Inquiring Attitudes and Erotetic Logic: Norms of Restriction and Expansion

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Abstract

A fascinating recent turn in epistemology focuses on inquiring attitudes like *wondering* and *being curious*. Many have argued that these attitudes are governed by norms similar to those that govern our doxastic attitudes. Yet, to date, this work has only considered norms that might *prohibit* having certain inquiring attitudes ("norms of restriction"), while ignoring those that might *require* having them ("norms of expansion"). We aim to address that omission by offering a framework that generates norms of expansion for inquiring attitudes. The framework draws on inferential erotetic logic, which we explain and augment with some theorems. We explore several of the norms that it yields—some sympathetically, others unsympathetically.

1 Introduction

You might *believe that* Plato wrote the *Republic*; alternatively, you might *wonder* or be *curious about* who wrote it. In the former case, your attitude would be *propositional*; in the latter it would be *inquiring*. This paper is about inquiring attitudes; those attitudes take questions (instead of propositions) as their objects and (in some sense) aim at answering or settling those questions. In addition to curiosity and wonder they include contemplation, deliberation, and perhaps more besides.[1] We'll henceforth use "wonder" as a general stand-in for them.

These attitudes are the subject of a rich debate.[2] In this debate, some argue that it is irrational to wonder a question Q (e.g. whether it's raining) while knowing a complete answer to Q. Others argue similarly but replace knowledge with other states, like belief. Remarkably, all these claims target people who *do* wonder something. What about people who *don't* wonder something? When are *they* being irrational? Consider a dialogue between Authades ("obstinate one") and Zetegetes ("leader of an inquiry"):

[1]Some theorists might include suspension of judgement, but we wouldn't. While suspension plausibly takes questions as its objects, it does not plausibly aim at answering or settling those questions. (Imagine suspending on whether the stars are even in number.) See Friedman (2017) and Archer (2019).

[2]Archer (2018, 2019); Carruthers (2018); Dover (Forthcoming); Falbo (2021, Forthcoming); Friedman (2013, 2019); Haziza (Forthcoming); Palmira (2020); Sapir and van Elswyk (2021); Whitcomb (2010); Woodard (2022). Mulligan (2018) documents similar earlier discussions among Brentano's heirs.

> Authades: I don't know who wrote the *Republic*, and I wonder who it was.
>
> Zetegetes: It's the same person who wrote the *Meno*.
>
> Authades: Who cares? I wonder who wrote the *Republic*, not who wrote the *Meno*.
>
> Zetegetes: Um... Did you believe what I just said?
>
> Authades: Yep. The writer of the *Republic* also wrote the *Meno*. I know that.
>
> Zetegetes: But then given what you know, the answer to *Who wrote the Meno?*—whatever it is—entails the answer to *Who wrote the Republic?*—whatever *it* is.
>
> Authades: I know. It's not that I don't understand logic. It's just that I don't care. I wonder who wrote the *Republic*, and I know that the writer of the *Republic* also wrote the *Meno*, but I still don't wonder who wrote the *Meno*.
>
> Zetegetes: That's irrational!

Zetegetes is right: Authades is being irrational.[3] But why? For an explanation, we might look to norms for wondering discussed in recent epistemology, e.g. the "Don't Believe and Inquire" norm DBI:

> "One ought not...have an interrogative attitude towards Q at t while believing [a complete answer to Q] at t." (Friedman 2019, 303).

Friedman here treats "one ought not" and "it is irrational for one to" interchangeably. Thus DBI says (e.g.) that it is irrational to *wonder whether P* when you *believe that P*.

DBI is what we'll call a "norm of restriction"—a principle saying it's irrational to *wonder* a question Q given certain conditions. Principles saying it's irrational *not* to wonder Q given certain conditions, are "norms of expansion". (Motivating this terminology: one kind of norm requires us to *restrict* our attitudes by *not wondering* certain things given certain conditions, while another requires us to *expand* our attitudes by *wondering* certain things given certain conditions.) For simplicity, we assume that all norms of both sorts are "wide scope" in the sense that they tell us *it is irrational to:* c_1, *and* $\ldots, c_n$ for some (possibly singleton) set of conditions $\{c_1, \ldots, c_n\}$. With norms of restriction, one of those conditions involves wondering something; with norms of expansion one of them involves not wondering something.

DBI is a norm of *pure* restriction because it is a norm of restriction *that isn't also* one of expansion. In fact, *all* of the norms for wondering thus far discussed in print are of this sort.[4] But there can be norms that are both. Indeed, many of the norms we'll discuss are; they say it's irrational to wonder certain things while not wondering others.

Norms of pure restriction don't explain Authades' irrationality. If they did, he wouldn't be able to ameliorate his condition by expanding his wondering. And, he *is*

[3]Other terms of criticism could be substituted for "irrational" here. See §4.
[4]Rosa (2022), an independently developed work in progress, helpfully discusses some norms of expansion.

able to do so—by coming to wonder who wrote the *Meno*. To explain his irrationality, then, we need a norm of expansion.

As it happens, a certain body of logical theory inspires such norms: *inferential erotetic logic* (IEL). We'll explain IEL and use it for two tasks: reverse-engineering some already-discussed norms of restriction, and forward-engineering some new norms of expansion. These tasks don't close down the territory. Rather, they *open it up* by starting a research program of connecting IEL to interrogative epistemology.

2 IEL Primer

IEL studies *erotetic arguments*: arguments whose conclusions (and sometimes premises) are questions.[5] Each question has a set of "direct answers" consisting in the propositions among which it calls for us to choose. For example, *Who won the race: Smith or Jones?* calls for us to choose among the propositions *Smith won the race* and *Jones won the race*; those are its direct answers. *Partial answers* are disjunctions of some but not all of a question's direct answers, and *eliminative answers* are negations of direct answers. Now to an erotetic argument:

1. Someone stole the tarts.
2. So, who was it?

Here, the conclusion-question in some sense follows from the premise. We might capture that sense by saying that *Someone stole the tarts* raises the question of who that person was. In IEL, this kind of raises-relation is called "evocation" and there is a standard attempt to define it. To state that definition, we'll need some additional vocabulary. A proposition P is *a presupposition* of Q just in case P is entailed by each of Q's direct answers. A question is *sound* just in case it has at least one true direct answer; and *sound relative to* a set of propositions Γ just in case Γ entails that it is sound. Notice that if a question is sound then all of its presuppositions are true.

For illustration, consider *Who wrote the Meno?*. Its direct answers are propositions like *Plato wrote the Meno* and *Aristotle wrote the Meno*. Its partial answers include such propositions as *Either Plato wrote the Meno or Aristotle wrote the Meno*. Its eliminative answers are propositions like *Aristotle didn't write the Meno*. Its presuppositions include *Someone wrote the Meno*. Since *Who wrote the Meno?* has a true direct answer, it's sound. In contrast, consider *Is the present king of France bald?*. Its direct answers are *The present king of France is bald* and *The present king of France is not bald*. Since there is no present king of France, neither of these is true; the question is therefore unsound. Still, it is sound relative to the proposition that there *is* a present king of France.

Many theorists—"partitioners"—think of questions as partitions of logical space, each element of which they call a "complete answer" and each proper subset of which they call a "partial answer".[6] IEL's framework is broader; it associates every (nonsingleton) set of

[5] Wiśniewski (1991, 1995, 2013); Leszczyńska-Jasion & Chlebowski (2015); Peliš (2016); Cordes (2020).

propositions with a question, whether or not its elements form a partition. Still, there is a workable translation scheme: roughly speaking, complete answers are direct answers to questions whose direct answers form a partition; and partial answers (in the partitioners' sense) are partial answers (in the IEL sense) to these questions.

Partitioners can thus make use of IEL, as can many others, including those with differing views about (declarative) entailment. While the latter figures in many of its definitions, IEL can treat it in numerous ways. We'll treat entailment in the manner most familiar to philosophers—i.e. as the relation between premises and conclusions in classically valid arguments. Logicians have explored several alternatives (see e.g. Leszczyńska-Jasion & Chlebowski (2015)). Perhaps entailment can even be usefully replaced by probabilistic relations. This flexibility is a feature, not a bug. IEL is an intentionally modular tool, designed to be embeddable into many theoretical frameworks.

Now we can state the standard definition of evocation in IEL. That definition has it that Q is evoked by Γ just if the following two conditions hold:[7]

Relative Soundness: Q is sound relative to Γ.

Affirmative Openness: Γ does not entail any direct answer to Q.

To illustrate this, notice that *Fiona is smiling* evokes only one of the questions below (for which one, see the footnote following them).

Q_1: Is Fiona smiling?

Q_2: Is the present king of France smiling?

Q_3: Is Fiona happy?[8]

From now on, we'll mean by "evocation" only what this definition picks out. We won't claim that this is a *good* explication of the notion of propositions "raising" questions. Rather, we'll take the relation as-defined and do some philosophical work with it.

Evocation connects questions to propositions. Another relation, *erotetic implication*, connects questions to questions (or to questions *and* propositions). Another argument illustrates it:

1. Who wrote the *Republic?*
2. The writer of the *Republic* also wrote the *Meno*.
3. So, who wrote the *Meno*?

[6]Partitionism traces back to Hamblin (1958).

[7]This definition comes from Wisniewski (1991, 1995) via Belnap (1969) and Bromberger (1971).

[8]It's Q_3 that's evoked. Q_1 violates affirmative openness; Q_2 violates relative soundness.

Erotetic implication comes in several varieties. The above example features "strong regular" erotetic implication, which we'll call *resolution*.[9] This relation requires three conditions. First, the premises must *secure the soundness* of the conclusion-question in the sense that, if the premise-propositions are all true and the premise-question is sound, then the conclusion-question is also sound.

Security: Q_2 is *secured* by Q_1 given Γ *iff*: if Q_1 is sound given Γ then so is Q_2.

Second, the conclusion-question must *effectively answer* the premise-question given the premise-propositions, in the sense that each direct answer to the conclusion-question entails (given the premise-propositions) a direct answer to the premise-question.

Effectiveness: Q_2 is *effective* for Q_1 given Γ *iff*, given Γ, each direct answer to Q_2 entails a direct answer to Q_1.

Third, the premise-propositions must leave the premise-question and (thus) the conclusion-question affirmatively *open* in the sense that they don't entail any of their direct answers. The definition of resolution is thus as follows:

Resolution: Q_2 *resolves* Q_1 given Γ *iff*

1. Q_2 is *secured* by Q_1 given Γ,
2. Q_2 is *effective* for Q_1 given Γ, and
3. Γ leaves Q_1 (and hence Q_2) *affirmatively open*.[10]

We've described IEL in terms of propositions. But, in full disclosure, it's usually formulated in terms of *wffs*—strings of symbols—some of which are *declarative* (the formulas of propositional logic, e.g. "p") and others *erotetic* (declarative wffs inside brackets with a question mark, e.g. "$?\{p, \neg p\}$"). In speaking of propositions, then, we impose a philosophical interpretation on IEL. Though not uncontroversial, this interpretation is traditional and modest; we assume only that propositions can be believed, known, and asserted (and adequately modeled by the kind of propositional logic that's in the Appendix). These assumptions are compatible with many theories about propositions' nature.

3 Proof of Concept for a Research Program

It's one thing to define logical relations like evocation and resolution; it's quite another to state norms of rationality and wondering. To move from the definitions to the norms we need "bridge principles" that connect the two. There's a tradition exploring principles that bridge *declarative* logic to rational *belief*. We'll explore some similar bridges from *erotetic* logic to rational *wondering*.

[9] See Wiśniewski (2013, 76) and Millson (2019, 2021).
[10] See Lemma 1.

In this section, we examine one such principle with two important features. First, it appeals to a central IEL relation: evocation. Second, it entails some norms of restriction for wondering that have appeared in recent debates among interrogative epistemologists. Our interest is not to adjudicate the plausibility of this principle (or the other norms it entails). Instead, we offer it up as evidence that IEL is a fruitful resource for theorizing norms of wondering—i.e. as our research program's proof of concept.

The aforementioned bridge principle is the Evoked Question Norm:

EQN
It is irrational to: wonder Q when your knowledge doesn't evoke Q.

Bridge principles have alterable parameters. EQN gives a *requirement* (rather than a permission or a *pro tanto* reason); its term of criticism is "*rational*" (instead of "moral," "epistemic," etc.); that term takes *wide*, rather than narrow, scope. EQN is *synchronic* (applying only at a given time) instead of diachronic (applying across times); it applies given our *knowledge* (rather than our beliefs, our certainties etc.); it enjoins us to *restrict* our wonderings (instead of expanding them); and it applies *irrespective* of whether a subject *knows about* the logical facts at issue.[11]

EQN offers *evaluations* rather than *blame*. All it claims is that certain people are being irrational—those who wonder a question their knowledge doesn't evoke. While irrationality may provide evidence of blameworthiness, it does not entail it. Maybe a person is being irrational but has an excuse, and so is blameless. Like the norms we'll discuss later, EQN concerns how we ought to *be* and not how we ought to *be held accountable*. Thus it brings out only one among several "ways in which logic might be normative".[12]

Now, there are two ways for Q to fail to be evoked by the propositions you know: it might be (affirmatively) unopen relative them or it might be unsound relative to them. Considering the first possibility, notice that EQN entails an "Open Question Norm":

OQN
It is irrational to: wonder Q when your knowledge entails a direct answer to Q.

One way to violate OQN is to *know* a direct answer to Q. Thus, OQN entails another norm—*it is irrational to: wonder Q when you know a direct answer to Q*. This other norm is a natural bridge from IEL to rationality and wondering. Interestingly, it's nearly identical to certain principles discussed in recent epistemology:

> "It is illegitimate to be curious about a question when you know its answer." (Whitcomb 2010, 674).
>
> "Necessarily, if one knows Q at t, then one ought not have [an interrogative attitude] towards Q at t". (Friedman 2017, 311).

[11] Compare MacFarlane (2004).
[12] Cf. Steinberger (2019).

> "Interrogative attitudes... are never compatible with knowledge of the question's answer." (Sapir and van Elswyk 2021, 1).

The similarity between bridge principles inspired by IEL, and principles discussed in recent epistemology, does not end there. Recall that the other way to violate EQN is to wonder Q when it's *unsound* relative to your knowledge—i.e. when that knowledge doesn't entail all of its presuppositions. Thus we have a "Sound Question Norm":

SQN
It is irrational to: wonder Q while your knowledge does not entail all of Q's presuppositions.

A nearby norm that's more demanding replaces entailment with inclusion, yielding

KNI
It is irrational to: wonder Q while your knowledge does not include all of Q's presuppositions.

Here again, the norm (the "knowledge norm of inquiry") is defended by an epistemologist: Willard-Kyle (forthcoming). So evocation, a relation broached decades ago by erotetic logicians, is closely related to several norms proposed recently by epistemologists.

None of those epistemologists mention IEL. How did that happen? We think the two literatures pursued similar issues and generated similar sets of ideas independently. As a result, each set can help "reverse engineer" the other. This point is not just historical. It is also *proof of concept* for a research program: the program of bringing new material into each of the literatures via the other. We'll now take that program's first steps.

4 A Research Program's First Steps

We seek an explanation of why Authades counts as being irrational. Thus consider a "Resolution Norm":

RN
It is irrational to: wonder Q_1 but not Q_2 when, given what you know, Q_2 resolves Q_1.

This norm leaves fixed all the parameters from EQN, save two: it is a norm of *expansion* and it focuses on cases where the logical relation *does* obtain. These changes yield an explanation of Authades' irrationality—he's being irrational *because he violates RN*. He (a) fails to wonder who wrote the *Meno* while also (b) wondering a question that *Who wrote the Meno?* resolves given his knowledge. RN deems these states jointly irrational.

RN doesn't deem irrational Authades' failure to wonder who wrote the *Meno*. That claim would "detach" one of the states RN targets jointly, invalidly inferring that it targets that state singly.[13] Still, RN entails that if Authades wonders a question that's resolved by *Who wrote the Meno* given his knowledge, he's doing *something* irrational if he fails to wonder who wrote the *Meno*: he's in jointly irrational states. So RN requires him—on pain of being in jointly irrational states—to wonder who wrote the *Meno* whenever he wonders a question resolved by *Who wrote the Meno?* given his knowledge.[14]

In sum, RN yields the explanation we seek. Similar explanations are available in declarative cases, as another dialogue illustrates:[15]

> Achilles: I'm agnostic about whether Plato wrote the *Meno*.
>
> Tortoise: Do you know that Plato wrote the *Republic*?
>
> Achilles: Of course.
>
> Tortoise: And do you know that the writer of the *Republic* also wrote the *Meno*?
>
> Achilles: Oh yeah. I've known that for a long time.
>
> Tortoise: Well there you go then. You should believe that Plato wrote the *Meno*.
>
> Achilles: Not so fast, turtle. I'm still unconvinced.

Achilles is being irrational. This can be explained by an "Entailed Belief Norm":

EBN
It is irrational to: know that P and not believe that R when, given what you know, P entails R.

Achilles and Authades fail to expand their attitudes in ways deemed irrational by the RN and EBN, respectively. What Achilles does with his *propositional* attitudes, Authades does with his *inquiring* attitudes. RN is an interrogative analogue of EBN.

It is an open question which *kinds* of irrationality Achilles and Authades exhibit. Perhaps they exhibit *epistemic* irrationality, like people whose beliefs are unjustified. Or perhaps it's *instrumental* irrationality, like people who don't take the means to their ends. Or perhaps it's *structural* irrationality, like people whose mental states don't "fit together" (e.g. people who want to ϕ while wanting to not want to ϕ). Or perhaps it's *zetetic* irrationality, a putative species of irrationality specific to inquiry.[16]

We suspect the irrationality at issue is, at least, structural. That's because it features incoherence. The relevant mental states (and lacks thereof) do not maximally fit together. This kind of fit, and its relationship to irrationality, are both subjects of extensive discussions.[17] We don't have developed analyses of either of them: hence our mere *suspicion* that the irrationality at issue is structural. You might view that irrationality differently; that would be fine for our purposes. So would be replacing irrationality *in toto* with something else, like incuriosity or uninquisitiveness or obtuseness. What's essential is

[13] Compare Broome (1999) and Whitcomb (2014).

[14] Since RN applies given what you know, it's weaker than (so at least as plausible as) a similar norm applying given what you *believe*. You might nonetheless prefer the belief version: belief, lacking some of the external conditions on knowledge, may better align with rationality.

[15] Compare Hawthorne (2004) and Carroll (1895).

[16] See Friedman (2020).

that there's *some* way in which our characters' mental states are defective or suboptimal or inappropriate.[18]

4.1 Problems for RN

RN faces two kinds of problems: old and new.

4.1.1 Old Problems

The old problems are versions of standard worries about declarative bridge principles.[19] We'll say two things about them. First, they aren't knock-down arguments against RN—if they were, they'd also knock down many popular declarative bridge principles, which they don't. Second, those who think the problems *do* knock down RN can avoid them by toggling its parameters.

For instance, here's an old problem for ("closure") principles like EBN. That principle deems irrational people who fail to believe a proposition entailed by what they know—*even if* they don't know the entailment obtains. Similarly, RN deems irrational those who fail to wonder a question that (given their knowledge) resolves a question they wonder—*even if* they don't know the *resolution* obtains. Call this *the problem of logical ignorance*.[20] We are unmoved by it, for two reasons. First, RN issues evaluations; it deems certain people irrational. It doesn't issue blame. If it did then we might include a logical knowledge requirement (since ignorance often mitigates blameworthiness); but again, it doesn't. Second, many plausible norms of evaluation are comparably independent of one's logical (or other formal) knowledge. Plausibly, people with inconsistent beliefs are being irrational even if they don't know about the inconsistency. Plausibly, people with probabilistically incoherent credences are being irrational even if they don't know about the incoherence. Similarly with RN.[21]

This isn't to say that RN is as plausible as the other norms, or that arguments *for* those other norms transfer over into arguments *for* RN. It's just to say that a certain problem that doesn't knock down those other norms, doesn't knock down RN either.

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[17] See e.g. Worsnip (2021).

[18] Compare (Friedman 2019, 303): "I'm happy to say that [incoherence] is irrational or unreasonable... some people mean...things by these terms that I don't...Either way...[incoherence] is defective or suboptimal or inappropriate."

[19] See Harman (1986), MacFarlane (2004), and Steinberger (2019).

[20] For example, EBN requires you to believe all the logical truths (even if you don't know they're logical truths). Similarly, RN requires you to wonder every question if you wonder any logical question (even if you don't know those questions resolve the logical one). Rosa's (2022) in-progress work discusses closely related issues. Discussions in the declarative case include Carr (2021).

[21] A related worry. "Coarse" theories of content deem *Batman is humorless* identical to *Bruce Wayne is humorless*. If your knowledge leaves affirmatively open *Is Batman humorless?* and *Is Bruce Wayne humorless?*, these theories make RN deem you irrational if you wonder one of them but not the other. (Irrational and also impossible: on these theories they're the same question.) That's worrisome, but here RN is in good company. Coarse content brings similar worries to many popular norms, for instance probabilism (see e.g. Christensen (2004, 16)). We submit that whatever solution applies to the other norms—perhaps it's to individuate content finely—applies to RN too.

While some theorists would restrict RN to cases where you know the relevant formal facts, others would restrict it to cases where your wonderings (and lacks-of-wonderings and knowledge) are "occurrent", rising to the level of consciousness. Call the resulting norm RNC. Applied generally, its internalism is objectionably silent. Unclosed or inconsistent beliefs, incomplete or otherwise-probability-theory-violating credences, unconnected or intransitive preferences: *whenever* these are backstage at the Cartesian theatre, the internalism behind RNC declines to deem them irrational. We think this loss of informativeness outweighs the plausibility added by dropping RN for (the logically weaker) RNC.

Some might weigh things differently and opt for RNC—especially since, in our dialogue, Authades knows about the formal facts and his mental states are easily construed as occurrent. If that's how you weigh things, don't stop reading! You can still profit from the discussion. Move forward with us *mutatis mutandis*, applying our points in your restricted domain.

Actually, you can toggle *all* of RN's parameters. For instance, you can (as we've said) replace irrationality with incuriosity. The resulting principle is sometimes very plausible, including in certain cases where your mental states are non-occurrent. But compatibly with this, we think those cases also feature irrationality. Here again we follow probabilism, on which similar cases would feature irrationality due to (non-occurent) credences in *Plato wrote the Republic* and *The writer of the Republic wrote the Meno* but no credences (not even a zero-valued ones) in *Plato wrote the Meno*. The cases here seem similar enough to merit at least one shared evaluation. We favor irrationality. Still, that parametric setting is (like the others) just a starting point. While we think it's defensible, we've chosen it partly because we've got to start *somewhere*. Feel free to start elsewhere: we've designed our discussion for easy retrofitting.

4.1.2 New Problems

At least two problems for RN are new. The first we'll call *the problem of eliminated conjuncts*. Suppose that a disease is afoot and that your student James wants to visit your office hours. Let $A =$ *James is allowed to visit*, $V =$ *James is vaccinated*, and $E =$ *James has an exemption from the vaccine requirement*. Suppose that nothing you know settles whether A, V, or E obtain, but that you do know that $[A \leftrightarrow (\mathrm{V} \vee \mathrm{E})]$. Finally, suppose that you wonder $Q_A =$ *Is James allowed to visit?*.

Holding these suppositions fixed, RN requires you to wonder the conjunctive question of whether James is vaccinated and whether he's exempt—the question $Q_V \& Q_E$ with direct answers $V \& E$, $V \& \neg E$, $\neg V \& E$, and $\neg V \& \neg E$. That's a welcome result. But now suppose you learn that James is not exempt ($\neg E$). RN then *still requires* you to wonder $Q_V \& Q_E$. That's *not* a welcome result. You know that James isn't exempt. Why should you have to wonder *Whether he's vaccinated <u>and</u> whether he's exempt?*.

RN also faces what we'll call *the problem of irrelevant conjuncts*. First a preliminary point. Conjunctive questions "resolve their evoked conjuncts" in the sense that, if Γ evokes both Q_1 and Q_2, then $Q_1 \& Q_2$ resolves Q_1 given Γ. We prove this in the Appendix (Theorem 1), but here an illustration should suffice. Suppose that your knowledge evokes *Is Ted alive?* and *Is Ted asleep?*. Then, given your knowledge, *Is Ted alive and is he asleep?* resolves *Is Ted alive?*.

To see the problem of irrelevant conjuncts, hold fixed the supposition that Authades' knowledge evokes *Who wrote the Republic?*, a question he wonders. Let Q_n be any other question his knowledge evokes. Since conjunctive questions resolve their evoked conjuncts, RN requires him to wonder *Who wrote the Republic and Q_n?*. To see how objectionable this can seem, add it to our stock of fixed suppositions that his knowledge evokes *What is the 104th digit in the phone book?*. RN then requires him to wonder *Who wrote the Republic and what is the 104th digit in the phone book?*.[22]

4.2 From RN to FRN

You might approach the problem of eliminated conjuncts by deeming it irrational to wonder questions while knowing any of their eliminative answers, and applying RN subject to that constraint. But why should we have to *stop* wondering a question just because we rule *out* one of its direct answers? Alternatively you might dig in, endorsing the requirement to continue wondering *whether James is vaccinated and whether he's exempt* after learning he's not exempt. But while rationality may *allow* that kind of continuation, we think *requiring* it is a bridge too far. A better approach narrows RN via the following notion:

Full Openness: Γ leaves Q *fully open iff* Γ does not entail any direct or eliminative answer to Q.

This notion yields limited versions of the resolution relation and RN:

Full Resolution: Q_2 *fully resolves* Q_1 given Γ *iff*

1. Q_2 resolves Q_1 given Γ, and
2. Γ leaves Q_2 *fully open.*

FRN
It is irrational to: wonder Q_1 but not Q_2 when, given what you know, Q_2 fully resolves Q_1.

FRN solves the problem of eliminated conjuncts via the full openness condition, which keeps it from requiring that we wonder questions with direct answers that are ruled out by our knowledge. But the problem of irrelevant conjuncts remains, at least when our knowledge leaves the questions at issue fully open.

[22]Concerns about excessive informativeness also motivated Groenendijk and Stokhof (1984), who argue that p is a *better* answer to (a partition question) Q than r just in case either p eliminates more of Q's cells than r, or they eliminate the same amount of cells but r is logically stronger (i.e. more informative) than p—an idea rendered quantitative by van Rooy (2003). Working in the program of *inquisitive semantics*, Groenendijk and Roelofsen (2011) describe questions given in response to questions as "noncompliant" when the former require more information to answer than the latter.

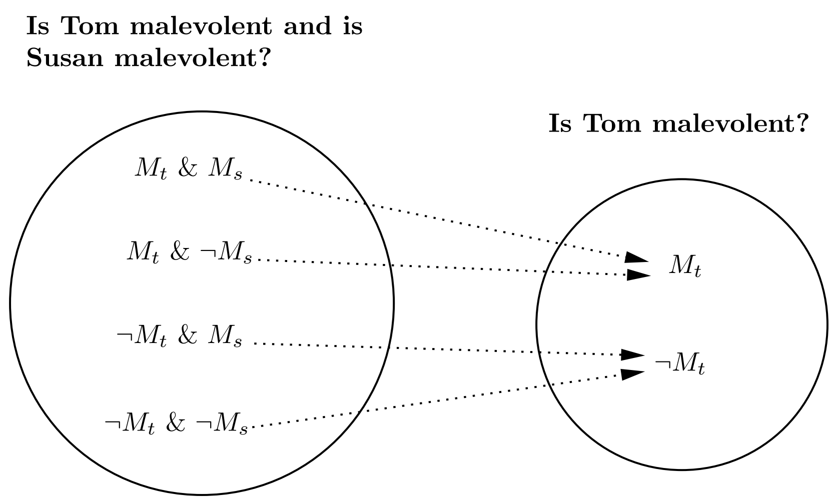
4.3 From FRN to MRN

The problem of irrelevant conjuncts applies to both RN and FRN. How bad is it? Some theorists might view it as a *reductio*. Others might find it unfortunate but tolerable. Others still might deny that it is even a problem. We aren't sure which of these responses is best. Assuming the problem is a genuine one, where should we go next?

One option is to rebuild RN and FRN using nonclassical consequence relations such as relevance entailment. Another is to continue using classical consequence and build from it norms that restrict RN even more than FRN restricts it. Both of these paths are worth navigating; we'll navigate the latter.

Suppose that you wonder whether Jerry has any malevolent friends, that you know that Jerry's only friend is Tom, and that your knowledge leaves fully open the questions of whether Tom is malevolent and whether Susan (Jerry's enemy) is malevolent. Under these assumptions, FRN requires you to wonder whether Tom is malevolent. It also requires you to wonder *whether Tom is malevolent and whether Susan is.* Here we see an instance of the problem of irrelevant conjuncts: why should you have to wonder the Tom-and-Susan question, and not just the Tom-question? We also see a way forward.

The Tom-question "weakens" the Tom-and-Susan question. In general, one question weakens another just if you can obtain the former by starting with the latter and replacing at least one of its direct answers with a proposition that is logically weaker given your knowledge.[23] In the case at hand, not just some but *all* of the direct answers get replaced in this way (see the figure below).



So the Tom-question weakens the Tom-and-Susan question. It has two other very important features (given your knowledge) as well. First, like the Tom-and-Susan question, the Tom-question *fully resolves* your original question—the question of whether

[23] For a similar notion see Wiśniewski (1995, 134).

Jerry has any malevolent friends. Second, and *unlike* the Tom-and-Susan question, the Tom-question *can't be further weakened* compatibly with *continuing to* fully resolve your original question. We'll combine these last two points by saying that the Tom-question *minimally resolves* your original question (given your knowledge). Here's the definition:

Minimal Resolution: Q_2 *minimally resolves* Q_1 given Γ *iff*, given Γ,

1. Q_2 fully resolves Q_1, and
2. No weakening of Q_2 fully resolves Q_1.

This relation suggests a new norm:

MRN:
It is irrational to: wonder Q_1 but not Q_2 when, given what you know, Q_2 minimally resolves Q_1.

To see MRN's payoff, return to Authades and his original question Q_R (*Who wrote the Republic?*) along with the irrelevant conjunct Q_D (*What is the 104th digit in the phone book?*) and the question Q_{RD} which conjoins them. MRN does *not* require Authades to wonder Q_{RD} when he wonders Q_R. For we can take any direct answer to Q_{RD} (say, *Aristotle wrote the Republic and the digit at issue is "7"*), replace it with one that is logically weaker given his knowledge (here, *Aristotle wrote the Republic*), and get a question that still fully resolves Q_R. Hence, Q_{RD} doesn't minimally resolve Q_R; and MRN doesn't require Authades to wonder the former when he wonders the latter.

Yet it *does* require him to wonder who wrote the *Meno* (Q_M) when he wonders Q_R. For Q_M, unlike Q_{RD}, *minimally* resolves Q_R given his knowledge—it fully resolves Q_R and no weakening of it does. MRN thus explains why Authades in our dialogue counts as being irrational, without objectionably requiring him to wonder questions like Q_{RD}.

This is a success story—and it generalizes over at least one large and interesting class of cases. These are the cases where (a) your knowledge leaves Q_1 affirmatively open and (b) there's at least one direct answer to Q_2 that isn't entailed by at least one direct answer to Q_1 (given your knowledge). In these cases, certain ways of settling Q_2 aren't delivered by certain ways of settling Q_1. The required disconnect is slim: *only one* way of settling Q_2 needs to be left out, and *only one* way of settling Q_1 needs to leave it out. In somewhat antiquated parlance, we might say that this disconnect obtains whenever a new question Q_2 "isn't universally contained in" an already-open inquiry.[24] In these cases, $Q_1\&Q_2$ can be weakened while still fully resolving Q_1; just eliminate from $Q_1\&Q_2$'s direct answers those conjuncts from Q_2 that aren't entailed (given your knowledge) by their corresponding conjuncts from Q_1. MRN thus permits you to wonder Q_1 without wondering $Q_1\&Q_2$. A large part of the problem of irrelevant conjuncts is duly boiled off.

There does remain a residue. Suppose we start with Q_R and add to one of its direct answers a conjunct consisting in a proposition Authades knows. Then MRN requires him

[24] See Theorem 4.

to wonder the resulting question if he wonders Q_R. This kind of case is a residue of the problem of irrelevant conjuncts, and a task for future work.

MRN makes significant progress on the problem of irrelevant conjuncts. And it solves the problem of eliminated conjuncts—in the same way as FRN. And it does these things while offering an explanation of why Authades counts as being irrational, at least if his knowledge leaves the questions at issue fully open. Thus, while not perfect, MRN has *bone fide* credentials. It merits our theoretical attention.[25]

5 Generalizing the Norms

MRN gives us the explanation we seek, solves the problem of eliminated conjuncts, and makes significant progress on the problem of irrelevant conjuncts. These are successes; but they come at a price. MRN is *narrower* than RN and FRN, targeting a smaller group of would-be violators than either of them. Narrow norms tend to avoid error. But they purchase their error-avoidance with reduced informativeness, rendering fewer (correct or incorrect) verdicts. Is there a way to lower the cost, to make MRN more informative?

5.1 From Resolution to Helps-resolve

Recall that each direct answer to a resolving question entails (given Γ) a *direct* answer to the resolved question. By replacing this *effectiveness* condition with the following *helpfulness* condition, we can generate a broadened version of the resolution relation:

Helpfulness: Q_2 is *helpful* for Q_1 given Γ *iff*, given Γ, each direct answer to Q_2 entails a *partial* answer to Q_1.

We can further broaden the resolution relation by removing its affirmative openness condition. The result is a new relation, the *helps-resolve* relation.

Helps-resolve: Q_2 *helps resolve* Q_1 given Γ *iff*

1. Q_2 is *secured* by Q_1 given Γ, and
2. Q_2 is *helpful* for Q_1 given Γ.

This relation is none other than *general erotetic implication*, the most extensively studied relation in IEL.[26] It inspires several norms that are more informative than those we've thus far discussed. For example:

HRN:
It is irrational to: wonder Q_1 but not Q_2 when, given what you know, Q_2 helps resolve Q_1.

[25] For a related discussion of minimality more generally see Yablo (Forthcoming).

[26] It underwrites Socratic Proofs and Erotetic Search Scenarios, methods for reducing one set of questions to another (Wiśniewski 2003, 2004; Leszczyńska-Jasion & Chlebowski 2015).

This norm attributes irrationality to everyone our previous resolution norms did, *plus* many about whom the latter are silent. It is thus a natural place to begin the search for more informative norms. It is also a natural norm to explore for practitioners of IEL, built as it is from IEL's most extensively studied relation.

Is HRN plausible? We'll argue that it is not, because it leads to a dilemma. Having done that, we'll ask whether the dilemma can be averted by reintroducing the affirmative or full openness conditions. We'll argue that it can't; then we'll draw out a general lesson.

5.2 The Difficult Dilemma

Two steps lead from HRN to an unhappy result. Let K be the propositions that you know, and let Q_{big} be The Big Question—*What are all the facts?*.

The first step is to note that Q_{big} helps resolve Q_{rain} (= *Is it raining?*) given K. Since the direct answers to Q_{big} form a partition, it has a true direct answer no matter what (and, thus, a true direct answer given K). So, Q_{big} is secured by Q_{rain} given K. Q_{big} is also helpful for Q_{rain} given K, since any direct answer to Q_{big} will (given K) either entail *it is raining* or entail *it is not raining* (each of those propositions being direct answers and, thus, partial answers to Q_{rain}). Since Q_{big} is secured by and helpful for Q_{rain} given K, it helps resolve Q_{rain} given K. Conditional on your wondering whether it's raining, then, HRN requires you to also wonder what all the facts are.

The second step is to note that Q_{even} (= *Are the sand grains even?*) helps resolve Q_{big} given K. Like Q_{big} itself, Q_{even} is a partitioning question and is thus secured by Q_{big} given K. Now notice that both of the direct answers to Q_{even} entail *partial* answers to Q_{big}. For instance, if there is an even number of sand grains, then the true direct answer to Q_{big} must be a disjunct in the disjunction of those direct answers (to Q_{big}) that contain the conjunct *the sand grains are even.* So Q_{even} is both helpful for and secured by, and thus helps resolve, Q_{big} given K. Conditional on your wondering what all the facts are, then, HRN requires you to also wonder whether the sand grains are even.

And now we've got the unhappy result: conditional on your wondering whether it's raining, HRN requires you to also wonder whether the sand grains are even. If you find yourself wondering whether it is raining, then, you've got two options: stop wondering that question, or start wondering whether the sand grains are even.

The dilemma generalizes. Conditional on your wondering *any question at all*, HRN requires you to also wonder *every other question* secured by Q_{big} given your knowledge. All partitioning questions, many of which are wildly trivial, meet those conditions no matter what you know. How many beige things crossed Poland's border a prime number of times in 1981? What proportion of those things were chihuahuas? Either you become a *dullard* by not wondering anything at all, or you become an *inquiry-pump* by wondering the foregoing questions and countless similar others.

Poisonous options both. And yet, if one were to start with IEL and try to glean norms of expansion from it, HRN would be a wholly sensible candidate. The helps-resolve relation on which it is built has been studied extensively and for good reason. On its face, that relation seems apt to make for a plausible norm of expansion.

Can the dilemma be averted by adding affirmative or full openness conditions to the helps-resolve relation and rebuilding HRN accordingly? Sadly, no: the resulting norms would still yield a version of the dilemma. To see why, just replace Q_{big} with the slightly smaller question *Which propositions, among those neither affirmed nor denied by what I know, are true?*. You'll then end up with (slightly restricted versions of) the same poisonous options we've described.

5.3 A Lesson

FRN and MRN don't lead—at least not via the path we've charted—to the choice between dullard and inquiry-pump. MRN blocks the path's first step, because Q_{big} can be weakened in the relevant way. And FRN blocks the path's second step, because Q_{even} does not resolve Q_{big} (it merely *helps* resolve it). Are there *other* paths from FRN or MRN to the difficult dilemma? We don't think so. Given any background knowledge whatsoever, Q_{big} helps resolve Q_{rain}, and Q_{even} helps resolve Q_{big}. But given that same background knowledge, it need not be the case that Q_{even} helps resolve Q_{rain}. The helps-resolve relation is therefore *intransitive*. This is likely why HRN leads to the difficult dilemma.

In contrast, we prove in the Appendix that resolution, full resolution, and minimal resolution are transitive. These proofs show that if it's only norms based on intransitive relations that lead to the difficult dilemma, RN and FRN and MRN evade its grip. Perhaps the dilemma can be reached in other ways, but we can't see how. Thus we conjecture that *no* path leads from RN or FRN or MRN to the difficult dilemma. This conjecture, if correct, is an important general lesson.

6 Outro

We've broached the research program of connecting IEL to interrogative epistemology, first by using IEL to reverse engineer several extant norms of restriction and second by using it to forward engineer several new norms of expansion. The new norms can explain why certain cases feature irrationality. Winner winner chicken dinner, but the food's not free. The strongest new norm, RN, yields implausible results. Some attempts to fix it gave us FRN and MRN. These norms add plausibility but subtract informativeness. The obvious way to bring some informativeness back, HRN, leads to a difficult dilemma.

Now to our conclusion. On brand, it consists in some questions. Given the foregoing points, it seems sensible to search for a minimal version of HRN. So, what would such a norm say? Would it evade the difficult dilemma? What other features would it have?

Finally, a metapoint. We've been, for philosophers, unusually noncommittal. Instead of staking out our ground and fortifying its defenses we've openly explored uncharted territory, traversing our preferred path while marking out other paths too. This kind of theorizing is not always called for, but sometimes it is.[27,28]

[27] Nozick (1974, vii): "The word 'exploration' is appropriately chosen...There is room for more words on subjects than last words."

[28] For helpful feedback we thank Taylor Dunn, Arianna Falbo, Joshua Habgood-Coote, Dan Howard-Snyder, Frances Howard-Snyder, Hud Hudson, Christian Lee, Dee Payton, Luis Rosa, Amites Sarkar, Mona Simion, Justyn Smith, Harald Thorsrud, David Thorstad, Neal Tognazzini, Nick Treanor, Peter Van Elswyk, Ryan Wasserman, Isaac Wilhelm, and Christopher Willard-Kyle.

A Appendix

A.1 Preliminaries

Our language, $\mathcal{L}$, is composed of $\mathcal{L}_d$ and $\mathcal{L}_e$. $\mathcal{L}_d$ consists of the declarative wffs of classical propositional logic (hereafter: "d-wffs" or "propositions"), built by applying the familiar connectives $(\neg, \wedge, \vee, \rightarrow, \leftrightarrow)$ to a countable set of atoms $(p, q, r, \ldots)$. We use A, B, C, D, sometimes with subscripts, for arbitrary propositions and Γ, Δ, Σ for (possibly empty) sets of propositions. Let $\vDash$ be the classical declarative consequence relation defined over $\mathcal{L}_d$, so that $\vDash \subseteq \mathcal{P}(\mathcal{L}_d) \times \mathcal{L}_d$. $\mathcal{L}_e$ consists of erotetic wffs (hereafter: "e-wffs" or "questions"), built by adding the question mark, left and right brackets, and the comma to a sequence of at least two syntactically distinct d-wffs constituting the resulting question's direct answers—for instance, $?\{p, \neg p\} \in \mathcal{L}_e$. (Note that the question mark is not a set-theoretic operator—e.g. $?\{p, q\}$ and $?\{q, p\}$ are distinct e-wffs.) Arbitrary questions are represented by Q, often with subscripts. We use $=$ for both set-theoretic identity and syntactic identity between wffs.

A.2 Basic Definitions

We begin with some basics.

Definition 1 (Syntax of $\mathcal{L}$)**.** $\mathcal{L}$ is the set such that

(i) If $A \in \mathcal{L}_d$, then $A \in \mathcal{L}$.

(ii) If $A_1, \ldots, A_n (n > 1) \in \mathcal{L}_d$ and $A_i \neq A_j$ for all $1 \leq i, j \leq n$, then $?\{A_1, \ldots, A_n\} \in \mathcal{L}$.

(iii) Nothing else is an element of $\mathcal{L}$.

Fact 1 (Properties of Classical Declarative Consequence)**.**

1. $\Gamma \cup \{A\} \vDash A$ *(Reflexivity)*

2. If $\Gamma \cup \{A\} \vDash B$ *and* $\Gamma \cup \{B\} \vDash C$*, then* $\Gamma \cup \{A\} \vDash C$ *(Transitivity)*

3. If $\Gamma \cup \{A\} \vDash B$*, then* $\Gamma \cup \{A\} \cup \{C\} \vDash B$ *(Monotonicity)*

Definition 2 (Direct Answers and the $d(\cdot)$-function)**.** Let dQ be the function that maps Q to the set of its direct answers. So, if $Q = ?\{A_1, \ldots A_n\}$, then $dQ = \{A_1, \ldots A_n\}$. We apply disjunction as follows: If $Q = ?\{A_1, \ldots, A_n\}$ then $\bigvee dQ = A_1 \vee \ldots \vee A_n$.

Definition 3 (Partial Answers)**.** Let $dQ = \{A_1, \ldots, A_n\}$. B is a partial answer to Q iff $B = A_i \vee \ldots \vee A_j$ where $\{A_i, \ldots, A_j\} \subset \{A_1, \ldots, A_n\}$.[29]

Definition 4 (Eliminative Answers)**.** B is an eliminative answer to Q iff $B = \neg A$ for some $A \in dQ$.

[29] This definition and the next slightly differ from the typical ones; see Wiśniewski (2013, 43–44).

Definition 5 (Presuppositions)**.** B is a presupposition of Q iff $A \vDash B$ for all $A \in dQ$.

Definition 6 (Sound Questions)**.** Q is *sound* iff at least one of its direct answers is true, i.e. iff $\bigvee dQ$ is true. Q is *sound relative* to Γ iff $\Gamma \vDash \bigvee dQ$. It follows from Definition 5 that Q is sound only if all of its presuppositions are true.

Definition 7 (Openness)**.** Γ leaves Q *fully open* iff it leaves Q affirmatively and negatively open—i.e. iff

(i) $\Gamma \nvDash A$ for all $A \in dQ$ *(affirmative openness)*, and

(ii) $\Gamma \nvDash \neg A$ for all $A \in dQ$ *(negative openness)*.

Definition 8 (Conjunctive Questions)**.** Let $Q_1 \& Q_2$ be the question such that $d(Q_1 \& Q_2) = \{A \wedge B \mid A \in dQ_1, B \in dQ_2\}$. Since disjunction distributes over conjunction, $\bigvee d(Q_1 \& Q_2)$ is equivalent to $\bigvee dQ_1 \wedge \bigvee dQ_2$.

A.3 Evocation, Helpful Resolution, and Resolution

Now to some inferential relations.

Definition 9 (Evocation)**.** Γ *evokes* Q iff Q is sound and (affirmatively) open relative to Γ—i.e. iff

(i) $\Gamma \vDash \bigvee dQ$ *(relative soundness)*, and

(ii) $\Gamma \nvDash A$ for all $A \in dQ$ *(affirmative openness)*.

Definition 10 (Helpful Resolution, a.k.a General Erotetic Implication)**.** Q_2 *helps resolve* Q_1 given Γ iff Q_2 is secured by Q_1 given Γ and every direct answer to Q_2 entails a partial answer to Q_1 given Γ—i.e. iff

(i) $\Gamma \cup \{\bigvee dQ_1\} \vDash \bigvee dQ_2$ *(security)*, and

(ii) for all $B \in dQ_2$ there is some $\Delta \subset dQ_1$ such that $\Gamma \cup \{B\} \vDash \bigvee \Delta$ (*helpfulness*).

Narrower than helpful resolution is the following relation.

Definition 11 (Resolution, a.k.a. Strong Regular Erotetic Implication)**.** Q_2 *resolves* Q_1 given Γ iff Q_2 is secured by and effective for Q_1 given Γ, and Γ leaves Q_1 affirmatively open—i.e. iff

(i) $\Gamma \cup \{\bigvee dQ_1\} \vDash \bigvee dQ_2$ *(security)*,

(ii) for all $B \in dQ_2$ there is some $A \in dQ_1$ such that $\Gamma \cup \{B\} \vDash A$ *(effectiveness)*,

(iii) $\Gamma \nvDash A$ for all $A \in dQ_1$ *(affirmative openness)*.

Definition 12 (Abbreviations)**.** When a relation obtains between a set of propositions Γ, a question Q_2, and a question Q_1, we say that Q_2 Γ-relates to Q_1. So, if Q_2 resolves Q_1 given Γ, we say that Q_2 Γ-*resolves* Q_1. Similarly, when $\Gamma \cup \{A\} \vDash B$ but $\Gamma \cup \{B\} \nvDash A$, we say that B is Γ-*weaker* than A, which we express as $A >_\Gamma B$.

On to some lemmas and theorems involving resolution and evocation.

Lemma 1 (Resolving Questions are Affirmatively Open)**.** *If Q_2 resolves Q_1 given Γ, then Γ leaves Q_2 affirmatively open.*

Proof. Suppose, for *reductio*, that Q_2 Γ-resolves Q_1 but that there is some $B \in dQ_2$ such that $\Gamma \vDash B$. From Definition 11, it follows that $\Gamma \nvDash A$ for all $A \in dQ_1$ and that for all $B \in dQ_2$ there is some $A \in dQ_1$ such that $\Gamma \cup B \vDash A$. So, if $\Gamma \vDash B$ for some $B \in dQ_2$, then, by transitivity (Fact 1), $\Gamma \vDash A$ for some $A \in dQ_1$—a contradiction. □

Theorem 1 (Conjunctive Questions Resolve their Evoked Conjuncts)**.** *If Γ evokes Q_1 and Q_2, then $Q_1 \& Q_2$ resolves Q_1 given Γ.*

Proof. Assume that Γ evokes Q_1 and Q_2. We now show that $Q_1 \& Q_2$ satisfies each of the three conditions in Definition 11 for Q_1. Since Q_1 and Q_2 are sound relative to Γ, we know that $\Gamma \vDash \bigvee Q_1$ and $\Gamma \vDash \bigvee Q_2$. So, $\Gamma \vDash \bigvee dQ_1 \wedge \bigvee dQ_2$. Distributing the conjunction we obtain $\Gamma \vDash \bigvee d(Q_1 \& Q_2)$, and from monotonicity it follows that $\Gamma \cup \{dQ_1\} \vDash \bigvee d(Q_1 \& Q_2)$. Thus, Q_1 Γ-secures $Q_1 \& Q_2$. By classical logic, we know that for any propositions A and B, $\Gamma \cup \{A \wedge B\} \vDash A$. So, $Q_1 \& Q_2$ is Γ-effective for Q_1. Since Γ evokes Q_1, it follows that it leaves Q_1 affirmatively open. Thus, $Q_1 \& Q_2$ resolves Q_1. □

Theorem 2 (Resolution is Transitive)**.** *If Q_2 resolves Q_1 given Γ and Q_3 resolves Q_2 given Γ, then Q_3 resolves Q_1 given Γ.*

Proof. Assume that Q_2 Γ-resolves Q_1 and Q_3 Γ-resolves Q_2. It follows that $\Gamma \cup \bigvee dQ_1 \vDash \bigvee dQ_2$ and $\Gamma \cup \bigvee dQ_2 \vDash \bigvee dQ_3$. By transitivity, $\Gamma \cup \bigvee dQ_1 \vDash \bigvee dQ_3$ and so Q_3 is Γ-secured by Q_1, satisfying condition (i) in Definition 11. Likewise, since Q_3 Γ-resolves Q_2, it follows that Γ leaves Q_3 affirmatively open, satisfying condition (iii) in Definition 11. Lastly, by hypothesis, Q_2 is Γ-effective for Q_1—i.e. for all $B \in dQ_2$ there is some $A \in dQ_1$ such that $\Gamma \cup B \vDash A$—and Q_3 is Γ-effective for Q_2—i.e. for all $C \in dQ_3$ there is some $B \in dQ_2$ such that $\Gamma \cup C \vDash B$. By transitivity, we obtain $\Gamma \cup C \vDash A$ for all $C \in dQ_3$ and some $A \in dQ_1$, and thus, Q_3 is Γ-effective for Q_1. So, Q_3 satisfies all the conditions in Definition 11 and Γ-resolves Q_1. □

A.4 Full Resolution

While resolution is narrower than helpful resolution, the following is narrower still.

Definition 13 (Full Resolution)**.** *Q_2 fully resolves Q_1* given Γ iff Q_2 resolves Q_1 given Γ and Γ leaves Q_2 negatively open—i.e. iff

(i) Q_2 *resolves* Q_1 given Γ, and

(ii) $\Gamma \nvDash \neg B$ for all $B \in dQ_2$ *(negative openness).*

Again we offer some proofs involving the relation at hand.

Lemma 2 (Fully Resolving Questions are Fully Open)**.** *If Q_2 fully resolves Q_1 given Γ, then Γ leaves Q_2 fully open.*

Proof. Assume that Q_2 fully Γ-resolves Q_1. From Definition 13 it follows that $\Gamma \not\models \neg B$ for all $B \in dQ_2$ and from Lemma 1, it follows that $\Gamma \not\models B$ for all $B \in dQ_2$. □

Lemma 3 (Full Resolution is Reflexive given Full Openness). *If Γ leaves Q fully open, then Q fully resolves itself given Γ.*

Proof. Assume that Γ leaves Q fully open. From reflexivity and monotonicity, it follows that every question is Γ-secured by itself—i.e. $\Gamma \cup \{A_1 \vee \ldots \vee A_n\} \models A_1 \vee \ldots \vee A_n$—and is Γ-effective for itself—i.e. $\Gamma \cup \{A_i\} \models A_i$ for all $A_i \in dQ$. So, Q Γ-resolves itself. Thus, Q meets both conditions in Definition 13 and fully Γ-resolves itself. □

Theorem 3 (Full Resolution is Transitive). *If Q_2 fully resolves Q_1 given Γ and Q_3 fully resolves Q_2 given Γ, then Q_3 fully resolves Q_1 given Γ.*

Proof. Suppose that Q_2 fully Γ-resolves Q_1 and that Q_3 fully Γ-resolves Q2. Since full resolution entails resolution, it follows from Theorem 2 that Q_3 resolves Q_1. So, all that remains is to establish that Γ leaves Q_3 negatively open, i.e. $\Gamma \not\models \neg C$ for all $C \in dQ_3$, and this follows from the hypothesis that Q_3 fully Γ-resolves Q_2. □

A.5 Minimal Resolution

Our final relation, *minimal* resolution, is narrower than even *full* resolution.

Definition 14 (Weakening Questions). Q_2 weakens Q_1 given Γ iff there is some subset Δ of dQ_1 such that dQ_2 is the result of replacing each element of Δ with a proposition that's logically weaker given Γ — i.e. iff, for some $\Delta \subseteq dQ_1$,

(i) there is some surjection $f\colon \Delta \to \Sigma$ such that $A >_\Gamma f(A)$ for every $A \in \Delta$, and

(ii) $dQ_2 = (dQ_1 \backslash \Delta) \cup \Sigma$.

Definition 15 (Minimal Resolution). Q_2 *minimally resolves* Q_1 given Γ iff

(i) Q_2 fully Γ-resolves Q_1, and

(ii) No Γ-weakening of Q_2 fully Γ-resolves Q_1.

The following notion of *counter-minimals* will become helpful momentarily.

Definition 16 (Counter-minimals). Q_3 is a Γ-*counter-minimal* to Q_2 for Q_1 iff

(i) Q_2 and Q_3 both fully resolve Q_1 given Γ, and

(ii) for some $B \in dQ_2$ and some $C \in dQ_3$, $B >_\Gamma C$.

Now we show that Definition 15 can be reformulated by substituting for (ii) a certain condition, (ii'), which is equivalent to it given condition (i).

Lemma 4 (Equivalencies). *If Q_2 fully Γ-resolves Q_1, then (ii) is equivalent to (ii'):*

(ii) *No Γ-weakening of Q_2 fully Γ-resolves Q_1,*

(ii') *There is no* Γ*-counter-minimal to* Q_2 *for* Q_1.

Proof. Assume that Q_2 fully Γ-resolves Q_1.

($ii \Rightarrow ii'$). For contraposition, let Q_3 be a Γ-counter-minimal to Q_2 for Q_1. Then there is some $B \in dQ_2$ and some $C \in dQ_3$ such that $B >_\Gamma C$. Now let $dQ_4 = (dQ_2 \backslash \{B\}) \cup \{C\}$. By Definition 14, Q_4 is a Γ-weakening of Q_2. We'll show that it also fully Γ-resolves Q_1. First note that, given how Q_4 is defined, it follows from $\Gamma \cup \{B\} \models C$ that $\Gamma \cup \{\bigvee dQ_2\} \models \bigvee dQ_4$. Thus, since Q_1 Γ-secures Q_2 (i.e. $\Gamma \cup \{\bigvee dQ_1\} \models \bigvee dQ_2$), transitivity ensures that $\Gamma \cup \{\bigvee dQ_1\} \models \bigvee dQ_4$. So, Q_1 Γ-secures Q_4. Next note that, since Q_2 and Q_3 are Γ-effective for Q_1, Q_4 is Γ-effective for Q_1. Finally, since Γ leaves Q_2 and Q_3 fully open, it leaves Q_4 fully open. Thus, Q_4 is a Γ-weakening of Q_2 that fully Γ-resolves Q_1.

($ii' \Rightarrow ii$). Assume, for contraposition, that some Q_3 fully Γ-resolves Q_1 and Γ-weakens Q_2. By Definition 14 then, $B >_\Gamma C$ for some $B \in dQ_2$ and some $C \in dQ_3$. □

Theorem 4 (Minimally resolving conjunctions require universal containment, if the original question is open)**.** *If* Γ *leaves* Q_1 *affirmatively open, then* $Q_1\&Q_2$ *minimally* Γ*-resolves* Q_1 *only if, for every* $A \in dQ_1$ *and* $B \in dQ_2$, $\Gamma \cup \{A\} \models B$.

Proof. Assume, for *reductio*, that Γ leaves Q_1 affirmatively open and that $Q_1\&Q_2$ minimally Γ-resolves Q_1, but that there is some $A \in dQ_1$ and some $B \in dQ_2$ such that $\Gamma \cup \{A\} \not\models B$. So, $\Gamma \cup \{A \wedge B\} \models A$, but $\Gamma \cup \{A\} \not\models A \wedge B$. Since $A \wedge B \in d(Q_1\&Q_2)$, it follows that Q_1 does not fully Γ-resolve itself, for otherwise, Q_1 would be a Γ-counter-minimal to $Q_1\&Q_2$ for Q_1. But every question is Γ-secured by itself and Γ-effective for itself, so it must be the case that Γ fails to leave Q_1 fully open. Thus, there is some $A_i \in dQ_1$ such that either $\Gamma \models A_i$ or $\Gamma \models \neg A_i$. If $\Gamma \models A_i$, then Γ does not leave Q_1 affirmatively open—a contradiction. If $\Gamma \models \neg A_i$, then $\Gamma \models \neg(A_i \wedge B_j)$ for some $B_j \in dQ_2$ and thus Γ does not leave $Q_1\&Q_2$ fully open—a contradiction. □

Theorem 5 (Minimal Resolution is Transitive)**.** *If* Q_2 *minimally resolves* Q_1 *given* Γ *and* Q_3 *minimally resolves* Q_2 *given* Γ*, then* Q_3 *minimally resolves* Q_1 *given* Γ.

Proof. Assume that Q_2 minimally Γ-resolves Q_1 and that Q_3 minimally Γ-resolves Q_2. By Definition 13, Q_2 fully Γ-resolves Q_1 and Q_3 fully Γ-resolves Q_2. By Theorem 3 then, Q_3 fully Γ-resolves Q_1. All that remains is to show that there is no Γ-weakening of Q_3 that fully Γ-resolves Q_1—which, by Lemma 4, is equivalent to showing that there is no Γ-counter-minimal to Q_3 for Q_1. To do so, observe that our hypothesis entails that Γ leaves Q_2 fully open. So, Lemma 3 ensures that Q_2 fully Γ-resolves itself. But since Q_2 cannot be a Γ-counter-minimal to Q_3 for Q_2, it follows from Definition 16 that, for all $C \in dQ_3$ and all $B \in Q_2$, if $\Gamma \cup \{C\} \models B$, then $\Gamma \cup \{B\} \models C$.

Now suppose, for *reductio*, that Q_4 is a Γ-counter-minimal to Q_3 for Q_1. Then there is some $C_i \in dQ_3$ and some $D_k \in dQ_4$ such that $\Gamma \cup \{C_i\} \models D_k$ but $\Gamma \cup \{D_k\} \not\models C_i$. By hypothesis, Q_3 is Γ-effective for Q_2; so, there is some $B_j \in Q_2$ such that $\Gamma \cup \{C_i\} \models B_j$. It follows from our points above that $\Gamma \cup \{B_j\} \models C_i$. Since $\Gamma \cup \{C_i\} \models D_k$, we obtain $\Gamma \cup \{B_j\} \models D_k$ by transitivity. If $\Gamma \cup \{D_k\} \models B_j$, then transitivity yields $\Gamma \cup \{D_k\} \models C_i$—a contradiction. If $\Gamma \cup \{D_k\} \not\models B_j$, then Q_4 is a Γ-counter-minimal to Q_2 for Q_1—a contradiction. □

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