are the lump sum taxes which she has to pay when she is young; θ is her subjective discount rate. As the representative household supplies one unit of labor when she is young, her labor income is equal to w_t, and her disposable income in her first period of life is $w_t - T_t$. Note that I assume that households do not leave bequests.

Utility maximization leads then to the following expressions for $c_{1,t}$ and $c_{2,t+1}$:

$$c_{1,t} = (1 - s_Y)(w_t - T_t) \tag{11}$$
$$c_{2,t+1} = (1 + r_{t+1})s_Y(w_t - T_t) \tag{12}$$

...where s_Y is the saving rate of the young generation:

$$s_Y = \frac{1}{2+\theta} \tag{13}$$

2.1.3 Equilibrium

In every period the goods market clears, such that aggregate saving is always equal to aggregate investment. Saving by the young generation in period t is $s_Y(w_t - T_t)$; saving by the old generation is zero, as they only have capital

income, which they completely consume. Equilibrium in the goods market therefore implies that

$$s_Y(w_t - T_t) = I_t \tag{14}$$

Substituting the firm's first-order condition (5) in equation (14), and using (7) to eliminate T_t, shows then how aggregate investment depends on aggregate output:

$$I_t = s_Y(1 - \alpha - \tau_t)Y_t \tag{15}$$

Aggregate consumption C_t, which is the sum of the consumption of the young generation and the elderly, can be found as follows: substitute equation (14) in the expression for the consumption of the elderly, equation (12), and recall that $I_t = K_{t+1}$ according to the law of motion (2); we then find that $c_{2,t+1} = (1+r_{t+1})K_{t+1}$; now rewrite this equation and the consumption equation for the young generation, equation (11), by exploiting the firm's first-order conditions (5) and (6), and eliminate T_t with equation (7). We then find how aggregate consumption depends on aggregate output:

$$
\begin{aligned}
C_t &= c_{1,t} + c_{2,t} \\
&= (1 - s_Y)(1 - \alpha - \tau_t)Y_t + \alpha Y_t \\
&= [(1 - s_Y)(1 - \alpha - \tau_t) + \alpha]Y_t \tag{16}
\end{aligned}
$$

The gross interest rate $1+r_{t+1}$ follows from the firm's first-order condition (6): use the fact that $I_t = K_{t+1}$, and use (15) to express I_t as a function of Y_t. We then find that

$$1 + r_{t+1} = \alpha \frac{1 + g_{t+1}}{s_Y(1 - \alpha - \tau_t)} \tag{17}$$

where g_{t+1} is the growth rate of aggregate output from period t to period $t+1$: $1 + g_{t+1} = Y_{t+1}/Y_t$. For the further discussion, it is useful (but not necessary) to assume that $\alpha/(s_Y(1 - \alpha)) > 1$, such that the interest rate is always higher than the growth rate of aggregate output.

I therefore conclude that aggregate investment and aggregate consumption are negatively affected by τ, the share of taxes in aggregate income. And as the tax share increases as the environmental quality goes down, we find that the share of aggregate income that is invested or consumed is lower the more the environment has been destroyed.

Similarly, given the growth rate of aggregate output, the interest rate is positively affected by τ - which means that environmental degradation increases the extent to which the interest rate is above output growth.

2.2 The steady state as a function of the emission level

I assume that the economy starts off in period 0 in a steady state where pollution is not a concern for policy makers. So in period 0, the environmental quality is optimal, emissions do not pollute the environment, and the government consequently sets the permitted emission level to its maximum value:

$$M_0 \;=\; E_0 \;=\; 1 \tag{18}$$

As the environmental quality is optimal, the share of aggregate income that goes to taxes is at its lower bound:

$$\tau_0 \;=\; z - \zeta \tag{19}$$

Substituting in equations (15) and (17) shows that the share of investment in aggregate income is at its highest possible level,

$$\frac{I_0}{Y_0} \;=\; s_Y(1 - \alpha - \tau_0) \tag{20}$$

while the interest rate is at its lowest possible level

$$1 + r_0 \;=\; \alpha \frac{1 + g}{s_Y(1 - \alpha - \tau_0)} \tag{21}$$

(where I use the fact that the steady state growth rate of aggregate output is equal to the technological growth rate g).

For future reference, I define $\bar{Y}_t$, the output level which the economy would attain in period t if it could grow along this initial steady state without suffering any environmental degradation or cuts in emission levels. The value for $\bar{Y}_t$ follows from substituting (21) in the first-order condition (6), combined with the production function (1) where E_t and M_t are assumed to be equal to 1:

$$\bar{Y}_t \;=\; \left(\frac{s_Y(1 - \alpha - \tau_0)}{1 + g} \right)^{\frac{\alpha}{1 - \alpha}} A_t \tag{22}$$

The economy will not attain $\bar{Y}_t$, however: from period 1 onwards, the dynamics of M are given by the law of motion (8), and the government has to weigh the costs and benefits of the emission levels which it allows. Suppose that the economy converges to a new steady state, which depends on the government's environmental policy, and let E^* be the emission level which the government allows in this new steady state. From (8) follows then the new steady state value of the environmental quality:

$$M^* \;=\; 1 - \frac{\psi}{1 - \phi} E^* \tag{23}$$

9

Substituting in equation (7) yields the new steady state share of taxes in aggregate income:

$$\tau^* = z - \zeta M^{*\mu_Z} \tag{24}$$

Substituting in equations (15) and (17), taking into account that aggregate output grows at rate g in the new steady state, gives the new steady state values of the investment share and the interest rate:

$$\frac{I_t^*}{Y_t^*} = s_Y(1 - \alpha - \tau^*) \tag{25}$$

$$1 + r^* = \alpha \frac{1+g}{s_Y(1 - \alpha - \tau^*)} \tag{26}$$

From (26), the firm's first-order condition (6) and the production function (1) follows then the new steady state level of aggregate output:

$$Y_t^* = \left(\frac{s_Y(1 - \alpha - \tau^*)}{1+g}\right)^{\frac{\alpha}{1-\alpha}} (E^{*\varepsilon} M^{*\mu_Y})^{\frac{1}{1-\alpha}} A_t \tag{27}$$

The percentage output loss compared with the case where the economy could move along the initial steady state, without suffering any environmental degradation or cuts in emission levels, follows from equations (22) and (27):

$$
\begin{aligned}
\triangle Y^* &= \frac{\bar{Y}_t - Y_t^*}{\bar{Y}_t} \\
&= 1 - \left(\frac{1 - \alpha - \tau^*}{1 - \alpha - \tau_0}\right)^{\frac{\alpha}{1-\alpha}} (E^{*\varepsilon} M^{*\mu_Y})^{\frac{1}{1-\alpha}} \tag{28}
\end{aligned}
$$

This expression identifies three reasons why the economy moves to a lower output level if the environment is affected by emissions: first, the higher tax share τ^* lowers the investment share, and therefore also the steady state capital stock; second, the steady state emission level E^* is lower; and third, the steady state level of the environmental quality M^* is lower.

2.3 Optimal environmental policy

Let us now consider a *social evaluator* (a private citizen, a government official, perhaps even an economist), who wants to figure out the optimal time path of emission allowances E once the environment starts getting polluted as of period 1.[6]

---

[6]I deliberately do not assume a social planner. The title of "social planner" suggests wide powers, including the power to determine government saving. The social evaluator, in contrast, takes the workings of the economy as given, including the assumption that the government budget is balanced in every period.

I assume that all social evaluators agree that the optimal path of emission allowances can be found by maximizing a social welfare function, given by the present discounted value of a stream of logarithmic felicity specifications of aggregate consumption:

$$W_t \;\; = \;\; \sum_{s=t}^{\infty} \left(\frac{1}{1+\rho} \right)^{s-1} \ln C_s \quad \text{with } \rho > 0 \qquad (29)$$

...where ρ is called *the social evaluator's subjective discount rate*. Unfortunately, it is not clear what an appropriate value for ρ would be: private citizens and government officials may or may not have an idea about this; and economists may or may not think that they have something to say about it.[7]

Note that ρ may well be different from the households' subjective discount rate θ. The households' subjective discount rate shows how households trade off consumption based felicity when they are young with consumption based felicity when they are retired. So it determines their personal consumption and saving decisions over their own lifetime. It does not say anything about how they would trade off aggregate consumption of the generations that are currently alive with aggregate consumption of the generations that are alive at some point in the future.

Therefore, the best thing we can do at this point is to derive the optimal path for emission allowances for a range of possible values of the social evaluator's subjective discount rate, given the firms' and the households' production and consumption behavior, and given the relations between the emission levels, the environmental quality, production, abatement costs and taxes. Note that I will thus assume full knowledge about the set-up of the economy as described in section 2.1. I will then use this analysis as a benchmark in sections 3 and 4, where I will assess the policy advice of neo-classical and new-classical economists who do not have full knowledge about the economy's set-up and who are therefore forced to make some simplifying assumptions in their models.

So let us maximize the social welfare function (29) as of period 1, subject to the aggregate investment and consumption functions (15) and (16), the aggregate production function (1), the tax function (7), the laws of motion for

---

[7]Actually, in the real world there is not even agreement about the social welfare function (29). First, the assumption that $\rho > 0$ implies that the well-being of the current generation matters more than then the well-being of a future generation - which is considered unethical by many philosophers and economists, including Ramsey (1928). Second, at least since the Brundtland Report (WCED, 1987), sustainability has been a prime policy objective around the globe; but sustainability is not a necessary nor a sufficient condition for discounted utilitarianism (of which the social welfare function (29) is an example), even not when $\rho = 0$ - see Pezzey and Toman (2002) for a literature review of the economics of sustainability.

the capital stock, the state of technology, and the environmental quality (2), (3) and (8), and taking as given the values for the state variables K, A and M in period 1:

$$
\begin{aligned}
W(K_t, A_t, M_t) & = \max_{E \in [0,1]} \left\{ \ln C_t + \frac{1}{1+\rho} W(K_{t+1}, A_{t+1}, M_{t+1}) \right\} \\
\text{subject to} \quad C_t & = [(1 - s_Y)(1 - \alpha - \tau_t) + \alpha] Y_t \\
I_t & = s_Y (1 - \alpha - \tau_t) Y_t \\
Y_t & = E_t^\varepsilon M_t^{\mu_Y} K_t^\alpha (A_t L_t)^{1-\alpha} \\
\tau_t & = z - \zeta M_t^{\mu_Z} \\
K_{t+1} & = I_t \\
A_{t+1} & = A_t (1 + g) \\
M_{t+1} & = 1 + \phi(M_t - 1) - \psi E_t \\
K_1 & = I_0, \quad A_1 = A_0(1 + g) \quad \text{and} \quad M_1 = 1
\end{aligned}
\tag{30}
$$

In appendix A, I solve this dynamic programming problem, I sketch a numerical procedure to derive the transitional dynamics if the model is parameterized, and I derive the new steady state to which the economy will converge. Note for future reference that the optimal steady state emission level, denoted by E^*, depends on the social evaluator's subjective discount rate ρ in such a way that

$$
\frac{\varepsilon}{E^*} \geq \frac{\psi}{1 + \rho - \phi} \frac{1}{M^*} (\mu_Y + \mu_Z \lambda^*) \quad \text{and} \quad E^* \leq 1 \quad \text{with c.s.} \tag{31}
$$

where "c.s." stands for "complementary slackness", M^* is the optimal steady state level of the environmental quality, and

$$
\lambda^* = \frac{1 + \rho - \alpha}{1 + \rho} (1 - s_Y) \lambda_c^* + \frac{\alpha}{1 + \rho} s_Y \lambda_I^* \tag{32}
$$

$$
\text{with} \quad \lambda_c^* = \frac{\zeta M^{*\mu_Z}}{(1 - s_Y)(1 - \alpha - z + \zeta M^{*\mu_Z}) + \alpha}
$$

$$
\text{and} \quad \lambda_I^* = \frac{\zeta M^{*\mu_Z}}{s_Y(1 - \alpha - z + \zeta M^{*\mu_Z})}
$$

Eliminating M^* with equation (23) yields an equation in E^*, which can be solved numerically if the model is parameterized. Once we have E^*, we can use equations (23), (24), (25), (26) and (28) to compute the optimal steady state values of the environmental quality, the tax share, the investment share, the interest rate, as well as the aggregate output loss compared with the case where emissions do not pollute the environment.

Let us illustrate this with a numerical example. Let us assume that one period lasts for 30 years. I set the capital share of aggregate income, α, to 1/3. Assuming 1.5% technological growth annually, I set $1 + g = 1.015^{30}$. I choose $\phi = 0.95$, such that the half-life of a shock in environmental quality is $30 * \ln 0.5 / \ln 0.95 \approx 400$ years. μ_Y and μ_Z are both set to 0.5. I assume that government spending in period 0 is 20% of aggregate output; and I assume that if the government forever keeps the emission allowance at its maximum level after period 0, the maximum output loss due to environmental degradation amounts to 10%, while the extra abatement costs for the government are another 10% of aggregate production. To satisfy these assumptions, I set $\psi = 0.0095$, $z = 1.2$ and $\zeta = 1$. I then choose s_Y such that aggregate investment is initially 20% of aggregate output. Finally, I assume that $\varepsilon = 0.01$: in this way, the optimal environmental policy if the social evaluator's subjective discount rate is 0, would lead the economy to a steady state where aggregate output is about 5% below what it would be if the economy could simply continue growing along the initial steady state without any environmental degradation or cuts in emission levels.

The graphs in figure 1 show then how the social evaluator's (annualized) subjective discount rate affects the optimal policy's transitional dynamics and steady state values of the emission level E, the environmental quality M, the tax share τ, the investment share I/Y, the (annualized) interest rate r, and the percentage output loss compared with the case where the economy could move along the initial steady state without suffering any environmental degradation or cuts in emission levels, which is given by $(Y - \bar{Y})/\bar{Y}$. The transitional dynamics are given for 50 periods; the new steady state values are projected on the back plane of the graphs. The transitional dynamics for some selected values of the subjective discount rate ($\rho \in \{0.005, 0.01, 0.015, 0.02, 0.025\}$) are traced out in bold.

The first graph shows that a higher subjective discount rate leads to higher emission levels, which, according to the second graph, causes faster deterioration of the environmental quality. The next three graphs show how this leads to a higher tax share, a lower investment share, and a higher interest rate. The last graph illustrates the trade-off which the social evaluator faces: the higher the social evaluator's subjective discount rate, the lower the impact is on aggregate output in the short run (because of higher emission levels), but the larger the output losses will be in the long run (because of more environmental degradation).

It is important to note that I do not claim that the parameter values which I choose and the graphs in figure 1 are realistic. As a matter of fact, the main motivation of this paper is precisely to point out that the micro-foundations

in macroeconomic models are almost always so much simplified that a clear relation with the available real-world micro-economic evidence is hard if not impossible to establish, and that it is dangerous to do as if these models *are* a positive description of the real world. In this respect, this paper is no exception: it is meant as a thought experiment, not as a positive model.

3   A neo-classical analysis

In the previous section, I derived the social evaluator's optimal environmental policy. But this required that he has a full understanding of how the economy works, which is typically not the case. So let us assume that the social evaluator turns to two experts for advise: a neo-classical and a new-classical economist.

I assume that both the neo-classical and new-classical economist know the interaction between production, pollution, environmental quality, abatement costs and taxes, as described in subsection 2.1.1; they are also aware that all markets always clear.[8] But unfortunately, they don't know much about the household sector: they lack sufficient micro-economic evidence to model the household sector in detail; and even if they had sufficient micro-economic evidence, they suspect that there is so much heterogeneity and behavioral complexity in the household sector that they would succumb to the curse of dimensionality if they tried to aggregate the consumption and saving decisions of all the individual households to model the macro-economy. So both the neo-classical and the new-classical economist are forced to make some drastic simplifications. In this section, I describe how a neo-classical economist does this. I will then describe in the next section the approach of a new-classical economist.

A neo-classical economist tries to find some stylized relations between aggregate variables. For instance, looking at data of aggregate consumption, aggregate output and government spending and taxes, he may find it reasonable to assume that in the long run aggregate saving is more or less a constant

---

[8]Of course, economists are not so lucky in the real world. There is in fact substantial heterogeneity among firms (which is the reason of existence of a trading system for emission allowances such as the SO_2-trading system of the Clean Air Act in the U.S. and the European Union's ETS); an aggregate production function may not exist (which is part of what the Cambridge-Cambridge controversy was about), and if it exists, deriving aggregate hiring and investment decisions by maximizing a profit function subject to such an aggregate production function may well lead to another fallacy of composition; there is substantial uncertainty about the impact of emissions on the environment; the political economy behind the government budget is complicated and arguably hard to predict; and markets don't always clear.

fraction of aggregate disposable income, just as in the Solow model (Solow, 1956). Let us assume that his estimate of this constant aggregate saving rate is such that it always perfectly matches the most recent data that are available.[9] As he knows that aggregate saving always equals aggregate investment, his period T estimate of the aggregate saving rate, $\hat{s}_T$, is then given by

$$\hat{s}_T = \frac{I_T}{Y_T - T_T} \tag{33}$$

Armed with this estimate of the aggregate saving rate and recalling that taxes are proportional with aggregate income according to equation (7), the neo-classical economist then designs a model of the economy for periods $t \geq T$ which features the following aggregate investment and consumption functions:

$$I_t = \hat{s}_T(1 - \tau_t)Y_t \tag{34}$$
$$C_t = (1 - \hat{s}_T)(1 - \tau_t)Y_t \tag{35}$$

Combining equation (34) with the capital stock's law of motion (2) and the firm's first-order condition (6) yields the interest rate equation in his model:

$$1 + r_{t+1} = \alpha \frac{1 + g_{t+1}}{\hat{s}_T(1 - \tau_t)} \tag{36}$$

Suppose now that the neo-classical economist wants to use his model to advise the social evaluator on the optimal path for emission allowances as of period 1. He would then take the social welfare function (29) as of period 1, and maximize it subject to the aggregate production function (1), the tax function (7), the laws of motion for the capital stock, the state of technology, and the environmental quality (2), (3) and (8) (which is all common knowledge), and subject to the investment and consumption functions (34) and (35) which he has estimated in period 0, starting from the values for the state variables K, A and M in period 1 (which are known at the end of period 0):

$$W(K_t, A_t, M_t) = \max_{E_t \in [0,1]} \left\{ \ln C_t + \frac{1}{1 + \rho} W(K_{t+1}, A_{t+1}, M_{t+1}) \right\}$$
$$\text{subject to} \quad C_t = (1 - \hat{s}_0)(1 - \tau_t)Y_t$$
$$I_t = \hat{s}_0(1 - \tau_t)Y_t$$
$$Y_t = E_t^\varepsilon M_t^{\mu_Y} K_t^\alpha (A_t L_t)^{1-\alpha}$$

---

[9]I make this assumption for convenience, but it may have a touch of realism: every period lasts very long (several decades), so if there are no good reasons to believe that there are long cycles, any change in the observed saving rate may well be perceived to be permanent.

15

$$\tau_t = z - \zeta M_t^{\mu_Z} \tag{37}$$
$$K_{t+1} = I_t$$
$$A_{t+1} = A_t(1 + g)$$
$$M_{t+1} = 1 + \phi(M_t - 1) - \psi E_t$$
$$K_1 = I_0, \quad A_1 = A_0(1 + g) \quad \text{and} \quad M_1 = 1$$

The neo-classical economist then does his computations, and derives the transitional dynamics and the new steady state as documented in Appendix B. Note that according to his policy advice, the economy will eventually settle down in a steady state which depends on the social evaluator's subjective discount rate in such a way that

$$\frac{\varepsilon}{E^*} \geq \frac{\psi}{1 + \rho - \phi} \frac{1}{M^*}(\mu_Y + \mu_Z \tilde{\lambda}^*) \quad \text{and} \quad E^* \leq 1 \quad \text{with c.s.} \tag{38}$$

where

$$\tilde{\lambda}^* = \frac{\zeta M^{*\mu_Z}}{1 - z + \zeta M^{*\mu_Z}} \tag{39}$$

He then eliminates M^* with equation (23), solves for E^*, and uses equations (23), (24), (25), (26) and (28) to compute the new steady state values of the environmental quality, the tax share, the investment share, the interest rate, as well as the aggregate output loss compared with the case where the economy could move along the initial steady state without loss in environmental quality and emission cuts.

Assuming the same parameter values as in the previous section, the neo-classical economist then goes back to the social evaluator in the beginning of period 1 with the graphs in figure 2, to show the implications of different values of the social evaluator's subjective discount rate ρ for the transitional dynamics and steady state values; the transitional dynamics for some selected values of the subjective discount rate are traced out in bold. Note, however, that the neo-classical economist does not take a stand about the subjective discount rate: he cannot, because he doesn't observe it in his macrocomomic data set.[10]

---

[10]Of course, he may have his own preferences for ρ, perhaps inspired by ethical or philosophical considerations. But then he typically acknowledges this, and admits that picking a suitable value for ρ is beyond what positive economics can accomplish, as in Dasgupta and Heal (1974) and Stiglitz (1974). (Solow, 1974, uses a Rawlsian social welfare function, not a discounted utilitarian one, so he does not need a value for ρ. But he also stresses throughout his paper that choosing an appropriate social welfare function and its parameterization is inherently an ethical and philosophical exercise.)

The transitional dynamics in figure 2 look very similar as the transitional dynamics of the optimal policy in figure 1: a higher discount rate leads to higher emission levels and faster environmental degradation, which drives up the tax share and the interest rate, and gradually depresses the investment share; and the neo-classical economist recognizes the trade-off which the government faces between setting low emission levels to avoid the impact of environmental degradation on aggregate output in the long run, or setting high emission levels which allow firms to produce more in the short run.

How well does the neo-classical economist do?

Figure 3 compares the steady state predictions of his advice with the steady states if the optimal policy is followed, for different values of the social evaluator's subjective discount rate ρ: the thin red curves are the steady states if the optimal policy is followed, which were also projected at the back plane of the graphs in figure 1; the broken green curves are the neo-classical economist's predictions as of period 1, which are taken from the back plane of the graphs in figure 2 - note that in the first three graphs in figure 3, the broken green curves coincide with full thick green curves (which will be introduced in a moment).

Naturally, the neo-classical economist makes some mistakes. His computation of the optimal steady state emission level is not completely correct, for instance: the green curve in the upper left graph in figure 3 deviates a little bit from the thin red curve, which is a consequence of the fact that $\tilde{\lambda}^*$ in equation (39) is not exactly the same as λ^* in equation (32). The reason for this mistake is that the neo-classical economist assumes that *all* income is taxed. But that is not true: in fact, only the income of the young generation is taxed - which has implications for the effect on aggregate consumption of a change in the tax rate due to environmental degradation. Because of equations (23), (24), (25), (26) and (28), this mistake spills over in all the other graphs of figure 3.

Furthermore, recall that environmental degradation causes higher abatement costs, such that the tax rate steadily increases until the new steady state is reached. But as only the income of the young generation is taxed, and as this is the only generation that saves, the *aggregate* saving rate will in fact go down as the economy moves to the new steady state - and not remain constant as the neo-classical economist assumes. As a result, the neo-classical economist, armed with his period 0 estimate of the aggregate saving rate $\hat{s}_0$, overestimates the steady state value of the investment share, and underestimates the steady state interest rate and the output loss - which follows immediately from equations (25), (26) and (28), and which is apparent from the last three graphs in figure 3.

But the neo-classical economist learns from his mistakes: in the next period, as environmental degradation will have pushed up the tax rate, he will observe

a lower aggregate saving rate and revise his estimate of the aggregate saving rate accordingly - and he will continue to do so in all subsequent periods, until the economy reaches a new steady state.

I illustrate this with the thick green curves in figure 3. Let us assume that the social evaluator's subjective discount rate ρ is 1.5% annually, and that the time path of emission levels is set according to the neo-classical economist's policy recommendations for this particular value of ρ.[11] As the neo-classical economist will steadily revise his estimate of the aggregate saving rate downwards, the curve that shows his predictions of the new steady state values for the investment share will steadily move downwards as well, while the curves that represent his predictions for the new steady state values of the interest rate and aggregate output loss will steadily move upwards. This goes on until the economy reaches a new steady state, where the neo-classical economist's estimate of the aggregate saving rate turns out to be correct. In this new steady state (reached after the emission levels have consistently been set following the neo-classical economist's policy advice for a subjective discount rate of 1.5% annually), the steady state predictions of the neo-classical economist for a *range* of values of ρ are given by the thick green curves in figure 3. So eventually, his steady state predictions turn out to be almost perfect, as long as the social evaluator does not suddenly prefer a totally different subjective discount rate.[12]

4   A new-classical analysis

New-classical economists, aware of the great critiques by Lucas (1976) and Sims (1980), find the neo-classical approach of the previous section *ad hoc*. Simply looking for some stylized relations between aggregate variables makes the neo-classical model vulnerable for the Lucas critique. Furthermore, they argue, as neo-classical economists remain silent about the households' objectives, they cannot carry out a proper welfare analysis: neo-classical economists have to

---

[11]The only reason why I choose a value for ρ of 1.5% is that this is right in the middle of the horizontal axis in figure 3, which helps to make the graphs more transparent. Note that if I would have set ρ to 0.1%, as in the Stern Review (Stern, 2007), the neo-classical economist's predictions of the steady state implications of the optimal environmental policy would have been almost perfectly right already at the end of period 0, using his period 0 estimate of the aggregate saving rate. This is because a low subjective discount rate minimizes environmental degradation, and therefore also the steady state implications for abatement costs, the tax share and the aggregate saving rate. But then again, the curves in figure 3 are not meant to be numerically realistic - they are part of a thought experiment.

[12]Note, however, that the optimal emission level will still be computed inaccurately, because of the discrepancy between $\tilde{\lambda}^*$ and λ^*.

impose a social welfare function, without being sure that it is somehow based on the preferences of the economic agents.

New-classical economists therefore propose to build macroeconomic models from micro-foundations. They assume households and firms with well-specified objective functions, and derive the behavior of the economic agents by maximizing their objectives subject to the constraints which they face; new-classical economists then aggregate the ecomomic decisions of the different agents, impose an equilibrium device such as market clearance, and derive the aggregate behavior of the economy.

Ideally, one would like to build the micro-foundations from the *bottom up* (De Grauwe, 2009): collect micro-economic evidence about households and firms and the institional set-up, and then carefully aggregate them making abstractions and simplifications where possible (as we don't want a map of scale 1 to 1). However, this strategy is mathematically often infeasible; and the available micro-economic evidence is a cheese with many, many holes (Hansen and Heckman, 1996).[13] New-classical economists therefore build their micro-foundations from the *top down*: they think about how much behavioral complexity, heterogeneity and institutional detail a model can swallow without becoming mathematically intractable, and then look at the data for calibration or (Bayesian) estimation procedures.

So a typical new-classical approach, especially for questions where the focus is on the supply side, is to assume away all heterogeneity among households, and to assume that the economy is populated by households who live infinitely long and all have the same preferences and budget constraints - which implies that their consumption and saving decisions can be derived by maximizing the utility of an immortal *representative* household.

Let us do this: let us assume that the economy is populated by a representative household who maximizes the utility function

$$U_t \;=\; \sum_{s=t}^{\infty} \left(\frac{1}{1+\theta^R} \right)^{s-t} \ln C_s \quad \text{with } \theta^R > 0 \tag{40}$$

subject to the budget constraint and the transversality condition

$$K_{t+1} \;=\; K_t(1+r_t) + w_t - T_t - C_t \tag{41}$$

$$\lim_{s \to \infty} \Pi_{s'=t+1}^{s} \frac{1}{1+r_{s'}} K_s \;=\; 0 \tag{42}$$

and taking K_t, the factor prices w and r and the lump sum taxes T as given. θ^R is the representative household's subjective discount rate. As in the previous section, the set-up of the supply side (the aggregate production function

---

[13]There are exceptions, as already noted in footnote 4.

(1), the tax function (7), and the laws of motion for the capital stock, the state of technology, and the environmental quality (2), (3) and (8)) is assumed to be common knowledge. The representative household uses this information to forecast factor prices and lump sum taxes, given the government's environmental policy.

The representative household's maximization problem leads then to the Euler equation

$$\frac{1}{C_t} = \frac{1 + r_{t+1}}{1 + \theta^R} \frac{1}{C_{t+1}} \tag{43}$$

Moving the dynamic budget constraint (41) forward and combining with the transversality condition (42) yields the household's lifetime budget constraint. By combining with the Euler equation (43), current consumption C_t can then be written as a function of K_t and the current and future factor prices and lump sum taxes.

The current and future factor prices and lump sum taxes depend on the time path of the emission levels, which are set by the government. How the government does this, was not specified in the set-up in section 2. But let us assume that the representative household thinks that the government's emission policy only depends on the state of the economy as described by the state variables K, A and M:

$$E_{t'} = E(K_{t'}, A_{t'}, M_{t'}), \quad \forall\, t' \geq t \tag{44}$$

Given the laws of motion of K, A and M, it then follows that current consumption C_t can be written as a function of the current state variables K_t, A_t and M_t:

$$C_t = C(K_t, A_t, M_t) \tag{45}$$

If the model is parameterized, the consumption function $C(\cdot)$ can be derived for any environmental policy $E(\cdot)$ by, for instance, a time iteration procedure.

The new-classical economist faces one unknown parameter, however: the representative households' subjective discount rate θ^R. But an estimate of θ^R follows immediately from the Euler equation (43). Let us assume that the new-classical economist always estimates θ^R in such a way that it perfectly matches the most recent data that are available.[14] His period T estimate of the representative household's subjective discount rate, $\hat{\theta}_T^R$, is then given by

$$\hat{\theta}_T^R = \frac{1 + r_T}{1 + g_T^c} - 1 \tag{46}$$

---

[14]A similar remark as in footnote 9 holds also here: I make this assumption for convenience, but given that every period lasts very long, it may have a touch of realism.

20

where $g_T^c = C_T/C_{T-1}$ is the growth rate of aggregate consumption between periods $T - 1$ and T.

Note that equation (46) is approximately equivalent with $r_T = \hat{\theta}_T^R + g_T^c$, which is the Ramsey rule (taking into account that the elasticity of the marginal social value of consumption is -1 because of the logarithmic felicity specification). So we find that the new-classical economist uses the market interest rate to calibrate the subjective discount rate with the Ramsey rule, where the Ramsey rule is derived from his macro-economic model with micro-foundations.

Suppose now that in the beginning of period 1, the social evaluator asks the new-classical economist for advice on the optimal path of emission levels E. Armed with his period 0 estimate of the representative household's subjective discount rate, $\hat{\theta}_0^R$, the new-classical economist then replies that he has a model that does not only describe how the economy works, but that has even revealed the population's preferences about the subjective discount rate which the social evaluator should use. So he tells the social evaluator not to worry about the subjective discount rate, and that he will use his model to figure out what environmental policy maximizes the utility of some representative household in the economy.[15]

The challenge which the new-classical economist then faces is to find an emissions function $E(\cdot)$ that maximizes the utility function (40), where the subjective discount rate is set to $\hat{\theta}_0^R$, and taking the representative household's consumption behavior as described in the consumption function (45) as given (and where this consumption function is derived by using the optimal emissions function $E(\cdot)$). He solves this problem as follows. He first computes the emissions function that maximizes the utility function (40) as of period 1 *for a given consumption function* (45), subject to the aggregate production function (1), the tax function (7), the laws of motion for the capital stock, the state of technology, and the environmental quality (2), (3) and (8), and starting from

---

[15]It is important to point out that this is not what is usually done in climate change economics, even not by environmental economists who favor the descriptive approach to cost-benefit analysis. Some integrated assessment models (IAMs), such as FUND (Tol, 2002a and 2002b), are not based on a utility maximizing immortal representative agent, but simply assume a constant saving rate - very much in the same spirit as the neo-classical analysis in the previous section; however, to justify the descriptive approach, it is necessary that the dynamics of aggregate consumption and saving can be derived by maximizing the utility function of a representative agent, which is inconsistent with a constant saving rate. Other IAMs, such as DICE (Nordhaus, 2008), do provide a consistent framework for the descriptive approach, but only by assuming a social planner (who determines both the time path of optimal emission allowances and the time path of consumption and investment).

the values for the state variables K, A and M in period 1:

$$
\begin{aligned}
U(K_t, A_t, M_t) \;&=\; \max_{E_t \in [0,1]} \left\{ \ln C_t + \frac{1}{1 + \hat{\theta}_0^R} U(K_{t+1}, A_{t+1}, M_{t+1}) \right\} \\
\text{subject to} \quad C_t &= C(K_t, A_t, M_t) \\
I_t &= (1 - \tau_t) Y_t - C_t \\
Y_t &= E_t^\varepsilon M_t^{\mu_Y} K_t^\alpha (A_t L_t)^{1-\alpha} \\
\tau_t &= z - \zeta M_t^{\mu_Z} \\
K_{t+1} &= I_t \\
A_{t+1} &= A_t(1 + g) \\
M_{t+1} &= 1 + \phi(M_t - 1) - \psi E_t \\
K_1 &= I_0, \quad A_1 = A_0(1 + g) \quad \text{and} \quad M_1 = 1
\end{aligned}
\tag{47}
$$

Once he knows the optimal emissions function *for a given consumption function,* he uses a simple iteration scheme to find the emissions and consumption functions that are *jointly* consistent with the maximization problem of the representative household and the maximization problem of the social evaluator, i.e. the emissions function which solves the social evaluator's problem (47), while the consumption function maximizes the utility function (40) subject to the budget constraint (41) and the transversality condition (42), where the factor prices and lump sum taxes (which the representative household takes as given) are consistent with the optimal environmental policy. Mathematical and numerical details are provided in Appendix C.

In Appendix C, I also show that according to this new-classical policy advice, the economy will eventually converge to a steady state which depends on the estimate of the representative household's subjective discount rate, $\hat{\theta}_0^R$, in such a way that

$$
\frac{\varepsilon}{E^*} \;\geq\; \frac{\psi}{1 + \hat{\theta}_0^R - \phi} \frac{1}{M^*} (\mu_Y + \mu_Z \tilde{\lambda}^*) \quad \text{and} \quad E^* \leq 1 \quad \text{with c.s.}
\tag{48}
$$

where $\tilde{\lambda}^*$ is a variable that also appeared in the neo-classical computations, and is defined in equation (39). The steady state values of the optimal emission level, the environmental quality, the tax share, the investment share, the interest rate and output loss can then be computed in a similar way as in the previous sections.

The graphs in figure 4 show the results of the computations of the new-classical economist (assuming the same parameter values as in the previous sections). Note that his policy advice is much more clear-cut than the advice of his neo-classical colleague. The neo-classical economist did not take a stand

about the subjective discount rate; so the best thing he could do is to show the implications of different values of the subjective discount rate, leaving it to the social evaluator to pick the value that he finds most appropriate. The new-classical economist, at the other hand, traces out the implications of just one value for the subjective discount rate, namely the subjective discount rate which - according to his analysis - reflects the preferences of the population.

How well does the new-classical economist do?

Figure 5 compares the steady state predictions of his advice with the steady states if the optimal policy is folowed. As in figure 3, the thin red curves are taken from the back plane of the graphs in figure 1, and represent the steady states if the optimal policy is followed for a range of values of the subjective discount rate ρ. The broken vertical line indicates the new-classical economist's estimate in period 0 of the representative agent's subjective discount rate, $\hat{\theta}_0^R$. The small green dots are taken from the right vertical axes in the graphs in figure 4, and give the steady state values which the economy will arrive at if the emission levels are set according to the new-classical economist's recommendations in period 0.

In computing the optimal emission levels, the new-classical economist makes exactly the same mistake as his neo-classical colleague, for essentially the same reason: by lumping all households together and not taking into account that in fact only the income of the young generation is taxed, the effect on aggregate consumption of a change in the tax rate due to environmental degradation is distorted - a mistake which affects all the other graphs in figure 5.

In addition (and unlike the neo-classical economist), the new-classical economist misses the point that environmental degradation leads to a higher interest rate: according to his advice in the beginning of period 1 (based on $\hat{\theta}_0^R$), the steady state interest rate will not change compared with period 0. As a result, his steady state projections for the investment share and output loss are totally off-track.

But his most serious mistake is that he claims to perceive the population's preferences for the subjective discount rate in the social welfare function. Of course that is not possible: how households trade off aggregate consumption today with aggregate consumption in the next period or in a period when they are not alive anymore, is not even specified in the set-up of section 2. It is an open question how households would want to do this. And suppose that the households wanted to make this trade-off in the same way as how they trade off their own consumption: even then the new-classical economist makes a mistake. His estimate of the subjective discount rate is based on the aggregate Euler equation (43). But the aggregate Euler equation suffers from a fallacy of composition: even though the Euler equation holds at the micro

level for individual households, there is no guarantee that it also holds at the macro level for aggregate consumption. The Euler equation only holds for aggregate consumption if the growth rate of aggregate consumption is equal to the growth rate of the consumption of individual households. But this is not the case with the parameterization which was used in the previous sections.[16] And it is not the case in the real world either, as is documented by Attanasio and Weber (1993) and Blundell and Stoker (2005).

To make matters even worse, the new-classical economist does not learn from his mistakes. At the end of period 1, he observes that the interest rate has gone up (which he did not expect in the beginning of period 1). But instead of revising his view on how macro-economic variables are related with each other, as the neo-classical economist did, the new-classical economist uses equation (46) to revise upwards his estimate of the subjective discount rate of the representative household. But as he increases his estimate of the subjective discount rate, his policy recommendations change as well, and he will now advise higher emission levels - which causes even more environmental damage and drives up the interest rate even further. Suppose now that in every period, the emission levels are set according to the latest update of the new-classical economist's policy advice. The economy will then eventually converge to a steady state where his prediction of the interest rate turns out to be correct. His recommended subjective discount rate in this new steady state is indicated by the full vertical lines in figure 5, leading to higher emission levels, lower environmental quality, a higher tax share, a lower investment share, a higher interest rate and more output loss than what he had predicted in the beginning of period 1 - as is shown by the big green dots.

The reason why the new-classical economist does not learn from his mistakes, is that his assumption that the world is populated by an immortal representative agent belongs to the maintained assumptions of his model. This assumption, unworldly as it is, is therefore not questioned and cannot be rejected. So when the new-classical economist is confronted with macroeconomic data that deviate from his predictions, he simply adjusts an unobservable preference parameter of the non-existent inhabitant in his imaginary world. So there is no scope for learning, and the parameter adjustments which he makes boil down to a strange way of data-mining - which results in misguided and time-inconsistent policy advice.

---

[16]With the parameterization presented in section 2, the growth rate of the consumption of individual households in the initial steady state is about 2.3% annually, while the growth rate of aggregate consumption is 1.5% annually; the households' subjective discount rate θ is about 1% annually, while the new-classical economist's period 0 estimate of θ is about 1.7% annually.

5   Some concluding remarks

I now present some concluding remarks, and an afterthought about modern macroeconomics.

5.1   Can the representative agent be saved?

Yes, but only if the wealth distribution across different generations is socially optimal: in that case, the marginal rate of return on capital is equal to the social evaluator's marginal rate of substitution with respect to consumption in consecutive periods for different generations.[17] There are two ways how this could be possible.

The first possibility is through bequests. If people care about the welfare of future generations, they can in principle transfer sufficient assets in order to compensate them for the loss in environmental quality. But it turns out that this only happens under very restrictive conditions for the households' utility function and the social evaluator's social welfare function (Bernheim, 1987). Furthermore, the welfare of future generations may have the characteristics of a public good (Marglin, 1963; and Sen, 1982): if people don't only care about the welfare of their own offspring but also about the welfare of the other members of future generations, leaving a bequest yields a utility spill-over to other households - a spill-over which is not reflected in the interest rate, such that bequests will be lower than what is socially optimal.

The second possibility is through government saving (or the level of public debt). But generating a socially optimal intergenerational wealth distribution requires a forward-looking, rational, benevolent and efficient government. I have not come across evidence of this.

5.2   Would more detailed micro-foundations be helpful?

This is an open question. As long as micro-foundations are empirically unfounded and we don't know the "true" model, we cannot check whether more detailed micro-foundations lead to better estimates of deep structural parameters and more accurate policy advice. One can only hope (Hoover, 2006).

But there are reasons to be sceptical. Perhaps it would be possible if the real world were as simple as the economy which I presented in section 2: sooner or later, the new-classical economist would probably discover that people don't live infinitely long, but that there are overlapping generations; he would then adapt his micro-foundations, and after some computations, he should be able

---

[17]See Howarth and Norgaard (1993) and Dasgupta (2008) for similar lines of reasoning.

to recover the correct value of the subjective discount rate in the individuals' utility function. But suppose that the real world is more complicated than the set-up in section 2: suppose that there are many more generations that overlap each other, and that households face aggregate and idiosyncratic income shocks (as documented by Storesletten, Telmer and Yaron, 2004, for instance). Then a new-classical economist would again fail to recover the true value of the subjective discount rate of the individuals, unless his micro-foundations somehow happen to be consistent with the real world. Furthermore, even if some day we end up with micro-foundations that are empirically supported, we may end up with an estimate of a parameter that is irrelevant for an environmental cost-benefit analysis. Why would the subjective discount rate that individuals use to trade off their current and future consumption based felicity be the same as the discount rate that they would use to trade off aggregate consumption between different generations? The evidence presented in Shane, Loewenstein and O'Donoghue (2002) casts serious doubts about this.

5.3 What is the role of the interest rate in social welfare analysis?

Lacking a model that describes well how the interest rate is related with the preferences of the population, the *descriptive* approach to distill a subjective discount rate out of market observations of the interest rate is doomed to fail. Choosing an appropriate subjective discount rate is therefore inescapably an ethical, *prescriptive* value judgement.[18]

This does not imply that market observations of the interest rate are irrelevant for social welfare analysis. First, the interest rate is the opportunity cost of projects that crowd out private investment, assuming that they do not involve important distributional aspects or externalities; so it can be used to compute the efficiency of such projects. And second, the interest rate gives information about marginal intergenerational redistribution possibilities by saving or dissaving in the capital market - which, for instance, is useful to evaluate the level of public debt or the government's tax policy to encourage private saving.[19]

---

[18]Furthermore, the assumption of a discounted utilitarian social welfare function is a value judgement as well. DeCanio (2003), Nelson (2008) and Ackerman et.al. (2009), among others, clarify how value judgements are more often than not interwoven in economic models of climate change and cost-benefit analyses.

[19]Note, however, that before advising the government, a social evaluator may first want to use the interest rate in a couple of thought experiments to check whether his social welfare function really reflects his ethical values. Dasgupta (2008), for instance, computes how much we should save to maximize the social welfare function (29) in a simple *classroom model*, and argues that the logarithmic specification causes some profound ethical puzzles.

5.4 The Ramsey and the Solow model, or thought experiments versus descriptive models

Modern macroeconomists often regard the Solow (1956) model as an example of technological regress compared with the Ramsey (1928) model, as the Solow model simply assumes that the saving rate is constant, while in the Ramsey model the saving rate is derived endogenously from its micro-foundations.

But this does not do justice to the Solow model. The Ramsey model was meant as a *thought experiment*: suppose that there is a social planner; what would then be his optimal consumption path? Solow, at the other hand, wanted to build a *descriptive or positive* model. The Ramsey model does not qualify as a descriptive model, because the maintained assumption that the world is ruled by a social planner is at odds with reality. So instead of starting from an unrealistic assumption that would undermine the model's empirical usefulness, Solow simply used as building-blocks some stylized relations between a few aggregate variables, which can easily be tested to get a sense of their reasonableness.[20] And successfully so: his approach proved to be fertile soil for a huge empirical literature on economic growth.[21]

So it is not surprising that also in this paper, Solow's approach turns out to be more useful than the Ramsey model to describe the economy presented in section 2: the neo-classical economist in section 3, whose model is based on the Solow model, indeed captures a key feature of the economy (the fact that a higher tax rate increases the interest rate), which is missed by the new-classical economist in section 4, whose model is based on the Ramsey model.[22]

...Which leads me to an afterthought about modern macroeconomics:

5.5 An afterthought about modern macroeconomics

Recall that the fundamental problem of the new-classical economist in section 4 is that even though the immortal representative agent in his model is empirically unfounded, he belongs to the maintained assumptions of the model. So

---

[20]It seems that this point is not always well understood. See Chari and Kehoe (2006), the comment by Solow (2008), and the reply by Chari and Kehoe (2008).

[21]One shivers at the thought of what would have become of the empirical growth literature if econometricians had devoted their efforts to estimating the preference parameters of immortal representative agents in various countries of the world based on the Ramsey model, rather than to growth regressions based on the Solow model.

[22]Of course, in the real world, a neo-classical model may also miss several essential features of the economy. Neo-classical models typically imply weak sustainability, for instance (Cabeza Gutés, 1996), while the question whether this is warranted in climate change models might very well turn out to be much more important than the question how we should discount future costs and benefits (Neumayer, 1999). See also DeCanio (2003).

it is an assumption which cannot be rejected. The consequence of this is that when the modern macro-economist uses this model to *describe* the real world (in contrast with Ramsey, who used his model to *think* about the real world), he does not learn from his mistakes: when he observes that the real world behaves somewhat differently than his model, he simply adjusts an unobservable preference parameter, instead of questioning the empirical foundation of his maintained assumptions. And to make matters even worse, he then goes to the social evaluator and tells him that this preference parameter reflects the preferences of real world people of flesh and blood.

Modern macroeconomic models, both of the new-classical and the new-keynesian type, as well as the dynamic stochastic general equilibrium models (DSGE's) that emerged after the new-neoclassical synthesis of the mid 1990s, are almost always based on empirically unfounded micro-foundations in at least some dimensions. So modern macroeconomists who use these models as *positive* models and not just as mere *thought experiments* risk to go astray on the same slippery route as the new-classical economist in section 4. First, when a modern macroeconomist is confronted with data that differ from what he had predicted, he may be tempted to ascribe this to an unexpected change in some unobservable structural parameter[23] - which easily degenerates into data-mining.[24] Second, having endowed the seriously defective humanoids in his models with utility functions, he may be tempted to confuse them with real world people of flesh and blood, and draw unwarranted welfare conclusions.[25]

This does not mean that modern macroeconomic models are useless, and it certainly does not imply that neo-classical models are flawless. Both may have their merits as thought experiments. And both have their flaws as descriptive models: neo-classical models because of *ad hoc* constraints that do not survive the critiques by Lucas and Sims, and modern macroeconomic models because of the constraints imposed by empirically unfounded micro-foundations.[26] So it is important to recognize and communicate the limitations and potential misuses of all these models, as was stressed by Lucas (1980) at the end of the neo-classical era and by Solow (2007, 2008), Colander et.al. (2009), and many others in the current wave of soul-searching among macroeconomists.

---

[23]Deep structural parameters, such as the households' subjective discount rate, are often assumed to be stochastic in DSGE models. See, for instance, Smets and Wouters (2003).

[24]The same point was raised by Solow (2008), and is even recognized by some advocates of DSGE's such as Kocherlakota (2010).

[25]Hoover (2006) expands on this issue.

[26]The set-up in section 2 has therefore clearly stacked the deck in favor of the neo-classical economist (as the great critiques by Lucas and Sims do not bite), and against the new-classical economist (as the key parameter is unobservable, leading the new-classical economist astray). Another set-up of the economy may shuffle the cards in a different way.

Appendix A: The optimal environmental policy

In this appendix, I derive the optimal environmental policy.

I first turn problem (30) in a stationary problem. Define $k_t = K_t/A_t$ and $c_t = C_t/A_t$. Problem (30) is then equivalent with the following stationary problem:

$$w(k_t, M_t) = \max_{E \in [0,1]} \left\{ \ln c_t + \frac{1}{1+\rho} w(k_{t+1}, M_{t+1}) \right\}$$

$$\text{subject to} \quad c_t = [(1 - s_Y)(1 - \alpha - z + \zeta M_t^{\mu_Z}) + \alpha] E_t^\varepsilon M_t^{\mu_Y} k_t^\alpha$$

$$k_{t+1} = \frac{1}{1+g} s_Y (1 - \alpha - z + \zeta M_t^{\mu_Z}) E_t^\varepsilon M_t^{\mu_Y} k_t^\alpha$$

$$M_{t+1} = 1 + \phi(M_t - 1) - \psi E_t$$

$$k_1 = k_0 \quad \text{and} \quad M_1 = 1$$

The first-order condition for the control variable E is:

$$\frac{\varepsilon}{E_t} \left(1 + \frac{1}{1+\rho} \frac{\partial w}{\partial k_{t+1}} k_{t+1} \right) \geq \frac{1}{1+\rho} \frac{\partial w}{\partial M_{t+1}} \psi \quad \text{and} \quad E_t \leq 1 \quad \text{with c.s.} \quad (A.1)$$

The envelope conditions for the state variables k and M are:

$$\frac{\partial w}{\partial k_t} = \frac{\alpha}{k_t} \left(1 + \frac{1}{1+\rho} \frac{\partial w}{\partial k_{t+1}} k_{t+1} \right) \tag{A.2}$$

$$\frac{\partial w}{\partial M_t} = \frac{1}{M_t} [\mu_Y + (1 - s_Y)\mu_Z \lambda_{c,t}] + \frac{1}{1+\rho} \frac{\partial w}{\partial k_{t+1}} \frac{k_{t+1}}{M_t} (\mu_Y + s_Y \mu_Z \lambda_{I,t})$$

$$+ \frac{1}{1+\rho} \frac{\partial w}{\partial M_{t+1}} \phi \tag{A.3}$$

with

$$\lambda_{c,t} = \frac{\zeta M_t^{\mu_Z}}{(1 - s_Y)(1 - \alpha - z + \zeta M_t^{\mu_Z}) + \alpha} \quad \text{and} \quad \lambda_{I,t} = \frac{\zeta M_t^{\mu_Z}}{s_Y(1 - \alpha - z + \zeta M_t^{\mu_Z})}$$

As $\alpha/(1 + \rho) < 1$, equation (A.2) implies that $(\partial w / \partial k_t) k_t$ must remain constant over time in order to rule out exploding paths:

$$\frac{\partial w}{\partial k_t} k_t = \frac{\alpha(1 + \rho)}{1 + \rho - \alpha}, \quad \forall t$$

Substituting in the first-order condition (A.1) and the law of motion (A.3) yields:

$$\frac{\epsilon}{E_t} \geq \frac{1 + \rho - \alpha}{1 + \rho} \frac{1}{1+\rho} \frac{\partial w}{\partial M_{t+1}} \psi \quad \text{and} \quad E_t \leq 1 \quad \text{with c.s.} \quad (A.4)$$

$$\frac{\partial w}{\partial M_t} = \frac{1 + \rho}{1 + \rho - \alpha} \frac{1}{M_t} (\mu_Y + \mu_Z \lambda_t) + \frac{1}{1+\rho} \frac{\partial w}{\partial M_{t+1}} \phi \tag{A.5}$$

with

$$\lambda_t = \frac{1 + \rho - \alpha}{1 + \rho}(1 - s_Y)\lambda_{c,t} + \frac{\alpha}{1 + \rho}s_Y\lambda_{I,t}$$

The optimal emission policy is therefore independent of the capital stock. Note also that if $E_t < 1$, combining equations (A.4) and (A.5) leads to an expression for $\partial w/\partial M_t$:

$$\frac{\partial w}{\partial M_t} = \left[\frac{\phi}{\psi}\frac{\epsilon}{E_t} + \frac{1}{M_t}(\mu_Y + \mu_Z\lambda_t)\right]\frac{1 + \rho}{1 + \rho - \alpha} \tag{A.6}$$

Expressions (A.4), (A.5) and (A.6) suggest then the following time iteration procedure to find the policy function $E(M_t)$:

1. Construct a grid $[M_1, M_2, ..., M_n]$ and choose associated starting values for the emissions $[E_1^{\{0\}}, E_2^{\{0\}}, ..., E_n^{\{0\}}]$.

2. Compute in each iteration $j = 1, 2, ...$ new values $[E_1^{\{j\}}, E_2^{\{j\}}, ..., E_n^{\{j\}}]$ as follows:

   (a) If $E_i^{\{j-1\}} < 1$, compute the implied value of E in the previous period by substituting equation (A.6) (moved forward with one period) in expression (A.4):

   $$\frac{\epsilon}{\tilde{E}_i^{\{j\}}} \geq \frac{\phi}{1 + \rho}\frac{\epsilon}{E_i^{\{j-1\}}} + \frac{\psi}{1 + \rho}\frac{1}{M_i}(\mu_Y + \mu_Z\lambda_i)$$

   $$\text{and} \quad \tilde{E}_i^{\{j\}} \leq 1 \quad \text{with c.s.}$$

   (b) If $E_i^{\{j-1\}} = 1$, derive the implied value of M in the next period from the law of motion (8) and compute the associated value of E by interpolating on $[E_1^{\{j-1\}}, E_2^{\{j-1\}}, ..., E_n^{\{j-1\}}]$. Continue until a value for E is found that is less than one - suppose this happens in period t'. Then use equation (A.6) to compute $\partial w/\partial M_{t'}$, and move backwards with the law of motion (A.5) until period $t + 1$. Then use expression (A.4) to find $\tilde{E}_i^{\{j\}}$. If no value for E is found that is less than one, set $\tilde{E}_i^{\{j\}}$ equal to one.

   (c) Find for each grid point i the value $\tilde{M}_i^{\{j\}}$ such that the law of motion (8) holds, i.e. such that

   $$M_i = 1 + \phi(\tilde{M}_i^{\{j\}} - 1) - \psi\tilde{E}_i^{\{j\}}$$

   (d) We now have a grid $[\tilde{M}_1^{\{j\}}, \tilde{M}_2^{\{j\}}, ..., \tilde{M}_n^{\{j\}}]$ and associated emission values $[\tilde{E}_1^{\{j\}}, \tilde{E}_2^{\{j\}}, ..., \tilde{E}_n^{\{j\}}]$, which define the policy rule in iteration j. Use then inter- and extrapolation to find the emission values $[E_1^{\{j\}}, E_2^{\{j\}}, ..., E_n^{\{j\}}]$ that are associated with $[M_1, M_2, ..., M_n]$ according to this policy rule.

30

3. Continue until $\max_i (|E_i^{\{j\}} - E_i^{\{j-1\}}|)$ is less than the tolerance level.

The steady state is found as follows. From equation (A.5) follows the steady state value of $\partial w/\partial M$:

$$\left(\frac{\partial w}{\partial M}\right)^* = \frac{1+\rho}{1+\rho-\phi}\frac{1+\rho}{1+\rho-\alpha}\frac{1}{M^*}(\mu_Y + \mu_Z\lambda_t^*)$$

Substituting in expression (A.4) yields then equation (31):

$$\frac{\epsilon}{E^*} \geq \frac{\psi}{1+\rho-\phi}\frac{1}{M^*}(\mu_Y + \mu_Z\lambda^*) \quad \text{and} \quad E^* \leq 1 \quad \text{with c.s.}$$

Appendix B: The neo-classical policy advice

The derivation of the neo-classical policy advice is very similar as for the optimal policy. First turn problem (37) in a stationary problem:

$$w(k_t, M_t) = \max_{E\in[0,1]} \left\{\ln c_t + \frac{1}{1+\rho}w(k_{t+1}, M_{t+1})\right\}$$

$$\text{subject to} \quad c_t = (1-\hat{s}_0)(1-z+\zeta M_t^{\mu_Z})E_t^\varepsilon M_t^{\mu_Y}k_t^\alpha$$

$$k_{t+1} = \frac{1}{1+g}\hat{s}_0(1-z+\zeta M_t^{\mu_Z})E_t^\varepsilon M_t^{\mu_Y}k_t^\alpha$$

$$M_{t+1} = 1 + \phi(M_t - 1) - \psi E_t$$

$$k_1 = k_0 \quad \text{and} \quad M_1 = 1$$

The first-order condition for the control variable E and the envelope condition for the state variable k are the same as for the optimal policy, and are given by expressions (A.1) and (A.2). The envelope condition for the state variable M is slightly different, however:

$$\frac{\partial w}{\partial M_t} = \frac{1}{M_t}\left[\mu_Y + \mu_Z\tilde{\lambda}_t\right] + \frac{1}{1+\rho}\frac{\partial w}{\partial k_{t+1}}\frac{k_{t+1}}{M_t}\left(\mu_Y + \mu_Z\tilde{\lambda}_t\right)$$

$$+ \frac{1}{1+\rho}\frac{\partial w}{\partial M_{t+1}}\phi \tag{B.1}$$

with

$$\tilde{\lambda}_t = \frac{\zeta M_t^{\mu_Z}}{1-z+\zeta M_t^{\mu_Z}}$$

Substituting the solution for $(\partial w/\partial k_t)k_t$, (A.4), in the first-order condition (A.1) and the envelope condition (B.1) for M yields:

$$\frac{\epsilon}{E_t} \geq \frac{1+\rho-\alpha}{1+\rho}\frac{1}{1+\rho}\frac{\partial w}{\partial M_{t+1}}\psi \quad \text{and} \quad E_t \leq 1 \quad \text{with c.s.} \tag{B.2}$$

$$\frac{\partial w}{\partial M_t} = \frac{1+\rho}{1+\rho-\alpha}\frac{1}{M_t}(\mu_Y + \mu_Z\tilde{\lambda}_t) + \frac{1}{1+\rho}\frac{\partial w}{\partial M_{t+1}}\phi \tag{B.3}$$

31

As the optimal policy, the neo-classical emission policy is independent of the capital stock. If $E_t < 1$, combining equations (B.2) and (B.3) leads to an expression for $\partial w / \partial M_t$, similar to equation (A.6):

$$\frac{\partial w}{\partial M_t} = \left[\frac{\phi}{\psi} \frac{\epsilon}{E_t} + \frac{1}{M_t} (\mu_Y + \mu_Z \tilde{\lambda}_t) \right] \frac{1 + \rho}{1 + \rho - \alpha} \tag{B.4}$$

Expressions (B.2), (B.3) and (B.4) suggest then a similar time iteration procedure to find the neo-classical policy advice $E(M_t)$ as the procedure that was used to derive the optimal policy.

The steady state is also found in the same way as for the optimal policy: from equation (B.3) follows the steady state value of $\partial w / \partial M$; substituting in expression (B.2) yields then equation (38).

Appendix C: The new-classical policy advice

I now derive the new-classical policy advice. I first solve the representative household's and the social evaluator's maximization problem. I then sketch a numerical procedure to find the emission and consumption functions that jointly solve the representative household's problem and the social evaluator's problem. I will conclude with a derivation of the new steady state values if emission levels are set according to the new-classical advice.

I first turn the representative household's problem in a stationary problem, using the same notation as in Appendix A:

$$u(k_t, M_t) = \max_c \left\{ \ln c_t + \frac{1}{1 + \hat{\theta}_0^R} u(k_{t+1}, M_{t+1}) \right\}$$

$$\text{subject to} \quad k_{t+1} = \frac{1}{1+g} \left(k_t(1 + r_t) + w_t/A_t - T_t/A_t - c_t \right)$$

$$\lim_{s \to \infty} \Pi_{s'=t+1}^s \frac{1+g}{1 + r_{s'}} k_s = 0$$

The first-order condition for the control variable c is:

$$\frac{1}{c_t} = \frac{1}{1 + \hat{\theta}_0^R} \frac{\partial u}{\partial k_{t+1}} \frac{1}{1+g} \tag{C.1}$$

The envelope condition for the state variable k is:

$$\frac{\partial u}{\partial k_t} = \frac{1}{1 + \hat{\theta}_0^R} \frac{\partial u}{\partial k_{t+1}} \frac{1 + r_t}{1+g} \tag{C.2}$$

Now substitute (C.1) in (C.2), move the resulting equation one period forward, and substitute it again in (C.1). This yields the consumption Euler equation:

$$\frac{1}{c_t} = \frac{1 + r_{t+1}}{(1 + \hat{\theta}_0^R)(1 + g)} \frac{1}{c_{t+1}} \tag{C.3}$$

32

I now turn the social evaluator's problem (47) in a stationary problem:

$$u(k_t, M_t) = \max_{E \in [0,1]} \left\{ \ln c_t + \frac{1}{1+\hat{\theta}_0^R} u(k_{t+1}, M_{t+1}) \right\}$$

subject to $\quad c_t = c(k_t, M_t)$

$$k_{t+1} = \frac{1}{1+g}[(1 - z + \zeta M_t^{\mu_Z}) E_t^\varepsilon M_t^{\mu_Y} k_t^\alpha - c_t]$$

$$M_{t+1} = 1 + \phi(M_t - 1) - \psi E_t$$

$$k_1 = k_0 \quad \text{and} \quad M_1 = 1$$

The first-order condition for the control variable E is:

$$\frac{1}{1+g} \frac{1}{1+\hat{\theta}_0^R} \frac{\partial u}{\partial k_{t+1}} (1 - z + \zeta M_t^{\mu_Z}) \varepsilon \frac{y_t}{E_t} \geq \frac{1}{1+\hat{\theta}_0^R} \frac{\partial u}{\partial M_{t+1}} \psi$$

$$\text{and} \quad E_t \leq 1 \quad \text{with c.s.} \tag{C.4}$$

where $y_t = Y_t/A_t = E_t^\varepsilon M_t^{\mu_Y} k_t^\alpha$.

The envelope condition for the state variable M is:

$$\frac{\partial u}{\partial M_t} = \frac{1}{1+\hat{\theta}_0^R} \frac{\partial u}{\partial k_{t+1}} \frac{1}{1+g} \left[(1 - z + \zeta M_t^{\mu_Z}) \mu_Y \frac{y_t}{M_t} + \zeta \mu_Z M_t^{\mu_Z} \frac{y_t}{M_t} - \frac{\partial c}{\partial M_t} \right]$$

$$+ \frac{1}{1+\hat{\theta}_0^R} \frac{\partial u}{\partial M_{t+1}} \phi + \frac{1}{c_t} \frac{\partial c}{\partial M_t} \tag{C.5}$$

Substituting the representative household's first-order condition (C.1) in expressions (C.4) and (C.5) yields:

$$\frac{\varepsilon}{E_t} \geq \frac{1}{1+\hat{\theta}_0^R} \frac{\partial u}{\partial M_{t+1}} \frac{c_t}{x_t} \psi \quad \text{and} \quad E_t \leq 1 \quad \text{with c.s.} \tag{C.6}$$

$$\frac{\partial u}{\partial M_t} = \frac{x_t}{c_t} \frac{1}{M_t} (\mu_Y + \mu_Z \tilde{\lambda}_t) + \frac{1}{1+\hat{\theta}_0^R} \frac{\partial u}{\partial M_{t+1}} \phi \tag{C.7}$$

with

$$x_t = (1 - z + \zeta M_t^{\mu_Z}) y_t \quad \text{and} \quad \tilde{\lambda}_t = \frac{\zeta M_t^{\mu_Z}}{1 - z + \zeta M_t^{\mu_Z}}$$

Note that if $E_t < 1$, combining equations (C.6) and (C.7) leads to an expression for $\partial u / \partial M_t$:

$$\frac{\partial u}{\partial M_t} = \left[\frac{\phi}{\psi} \frac{\varepsilon}{E_t} + \frac{1}{M_t} (\mu_Y + \mu_Z \tilde{\lambda}_t) \right] \frac{x_t}{c_t} \tag{C.8}$$

Expressions (C.3), (C.6), (C.7) and (C.8) suggest then the following time iteration procedure to find the policy functions $c(k_t, M_t)$ and $E(k_t, M_t)$:

33

1. Construct grids $[k_1, k_2, ..., k_n]$ and $[M_1, M_2, ..., M_m]$ and construct two (n,m)-matrices with associated starting values for consumption c and emission levels E.

2. Perform a time iteration procedure with expressions (C.6), (C.7) and (C.8) in a similar way as explained in Appendix A; this yields a new (n,m)-matrix with the optimal emission levels associated with the k- and M-grids, assuming that the consumption function $c(k_t, M_t)$ is described by the (n,m)-matrix with consumption levels.

3. Perform a time iteration procedure based on (C.3) to determine the optimal consumption levels associated with the k- and M-grids, assuming that the emissions function $E(k_t, M_t)$ is described by the (n, m)-matrix with emission levels.

4. Repeat steps 2 and 3 until convergence, which is reached when both time iteration procedures are finished after one step.

The steady state is found as follows. From equation (C.7) follows the steady state value of $\partial u / \partial M_t$:

$$
\left(\frac{\partial u}{\partial M} \right)^* = \frac{1 + \hat{\theta}_0^R}{1 + \hat{\theta}_0^R - \phi} \frac{1}{M^*} (\mu_Y + \mu_Z \tilde{\lambda}_t^*) \frac{x_t}{c_t}
$$

Substituting in expression (C.6) yields then equation (48):

$$
\frac{\epsilon}{E^*} \geq \frac{\psi}{1 + \hat{\theta}_0^R - \phi} \frac{1}{M^*} (\mu_Y + \mu_Z \tilde{\lambda}^*) \quad \text{and} \quad E^* \leq 1 \quad \text{with c.s.}
$$

References

Ackerman, Frank, Stephen J. DeCanio, Richard B. Howarth, and Kristen Sheeran (2009), "Limitations of integrated assessment models of climate change", Climatic Change, 95, 297-315.

Attanasio, Orazio P., and Guglielmo Weber (1993), "Consumption Growth, the Interest Rate and Aggregation", Review of Economic Studies, 60, 631-649.

Bartelsman, Eric J., John C. Haltiwanger, and Stefano Scarpetta (2009), "Cross-Country Differences in Productivity: the Role of Allocation and Selection", NBER Working Paper Series, Working Paper 15490.

Bernheim, B. Douglas (1989), "Intergenerational Altruism, Dynastic Equilibria and Social Welfare", Review of Economic Studies, 56, 119-128.

Buiter, Willem (2009), "The unfortunate uselessness of most 'state of the art' academic monetary economics", www.voxeu.org, March 6, 2009.

Blundell, Richard, and Thomas M. Stoker (2005), "Heterogeneity and Aggregation", Journal of Economic Literature, Vol. XLIII (June), 347-391.

Cabeza Gutés, Maite (1996), "The concept of weak sustainability", Ecological Economics, 17, 147-156.

Chari, V.V., and Patrick J. Kehoe (2006), "Modern Macroeconomics in Practice: How Theory is Shaping Policy", Journal of Economic Perspectives, 20(4), 3-28.

Chari, V.V., and Patrick J. Kehoe (2008), "Response", Journal of Economic Perspectives, 22(1), 247-249.

Cline, William R. (1992), *The Economics of Global Warming*, Washington D.C.: Institute for International Economics.

Colander, David, Hans Fölmer, Armin Haas, Michael Goldberg, Katarina Juselius, Alan Kirman, Thomas Lux and Brigitte Sloth (2009), "The Financial Crisis and the Systemic Failure of Academic Economics", Critical Review, 21.

Dasgupta, Partha S. (2008), "Discounting climate change", Journal of Risk and Uncertainty, 37, 141-169.

Dasgupta, Partha S., and Geoffrey M. Heal (1974), "The Optimal Depletion of Exhaustible Resources", Review of Economic Studies, Symposium on the Economics of Exhaustible Resources.

DeCanio, Stephen J. (2003), *Economic Models of Climate Change: a Critique*, Palgrave-Macmillan.

De Grauwe, Paul (2009), "Top-down versus bottom-up macroeconomics", www.voxeu.org, November 19, 2009.

DeLong, Bradford J., and Konstantin Magin (2009), "The U.S. Equity Return Premium: Past, Present, and Future", Journal of Economic Perspectives, 23(1), 193-208.

Frederick, Shane, George Loewenstein, and Ted O'Donoghue (2002), "Time Discounting and Time Preference: A Critical Review", Journal of Economic Literature, Vol. XL (June), 351-401.

Hansen, Lars Peter, and James J. Heckman (1996), "The Empirical Foundations of Calibration", Journal of Economic Perspectives, 10(1), 87-104.

Hoover, Kevin D. (2006), "A Neowicksellian in a New Classical World: the Methodology of Michael Woodford's *Interest and Prices*", Journal of the History of Economic Thought, 28(2), 143-149.

Hoover, Kevin D. (2009), "Microfoundational Programs", mimeo, Department of Economics and Department of Philosophy, Duke University.

Howarth, Richard B., and Richard B. Norgaard (1993), "Intergenerational Transfers and the Social Discount Rate", Environmental and Resource Economics, 3, 337-358.

Johansen, Soren (2006), "Confronting the economic model with the data", in David Colander, *Post Walrasian Macro: Beyond the Dynamic Stochastic General Equilibrium Model*, Cambridge University Press, Cambridge, 287-300.

Juselius, Katarina, and Massimo Franchi (2007), "Taking a DSGE Model to the Data Meaningfully", The Open-Access, Open-Assessment E-journal, 2007-4.

Kocherlakota, Narayana (2010), "Modern Macroeconomic Models as Tools for Economic Policy", The Region, May, Federal Reserve Bank of Minneapolis.

Krugman, Paul (2009), "How Did Economists Get It So Wrong?", The New York Times, September 6, 2009.

Lucas, Robert E., Jr. (1976), "Econometric Policy Evaluation: A Critique", Carnegie-Rochester Conference Series on Public Policy 1: 19-46.

Lucas, Robert E., Jr. (1980), "Methods and problems in business cycle theory", Journal of Money, Credit, and Banking, 12, 696-715.

Mehra, Rajnish (2006), "The Equity Premium Puzzle: A Review", Foundations and Trends in Finance, 2(1), 1-81.

Mendelsohn, Robert O. (2007), "A Critique of the Stern Report", Regulation, 29(4):40-47.

Marglin, S.A. (1963), "The Social Rate of Discount and the Optimal Rate of Investment", Quarterly Journal of Economics, 77, 95-111.

Nelson, Julie A. (2008), "Economists, value judgments, and climate change: A view from feminist economics", Ecological Economics, 65, 441-447.

Neumayer, Eric (1999), "Global warming: discounting is not the issue, but substitutability is", Energy Policy, 27, 33-43.

Nordhaus, William D. (2007), "A Review of the *Stern Review on the Economics of Climate Change*", Journal of Economic Literature, Vol. XLV (Sept.), 686-702.

Nordhaus, William D. (2008), *A Question of Balance: Economic Modeling of Global Warming*, New Haven, Yale University Press.

Pezzey, John C.V., and Michael A. Toman (2002), "The Economics of Sustainability: a Review of Journal Articles", Discussion Paper 02-03, Resources for the Future, Washington D.C.

Ramsey, Frank P. (1928), "A mathematical theory of saving", Economic Journal, 38(152), 543-59.

Sen, Amartya K. (1982), "Approaches to the choice of discount rate for social benefit-cost analysis", in R.C. Lind (ed.), *Discounting for Time and Risk in Energy Policy*, Washington D.C.: Resources for the Future, 325-353.

Sims, Christopher A. (1980), "Macroeconomics and Reality", Econometrica 48(1): 1-48.

Smets, Frank, and Raf Wouters (2003), "An estimated dynamic stochastic general equilibrium model of the Euro area", Journal of the European Economic Association, 1(5), 1123-1175.

Solow, Robert M. (1956), "A Contribution to the Theory of Economic Growth", Quarterly Journal of Economics, 70(Feb.), 65-94.

Solow, Robert M. (1974), "Intergenerational Equity and Exhaustible Resources", Review of Economic Studies, 41: Symposium Volume, 29-46.

Solow, Robert M. (2007), "Reflections on the Survey", in David Colander, *The Making of an Economist, Redux*, Princeton University Press, chapter 12, 234-238.

Solow, Robert M. (2008), "The State of Macroeconomics", Journal of Economic Perspectives, 22(1), 243-249.

Stern, Nick (2007), *The Economics of Climate Change: The Stern Review*, Cambridge: Cambridge University Press.

Stiglitz, Joseph E. (1974), "Growth with Exhaustible Natural Resources: Efficient and Optimal Growth Paths", Review of Economic Studies, Symposium on the Economics of Exhaustible Resources.

Storesletten, Kjetil, Chris I. Telmer, and Amir Yaron (2004), "Consumption and Risk Sharing over the Life Cycle", Journal of Monetary Economics, (51), 609-633.

Tol, Richard S.J. (2002a), "Estimates of the damage costs of climate change. Part 1: Benchmark estimates", Environmental & Resource Economics, 21(1),47-73.

Tol, Richard S.J. (2002b), "Estimates of the damage costs of climate change. Part 2: Dynamic estimates", Environmental & Resource Economics, 21(2), 135-160.

Tol, Richard S.J., and Gary W. Yohe (2006), "A Review of the *Stern Review*", World Economics, 7(4):233-50.

Weitzman, Martin L. (2007), "A Review of *The Stern Review on the Economics of Climate Change*", Journal of Economic Literature, Vol. XLV (Sept.), 703-724.

World Commission on Environment and Development (1987), *Our Common Future*, Oxford University Press, Oxford.

Figure 1: *Transitional dynamics according to the optimal policy*

Emissions E

Environmental quality M

Figure 1 (continued)

Tax share τ

Investment share (I/Y)

Figure 1 (continued)

Interest rate r (annualized)

Output loss
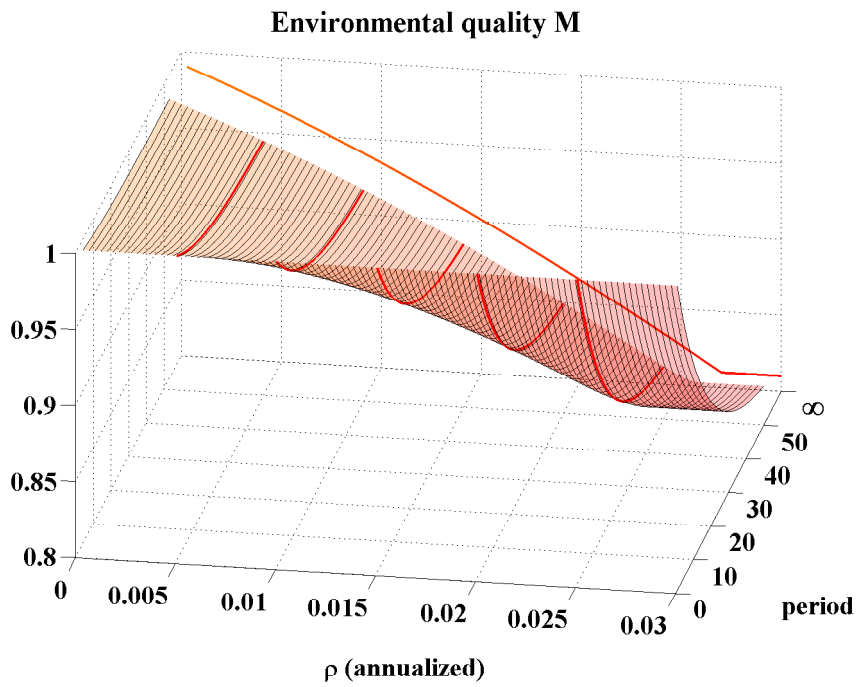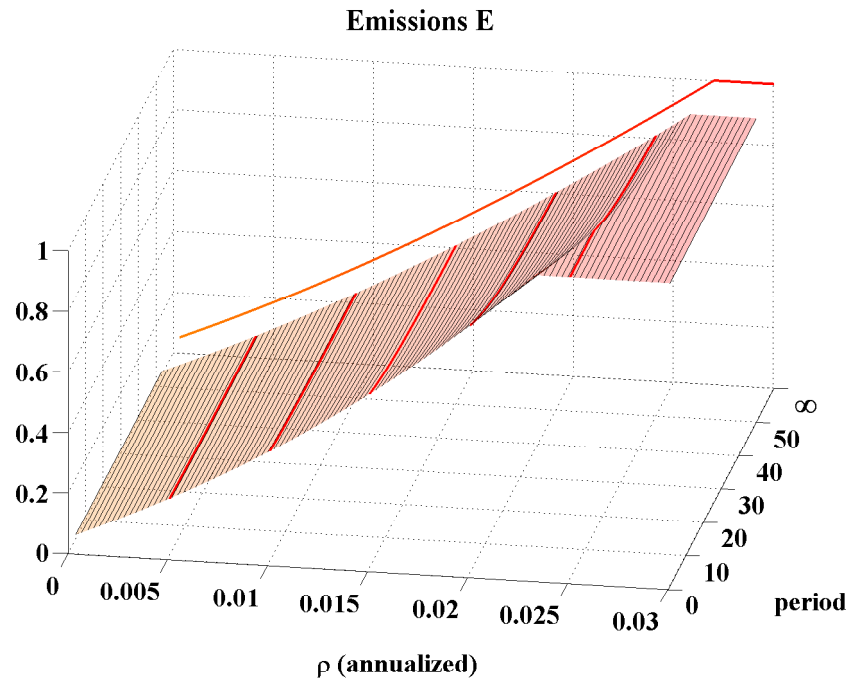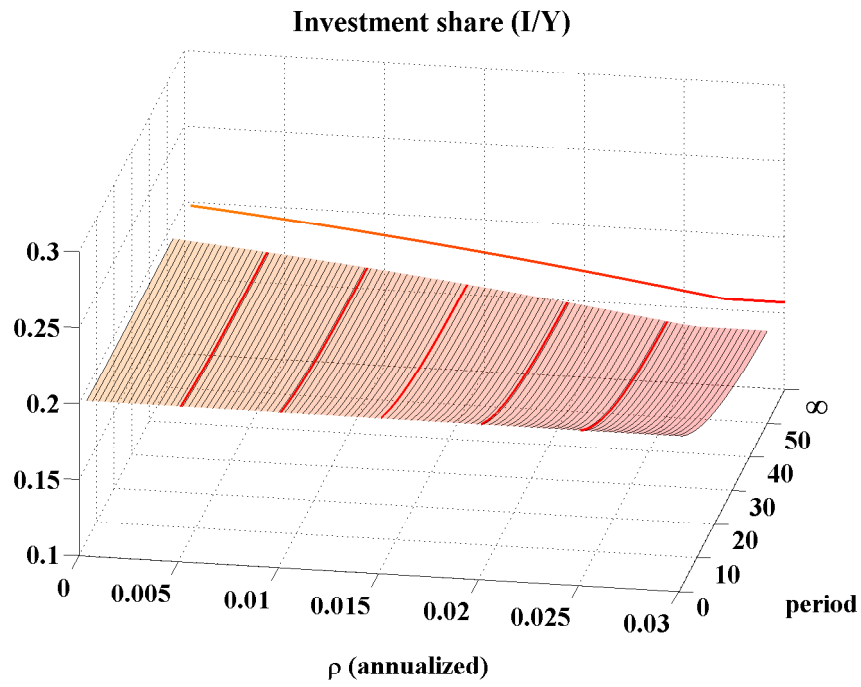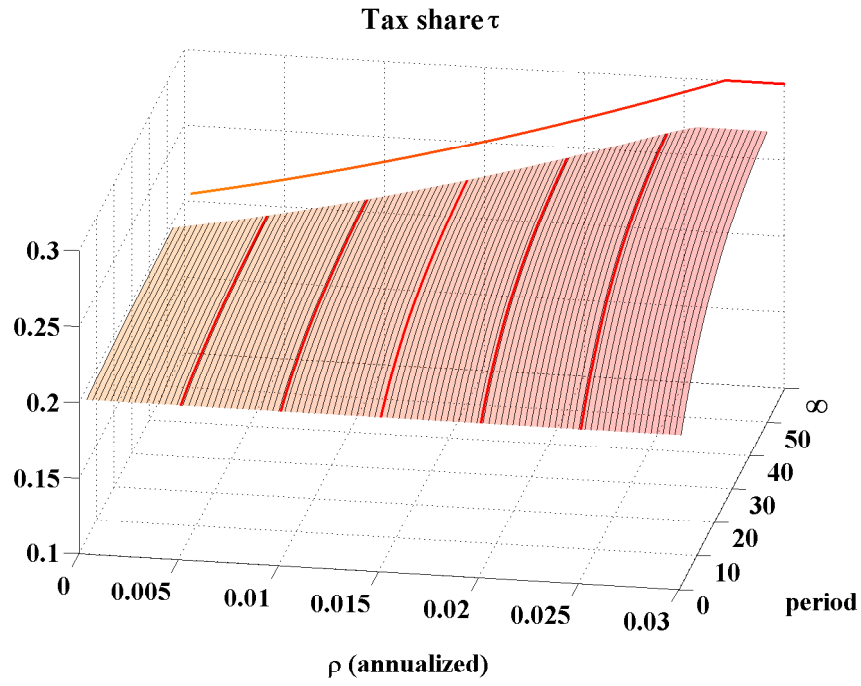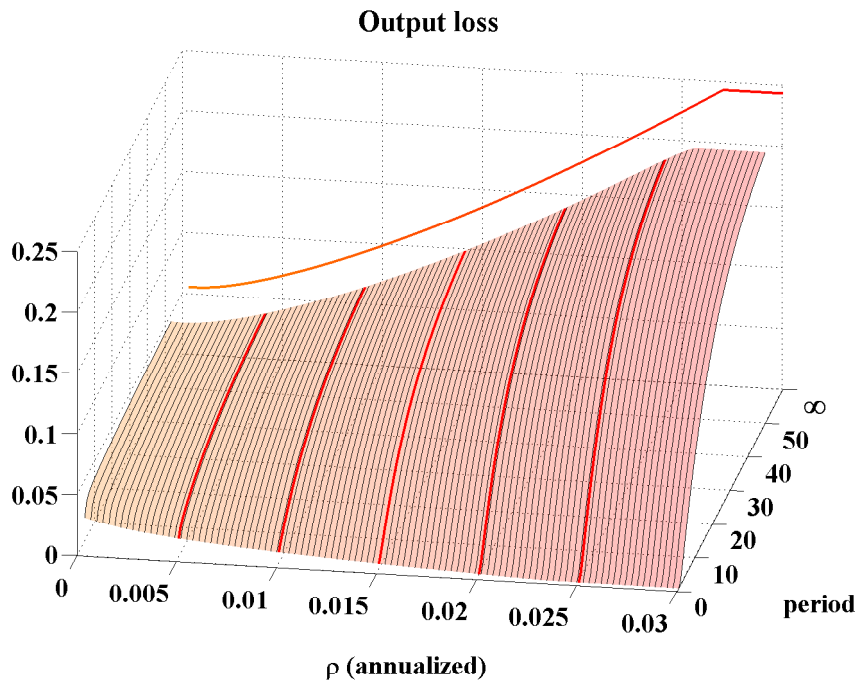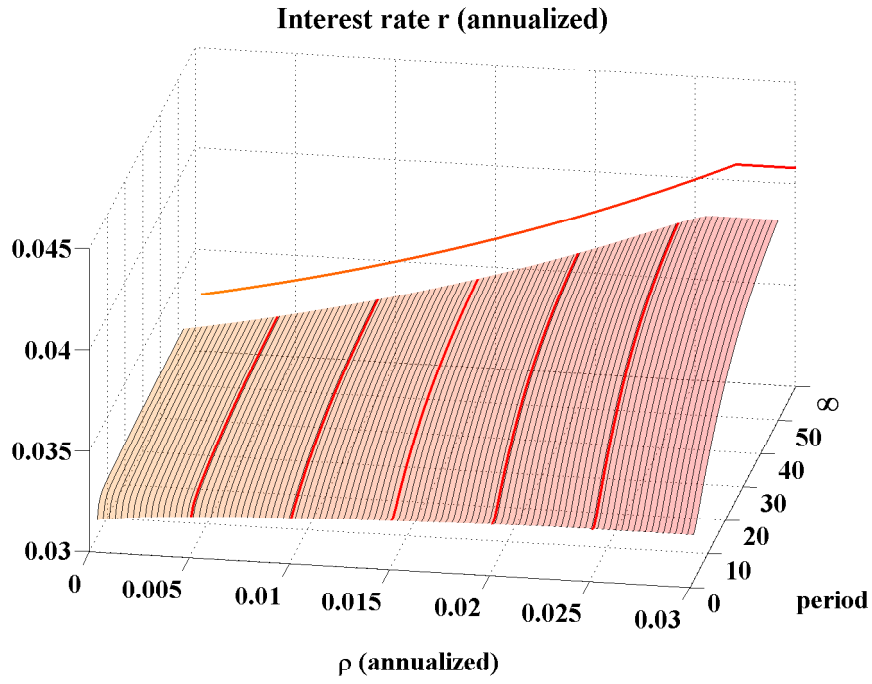
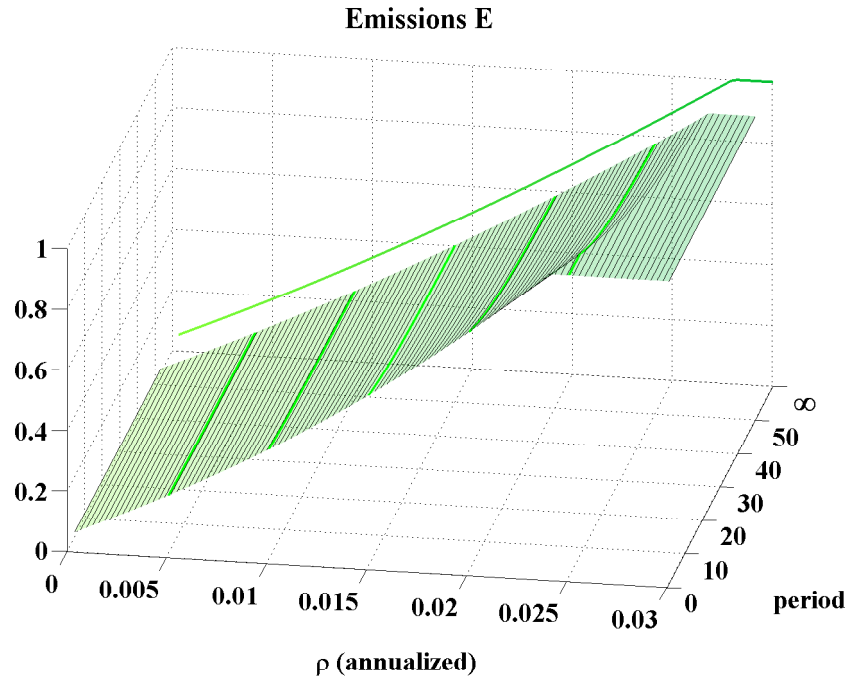**Figure 2:** *Transitional dynamics according to the neo-classical policy advice*

Emissions E

Environmental quality M

Figure 2 (continued)

Tax share τ

Investment share (I/Y)

Figure 2 (continued)

Interest rate r (annualized)

Output loss
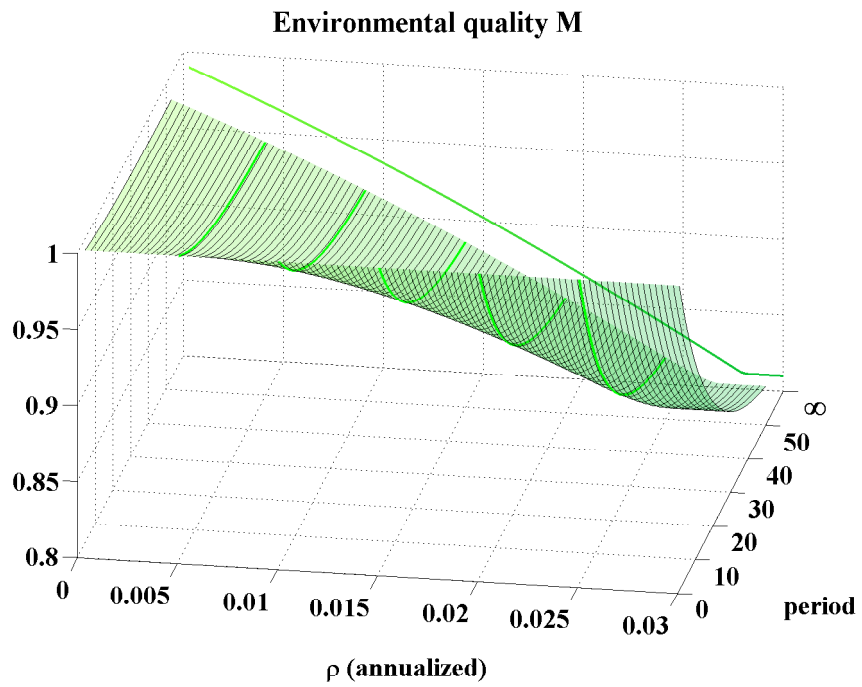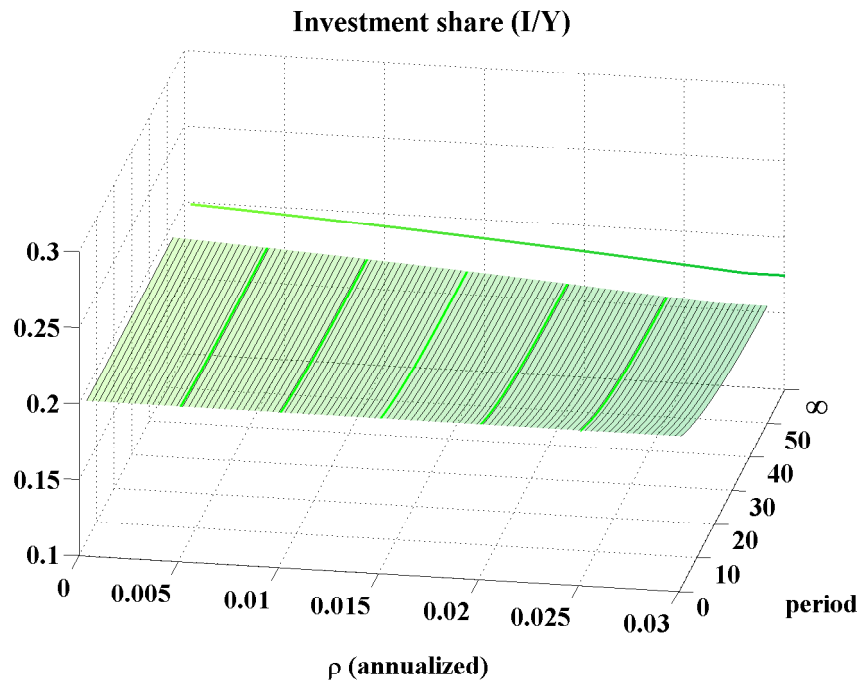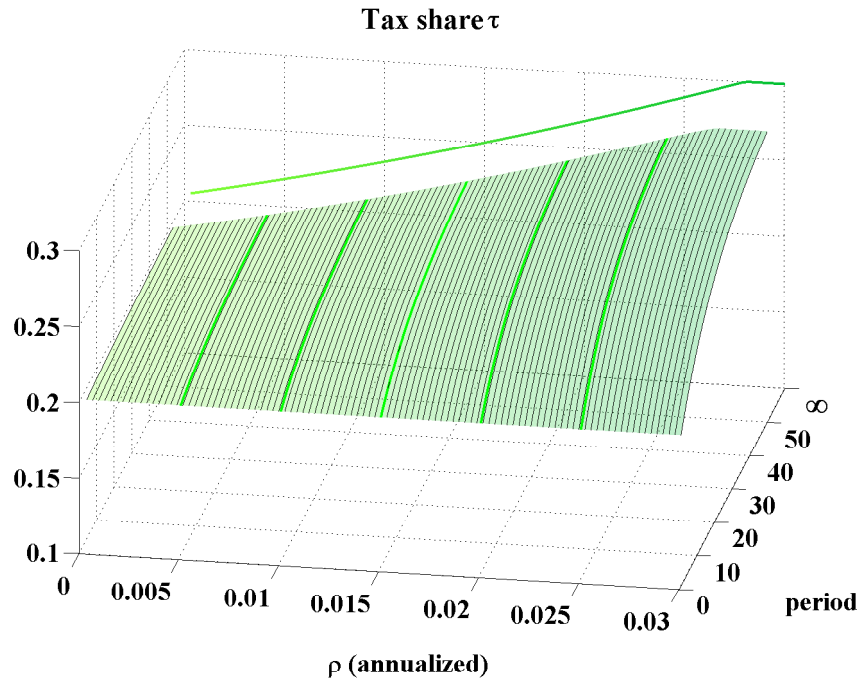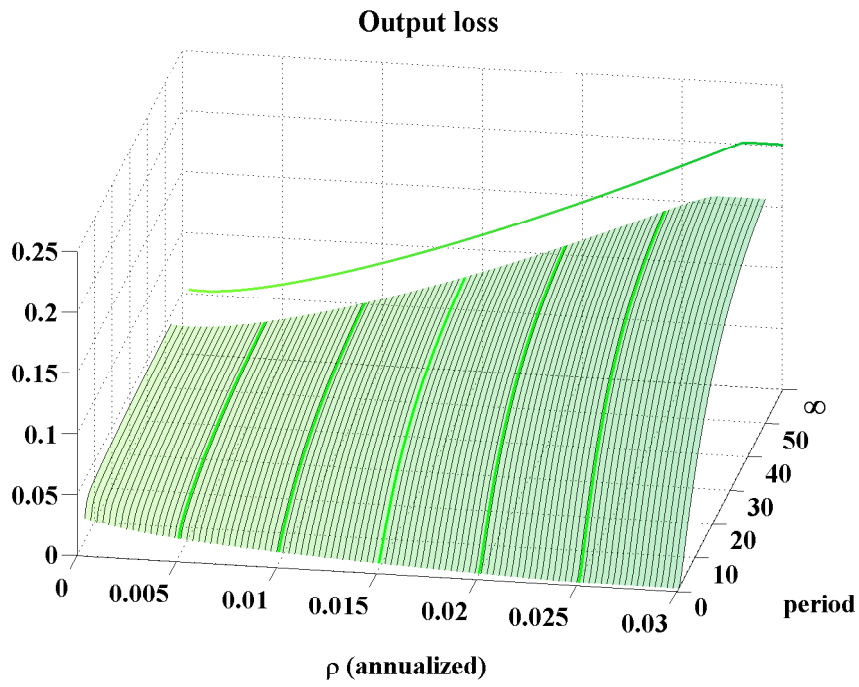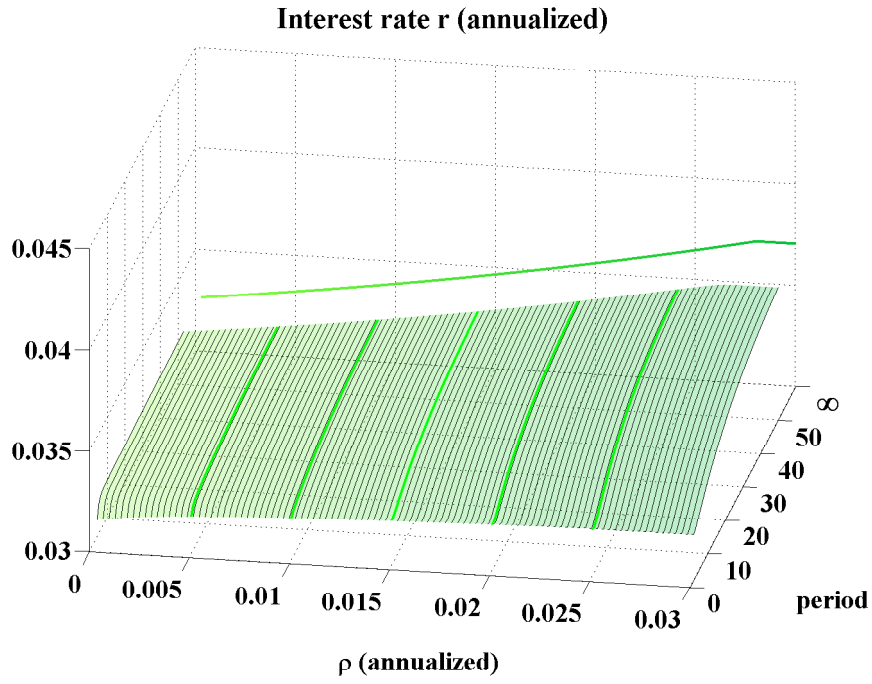
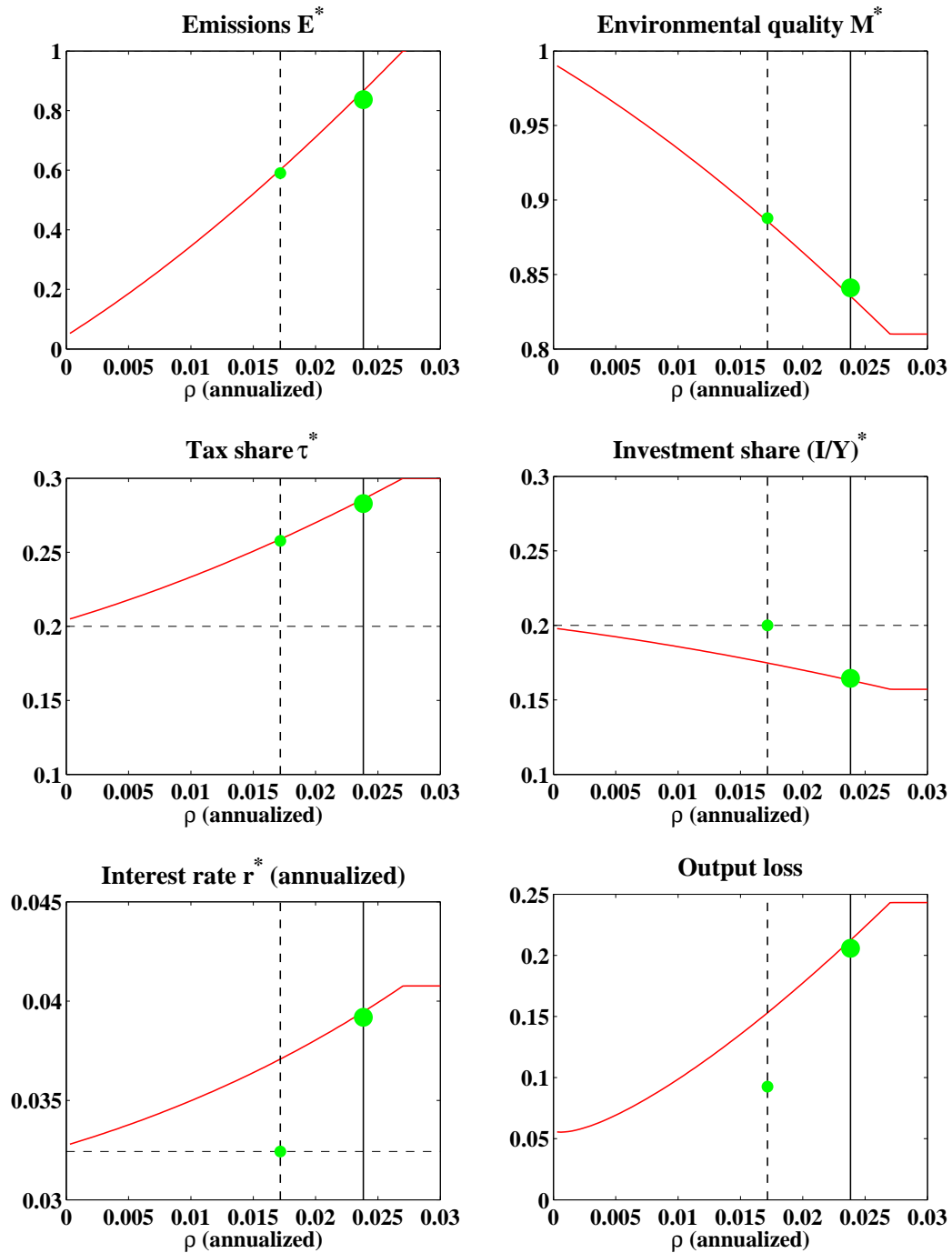**Figure 3:** *Steady state according to the neo-classical policy advice*
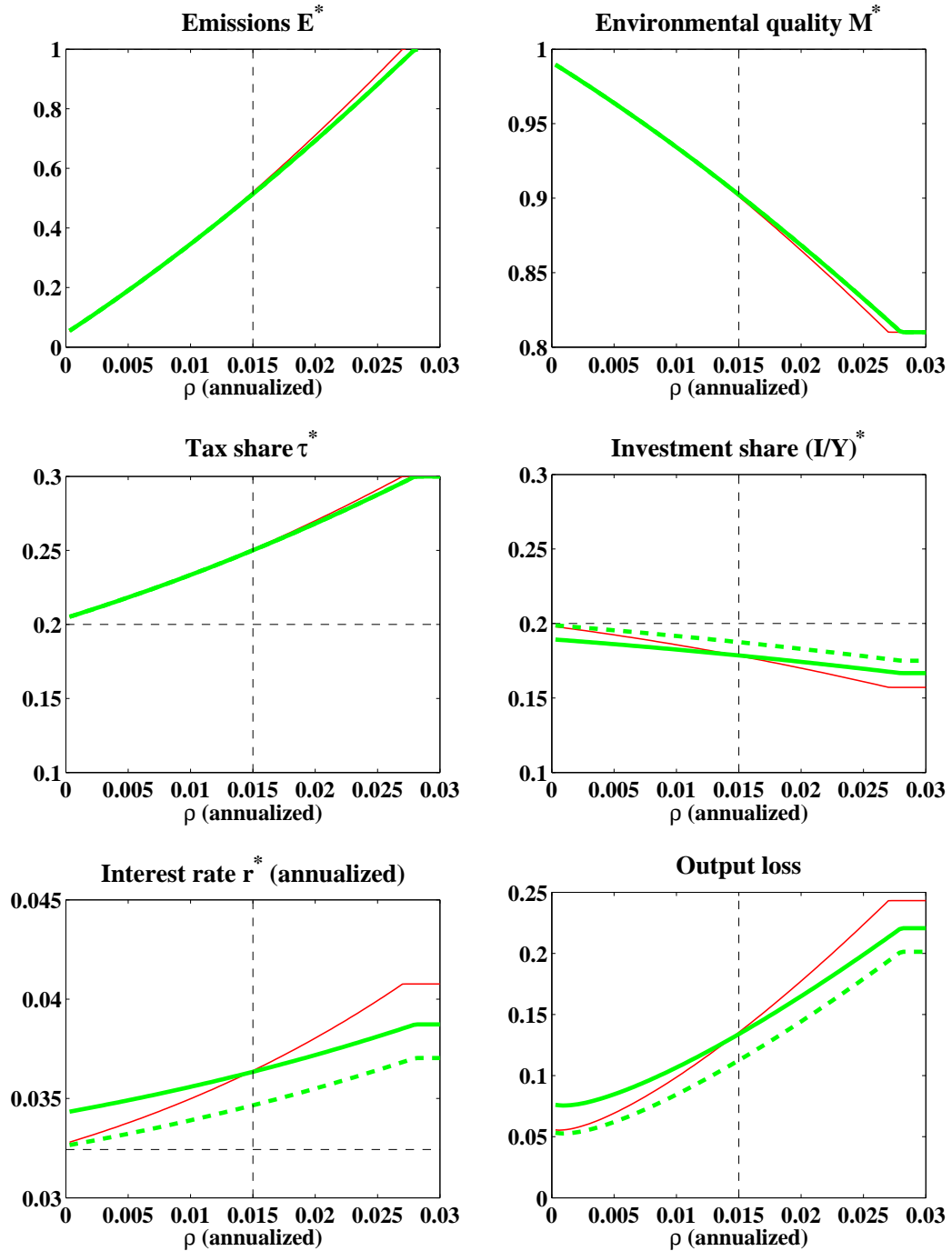
Thin red line: steady state according to the optimal policy. *Broken black line, vertical*: social evaluator's subjective discount rate. *Broken green line and thick green line*: steady state as predicted by a neo-classical economist at time 0 and in steady state. *Broken black line, horizontal*: values in period 0.

45

Figure 4: *Transitional dynamics according to the new-classical policy advice*

Emissions E

Environmental quality M

Tax share τ

Investment share (I/Y)

Interest rate r (annualized)

Output loss

Note: the green dots on the vertical axes are the steady state levels as predicted at time 0.

46

Figure 5: *Steady state according to the new-classical policy advice*