The Logical and Philosophical Foundations for the Possibility of True Contradictions

by

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I, Benjamin Joseph Lewis Martin confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

Abstract

The view that contradictions cannot be true has been part of accepted philosophical theory since at least the time of Aristotle. In this regard, it is almost unique in the history of philosophy. Only in the last forty years has the view been systematically challenged with the advent of dialetheism. Since Graham Priest introduced dialetheism as a solution to certain self-referential paradoxes, the possibility of true contradictions has been a live issue in the philosophy of logic. Yet, despite the arguments advanced by dialetheists, many logicians and philosophers still hold the opinion that contradictions cannot be true.

Rather than advocating the truth of certain contradictions, this thesis offers a different challenge to the classical logician. By showing that it can be philosophically coherent to propose that true contradictions are metaphysically possible, the thesis suggests that the classical logician must do more than she currently has to justify her confidence in the impossibility of true contradictions. Simply fighting off the dialetheist's putative examples of true contradictions at the actual world isn't enough to justify the classical logician's conclusion that true contradictions are impossible.

To aid the thesis dialectically, we introduce a new position, absolutism, which hypothesises that it's metaphysically *possible* for *at least* one contradiction to be true, contrasting with the dialetheic hypothesis that *some* contradictions are true *in the actual world*. We demonstrate that absolutism can be given a philosophically coherent interpretation, an appropriate logic, and that certain criticisms are completely toothless against absolutism. The challenge put to the classical logician is then: On what logical or philosophical grounds can we rule out the metaphysical possibility of true contradictions?

'What you say is absurd,' I expostulated. 'You proclaim that non-existence is the only reality. You pretend that this black hole which you worship exists. You are trying to persuade me that the non-existent exists. But this is a contradiction: and, however hot the flames of Hell may become, I will never so degrade my logical being as to accept a contradiction.'

From Russell's (2010a) retelling of
Andrei Bumblowski's dream

Preamble

This thesis began life as an attempted survey of the criticisms of dialetheism. However, that project was swiftly abandoned when I recognised that any such survey would better suit a life long project on the scale of Bayle's *Dictionnaire Historique et Critique*, rather than a PhD thesis. The revised project, realised to fruition here, was born out of a conviction that the classical logician has failed to appropriately answer the challenges that dialetheism has raised. While there have been many attempts to demonstrate that the putative true contradictions that the dialetheist advances are not, in fact, true, little has been done to establish on logical or philosophical grounds that it's *impossible* for contradictions to be true. Once we recognise dialetheism's potential as a challenge akin to scepticism, we come to appreciate how far away classical logic actually is from adequately answering such a challenge. My hope, reiterated on several occasions in these pages, is that this thesis encourages classical logicians to seek out arguments for their principle that contradictions cannot be true, by demonstrating the seriousness and philosophical coherence of this challenge.

A few words on terminology are necessary. We will use the convention of abbreviating 'if and only if' as 'iff' in definitions, and in general use propositions, conceived as the meanings of declarative sentences, as our truthbearers. This latter choice, however, doesn't represent any stance on which type of truthbearer is philosophically fundamental, if indeed any is. Many of our conclusions should hold regardless of the truthbearer we choose to use. When distinctions between the types of truthbearer are pertinent to the discussion at hand, we will mention it. We will use standard propositional logic notation: italicised lower-case Roman letters (p, q, r…) will represent propositional parameters, italicised upper-case Roman letters (A, B, C…) will represent metavariables, and upper-case Greek letters (Σ, Γ, Π…) will represent sets of formulae. The symbols $\sim$, $\wedge$, $\vee$, $\rightarrow$, and $\leftrightarrow$, represent negation, conjunction, disjunction, a detachable conditional, and a detachable bi-conditional, respectively. For first-order formulae, we will use x, y and z, for variables, a, b and c, for constants, and italicised upper-case Roman letters (Pn, Qn, Rn…) for *n*-placed predicates. Our first-order notation includes the logical constants of propositional logic plus the universal quantifier ($\forall$), the existential quantifier ($\exists$), and identity ($=$), and we will use $\Box$ and $\Diamond$ for our necessity and possibility operators, respectively. If any quoted author's notation differs from these conventions, it is suitably brought into line. Standard set-theoretic notation is also used, and the metasymbols $\vdash$ and $\vDash$ express proof-theoretic consequence (derivability) and semantic consequence (validity), respectively. Any other use of notation is clearly introduced.

I owe thanks to many people for the completion of this thesis. José Zalabardo and Marcus Giaqunto have been nothing other than supportive and attentive in their roles as supervisors, and this thesis would have contained far more faults if it had not been for their keen eyes and insistence on particular points. Any errors still contained in these pages are, of course, my sole responsibility.

I have also benefited from the comments of many colleagues, including those at Braga, Edinburgh, Leeds, London, Madrid, Oxford and Warwick, where I've been given the kind opportunity to present material contained in this thesis. Particularly, I would like to thank my colleagues and friends at University College London, with whom I have shared numerous helpful and insightful conversations.

Finally, I must thank my family, for their constant support and love. Without them, none of this, and much more, would have been possible.

This thesis is dedicated to the memory of my grandfather, Albert Martin.

Contents

1. Introduction

> [T]he most indisputable of all beliefs is that contradictory statements are not at the same time true.
>
> (Aristotle (1971a) Γ 1011[b] 12-13)

The view that contradictions cannot be true has been part of accepted philosophical theory since at least the time of Aristotle. In this regard it's almost unique in the history of philosophy. Only in the last forty years has the view been systematically challenged, with the introduction of dialetheism as a solution to certain self-referential paradoxes. With dialetheism, the possibility of true contradictions is a live issue in the philosophy of logic. However, despite the arguments advanced by dialetheists, most logicians and philosophers are still of the opinion that contradictions cannot be true.

Rather than advocating the truth of certain contradictions, this thesis offers a different challenge to the classical logician. By showing that it can be philosophically coherent to propose that true contradictions are metaphysically possible, the thesis suggests that the classical logician must do more than she currently has to justify her confidence in the impossibility of true contradictions. Simply fighting off the dialetheist's putative examples of true contradictions in the actual world isn't enough to justify the classical logician's conclusion that true contradictions are impossible. While demonstrating that a competing position is coherent does not in itself challenge the truth of classical logic, it does put a burden on the classical logician to justify her confidence in the impossibility of true contradictions on grounds other than the simple incoherence of true contradictions.

To aid the thesis dialectically we will introduce a new position, absolutism, which hypothesises that it's metaphysically *possible* for *at least* one contradiction to be true, contrasting with the dialetheic hypothesis that *some* contradictions are true *in the actual world*. By showing that absolutism can be given a philosophically coherent interpretation, with an appropriate logic, the challenge put to the classical logician is then: On what logical or philosophical grounds can we rule out the metaphysical possibility of true contradictions?

Our decision to concentrate on a position that differs from what most classical logicians will consider as the real challenge, dialetheism, is no slight of hand. It is a decision grounded in at least six considerations:

1. By using a philosophical theory that hypothesises the possibility of true contradictions, rather than putting a certain set of true contradictions forward, we are assured to concentrate on the philosophical and logical coherence of true contradictions. There will be no temptation to go back to

the proposed cases and simply say, 'But none of them work anyway!' Given that in this thesis we are challenging the classical logicians' reasons for their rejection of the possibility of true contradictions, it's vital that we concentrate on these philosophical and logical issues rather than on the dialetheist's proposed case.

2. The classical logician not only states that there are no true contradictions in the actual world, but that it's *impossible* for there to be true contradictions. It isn't only in zero and first-order classical logics that contradictions can't be assigned the truth-value true. The formula $\sim\Diamond(p \wedge \sim p)$ is a theorem of every classical modal logic. Even if the classical logician succeeds in disposing of all of the dialetheist's putative examples of true contradictions, this won't necessarily demonstrate that it's impossible for there to be true contradictions. Yet, all classical logicians are committed to this latter thesis. Thus, by using absolutism as the target position in this thesis, we can hope to refocus the classical logician's goals. We are looking for reasons not only to think that there happen not to be any true contradictions, but to think that there *cannot* be any true contradictions due to philosophical or logical considerations.

3. Absolutism comes in different strengths, ranging from the weaker position that *at least one* contradiction is true at a possible world w, to the stronger position that for every set of contradictions Σ, there is a possible world w at which $\forall C \in \Sigma$, C is true. This gives the position flexibility, which is a virtue when it's being used as a challenge to classical logic. Separating these different positions will give the field some clarity and allow us to appreciate that a knockdown argument for one position isn't necessarily a knockdown for another.

4. Dialetheists mainly concentrate on those true contradictions that are putatively produced by self-referential paradoxes, as these are considered to be the most reasonable examples of true contradictions.[1] However, by

[1] There are other putatively true contradictions however. Priest (2006b), for example, has suggested that one can derive true contradictions from the concept of change and that obligations can produce true contradictions. Additionally, if it turns out that there is a paraconsistent solution to the problem of vagueness, then this may be convincing evidence that instances of vagueness entail true contradictions. For more on proposed paraconsistent solutions to the problem of vagueness see Hyde (1997), Akiba (1999), Hyde (1999), Odrowaz-Sypneiwska (1999), Beall & Colyvan (2001), Hyde

concentrating on contradictions *qua* contradictions, the important question of whether contradictions cannot be true *tout court* or whether certain contradictions cannot be true because of their particular content becomes clearer. For this reason, it's better to use a position that is indiscriminate about contradictions, facilitating discussion about the whole variety of contradictions.

5. By not focusing on dialetheism, we may come to appreciate that there are other plausible justifications for the thesis that there are possibly true contradictions. By concentrating on the concepts and issues surrounding the possible truth of contradictions, and temporarily ignoring the dialetheist's examples, we may find even more plausible reasons to think that contradictions can be true.

6. Sometimes it's necessary to start afresh. Dialetheism has taken on certain theoretical commitments that are unnecessary baggage for our project. While we have the choice to use dialetheism without this excess baggage, this would likely lead to wastage of time on exegesis, sorting out what needs to stay and what needs to go. Additionally, by stripping dialetheism of what we consider in this project to be excess baggage, we may be in the dialetheist's eyes stripping the theory of what makes it dialetheic. Yet, a disagreement over whether our position here is dialetheic or not isn't one we want to be concentrating on. While we are building on some of the foundations laid by dialetheism, only those elements that are relevant to our purpose, and don't unnecessarily hinder our project, will be used.

Proposing absolutism as a challenge to classical logic emphasises the thesis's similarities to the global sceptic's challenge to orthodox philosophy. That challenge, most forcibly put by the Pyrrhonian sceptics with Agrippa's tropes, had an almost unrivalled influence on the history of philosophy, from being one of the primary motivations behind Descartes's (1996) *Meditations* and the inspiration for much of Hume's own sceptical tendencies, to at least partially motivating some modern externalist theories of knowledge.[2] Even if, as many believe, the Pyrrhonian sceptic's challenge was unsuccessful, whether because the Pyrrhonist's position was self-defeating or because we can have knowledge without having justified beliefs of the type

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(2001), Hyde & Colyvan (2008), Hyde (2010), Priest (2010), Cobreros (2011), Weber (2011), and Ripley (2013).

[2] See Popkin (1951 & 2003, Ch. 9) and Zalabardo (2012).

Pyrrhonism rules out, there's no doubt that philosophy learnt a great deal from the challenge. Our theory of knowledge is much the better for the challenge that Pyrrhonian scepticism instigated, creating fertile ground from which new philosophical concepts and arguments flourished. It's a motivating assumption of this thesis that the challenge raised by the absolutist can motivate responses as philosophically insightful as those to the Pyrrhonian challenge.

The hope for this thesis is fourfold: 1) That it makes clearer the challenge set by the absolutist in clarifying the notion of a true contradiction; 2) That it tackles to some extent the question of whether the absolutist's position can be stated in a philosophically and logically meaningful and coherent way (just as any commentator of Pyrrhonian scepticism must); 3) That it removes from the field of debate some arguments against the possibility of true contradictions, clearing the way for better ones; and, 4) That through careful consideration of the concepts and issues relevant to the (im)possibility of true contradictions, we can point the way towards future research that may produce evidence for either the possibility or impossibility of true contradictions.

In all, the goal of this thesis is to do some necessary groundwork for the debate over the (im)possibility of true contradictions. Even if we find in this thesis good reason to believe that it's impossible for there to be true contradictions, it's hoped that we have made the absolutist's position clearer and that certain philosophical concepts are also clearer because of our discussions.

The goals of this thesis shouldn't, however, distract completely from its relevance to dialetheism. If we succeed in clarifying some of the concepts and issues surrounding absolutism then the same will often be true of dialetheism. A main criticism of the dialetheic solution to paradoxes is that the solution comes at too high a price, even if it's partly successful, by violating the law of non-contradiction. If we succeed to some extent in undermining the philosophical incoherence of violations of the law of non-contradiction then this will be to both the absolutist's and dialetheist's benefit.[3]

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[3] It's important to alert the reader here of well-known criticisms of the possibility of true contradictions that we won't consider in this thesis, because they have been sufficiently dealt with elsewhere. This should dissolve any concerns that we've been neglectful in our consideration of the available arguments against true contradictions.

Aristotle's (1971a, Γ 1005^b-1008^b) arguments against the possibility of true contradictions are thoroughly critiqued in Dancy (1975), Łukasiewicz (1971), and Priest (1998); the argument that the concept of *truth* precludes true contradictions is dealt with in Priest (2000) and Armour-Garb & Beall (2001); and, the argument that contradictions cannot be true (or believed) because they are meaningless, is suitably answered in Cooper (1966), Williams (1982), and Priest (2004).

Additionally, although not directly arguments against the possibility of true contradictions, transcendental arguments that the possibility of true contradictions would preclude the possibility of rational debate, or individuals' ability to effectively express their disagreement of a position, is considered in depth in Priest (2004 & 2006a, ch. 6-8).

To reiterate in this thesis the responses to these well-worn criticisms would unnecessarily expend space that can be better used for other important topics.

2. Dialetheism and Absolutism

No one draws conclusions from the Liar.
(Wittgenstein (1967) p. 170)

In this chapter we introduce absolutism, giving precision to our prospective challenge to classical logic, and contrast it with the well-established position of dialetheism.

2.1 Absolutism

Absolutism is the thesis that it's metaphysically possible for at least some contradictions to be true. Formalised, let W be the set of possible worlds, w_a be the actual world, and R be an accessibility relation:

Ab) $\exists\Sigma\ (\Sigma \subseteq \{x \mid x \text{ is a contradiction}\} \wedge (\exists w \in W, w_a R w, \forall B \in \Sigma, v(B_w) = 1))$.

Two features of Ab) require clarification. Firstly, the absolutist requires the accessibility relation R in Ab) to adequately model *metaphysical* possibility, given that she proposes that it's *metaphysically* possible for at least some contradictions to be true. However, which properties exactly R must possess to so model *metaphysical* possibility isn't a question that we will broach in this thesis. It is a substantive question in its own right, and its correct answer shouldn't influence the (im)plausibility of absolutism. Therefore, we can safely leave the precise identity of R alone and assume that there is an accessibility relation which adequately models *metaphysical* modality. The most we will require of R in this thesis is that i) R is reflexive and ii) some non-actual possible worlds are accessible from the actual world.

Secondly, we haven't specified in Ab) what a contradiction is. This isn't because the concept of *contradiction* is unanalysable, for it certainly is.[4] Instead, it's because there are far too many non-equivalent accounts of contradiction available in the literature to simply plump for one in Ab). Establishing that there's a philosophically plausible account of contradiction which can be embedded within Ab), without consigning the thesis to nonsense, is one of the

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[4] Heyting (1966, p. 98) disagrees with us on this point:

> [C]ontradiction must be taken as a primitive notion. It seems very difficult to reduce it to simpler notions, and it is always easy to recognise a contradiction as such. In practically all cases it can be brought into the form 1=2.

Our discussion of multiple non-equivalent accounts of contradiction in the next chapter should be enough to show that 'contradiction' *can* be analyzed. This will also save us from accepting Heyting's historically baffling view that it's always easy to recognise a contradiction.

main challenges facing absolutism. For this reason, consideration of the available accounts of contradiction is the purpose of the next chapter, and we shall have to wait until then to fully state the absolutist's thesis.[5] To be clearer on the content of absolutism it might be helpful to compare the position with dialetheism, an already well-established philosophical thesis. With this aim, let us outline dialetheism.

2.2 Dialetheism

According to dialetheism's most avid and eloquent proponent Graham Priest,

> [D]ialetheism is the view that *some* contradictions are true: there are sentences (statements, propositions, or whatever one takes truth-bearers to be), p, such that both p and $\sim p$ are true, that is, such that p is both true and false.
>
> (Priest (2006a) p. 1. Emphasis mine)

As the quote implies, but quickly passes over, there are two possible defining features of dialetheism:

D1) Some contradictions are true.

D2) Some propositions are both true *and* false.

There is some lack of clarity in the dialetheic literature as to whether both of D1) and D2) are equally defining features of dialetheism, or whether one is theoretically primary for the dialetheist.[6] Priest (1989, p. 141) himself has emphasised that dialetheism rejects "[t]he fundamental classical postulate that truth and falsehood are mutually exclusive," although when defining dialetheism he often gives first billing to the truth of contradictions.[7] While it's unimportant for our purposes here to determine whether, and if so which, one of the theses is theoretically primary for the dialetheist, as the absolutist is unquestionably primarily concerned with the possible truth of contradictions, understanding the relation between D1) and D2) will bring into focus a restriction on any successful absolutist logic.

Given that the only way to block the equivalence between D1) and D2) is to give *non*-normal semantics for conjunction or negation,[8] the dialetheist must choose either to endorse

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[5] We also haven't specified here the theory of truth that the absolutist endorses. However, given that Priest (2000) has shown that advocating the (possible) truth of contradictions is compatible with multiple theories of truth, we can treat the question of which theory of truth the absolutist endorses as independent of her endorsement of the possibility of true contradictions.

[6] For a survey of the definitions of dialetheism in the literature see Appendix A.

[7] See Priest (2004, p. 29). Priest (2006b, p. 4), when explaining the etymology of 'dialetheia', gives a conditional with a true contradiction, $p \wedge \sim p$, as the antecedent, and the simultaneous truth *and* falsity of p as the consequent, which could suggest that the truth of contradictions is primary for Priest. Similarly, if we conceive of a contradiction as the conjunction of a proposition and its negation, then Priest's (2004, p. 33 & 2006a, p. 1) use of 'false' to mean simply 'has a true negation' implies that the truth of contradictions is primary for Priest.

[8] The normal semantics being:

both D1) and D2), if she wishes to endorse either, or to use *non*-normal semantics for conjunction or negation. So far, dialetheists have chosen the former option and rejected the latter. For example, Priest has endorsed the *Logic of Paradox* (**LP**) which only permits true contradictions, $A \wedge \sim A$, when A is both true and false,[9] while criticising the negation operator in da Costa's **C-Systems** which permit true contradictions, $A \wedge \sim A$, without A being assigned both truth-values.[10] Given that the absolutist also allows for true contradictions, she too must decide whether to allow for propositions to be both true and false, or to accept *non*-normal semantics for conjunction or negation. We will consider the logic the absolutist requires in chapters 4 and 5.

The initial motivation for the dialetheist's position derived from self-referential semantic and set-theoretic paradoxes. According to dialetheists, these paradoxes have so far evaded successful non-dialetheic solutions, not because of a lack of effort or rigour on the logician's part, but due to an inherent flaw that all non-dialetheic solutions share.[11] Take as an example the most famous of the self-referential paradoxes, the Liar,

(λ) λ is false.

According to Tarski (1944, pp. 348-349), the Liar paradox is a consequence of four conditions holding in a language L:

T1) That any proposition p in L can be named by a term t belonging to L.

T2) That L has the resources to express its own semantics (e.g. in English we can express 'Proposition p is true').

T3) The T-schema ('p is true' iff p) is universally applicable to the truth-predicate in L.

T4) The "ordinary", i.e. classical, laws of logic hold in L.[12]

Conjunction: $v(A \wedge B) = \min\{v(A), v(B)\}$
Negation: $v(\sim A) = 1 - v(A)$.

[9] See section 4.4.1 below. Given the normal semantics for conjunction and negation in **LP**, this also ensures that contradictions are false under every truth-value assignment, and consequently that propositions of the form $\sim(A \wedge \sim A)$ are logical truths in **LP**. Beall (2009) also favours a logic **LP*** with the normal semantics for conjunction and negation.
[10] See section 4.4.2 below. Priest & Routley (1989, pp. 165-166) criticise the **C-Systems**' negation for being a subcontrary-forming, rather than a contradictory-forming, operator.
[11] Priest (2006b), Ch. 1-2.
[12] The inclusion of T4) has caused some confusion in the interpretation of Tarski's position regarding the Liar. For example, Hugley and Sayward (1980) incorrectly interpret Tarksi as attempting to demonstrate that natural languages include true contradictions, and criticise his (putative) argument for failing to recognise that the inclusion of T4) ensures that a true contradiction cannot be validly deduced. Indeed, the inclusion of T4) demonstrates that the Liar is a *reductio* of the joint truth of T1-T3). For the more plausible interpretation that Tarski used the Liar to demonstrate that T1-T3) were inconsistent with classical logic see Howson (1982) and Ray (2003). Given this interpretation, the

Conditions T1-T2) are known as the *semantic closure* of a language, and it was Tarski's view that natural languages are inherently semantically closed:[13]

> A characteristic feature of colloquial language (in contrast to various scientific languages) is its universality. It would not be in harmony with the spirit of this language if in some other language a word occurred which could not be translated into it… If we are to maintain this universality of everyday language in connexion with semantic investigations, we must, to be consistent, admit into the language, in addition to its sentences and other expressions, also the names of these sentences and expressions, and sentences containing these names, as well as such semantic expressions as 'true sentence', 'name', 'denote', etc. But it is presumably just this universality of everyday language which is the primary source of all semantic antinomies.
>
> (Tarski (1983b) pp. 164-165)

Thus, on this diagnosis of the Liar paradox, a natural language *L* will contain true contradictions as long as i) *L* is semantically closed and ii) the T-schema is valid for all applications of the truth predicate in *L*. Consequently, if one wished to resolve the Liar paradox without rejecting T4), one must somehow restrict the semantic closure of all natural languages or the applicability of the T-schema in those languages.

According to the dialetheist, all attempts to reject either semantic closure or T3) for a natural language *L* by restricting *L*'s expressive power either involve *ad hoc* manoeuvres or are susceptible to revenge paradoxes, or both.[14] In other words, non-dialetheic solutions to the Liar are essentially flawed. The only available solution to the paradox, therefore, is to reject T4), and particularly the classical tenets that no propositions can be both true and false, and that no contradiction can be true. Thus, the dialetheist resolves the paradox by accepting its conclusion, which consequently requires her to admit that the conclusion is a true contradiction, facilitated by a rejection of some of the tenets of classical logic. As such, dialetheism offers a novel type of solution to paradoxes; admit their soundness.[15]

2.3 Comparing Absolutism and Dialetheism

Now that we are clearer on what absolutism and dialetheism postulate, we can highlight some of the (dis)similarities between the theses.

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important question for Tarski is then which of T1-T3) should go, given that the validity of classical logic is indisputable; see Tarski (1944, p. 349). Ray (2003) interprets Tarski as rejecting T3), the applicability of the T-schema to the Liar.

[13] Cf. Priest (1984 & 2006b, Ch. 1).

[14] Priest (2006b), Ch. 1.

[15] For more on how dialetheic solutions to paradoxes require a reevaluation of the standard definition of paradoxes see Armour-Garb (2004).

2.3.1 Actuality/Possibility of True Contradictions

Dialetheists maintain that there are true contradictions at the *actual* world. Absolutists, in contrast, hypothesise that there are true contradictions at a *possible* world. While it's perfectly consistent with the absolutist's position for her to believe that there are some true contradictions at the actual world, her absolutism shouldn't commit her to this thesis. If we discover that the truth of contradictions at a possible world entails the truth of a contradiction at the actual world, then absolutism will fail to be a philosophical position independent from dialetheism.[16]

2.3.2 Ontological/Semantic Dialetheism

Mares (2004) has proposed two forms of dialetheism, *ontological* and *semantic* dialetheism, which differ in how they account for true contradictions. While *ontological* dialetheism hypothesises that there are true contradictions because the world contains non-propositional contradictory elements, so that the very nature of the objects in the world cause there to be true contradictions, *semantic* dialetheism claims that it's *merely* semantic facts that cause true contradictions.[17] Some *semantic* dialetheists propose differing linguistic causes for these true contradictions. Mares (2004, pp. 266-270) suggests that just as vagueness is commonly considered to be a property of language and not of the world,[18] caused by *under*determined predicates, so true contradictions are a property of language caused by *over*determined predicates.[19] Beall (2009, p. 6), in comparison, proposes that true contradictions are an unforeseen consequence of our use of semantic concepts such as 'truth', 'satisfaction', and 'exemplifies'.[20]

In contrast to some dialetheists, the absolutist doesn't commit herself on the cause of the true contradictions at possible worlds. She is sure that if there are true contradictions then this requires certain propositions to be true, but she suspends judgement as to whether those propositions are true because they accurately reflect the world or because of certain idiosyncrasies in a language. Postulating a cause for those contradictions that are true at a possible world isn't an essential element of absolutism.

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[16] A problem we take up in chapter 5.

[17] Beall (2004, pp. 197 & 213), Kroon (2004, p. 262), and Mares (2004) all endorse *semantic* dialetheism. Mares (2004, p. 265) attributes ontological dialetheism to Priest, however Priest (2006b, pp. 299-302) himself is unsure whether he endorses ontological or semantic dialetheism.

[18] Contrary to those who propose *ontological* vagueness: see Tye (1990), Zemach (1991), Merricks (2001) and Morreau (2002). The *ontological* vagueness research programme is criticised in Russell (1923), Evans (1978), Pelletier (1989), Sainsbury (1995), Heller (1996), Sorenson (1998), Eklund (2008), and Moore (2008).

[19] A predicate P is *over*determined at a world w iff P's extension and anti-extension overlap at w.

[20] Beall (2009, p. 6) calls his position "deflated dialetheism".

2.3.3 All/Some Contradictions

Dialetheists only believe *some* contradictions to be true, and claim to have good reasons for holding these particular contradictions to be true.[21] Absolutists, in contrast, are not restricted in the number of true contradictions they can admit at a possible world. How many true contradictions they are willing to admit is dependent simply on the cardinality of the set Σ in Ab). In the extreme case that $|\Sigma| = |C|$, with $C = \{x \mid x \text{ is a contradiction}\}$, absolutism merges into *modal* trivialism, the thesis that,

MT) For some possible world *w*, every proposition *p* is true at *w*.

on the assumption that every proposition *p* can partially constitute a contradiction.[22] Thus, absolutism can be more extreme in the number of true contradictions it postulates than dialetheism.[23]

2.3.4 Paraconsistency

Neither the dialetheist nor absolutist, save when she hypothesises that $|\Sigma| = |C|$, wish for their theses to entail a form of trivialism. This requires both the dialetheist and the absolutist to use a logic that invalidates explosion, $\{A, \sim A\} \vDash B$. In other words, both parties require a paraconsistent logic.[24]

In addition, as both the dialetheist and absolutist want to allow for at least some contradictions, $A \wedge \sim A$, to be true, they must use a logic that either allows for propositional parameters to be assigned both truth-values, or gives *non*-normal semantics for conjunction or

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[21] Priest (1995) has argued that those contradictions entailed by self-reference which we have good reason to believe are true fulfill certain conditions, which he formulates as the *Inclosure Schema* (pp. 147-149). This schema may give the dialetheist a principled distinction between those contradictions we have good reason to believe are true, and those that we don't, although it fails to capture putatively true contradictions not derived from self-reference sentences, such as those concerning change.

[22] If we conceive of a contradiction as the conjunction of a proposition and its negation, then this reduces to the assumption that every proposition has a negation.

[23] There are other interesting forms of absolutist that are even more extreme in their requirements. For example, letting $C = \{x \mid x \text{ is a contradiction}\}$ and c be members of C:

$$\text{Ab}_{\text{EX}})\ \forall\Sigma\,((\Sigma \in \wp(C) \wedge \Sigma \neq \varnothing) \rightarrow ((\exists w \in W, w_aRw, \forall c \in \Sigma, v(c_w) = 1) \wedge (\forall c'\,(v(c'_w) = 1 \rightarrow c' \in \Sigma)))).$$

So, according to Ab_{EX}), for every non-empty set of contradictions Σ, there is a possible world *w* such that every member of Σ is true at *w*, but no other contradictions are true at *w*. We won't be spending time on these extreme versions of absolutism here.

[24] For more on paraconsistent logics see chapter 4. Without a paraconsistent logic dialetheism would entail *trivialism*, the thesis that every proposition *p* is true at the actual world, and absolutism would entail *modal trivialism*, the thesis that for some possible world *w*, every proposition *p* is true at *w*. Arguments for trivialism can be found in Azzouni (2003, 2007 & 2013) and Kabay (2008).

negation. As we have already noted, the norm in dialetheic circles is to accept the former option, possibly because it allows the derivation of true contradictions from Liar-like sentences. While we won't exclude the possibility of the absolutist constructing a suitable logic for her position using *non*-normal semantics for conjunction or negation, we will be assuming here that she is better off endorsing a logic with the truth-functors' normal semantics. In constructing a challenge to classical logic, the absolutist must be wary of fighting too many battles at once, and for this reason she will be better served attempting to build a logic with the normal semantics for conjunction and negation. Thus, in constructing a logic for the absolutist, we will place upon ourselves the restriction that she must allow for propositional parameters to be assigned both truth-values.

2.4 The Way Forward

We have introduced absolutism and highlighted some of its (dis)similarities to dialetheism. As it stands though, absolutism is still an incomplete philosophical position. We have yet to establish that there are available accounts of contradiction which are both philosophically adequate and compatible with the absolutist's thesis. If we can find no such account of contradiction then the absolutist's hypothesis will falter at the earliest possible stage, and the classical logician's view that contradictions cannot be true will be vindicated. For this reason, in the next chapter we consider the available accounts of contradiction for their suitability.

3. Accounts of Contradiction

> The talk of making more exact a vague or not quite exact concept used in everyday life or in an earlier stage of scientific or logical development, or rather of replacing it by a newly constructed, more exact concept, belongs among the most important tasks of logical analysis and logical construction.
>
> (Carnap (1958) p. 8)

The absolutist speaks about the truth of *contradictions*. So far we have left that concept unanalyzed, but now we must decide what the absolutist means by contradictions if her thesis is to suitably challenge classical logic. Given that there numerous non-equivalent definitions of contradictions in the philosophical literature, our task isn't the simple one of replacing the term 'contradiction' in Ab) with an agreed definition. We must choose from among the candidate definitions. Of course, not just *any* definition of contradiction can be embedded into Ab). If absolutism is to be a plausible challenge to classical logic then the position needs to use a definition of contradiction that respects the theoretical and inferential roles that contradictions play within philosophy and logic. In this regard, her situation is no different to her opponent's. Both parties must ensure that their definition of contradiction respects these roles and can explain the majority of the important references we make to contradictions in philosophical practice. If both sides are to engage in a debate over the possible truth of contradictions it's vitally important that their definitions of 'contradiction' respect what is at stake philosophically. If either party misrepresents the concept of *contradiction* in their arguments, then what promised to be an important philosophical debate could easily become completely philosophically immaterial.

This chapter is dedicated to evaluating the available non-equivalent accounts of contradiction, for both their philosophical *plausibility*, i.e. whether the account can explain philosophical uses of the concept of contradiction, and their *suitability* to be embedded within Ab). The accounts, therefore, have two criteria to meet in order to be considered adequate for use in the debate between the absolutist and the classical logician: *plausibility* and *suitability*. It's essential that the absolutist succeeds in finding a definition of contradiction which can suitability be embedded into Ab). Without such a definition, her thesis cannot hope to be taken seriously. If one advances the thesis that it's possible for contradictions to be true, there better be an available definition of 'contradiction' such that the truth of the thesis isn't instantly precluded by the definition. Consequently, to ensure that absolutism is a genuine challenge to classical logic, we must find a definition of contradiction that is both philosophically plausible and suitable to embed within Ab). In surveying the available accounts of contradiction we will keep, for the sake of continuity, the four broad categories for the accounts that Grim (2004) used in a previous survey: semantic, syntactic, pragmatic and ontological. On their own, however, these categories are too broad, ensuring that some accounts of contradiction were unrepresented in Grim's survey. With this concern in mind, we have included additional sub-categories in our survey.

3.1 Semantic Accounts

Semantic accounts of contradiction define contradictions, unsurprisingly, in terms of semantic properties. Examples of these properties are certain truth-conditional properties that contradictions uniquely possess, certain semantic entities (or combinations of) that only contradictions are constituted of, and sets of propositions that only contradictions entail. There are three main distinct semantic accounts of contradiction: *Classically-assumed*, *explosion*, and *truth-value neutral* accounts.

3.1.1 Classically-Assumed Accounts

Classically-assumed accounts of contradiction define contradictions in terms of certain truth conditions. Particularly, they define contradictions in terms of the truth-conditional property of always being false.[25] Some varieties of the classically-assumed account are slightly more subtle, defining contradictions not just as those propositions that are always false, but as a set of propositions Σ such that the conjunction of all $B \in \Sigma$ is always false, but, necessarily, one and only one member of Σ is true. Here are some versions of the account found in the literature:

Bonevac (1987, p. 25): "A sentence is *contradictory* if and only if it's impossible for it to be true."

Sainsbury (1991, p. 369): "Two propositions are contradictories if and only if it is logically impossible for both to be true and logically impossible for both to be false."

Detlefsen *et al.* (1999, pp. 27-28): "A proposition is said to be a contradiction when it is logically impossible that it be true or, equivalently, when it is logically necessary that it be false."

[25] Given that in classical semantics 'false' and 'not true' are equivalent, we will treat 'always false' and 'never true' to be equivalent when discussing versions of the classically-assumed account within a classical framework. Although the equivalence doesn't hold for some glutty and gappy logicians, for the most part this will be unimportant for our discussion. When it is important, we will make the relevant distinctions.

Prior (1967, p. 458): "Contradictory negation, or contradiction, is the relation between statements that are exact opposites, in the sense that they can be neither true together nor false together."

Guttenplan (1997, p. 315): "A sentence which has F [false] in every row is a *contradiction*."

Copi (1979, p. 28): "When it is logically impossible for a particular statement to be true, that statement itself is said to be self-contradictory, or a self-contradiction. Such statements are also said more simply to be contradictory or contradictions."

Within this small sample there are interesting and important differences between the definitions. It's worth us here highlighting two variations of particular importance. Firstly, while Bonevac, Detlefsen *et al.*, Guttenplan and Copi only require that contradictions are never true (or, always false), Sainsbury and Prior also require that a contradiction is a molecular proposition, entailed by the constraint that *contradictories* can never be false together. Without envisaging a contradiction as a molecular proposition, there would be no constituent propositions to be contradictories.

Secondly, the authors vary on their interpretations of 'never true'. While Bonevac talks about the impossibility of being true, giving us no extra information with which to interpret his use of 'impossibility', all of Sainsbury, Detlefsen *et al.* and Copi talk in terms of *logical* impossibility. In contrast, Guttenplan uses rows of a truth table, rather than the term 'logical', to explain that contradictions are never true, and Prior uses none of these modal or truth-functional devices, sticking to the unanalyzed 'they can be neither true together nor false together'.

These two variations are not only of sociological interest but affect the degree of philosophical plausibility the different versions of the account possess. While we will argue that all of the versions of the account are philosophically inadequate, the extent of their philosophical inadequacy is a function of both the variables mentioned above.

Those accounts, such as Bonevac's, that define contradictions as those propositions that are necessarily false, without specifying that their being necessarily false is in any way a *logical* constraint, are the worse off.[26] For these accounts are committed to admitting that those propositions that are necessarily false, *not* because of logical constraints, but because of certain metaphysical constraints, are contradictions. For example, those propositions that are necessarily false because they assign a property to a particular that it possesses at no possible world ('Aristotle is a platypus'), or propositions that are the negation of an identity relation

[26] For another instance of this version of the classically-assumed account, see Cavender & Kahane (2009, pp. 65 & 386).

between two co-referring rigid designators ('Hesperus is not identical to Phosphorus'). Given that these propositions are necessarily false, Bonevac, and those who advocate the same account, are committed to asserting that these propositions are contradictions. That they are not cases of what a philosopher would consider to be a contradiction, however, is pretty clear. It was an element of Kripke's (1981) argument for the impossibility of knowing *a priori* certain necessary truths that the truths' negations were *not* reducible to a contradiction. Even those who oppose Kripke's conclusion that there are certain necessary truths that can only be known *a posteriori* agree with his contention that 'Water is not H_2O' isn't a contradiction. As long as we allow for *broad* impossibility to outstretch *logical* impossibility (and, conversely, *logical* possibility to outstretch *broad* possibility), Bonevac's account will suffer from categorising as contradictions propositions that most philosophers wouldn't recognise as such.[27]

There are of course counterarguments available to Bonevac. The costs of these counters, however, seem to drastically outweigh their benefits. One could propose that we extend the set of propositions that 'contradiction' refers to so that it includes these (merely) metaphysical impossibilities. Revising our application of a concept is always an option when we are grappling with a definition; however, it is only plausible when the definition has explanatory power and we have thoroughly considered competing definitions. Given that, as we will suggest later, this account lacks explanatory power in comparison to at least some of its competitors, this isn't a viable option. Alternatively, Bonevac could propose that there are *no* propositions expressing metaphysical impossibility that don't also express logical impossibility. From this it could be argued that all logical impossibilities are contradictions. This proposal though still requires Bonevac to argue for, and decide on, the modal status of the troublesome propositions. We haven't simply got rid of them by re-aligning metaphysical impossibility so that it is identical to logical impossibility. Are propositions such as 'Hesperus is not identical to Phosphorus' and 'Water is not H_2O' logically impossible, or are they not impossible at all? Bonevac can hardly go for the former option, as this would put him in the same position as before, having to argue that the propositions were contradictions. Being logical impossibilities, he would be committed to categorising them as contradictions. The latter option is hardly appealing either. Without any counterargument to the necessary falsity of these propositions, it would be *ad hoc* for us to revise our opinion on their modal status, an opinion based on good reasons, for the sake of an account of contradiction that has seemingly little going for it. Neither option then has much plausibility. Bonevac's account suffers from admitting too many propositions into the category of contradictions.

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[27] The extent to which *broad* impossibility outstretches *logical* impossibility isn't solely down to one's views on topics such as essentialism, rigid designation or the laws of nature. Instead, it's a function of these views *and* how flexible one's concept of *logicality* is. If one allows logicality to only stretch to the first-order plus identity, then there will be more propositions that couldn't count as *logically* impossible than if one admitted, say, second-order and tense logics.

The most plausible solution to this miscategorisation of the above metaphysical impossibilities as contradictions is to exclude them explicitly in one's definition of contradiction. This one could do by prefacing impossibility with 'logical'. The propositions are thus no longer troublesome, for although they are clearly not contradictions, they are also not logically false. If they *were* to turn out to be logically false, then this would present a problem for the account. However, given that we are nowhere near constructing a logical system that captures the properties of the said propositions and demonstrates their necessary falsity, we have no reason for the advocate of the account to worry about this (epistemically) possible eventuality.

We have progressed to the accounts of Guttenplan, Detflesen *et al.*, and Copi. Call them the Guttenplan *et al.* accounts. Contradictions are still those propositions that are always false, but now we have added the condition that they are always false for *logical* reasons.[28] We need to be careful with what we mean here by *logical* reasons. We don't wish to endorse any controversial thesis on the relationship of logic to the world, and what combination of the two might ground the truth of a particular proposition; a question that we couldn't possibly do justice to so briefly. Instead, what we mean by logical reasons is that the role which logical vocabulary plays within the propositions allows those of us who understand the logical vocabulary to deduce whether that proposition is always true/false, or whether the logical vocabulary underdetermines its truth or falsity. What counts as this logical vocabulary is difficult to determine, and there is currently no agreed definition or extension of the term.[29] That there are some, however, and that they play a special role in the truth-conditions of a proposition they occur in, is a fundamental tenet of modern logic. In what follows, to remain on uncontroversial ground, we will only include the Boolean extensional connectives into our set of such logical vocabulary, leaving it an open question whether quantifiers, identity, and intensional operators should also be included in the set. This ensures that the proponents of the Guttenplan *et al.* account cannot simply deflect our counterexamples by disagreeing with us over the membership of the set.

With our stock of logical connectives in hand, there seem to be propositions that are logically false but which philosophers and logicians wouldn't commonly consider contradictions. Given that we have restricted ourselves to the Boolean extensional connectives here, we will focus on logical falsehoods in propositional logic, but examples at the first-order level are forthcoming through some natural extension. A logical falsehood in propositional logic is a proposition that has a propositional form F which outputs the truth-value false for every truth-value assignment to its propositional parameters. This logical falsity is often

[28] Wittgenstein (1961, 4.6-4.61) seems to have endorsed a definition of contradiction very similar to the Guttenplan *et al.* accounts.
[29] See Gómez-Torrente (2002) for a survey and critique of the available definitions of logical constants.

demonstrated and defined in terms of false on every line of a truth table, but can also be defined in terms of truth-trees.

Now, there are propositions in classical propositional logic that are translated into a logical form F, such that F receives the truth-value false whatever truth-value is assigned to its propositional parameters, but which don't seem to be contradictions. For example, the proposition 'It's not the case that either it's raining or it's not raining' is an instance of the propositional schema $\sim(p \vee \sim p)$, the negation of the law of excluded-middle, which receives the truth-value false whichever truth-value one assigns p. Thus, the proposition is logically false according to classical propositional logic. The status of the proposition as a contradiction is somewhat doubtful, however. The law of excluded-middle, after all, has historically been postulated as a logical law independently of the law of non-contradiction. While De Morgan's laws demonstrate that the proposition *entails* 'It's raining and it's not raining', a clear contradiction, to admit that a proposition is a contradiction because it *entails* a contradiction requires us to subsequently admit that entailment backwardly preserves the property of being a contradiction, a thesis we have at least two reasons to deny.

Firstly, if we admit that the property of being a contradiction *is* backwardly preserved then this would lead to a restriction in the expressive power of the term 'contradiction'. This restriction in expressive power would manifest itself in at least two respects. Firstly, we wouldn't be able to distinguish between those sets of beliefs or theories that *contained* a contradiction, such as the set $\{q, p \wedge \sim p, r\}$, and those that *entailed* a contradiction, such as the set $\{q, q \rightarrow \sim p, p\}$. However, in our assessment of an individual's beliefs or a theory we seem to value this distinction. If we view endorsing contradictions as rationally blameworthy to some extent, then we will be less damning of the individual whose beliefs *entail* a contradiction than the individual whose beliefs *contain* a contradiction. After all, unless we are unrealistically conservative with our beliefs, they are sure to entail a contradiction somewhere. While we don't consider it to be epistemically irresponsible to endorse a set of beliefs that entail a contradiction, until it is brought to our attention, most of us do consider the endorsement of a contradiction to be epistemically irresponsible to some extent. This valuable distinction that we use in our evaluation of beliefs and theories, however, would disappear if we allowed for the property of being a contradiction to be backwardly preserved. Secondly, we want to be able to distinguish between someone endorsing a contradictory *theory*, and their *statement* that endorses that theory being itself a contradiction. However, if the property of being a contradiction were backwardly preserved then this would result in *any* proposition that states the truth of a contradictory theory or set of beliefs being a contradiction. The scientist's statement that 'Scientific theory T is true' would be a contradiction if T included a contradiction, and someone's endorsement of Clinton with 'Whatever Bill Clinton says is true' would be a contradiction if Bill stated a contradiction. Again, we lose important expressive resources if we allow for the property of being a contradiction to be backwardly preserved. When we state that a set of beliefs or theory *entails* a contradiction, we mean to communicate more than simply that our consequence relation is reflexive, yet this is exactly what the statement reduces to if we admit that the property of being a contradiction is backwardly preserved.

Secondly, when constructing a formal *reductio* argument we attempt to demonstrate that a contradiction is entailed by an assumed premise, perhaps in conjunction with other premises.[30] From this, the truth of the negation of the assumed premise is entailed, given the truth of the other premises. Now, if we admit that those propositions that entail a contradiction are contradictions themselves, we would have no need, according to the *reductio* rule, to carry out any more derivations before we derive the negation of the premise that *is a contradiction.* Our derivation systems don't allow this however. We have to infer those formulae that are given as representing contradictions in the derivation system, commonly of the form $A \wedge \sim A$ or $\sim A \wedge A$. That our derivation systems don't allow us to directly derive the negation of the assumed premise that entails a contradiction gives us good reason to doubt that the property of being a contradiction is backwardly preserved by entailment. It may be that the derivation systems are at fault here, and that they should possess a mechanism that allows us to derive the negation of the premise as soon as we are aware that it's a contradiction (as defined by the Guttenplan *et al.* account). However, given the utility of the present derivation systems, and the lack of explanatory power that the Guttenplan *et al.* account possesses, this discrepancy between the two should be considered a weakness of the Guttenplan *et al.* account and not of the derivation systems.

It should be noted that although formal *reductios* are deductions of a contradiction from an assumption, in practice we don't always need to reason all the way to the contradiction. Whether we do or not will be dependent on the logical abilities of those involved in the reasoning and the difficulty and amount of the inferential steps required to reach the contradiction. This occurs most readily when we present a *reductio* in discussion, but it can also occur when using a derivation system, such as when the system allows us to complete the *reductio* from the occurrence of A and $\sim A$ on separate derivation lines. This shortcut is harmless given the admissibility of conjunction introduction in classical logic, but would be reckless if we had reason to doubt the universal validity of conjunction introduction. Neither of these cases show that the status of being a contradiction isn't reserved for the particular propositions that we recognise as the culmination of a *reductio*, allowing us to infer the negation

[30] *Formal reductios* should be distinguished from *informal reductios*. A formal *reductio* demonstrates that an assumed premise entails a contradiction and can be represented formally in a derivation system. In comparison, informal *reductios* demonstrate that a proposition entails a rather unsavoury consequence that isn't a contradiction, and these *reductios* cannot be represented within a derivation system (at least, we don't have a derivation system that models unsavoury consequences yet). Both could be as effective at demonstrating that the premise is false, or shouldn't be believed, but they are quite distinct routes to this conclusion.

of the assumption, commonly represented formally by formulae of the form $A \wedge {\sim}A$ or ${\sim}A \wedge A$. Instead, the cases simply show that in practice we needn't go through all the steps of a formal *reductio* to complete it. It's enough that we can see that a contradiction follows from the proposition at hand. If this isn't recognised by all parties, we can go further in our reasoning, making the derivation of the contradiction more explicit. The status of the contradiction in a formal *reductio* is clear by it being the proposition *p* that affords no extra inferential step in the *reductio* before discharging the assumption. If one of the parties fails to find the argument troubling at this stage, then there must be some more fundamental philosophical disagreement between the parties regarding the admissibility of contradictions.

In addition to these concerns regarding the backwards preservation of the property of being a contradiction, our inclinations about the de Morgan case are revealing of the Guttenplan *et al.* account's plausibility. It is telling that it's only once we realised, through de Morgan's laws, that propositions of the form ${\sim}(A \vee {\sim}A)$ were truth-functionally equivalent to propositions of the form $A \wedge {\sim}A$, that we had doubts that the propositions were not contradictions. If the Guttenplan *et al.* account is correct, then it is by virtue of a proposition's being logically false that it's a contradiction. The proposition being logically false is wholly constitutional of its being a contradiction. We certainly don't need de Morgan's laws, however, to show that propositions of the form ${\sim}(A \vee {\sim}A)$ are logically false. This we can do with a truth table. But, from the consideration above, what seems to have given us reason to doubt that the proposition *isn't* a contradiction, is that propositions of a certain logical form can be shown to be truth-functionally equivalent to propositions of a differing form. That is, we can show that propositions that don't seem to be of a 'contradictory form' can be shown to be truth-functionally equivalent to propositions of a 'contradictory form'. If it is this which persuades us to think that propositions of the form ${\sim}(A \vee {\sim}A)$ are contradictions, it's not by virtue of their being logically false, but instead the propositional logical form they instantiate having a certain relation to another propositional logical form. This suggests that the Guttenplan *et al.* account has left something out of the picture that it shouldn't have. There is something pertinent about the logical form $A \wedge {\sim}A$, such that when we are shown that a proposition *B* is semantically equivalent to propositions of this form, we take this to be a good reason to consider *B* a contradiction. What de Morgan's laws actually show us then, rather than giving the Guttenplan *et al.* a means to explain how propositions of the form ${\sim}(A \vee {\sim}A)$ are contradictions, is that there's something about the propositional form a proposition instantiates which gives us reason to think that the proposition is a contradiction.

It seems then that advocates of the Guttenplan *et al.* account cannot rely on the counter-argument that we have good reason to consider propositions of the form ${\sim}(A \vee {\sim}A)$ contradictions because they *entail* propositions of the form $A \wedge {\sim}A$. Firstly, we have good reasons to reject the principle underlying the counterargument, that entailment backwardly preserves the property of being a contradiction. Secondly, the example undermines the Guttenplan *et al.* account by suggesting that there's more to being a contradiction than just being logically false. If a contradiction were simply a proposition that's logically false then we shouldn't need de Morgan's laws to persuade us that propositions such as 'It's not the case that either it's raining or it's not raining' are contradictions. That the proposition is an instance of the formula $\sim(p \vee \sim p)$, which outputs the truth-value false whichever truth-value one assigns its propositional variables, should be enough information. Thus, the criticism that the Guttenplan *et al.* account miscategorises some logical falsehoods as contradictions still holds.[31]

We have found a good reason then to consider the Guttenplan *et al.* accounts implausible. This leaves us with the accounts of Sainsbury and Prior, which differ from the Guttenplan *et al.* accounts in not talking directly of contradictions, but instead propositions being the *contradictories* of each other. Talking in terms of the relation between propositions allows for the accounts to state that contradictory propositions cannot be false together, in addition to not being true together. The two accounts are worth repeating here, as they are full of detail:

Sainsbury (1991, p. 369): "Two propositions are contradictories if and only if it is logically impossible for both to be true and logically impossible for both to be false."

Prior (1967, p. 458): "Contradictory negation, or contradiction, is the relation between statements that are exact opposites, in the sense that they can be neither true together nor false together."

Only Prior's account mentions contradictions at all, with Sainsbury giving contradictories his full attention, and even Prior seems more concerned with the *relation* between contradictories ("contradictory negation") than contradictions themselves.

Neither Prior nor Sainsbury have so far then given us a definition of contradiction. Rather, they have given us a definition of a relation between propositions, that of being *contradictories*. A contradiction, though, isn't a relation. Some propositions are self-contained contradictions. It makes sense to say '*that* proposition is a contradiction'. Consequently, if the Prior and Sainsbury accounts are going to be extended to plausible accounts of *contradiction*, they need to respect the difference between the relation of being a contradictory, facilitating talk of *contradictories*, and contradictions themselves. The accounts, at least, allow for talk of *contradictories*, which is more than the Bonevac and Guttenplan *et al.* accounts facilitated. It made

[31] We could have used other logical falsehoods as examples, such as propositions of the form $\sim(A \rightarrow A)$. However, using the example of the law of excluded middle brought out some important points regarding the possible counterargument built around the De Morgan laws.

no sense in either of these accounts to talk of *contradictory pairs*, which we seem to make complete sense of. This then is an advantage that the Prior and Sainsbury accounts have, but they need to be extended from an account of contradictories to one of contradictions. At the start of the section we noted a rather natural way of doing this, by just allowing contradictions to be those sets of propositions constituted by contradictories:

CL) Contradictions are a set of propositions Σ such that the conjunction of all $B \in \Sigma$ is always false, but, necessarily, one and only one member of Σ is true.[32]

This extension of the definition of contradictories to contradictions has some plausibility in that it precludes those merely metaphysically necessarily false propositions considered earlier from being contradictions. Each, after all, was a singular proposition.

That CL) contains no mention of the requirement that the necessary falsity of the conjunction of the propositions is for *logical* reasons, however, still causes problems. Consider the set of propositions {Water is H_2O and there are white swans, Water is H_2O and it's not the case that there are white swans}. The set contains two propositions, each with two conjuncts. The second conjunct of each proposition has a truth-value contingently. There are possible worlds at which it is true and possible worlds at which it is false. The logical forms of the second conjunct of both conjunctions, however, are such that they are the negation of each other, which in classical logic entails that they cannot be true together but one of the pair must be true. Thus, although logic doesn't dictate which of the pair is true at each possible world, it dictates that one and only one is true at each possible world. The first conjunct of both conjunctions, 'Water is H_2O', in comparison, is true at *every* possible world, but for non-logical reasons. It is a merely metaphysically necessary truth. Given that a conjunction is true if both conjuncts are true, and false otherwise, the set of propositions given above is a contradiction according to CL). That 'Water is H_2O' is necessarily true ensures that either of the two propositions will be true if their second conjunct is true (and false otherwise). Given that one and only one of 'There are white swans' and 'It's not the case that there are white swans' is true at a possible world *w*, the set of propositions given above fulfils the criteria for being a contradiction given by CL).

This result provokes two doubts, the first formal and the second epistemic. Firstly, when we carry out a formal *reductio* within a derivation system for classical logic, we are only allowed to derive the negation of the assumed premise from which the contradiction was

[32] This definition of contradiction can be found in Dutilh Novaes (2007, p. 478). We are using the somewhat unsatisfactory pre-theoretical language of 'always false' here, as we don't want to commit advocates of the definition to a particular theory of how broad possibility should be modelled. If she is happy with talk of possible worlds, then a more precise version of CL) is easy to produce.

derived *once* we derive a formulae of the form $A \wedge {\sim}A$ or ${\sim}A \wedge A$ (or, assuming the validity of conjunction introduction, when both A and ${\sim}A$ occur in the sub-proof). These formulae are supposed to be symbolisations of contradictions. The putative contradiction which is the propositions contained in the set {Water is H_2O and there are white swans, Water is H_2O and it's not the case that there are white swans}, however, isn't of this logical form. Even if the propositions were conjoined in a derivation with conjunction introduction, their form would be $(q \wedge p) \wedge (q \wedge {\sim}p)$, which isn't identical in form to either $A \wedge {\sim}A$ or ${\sim}A \wedge A$. For the propositions to be of the right logical form to be instances of either $A \wedge {\sim}A$ or ${\sim}A \wedge A$, both conjunctions would have to be split and the two latter conjuncts of p and ${\sim}p$ conjoined to form $p \wedge {\sim}p$. We would then be able to complete a formal *reductio*. However, given that for the classical logician it is from formulae symbolising contradictions that we can derive the negation of the assumption from which we derived the contradiction, the *reductio* case suggests that the propositions in our set above are *not* themselves a contradiction, but instead can be used to *derive* a contradiction. Unless we have good reason, again, to think that the property of being a contradiction is preserved backwards through entailment, we have good reason to think the propositions are not a contradiction. The propositions are no more a contradiction than the propositions symbolised by the set of formulae $\{{\sim}p, q, q \rightarrow p\}$. We can easily derive a formula of the form $A \wedge {\sim}A$ from these premises, but this doesn't mean the propositions symbolised by the formulae without such derivation have the same properties *as the proposition symbolised by the derived formula*. To assert that the premises from which one can derive what the classical logician considers to be a contradiction in a formal *reductio* are also contradictions would be failing to make an important distinction between being *able to derive a contradiction* from a proposition and from the proposition *being a contradiction*. If CL) is to succeed as an account of contradiction, we would need to revise our use of the *reductio* rule in derivations and allow for the rule to be completed on the occurrence of formulae other than formulae of the form $A \wedge {\sim}A$ or ${\sim}A \wedge A$. Otherwise, we must admit that our formal methods for *reductios* don't allow us to complete *reductios* whenever a contradiction occurs. Given that the CL) account hasn't much to recommend it at present, we have good reason to see this result as a weakness of the account rather than of our derivation methods.

Before we move on to our second concern with the result, it's important to mention that the first is no less troublesome if we adapt CL) so that it includes the restriction that 'the conjunction of all $B \in \Sigma$ is *logically* false, but, *the occurrence of logical constants in the propositions ensure* that one and only one member of Σ is true'. By adding this restriction to CL), the set of propositions above no longer causes any problems, as there's nothing logically which stops 'Water is H_2O' from being false. Thus, CL) would no longer categorise the propositions in the set as contradictions. However, we can recapture the troublesome set by replacing 'Water is H_2O' with a logical truth as the first conjunct. For example, from a conjunction of the formulae $(q \rightarrow q) \wedge p$ and $(q \rightarrow q) \wedge \sim p$, we cannot directly apply *reductio*. The *reductio* case, therefore, seems to suggest not only that we need to ensure that the propositions are contradictions by virtue of possessing *logical properties*, but that we also need to ensure that we don't give the status of contradictions to propositions that our formal methods don't recognise as such. Unless, that is, we can show that there are no other accounts of contradiction that fit our formal derivation practices while possessing the same amount of explanatory power as CL).

The second concern that the result raises is epistemic. We should be able to work out whether a proposition is a contradiction without any commitments to metaphysical modality. Contradictions are supposed to be a matter settled by logic. However, with the set {Water is H_2O and there are white swans, Water is H_2O and it's not the case that there are white swans}, it's only by making a choice on the metaphysical modal status of 'Water is H_2O' that we can come to the conclusion that the set of propositions is a contradiction according to CL). Without the conviction that 'Water is H_2O' is metaphysically necessarily true, both conjunctions may be simultaneously false and thus the set of propositions will fail to fulfill the criteria of being a contradiction according to CL). Given that we shouldn't be required to take a theoretical position on the metaphysical modal status of certain propositions in order to decide whether a proposition is a contradiction, this demonstrates that, in some cases, CL) requires us to take a stand on topics that we shouldn't have to before we can decide that a proposition is a contradiction or not.

Fortunately for the CL) account, this concern *can* be rectified if we include the restriction that the conjunction of all $B \in \Sigma$ is *logically* false, but, *the occurrence of logical constants in the propositions ensure* that one and only one member of Σ is true. The 'Water is H_2O' case would no longer pose a problem, as the conjunction of the propositions isn't *logically* false, given that it's logically possible for 'Water is H_2O' to be true, and thus CL) wouldn't classify the propositions as a contradiction. Additionally, if we substitute the solely metaphysically necessary truth for a logical truth then we *can* infer the truth of the conjunct by logic alone, and thus CL) wouldn't require us to take a stand on topics that we shouldn't have to before we can decide that a proposition is a contradiction. By adding the restriction that it's *logically* false propositions we are interested in, the CL) account can sidestep this epistemic concern.

CL) doesn't only suffer from the deficiency of allowing too many propositions to be contradictions, it omits many propositions from the set of contradictions that it shouldn't. Given that CL) requires for there to be *at least two* members of the set, such that the conjunction of the members is false but one and only one of the conjuncts is true, this would rule out propositions such as 'There are white swans and it's not the case that there are white swans' from being a contradiction on its own. There are no two propositions such that one must be true and the other false. There is only one. Of course, we can derive two propositions from the proposition by conjunction elimination, but this means that the proposition only becomes a contradiction *once we have carried out* conjunction elimination (unless, again, we want to suppose that the property of being a contradiction is preserved backwards through entailment). This result is completely counter-intuitive. Propositions that have the propositional form $p \wedge {\sim}p$ are paradigm cases of contradictions. For one's account to have the consequence that a set of propositions of the form $\{p, {\sim}p\}$ is a contradiction, but that propositions of the form $p \wedge {\sim}p$ are not, seems a decent indication that the account has gone wrong somewhere. Allowing for a set of propositions such as {There are white swans, It's not the case that there are white swans} to be a contradiction has its concerns, but allowing for that set of propositions to be a contradiction and *not* allowing for the proposition 'There are white swans and it's not the case that there are white swans' to be a contradiction is without any justification. If CL) is to have any plausibility, we will need to adapt it to ensure that propositions of the form $p \wedge {\sim}p$ are included as contradictions.

Unfortunately for those wishing to preserve both the status of sets of propositions of the form $\{p, {\sim}p\}$, and propositions of the form $p \wedge {\sim}p$, as contradictions, while defining contradictions in terms of its truth-conditions, this can only be achieved by endorsing a disjunctive definition. The original CL) definition with the added restriction on logical falsehood,

> CL_{Log}) Contradictions are a set of propositions Σ, such that the conjunction of all $B \in \Sigma$ is logically false, but, necessarily, the occurrence of logical constants in the members of Σ ensure that one and only one member of Σ is true,

doesn't allow for propositions of the form $p \wedge {\sim}p$ to count as contradictions, for the reasons already given. If we make the necessary changes to CL_{Log}) to ensure that propositions of the form $p \wedge {\sim}p$ *can* count as contradictions, we are going to have to add that contradictions are a *conjunction* of propositions:

> CL_{Conj}) Contradictions are a conjunction of two propositions of A and B, such that the conjunction of A and B is logically false, but, the occurrence of logical constants in A and/or B ensure that one and only one of A and B is true.[33]

[33] CL_{Conj}) has the advantage over its predecessors, CL) and CL_{Log}), that it doesn't require one to speak of contradictions as *sets*. Sets themselves cannot be contradictions. After all, sets cannot be false (or true), and contradictions certainly can be. This is a concern with CL) and CL_{Log}) that we have, and shall continue to, bracket for the sake of our discussion.

CL_{Conj}), however, rules out unconjoined propositions from being contradictions, even if one can *derive* a contradiction from them. To allow for both types of (sets of) propositions to be contradictions, we would require a disjunctive definition:

CL_{Disj}) For a proposition(s) to be a contradiction it must meet the criteria of CL_{Log}) or CL_{Conj}).

This disjunctive definition solves the problem for the advocate of the classically-assumed account who wants to allow both the set of propositions {There are white swans, It's not the case that there are white swans}, and the proposition 'There are white swans and it's not the case that there are white swans', to be contradictions. What it doesn't show us, however, is whether the definition is *plausible*. Given that we have already considered some of the faults with CL) and CL_{Log}), we should now consider the plausibility of CL_{Conj}) with the knowledge that if CL_{Conj}) is an implausible account of contradiction, then the disjunctive definition that it's a disjunct of, CL_{Disj}), will be at least as equally implausible. Thereby only adding to the implausibility of CL_{Disj}) contributed by the weaknesses of its other disjunct, CL_{Log}). Equally, we should be aware of the plausibility of CL_{Conj}) independently of its status as a disjunct of CL_{Disj}). It could, after all, serve as a separate definition of contradiction.

Before we move on to consider any new problems with CL_{Conj}), it's worth revisiting those criticisms we had of the previous classically-assumed accounts and asking whether they also burden CL_{Conj}). We found that the Bonevac account categorised metaphysical falsehoods as contradictions, but given the logical restrictions placed on contradictions by CL_{Conj}), it doesn't suffer from this concern. Additionally, the account doesn't miscategorise propositions of the form $\sim(A \vee \sim A)$ or $\sim(A \rightarrow A)$ as contradictions, as the Guttenplan *et al.* account does, for the propositions don't fulfill the conditions of logical form required by the definition. Thus, the account ensures that not all logical falsehoods are contradictions, respecting the intuitive distinction between the two categories. Finally, CL_{Conj}) avoids the criticism of its predecessor, CL_{Log}), in not precluding paradigm cases of contradictions, such as propositions of the form $p \wedge \sim p$.

The account, however, does suffer from our other criticism of CL_{Log}), that the definition mischaracterises certain propositions as contradictions when we have good reason to believe they aren't. Consider propositions of the form $(p \wedge (p \vee \sim p)) \wedge (\sim p \wedge (p \vee \sim p))$. Propositions of this form, given classical semantics, fulfill the definition of contradiction given by CL_{Conj}). The second conjunct of the two embedded conjunctions is a logical truth, and thus the truth of each embedded conjunction depends on the truth of the first conjunct. Given that in classical logic one and only one of p and $\sim p$ can be assigned the truth-value true, propositions of this form have the truth-functional properties required to be a CL_{Conj})

contradiction. Again, however, the occurrence of this formula in a proof wouldn't permit the completion of a formal *reductio*. Either our derivation systems are faulty, or CL_{Conj}) isn't stringent enough. As before, given that we haven't been provided yet with any reasons to consider the CL_{Conj}) account to have greater explanatory power than its competitors, we can hardly conclude that our derivation systems are faulty to rescue an unproven account of contradiction. To further substantiate this point, in section 3.1.3 we will consider an account of contradiction that doesn't suffer from this or any of the other criticisms that inflict versions of the classically-assumed account.

This criticism of both CL_{Log}) and CL_{Conj}) demonstrates that a truth-conditional definition of contradictions is never going to be adequate without some restrictions being placed on the form of the propositions that count as contradictions. Without this restriction our semantic definition of contradiction is always going to outrun our use of contradictions in a formal *reductio*. CL_{Conj}) already went some way to putting formal restrictions in place, ensuring that a contradiction is a conjunction rather than just a set of propositions. What CL_{Conj}) has not done, however, is to place restrictions on the properties of the conjuncts besides that one and only one is true. This ensures that the criteria given by CL_{Conj}) can be fulfilled by a multitude of propositions, given the logical constants we have at our disposal, with unacceptable consequences. Unfortunately for the advocate of the classically-assumed account, the only method by which to ensure that a definition in terms of truth-conditions doesn't outrun our use of contradictions in derivations and in applications of formal *reductios* is to place restrictions on the conjuncts of the contradictions such that one conjunct is the negation of the other. Now, this doesn't entail that there are no successful *semantic* account of contradictions, for these constituents of contradictions may end up being semantic entities. However, it does entail that there can be no successful definition of contradictions in *solely truth-conditional terms*. Reference will have to be made directly to the form of the proposition. This then seems to spell the end for the classically-assumed account of contradiction.[34]

CL_{Conj},) therefore, brings with it a criticism of its predecessors, and this criticism seems strong enough on its own, not only to cause us to doubt the plausibility of CL_{Conj}), and thus CL_{Disj}), but to doubt the possibility of any plausible classically-assumed account of contradiction. This doubt will be compounded later by the realisation that at least one of its competitors fails to succumb to the same criticisms.

There is one final criticism of CL_{Conj}), and this criticism applies equally to all the versions of the classically-assumed account mentioned. Additionally, it is hard to see how any

[34] This result, and our criticism that the membership criteria of the accounts are too lax, is symptomatic of a general failing to account for logical form in terms of truth-conditions. If logical form plays any part in the identity criteria for contradictions, which it seems to, then more than truth-conditions is required for any suitable definition.

classically-assumed account could escape its force. This criticism is that CL_{Conj}), and all its predecessors, trivialise the law of non-contradiction.

One of the main criteria for the plausibility of any account of contradiction should be whether the account can be meaningfully embedded into the law of non-contradiction (LNC) whilst ensuring the law maintains its philosophical importance. The pertinence of this feature of contradictions, as a criterion for an account's plausibility, is ensured by a) the perceived philosophical importance of the LNC, and b) the fact that the LNC contains the concept of contradiction within it.

On the first point, historically the LNC has held an elevated position as one of the three most philosophically important logical laws (along with the laws of excluded middle and identity), with Aristotle (1971a, Γ 1005^b 22) considering the LNC to be the most certain of all principles. The continued vaulted status of the law in contemporary philosophy can be demonstrated by the vast amount of simultaneous puzzlement and quick rejections that the dialetheist's assertion of true contradictions receives. Even a philosopher such as Lewis, who was willing to endorse a position as radical as modal realism, didn't feel that he could entertain the possibility of true contradictions.[35] Where Lewis's (1973, p. 86) thesis gained "incredulous stares", the dialetheist's thesis gained confused looks and giggles. The law is still given mention in introductory logic textbooks as a fundamental logical law and philosophers, as we shall see below, still bother to put forward formulations of it. The *prima facie* plausible position, therefore, is that the LNC does have philosophical importance.

Our second point, that the LNC contains the concept of contradiction within it, if not obvious from the occurrence of the term 'contradiction' in its name, can be shown by natural-language formulations of the law in the literature:

Aristotle_1 (1971a, Γ 1011^b 13-14): Contradictory statements are not at the same time true.

Aristotle_2 (1971a, Γ 1005^b 19-20): The same attribute cannot at the same time belong and not belong to the same subject in the same respect.

Armstrong_1 (2004, p. 107): It is impossible for *p* and not-*p* both to be true.

Brown_1 (2004, p. 126): No contradictory sentence is ever correctly assertable.

Dummett_1 (1978, p. xix): Not both *A* and not *A*.

[35] Lewis (2004) felt the LNC was so philosophically and logically entrenched that he couldn't say anything insightful about the law, rejecting the opportunity to contribute a paper to the Priest *et al.* (2004) collection.

Englebretsen$_1$ (1981, p. 5): A sentence and its negation cannot both be true.

Goldstein$_1$ (2004, p. 295): The conjunction of a proposition and its negation is never true.

Grim$_1$ (2004, p. 49): Both sides of a contradiction cannot be true.

Hymers$_1$ (2009, p. 198): No proposition is simultaneously and unambiguously both true and false.

Price$_1$ (1990, p. 224): A proposition and its negation cannot both be accepted.

Priest$_1$ (2004, p. 29): Nothing is both true and false.

Prior$_1$ (1967, p. 461): A statement and its direct denial cannot be true together.

Prior$_2$ (1967, p. 461): Nothing can both be and not be the same thing at the same time ('Nothing is at once A and not-A').

Quine$_1$ (1965, p. 51): No statement is both true and false.

Read$_1$ (1995, p. 250): No proposition and its contradictory can be true together, that is, no proposition of the form 'A and not-A' is true.

Russell$_1$ (2001, pp. 46-47): Nothing can at the same time have and not have a certain property.

Smith$_1$ (2003, p. 101): A proposition and its negation cannot both be true.

von Wright$_1$ (1957, pp. 8 & 22): No one proposition has more than one truth-value (is not true *and* false).

von Wright$_2$ (1957, p. 29): No property in a corresponding universe is present and absent in a thing.

Woodhead$_1$ (2006, p. 24): A statement cannot be true and false at the same time.

Although only four of the formulations of the law explicitly use the terms 'contradiction' or 'contradictory', those which don't can be shown to include the concept of contradiction under the guise of a *particular* account of contradiction, or under the guise of an account of contradiction plus some background assumptions. So, while Aristotle$_2$, Prior$_2$, Russell$_1$ and von Wright$_2$ all speak of a subject both possessing a property and not possessing a property, rather than directly of contradictions, we find Routley & Routley (1985, p. 204) defining contradictions in similar terms. Englebretsen$_1$ speaks of "a sentence and its negation", and in Kalish *et al.* (1980, p. 18) we find very similar language being used to explicate the concept of a contradiction. All of Goldstein$_1$, Price$_1$ and Smith$_1$ similarly contain Kalish *et al.*'s account of contradiction in their formulations of the LNC. The "statement and its direct denial" of Prior$_1$ can either be linked to the Kalish *et al.* account of contradiction, if Prior's "denial" is interpreted as negation with the background assumption that negation is identical to the force-operator denial, or it can be related to the pragmatic accounts of contradiction found in Kahane (1995, p. 308) and Peter Strawson (1993, p. 21), interpreting Prior's "statement" as a communicative act. The "*p* and not-*p*" of Armstrong$_1$ and "*A* and not *A*" of Dummett$_1$ could equally be versions of the account of contradiction found in Kalish *et al.*, or be partial natural language translations of the formula $p \wedge \sim p$, which is used as a definition of contradiction in Haack (1978, p. 244) and Arruda (1980, p. 3). Lastly, all of Hymers$_1$, Priest$_1$, Quine$_1$, von Wright$_1$ and Woodhead$_1$, which talk of truthbearers being both true and false, can reasonably be considered as products of an inference from the law stated in Englebretsen$_1$, Goldstein$_1$ and Smith$_1$. If a contradiction is the conjunction of a proposition and its negation then, under the classical assumptions that negation is a truth-reversing truth-functor and a conjunction is only true when both conjuncts are, a contradiction is true only when a proposition is both true and false. Thus, given the classical meaning of negation and conjunction, to say that the conjunction of a proposition *p* and *p*'s negation cannot be true is equivalent to saying that no proposition *p* can be both true and false. It seems, therefore, that the concept of contradiction is contained in formulations of the LNC. Granted, we have only considered twenty examples here, out of the hundreds or thousands that could be found in the literature. However, we should be confident that any other plausible articulation of the law can be shown to include the concept of contradiction within it, whether explicitly or under the guise of a particular account.

The LNC, therefore, is both a logical law of great philosophical import and contains the concept of contradiction. Consequently, if an account of contradiction is going to be philosophically plausible, it must respect both of these facts either by ensuring that it is meaningfully embeddable into the LNC, *or* by the account's advocates giving us good independent reasons to no longer consider the LNC to be of any philosophical import.

Now, unfortunately for advocates of the classically-assumed account, given that all versions of the account define contradictions as being necessarily false, the account seems to make the LNC completely philosophically redundant. Whatever other restrictions a version of the classically-assumed account places on its definition of 'contradiction', all the definitions include the condition that contradictions *are necessarily false*, whether that's explained in terms of being *logically* false or just *broadly necessarily* false. However, if it's a given that contradictions are necessarily false, what work does the LNC do? What new fact does it state? Surely the LNC is saying something *about* contradictions that we didn't know before, but what new fact could the LNC be stating about contradictions? It can hardly be stating a new *semantic* fact about contradictions, as defined by the classically-assumed accounts, for these accounts already define contradictions as being necessarily false. One adds nothing by stating that a proposition that's necessarily false *is false*. If we use a classically-assumed account of contradiction, all we need to state to retain the content of the LNC is 'contradiction', given that reiteration adds no extra content.

Strictly speaking, this formulation of the LNC as 'All necessarily false propositions are false' transforms the law into an instance of the reflexivity axiom ($\Box A \rightarrow A$). This might lead one to conclude that it does tell us a fact about contradictions after all, namely that our world is one of the possible worlds, and thus that contradictions are false at our world, the *actual* world. In this case, the LNC acts as a demonstration for the truth of the reflexivity axiom for contradictions. Any modal logic that didn't include the reflexivity axiom, at least for contradictions, would be shown to be inadequate because of the LNC. Yet, it seems just plainly incorrect to suggest that we need the LNC to demonstrate that modal logics without the reflexivity axiom have gone wrong somewhere. Instead, we can rely simply upon our concept of necessity to demonstrate that necessity entails actuality. Additionally, if the LNC were a demonstration of the validity of the reflexivity axiom for contradictions, then the law should only be formally expressible within a modal extension of propositional logic as,

$$\Box{\sim}(A \wedge {\sim}A) \rightarrow {\sim}(A \wedge {\sim}A).$$

Yet, this isn't the case. The LNC is symbolised in propositional logic as ${\sim}(A \wedge {\sim}A)$, which expresses nothing about the reflexivity axiom.[36] Therefore, by interpreting the LNC as stating a semantic fact about contradictions, the classically-assumed accounts *do* seem to trivialise the LNC.

If the LNC is stating a non-trivial fact about contradictions, as defined by a classically-assumed account, then it must be stating a *non*-semantic fact. The only two available choices for this role seem to be pragmatic laws stating facts about either belief or assertion. Thus, the

[36] Equally, attempts to rescue the LNC by suggesting that the semantic property being assigned to contradictions is *untruth*, in which case the law becomes 'All necessarily false propositions are untrue', fail to respect our formalisation of the law in propositional logic. If this really were the content of the law, then its truth would be ensured by the axiom $Tr({\sim}p) \rightarrow {\sim}Tr(p)$ in our truth theory, not by the law being a theorem of our propositional logic. Thus, this formulation of the LNC would require us to reconsider our formalisation of the law, a move that the classically-assumed account would require strong motivation for, which at present it doesn't possess.

LNC could be expressing either 'One shouldn't believe contradictions' or 'One shouldn't assert contradictions'. Given that the classically-assumed accounts define contradictions as being necessarily false, these interpretations of the law ensure that it's reduced to the rather bizarre 'One shouldn't believe/assert necessarily false propositions', which suggests the LNC isn't such the fundamental philosophical law. Believing or asserting contingent falsehoods is just as unwelcome as believing or asserting necessary falsehoods. A falsehood is a falsehood. The LNC, therefore, becomes a special instance of the more fundamental principle, 'One shouldn't believe/assert false propositions'. Interpreting the LNC as a pragmatic law, embedding the classically-assumed account of contradiction, demotes it from the status of a fundamental logical law to a particular instance of a more general pragmatic law of not believing or asserting falsehoods.

A second problem with interpreting the LNC as a pragmatic law is also forthcoming. The interpretation requires us to revise the established formalisation of the LNC, $\sim(A \wedge \sim A)$. Given that tilde ('~') occurs twice in the formula, we are required by formal rules to interpret the symbol consistently throughout the formula. The pragmatic interpretations of the LNC would require us to interpret the external tilde, however, as a force operator. Yet, interpreting tilde universally in the formula as a force operator would make the formula nonsensical. Force operators cannot be meaningfully embedded. If we want the LNC to be stating a pragmatic law, we shall have to change our formalisation of the law. While it's possible to change the current formalisation, the fact that it's well established ensures that we need very good reasons for doing so, and the current proposal hasn't provided us with those.[37] The pragmatic interpretation of the LNC, therefore, seems an unpromising solution to the classically-assumed account's troubles.

There is one reply to our second criticism of the pragmatic interpretation worth mentioning here. It might be thought that in interpreting tilde as the force operator in the symbolisation we have been unfair on the pragmatic interpretation. The formula symbolising the LNC, after all, is a theorem in classical logic and is preceded by the turnstile, which was originally conceived by Frege as a *judgement stroke*. It is, so the reply goes, the turnstile that is the force operator and not the external tilde. Thus, by considering the formalisation of the law as

[37] Brady (2004) has argued quite persuasively for a change to our standard formalisation of the LNC, as the formula $\sim(A \wedge \sim A)$ fails to serve the function in formal systems expected of the LNC. The point being made here, however, isn't that the standard formalisation of the LNC *is* the correct formalisation, but that it's the well-established formalisation, and that the pragmatic interpretation of the LNC is incompatible with it. For the classically-assumed account, it isn't enough that the standard formalisation of the LNC can be criticised for reasons unrelated to their account. If advocates of the classically-assumed account are to successfully explain away the incompatibility of the standard formalisation of the law and the pragmatic interpretation of the LNC, then they need to rely upon the theoretical virtues of both their account of contradiction and the pragmatic interpretation of the law. Neither, however, seems forthcoming.

$\vdash\sim(A \wedge \sim A)$, rather than $\sim(A \wedge \sim A)$, it can be shown that the pragmatic account isn't incompatible with the standard formalisation of the LNC.

There are two broad points to make in response to this suggestion. Firstly, it would be a mistake to equate Frege's *judgement stroke* with the turnstile that we find in modern calculi and, secondly, irrespective of the analogy with the *judgement* stroke, the turnstile cannot plausibly be interpreted as a force operator.

We find Frege's (1970, pp. 11-12) most detailed explanation of the judgement stroke in the *Begriffsschrift*:

> A judgement will always be expressed by means of the sign
> $|$—,
> which stands to the left of the sign, or combination of signs, indicating the content of the judgement. If we *omit* the small vertical stroke at the left end of the horizontal one, the judgement will be transformed into a *mere combination of ideas*, of which the writer does not state whether he acknowledges it to be true or not. For example, let
> $|$—A
> stand for the judgement 'Opposite magnetic poles attract each other'; then
> —A
> will not express this judgement; it is to produce in the reader merely the idea of the mutual attraction of opposite magnetic poles, say in order to derive consequences from it and to test by means of these whether the thought is correct. When the vertical stroke is omitted, we express ourselves *paraphrastically*, using the words 'the circumstance that' or 'the proposition that'… *The horizontal stroke* that is part of the sign $|$— *combines the signs that follow it into a totality, and the affirmation expressed by the vertical stroke at the left end of the horizontal one refers to this totality*. Let us call the horizontal stroke the *content stroke* and the vertical stroke the *judgement stroke*.

The judgement stroke, therefore, "converts the content of *possible* judgement into a judgement," and is "placed vertically at the left hand end of the content-stroke," (Frege (1979a) p. 11. My emphasis).[38] The sign '$|$—' is a combination of the *content* stroke '—' and the *judgement* stroke '$|$'. The content stroke on its own does not express a judgement or carry any assertoric force.[39] Only when the content stroke is combined with the judgement stroke is a judgement about the content following the strokes expressed. The judgement stroke, therefore, can be seen to be enacting some assertoric force on the content following it, and thus could plausibly act as the foundation of a counterargument against our criticism of the pragmatic LNC. Given that the judgement stroke only occurs in combination with the content stroke, and not on its own, we need to consider the judgement stroke and content stroke combined, symbolised by the sign '$|$—', if we are going to adequately compare the function of the modern

[38] Cf. Frege (1964, pp. 37-38; 1972, p. 94; 1979b, p. 51; and 1984a, p. 149).
[39] There is an obvious distinction to be made between a judgement and an assertion, with one being an internal act and the other being an external act. However, given that assertion for Frege just is "the outer expression of the inner act of judgement," (Smith (2009) p. 644) we don't need to worry about making any distinction between the two acts for our purposes here.

turnstile and Frege's force operator. Call this force operator the *Frege stroke*. It is the Frege stroke, and not the judgement stroke on its own, that must be appealed to in demonstrating that the turnstile is a force operator.

For the counterargument to be successful, it must be demonstrated that it's plausible that the modern turnstile symbol '⊢', which we use in the expression of our logical systems, is a force operator. *Prima facie* evidence for this view may derive from the typographic similarity of the turnstile and Frege stroke symbols, and the fact that many features of our modern calculi originated in Frege's system. This *prima facie* evidence, however, is no real evidence at all. For just as homonyms and homographs can be typographically identical without sharing meaning, so two typographically similar formal symbols can be given distinct meanings, and the heritage of a sign doesn't ensure heritage of its meaning. A tool can be put to different uses over time. Once we consider the respective uses that the Frege stroke and the turnstile are put to, we quickly realise both that the two symbols cannot be equated and that the turnstile cannot plausibly be construed as a force operator. Let us take each of these concerns in turn, starting with the former.

There at least six differences between Frege's '|—' and the turnstile:[40]

1) While the turnstile can only meaningfully occur before the conclusion of a proof, whether the premise set is empty or not, the Frege stroke can occur before a premise or conclusion to indicate that what follows is a judgement, and not just a potential judgement. The Frege stroke indicates that what follows is *being asserted*, rather than just being considered.

2) While the turnstile is an indivisible symbol, the Frege stroke is a combination of the judgement stroke and the content stroke. While the judgement stroke cannot occur on its own, the content stroke can.

3) While the turnstile is indexed to a particular logical system, telling us something about *that* system, the Frege stroke is not indexed to any system. It simply informs us that what follows constitutes a judgement.

4) While the turnstile can be used in conjunction with semantic validity '⊨' to express the meta-mathematical proofs of soundness and completeness, the same isn't true of the Frege stroke.

[40] These points have been heavily influenced by Smith's (2009) excellent discussion of the function of the judgement stroke.

5) While the turnstile can be struck through with a line to communicate that the formula following the turnstile cannot be proved in that system, there is no such device for the Frege stroke to communicate that the content following cannot be the subject of judgement or asserted.

6) While the turnstile is a metasymbol within logics of consequence, the Frege stroke is a symbol within a logic of inference. While the common modern conception of logic is that of working out what follows from what, in other words *consequence*, Frege believed that logic should be concerned with *inference* and not just with consequence.[41] The Frege stroke was required by a logic of inference to communicate that the content following the stroke was being asserted, and not just being merely considered, and thus could be used as the premise of an inference.[42] Without the Frege stroke there are no means by which one can communicate that the content following the stroke is being asserted and, therefore, that inferences can be made from the proposition. The turnstile, in contrast, communicates that what follows it can be derived within a particular formal system, not that it's being asserted. The appearance of a turnstile before a conclusion but *not* before the premise set is conclusive evidence that the turnstile is not a symbol of any logic of inference.

These six differences between the behaviour of the signs constitute more than enough evidence to reasonably conclude that the meanings of the two symbols cannot be equated; they serve very different functions.

The strength of the reply to our criticism, however, doesn't depend solely on the analogy with the Frege stroke. The turnstile may share very few similarities with the Frege stroke while still plausibly being a force operator. For example, the turnstile may not communicate what has been judged or asserted, but rather what *should be* judged to be true or asserted. This would make more sense, given that modern logical systems are logics of consequence. While one cannot plausibly suggest that the turnstile communicates that a premise set or conclusion has been judged or asserted, we can make sense of our logics possessing normative power. Rather than communicating that the premises or conclusions have been asserted, the logics might have the conditional normative force that *if* one has

[41] This is demonstrated most clearly by passages in which Frege states that logic is only concerned with judgements that are true; see Frege (1979c, p. 3; 1980, pp. 16-17 'Letter to Dingler' (31/01/1917); 1984b, pp. 375 & 402; and 1984c, p. 318). For more on Frege's conception of logic see Currie (1987).

[42] "This separation of the act from the subject matter of judgement seems to be indispensable; for otherwise we could not express a mere supposition – the putting of a case without a simultaneous judgement as to its arising or not. We thus need a special sign in order to be able to assert something," (Frege (1984a) p. 149).

asserted a premise set Σ then one *should assert p*, with p being a logical consequence of Σ according to the logic. Thus, if we are going to fully dispose of this reply, we must argue directly against the view that the turnstile can be plausibly interpreted as a force operator. There are at least four reasons to consider the force operator interpretation of the turnstile implausible.

Firstly, the turnstile appears in the expression of theorems of formal systems where there are uninterpreted propositional variables at the propositional level and uninterpreted variables and functions at the first-order level. To interpret the turnstile of $\vdash\sim(A \wedge \sim A)$ as 'assert that' is to require not that one assert certain propositional content but instead that that one assert a string of symbols. Yet, logics do more than suggest one asserts an uninterpreted formula. We take $\vdash\sim(p \wedge \sim p)$ to be communicating that the formula $\sim(p \wedge \sim p)$ can be derived within a logic L, according to L's derivation rules, without any undischarged assumptions. Interpreting the turnstile as a force operator implies that logics require one to assert uninterpreted formulae, yet this certainly isn't what they require of us.

Secondly, if one establishes a completeness theorem for a logic L then, if it can be shown that an argument form is truth-preserving under all interpretations in L, one can conclude that there is a successful proof for the derivability of the conclusion from the premises in L. By interpreting the turnstile as a force operator, a completeness theorem for L entails that one should assert a theorem T of L because T has been shown to be true under all interpretations in L. The truth of T under all interpretations entails that one should assert T, given a completeness theorem.[43] This consequence of the force operator interpretation of the turnstile, however, requires us to accept a particular norm for assertion: The truth of p entails that one should assert p. While this norm may indeed be constitutive of the concept of assertion, we need evidence for the plausibility of a *truth-based norm* account independently of the force operator interpretation's commitment to it. If such a justification isn't forthcoming, then that will be a black mark against the force operator interpretation of the turnstile. Although this isn't the place to critique a *truth-based norm* account of assertion, being committed to this particular account of assertion, when there are so many available in the literature, is a large burden for the interpretation to carry; especially when some of the other accounts, such as the *knowledge-based* and *justified belief-based* norm accounts, seem to have greater plausibility.[44] Additionally, given the multitude of available accounts of assertion, it seems a theoretically weak position that requires us to take a stand in the debate without any direct evidence for the truth-based norm account. It would be better to have an interpretation of our formal symbols that doesn't require us to take on substantive commitments on other philosophical topics.

[43] We are passing over the difficulty presented in the first criticism that it makes little sense to assert formulae.

[44] For more on the available accounts of assertion see section 3.3 below.

The third criticism is heavily related to the second. Any mathematical statement that includes the turnstile symbol is a statement *about* a system, not a statement from within a system. Equally, the turnstile expresses the same content about the system whatever system it's being used in conjunction with. For this reason, the turnstile is considered a metasymbol of any logic; its meaning isn't fixed by a particular logic, but can be used to express facts about many different logics. For this reason logicians will often add a subscript to the turnstile to specify which logic they are talking about. This property of the turnstile is problematic for the force operator interpretation, given that we can use the turnstile to express facts about logics that preserve properties other than truth with their consequence relation. So, for example, the preservationist logic of Jennings and Schotch (1984) preserves *levels of coherence* with its consequence relation.[45] Now, if Jennings and Schotch's logic had a derivation system, with a completeness proof, then we would be able to deduce which formulae could be derived from which from facts about the semantic consequence relation. Thus, we would be able to conclude that $\Sigma \vdash_{JS} p$, which under the force operator interpretation translates as 'Given Σ, *one should assert p*', if it could be shown that $\Sigma \vDash_{JS} p$. Yet, this entailment, given a completeness proof, requires us to adhere to a different norm of assertion. The consequence relation that justified our conclusion of $\Sigma \vdash_{JS} p$ didn't preserve truth, but rather levels of coherence. It's bad enough that the force operator interpretation committed us to a *truth-based norm* account of assertion. Yet now it's also committing us to a *levels of coherence-based norm* account of assertion, in addition to the multitude of other norms of assertion that will be related to every single property P preservable through a consequence relation for which a derivation system can be devised and a completeness proof given. Again, it seems highly implausible to accept an interpretation of the turnstile that commits us to a new norm for assertion every time a formal system that preserves a new property P, with a derivation system and completeness proof, is constructed. Formal systems seem to place no restriction on our theories of assertion.

Lastly, the force operator interpretation of the turnstile entails that for any theorem T, one *can* assert T. This is merely an instance of the principle that ought implies can. If one *ought* to assert T, then one *can* assert T. Now, this principle casts doubt on the force operator interpretation not only because it's incoherent to assert a formula, our first criticism, but also because there are logics in which theorems can be of an infinite length, and thus cannot possibly be asserted. These infinitary logics are incredibly theoretically interesting, yet if the force operator interpretation is correct, the turnstile cannot correctly be used in any such logic. This conclusion, however, is contrary to the practices and opinions of the logicians who work with infinitary logics. They can make perfect sense of the turnstile as it is applied in these logics. The force operator interpretation, therefore, seems to rule out certain uses of the turnstile when there's no independent justification to do so. Given that it's theoretically

[45] See section 4.3.2 below.

dubious to rule out technically interesting logics because of a controversial interpretation of the turnstile, this consequence of the interpretation gives us good reason to reject it.

These four consequences of the force operator interpretation, when combined, give us good reason to think that the turnstile cannot plausibly be interpreted as a force operator. Thus, we can dismiss the interpretation as a counterargument to our second criticism of the pragmatic version of the LNC. As it stands then, the pragmatic LNC is both uninteresting and incompatible with our current formalisation of the law.

Neither the semantic nor pragmatic interpretations of the law are suitable when the classically-assumed account is embedded within them. Yet, there don't seem to be any other viable interpretations of the law. It seems that if we want an available interpretation of the LNC in which it's a substantial philosophical law, we are going to require a different account of contradiction to embed within it.

Given that there seem to be no substantial interpretations of the LNC when the law is embedded by a version of the classically-assumed account, the advocate of the account has only one option with which to rescue her theory from the accusation that she has trivialised the LNC or made it insubstantial. She must give us reason to believe that the LNC should no longer have philosophical import. Unfortunately for the classically-assumed account, there are no obvious reasons to consider the law trivial except that the account makes it trivial. Additionally, as we shall see later, there are accounts of contradiction that do *not* trivialise the law. This gives us good reason to think that the account has less explanatory power in this regard, at least, than some of its competitors.

We have considered several versions of the classically-assumed account of contradiction. The Bonevac account, the Guttenplan *et al.* account, Prior and Sainsbury's account, and the CL_{Log}) and CL_{Conj}) extensions of Prior and Sainsbury's account. We found good reason to think that all of the versions are philosophically implausible, and the final criticism, at least, has given us good reason to believe that all further variations of the account will be implausible. Whether there are any versions of the account that can escape the other criticisms levelled against its predecessors is a matter left open for debate

Although the classically-assumed account is philosophically *implausible*, and thus automatically *unsuitable* to embed within Ab) to express the absolutist's position, it is worth noting here an extra reason to consider the account unsuitable for the absolutist's purposes. Given that some dialetheic logics, such as **LP**, propose a glutty semantics, two interpretations of the classically-assumed account can come apart:[46]

CA_F: A proposition *p* is a contradiction iff *p* is always false.

[46] For more on **LP** see section 4.4.1 below. Whether CA_F and CA_T do come apart will depend on whether the glutty logician's truth theory includes the axiom $F(p) \rightarrow {\sim}Tr(p)$. Given that $F(p) \leftrightarrow Tr({\sim}p)$, this axiom is equivalent to $Tr({\sim}p) \rightarrow {\sim}Tr(p)$. Without this axiom, a proposition can be false without being *un*true.

CA_T: A proposition *p* is a contradiction iff *p* is never true.

In discussing the classically-assumed account's suitability for the absolutist's purposes, we can concentrate on CA_F, as CA_T entails CA_F in any glutty logic, given that in a glutty truth theory $\sim Tr(p) \rightarrow F(p)$. Thus, if CA_F is unsuitable for the absolutist, then so is CA_T.

The suitability of CA_F for the absolutist's purposes is dependent on whether the *falsity* of a proposition *p* precludes its *truth*. If truth and falsity are mutually exclusive, then the stipulated falsity of all contradictions would ensure that they cannot be true, which would preclude the truth of the absolutist's thesis. Yet, if truth and falsity are not mutually exclusive, then the necessary falsity of a proposition *p* doesn't preclude *p*'s truth, which ensures that CA_F doesn't preclude the truth of absolutism.

Now, unfortunately for the absolutist, this state of affairs is very unsatisfactory. Embedding CA_F into the absolutist's thesis we get:

Ab_{CA}) Some necessarily false propositions are true at a possible world *w*.

Thus, establishing the *suitability* of CA_F for the absolutist's purposes, by demonstrating that truth and falsity can intersect, would be tantamount to establishing absolutism's truth! Establishing the suitability of CA_F is equivalent to showing that contradictions can be true. Yet, the absolutist needs a definition of contradiction that she can rely upon in her disagreement with the classical logician, and CA_F hardly gives her this firm ground. If possible, she would undoubtedly prefer an account of contradiction that was indisputably suitable to embed within Ab). In contrast, CA_F ensures that either absolutism is true or nonsense, by contravening the rules of an accepted definition. Thus, CA_F is somewhat paradoxically unsuitable for the absolutist's thesis given that demonstrating its suitability would be tantamount to showing that absolutism is true. Consequently, CA_T is *at least* as unsuitable for the absolutist's purposes as CA_F.

After careful consideration of versions of the classically-assumed account, we have been given good reasons to consider the account both philosophically implausible and unsuitable for the absolutist's purposes.

3.1.2 Explosion Accounts

Call a set of propositions Σ *explosive* if and only if, for any proposition B, $\Sigma \models B$. According to explosion accounts, a set of propositions Σ is a contradiction if and only if Σ is explosive. Endorsement of the explosion account is found in Field (2005a, p. 24),

> I think a better use of the term 'contradiction' would be: sentence that implies every other.

To fully appreciate the implausibility of the explosion account, it's necessary to be clear on why contradictions are considered to be explosive in classical logic. The standard semantic account of logical consequence (SSC), used in classical logic, states that,

$$\Sigma \vDash B \text{ iff for any } v, \text{ if } \forall A \in \Sigma, v(A)=1, \text{ then } v(B)=1.$$

Thus, according to SSC, unless the logic in question is trivial,[47] a premise set Σ is explosive if and only if there is *no* truth-value assignment under which every member of Σ receives the truth-value true. Given that in classical logic a proposition A being false is equivalent to A not being true, a premise set Σ is explosive if and only if the conjunction of all the members of Σ is a logical falsehood. Therefore, under the assumption that truth and falsity cannot intersect, a proposition A is explosive if and only if A is a logical falsehood. Consequently, there are two circumstances in which a *set of* propositions Σ is explosive:

E_C) The conjunction of all the members of Σ is a logical falsehood.

E_S) A member of Σ is a logical falsehood.

E_S) covers those singleton cases that E_C) doesn't. Thus,

A set of propositions Σ is *explosive* iff E_C) or E_S) are true of Σ.

Consequently, assuming the SSC, the explosion account entails:

A set of propositions Σ is a *contradiction* iff E_C) or E_S) are true of Σ.

Given that E_C) and E_S) are *definiens* of versions of the classically-assumed account of contradiction,[48] the explosion account seems to be a simple consequence of versions of the classically-assumed account and the SSC. Thus, assuming the SSC, the explosion account categorises as contradictions *at least* all of those sets of propositions categorised as contradictions by these versions of the classically-assumed account. With this in mind, it's worth revisiting some of the criticisms of the classically-assumed account and noting their relevance to the explosion account.

Firstly, as with the Guttenplan *et al.* account, the explosion account counts all propositions that are logically false as contradictions. So, for example, propositions of the form $\sim(A \vee \sim A)$ are categorised as contradictions, because they are instances of a logical

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[47] A logic L is trivial iff $\forall B\ \varnothing \vDash_L B$, which given weakening ensures $\forall A \forall B\ A \vDash_L B$.

[48] Specifically, E_C) is the *definiens* of CL_{Log}) without the proviso that one (and only one) member of Σ is true, and E_S) is the *definiens* of the Guttenplan *et al.* account.

form F that outputs the truth-value false under all truth-value assignments. We have already considered the counter-intuitive nature of this commitment and how it counts against any account that possesses it.

The failure of the explosion account to adequately differentiate between contradictions and logical falsehoods is manifested in its inability to distinguish two forms of explosion that we can distinguish in our formal systems:

C-explosion: $\{A \wedge \sim A\} \vDash B$.

F-explosion: $\bot \vDash B$.

Not only are C-explosion and F-explosion non-equivalent, but C-explosion isn't even a special case of F-explosion in some logics. With the introduction of the bottom particle into da Costa's **C-Systems**, for example, the logics would contain F-explosion and not C-explosion, for the **C-Systems** use the SSC while assigning some contradictions the truth-value true.[49] Thus, we are able to formally distinguish and model cases of explosion involving contradictions, as they are normally formally conceived, and cases of explosion involving arbitrary logical falsehoods. By defining contradictions as explosive sets of propositions, however, the explosion account is unable to accommodate this formally interesting distinction. As far as the explosion account is concerned, the premise sets of both forms of explosion are contradictions, and thus there's no distinction to be made between them. The explosion account, therefore, suffers from the weakness of blinding us to theoretically interesting formal results.

Secondly, the explosion account seems to entail that the LNC is insubstantial, although not as obviously as the classically-assumed account. Our previous discussion of the law gave us good reason to believe that the LNC is stating a *semantic* fact about contradictions, as pragmatic interpretations of the law required a total revision of our standard formalisation of the LNC, something which we would need very good reasons to undertake. Thus, let us assume for the moment that the LNC is stating a semantic fact about contradictions; namely, the fact that they are all false.

As it stands, the semantic interpretation of the LNC embedded by the explosion account of contradiction is nonsensical,

LNC_{EX}) All explosive sets of propositions are false.

Sets themselves cannot be false. So we must rephrase LNC_{EX}) by using E_C) and E_S) above,

LNC_{EX}') All explosive propositions are false, and for every explosive set of propositions $\mathbf{\Sigma}$, the conjunction of all $A \in \mathbf{\Sigma}$ is false.

[49] See section 4.4.2 below.

While LNC_{EX}') makes sense, it is totally uninformative. We already knew that explosive propositions are false. After all, a proposition A is explosive *because* it is *logically* false, assuming the SSC, and similarly for explosive sets of propositions.[50] Thus, by understanding the SSC we already know LNC_{EX}'). The law of non-contradiction, embedded with the explosion account, simply reiterates a result of the SSC – hardly the fundamental logical law.[51] While the LNC could be retained in its form LNC_{EX}') for its pedagogic form, expressing succinctly a surprising logical fact, this isn't how the law has been historically viewed. To move the law from the status of substantial logical principle, which Aristotle (1971a, Γ 1005[b] 22) once called the most certain of all principles, to a mere shorthand for a chain of reasoning that can be easily explained seems ill-motivated.[52] Demonstrating the truth of the LNC isn't synonymous with explaining the SSC and the fact that not every proposition is logically true. By embedding the explosion account of contradiction into the LNC, the law seems to lose all philosophical significance.

In addition, it's worth noting that LNC_{EX}') creates problems when one attempts to find a formalisation of the law within a logical system. Consider, firstly, the first conjunct of LNC_{EX}'): 'All explosive propositions are false'. What the standard formalisation of the law tells us is that propositions of the form $\sim(A \wedge \sim A)$ are logically true, which given the normal semantics for negation entails that propositions of the form $A \wedge \sim A$ are logically false. This formalisation tells us nothing directly about explosive propositions. It's only through another principle, the conjoined form of explosion $\{A \wedge \sim A\} \vDash B$, that we become informed that formulae of the form $A \wedge \sim A$ symbolise (some) explosive propositions. Given that not every explosive proposition, according to the SSC, is of the form $A \wedge \sim A$, even the combination of $\vDash \sim(A \wedge \sim A)$ and $\{A \wedge \sim A\} \vDash B$ doesn't adequately formalise 'All explosive propositions are false'.[53] While the advocate of the explosion account may be able to find an adequate formalisation of the first conjunct of her interpretation of the law, LNC_{EX}'), two points are

[50] There is one potential exception to this rule, a proposition which explodes although it might not be considered logically false, which we consider below. This one potential exception, however, hardly affects the strength of this criticism.

[51] As the SSC only informs us that a proposition A is explosive if and only if A is either logically false or every proposition B is logically true, we might think that LNC_{EX}') informs us of something which the SSC doesn't. Namely, that not every proposition B is a logical truth. However, this clearly isn't the perceived content of the LNC by the philosophical community, nor do we need a philosophical principle to inform us that there are propositions of the form p that aren't logical truths (a simple consequence of the SSC).

[52] While Barnes (1969) has argued that Aristotle (1971b, A 74[b] 5-38) considered syllogistic demonstrations to have a solely pedagogical use, there's no evidence that Aristotle equally considered his presentation of the logical laws to be solely pedagogical. For example, the fact that Aristotle offers justification for the LNC suggest that he wasn't merely presenting an already well-established logical or metaphysical fact. For more on Aristotle's arguments for the LNC see Łukasiewicz (1971), Dancy (1975), Code (1986), Cohen (1986), Inciarte (1994), Priest (1998), and Wedin (2004).

[53] Nor does $\perp \vDash B$, as this only informs us that every logically false proposition is explosive, not that every explosive proposition is false.

clear. Firstly, her interpretation of the law requires us to revise the standard formalisation of the LNC, something that the explosion account would have to offer a substantial justification for. Secondly, we will not be able to formalise LNC_{EX}') as a theorem of any propositional logic; at best, the law will have to be formalised as a conjunction of theorems. Given that the explosion account hasn't demonstrated the theoretical virtues necessary to both justify a change in our formalisation of the LNC, and deny LNC the status of theoremhood in propositional logic, both of these consequences of the attempt to formalise LNC_{EX}') seem damaging for the account.

Attempts to formalise the second conjunct of LNC_{EX}'), 'For every explosive set of propositions Σ, the conjunction of all $A \in \Sigma$ is false', are even more damaging for the account. Given that propositional logic contains no quantifiers or set-theoretic apparatus, no propositional logic can formalise the falsity of the conjunction of all $A \in \Sigma$ for any explosive set Σ, even if the logic could formalise Σ's explosiveness, $\Sigma \vDash B$. Thus, not only does the explosion account's interpretation of the LNC, LNC_{EX}'), require a formalisation of the law that denies the law theoremhood in any propositional (and, indeed, higher-order) logic, but it cannot be formalised in a propositional logic at all. Again, given that we conventionally conceive of the LNC as a theorem of propositional logic, it is quite a burden the explosion account places upon itself by ensuring that the law cannot even be formalised within a propositional logic.

Our third criticism of the classically-assumed accounts that's relevant to the explosion account is its failure to effectively distinguish between contradictory sets of propositions and contradictions. The account, therefore, fails to facilitate a philosophically important distinction between those set of propositions that *entail* a contradiction, those that *contain* a contradiction, and those that are *constituted exclusively* by a contradiction. We want to be able to say that a theory contains or entails a contradiction, and thus is contradictory, while also picking out particular elements of a theory that constitute a contradiction.

We can see that the explosion account fails to distinguish between a set of propositions containing a contradiction and a set constituted exclusively by a contradiction through weakening: for any proposition A, if $A \vDash B$, then for any set of propositions Γ, $\{\Gamma, A\} \vDash B$. Thus, if a proposition A is explosive, any sets containing A are also explosive. Consequently, any set Γ containing an explosive proposition is, according to the explosion account, a *contradiction.* Additionally, the failure of the explosion account to distinguish between sets of propositions that entail a contradiction, and sets constituted exclusively by a contradiction, can be demonstrated by considering a set of propositions of the form $\{B, A \rightarrow \sim B, A\}$. While this set of propositions doesn't *contain* a contradiction formally conceived, it does *entail* a contradiction, and this ensures that it's explosive. Therefore, according to the explosion account the set of propositions is as much a contradiction as the proposition of the form $A \wedge \sim A$ that can be derived from the proposition set. Alterations will need to be made to the explosion account if it is to allow a distinction between these three intuitively distinct types of proposition sets.

Two possible amendments to the explosion account, proposing a rescue of the distinctions, are worth mentioning here. Firstly, the account could restrict itself to explosive propositions, rather than explosive sets of propositions. So, a proposition A is a contradiction if and only if A is explosive, and no non-singleton sets of propositions are contradictions. This would ensure that although a proposition A would be a contradiction if $\forall B\ A \vDash B$, the set of propositions $\{\Gamma, A\}$ derived through weakening wouldn't be. Instead, $\{\Gamma, A\}$ could be said to *contain* a contradiction. Similarly, sets of propositions of the form $\{B, A \rightarrow \sim B, A\}$ could be said to *entail* a contradiction, as they entail an explosive proposition. Thus, by restricting the account to explosive *propositions*, it is able to distinguish between the three distinct types of proposition sets. This, however, doesn't stop the account from being inadequate for other reasons. As we mentioned earlier, there are many explosive propositions that wouldn't normally be considered contradictions in the philosophical community.

The second possible amendment retains the original definition of the explosion account with the additional clause that a set of propositions Σ is a contradiction only if Σ is explosive but *no* proper subset Γ of Σ is explosive. Contradictions and sets of propositions that *contain* contradictions now can be distinguished. However, this amendment fails to deal with those proposition sets, such as those of the form $\{B, A \rightarrow \sim B, A\}$, that fulfil the new criteria to be a contradiction but, according to our usual formal standards, only *entail* a contradiction, rather than being constituted of, or containing, a contradiction. Therefore, only the first possible amendment is wholly successful in allowing a distinction to be made between the three intuitively distinct types of proposition sets; the criticism can only be answered by restricting the account to contradictions as explosive *propositions*.

In addition to these three revisited criticisms, there are two fresh reasons to doubt the plausibility of the explosion account. Firstly, there is at least one explosive proposition that is neither commonly considered to be a contradiction or a logical falsehood in propositional or first-order logic:

> 'Every proposition is true'.

Thus, it seems the explosion account categorises at least one non-apparent contradiction as a contradiction. This criticism is putatively distinct from the previous, that the explosion account doesn't adequately distinguish contradictions and logically false propositions, as 'Every proposition is true' putatively isn't a logical falsehood. Indeed, for this troublesome proposition to count as a logical falsehood, we would need to admit a logic that both allowed us to quantify over propositions and included truth and falsity predicates.[54] Therefore, if one didn't admit any logic beyond first-order plus identity, this proposition would constitute a new challenge to the explosion account. Two responses to this troublesome case are available to the explosion account.

Firstly, the advocate of the explosion account could appeal to the principle of reflective equilibrium and bite the bullet with this example, thereby considering the proposition to be a contradiction. It is, after all, only one example. Yet, for a theory to be justified in absorbing anomalies we need to have some independent support for the theory. There is no justification at all for an unsuccessful theory to absorb anomalies. Given that the explosion account possesses at present none of this independent support, the account would be unjustified in absorbing the anomaly to deflect the counterevidence against it.[55]

Secondly, the account could adapt its definition slightly so that contradictions were not just those explosive sets of propositions Σ or propositions A, but were those sets of propositions Σ, or propositions A, that exploded *due to their logical falsity*. Thus, a (set of) proposition(s) would only be categorised as a contradiction by this new definition if it were explosive because of its logical form and the SSC. This response would successfully block the proposition above from being categorised as a contradiction by the explosion account for, as stipulated by the counterexample, it isn't a logical falsehood. In giving this reply, however, the advocate of the explosion account is entering dangerous ground. By adding the condition that contradictions must explode because of their logical form, she ensures that contradictions are of a logical form that no other (sets of) proposition(s) are. Now, if contradictions are of this unique logical form, it's bemusing why we should define contradictions as those (sets of) proposition(s) that explode due to their logical form, rather than directly in terms of their unique logical form. After all, if we defined contradictions in terms of this unique logical form, we would be able to explain *why* contradictions are often considered to be explosive, rather than simply stipulating that they are. When it is a choice between stipulation and explanation,

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[54] With such a logic, we can easily show that the proposition was a logical falsehood by *reductio*. Let T be the truth predicate and F the falsity predicate:

1.	$\forall pTp$		Assumption
2.	Tp	{1}	Universal Elimination
3.	p	{2}	T-predicate Release [$Tp \rightarrow p$]
4.	$T(Fp)$	{1}	Universal Elimination
5.	Fp	{4}	T-predicate Release
6.	$\sim p$	{5}	F-predicate Release [$Fp \rightarrow \sim p$]
7. $\sim\forall pTp$		{1-6}	*Reductio*

[55] Note that *arguing* for the proposition 'Every proposition is true' to be considered a contradiction isn't an option for the advocate of the explosion account. If contradictions are *just* sets of propositions Σ, or propositions A, that explode, then there shouldn't be any other properties for the advocate of the explosion account to appeal to in arguing that the proposition is a contradiction. If the (set of) proposition(s) is explosive, then it's a contradiction as far as the explosion account is concerned; there are no further identifying properties of contradictions.

the account that offers the explanation seems the more theoretically fruitful option. Therefore, in adding this condition to the explosion account, its advocate is almost admitting that there are other more explanatorily fruitful accounts of contradiction available. To this extent, it hardly seems a viable solution to the troublesome proposition for the explosion account. Consequently, neither proposed solution to the troublesome case of 'Every proposition is true' has been successful. It still seems the explosion account has miscategorised at least one proposition as a contradiction.

Our second, and final, new criticism of the explosion account is that it precludes the possibility of some *paraconsistent* logics through stipulation. Paraconsistent logics invalidate an unconjoined form of C-explosion $\{A, \sim A\} \vDash B$, and some, such as Priest's **LP** and da Costa's **C-systems**, also invalidate C-explosion $\{A \wedge \sim A\} \vDash B$.[56] Now, these latter paraconsistent logics, in invalidating C-explosion, are ordinarily understood to be showing that *contradictions* are not explosive.[57] Yet, by defining contradictions as those (sets of) proposition(s) that are explosive, the explosion account precludes the possibility of this very serious research area on purely definitional grounds. There can, according to the explosion account, be no logics that model non-explosive contradictions, as this would contravene the meaning of 'contradiction'. This consequence of the account seems totally inappropriate. Paraconsistent logics that invalidate C-explosion have been successfully constructed, and they cannot be simply disregarded on definitional grounds.

For a potential solution to this concern, we can look to an actual advocate of the explosion account. Field (2005a) has suggested that paraconsistent logicians aren't in fact talking about contradictions when they state that 'Contradictions are not explosive'. After all, they can't be, because *contradictions explode*:

> Priest revels in saying that we should accept contradictions. Here 'contradiction' is used either in the sense indicated above (a pair of a sentence and its negation) or in the sense of a sentence of the form $B \wedge \sim B$… Talk of accepting contradictions shows a flair for the dramatic, but I think it tends to put people off for bad reasons. Given the kind of logic Priest advocates, I think a better use of the term 'contradiction' would be: sentence that implies every other. On this alternative usage, the way to put Priest's view is that sentences of form $B \wedge \sim B$ (or pairs $\{B, \sim B\}$) aren't in general contradictory: they don't imply everything.
>
> (Field (2005a) p. 24)

Field, therefore, allows for paraconsistent logicians to articulate their point while requiring they don't use the term 'contradiction'. The *content* of the paraconsistent logician's theories remain, while their terminology is revised. The problem, one might think, has been avoided.

[56] For more on paraconsistent logics see chapter 4.

[57] Indeed, the whole point of Priest's **LP** is to support the dialetheic hypothesis that some contradictions are true without triviality ensuing.

The possibility of paraconsistent logics that invalidate C-explosion has no longer been precluded.

Unfortunately, the proposed solution has some unsavoury consequences. Dialetheists use paraconsistent logics that invalidate C-explosion, while challenging the necessary *un*truth of contradictions, because they wish to admit the truth of contradictions *without* triviality. Yet, according to the explosion account, if a (set of) proposition(s) isn't explosive then it isn't a contradiction. It turns out then that those propositions dialetheists propose as contradictions, and propose the truth of, aren't actually contradictions (as Field admits in the quote). What then are dialetheists challenging if not the necessary *un*truth of contradictions? According to Field, it would be the necessary *un*truth of the conjunction of a proposition and its negation. Yet, there is an available account of contradiction that defines contradictions just as the conjunction of a proposition and its negation.[58] What reasons then does Field give us to prefer the explosion account of contradiction to this rival definition, when it seems the explosion account requires us to unnecessarily distort the debate between the classical logician and the dialetheist? He proposes that by defining contradictions as the conjunction of a proposition and its negation, we "put people off [the dialetheist's thesis] for bad reasons." But, one of reasons that philosophers *are* concerned by dialetheism is that it challenges the historic LNC in some form, and what is the LNC about if not *contradictions*? Dialetheism could hardly be offering violations of the LNC if it were not talking about contradictions. Given that we possess available accounts of contradiction that can explain how dialetheists are proposing the truth of *contradictions*, and thus proposing violations of the LNC, we need to be given some good reason to consider the term 'contradiction' inappropriate for use by a dialetheist. Field's reason isn't one.[59]

There is something theoretically damaging about an advocate of explosive logics precluding the possibility of 'contradiction' entering the vocabulary of non-explosive logics. While it's an option for advocates of explosive logics to insist that non-explosive logics just don't talk about contradictions, that contradictions don't enter those logics' semantics, it seems theoretically unfruitful for explosive logics to have a monopoly on the term. It promotes the appearance of incommensurability between the logics when there is none. We understand perfectly what the paraconsistent logician means when she says that contradictions

[58] See section 3.1.3 below.

[59] The same point can be made with regards to non-dialetheic paraconsistent logicians who propose the invalidation of C-explosion. Given that the paraconsistent logician isn't talking about contradictions, their position would be correctly stated as:

> Those propositions the classical logician considers to be contradictions *aren't* contradictions.

No one speaks in these terms though, not even classical logicians. The great challenge that paraconsistent logics propose is that *contradictions (or, contradictory sets of propositions) might not be explosive*.

don't entail every proposition. Any account of contradiction that demonstrates there is no incommensurability between explosive and non-explosive logics has a theoretical advantage over the explosion account.

We have proposed several weaknesses of the explosion account, some new and some of which we had previously noted as weaknesses of the classically-assumed account. It seems unlikely then, given the sheer number of weaknesses that the account possesses, that it will succeed in being a plausible account of contradiction. This, consequently, ensures that it's also unsuitable for the absolutist's purposes. Yet, additionally, even if the account *were* philosophically plausible, it would still be unsuitable to embed within Ab). According to the explosion account, every contradiction explodes by definition. Therefore, any theory that proposes the (possible) truth of a contradiction so understood is also committed to the (possible) truth of every proposition. Embedding the explosion account of contradiction into Ab) would commit the absolutist to modal trivialism, a position she wishes to distance herself from.[60] Hence, the explosion account is totally unsuitable for the absolutist's purposes.

3.1.3 Truth-Value Neutral Accounts

Our third and final semantic account of contradiction is the truth-value neutral account. Instances of this account define contradictions in terms of the conjunction of a proposition and its negation, without taking a stand on the truth-value(s) of the conjunction of a proposition and its negation. An example of the account is found in Kalish *et al.* (1980, p. 18),

> A contradiction consists of a pair of sentences, one of which is the negation of the other.[61]

While Grim (2004, p. 53) defines the account as *syntactic*, this seems a mistake. Before we consider the account's plausibility, it's worth considering why the account is semantic and not syntactic. As versions of the account are about a truthbearer and its negation, the account's status as semantic or syntactic is dependent upon whether it defines contradictions in terms of semantic or syntactic objects, or, at least, whether the account is concerned with these objects' semantic or syntactic properties. Let us then briefly consider the objects that the account defines contradictions in terms of.

Both a truthbearer and its negation are, obviously, truthbearers. Now, whether a truthbearer *p* is *both* a semantic and syntactic object, or *solely* a syntactic object, is dependent on

[60] As we noted in section 2.3.4, and we note again in chapter 4.

[61] Similar accounts of contradiction are embedded in the Price$_1$, Englebretsen$_1$ and Goldstein$_1$ accounts of the LNC. In stating that "[a] dialetheia is a true contradiction, a proposition (or, again, truth bearer), *p*, such that both *p* and its negation, ~*p*, are true," Armour Garb (2004, p. 114) also seems to endorse the truth-value neutral account.

the type of truthbearer *p* is. While some types of truthbearer, such as declarative sentences, have both syntactic and semantic properties, others, such as propositions, are commonly conceived to possess solely semantic properties. Given that some truthbearers are both semantic and syntactic objects, the truth-value neutral account isn't a semantic account directly in virtue of the nature of truthbearers. However, two additional considerations should persuade us that the account is concerned with the *semantic properties* of the truthbearers.

Firstly, what makes a truthbearer *q* the negation of a truthbearer *p* are the respective semantic properties that the two truthbearers possess. Therefore, in defining a contradiction in terms of a truthbearer and its negation, the truth-value neutral account is interested in the semantic relationship between the two truthbearers and less so with the truthbearers' syntactic properties. To demonstrate this, consider the many possible grammatical devices that exist in English for one to *negate* a truthbearer. In negating one's friend's statement that 'Emma believes that the Earth is flat' one could choose from at least: 'You're mistaken', 'Emma doesn't believe that', 'That simply isn't the case', or 'If you think Emma believes that, you're wrong'. Thus, faced with the task of negating a proposition in English one has a multitude of words and sentence constructions available. What makes all these sentences *instances* of the negation of 'Emma believes that the Earth is flat' isn't a set of syntactic properties $\mathfrak{I}$ that they all possess. After all, natural languages are so malleable that the formulation of new well-formed formulae expressing the negation of this statement, while not possessing any of the properties in $\mathfrak{I}$, is always possible. Instead, the sentences are all instances of the negation of the proposition 'Emma believes that the Earth is flat' because they all have the same semantic *relationship* to that proposition. All of the sentences, regardless of their different syntactic properties, express the negation of our friend's statement. That we even speak of a sentence *expressing* the negation of a proposition ensures that negations of propositions are defined by their semantic properties. Syntactic entities are not expressed, meanings are. At most, syntactic entities are the vehicles for expressing meaning. While the syntactic properties of the sentences with which we negate a proposition *p* can be studied, a sentence *s* expresses the negation of *p* in virtue of its semantic content and not its syntactic properties. The negation *q* of a proposition *p*, therefore, is defined by the semantic properties that *q* possesses in relation to *p*, whatever these properties are.[62] Consequently, the account is concerned with the semantic properties of truthbearers.

Secondly, and on a similar note, it's perfectly meaningful to speak of a proposition and its negation, with 'proposition' interpreted as a truthbearer with no syntactic properties. Given that the account can use a purely semantic truthbearer without any loss of

[62] We must be careful to distinguish a proposition's negation and the unary truth-functor that we use to model a proposition's negation. The negation of a proposition *p* is another proposition *q*, and *q* is *p*'s negation in virtue of the semantic properties both possess. The unary truth-functors called 'negations' found in propositional logics, normally symbolised by '~' or '¬', are formal logic's attempt to model the semantic relationship between a proposition and its negation.

meaningfulness, this suggests that the properties of truthbearers that are primarily relevant to the account are their semantic properties. Both considerations give us good reason to categorise the account as semantic. To categorise it in any other terms would be misleading. We can now move on to consider the philosophical plausibility of the account.

In outlining the account's plausibility let's begin by noting how it doesn't fall foul of the criticisms levelled against the previous two accounts. Firstly, the account doesn't trivialise the law of non-contradiction. The claim that 'All conjunctions of a proposition and its negation are false' is substantial indeed. It tells us something about the meaning of natural-language negation and conjunction. By being informative the law leaves itself open to challenge.

Secondly, the account does not define counter-intuitive cases as contradictions, unlike the previous accounts. Neither propositions of the form $\sim(A \vee \sim A)$, nor the proposition 'Every proposition is true', are defined as contradictions, as they are not the conjunction of a proposition and its negation. The account can explain, however, the temptation to call some of these propositions contradictions when we realise that they entail the conjunction of a proposition and its negation. The De Morgan laws give us just this temptation with propositions of the form $\sim(A \vee \sim A)$. The account manages to explain this without committing us to the theory that the property of being a contradiction is backwardly preserved through the entailment relation. The account can see, and explain, the temptation of categorising these propositions as contradictions, but resists the temptation and gives us the explanatory power with which to distinguish contradictions and (sets of) propositions that *entail* contradictions.

Similarly, the account possesses the explanatory power to adequately distinguish between contradictions, contradictories and sets of contradictory propositions. By defining contradictions as a conjunction of a propositions and its negation, the account can identity contradictories as the two conjuncts of a contradiction, which remain as *just* contradictories until they are conjoined.[63] Of course, in a logic in which adjunction is valid, two contradictories can always be conjoined to form a contradiction but, again, the account doesn't commit us to the backward preservation of the property of being a contradiction through entailment. There is, then, a clear distinction between contradictories and contradictions that the truth-value neutral account facilitates, giving us yet more expressive power. The account deals with sets of contradictory propositions comparably, except that the concept of a set of contradictory propositions ambiguously stands for four distinct types of proposition sets. The first is a set constituted solely of a contradictory pair $\{A, \sim A\}$, the second is a set that contains a contradictory pair in addition to other members

[63] Note that the Kahane *et al.* version of the truth-value neutral account is inadequate for not defining contradictions as the *conjunction* of a truthbearer and its negation, and therefore failing to adequately distinguish *contradictions* and *contradictories*.

$\{A, B \wedge C, \sim A\}$, the third is a set that contains a *contradiction* in addition to other members $\{B, C \vee B, A \wedge \sim A\}$, and lastly it's a set that entails a contradiction, normally assuming classical semantics, such as $\{A \rightarrow \sim B, A, B\}$. It's common for all four types to be called sets of contradictory propositions. The truth-value account explains the reason for the first and second sets to be called 'sets of contradictory propositions', for the sets contain contradictories. If we wished to distinguish the two cases we could easily achieve this by calling the former 'a set of contradictory *pairs*' and the latter 'a set of contradictory *propositions*'. While the use of the term 'sets of contradictory propositions' to denote the third and fourth types of proposition sets is understandable, the truth-value neutral account allows us to make a reasoned distinction between these and the first two cases. In the third case, the set isn't just that of contradictory propositions, but it instead *contains* a contradiction, and in the fourth case, there are no contradictories or contradictions present in the set; instead, a contradiction is *derivable* from the set of propositions. Our ability to distinguish these four cases, facilitated by the truth-value neutral account, isn't just a naming exercise. By considering the sets above as not just sets of propositions, but instead sets of beliefs, the distinctions become a lot more important.[64]

Thirdly, the account can explain why the previous two semantic accounts we considered have gained support. It can achieve this by demonstrating how they follow from a combination of the truth-value neutral account and well-established background assumptions. By demonstrating this, the account shows itself to be somehow more philosophically basic, untouched by these background assumptions embedded in the other accounts.[65] The pull of the classically-assumed account can be explained by the truth-conditions given to negation and conjunction in classical semantics. Given that the negation of a proposition p is true if and only if p is false, and a conjunction is true if and only if both conjuncts are true, the conjunction of a proposition and its negation in classical semantics will always receive the truth-value false. Additionally, the truth-value neutral account can explain Prior and Sainsbury's version of the classically-assumed account by showing that although the conjunction of a proposition and its negation is always false in classical semantics, necessarily one and only one of a proposition and its negation is true. Similarly, it can explain the explosion account in terms of the truth-value neutral definition of contradiction, classical

[64] Given that beliefs aren't closed under entailment, it's important to distinguish cases in which an individual believes contradictories, $B(p)$ and $B(\sim p)$, and cases in which an individual believes a contradiction, $B(p \wedge \sim p)$. The same is true of the distinction between an individual believing a set of propositions that entail a contradiction, and an individual believing a contradiction. Additionally, distinguishing between believing a set of propositions that entail, but don't contain, a contradiction, and believing a contradiction, is necessary for making sense of claims that we can rationally hold a set of beliefs that entail a contradiction but *not* a contradiction itself; see Foley (1979 & 1992).

[65] This isn't to say that the truth-value neutral account doesn't have its own assumptions embedded, as it's natural to think any philosophical account will. Rather, the suggestion is that the account can show it has fewer assumptions embedded into it by demonstrating that the other accounts are a product of it and other assumptions.

semantics, and the SSC. By being able to give reductive explanations of the previous semantic accounts, the truth-value neutral account seems to show itself to be explanatorily prior to them. It can *account* for more.

Lastly, by ensuring that there's a distinction between contradictions and logical falsehoods, the account supports the distinction between C-explosion and F-explosion made in certain paraconsistent systems, while respecting the paraconsistent logician's research programme by not defining contradictions as explosive propositions.

Beyond the truth-value neutral account not falling foul of many of the criticisms levelled against the previous accounts, it also points in the direction of a real debate between those who disagree over the truth of contradictions. Given that, according to the truth-value neutral account, a contradiction is the conjunction of a proposition and its negation, it would seem sensible to suppose that those who disagree over the possible truth of contradictions disagree over the semantics of conjunction or negation. Impressively, this is exactly what we find in the literature, disagreements over the semantics of negation.[66] Therefore, the account has successfully predicted a prominent disagreement between those involved in the debate over the possible truth of contradictions. Additionally, by not assuming any particular theory of negation, the account facilitates disagreement between the parties over the semantic properties of negation while giving the parties that solid basis of agreement, a common definition of contradiction. Here then is an account of contradiction that both captures the intuitive meaning of contradiction, and signals a direction of debate between those who accept the possibility of true contradictions and those who don't. Any account that can predict a prominent question in the debate, while clarifying what the two parties agree over, has great promise.

Although the truth-value neutral account has great plausibility, we need to be careful over the truthbearers we choose to use in the account. Kalish *et al.*'s version of the account uses *sentences*, and this can lead us to calling a conjunction of truthbearers a contradiction when they aren't. Consider two complications. Firstly, some sentences can have the appearance of being the conjunction of a sentence and its negation while not being a contradiction because of implicature. We can correctly understand the sentence as the conjunction of a sentence and its negation, while also appreciating that the speaker doesn't intend a contradiction with the sentences due to context. Two contexts where we would be wary to assign a contradiction to a speaker are: Firstly, in cases of indeterminable situations where the individual is attempting to express the indeterminacy of the situation; a famous example of this being the use of 'It's raining and it's not' to describe drizzly weather conditions. Secondly, when the sentence and negation are used for emphasis, such as a foul sports coach shouting 'My grandmother swims

[66] See Priest (1990).

faster than you, AND she can't swim', in which case the speaker doesn't actually endorse both conjuncts, but pretends to for dramatic effect.

Secondly, Kalish *et al.*'s account allows for a conjunction of a sentence and its negation to be a contradiction even when the two conjuncts are spoken by different individuals, and thus isn't intuitively a contradiction. For example, individual A can claim 'I am Swedish', while individual B can claim 'I am not Swedish', and although we have a conjunction of a sentence and its negation here, it isn't a contradiction. This is an example of a more general problem sentences face with regards to indexicals.

Declarative sentences, therefore, aren't suitable to serve as the truthbearer for the truth-value neutral account. The account needs a truthbearer that effectively cashes out indexicals, while allowing for a sensitivity of context. Which truthbearers are best suited to this job isn't a subject that we will take up here. Whether there are any truthbearers that aren't plagued by the context problem, or whether allowances for context can ever effectively be built into an account of contradiction, are questions that we will leave open.

In addition to the account's philosophical plausibility, it has the benefit of being suitable for the absolutist's needs. By not assuming classical semantics, or the explosiveness of contradictions, the account doesn't preclude the truth of absolutism. If it turns out that the correct semantics for conjunction and negation are their proposed classical semantics then so be it, it will turn out that contradictions are never true, but at least the definition of contradiction itself won't have precluded the truth of absolutism. The account also acts as a neutral foundation on which the absolutist and classical logician agree, thereby giving them propositions over which they can meaningfully disagree. They agree over what a contradiction is, but disagree over other matters that relate to the possible truth of contradictions, such as the semantics of negation. This is overall beneficial for the absolutist, as it indicates that there can be meaningful debate over the possible truth of contradictions. Their theory isn't nonsense after all. The truth-value neutral account is both philosophically plausible and suitable to embed within Ab).

3.2 Syntactic Accounts

Syntactic accounts define contradictions in formal terms. We are considering here two varieties of a syntactic account. The first, *form* accounts, define contradictions in terms of logical form, and the second, *reductio* accounts, define contradictions in terms of certain rules of logical inference.

3.2.1 Form Accounts

Form accounts identify contradictions with a certain logical form. Versions of the account are found in:

Arruda (1980, p. 3): "[A] formula of the form $A \wedge {\sim}A$ is called a contradiction."

Haack (1978, p. 244): "Contradiction: Wff* [well-formed formulae] of the form '$A \wedge {\sim}A$'; statement of the form '*A* and not *A*'."

Beall (2001, p. 114): "[A sentence] *p* is a *contradiction* iff *p* is of the form $p \wedge {\sim}p$."[67]

Olin (2003, p. 21): "Contradictions (statements of the form '$A \wedge {\sim}A$')."

There are two interpretations of what's being stated in form accounts. The first, the *formulae* interpretation, following Arruda's and Haack's suggestion that contradictions are formulae of the form A & ${\sim}A$, defines contradictions as formulae of a certain form, and the second, the *proposition* interpretation, defines contradictions as propositions of a certain form. Before we move on to discuss the *proposition* interpretation, we should briefly note why the *formulae* interpretation is so implausible.

Whatever contradictions are, they are the right kind of objects to be true or false. It's certainly meaningful to say that a contradiction is false. Formulae, however, aren't truthbearers; they can't be true or false. Additionally, one can *believe* or *assert* a contradiction, yet it makes no sense to have a doxastic attitude towards, or endorse, a purely syntactic object. By defining contradictions as formulae, we neglect both properties we regularly apply to contradictions and attitudes we sometimes have towards them.

This leaves us with the *proposition* interpretation. From now on, to avoid ambiguity, let's call this account of contradiction the *proposition-form* account. According to this account, contradictions are those propositions of the logical form *F*, where *F* in the definitions above has been replaced with '$A \wedge {\sim}A$'.[68] What, though, is it for a proposition to be of the logical form $A \wedge {\sim}A$? What properties must it possess? There are two distinct plausible answers to

[67] Cf. Beall (2004a) p. 4.

[68] We are assuming here, as we have throughout, standard propositional notation. So, given that contradictions *are* symbolised by formulae of the form $A \wedge {\sim}A$ in standard propositional notation, it's beyond doubt that if contradictions are *defined* by being of a logical form *F*, then *F* will be $A \wedge {\sim}A$. The question is whether contradictions are *defined* by being of this logical form or not. If one were using Polish notation then the situation would be exactly the same, except that '$A \wedge {\sim}A$' would be replaced with 'NK*p*N*p*'.

this question.[69] The first, the *truth-conditional* account of logical form, defines the logical form of a proposition as the proposition's truth-conditions, which are regimented by a logical system in the shape of well-formed formulae.[70] The second, the *constituent* account of logical form, defines the logical form of a proposition as the sum of the proposition's constituent semantic parts, which is regimented by a logical system in the shape of well-formed formulae.[71] To demonstrate why the choice between these two accounts of logical form matters, it may be useful to show how the accounts diverge over the logical form they assign certain propositions. Consider the three propositions,

P1) It's not the case that if it's raining then it's raining.

P2) It's both raining and not raining.

P3) It's not the case that it's raining or it's not raining.

Now, according to the *truth-conditional* account of logical form, under classical semantics all of the propositions above have the same logical form because they are all logical falsehoods.[72] All, therefore, can be symbolised by formulae of the form $A \wedge {\sim}A$, as they will output the truth-value false for all truth-value assignments to their variables. In contrast, according to the *constituent* account of logical form, all of the propositions P1-P3) have different logical forms because they are composed of different semantic constituents: P1) is the negation of a conditional, P2) is the conjunction of a proposition and it's negation, and P3) is the negation of the disjunction of a proposition and it's negation. Thus, the distinct formalisations of the propositions would be:

$P1_F$) ${\sim}(p \rightarrow p)$

$P2_F$) $p \wedge {\sim}p$

[69] We won't be considering here the *syntactic* account of logical form found in current grammatical theory, *Logical Forms* (**LF**), as an option for the advocate of the proposition-form account. This is for two reasons. Firstly, we are interested in accounts of logical form that aim to explain why propositions possess particular logical forms in propositional and first-order logics (at least), and **LF** doesn't deliver this with its *syntactic* categories. Secondly, it's a presumption of modern logic that even purely semantic truthbearers can have a logical form, and thus we require *semantic* accounts of logical form to explain the logical form of these *solely semantic* truthbearers. For more on **LF** see May (1985) and Chomsky (1995).

[70] The truth-conditional account of logical form is discussed in LePore & Ludwig (2002) and Iacona (2013).

[71] Examples of the constituent account of logical form are actually found in definitions of contradiction. For example, Beall (2004a, p. 4) defines contradictions as "sentences *of the form* $A \wedge {\sim}A$, where $\wedge$ is conjunction and… ~ is negation. In other words, a contradiction, on the formal usage, is the conjunction of a sentence and its negation."

[72] It's harmless to assume classical semantics here. Most philosophers accept classical semantics and, thus, we are demonstrating that for *at least* most philosophers the two accounts of logical form diverge in the logical form they assign some propositions.

$P3_F$) $\sim(p \vee \sim p)$.

Although all of P1-P3) have the same truth-conditions under classical semantics, this doesn't suffice for the propositions to share the same logical form according to the *constituent* account.[73]

If we return to the definition of contradiction given by the proposition-form account,

PF) A proposition is a contradiction iff it's of the logical form $A \wedge \sim A$,

we see disagreement between the two accounts of logical form over which of the propositions above are contradictions. While the *truth-conditional* account defines all of P1-P3) as contradictions, the *constituent* account only categorises P2) as a contradiction. The plausibility of the proposition-form account, therefore, is bound to vary depending upon the account of logical form embedded within it. We should consider then both versions of the account:

PF_T) A proposition is a contradiction iff it possesses the same truth-conditions as are assigned to formulae of the form $A \wedge \sim A$.

PF_C) A proposition is a contradiction iff it possesses the same constituent semantic parts as are assigned to formulae of the form $A \wedge \sim A$.

Beginning with PF_T), there are two available interpretations of the account. The first assumes a classical semantics for the account so that it becomes,

PF_T^C) A proposition is a contradiction iff it possesses the same truth-conditions as are assigned to formulae of the form $A \wedge \sim A$ *in classical logic*,

and the second doesn't assume any particular truth-conditional semantics. Unfortunately, both interpretations leave the account implausible.

PF_T^C) categorises as contradictions the same set of propositions as the Guttenplan *et al.* account discussed above;[74] formulae of the form $A \wedge \sim A$ output the truth-value false for all truth-value assignments. The only difference between the two accounts is that PF_T^C) articulates the truth-conditional properties of contradictions via formulae with the same truth-

[73] Note that although the accounts disagree over the logical form of certain truthbearers, both agree that the logical form of a truthbearer cannot simply be read off its surface grammar. Indeed, solely semantic truthbearers don't possess any surface grammar to read logical form off.
[74] Section 3.1.1.

conditional properties.[75] Consequently, PF_T^C) suffers from at least the same faults as the Guttenplan *et al.* account, which we needn't revisit here.

The second interpretation of PF_T) is even more implausible. By not assuming a particular semantics, PF_T) either defines contradictions universally for all logics so that,

> PF_T^A) A proposition is a contradiction iff it possesses the same truth-conditions as are assigned to formulae of the form $A \wedge {\sim}A$ in *any* logic L,

which has obviously absurd consequences, given the possible meanings that a logic can assign its logical constants. Or, PF_T) can define contradictions relative to a logic L,

> PF_T^R) A proposition is a contradiction in a logic L iff it possesses the same truth-conditions as are assigned to formulae of the form $A \wedge {\sim}A$ in L.

To demonstrate PF_T^R)'s implausibility, let's concentrate on the effect it has on the debate between the classical logician and the dialetheist. By allowing for the definition of a contradiction to be relative to a logic, PF_T^R) ensures that the dialetheist using **LP** and the classical logician possess divergent definitions of 'contradiction'. Making the required changes to PF_T^R), by clarifying the truth-conditions formulae of the form $A \wedge {\sim}A$ are assigned in the respective logics, we generate one definition of contradiction, Con_{CL}), for classical logic, and another, Con_{LP}), for **LP**:

> Con_{CL}) A proposition is a contradiction iff it's always false and never true.

> Con_{LP}) A proposition is a contradiction iff it's always false and can be true.[76]

This divergence in definitions of contradiction between the dialetheist and classical logician produces two rather bizarre consequences for their debate over the truth of contradictions.

Firstly, PF_T^R) allows for both the classical logician and dialetheist to be simultaneously correct with their thesis regarding the possible truth of contradictions, *given their own respective definitions of contradictions.* As far as Con_{CL}) is concerned, it's true by definition

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[75] In some logic textbooks one finds the two accounts of contradiction being used interchangeably. Brenner (1993, p. 201), for example, states the accounts one after the other without any distinction.
[76] For more on **LP**'s semantics see section 4.4.1 below.

that contradictions *cannot* be true, and as far as Con_{LP}) is concerned, it's true by definition that contradictions *can* be true. PF_T^R), therefore, allows both parties to prove too much in what is taken to be a substantive debate within the philosophical community. Secondly, by ensuring that the dialetheist and classical logician have different definitions of contradictions, PF_T^R) is at a total loss to explain why both parties generally agree over whether a proposition is a contradiction or not.[77] With Con_{CL}) and Con_{LP}) defining contradictions in terms of different truth-conditional properties, one would expect some divergence in the propositions the parties categorise as contradictions. Given that we find no such divergence, PF_T^R) owes us an explanation for the parties' unexpected agreement over the set of contradictions.[78] Yet, by making the definition of contradiction relative to a logic's semantics, and defining contradictions *solely* in terms of these truth-conditional properties, PF_T^R) has no mechanism at all by which to explain how logics which define contradictions in terms of different truth-conditional properties agree on which propositions are contradictions. In contrast, other accounts of contradiction can explain how the two parties agree over which propositions are contradictions, while disagreeing over their truth-conditional properties. Indeed, any definition that doesn't define contradictions in terms of truth-conditional properties, or consequences of these properties, can do so. For example, the truth-value neutral account can explain their agreement by appealing to the fact that both parties can spot instances of propositions and their negation together, while also disagreeing over the truth-conditional properties of the conjunction of these propositions. The acts of agreeing over which propositions are contradictions, and agreeing over the truth-conditional properties of these propositions, become distinct. PF_T^R), therefore, possesses less explanatory power than, at least, some of its competitors.

Thus, regardless of whether PF_T) takes the form PF_T^C), PF_T^A) or PF_T^R), its an implausible account of contradiction. In addition, PF_T) assumes an account of logical form which is at odds with our actual formalisation practices. If the logical form of a proposition were dictated *solely* by its truth-conditions then, when formalising a proposition, we should *only*

[77] Although there may be more disagreement over whether a (set of) proposition(s) *entails* a contradiction or not, this is a completely different matter as we established earlier.

[78] If one needs convincing of this, consider the analogous case of PF_C). If we adapt PF_C) so that it doesn't assume standard propositional notation,

PF_C^R) A proposition is a contradiction in a logic *L* iff it possesses the same constituent semantic parts as are assigned to formulae of the form $A \wedge {\sim}A$ in *L*,

we would be incredibly surprised if the respective proponents of two logics *L* and *L'* agreed on which propositions were contradictions if '$A \wedge {\sim}A$' denoted completely different logical forms in *L* and *L'*. We would at least expect an explanation of this coincidence from any proponent of PF_C^R). This requirement for an explanation isn't diminished by replacing PF_C^R) with PF_T^R).

be concerned with the formula, that we symbolise the proposition as, sharing its truth-conditions. Yet, it's clear this isn't a principle we endorse when actually engaged in the practice of formalising an argument. It would make any argument that relies upon an equivalence of logical operators unacceptably trivial. For example, we wouldn't think that a formalisation of,

> Q1) All humans are not immortal
> Q2) Therefore, there is no human that is immortal,

as $\{\sim\exists x(Hx \wedge Ix)\} \vdash \sim\exists x(Hx \wedge Ix)$ was acceptable, although the formulae $\sim\exists x(Hx \wedge Ix)$ and $\forall x(Hx \rightarrow \sim Ix)$ are truth-conditionally equivalent in classical logic. We want to show in our formalisation, and subsequent derivations, *how* Q2) follows from Q1) formally. This indicates that when we symbolise a proposition, attempting to capture its logical form, we don't only attempt to preserve its truth-conditions. There are other properties of the proposition that we attempt to preserve in the symbolisation process, whatever these other properties are. Therefore, not only is PF_T) implausible as an account of contradiction because of the individual failures of $PF_T{}^C$), $PF_T{}^A$) and $PF_T{}^R$), but it's built upon an account of logical form which is contrary to our actual practice of formalising.[79] If the proposition-form account is to offer us any plausible account of contradiction, it will have to be in the form of PF_C).

PF_C) denies that the truth-conditions of a proposition define its logical form. Instead, a proposition's constituent semantic parts define its logical form. As with PF_T), there are two broad interpretations of PF_C). In the first, the semantics of a particular logic are assumed, so versions of this interpretation of PF_C) fit the schema,

> $PF_C{}^L$) A proposition is a contradiction iff it possesses the same constituent semantic parts as are assigned to formulae of the form $A \wedge \sim A$ in logic L,

and in the second no particular logic's semantics are assumed,

> $PF_C{}^W$) A proposition is a contradiction iff it possesses the same constituent semantic parts as are assigned to formulae of the form $A \wedge \sim A$, assuming no particular logic L's semantics.[80]

The motivation for the $PF_C{}^L$) interpretation of PF_C) is the principle that formulae are *only* assigned semantic content *within a logic*. Therefore, according to the $PF_C{}^L$) interpretation, PF_C)

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[79] For similar concerns over the compatibility of the *truth-conditional* account of logical form and our formalisation procedures, see Brun (2008, pp. 10-13).

[80] We aren't considering here a version of PF_C) that defines contradictions relative to a logic, as $PF_T{}^R$) does, for it would clearly suffer at least the same deficits as $PF_T{}^R$).

is an incomplete definition of contradiction, even assuming standard propositional notation, because it doesn't detail which logic is assigning the semantic content to the formulae. However, we can demonstrate that, by requiring that PF_C) assumes a particular logic's semantics, PF_C^L) entails two unsavoury consequences.

Firstly, PF_C^L) produces too simple a solution to the debate between the dialetheist and the classical logician. For example, if we assume a classical semantics we generate the definition,

> PF_C^C) A proposition is a contradiction iff it possesses the same constituent semantic parts as are assigned to formulae of the form $A \wedge {\sim}A$ in *classical logic*.

Now, given that the constituent semantics parts assigned to formulae of the form $A \wedge {\sim}A$ in classical logic are the *classical* conjunction of a proposition and its *classical* negation, PF_C^C) entails that the classical logician is correct in precluding the truth of contradictions. After all, if a proposition is the *classical* conjunction of a proposition *p* and *p*'s *classical* negation, then *by definition* it is false (and not true). If PF_C^C) is the correct definition of contradiction, one must wonder why classical logicians do not simply offer a proof of the logical falsity and untruth of propositions of the form $A \wedge {\sim}A$ in classical logic as conclusive evidence against dialetheism.

Alternatively, if we assume the semantics of a dialetheic logic, such as **LP**, then we generate a different definition,

> PF_C^D) A proposition is a contradiction iff it possesses the same constituent semantic parts as are assigned to formulae of the form $A \wedge {\sim}A$ in **LP**.

Yet PF_C^D), as PF_C^C) did, produces too simple a victory for one of the parties involved in the debate; on this occasion the dialetheist. Given that the constituent parts assigned to formulae of the form $A \wedge {\sim}A$ in **LP** are the *dialetheic* conjunction of a proposition and its *dialetheic* negation, PF_C^D) entails that the dialetheist is correct in *not* precluding the truth of contradictions. If a proposition is the *dialetheic* conjunction of a proposition *p* and *p*'s *dialetheic* negation, then *by definition* it *can* be true, although it's also false.

The same unsavoury consequence results whichever logic we substitute for '*L*' in PF_C^L). A logic's semantics must take a stand on whether formulae of the form $A \wedge {\sim}A$ ever output the truth-value true or not, given the meanings the logic assigns to the elements of the formulae. Therefore, whichever logic we substitute for '*L*' in PF_C^L), the question of whether any contradictions can be true or not is settled by definition. Consequently, PF_C^L) requires that we settle far more in virtue of a logic's semantics than we are actually able to. When a classical logician argues for the logical untruth of contradictions, she does so by arguing *for* classical logic, and not simply arguing *from* classical logic. $PF_C{}^{C})$ makes her demonstration of the logical untruth of contradictions the simple procedure of setting out the consequences of classical semantics, whereas in reality, to demonstrate that contradictions are logically untrue, the classical logician must argue for classical logic's theoretical virtues. Whichever logic we substitute for '*L*', $PF_C{}^{L})$ trivialises a substantive debate.

Secondly, $PF_C{}^{L})$ could entail that paradigm cases of natural language contradictions are not categorised as contradictions, whichever logic we substitute for '*L*'. To demonstrate this point, we will use the examples of $PF_C{}^{C})$ and $PF_C{}^{D})$. Some propositions expressed in natural language, such as 'The table's both red and not red' and 'Emma's going to the beach and she isn't', are paradigm examples of contradictions. If any propositions are contradictions, then these propositions are. Now, it's an epistemic possibility that we will gather good evidence to believe that neither classical logic nor **LP** adequately model the semantics of propositional negation. After all, the properties of propositional negation, properly understood as a phenomenon of natural languages, are still hotly contested. If we did possess such evidence, then we would have good reason to believe that there were no propositions expressed in the natural language that had the same semantic constituents as are assigned to formulae of the form $A \wedge \sim A$ in either classical logic or **LP**, given that both logics assign their own semantics for the negation operator to tilde. In such a case, regardless of whether one endorsed $PF_C{}^{C})$ or $PF_C{}^{D})$, one would be required to either denounce one's definition of contradiction or to admit that no contradictions were expressed in the natural language under discussion. The latter option, though, is absurd. Contradictions *are expressed in* our natural language. Both $PF_C{}^{C})$ and $PF_C{}^{D})$, therefore, are inadequate because we have no assurances at present that classical logic or **LP**, respectively, adequately model propositional negation and, consequently, no assurances that any propositions expressed in our natural language possess the same semantic constituents as are assigned to formulae of the form $A \wedge \sim A$ in classical logic or **LP**. By defining contradictions in terms of the semantics of classical logic or **LP**, we are leaving it to chance whether any contradictions are expressed in our natural language, when no definition of contradiction should be leaving this question up for debate. Given that it's epistemically possible for any logic L to fail to accurately model propositional negation, whatever logic we substitute for '*L*' in $PF_C{}^{L})$ the account will still have this absurd consequence. $PF_C{}^{L})$ simply concedes a possibility that shouldn't be conceded.

While it may be possible for a plausible definition of contradiction of the form $PF_C{}^{L})$ to be formulated *once* we know which logic accurately models propositional negation, by this point we will already know whether contradictions can be true or not. Thus, previous to our knowing whether contradictions can be true or not, $PF_C{}^{L})$ isn't a viable definition of contradiction, whichever logic we substitute for *L*. No definition of contradiction of the form PF_C^L) can ever serve as a workable definition upon which the classical logician and the absolutist/dialetheist can argue over the (possible) truth of contradictions. Consequently, for a version of PF_C) to be plausible, it must be possible to formulate the account adequately without any mention of a particular logic's semantics.

Whether this can be achieved is moot. It requires us to assign meaning to a formula without assuming any particular semantics for the logical constants. Now, our apparent ability to formalise propositions without assuming a particular logic's semantics would suggest that we can do just this. For example, one can formalise the argument,

R1) Stockholm is the capital city of Sweden
R2) It's not the case that Stockholm is the capital city of Sweden
R3) London is the capital city of Sweden

in standard propositional notation as $\{p, \sim p\} \vdash q$, without deciding which propositional logic one endorses.[81] One knows that tilde denotes the negation operator in standard propositional notation, and that the proposition contained in R2) is the negation of the proposition in R1), without committing oneself to a particular semantics for the negation operator. Thus, it seems one can formalise the negation of a proposition *A* as '$\sim A$' without taking a stand on the semantic properties of the negation operator.[82] Consequently, our apparent ability to formalise propositions, without taking a stand on the semantics of the logical constants, gives *prima facie* plausibility to the possibility of assigning meaning to formulae without assuming a particular logic's semantics.

However, even if it appears possible to formalise propositions without assuming a particular logic's semantics, we are still left with the mystery of *how* we assign meaning to a formula in this instance. Consequently, given PF_C^W) as a definition of contradiction, it's a mystery *how* we determine the semantic constituents of a contradiction. While it may appear that we have the ability to formalise propositions without assuming a particular semantics for the logical constants, appearances can always be misleading. It may turn out that, contrary to appearances, we implicitly assume a particular logic's semantics when engaged in the formalisation process, as there's no other reasonable explanation for how we assign meaning to a formula. In this eventuality, PF_C) would be an incomplete definition of contradiction without any reference to a logic's semantics, as suggested by advocates of PF_C^L), and *no*

[81] The situation is perhaps even clearer with regards to modal propositions. One's ability to formalise the proposition 'It's possible that water is H_2O' as $\Diamond p$, isn't dependent on one's endorsement of a particular semantics for the possibility operator. After all, most of us learnt how to formalise modal propositions before endorsing a particular relational semantics for the modal operators.
[82] Of course, whether one considers the argument to be (in)valid will be dependent on the semantics for the logical operators one accepts, but this is a separate question from whether one can formalise the argument without assuming a particular logic's semantics.

proposition would fulfill the criteria of $PF_C{}^{W}$)). Without any detailed account of *how* we assign meaning to formulae without assuming a particular logic's semantics, we lack the assurance that we *can* assign meaning to formulae without assuming a particular logic's semantics. After all, we certainly *can* assign meaning to formulae when assuming a particular logic's semantics, and therefore we don't need to rely upon the hypothesis that formulae can be assigned meaning *without* assuming a particular logic's semantics to reasonably conclude that formulae *can* be assigned meaning. Now, given that we lack at present any detailed account of how we could assign meaning to formulae without assuming a particular logic's semantics, and offering such an explanation is far beyond the scope of this chapter, any endorsement, or full evaluation of the prospects, of $PF_C{}^{W}$) must be delayed.[83] For the account to be plausible, we must have assurances that we can perform what the definition requires. Yet, without an explanation of how we could assign meaning to formulae without assuming a logic's semantics, we lack these assurances. Conversely, as we have no reason at present to preclude the possibility of such an explanation, we cannot reasonably conclude that the account is implausible.

In failing to be in a reasonable position to fully evaluate $PF_C{}^{W}$), we have consequently failed to find any plausible version of the form account. We initially distinguished between two interpretations of the form account, the *formulae* and *proposition* interpretations. The formulae interpretation of the account was swiftly rejected because contradictions are not formulae. This left us with the proposition-form account of contradiction, which defined contradictions as propositions of a certain logical form *F*. Two interpretations of logical form were then proposed, subsequently producing two versions of the account, PF_T) and PF_C). PF_T) itself was found to be open to three distinct interpretations, and all three of the resulting

[83] One possible explanation is that we can fix the semantic referents of the elements of a formula without fully committing on the properties of these semantic objects. So, we can say that '$\sim A$' symbolises the negation of a proposition *A* and that, although we don't yet have a settled account of the semantic properties of propositional negation, we can fix what we mean by a proposition's negation by giving *examples*: 'All men are mortal' is the *negation* of 'Some men are immortal', 'Stockholm isn't the capital of Sweden' is the *negation* of 'Stockholm is the capital of Sweden', and so on. This would allow for all the logics that use the same notation to agree on the logical form of a proposition and then subsequently disagree on the truth-conditions of the propositions, due to a divergence in the semantic properties that are assigned to the logical constants by the respective logics. This, though, is a mere suggestion, and we cannot base our evaluation of $PF_C{}^{W}$) on it.

Interestingly, if one were to accept this, or some similar, explanation, then given that in standard propositional notation tilde symbolises negation and wedge symbolises conjunction, $PF_C{}^{W}$) would produce a definition of contradiction extensionally equivalent to the truth-value neutral account, given the meanings assigned to elements of formulae of the form $A \wedge \sim A$. This accords with Beall's (2004a, p. 4) claim that if contradictions are "sentences *of the form* $A \wedge \sim A$," then "a contradiction, on the formal usage, is the conjunction of a sentence and its negation." Therefore, in failing to produce an adequate explanation of how we can assign meaning to formulae without assuming a logic's semantics, we may not be doing any serious damage to our survey of the available accounts of contradictions. Plausibly, any such adequate explanation would only entail that contradictions, according to $PF_C{}^{W}$), are the conjunctions of a proposition and its negation, with the added complication of referring to these conjunctions via their sharing certain semantic properties with certain formulae.

definitions of contradiction PF_T^C), PF_T^A), and PF_T^R), were found to be inadequate. This left us with PF_C), which we interpreted in two forms. The first, PF_C^L), insisted that we assume a particular logic's semantic, and the second, PF_C^W), did not. PF_C^L) had unsavory consequences for the debate between the classical logician and the dialetheist, and was consequently condemned as implausible. In contrast, the plausibility of PF_C^W) was left unclear, as we don't yet possess any credible and detailed account of how we could assign meaning to formulae without assuming a particular logic's semantics. Thus, in evaluating the form account, we have failed to find any plausible version of it, and we have good reason to believe that all the versions of the account we have considered, except PF_C^W), are *implausible*. Consequently, we've also failed to find a version of the form account of contradiction that can be suitably embedded into Ab).

3.2.2 *Reductio* Accounts

Reductio accounts define contradictions in terms of certain proof-theoretic rules, *reductio ad absurdum* rules. While no one in the literature appears to endorse the *reductio* account, the fact that contradictions permit the application of *reductios* in classical logic makes this account a sensible suggestion to briefly explore.[84]

Now, as we apply proof-theoretic rules to formulae, and contradictions are *not* formulae, as demonstrated in the previous section, the *reductio* account must define contradictions parasitically as,

R) Propositions of the logical form *F*, where formulae of the form *F* permit the application of a formal *reductio*.

We have two interpretations of R) open to us. We can either interpret R) broadly, so that *F* stands for any formulae that permit the application of a formal *reductio in any logic*. In other words, as long as *F* fulfils the proof-theoretic role given by the rule,

[84] The closest anyone in the literature gets to endorsing the *reductio* account is Rumfitt (2010, p. 36):

> The criterion for being an overt contradiction relates to our practice of deducing consequences from hypotheses. Sometimes, in making such deductions, we reach a point from which no forward inferential step can be taken: all we can do is to retrace our steps and ask which of our hypotheses led us to the impasse. An overt contradiction marks an impasse of this kind. Thus, if a hypothesis yields the consequence 'This is red and not coloured', we know that the hypothesis must be wrong; deduction can proceed only by discharging it, and exploring the consequences of its negation.

Rumfitt's account of contradiction is implausible for failing to recognise the distinction between formal and informal *reductios* that we mentioned in section 3.1.1 above. There are many absurd propositions that are blatantly not contradictions, but which would also mark "an impasse of this kind" if we were to derive them. Paradoxes with non-contradictory conclusions are special instances of this phenomenon. This demonstrates why one couldn't plausibly define contradictions in terms of *informal reductios*.

$$\frac{\Gamma, B \Rightarrow F}{\Gamma \Rightarrow \sim B} \qquad B\text{-RAA}$$

in any logic, then propositions of the logical form F will be categorised as contradictions. Or, we can interpret R) narrowly, so that the formulae must be of the form that permits the application of the standard *reductio* rule as it's found in classical logic,

$$\frac{\Gamma, B \Rightarrow A \wedge \sim A}{\Gamma \Rightarrow \sim B} \qquad S\text{-RAA}$$

in which case we substitute '$A \wedge \sim A$' for 'F' in R). So, if we interpret R) broadly we produce the definition of contradiction,

R_B) Propositions of the logical form F, where formulae of the form F permit the application of a formal *reductio* in some logic L,

and if we interpret R) narrowly we produce the definition,

R_N) Propositions of the logical form $A \wedge \sim A$, where formulae of the form $A \wedge \sim A$ permit the application of a formal *reductio* in classical logic.

Two points regarding R_B) and R_N) are particularly important. Firstly, R_N) is an instance of R_B), as formulae of the form $A \wedge \sim A$ fulfill the proof-theoretic role of F in B-RAA in classical logic. Therefore, if R_N) has unsavoury consequences then so will R_B). The only possible situation in which R_B) and *not* R_N) will be a plausible definition of contradiction, is if R_N) is both too strict and not too lax in it's criteria.

Secondly, both R_B) and R_N) include talk of the *logical form* of a proposition. Therefore, both versions of the *reductio* account owe us a clarification of which theory of logical form is embedded within the definition. In establishing whether either of the accounts are plausible when embedded with a theory of logical form, we will concentrate on R_N) for the reason already given. If we can establish that R_N) embedded by each available theory of logical form possesses unsavoury consequences, then we can justifiably reach the same conclusions regarding R_B).

R_N) just is a proposition-form account of contradiction, *plus* the additional claim that 'formulae of the form $A \wedge \sim A$ permit the application of the formal *reductio* in classical logic'. Therefore, the fact that PF) is embedded within R_N) entails that R_N) suffers from the same theoretical concerns as PF) when we insert a theory of logical form into the definition. Irrespective of whether we embed the *truth-conditional* theory of logical form into R_N),

> R_{NT}) Propositions with the same truth-conditions as are assigned to formulae of the form $A \wedge \sim A$, where formulae of the form $A \wedge \sim A$ permit the application of a formal *reductio* in classical logic,

or the *constituent* theory of logical form,

> R_{NC}) Propositions with the same semantic constituents as are assigned to formulae of the form $A \wedge \sim A$, where formulae of the form $A \wedge \sim A$ permit the application of a formal *reductio* in classical logic,

R_N) is implausible.[85] In short, the implausibility of the versions of the proposition-form account entail the implausibility of R_N).

Additionally, because the implausibility of the versions of R_N) are not constituted by the strictness of their criteria, their implausibility entails the implausibility of the respective versions of R_B). Therefore, by demonstrating in the previous section that none of the versions of the proposition-form account are plausible definitions of contradiction, we have subsequently shown that the same is true for both of the versions, R_N) and R_B), of the *reductio* account. The only potentially saving grace for R_N) and R_B) is the possibility of PF_C^W)'s plausibility. Yet, until we have good reason to believe that we can assign meaning to formulae without assuming a particular logic's semantics, we have no reason to believe that any version of R_N) or R_B) is plausible. Consequently, as with versions of PF), the lack of any plausible version of R) entails that none of the versions of the account at present are suitable to embed within Ab).

3.3 Pragmatic Accounts

Pragmatic accounts define contradictions in terms of the communicative acts of assertion and denial. Instances of the account are found in,

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[85] Or, in the case of the version of R_N) which results from inserting the theory of logical form embedded within PF_C^W),

> R_{NC}^W) Propositions with the same semantic constituents as are assigned to formulae of the form $A \wedge \sim A$, *assuming no particular logic L's semantics*, where formulae of the form $A \wedge \sim A$ permit the application of a formal *reductio* in classical logic,

we have no reason to believe at present that the account is plausible. PF_C^W)'s plausibility is a prerequisite for R_{NC}^W)'s plausibility.

Kahane (1995, p. 308): A *contradiction* both makes a claim and denies that very claim,

and

Strawson (1993, p. 21): We would not say that a man could, in the same breath, assert and deny the same thing without contradiction.

Thus, instead of expressing the content that constitutes a contradiction, the communicative acts themselves constitute the contradiction. The assertion is not an assertion *of a* contradiction, but the *act of* assertion and denial is itself a contradiction.

Removing the unnecessary ambiguities from the Kahane and Strawson accounts by replacing 'makes a claim' with '*asserts a proposition*' and 'assert and deny the same thing' with 'assert and deny the same *proposition'*, we arrive at a precise pragmatic definition of contradiction:

P) A contradiction is the simultaneous assertion and denial of a proposition *A*.

The condition in P) that the assertion and denial of *A* must be simultaneous is included to ensure that assertions of *A* that are renounced and replaced by denials of *A*, and vice versa, aren't categorised as instances of contradiction. P) doesn't require that the acts of assertion and denial *occur* simultaneously, only that the assertion/denial of *A* doesn't annul the previous denial/assertion of *A*, in the case of contradictions.

Before we move onto assess P)'s plausibility, two clarifications of the definition are required. Firstly, both assertion and denial are communicative acts that only *agents* perform. Non-agents do not themselves assert or deny.[86] Truthbearers, for example, are the *contents* to be asserted or denied, and are not the acts themselves. Thus, the occurrence of a sentence *s* expressing a proposition *p* does *not* constitute an assertion of *p*, or a denial of another proposition *q*. Three points should clarify why. Firstly, treating *s* as an assertion of *p* would ensure that one could never just mention or hypothesise *p* with the use of *s*, as by using *s* one would be automatically asserting *p*. Thus, the fact that one can mention or hypothesise the truth of a proposition *p* with the use of a sentence *s* expressing *p* demonstrates that a sentence *s*'s expressing a proposition *p* doesn't constitute assertion of *p*. Secondly, one can *assert* a proposition *p* by asserting a sentence *s* that expresses *p*, yet if *s* just is the *assertion of p* then we would be asserting an *assertion of p*. This seems to misrepresent what we are doing when we use a sentence to assert a proposition however, and it requires us to admit that force operators can

[86] Contrary to Marciszewski's (1981, p. 70) definition of contradictions as "a pair of formulae such that one of them is the denial of the other." Formulae either exist as solely syntactic objects that can be manipulated according to certain syntactical rules, or they can be part of a calculus that models certain phenomena, in which case the formulae are *symbolising* phenomena as dictated by the rules of the calculus. In neither case are the formulae themselves asserting or denying anything.

be embedded, which is a commitment we have good reasons to reject.[87] The same point holds if we hypothesise that a sentence *s* denies a proposition *q*. Lastly, the acts of assertion and denial bring with them some form of social commitment, a commitment which sentences aren't the right kind of objects to have.[88] To have social commitment, an object must be a social entity. Therefore, a contradiction, according to P), must be a communicative *act* by an *agent*.

Secondly, to fully evaluate the plausibility of P), we would have to be sure of the correct theory of assertion. Given the plethora of substantive theories of assertion, however, this clearly isn't possible for us to achieve here. Our only reasonable option is to acknowledge the available theories in the literature, and suspend judgement on which is correct. The available theories of assertion can be sorted into three categories:[89]

> **Normative accounts**: Assertion is a speech act constituted by certain rules.[90]
>
> **Information accounts**: Assertion is a proposal to add propositions to the information being presupposed in a conversation.[91]

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[87] Geach (1965).
[88] For more on assertion and denial bringing with them social commitments see Brandom (1994, pp. 157-180) and Williamson (2000, pp. 266-269).
[89] For alternative categorisations of the available theories of assertion, see Cappelen (2011) and MacFarlane (2011). Cappelen's (2011) own theory of assertion, the *No-Assertion* theory, doesn't fit any of our categories, as it's tantamount to a rejection of any philosophically insight theory of assertion:

> Sayings are governed by variable norms, come with variable commitments and have variable causes and effects. What philosophers have tried to capture by the term 'assertion' is largely a philosophers' invention. It fails to pick out an act-type that we engage in and it is not a category we need in order to explain any significant component of our linguistic practice.
> (Cappelen (2011) p. 21)

[90] Williamson (2000) considers five different possible constitutive rules for assertion, all of which are instances of the 'C rule' (One must: assert *p* only if *p* has C):

The truth rule:	One must: assert *p* only if *p* is true. (p. 242)
The warrant rule:	One must: assert *p* only if one has warrant to assert *p*. (p. 242)
The knowledge rule:	One must: assert *p* only if one knows *p*. (p. 243)
The BK rule:	One must: assert *p* only if one believes that one knows *p*. (p. 260)
The RBK rule:	One must: assert *p* only if one rationally believes that one knows *p*. (p. 261)

The truth rule is endorsed in Weiner (2005); the warrant rule in Lackey (2007); the knowledge rule in Unger (1975, p. 253), Williamson (1996 & 2000, ch. 11), DeRose (1996 & 2002), Reynolds (2002), and Hawthorne (2004, pp. 21-27); and the RBK rule in Douven (2006).
[91] Stalnaker's (1978 & 2009) theory of assertion is the prime example of *information* accounts of assertion:

> An assertion should be understood as a proposal to change the context by adding the content to the information presupposed. This is an account of the force of an assertion, and it respects the traditional distinction between the content and the

> **Commitment accounts**: Assertion is an expression of the undertaking of a commitment on the part of the asserter.[92]

While our lack of a commitment on the correct theory of assertion and denial to embed within P) ensures that our evaluation of the account is bound to be incomplete, there are still enough properties that all of the plausible accounts of assertion apply to assertion (and denial) to enable us to suitably evaluate P). We offer here four reasons to believe that P) is implausible, whichever plausible theory of assertion and denial we embed within it.

Firstly, P) precludes the possibility of an individual asserting/denying or (dis)believing a contradiction. One (dis)believes or asserts/denies truthbearers, which have propositional content, and not communicative acts, which assertion and denial are. Communicative acts can express truthbearers, but *not* other communicative acts. It makes no sense to say that one has asserted an assertion, denied a denial, asserted a denial, or the inverse. Similarly, to say that someone believes an assertion only makes sense if we interpret the claim loosely as 'Someone believes a proposition *p*, which was previously asserted'. Yet, we believe that individuals can both assert (or deny) a contradiction and (dis)believe a contradiction. Additionally, we believe that these acts can be meaningfully expressed when we use the term 'contradiction' instead of the intended pragmatic *definiens*. While the propositions 'Someone has asserted a contradiction' and 'Someone believes a contradiction' are perfectly meaningful, substituting 'contradiction' for the *definiens* of P) in either proposition ensures that they become meaningless. P), therefore, fails the criterion of eliminability that is so crucial to any good definition of a term. If a *definiens* cannot replace the *definiendum* in a proposition without retaining the proposition's meaningfulness, then the proposed *definiens* is inadequate.[93]

Secondly, assertions and denials cannot themselves be true or false, yet it's perfectly meaningful (at least) to say of a contradiction that it's false. Assertion and denial are not truthbearers, as we have already noted. Therefore, they cannot be assigned a truth-value. Instead, they are communicative acts that have truthbearers as their content. The *definiens* of P) fails again to preserve the meaningfulness of certain propositions when it replaces 'contradiction'. This failure brings us suitably onto our third criticism of P), P)'s inability to produce a plausible version of the LNC.

> force of a speech act. Propositional content is represented by a (possibly partial) function from possible worlds to truth-values; assertive force is represented by the way in which any such function is used to change the context that the speaker shares with those to whom he is speaking.
>
> (Stalnaker (1999) pp. 10)

[92] This account is endorsed in Searle (1969, p. 29 & 1979, p. 12), Brandom (1983 & 1994, pp. 199-200), Davidson (1984, pp. 5-6), Wright (1992, p. 23), Alston (2000, pp. 116-120), Williams (2002, p. 74), Watson (2004), MacFarlane (2003 & 2005), Priest (2006a, p. 104), and Hindricks (2007).
[93] For more on the criteria for rigorous definitions see Belnap (1993).

From our previous criticisms of P), it's clear that the two obvious and *prima facie* plausible functions on contradictions that constitute the LNC, the pragmatic 'Do not assert contradictions' and the semantic 'All contradictions are false', cannot meaningfully embed P). Thus, we will need to extend our reaches to more obscure interpretations of the law. The difficulty with these interpretations, such as,

LNC_W) All contradictions are wrong,

LNC_I) All contradictions are incorrect,

or

LNC_M) All contradictions are mistaken,

is that their suitability to capture the LNC's content is dubious. While we can meaningfully embed the *definiens* of P) into them all,

LNC_W^P) All simultaneous assertions and denials of a proposition are wrong,

LNC_I^P) All simultaneous assertions and denials of a proposition are incorrect,

LNC_M^P) All simultaneous assertions and denials of a proposition are mistaken,

they fail as plausible interpretations of the LNC for other reasons.

Any adequate version of the LNC must fulfill *at least* two criteria. Firstly, it must be meaningful for 'contradiction' to replace the proposed *definiens* of contradiction embedded into the version of the LNC. Secondly, the version of the LNC must either respect the standard formalisation of the law, or a new formalisation of the law must be presented while explaining away the success of the previously accepted formalisation.

All of LNC_W^P), LNC_I^P), and LNC_M^P), fail to fulfill the first criterion. As we can see from LNC_W), LNC_I), and LNC_M), 'contradiction' cannot meaningfully replace the *definiens* of P) in any of these versions of the LNC. Even disambiguating both LNC_W) and LNC_I) so that they become,

LNC_R) All contradictions are irrational,

doesn't rescue either version. It doesn't make sense to say that contradictions themselves are irrational (nor wrong, incorrect, or mistaken). Contradictions have to be truthbearers of some kind, as it must make sense for them to be false (and, perhaps, true). Truthbearers themselves, however, are the wrong kind of entity to be irrational, wrong, incorrect, or mistaken.

The only way for P) to ensure that one of LNC_W), LNC_I), or LNC_M), is meaningful, is to interpret them as the standard interpretation of the LNC,

$\mathrm{LNC_F}$) All contradictions are false.

Yet, in this case, while $\mathrm{LNC_F}$) is meaningful, the respective version of the LNC for the *definiens* of P),

$\mathrm{LNC_F}^{\mathrm{P}}$) All simultaneous assertions and denials of a proposition are false,

isn't. Thus, all of $\mathrm{LNC_W}^{\mathrm{P}}$), $\mathrm{LNC_I}^{\mathrm{P}}$), and $\mathrm{LNC_M}^{\mathrm{P}}$), can only fulfill the first criterion by being interpreted as $\mathrm{LNC_F}^{\mathrm{P}}$), which itself is meaningless. Therefore, none of the versions of the LNC above fulfill the first criterion while being suitable to embed the *definiens* of P).

All of $\mathrm{LNC_W}^{\mathrm{P}}$), $\mathrm{LNC_I}^{\mathrm{P}}$), and $\mathrm{LNC_M}^{\mathrm{P}}$), equally fail to fulfill the second criterion for any plausible version of the LNC. The standard formalisation of the LNC, $\sim(A \wedge \sim A)$, places two restrictions on any informal interpretations of the law, both of which are problematic for the interpretations discussed above. Firstly, the law must contain propositions that can be embedded into more complex propositions, due to the presence of embedded sub-formulae in $\sim(A \wedge \sim A)$. Secondly, the law must contain two instances of the same truth-function, given that $\sim(A \wedge \sim A)$ contains two tildes.

All three interpretations of the LNC above fail to meet the first restriction. They define contradictions as a combination of acts, formalised as force operators, which cannot be meaningfully embedded, rather than as a combination of propositions, which *can* be meaningfully embedded. Similarly, the interpretations fail to meet the second restriction, as there are no two instances of the same truth-function in any of the three interpretations of the LNC to account for the two tildes in the formula. Even if we assume that $A \wedge \sim A$ is a plausible formalisation of the assertion and denial of a proposition, which we know it isn't for the reasons already given, we would be required by formal constraints to interpret the outer tilde as a denial operator, given that the internal tilde would symbolise denial. Thus, the only available interpretation of the LNC embedding the *definiens* of P), while fulfilling this formal requirement, is,

$\mathrm{LNC_D}^{\mathrm{F}}$) One should deny all simultaneous assertions and denials of a proposition,

which is meaningless for the reason already given; force operators cannot be meaningfully embedded. It makes no sense to deny an assertion or denial. Thus, none of the interpretations of the LNC discussed above are reasonable informal representations of the standard formalisation of the LNC.

Now, given how established $\sim(A \wedge \sim A)$ is as the formalisation of the LNC, any account of contradiction that cannot respect this formalisation must possess substantial theoretical virtues to justify its replacement. At present, however, P) doesn't possess any of the theoretical virtues required to justify such a divergence from the established path. In fact, it seems a relatively theoretically weak account of contradiction. Thus, we can reasonably conclude that none of the interpretations of the LNC above that can meaningfully embed the *definiens* of P) are adequate, because they all fail to fulfill at least two criteria for any adequate version of the law. P)'s failure to produce an adequate version of the LNC must count heavily against the account.[94]

There is a clear connection between all three of the criticisms of the pragmatic account mentioned so far. All three are based on the inability of pragmatic notions such as assertion and denial to be embedded meaningfully within certain complex propositions, which unnecessarily restricts our ability to talk about properties of, or attitudes towards, contradictions. Talk that we find both very meaningful and necessary. Given that P)'s inability to deal with meaningful propositions such as 'Some people believe contradictions' and 'All contradictions are false' is a symptom of a more general ailment, it's extremely likely that there are other philosophically important propositions about contradictions that cannot be accommodated by the pragmatic definition of contradiction.

Our final criticism of P) is that we often want to talk about the occurrence of contradictions *without* the occurrence of any assertions or denials. The theoretical role that contradictions play is such that we do, and want to, allow them to occur without anyone asserting or denying a proposition. Yet, if contradictions are simply the simultaneous assertion and denial of the same proposition, as P) claims, then this possibility is precluded. Our use of contradictions without the occurrence of assertions and denials demonstrates that P) is inadequate at explaining our philosophical use of the concept of *contradiction*. We can demonstrate these important uses of contradictions, without the presumption of the occurrence of an assertion or denial, with four examples.

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[94] We can also show that it's unlikely any version of the LNC that meaningfully embeds the *definiens* of P) can produce an acceptable formalisation of the LNC, which, given the criteria for any adequate version of the LNC above, subsequently demonstrates that it's unlikely any adequate version of the LNC can meaningfully embed the *definiens* of P). For a version of the LNC that meaningfully embeds the *definiens* of P) to produce an acceptable formalisation of the LNC, at least one of these three very plausible principles must be rejected:

For) Assertion and denial are formalised as force operators.

Em) Force operators cannot be meaningfully embedded.

Th) The LNC should be formalised as a function on contradictions.

Given the plausibility of all three, the prospects for P) to produce an adequate version of the LNC seem very grim.

Firstly, it is common in philosophy for a researcher to take a set of propositions Σ, that no individual endorses all the members of, and to infer some of Σ's interesting consequences. One of these interesting consequences could be that the members of Σ jointly entail a contradiction *C*. Yet, in demonstrating that the members of Σ jointly entail *C*, the researcher won't have presupposed *any* occurrences of assertions or denials.

Secondly, we sometimes state of a set of propositions Σ, which some individual *does* assert all the members of, that its members jointly entail a contradiction *C*, without presupposing that the individual asserts or denies any proposition *A* that constitutes *C*. Even if beliefs are closed under logical consequence, which seems dubious, we don't require that acts such as assertions and denial are closed under logical consequence. So, we often speak about an individual's rational commitments, including contradictions, without presupposing that those commitments are accompanied by the assertions or denials of the individual.

Thirdly, we have no problem speaking about contradictions as the antecedent of a conditional without presupposing that the antecedent is endorsed by anyone. This is demonstrated by our ease in writing, 'Assume that a contradiction *C* occurs in a set of propositions Σ'. This is exactly the kind of assumption we might make when discussing the principle of explosion, yet we don't require that the antecedent is a proposition being asserted and denied by someone. We can merely assume the contradiction, as a proposition, and work from there.

Lastly, sometimes we just *mention* a contradiction. This could be because one wants to comment on some particular interesting grammatical properties of a contradiction *C*, or because one wants to demonstrate to a class how one could go about formalising *C*. Writing a sentence such as 'The table is both red and not red' on a whiteboard in a logic class and asking the class whether it's a contradiction shouldn't, according to P), elicit a positive response, given that there is no one performing any simultaneous assertion and denial of a proposition. However, the sentence clearly is a contradiction, and we would expect a positive response from the class.

In conjunction with our other criticisms of P), the account's inability to accommodate these important uses of contradictions without the presupposition of the simultaneous assertion and denial of a proposition, demonstrates P)'s implausibility as an account of contradiction. In addition, it's clear that none of the account's weaknesses are suffered by at least one of its competitors, the truth-value neutral account. Thus, the pragmatic account is a weaker candidate than at least one of its competitors for the definition of contradiction.

The weaknesses of the pragmatic account are enough in themselves to demonstrate the account's unsuitability for the absolutist's purposes. Embedding the *definiens* of P) into Ab) would ensure that the absolutist is making a category mistake, as the classical logician would be in stating that 'All contradictions are false'. Assertions and denials are not truthbearers. The pragmatic account of contradictions is both philosophically implausible and unsuitable to embed within Ab).[95]

3.4 Ontological Accounts

Our final accounts of contradiction, ontological accounts, define contradictions in terms of truthmakers or properties and *not* truthbearers. Examples of the ontological account are found embedded in the $Aristotle_2$, $Prior_2$, $Russell_1$, and von $Wright_2$ definitions of the LNC,[96] and in Routley & Routley (1985, p. 204),

> A contradictory situation is one where both *B* and ~*B* (it is not the case that *B*) hold for some *B*.[97]

A striking consequence of the ontological account is that it converts the LNC from a *logical* law, as it's commonly conceived, to a *metaphysical* law. As the subjects of ontological contradictions are truthmakers, particulars and properties, it would be unsuitable to embed the account into a version of the LNC that claimed the falsity or untruth of contradictions. Truthmakers, particulars and properties, after all, are not truthbearers. Thus, for a version of the LNC to state anything meaningful about ontological contradictions, it must be a law that decrees which objects can and do exist, rather than which propositions can be true. While it's beyond the scope of this section, any complete evaluation of the ontological account's plausibility would need to incorporate an evaluation of the metaphysical version of the LNC which follows from the ontological account. Even if we find that the ontological account itself is plausible, we may find that the metaphysical version of the LNC places too great a burden on modern philosophy. While Aristotle may have originally conceived of the LNC as a metaphysical principle, we cannot allow these ancient sources of the law to force us into

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95 It's interesting to note that if the absolutist follows Priest (2006a, p. 103) in precluding the possibility of the simultaneous assertion and denial of a proposition *p*, then P) would commit her to the impossibility of contradictions. Yet, it seems highly questionable whether one should have to assume the possibility of the simultaneous assertion and denial of a proposition to admit the possibility of contradictions simply *occurring*. It seems perfectly reasonable to believe that contradictions occur without taking any stand whatsoever on whether it's possible to simultaneously assert and deny a proposition. Here then is another theoretical weakness of P).

96 See section 3.1.1 above.

97 There are two available interpretations of the Routleys' definition of contradiction. According to the first, contradictions are a conjunction of truthmakers and, according to the second, contradictions are a conjunction of a proposition and its negation *and* the conjunction's being true in a situation *s* makes *s* contradictory. As the Routleys' definition under the second interpretation is a semantic account of contradiction, being a version of the truth-value neutral account, we are using here the first interpretation. If one has any doubt that interpreting the Routleys' definition as an ontological account in terms of truthmakers is compatible with their use of situation semantics, see Beall (2000) and Priest (2000), where situations are used as models for truthmakers.

accepting a version of the LNC that's damaging to highly plausible modern philosophical theories.[98]

On this cautious note, let us move on to consider versions of the ontological account. The Routleys' account is the only version we possess not embedded within a definition of the LNC. Interpreted as an account in terms of truthmakers, we are required to interpret the tilde in the definition unconventionally. Tilde is traditionally used to symbolise a unary truth-functional connective, yet as the hypothesised subjects of the Routleys' account are truthmakers, and not truthbearers, we must interpret the tilde to mean something other than the traditional truth-functor. As the symbol is representing an operation on, or modification of, a truthmaker, the only plausible interpretation of the tilde sign, given its connotations, is that it represents a *negative truthmaker*. If tilde prefixed to a propositional parameter p represents p's negation, then tilde prefixed to a symbol representing a truthmaker T most plausibly represents the corresponding negative truthmaker of T. Thus, by interpreting the Routleys' definition of contradiction as an ontological account in terms of truthmakers, we must parasitically interpret the tilde of their definition as representing a negative truthmaker.

As we have introduced the concept of negative truthmakers to help explain the Routleys' account of contradiction, it is worth briefly mentioning what these negative truthmakers are, and why they were initially hypothesised as a part of truthmaker ontologies. Truthmaker theories historically have a problem accounting for negative existential propositions, such as 'No unicorns exist', and propositions attributing the *lack* of a property to a particular, such as 'Emma cannot breath fire'.[99] There are two basic methods for the truthmaker theorist to account for the truth of these types of propositions (unless she's willing to admit that they don't have truthmakers):

1) Explain their truth using *positive* truthmakers, the same type of truthmakers that explain the truth of positive existential propositions and propositions that attribute properties to particulars.[100]

2) Explain their truth using *negative* truthmakers, creating symmetry between truthbearers and truthmakers. *Positive* truthmakers account for the truth of

[98] A metaphysical version of the LNC is also found in Plato (1997, 436b), during his discussion of the soul:

> It is obvious that the same thing will not be willing to do or undergo opposites in the same part of itself, in relation to the same thing, at the same time. So, if we ever find this happening in the soul, we'll know that we aren't dealing with one thing but many.

[99] See Molnar (2000). It's important to note that *negative* propositions aren't equivalent to a proposition's *negation*. The negation of 'Some gods don't exist' is 'All gods exist', yet 'All gods exist' isn't a *negative* proposition. Numerous other examples demonstrate the same point.
[100] For attempts to explain these *negative* propositions in terms of *positive* truthmakers see Molnar (2000, pp. 72-76) and Armstrong (2004, pp. 70-73).

> positive existential propositions and propositions attributing properties to particulars, and *negative* truthmakers account for the truth of negative existential propositions and propositions attributing the *lack* of properties to particulars.

Both of these options have their problems, and it's important to recognise that in accepting an ontological account of contradiction in terms of truthmakers, one is buying into the problem and the task of adequately solving it. This is a consequence many would undoubtedly be unwilling to commit to. After all, it seems slightly strained to suggest that we need to solve the problem of truthmakers for negative existential propositions to give an adequate account of contradictions. However, if all available versions of the ontological account require truthmakers, then giving an adequate solution to this problem will be tantamount to accepting an ontological account of contradiction. This should give us even more reason to be cautious of endorsing an ontological account.

Finally, it's worth bearing in mind that the more coherent option to deal with truthmakers for these *negative* propositions for the general truthmaker theorist, may *not* be the most coherent option for the advocate of the ontological account of contradiction. There may be a very successful solution to the problem for the general truthmaker theorist which is incompatible with the ontological account of contradiction. Indeed, there's good reason to think that any ontological account defining contradictions in terms of two truthmakers obtaining should posit *negative* truthmakers to account for negative propositions.

If an ontological account defines contradictions in terms of two truthmakers obtaining, then it has the option of either defining contradictions in terms of two positive truthmakers or a positive truthmaker and its corresponding negative truthmaker.[101] If one takes the former option, then one will have to explain how we can be sure that the two positive truthmakers in question constitute a contradiction, as many combinations of positive truthmakers certainly don't. For example, neither of the states of affairs 'London's being the capital of England *and* Stockholm's being the capital of Sweden' or 'Emma's being at the beach *and* Emma's carrying a towel' are in any sense contradictory. Yet, the only means by which we can ensure that we only pick out those pairs of positive truthmakers that constitute *prima facie* contradictions is to make reference to the *contents* of the truthmakers, and thus the

[101] The theoretical possibility of having an account constituted of two *negative* truthmakers isn't of interest to us here, based on two considerations. Firstly, an account constituted of two *negative* truthmakers suffers the same shortcoming as the option of using two *positive* truthmakers in the definition, *without* the theoretical advantage of not having to posit negative truthmakers. Secondly, if one can show that it's reasonable to posit negative truthmakers, then one has the option of defining contradictions in terms of a positive truthmaker and its corresponding negative truthmaker without any ontological concerns. Thus, *if* it's reasonable to posit negative truthmakers, the advocate of the ontological account gains no benefit in defining contradictions in terms of two *negative* truthmakers rather than in terms of a positive truthmaker and its corresponding negative truthmaker.

propositions that the truthmakers make true. Consequently, given what we've noted previously about the plausible semantic accounts of contradiction, an ontological contradiction according to this interpretation would be the simultaneous obtaining of two truthmakers ***P*** and ***Q***, such that ***P*** is the truthmaker for some proposition *A* and ***Q*** is the truthmaker for *A*'s negation. The identity of the positive truthmakers would, therefore, be fixed by reference to a proposition and its negation.

Given that it's only by making reference to the propositions that the truthmakers make true, that we can ensure that the account *exclusively* categorises as contradictions those pairs of positive truthmakers that plausibly constitute contradictions, it seems extravagant to use truthmakers rather than propositions in our definition of contradictions. If we have a choice between two ontological types, O_1 and O_2, to define a concept in terms of, and defining the concept in terms of O_2 doesn't require any mention of O_1 whatsoever, but the definition in terms of O_1 does require us to include talk of O_2, this would indicate that it's theoretically preferable to define the concept in terms of ontological type O_2. This is exactly the situation we find ourselves in when attempting to define contradictions in terms of two positive truthmakers, with O_2 representing truthmakers and O_1 representing propositions. Thus, for the advocate of the ontological account, defining contradictions in terms of two positive truthmakers is ultimately unsatisfying.[102]

In contrast, by defining contradictions in terms of a positive truthmaker and a negative truthmaker, we don't face the same problem. By introducing a polarity into the truthmaker ontology, we nullify the need to make reference to propositions. In positing a positive truthmaker and its corresponding negative truthmaker, the truthmakers themselves fix their relative content, and thus their divergences in content. While the positive truthmaker will posit that some object(s) *O* possess some property(s) *P* or exist, the negative truthmaker posits that the same object(s) *O don't* possess *P* or *don't* exist. Thus, it seems that the only reasonable option available for a truthmaker account of contradiction is to define contradictions in terms of a positive truthmaker ***T*** and its corresponding negative truthmaker $\ominus$***T***, requiring the truthmaker theorist to admit negative truthmakers into her ontology.[103] Here, then, is our first version of the ontological account, in terms of truthmakers:

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[102] The same problem is faced by any truthmaker account of contradiction that defines contradictions in terms of *one* truthmaker, whether it's a positive or negative truthmaker.

[103] It's unclear, even if we manage to produce a suitable ontology for negative truthmakers, whether the set of every pair of a positive truthmaker ***T*** and its corresponding truthmaker $\ominus$***T*** will contain no pair that's blatantly *not* a contradiction, *while* containing a pair for every obvious contradiction. Negative truthmakers don't map onto the negations of propositions, as some negations of propositions are *not* negative propositions. Thus, there are no assurances that the set of every pair of a positive truthmaker ***T*** and its corresponding truthmaker $\ominus$***T*** will fulfill these conditions. In fact, if one could show that some contradictions are constituted of two *positive* propositions, then this would spell trouble for the truthmaker account of contradiction, as no pair of a positive truthmaker ***T*** and its corresponding truthmaker $\ominus$***T*** would be able to accommodate the contradiction. We won't speculate

O_{TM}) A contradiction is the simultaneous obtaining of a positive truthmaker *T* and *T*'s corresponding negative truthmaker $\ominus T$.

Before assessing O_{TM})'s plausibility, we should consider whether the ontological accounts embedded within the definitions of the LNC noted above offer us a distinct version of the account. For the sake of simplicity, let's consider the version of the account embedded within $Aristotle_2$. The same conclusions will follow for the other accounts embedded within $Prior_2$, $Russell_1$, and von $Wright_2$, given their relevant similarities.

In $Aristotle_2$, contradictions are conceived as the same attribute belonging and not belonging to the same subject, at the same time and in the same respect. The account of contradiction embedded within $Aristotle_2$, therefore, is formulated in terms of attributes and not truthmakers. Yet, it is the attributes' *belonging* and *not belonging* to a subject which is contradictory, and not the attributes themselves.[104] This, however, is just disguised truthmaker talk, the obtaining of certain states of affairs. Therefore, plausibly interpreted as an account of contradictions, the account becomes:

O_A) A contradiction is the simultaneous obtaining of both a state of affairs 'object *O*'s possessing attribute ϕ' *and* a state of affairs 'object *O*'s not possessing attribute ϕ'.[105]

As O_A) is a truthmaker version of the ontological account, advocates of O_A) are committed to the existence of *negative* truthmakers for the reasons already given. In this respect, there's no difference between the ontological baggage of O_{TM}) and O_A). The accounts do, however, differ in their generality. As it stands, O_A) is unable to account for certain paradigm contradictions. Given that O_A) only includes pairs of states of affairs of the form 'object *O*'s (not) possessing attribute ϕ', the account won't accommodate any obvious cases of contradictions that fail to fit this form, such as those formed by an existential claim and its negation, 'Emma exists and doesn't exist'. Contradictions constituted of existential propositions and their negations will only fit the schemas in O_A) if we commit ourselves to the thesis that existence is a first-order property, in which case 'exists' would be an appropriate substitution of 'ϕ' in O_A). However, it's both peculiar and unwarranted for a definition of

here on whether there are any such contradictions, but it's a challenge the truthmaker account of contradiction must face.

104 Once we start talking of *contradictory* or *incompatible* attributes, we can only make sense of these concepts in terms of the possession of one of these attributes *F* precluding the possession of the other attribute *G*, and *vice versa*. That is, possessing *F* is tantamount to *not* possessing *G*, and possessing *G* is tantamount to *not* possessing *F*. Thus, possessing two contradictory attributes is equivalent to both possessing and *not* possessing both attributes.

105 We aren't suggesting that this *was* Aristotle's account of contradiction, only that the account embedded in $Aristotle_2$ must be translated into talk of truthmakers if it's to be plausible.

contradiction to require us to take a stand on such a substantive debate to handle these obvious cases of contradictions. As it stands then, O_A) is an inadequate version of the ontological account. Either it fails to capture some obvious contradictions, or it requires one to commit to a thesis that a definition of contradiction shouldn't require.

While we can alter O_A) so that it accommodates these existential contradictions,

> $O_{A\text{-}Ex}$) A contradiction is the simultaneous obtaining of both a state of affairs 'object *O*'s possessing attribute *ϕ*' *and* a state of affairs 'object *O*'s not possessing attribute *ϕ*', *or* the simultaneous obtaining of both a state of affairs 'object *O*'s existing' *and* a state of affairs 'object *O*'s not existing',

the solution seems pretty *ad hoc* and flags up a general concern with the Aristotelian version of the ontological account. If we are willing to add further conditions into O_A), so that more conjunctions of states of affairs are categorised as contradictions, what's to stop us from adding yet further conditions into $O_{A\text{-}Ex}$)? The sensible answer is that we are limited to postulating as contradictions those states of affairs that serve as truthmakers for propositions that we consider to be contradictions. In other words, we only include schemas that states of affairs which serve as truthmakers for *semantic* contradictions fit. However, then it begins to seem that contradictions are primarily truthbearers and *not* truthmakers, and that our criteria for contradictions is at base semantic and not ontological. The states of affairs become *parasitically* contradictory because they are truthmakers *for contradictions*. This is a general concern for all ontological accounts of contradiction, but it's particularly pressing for the Aristotelian account.

Although it may be possible for an advocate of an ontological account to argue for the primacy of ontological accounts over suitable semantic accounts, due to metaphysical considerations, any such ontological account would need to possess the same explanatory power as its semantic competitors.[106] This requires the ontological account to not only accommodate the same data as the semantic accounts, by explaining why some propositions are contradictions and others not, but to account for this data in a non-piecemeal fashion. If an ontological account can achieve this, then it may be possible to demonstrate that an ontological definition of contradiction has some theoretical primacy over its semantic

[106] One potential argument for the primacy of ontological accounts over semantic accounts of contradiction is via a metaphysical version of the LNC. If one could demonstrate that a metaphysical LNC is, in some sense, more theoretically fundamental than logical versions of the law, then this would plausibly give us reason to believe that ontological accounts of contradiction, which are embedded in a metaphysical LNC, are philosophically primary. While Takho (2009) has proposed arguments for a metaphysical version of the LNC, a full evaluation of those arguments is both well beyond the scope of this section and unnecessary for our purposes here, given that our criticisms of the ontological accounts should demonstrate that they are philosophically implausible.

competitors. $O_{A\text{-}EX}$), however, fails these conditions. It can only explain its inclusion of the two separate schemas 'Object *O*'s (not) possessing attribute ϕ' and 'Object *O*'s (not) existing', and *not* other schemas, by appealing to our intuitions regarding contradictory propositions, which semantic accounts such as the truth-value neutral account can both explain and accommodate far more simply. Given that any plausible ontological account must demonstrate the primacy of ontological accounts of contradiction over semantic accounts, $O_{A\text{-}EX}$) is inadequate for the ontological account advocate's purposes. Given the unity of explanation it offers, the generalised truthmaker version of the ontological account, O_{TM}), offers much more potential.

Consideration of the available versions of the ontological account has produced three important conclusions:

1) Unless we gain new conceptual resources, the only viable option for an ontological account of contradiction is a truthmaker account.

2) Any *prima facie* plausible truthmaker versions of the account will be committed to *negative* truthmakers.

3) The Aristotelian ontological account, $O_{A\text{-}Ex}$), lacks the explanatory power to demonstrate that ontological accounts are theoretically primary to semantic accounts.

Now that we've clarified the available ontological accounts of contradiction, we can consider their plausibility. Two shortcomings plague any ontological account.[107]

Firstly, there are no means for a nominalist regarding truthmakers to reasonably accept a version of the ontological account, given that it seems truthmaker versions of the ontological account are its only *prima facie* plausible versions. The nominalist has two options when attempting to formulate a nominalist friendly version of the ontological account, neither of which are acceptable.

Firstly, she could accept O_{TM}) whilst treating truthmakers as fictions. This option though suffers from the weakness that it attempts to explain contradictions in terms of an ontological kind that one admits doesn't exist, in which case it's hardly accurate to call the account an *ontological* account. Additionally, as this interpretation of O_{TM}) defines contradictions in terms of something that doesn't exist, the interpretation entails that contradictions themselves don't exist. They become mere fictions. In addition to being totally

[107] In addition to the concerns raised here, there is the more fundamental criticism of ontological accounts that speaking of *ontological* contradictions is a category mistake; a view expressed in both Berto (2007) and Bobenrieth (2007). A full evaluation of this criticism of ontological accounts is well beyond the scope of this section. Fortunately, we can base our conclusion that ontological accounts are implausible on other considerations.

bizarre, this consequence of the nominalist interpretation of O_{TM}) seems theoretically unpalatable. It should take more than being sceptical about the existence of truthmakers to convince one that contradictions are fictions. We can, and *do*, separate these ontological questions. Needless to say, this interpretation of O_{TM}) isn't a plausible option for the nominalist. Unless she's willing to admit that contradictions are fictions, she will need to ground her definition of contradictions in an ontological kind that she admits the existence of.

The nominalist's second option is to talk in terms of the extension and *anti*-extension of a predicate:

> O_{NOM}) A contradiction is the intersection of a predicate *P*'s extension and *P*'s anti-extension.

Interpreting an ontological account as O_{NOM}) has at least two shortcomings. Firstly, for O_{NOM}) to account for all obvious cases of contradictions, 'exists' will have to be considered a predicate. However, this is a substantive philosophical question and shouldn't have to be settled to accommodate troublesome cases into a definition of contradiction. What counts as a contradiction shouldn't be dictated by one's views on whether 'exists' is a predicate or not. There are very few philosophers who would admit that 'Emma both exists and doesn't' isn't a contradiction, whatever their stance on 'exists' was. This demonstrates that O_{NOM}) lacks a generality which cannot be recaptured other than through *ad hoc* means, as we saw with O_A). By concentrating on the intersection of a predicate's extension and anti-extension, the account fails to admit some paradigm cases of contradictions.

O_{NOM})'s second shortcoming is that it isn't an ontological account of contradiction at all. The account talks about a predicate's extension and anti-extension, which is a matter of semantics and not ontology. Predicates are an element of a language, and there are no assurances that predicates carve up the world at its natural ontological joints. By defining contradictions in terms of linguistic elements, one is abandoning the ontological account. Yet, of course, not talking directly in terms of ontological kinds, such as properties and truthmakers, is exactly what the nominalist wants to achieve; being a nominalist regarding truthmakers is tantamount to rejecting any truthmaker account of contradiction.

In failing to produce a plausible nominalist interpretation of the ontological account, we have shown a failure in the account. A plausible account of contradiction should be employable by as many individuals as possible, however divergent the individuals' philosophical views are. The concept is fundamentally important to both logical and philosophical discussion, and its application of use shouldn't be restricted to those areas of philosophy in which certain ontological commitments are commonly admitted. To encourage such a restriction, by advancing an account of contradiction that's only suitable for philosophers with certain ontologies, is theoretically imprudent. Perhaps this concern wouldn't be fatal for the account if it weren't for other available accounts that don't unnecessarily restrict the concept's use. Yet, as there are accounts, such as the truth-value neutral account, which place no ontological restrictions on the concept's use, the failing does seem fatal for any ontological account.

The ontological account's second failure, related to the first, is its inability to accommodate any talk of contradictions not involving truthmakers. Even if one is willing to admit truthmakers into one's ontology, one may reasonably be undecided about whether certain areas of discourse require truthmakers, whilst being sure that we can still talk about contradictions occurring in these discourses. This view could have at least two good independent motivations. Firstly, there may be certain objects, such as mathematical objects, which one is either a fictionalist about, or unsure of the existence of. In this case, one would feel able to speak of contradictions involving propositions about these objects, while not believing that propositions about these objects had truthmakers. Secondly, one may believe that certain propositions don't have truthmakers because of particular modal properties they possess, such as being necessarily true, while also believing that these propositions can constitute part of a contradiction.[108] So, if one thinks that arithmetical propositions are either necessarily true or necessarily false, the proposition 'Two plus two equals four and two plus two doesn't equal four' won't be constituted of two truthmakers, for at least one of its conjuncts doesn't have a truthmaker (on the assumption that negation has its usual semantics). Yet, this wouldn't stop one from considering the proposition a contradiction.

The ontological account requires one to commit to the existence of an object *O* to talk about contradictions involving *O*. This, though, seems theoretically unpalatable. An account of contradiction should allow one to recognise a proposition as a contradiction without committing oneself to the existence of the subject of the proposition. We do, after all, seem to achieve this task. Any account of contradiction that didn't require ontological commitment to the subject of a contradiction would be theoretically preferable in this regard. Fortunately, we have an account that offers this ontological neutrality in the form of the truth-value neutral account.

From our discussion of the ontological account, we can stress two conclusions. Firstly, we have found good reasons to believe that the account is philosophically implausible because it requires individuals to take on ontological commitments that they shouldn't have to just to mention a contradiction. Secondly, the truth-value neutral account doesn't suffer from the ontological account's faults. Yet more evidence for the plausibility of the truth-value neutral account.

In demonstrating the ontological account's implausibility, we have also shown its unsuitability for the absolutist's purposes. We should assume neither that the absolutist

[108] Mellor (2003) p. 213.

endorses a truthmaker ontology or that she, implausibly, precludes the possibility of talking about propositions being contradictions when she isn't sure as to the existence of the subjects of the propositions. In addition, no ontological account can be simply embedded into Ab), given that Ab) proposes the possible *truth* of contradictions. Truthmakers cannot be true. Therefore, by using an ontological account, the absolutist would have to adapt Ab) to an explicitly more metaphysical thesis, something we have no reason to think she has any inclination to do. This final point brings into focus the importance of conceiving of contradictions as truthbearers, for both the absolutist's and classical logician's sakes. Both believe that we can assign contradictions truth-values, and our ability to do so is incredibly important in logical discourse. Thus, any ontological account must explain our talk of truthbearers as contradictions, and it must do so while demonstrating why an ontological account is more plausible than any semantic account that explains the same data. Given the additional, and seemingly unnecessary, ontological baggage that an ontological account carries, the prospects of the account demonstrating its suitability over semantic accounts seem slim.

3.5 Ab) Revisited

In this chapter we have considered seven distinct accounts of contradiction and their variations, evaluating each for their philosophical adequacy as definitions of contradiction and their suitability to be embedded within Ab). Following Grim (2004), we categorised the accounts of contradiction under the headings of *semantic*, *syntactic*, *pragmatic*, and *ontological* accounts.

Within the *semantic* accounts category we considered three accounts of contradiction. The first, the classically-assumed account, came in three distinct versions: the Bonevac, the Guttenplan *et al.*, and the Prior & Sainsbury versions. While each version was more plausible than its predecessor, none were ultimately found to be philosophically plausible or suitable for the absolutist's purposes. The second semantic account, the explosion account, endorsed by Field, was found to be both highly implausible and unsuitable to embed within Ab). The final semantic account, the truth-value neutral account, which defined contradictions as the conjunction of a proposition and its negation, was found to have the theoretical virtues of both being unsusceptible to the criticisms of the previous semantic accounts, and successfully predicting an important area of debate between those who disagree over the truth of contradictions. The truth-value neutral account was found to be both philosophically adequate and suitable to embed within Ab).

We then considered the *syntactic* accounts, of which there were two. The first, the form account, was open to two interpretations: the formulae interpretation and the proposition interpretation. While the former interpretation was swiftly rejected as being implausible, the latter was further investigated. We found that the proposition-form account itself could be interpreted in two forms, PF_T) and PF_C), depending on one's theory of logical form. While all versions of the PF_T) interpretation of the account were found to be implausible, we were forced to suspend judgement on the plausibility of one version of PF_C), PF_C^W), given our inability to suitably assess whether it's possible to assign meaning to formulae without assuming a particular logic's semantic. Thus, though we suspended judgement on the plausibility of PF_C^W), we concluded that we had no good reason to believe that any of the versions of the form account were philosophically plausible, and good reason to believe that all the versions except PF_C^W) were both philosophically implausible and unsuitable to embed within Ab). The second syntactic account, the *reductio* account, was found to contain the proposition-form account of contradiction. Therefore, the (im)plausibility of the versions of the *reductio* account were parasitic upon the (im)plausibility of the versions of proposition-form account embedded within them. Consequently, only R_{NC}^W), which embedded PF_C^W), wasn't condemned as implausible and, following our findings with regards to the proposition-form account, we concluded that we had no good reason to believe that any version of the *reductio* account is philosophically plausible.

Finally, we considered the pragmatic and ontological accounts. The pragmatic account, P), defined contradictions as the simultaneous assertion and denial of a proposition. We found that the pragmatic account, whichever theory of assertion it embedded, failed to accommodate some very basic philosophical uses of the concept of contradiction, and therefore concluded that the account was philosophically inadequate as well as being wholly unsuitable to embed within Ab).

The ontological account came in two varieties: O_{TM}), which defined contradictions in terms of truthmakers, and O_A), which defined contradictions in terms of an attribute both belonging and not belonging to a subject. While O_A) was found to be implausible because it failed to account for certain paradigm cases of contradictions without unnecessarily committing one to substantial philosophical theories, O_{TM}) is implausible for requiring one to take on ontological commitments which shouldn't be required by a definition of contradiction. Consequently, both versions of the ontological account were found to be philosophically implausible and unsuitable to embed within Ab).

Of all the accounts of contradiction we have considered, only the truth-value neutral account was found to be both philosophically plausible and suitable to embed within Ab). Thus, based on our consideration of the available accounts, the truth-value neutral account seems the only viable candidate for the absolutist to embed within her theory. By embedding the truth-value neutral account into Ab),

Ab_C) $\exists\Sigma$ ($\Sigma \subseteq \{x \mid x$ is a conjunction of a proposition and its negation$\} \wedge (\exists w \in W, w_a R w, \forall B \in \Sigma, v(B_w) = 1)$), we give the absolutist's thesis the substantive content it requires. We now seem to know, firstly, exactly what the absolutist is hypothesising and, secondly, that the concept of contradiction itself doesn't ensure that the absolutist's position is nonsense.

However, absolutism may still turn out to be nonsense, or fail to be a distinct philosophical position from modal trivialism or dialetheism, regardless of our finding a suitable philosophically plausible definition of contradiction to embed within Ab). The absolutist requires more than a coherent statement of her thesis; she requires a suitable logic for her thesis. Establishing that there is just such a logic is the topic of the next three chapters.

4. Trivialisation and Paraconsistency

> Once a contradiction were admitted, all science would collapse.
>
> (Popper (1940) p. 410)

The absolutist hypothesises that there are at least some true contradictions at a possible world. In doing so, she intends her hypothesis to be distinct from *modal* trivialism; the thesis that there's a metaphysically possible *trivial* world. While the absolutist's hypothesis entails *modal* trivialism if she interprets the set of contradictions Σ in Ab) to be identical to $\{x \mid x \text{ is a contradiction}\}$, this is only one possible interpretation of Σ. Under all but one interpretation of Σ in Ab), the absolutist wishes to allow for non-trivial worlds at which some contradictions are true.

In hypothesising these contradictory but non-trivial worlds, the absolutist comes into conflict with a principle of classical logic, the principle of explosion. Explosion ensures that an inconsistent premise set implies any arbitrary proposition, $\{A, \sim A\} \vDash B$. To resist any accusations that her thesis reduces to *modal* trivialism, the absolutist must demonstrate there are logics available that invalidate explosion while being otherwise adequate to model her theory.

4.1 The Argument from Triviality

With the principle of explosion, classical logic threatens any theory that allows for true contradictions with triviality. According to Bobenrieth (2010, pp. 113-115), the first use of the principle of explosion to demonstrate the absurdity of allowing for true contradictions is found in Hilbert and Ackermann's (1928) *Grundzüge der Theoretischen Logik*:

> [T]he occurrence of a formal contradiction, *i.e.* the provability of two formulas A and $\sim A$, would condemn the entire calculus as meaningless; for we have observed above that if two sentences of form A and $\sim A$ were provable the same would be true of any other sentences whatsoever.
>
> (Hilbert & Ackermann (1950) p. 383)

Not much later than Hilbert and Ackermann's argument from triviality, we find in Lewis and Langford (1932, p. 252) a putative proof of explosion via the disjunctive syllogism:

1.	$p \wedge \sim p$		Assumption
2.	p	{1}	Simplification
3.	$p \vee q$	{2}	Addition
4.	$\sim p$	{1}	Simplification
5.	q	{3, 4}	Disjunctive Syllogism

In both cases, we are given assurance that the proof of any contradiction permits us to prove any formula. Every formula becomes a theorem. Theories that postulate true contradictions are consequently absurd, for they are committed to postulating the truth of *every* proposition.

The modern classical logician can also provide a semantic demonstration for why an inconsistent premise set entails every proposition, by simply appealing to the standard semantic consequence relation. Given the classical consequence relation,

$$\Sigma \vDash B \text{ iff for any } v\text{, if } \forall A \in \Sigma,\ v(A)=1\text{, then } v(B)=1,$$

one can demonstrate that a premise set Σ containing contradictory propositions entails any proposition B. As each proposition A can only be assigned one truth-value in classical logic and negation is a truth-reserving truth-functor, no pair of contradictory propositions $\{A, \sim A\}$ can both be assigned the truth-value true in a valuation in classical logic. Therefore, if a premise set Σ contains a contradictory pair, there will be no valuation v in which every member of Σ is true, ensuring that there is no valuation v such that every member of Σ is true and an arbitrary proposition B is false.

For the absolutist to accommodate contradictory, but non-trivial, worlds in her logic, she will have to resist the argument from triviality, which requires a rejection of explosion and, subsequently, a rejection of classical logic. To accommodate non-trivial contradictory worlds, the absolutist will need to enter the non-classical world of paraconsistent logics.

4.2 Paraconsistent Logics

The common definition of paraconsistent logics is that they invalidate explosion, $\{A, \sim A\} \vDash B$, which just means that the logics allow for inconsistent non-trivial theories.[109] Although it's doubtful that the invalidation of explosion is sufficient for paraconsistency, with a finer-grained definition being required, the necessary alterations to the definition are irrelevant for our purposes here.[110] Therefore, we can safely use 'Logics that invalidate explosion' as our definition of paraconsistency here.[111]

[109] See Arruda (1980, p. 2 & 1989, p. 100), Priest & Routley (1984, p. 3), and da Costa & Doria (1995, p. 34). The term 'paraconsistent logic' was first used in a paper by Miró Quesada (1978), at the Third Latin American Congress of Mathematical Logic in 1976.

[110] There are logics such as Johansson's (1936) 'Minimal Calculus', a positive fragment of intuitionistic logic, in which although explosion isn't a theorem, a special instance of explosion such as $\{A, \sim A\} \vdash \sim B$ is. This isn't ideal. In principle one wants a definition of paraconsistency that ensures for any –nary placed connective $\circ$ and any formulae A, B and C, both $\{A, \sim A\} \nvdash \circ B$ and $\{A, \sim A\} \nvdash B \circ C$. After all, the underlying philosophical motivation for paraconsistent logics is that they allow for substantive inferences to be made from inconsistent premise sets. For more on how this might be achieved, see Urbas (1990).

[111] Other restrictions on paraconsistency have been hypothesised. Notably, da Costa (1974, p. 498), in his pioneering work on **C-Systems**, suggested that calculi serving the foundation of non-trivial inconsistent theories must meet four conditions:

Our goal in this chapter isn't to produce a survey of the available paraconsistent logics. After all, there are already many good introductions to paraconsistency, and surveys of the literature, available.[112] Instead, it is two-fold:

1) To demonstrate that some paraconsistent logics don't entail dialetheism or absolutism.

2) To demonstrate that the absolutist requires a particular type of paraconsistent logic to adequately model her theory.

In clarifying both of these matters, we will be clearer on those logics suitable for the absolutist's purposes.

4.3 Paraconsistency Isn't Dialetheism or Absolutism

Not only have some prominent paraconsistent logicians professed their agnosticism towards, or outright rejection of, the possibility of true contradictions,[113] but there are also certain paraconsistent logics that invalidate explosion without allowing for the truth of contradictions. Therefore, there are logics available for those who don't wish to permit true contradictions, but do wish to allow for non-trivial inconsistent theories. Two prominent cases of such non-dialetheic paraconsistent logics are the non-adjunctive *discursive* logic of Jaśkowski and the preservationist logic of Jennings and Schotch.[114]

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1) The principle of contradiction $\sim(A \wedge \sim A)$ must not be a theorem.
2) From two contradictory formulae it must be impossible to deduce an arbitrary formula.
3) The logic must have a simple first-order extension.
4) The logic must contain as much of classical logic that doesn't impede the fulfillment of conditions 1-3) as possible.

While da Costa admits that the last two conditions are vague, it's the irrelevance of condition 1) to logics admitting non-trivial inconsistent theories which is particularly striking. The law of non-contradiction ensures that all contradictions are false, yet there are logics, as we shall see, that successfully invalidate explosion while still validating the law of non-contradiction. It seems theoretically fruitless to preclude these logics from the set of paraconsistent logics, when it's clear they can model non-trivial inconsistent theories.

[112] A good starting point is still the landmark Priest *et al*. (1989), with more recent surveys available in Priest (2002) and da Costa *et al.* (2007).

[113] While agnosticism towards true contradictions is expressed in da Costa (1982), da Costa *et al.* (1995), and da Costa & Bueno (2001), an outright rejection of the possibility of true contradictions is found in Schotch (1992).

[114] Throughout this chapter we will only be interested in the zero-order fragment of a logic. If the logic has a first-order extension we will reference it in a footnote.

4.3.1 Jaśkowski's Discursive Logic

In Jaśkowski's (1969, p. 149) non-adjunctive *discursive* logic $\mathbf{D_2}$, possible worlds act as participants in a discourse, due to $\mathbf{D_2}$ treating discursive assertion as equivalent to possible truth. Thus, the sum of possible worlds becomes the discourse between individuals. A proposition A is true in a model $\mathcal{M}$ of $\mathbf{D_2}$ if and only if A is true at some world w in $\mathcal{M}$. So, letting W be the set of worlds, $\mathcal{M}$ be a model for the possible worlds, and $w \vDash A$ mean that A is true at w:

$$\mathcal{M} \vDash_{\mathrm{D}} A \text{ iff for some } w, \text{ such that } w \in W, w \vDash A \text{ in } \mathcal{M}.$$

In combination with $\mathbf{D_2}$'s consequence relation,

$$\Sigma \vDash_{\mathrm{D}} B \text{ iff for all } \mathcal{M}, \text{ if } \forall A \in \Sigma, \mathcal{M} \vDash_{\mathrm{D}} A, \text{ then } \mathcal{M} \vDash_{\mathrm{D}} B,$$

this definition of truth-in-a-model allows us to produce a countermodel to explosion. Imagine a model $\mathcal{M}$ constituted of two worlds w and w', both representing participants in a discourse. At w A holds, and at w' $\sim A$ holds, but B holds at neither. In which case, $\{A, \sim A\} \nvDash_{\mathrm{D}} B$. Explosion fails in Jaśkowski's $\mathbf{D_2}$.

Despite explosion failing in $\mathbf{D_2}$, there's no model $\mathcal{M}$ in $\mathbf{D_2}$ such that a contradiction $p \wedge \sim p$ is true. This is ensured, firstly, by there being no possible world w in $\mathbf{D_2}$ at which a contradiction $p \wedge \sim p$ is true and, secondly, by the rule of adjunction being invalid in $\mathbf{D_2}$,

$$\{A, B\} \nvDash_{\mathrm{D}} A \wedge B.$$ [115]

Thus, while the contradictories p and $\sim p$ can be true at distinct worlds w and w', there is neither any world w'' at which $p \wedge \sim p$ is true, nor any way to deduce the truth of $p \wedge \sim p$ in a model $\mathcal{M}$, given that adjunction is invalid. Consequently, $\mathbf{D_2}$ validates the conjoined form of explosion,

$$\{A \wedge \sim A\} \vDash_{\mathrm{D}} B,$$

and contains the law of non-contradiction,

$$\vDash_{\mathrm{D}} \sim(A \wedge \sim A),$$

[115] Once one recognises that possible worlds in $\mathbf{D_2}$ represent participants in a discourse, it's clear why one shouldn't be allowed to conjoin propositions that are true at distinct worlds. Treating the assertions a of an individual i as possibly true and the assertions a' of an individual i' as possibly true doesn't entail that one should then treat the conjunction of the assertions as possibly true. Validating the rule of adjunction in $\mathbf{D_2}$ is tantamount to validating the absurd distribution rule $\Diamond A \wedge \Diamond B \leftrightarrow \Diamond(A \wedge B)$.

as a logical truth.[116] Clearly then, $\mathbf{D_2}$ is a paraconsistent logic that doesn't entail true contradictions at either the actual, or a possible, world.[117]

4.3.2 Jennings and Schotch's Preservationist Logic

Jennings and Schotch's (1984 & 1989) preservationist logic is produced by introducing a new consequence relation, *level forcing*, into an underlying logic L.[118] To understand this *L-level forcing* relation, we first need to introduce some definitions.

Logical Cover

Let $\mathfrak{I}$ be an indexed set beginning with $\varnothing$, I be the index set of $\mathfrak{I}$, and $\mathbb{C}_L(a)$ be the deductive closure of a in L. Now, $\mathfrak{I}$ is a *logical cover* of the set Σ in L, $\mathrm{COV}_L(\mathfrak{I}, \Sigma)$, iff,

i) For every element $a_i \in \mathfrak{I}$, $\exists B\ a_i \nvdash_L B$.[119]

ii) $\Sigma \subseteq \bigcup_{i \in I} \mathbb{C}_L(a_i)$.

So, a *logical cover* for a set Σ in L is an indexed family of sets $\mathfrak{I}$, beginning with $\varnothing$, such that every $a_i \in \mathfrak{I}$ is consistent, and every $A \in \Sigma$ is a member of the union of the deductive closure of each a_i in L.

The Width of a Logical Cover

Let $w(\mathfrak{I}_\Sigma)$ stand for the *width* of a *logical cover* $\mathfrak{I}_\Sigma$ for the set Σ:

$$w(\mathfrak{I}_\Sigma) = \text{The cardinality } |\, I \,| - 1 \text{ of the index set } I \text{ of } \mathfrak{I}_\Sigma.$$

The definitions of *logical cover* and *width* allow us to introduce the all-important concept of *level*.

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[116] Jaśkowski (1969) p. 152.

[117] For a first-order extension of $\mathbf{D_2}$, see da Costa & Dubikajtis (1977). Other paraconsistent non-adjunctive logics can be found in Rescher & Manor (1970) and Rescher & Brandon (1980).

[118] It's unclear which logics can serve as the underlying logic L of a preservationist logic. Two necessary properties of L mentioned in Payette & Schotch (2009, p. 102) are: i) That L is non-trivial, $\exists B\ \varnothing \nvdash B$, and ii) That L contains *denial*, the requirement that $\forall A \exists B \forall C\ \{A, B\} \vdash C$. This second condition is fulfilled by any logic L containing an explosive negation. Whether there are any other necessary conditions a prospective underlying logic L must fulfill is an important topic for preservation logics, but beyond the scope of this chapter.

[119] Given that the underlying logic L of a preservationist logic must contain *denial* and, therefore, a set Γ is *inconsistent* in such a logic L if and only if $\forall B\ \Gamma \vdash B$, this condition is tantamount to every element $a_i \in \mathfrak{I}$ being *consistent*.

Level

Let $\ell^L(\Sigma)$ stand for the *level* of the set Σ in L:

$$\ell^L(\Sigma) = \begin{cases} \min\limits_{w(\mathfrak{I})} [\mathrm{COV}_L(\mathfrak{I}, \Sigma)] & \textit{if this limit exists} \\ \infty & \textit{otherwise.} \end{cases}$$

The *level* of a set Σ in L, therefore, is the width of the narrowest logical cover of Σ in L. Hence, if Σ is the empty set, or contains only theorems of L, then the $\ell^L(\Sigma) = 0$, for $\{\varnothing\}$ will be the narrowest *logical cover* for Σ in L. Similarly, if Σ is a *consistent* set of formulae in L, such that $\exists B\ \Sigma \nvdash_L B$, then the $\ell^L(\Sigma) = 1$, as there will be a set $a \in \mathfrak{I}$ such that a is both consistent and $\Sigma \subseteq \mathbb{C}_L(a)$. Thus, only *inconsistent* sets have a *level* greater than 1. Sets that receive a *level* of ∞ in L are those that have no *logical cover* in L. Such sets Σ contain at least one member A, such that $\forall B\ A \vdash_L B$. In other words, A is *self-inconsistent* (given that L is non-trivial).

To see why such a set Σ, containing a *self-inconsistent* member A, has no *logical cover* in L, one only has to look to the two conditions on *logical cover*. For $\mathrm{COV}_L(\mathfrak{I}, \Sigma)$, every $A \in \Sigma$ must be a member of the union of the deductive closure of each $a_i \in \mathfrak{I}$. Yet, this requires there to be some $a_i \in \mathfrak{I}$ such that $A \in \mathbb{C}_L(a_i)$, which subsequently requires some $a_i \in \mathfrak{I}$ to be *inconsistent*, given that A is *self-inconsistent*. This is a consequence of consistency being closed under entailment in a non-trivial logic L. So, if the deductive closure, $\mathbb{C}_L(\Sigma)$, of a set Σ contains a (self-)inconsistency, then so does Σ. However, given that condition i) of *logical cover* requires every $a_i \in \mathfrak{I}$ to be consistent, there are *no* inconsistent members of $\mathfrak{I}$. Therefore, if A is *self-inconsistent*, there is no $a_i \in \mathfrak{I}$ such that $A \in \mathbb{C}_L(a_i)$ and, consequently, no set Σ containing A will have any *logical cover*.

In giving the *level* of a set Σ in L, preservationist logics convey two properties of Σ in L:

> First, that there is a way to divide the logical resources of Σ into k distinct [consistent] subsets [known as *cells*] … Second, that any way of thus dividing Σ, must have at least k cells.
>
> (Payette & Schotch (2009) p. 99)

With a measure of the inconsistency of a set Σ in terms of the number of cells required to logically cover Σ, preservationist logics can produce a new consequence relation in terms of the *level* of a set Σ in L:

The level forcing consequence relation (for some logic L)

$\Sigma\ [\Vdash_L B$ iff for every division of Σ into $\ell^L(\Sigma)$ cells, for at least one of the cells a, $a \vdash_L B$.

Thus, the *level forcing* relation defines the consequences of a set Σ in L in terms of the implications of those cells a_i that fulfil the conditions, i) $a_i \in \mathfrak{I}$, ii) $COV_L(\mathfrak{I}, \Sigma)$, and iii) $w(\mathfrak{I}_\Sigma)$ in $L = \ell^L(\Sigma)$. So, if for every $\mathfrak{I}$ that fulfils conditions ii-iii), there is some $a_i \in \mathfrak{I}$ such that $a_i \vdash_L B$, then Σ $[\Vdash_L B$.

Now, the *level-forcing* consequence relation invalidates explosion without entailing that some contradictions are (possibly) true. Substitute 'classical logic' for L. In classical logic (CL), no inconsistent pair of propositions can be true. However, with the *level forcing* consequence relation, explosion is invalidated when classical logic is the underlying logic. Let $\Sigma = \{p, \sim p\}$, and let $\mathfrak{I}$ be an indexed set such that $COV_{CL}(\mathfrak{I}, \Sigma)$, and the $w(\mathfrak{I}_\Sigma)$ in $CL = \ell^{CL}(\Sigma)$. Given that the $\ell^{CL}(\Sigma) = 2$, there is a $\mathfrak{I}$ such that $\mathfrak{I} = \{\varnothing, \{p\}, \{\sim p\}\}$. Given that in classical logic not every proposition follows from the null set, p, or $\sim p$, explosion is invalidated – $\{A, \sim A\}$ $[\nVdash_{CL} B$. The same, however, isn't true of conjoined explosion, $\{A \wedge \sim A\} \vDash B$. Let $\Sigma = \{p \wedge \sim p\}$. In this case $\ell^{CL}(\Sigma) = \infty$, which ensures that, for every division of Σ into ∞ cells, $\forall B \exists A\ A \vDash_{CL} B$, due to reflexivity. Therefore, $\{A \wedge \sim A\}$ $[\Vdash_{CL} B$.[120]

We have seen that there are at least two paraconsistent logics that entail neither true contradictions, nor the possibility of true contradictions. This ensures that one can use paraconsistent logics without the fear of being committed to (possibly) true contradictions.

4.4 The Absolutist Needs More Than Just Any Paraconsistent Logic

Not all paraconsistent logics entail true contradictions or the possibility of true contradictions. Therefore, the absolutist can't use just any paraconsistent logic to model her theory. If there is any logic L suitable for modelling the absolutist's thesis, then L must at least:

A1) Invalidate the unconjoined form of explosion $\{A, \sim A\} \vDash B$.

A2) Invalidate the conjoined form of explosion $\{A \wedge \sim A\} \vDash B$.

A3) Allow for contradictions, formulae of the form $A \wedge \sim A$, to be assigned the truth-value true.

A4) Ensure that true contradictions at possible worlds don't entail true contradictions at the actual world.

[120] For a preservationist logic that also invalidates $\{A \wedge \sim A\} \vDash B$, see the work of Brown (1999, 2000 & 2009), where measures of *ambiguity* are preserved. For an overview of the preservationist research programme, see the collection of papers in Schotch *et al.* (2009).

Any logic that fulfills conditions A1-A3) we will call a *dialetheic* logic, and any logic fulfilling all the conditions an *absolute* logic. So, all absolute logics are dialetheic logics, but the inverse isn't true. While dialetheists only require a dialetheic logic, absolutists need an absolute logic. If a true contradiction at a possible world w entails true contradictions at the actual world w_a, then absolutism merely becomes a subspecies of dialetheism, contrary to absolutism presenting itself as a thesis independent of the truth of contradictions at the actual world. Therefore, if absolutism is to be taken seriously as an independent philosophical theory, it requires an absolute logic. Whether there are any absolute logics is a question we will broach in the next chapter. The rest of this chapter is dedicated to highlighting two available dialetheic logics.

4.4.1 Priest's Logic of Paradox

Priest's (1979) *Logic of Paradox* (**LP**) is a three-valued logic that allows for truth and falsity to intersect.[121] In **LP**, valuations of propositional parameters are relations ε between the parameters and the set of truth-values $\{1, 0\}$, with each parameter taking *at least* one truth-value. There are no truth-value gaps in **LP**.[122] Thus, propositional parameters may be assigned the truth-value *true*, *false* or *both* (which is just truth *and* falsity). The conjunction, disjunction, and negation of **LP** are those of Kleene's (1971, p. 334) strong matrix:

$(A \wedge B)\varepsilon 1$ iff $A\varepsilon 1$ and $B\varepsilon 1$
$(A \wedge B)\varepsilon 0$ iff $A\varepsilon 0$ or $B\varepsilon 0$

$(A \vee B)\varepsilon 1$ iff $A\varepsilon 1$ or $B\varepsilon 1$
$(A \vee B)\varepsilon 0$ iff $A\varepsilon 0$ and $B\varepsilon 0$

$({\sim}A)\varepsilon 1$ iff $A\varepsilon 0$
$({\sim}A)\varepsilon 0$ iff $A\varepsilon 1$.[123]

With the consequence relation defined as,

$$\Sigma \vDash_{\mathbf{LP}} B \text{ iff for any } \varepsilon\text{, if } \forall A \in \Sigma, A\varepsilon 1\text{, then } B\varepsilon 1,$$

such that both *true* and *both* are treated as designated values, we can see that **LP** fulfils conditions A1-A3).

[121] **LP** is just Asenjo's (1966) *Calculus of Antinomies*.
[122] For a logic that allows for truth-value gaps, as well as gluts, see Dunn's (1976) four-valued semantics for Anderson & Belnap's (1975) *First-Degree Entailment* **(FDE)**.
[123] We haven't included a conditional for **LP** here, as it would require an unnecessary digression. See Priest (2006b, Ch. 6) for a discussion of a suitable conditional that evades Curry's (1942) paradox. The conditional-free fragment of Beall's (2009) **LP*** is identical to the conditional-free fragment of **LP**; see Beall (2009, p. 20).

Beginning with A3), there are valuations in **LP** such that a formula A is assigned both truth and falsity, $A\varepsilon 1$ and $A\varepsilon 0$, respectively. Now, given the meaning of negation in **LP**, the truth-value *both* is treated as a fixed-point by negation. This ensures that $\sim A$ takes both truth-values if A does. As both A and $\sim A$ are true, as well as false, their conjunction is true in **LP**, as well as being false. Therefore, it's possible for contradictions to be assigned the truth-value true in **LP**. Interestingly, given that every contradiction in **LP** is false, the meaning of negation in **LP** entails that $\sim(A \wedge \sim A)$ is true in every valuation. Thus, the law of non-contradiction is a logical truth in **LP**.[124]

The satisfaction of A1) and A2) is ensured by **LP**'s consequence relation and the satisfaction of A3). Given that A and $\sim A$ can both be assigned the truth-value *both* in a valuation in **LP**, and thus the contradiction $A \wedge \sim A$ can be assigned both truth-values, there will be valuations in which both $\{A, \sim A\} \nvDash_{\mathbf{LP}} B$ and $\{A \wedge \sim A\} \nvDash_{\mathbf{LP}} B$. Consider a valuation in which $A\varepsilon 1$ and $A\varepsilon 0$, and $B\varepsilon 0$. In this case, both A and $\sim A$ will be assigned truth and falsity, given the meaning of negation in **LP**, while B is only assigned the truth-value false. Therefore, $\{A, \sim A\} \nvDash_{\mathbf{LP}} B$. Similarly, given that $(A \wedge \sim A)\varepsilon 1$ and $(A \wedge \sim A)\varepsilon 0$, while B is only assigned the truth-value false, we also have $\{A \wedge \sim A\} \nvDash_{\mathbf{LP}} B$. Thus, as a consequence of assigning some contradictions both truth-values and treating *both* as a designated value, **LP** fulfils conditions A1-A3). **LP** is a dialetheic logic.[125]

4.4.2 da Costa's C-Systems

Our second dialetheic logic is da Costa's family of **C-Systems**, which contains a logic $\mathbf{C}_n$ for every $1 \leq n < \omega$. For present purposes we will concentrate on $\mathbf{C}_1$, and note how to extend $\mathbf{C}_1$ to the other $\mathbf{C}_n$ logics later. A valuation v in $\mathbf{C}_1$ is a function from the propositional parameters into the truth-value set $\{1,0\}$, satisfying the conditions:[126]

C1) $v(\sim A) = 1$ if $v(A) = 0$
C2) $v(A) = 1$ if $v(\sim\sim A) = 1$
C3) $v(A \rightarrow B) = 1$ iff $v(A) = 0$ or $v(B) = 1$
C4) $v(A \wedge B) = 1$ iff $v(A) = 1$ and $v(B) = 1$
C5) $v(A \vee B) = 1$ iff $v(A) = 1$ or $v(B) = 1$.

Introducing the abbreviation,

A° for $\sim(A \wedge \sim A)$,

[124] Indeed, **LP** upholds all of the logical truths of classical propositional logic; see Priest (2006b, p. 80).
[125] For a first-order extension of **LP** see Priest (2006b) pp. 76-78.
[126] da Costa's (1974) **C-Systems** were originally presented axiomatically, but we are giving the logic's formal semantics here to keep in line with the rest of the chapter. See da Costa & Alves (1977, p. 624) for soundness and completeness proofs for the **C-Systems**.

and the definition,

$$\sim^* A =_{df} \sim A \wedge A^{o},$$

we have the additional conditions:

C6) $v(A) = 0$ if $v(B^{o}) = 1$ and $v(A \rightarrow B) = 1$ and $v(A \rightarrow \sim B) = 1$
C7) $v((A \vee B)^{o}) = 1$ if $v(A^{o}) = 1$ and $v(B^{o}) = 1$
C8) $v((A \wedge B)^{o}) = 1$ if $v(A^{o}) = 1$ and $v(B^{o}) = 1$
C9) $v((A \rightarrow B)^{o}) = 1$ if $v(A^{o}) = 1$ and $v(B^{o}) = 1$
C10) $v(A) = 1$ iff $v(\sim^* A) = 0$
C11) $v(A) = 0$ iff $v(\sim^* A) = 1$
C12) $v(A) = 0$ iff $v(A) = 0$ and $v(\sim A) = 1$
C13) $v(A) = 1$ iff $v(A) = 1$ or $v(\sim A) = 0$.

Although C1) ensures that for every A at least one of A and $\sim A$ is assigned the truth-value true, none of the conditions on $\mathbf{C_1}$ valuations ensure that A and $\sim A$ are *not both* assigned the truth-value true. Thus, there are interpretations in $\mathbf{C_1}$ such that for some A, both $v(A) = 1$ and $v(\sim A) = 1$. In combination with the conditions on conjunction in C4), this ensures that contradictions can be assigned the truth-value true in $\mathbf{C_1}$. $\mathbf{C_1}$ fulfils condition A3).

As with **LP,** we can demonstrate that $\mathbf{C_1}$ fulfils conditions A1) and A2) by appealing to $\mathbf{C_1}$'s fulfillment of condition A3) and its consequence relation,

$$\Sigma \vDash_{\mathbf{C}} B \text{ iff for any } v, \text{ if } \forall A \in \Sigma, v(A) = 1, \text{ then } v(B) = 1.$$

Given that there is some $\mathbf{C_1}$ valuation in which, for some A, both A and $\sim A$ are assigned the truth-value true, and some other formula B is assigned the truth-value false, $\mathbf{C_1}$'s consequence relation ensures that unconjoined explosion is invalidated. Additionally, the conditions on conjunction in C4) equally ensure that conjoined explosion is invalidated. $\mathbf{C_1}$ fulfils conditions A1-A3), and thus qualifies as a dialetheic logic.

While **LP** contains no explosive negation, and therefore no valid form of explosion, $\mathbf{C_1}$ *does* contain an explosive negation $\sim^* A$ in addition to its non-explosive negation, as demonstrated in conditions C10) and C11). In fact, $\sim^* A$ just is classical explosive negation.[127] Consequently, the form of explosion,

$$\{A, \sim^* A\} \vDash_{\mathbf{C}} B,$$

is valid in $\mathbf{C_1}$. This highlights the potentially interesting property of some paraconsistent logics that they needn't contain *only* non-explosive negations.[128]

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[127] da Costa & Alves (1977, p. 622).

[128] The inclusion of the symbol '^{o}' in da Costa's **C-Systems** ensures that they are *Logics of Formal Inconsistency* (**LFI**s); that is, they are paraconsistent logics that can recapture classical inferences through certain consistency assumptions within the logic's own object-language. For more on **LFI**s see Marcos (2005).

Almost everything that has been said here of $\mathbf{C_1}$ is the case for the other $\mathbf{C_n}$, $1 \leq n < \omega$, systems. The only differences occur with the conditions including A^o and $\sim^* A$. Let A^n abbreviate $A^{oo\ldots o}$, where ^{o} occurs n times, $n \geq 1$, and $A^{(n)}$ abbreviate $A^1 \wedge A^2 \wedge \ldots \wedge A^n$. Similarly, the $\sim^* A$ of $\mathbf{C_1}$ becomes $\sim^{(n)} A$ in $\mathbf{C_n}$, $1 \leq n < \omega$, abbreviating $\sim A \wedge A^{(n)}$. Consequently, the conditions C6-C11) from $\mathbf{C_1}$ become,

C6^n) $v(A) = 0$ if $v(B^{(n)}) = 1$ and $v(A \rightarrow B) = 1$ and $v(A \rightarrow \sim B) = 1$
C7^n) $v((A \vee B)^{(n)}) = 1$ if $v(A^{(n)}) = 1$ and $v(B^{(n)}) = 1$
C8^n) $v((A \wedge B)^{(n)}) = 1$ if $v(A^{(n)}) = 1$ and $v(B^{(n)}) = 1$
C9^n) $v((A \rightarrow B)^{(n)}) = 1$ if $v(A^{(n)}) = 1$ and $v(B^{(n)}) = 1$
C10^n) $v(A) = 1$ iff $v(\sim^{(n)} A) = 0$
C11^n) $v(A) = 0$ iff $v(\sim^{(n)} A) = 1$,

for a logic $\mathbf{C_n}$, $1 \leq n < \omega$. As none of the alterations in the conditions C6-C11) have any bearing on conditions A1-A3), every $\mathbf{C_n}$, $1 \leq n < \omega$, system is a dialetheic logic.[129]

4.5 From Dialetheic Logics to Absolute Logics

We have established in this chapter that there are both non-dialetheic and dialetheic paraconsistent logics. For the absolutist to adequately model her thesis, however, she requires an *absolute* paraconsistent logic – a paraconsistent logic that fulfils conditions A1-A4). The possibility of absolute paraconsistent logics is the subject of the next chapter

[129] For first-order extensions of the **C-systems** see da Costa (1974).

5. The Possibility of an Absolute Logic

> Each [philosophical] position can furnish viable, equally formal, but competing logical theories, and the differences between these positions will come down to *philosophical* differences.
>
> (Routley & Meyer (1976) p. 1)

In the previous chapter we demonstrated the availability of dialetheic logics. This chapter's task is to produce an absolute logic that respects the normal semantics for conjunction and negation. We will first offer an intuitive modal extension of dialetheic logics respecting the normal semantics for conjunction and negation. This will highlight the difficulties that arise in attempting to build an absolute logic. In addition, we will demonstrate that this intuitive modal extension of a dialetheic logic is as unpalatable for the dialetheist as it is for the absolutist. Both parties, therefore, require a different modal extension of dialetheic logics. We then propose a new modal logic **AV** that succeeds in both being an absolute logic and avoiding the unsavoury consequences of the previous modal extension, albeit at the cost of the comprehensiveness of the dialetheist's research programme.

5.1 The Semantics

Given that absolutism is a modal thesis, a thesis about what is true at *possible* worlds, the absolutist requires a modal extension of a dialetheic logic. In building a dialetheic modal logic for the absolutist, for dialectical purposes, we will use **LP** as our dialetheic propositional logic. However, all the same conclusions follow from any zero-order dialetheic logic that has the normal semantics for conjunction and negation.

An interpretation for our modal extension of **LP** is a quadruple $<W, w_a, R, \varepsilon>$. W is a set of possible worlds, w_a is a distinguished member of our domain known as the *actual world*, R is a binary relation between sets of worlds known as the *accessibility relation*, and ε is a valuation relation assigning truth-values to a world-indexed proposition (or, to put the point another way, to a proposition-world pair $<w, p>$). As in **LP**, the modal extension allows for world-indexed propositions to receive both truth-values, thus a world-indexed proposition can take the truth-value *true*, *false*, or *both*. The truth-value of complex world-indexed propositions are then defined so as to mirror those of **LP** ($w \in W$):

$(A \wedge B)_w \varepsilon 1$ iff $A_w \varepsilon 1$ and $B_w \varepsilon 1$
$(A \wedge B)_w \varepsilon 0$ iff $A_w \varepsilon 0$ or $B_w \varepsilon 0$

$(A \vee B)_w \varepsilon 1$ iff $A_w \varepsilon 1$ or $B_w \varepsilon 1$

$(A \vee B)_w \varepsilon 0$ iff $A_w \varepsilon 0$ and $B_w \varepsilon 0$

$(\sim A)_w \varepsilon 1$ iff $A_w \varepsilon 0$
$(\sim A)_w \varepsilon 0$ iff $A_w \varepsilon 1$

To give the semantics of the modal operators we need to assume a certain accessibility relation *R*. However, we don't want to take a stand here on which accessibility relation is the most suited to the absolutist's philosophical needs. Therefore, let us assume an arbitrary accessibility relation *R*. The cogency of our point will hold whichever accessibility relation we use, as long as *some* possible but non-actual worlds are accessible from the actual world. Now, by interpreting the usual semantics for necessity and possibility into our talk of a valuation as a relation between a proposition-world pair and the set of truth-values {1, 0}, we achieve:

$\Box A_w \varepsilon 1$ iff, for all $w' \in W$ such that wRw', $A_{w'} \varepsilon 1$
$\Box A_w \varepsilon 0$ iff, for some $w' \in W$ such that wRw', $A_{w'} \varepsilon 0$
$\Diamond A_w \varepsilon 1$ iff, for some $w' \in W$ such that wRw', $A_{w'} \varepsilon 1$
$\Diamond A_w \varepsilon 0$ iff, for all $w' \in W$ such that wRw', $A_{w'} \varepsilon 0$

These definitions retain the intuition that a proposition *p* is necessary at a possible world *w* if and only if *p* is true at every world accessible from *w*, that *p* isn't necessary at *w* if and only if *p* is false at some world(s) accessible from *w*, that *p* is possible at *w* if and only if *p* is true at some world(s) accessible from *w*, and that *p* isn't possible at *w* if and only if *p* is false at every world accessible from *w*. Here then we have a modal extension of **LP** that retains the intuitive semantics of the modal operators.

5.2 Consequence One: From Possibility to Actuality

Unfortunately for the absolutist, we can demonstrate that this modal extension of **LP** fails to be an absolute logic because contradictions at a possible world permeate into the modal level. This entails that if a contradiction is true at a possible world accessible from the actual world, then there is a true contradiction at the actual world.

Consider a possible world w_1 accessible from the actual world w_a. Following the absolutist's thesis, allow for there to be a proposition *p* at w_1 that is assigned both truth-values. Given the meaning of conjunction and negation above, $p \wedge \sim p$ takes the truth-value true at w_1 as well as the truth-value false. There is nothing new here. In **LP** any proposition that is the conjunction of a proposition *p* and *p*'s negation, when *p* is assigned both truth-values, also has both truth-values. We, therefore, have a world accessible from w_a at which a contradiction is true. Irrespective of the truth of any contradiction at any other possible world $w \in W$, we have at the actual world w_a,

$$\Diamond(p \wedge \sim p)_{wa}\varepsilon 1.$$

This isn't the end of the story though. For at *all* possible worlds, accessible or inaccessible from w_a, for every proposition p, the conjunction of p and its negation $\sim p$ takes the truth-value false, whatever truth-value p is assigned. Therefore, given the semantics of the possibility operator above, it's also going to be false at the actual world w_a that $\Diamond(p \wedge \sim p)$. This entails, given the meaning of negation above, that at the actual world w_a,

$$\sim\Diamond(p \wedge \sim p)_{wa}\varepsilon 1.$$

A contradiction at the modal level, as the rule of adjunction is valid. The same point can be demonstrated with the necessity operator, although we needn't go through the same process given the interdefinability of the modal operators:

$$\Diamond(p \wedge \sim p) = \sim\Box\sim(p \wedge \sim p),$$

and

$$\sim\Diamond(p \wedge \sim p) = \Box\sim(p \wedge \sim p).$$

Therefore, given that absolutism entails,

$$(\Diamond(p \wedge \sim p) \wedge \sim\Diamond(p \wedge \sim p))_{wa}\varepsilon 1,$$

in this modal extension of **LP**, it also entails,

$$(\sim\Box\sim(p \wedge \sim p) \wedge \Box\sim(p \wedge \sim p))_{wa}\varepsilon 1.$$

Contradiction.

Thus, this intuitive modal extension of **LP** isn't an absolute logic, for it fails to fulfil condition A4), the condition that true contradictions at a possible world w don't entail true contradictions at the actual world. Consequently, this modal logic is inadequate for the absolutist's purposes.

This consequence of the intuitive modal extension of **LP** is less troubling for the dialetheist. Indeed, it gives her a new avenue to establish that there are true contradictions at the actual world. Rather than relying on semantic or set-theoretic paradoxes, she can conclude that there are true contradictions at the actual world *if* she can establish that there are true contradictions at non-actual possible worlds accessible from the actual world. Unlike the absolutist, therefore, who cannot use this modal extension of **LP** to model her thesis, the dialetheist should be encouraged by this consequence of the semantics. She would no longer need to depend on the paradoxes to justify her claims that there are true contradictions; she could look to the fruitful grounds of possibility. Unfortunately for the dialetheist, the second consequence of the semantics is far less encouraging.

5.3 Consequence Two: The Possibility of Impossibility

Interpreted naturally, a consequence of the modal extension of **LP** above is that the absolutist would be committed to saying that it's *im*possible for contradictions to be true. By necessitation of the theorem $\sim(p \wedge \sim p)$ we can derive $\Box\sim(p \wedge \sim p)$, and given the interdefinability of the modal operators we can derive $\sim\Diamond(p \wedge \sim p)$. Now, interpreted naturally, this formula reads as 'It's impossible for the conjunction of a proposition *p* and *p*'s negation to be true', or 'It's impossible for a contradiction to be true'. Given that an impossible world is a world *w* at which propositions it's impossible to be true *are true*, any world *w* at which a contradiction is true is going to be an impossible world. In conjunction with the absolutist's hypothesis that there are possible worlds at which contradictions are true, this entails that the absolutist is committed to the *possibility* of *impossible* worlds. The whole notion of an impossible world, however, is of a world that could never obtain, and such worlds are only ever used in reasoning about peculiar situations.[130] If possible worlds are ways the actual world could be (or could have been), then impossible worlds are ways the actual world cannot be (and could not have been).

The absolutist is using possible worlds semantics to model her theory that there (metaphysically) could be true contradictions. That is, it's metaphysically possible for there to be true contradictions. Therefore, she shouldn't wish for the semantics of the logic she uses to entail that worlds at which contradictions are true are impossible worlds. By both maintaining that it's metaphysically possible for there to be true contradictions, and using the modal extension of **LP** given above, the absolutist is committing herself to the proposition that at least some *impossible* worlds are *possible* worlds. Yet, this just seems to strip impossibility of the theoretical role that the concept plays. If a world *w*'s being an impossible world doesn't preclude that it's also a possible world, then it's unclear what function the concept of impossibility serves. It certainly doesn't serve the role that we commonly require it to.

In fact, the situation gets worse for the absolutist. By allowing possibility and impossibility to intersect, the modal extension of **LP** given above entails that absolutism cannot logically preclude the actual world being an *impossible* world. Imagine that the absolutist

[130] Nolan (1997).

accepts the modal extension of **LP** above, and that we find good reason to believe that *at the actual world* there is a true contradiction, which isn't precluded by the absolutist's theory. This would lead us to conclude that the actual world, as well as being a possible world, is an impossible world, given that a contradiction would be true at it. This is incoherent. An impossible world is one that couldn't be realised, whereas the actual world is. Whatever the actual world is, it isn't an impossible world. However, the absolutist's position doesn't preclude a contradiction being true at the actual world, although it doesn't require it, and thus it wouldn't preclude the actual world being an impossible world. Any serious modal theory though should preclude the impossibility of the actual world. By failing to do so, we would be losing our notion of what an impossible world is. This result would seem to be a *reductio* on the plausibility of allowing for the intersection of possibility and impossibility, at least when one's logic allows for such intersection to occur at the actual world.

To avoid the consequence of allowing the actual world to be an impossible world the absolutist must either,

a) Refuse to refer to propositions of the form $\sim\Diamond A$ as impossibility claims,

or

b) Change the semantics of the modal extension of **LP** to ensure that propositions of the form $\sim\Diamond(p \wedge \sim p)$ aren't logical truths in the logic. Thus, even if propositions of the form $\sim\Diamond A$ are read as impossibility claims, this doesn't entail that all worlds at which contradictions are true are impossible worlds.

Option a) seems unviable. By insisting that propositions of the form $\sim\Diamond A$ are not impossibility claims, the absolutist is left requiring a new operator to express impossibility claims. After all, a modal semantics is somewhat incomplete if it cannot express impossibility claims. Yet, if a new modal operator M were introduced, acting as an impossibility operator, the absolutist would require a semantics in which M had a logical relation to the possibility operator but possibility and impossibility couldn't intersect. If she produces a semantics in which M is an impossibility operator, but both $\Diamond A_w$ and MA_w can be true in an interpretation, then the absolutist will find herself in the same situation as before. Given our strong intuition, however, that possibility and impossibility are the negation of one another, and negation is a truth-reserving truth-functor, ensuring that $\Diamond A_w$ and MA_w cannot be true together in an interpretation would be the same as demonstrating that $\Diamond A_w$ cannot be both true and false.

One would simply be replacing the formula $\sim\Diamond A$ with MA. Given that it's theoretically extravagant to postulate a new modal operator when the same result could be produced by changing the semantics of the modal operators we already have, option b) seems more viable.

In favour of option a), one could reject the intuition that possibility and impossibility are the negation of one another, thus resisting the equivalence of establishing that $\Diamond A_w$ and MA_w cannot both be true in an interpretation and establishing that $\Diamond A_w$ cannot receive both truth-values. This is an interesting proposal that requires more consideration than we can give it here. However, there are still two reasons to favour option b) over option a) for our purposes. One is a general concern with option a), and the other is a reason particular to the absolutist.

Firstly, if one rejects the intuition that possibility and impossibility are the negations of one another, then one must give a reasonable account of the natural language meanings of $\sim\Diamond A$ and $\sim MA$, given that $\sim\Diamond A \neq MA$ and $\sim MA \neq \Diamond A$ under these new semantics. Yet, as possibility and impossibility exhaustively sort worlds, it's unclear what extra content is produced by prefixing the modal operators with negation. Thus, using two separate modal operators for possibility and impossibility generates the problem of giving reasonable interpretations of formulae containing negated modal operators. To produce a semantics that doesn't equate MA and $\sim\Diamond A$ or $\Diamond A$ and $\sim MA$, while both respecting the normal semantics for negation and giving a reasonable natural language reading of the formulae, seems a monumental task. Until someone succeeds, we should consider option a) unviable.

Secondly, introducing the new modal operator wouldn't remove the unwanted modal contradictions entailed by the semantics. Without changing the semantics for the possibility operator, the absolutist would still have true contradictions at the actual world. Option b) is the only candidate that blocks both of the unsavoury consequences for the absolutist. Here then is a second motivation for the absolutist to alter the semantics of the modal extension of **LP**.

This second unsavoury consequence of the modal extension of **LP** is equally, if not more, damaging for the dialetheist. The dialetheist, like all of us, needs to be able to model modal claims, and given that she allows for true contradictions at the actual world, she needs to allow for true contradictions at possible worlds. The most obvious way of her achieving this, however, is by giving the modal extension of a dialetheic logic as presented above. A consequence of this modal extension is that all worlds at which contradictions are true are impossible worlds. Given that the dialetheist is committed to there being true contradictions at the actual world, the logic commits her to the actual world being an impossible world. This is patently absurd.[131] The modal dialetheic logic given above, therefore, is not only inadequate for

[131] Priest makes it clear that he doesn't want to say that true contradictions at the actual world ensure that the actual world is an impossible world. See his added footnote in Lewis (2004).

the absolutist's purposes, but for the dialetheist also. The dialetheist requires an alternative dialetheic modal logic as much as the absolutist.[132]

We need then a different modal extension of **LP**, for both the absolutist's and dialetheist's sake. If no alternative modal extension can be found then, 1) Absolutism will be condemned to the status of being a sub-species of dialetheism, contrary to the position's hypothesis; 2) The dialetheist will be committed to the actual world being an impossible world. Thus, if no alternative logic that solves these problems is available, both positions face substantial philosophical challenges.

5.4 A Possible Solution to the Problem

If the absolutist is going to block both the occurrence of contradictions at the modal level and the possibility of impossible worlds, then she will need to invalidate the interdefinability rules for the modal operators. While, through necessitation, $\Box{\sim}(p \wedge {\sim}p)$ must be a theorem of the absolutist's logic, and she must be able to derive $\Diamond(p \wedge {\sim}p)$ to allow for true contradictions at possible worlds, these commitments entail devastating consequences when conjoined with the interdefinability of the modal operators. As the absolutist cannot sanction the rejection of either commitment, the interdefinability of the modal operators must be invalidated somehow.

The occurrence of true contradictions at the modal level is caused by both interdefinability rules,

$$\Diamond A = {\sim}\Box{\sim}A$$

$$\Box{\sim}A = {\sim}\Diamond A.$$

If either of these interdefinability rules is valid, a contradiction occurs at the modal level in the absolutist's logic. Thus, any modal extension of a dialetheic logic that respects both the normal semantics for conjunction and negation, and validates either of the interdefinability rules for the modal operators, isn't an absolute logic. Both of the interdefinability rules need to go. Additionally, the second consequence of the modal semantics given above, that some impossible worlds are possible worlds, is a result of the second interdefinability rule. Both of the unsavoury consequences can be avoided, therefore, by invalidating the interdefinability rules for the modal operators.

[132] Post-Viva Note: I have now come to recognise that a logic's entailing, in conjunction with dialetheism, the impossibility of the actual world is not conclusive evidence that the logic is unsuitable for the dialetheist's purposes. However, rather than substantially changing this chapter I direct readers to Martin (forthcoming), where a discussion of this point can be found with more moderate conclusions. The general conclusions of this chapter, however, I believe still hold, as is made clear in Martin (forthcoming).

The absolutist requires a semantics that both invalidates the interdefinability rules for the modal operators and for which a decent philosophical motivation can be given. If negation is to keep its usual truth-reversing properties, then the only way to block the occurrence of contradictions involving the modal operators is to ensure that modal formulae are never assigned both truth-values. In other words, the absolutist needs to enforce a classical-like semantics onto the modal operators, while keeping dialetheic semantics for the propositional parameters and truth-functional connectives. Making this move would have some plausibility for the absolutist, for not only would it allow her to avoid modal contradictions, but it would also block the consequence that some impossible worlds are possible. If truth and falsity are mutually exclusive for modal formulae, then there will be no fixed-point for negation when it prefixes modal formulae. Possibility and impossibility would be mutually exclusive, which is also good news for the dialetheist.

How we come to ensure this exclusivity can be guided by an intuitive reading of possibility and impossibility under the assumption of a dialetheic propositional logic. If $\sim\Diamond(p \wedge \sim p)$ is read naturally as 'It's impossible for a contradiction to be true', then the semantics given above for the possibility operator are wrong, given that formulae of the form $\sim\Diamond A$ can be assigned the truth-value true even when A is true at some possible worlds. In other words, $\sim\Diamond A$ can be assigned the truth-value true simply because $\Diamond A$ is assigned the truth-value false, and not because $\Diamond A$ *fails to be true*. The concept of impossibility implies exclusion where the logic's semantics fail to deliver exclusion. While it's both totally acceptable and understandable for the classical logician to read the formula $\sim\Diamond(p \wedge \sim p)$ as 'It's impossible for a contradiction to be true', given that in classical semantics p falsity at every possible world entails immediately that p isn't true at any possible world. For the absolutist and dialetheist, it's inappropriate to translate formulae of the form $\sim\Diamond A$ as 'It's impossible that…' when the possibility operator is assigned the semantics above, given that assigning formulae of the form $\sim\Diamond A$ the truth-value true only entails that A takes the truth-value false at every world, and not that A fails to be true at every world. This shouldn't mean that the absolutist gets rid of the concept of impossibility, and even if she did it wouldn't stop absolutism from sliding into dialetheism. Rather, it shows that she should change the semantics of $\Diamond A$ so that if $\sim\Diamond A$ takes the truth-value true, it's because for every world w, A fails to be true at w. This requires, given that propositional parameters in **LP** can be assigned both truth-values, for a formula prefixed by the possibility operator not only to take the truth-value false, but to *not* take the truth-value true. Therefore, the absolutist requires the logical apparatus to ensure that the truth-value true is *not* a member of the truth-values a propositional parameter takes.

We will achieve this by positing two primitive valuation relations in our modal logic rather than one. Our new logic **AV** is a quintuple, $<W, w_a, R, \varepsilon^+, \varepsilon^->$. Only the final element differs from our previous logic, with 'ε^{+}' symbolising the valuation relation for the logic as 'ε' did before. This new element, ε^{-}, rather than symbolising the *valuation* relation for the logic, symbolises the logic's *anti-valuation* relation. Although the idea of having two valuation relations in a logic may seem bizarre, it's worth persevering with as its results are fruitful for both the absolutist and dialetheist. There is an obvious analogy at work here between the valuation and anti-valuation relations and the extension and anti-extension of a predicate. Thus, a propositional parameter p will have both a *valuation* set and an *anti-valuation* set. The *valuation* set of p is dictated by the truth-values that p has the relation ε^{+} to, and the *anti-valuation* set of p is dictated by the truth-values that p has the relation ε^{-} to. We will come on to discuss some of the properties that the two valuation relations possess presently. On the subject of our accessibility relation R, again we don't want to make too many assumptions about which accessibility relation would best suit the absolutist and dialetheist. All we require for our purposes here are the weak requirements that the relation is reflexive, and that the distinguished world w_a has some non-distinguished possible worlds accessible from it.

As before, our valuations are relations from proposition-world pairs to the set of truth-values {1, 0}, and our anti-valuations are similarly relations from proposition-world pairs to the set of truth-values {1, 0}. Now, for the semantics to deliver the results that both the absolutist and dialetheist need, we must make two assumptions about the valuation and anti-valuation sets for each proposition-world pair. Firstly, we need to assume that the valuation and anti-valuations sets partition the set of truth-values {*true*, *false*} for each proposition-world pair. Thus, for any proposition-world pair p_w and truth-value t:

Either $p_w \varepsilon^{+} t$ or $p_w \varepsilon^{-} t$,

and

It's not the case that both $p_w \varepsilon^{+} t$ and $p_w \varepsilon^{-} t$.

These conditions ensure that for every proposition-world pair p_w, each truth-value t is a member of p_w's *valuation* or *anti-valuation* set, but not both.[133] The second assumption, to ensure that **AV**'s semantics aren't gappy, is that the *valuation* set for every proposition-world pair p_w must be non-empty. If the absolutist wants to allow for truth-value gaps, for whatever reason, this condition can be dropped.

[133] One might rightly wonder whether we can simply stipulate that a truth-value t cannot be a member of both the valuation and anti-valuation set of a proposition-world pair p_W. After all, the possibility of such stipulated mutual exclusivity failing is one of the lessons learnt from dialetheism. We will move on to consider this possibility later.

These assumptions ensure that although a proposition-world pair p_w can have the valuation relation to both true and false, p_w can only have the *anti*-valuation relation to either true or false. If we dropped the second assumption, and allowed **AV** to be gappy, then p_w could have the *anti*-valuation relation to both true and false, as it can with the valuation relation.

Given these restrictions on the anti-valuation relation, it seems reasonable to consider the relation to be communicating which truth-values are *not* members of the valuation set of a proposition-world pair p_w. Thus, the anti-valuation set of a proposition-world pair p_w is hypothesised, at least, to be able to communicate when p_w is *un*true ($p_w\varepsilon^+0$ and $p_w\varepsilon^-1$) and *un*false ($p_w\varepsilon^+1$ and $p_w\varepsilon^-0$), as well as both true and false ($p_w\varepsilon^+\{1,0\}$ and $p_w\varepsilon^-\varnothing$). This makes the anti-valuation relation similar to the classical logic negation used when claiming that a proposition p is '**not** true' or '**not** false', attempting to preclude p's falsity and truth, respectively. Priest (1990 & 2007, pp. 469-471) has argued that we cannot interpret these **not**s classically without begging the question against the dialetheist. **AV** makes this hypothesised mutual exclusivity of the relations explicit by stipulating that for no proposition-world pair p_w and truth-value t is it the case that both $p_w\varepsilon^+t$ and $p_w\varepsilon^-t$. Again, whether we can ensure such mutual exclusivity through stipulation is something we will discuss later. The hope for **AV** is that it can avoid accusations of begging the question against the dialetheist because the dialetheist's endorsement of the logic is ultimately in her best interests.

Critically for the absolutist and dialetheist, **AV**'s semantics allow for contradictions to be true at a world. We can see this by giving the dual truth-conditions of the truth-functional connectives using both valuation relations:

$(A \wedge B)_w\varepsilon^+1$ iff $A_w\varepsilon^+1$ and $B_w\varepsilon^+1$
$(A \wedge B)_w\varepsilon^+0$ iff $A_w\varepsilon^+0$ or $B_w\varepsilon^+0$
$(A \wedge B)_w\varepsilon^-1$ iff $A_w\varepsilon^-1$ or $B_w\varepsilon^-1$
$(A \wedge B)_w\varepsilon^-0$ iff $A_w\varepsilon^-0$ and $B_w\varepsilon^-0$

$(A \vee B)_w\varepsilon^+1$ iff $A_w\varepsilon^+1$ or $B_w\varepsilon^+1$
$(A \vee B)_w\varepsilon^+0$ iff $A_w\varepsilon^+0$ and $B_w\varepsilon^+0$
$(A \vee B)_w\varepsilon^-1$ iff $A_w\varepsilon^-1$ and $B_w\varepsilon^-1$
$(A \vee B)_w\varepsilon^-0$ iff $A_w\varepsilon^-0$ or $B_w\varepsilon^-0$

$(\sim A)_w\varepsilon^+1$ iff $A_w\varepsilon^+0$
$(\sim A)_w\varepsilon^+0$ iff $A_w\varepsilon^+1$
$(\sim A)_w\varepsilon^-1$ iff $A_w\varepsilon^-0$
$(\sim A)_w\varepsilon^-0$ iff $A_w\varepsilon^-1$

If a proposition-world pair p_w has the valuation relation to both truth and falsity, $p_w\varepsilon^+\{1,0\}$, then both $p_w\varepsilon^+1$ and $\sim p_w\varepsilon^+1$. Consequently, we have $(p \wedge \sim p)_w\varepsilon^+1$, which ensures that we can have a true contradiction at a world. Additionally, we retain the intuitive consequence that all contradictions are false, $(p \wedge \sim p)_w\varepsilon^+0$. Treating the extensional fragment of **AV** as a conditional-free propositional logic, **AVp**, with the consequence relation,

$$\Sigma \vDash_{\mathrm{AVp}} B \text{ iff, for every } \varepsilon^+ \text{ and } \varepsilon^-, \text{ if } \forall A \in \Sigma,\ A\varepsilon^+1, \text{ then } B\varepsilon^+1,$$

we can see that propositions of the form $\sim(A \wedge \sim A)$ maintain their status as logical truths. Indeed, all the conditional-free logical truths of **LP**, and thus of classical logic, are maintained in **AVp**. I leave the proof of this as an exercise for the reader. Whether all the logical truths of **LP** and classical propositional logic are preserved in **AVp** will be dependent on the choice of a suitable conditional, something which we won't discuss here. Therefore, given that **AVp** is the extensional fragment of **AV**, all the conditional-free logical truths of classical propositional logic are preserved in **AV**. This gives **AV** the conservatism and plausibility that **LP** possesses. Indeed, **AVp** is just a conditional-free **LP** with an additional anti-valuation relation that plays no role in the consequence relation.

What we now need are semantics for the modal operators that invalidate the interdefinability rules. We can achieve this by giving the modal operators' semantics exclusively in terms of truth:

$$(\Box A)_w\varepsilon^+1 \text{ iff, for all } w' \in W \text{ such that } wRw',\ A_{w'}\varepsilon^+1$$
$$(\Box A)_w\varepsilon^+0 \text{ iff, for some } w' \in W \text{ such that } wRw',\ A_{w'}\varepsilon^-1$$
$$(\Diamond A)_w\varepsilon^+1 \text{ iff, for some } w' \in W \text{ such that } wRw',\ A_{w'}\varepsilon^+1$$
$$(\Diamond A)_w\varepsilon^+0 \text{ iff, for all } w' \in W \text{ such that } wRw',\ A_{w'}\varepsilon^-1$$

We don't require the dual anti-valuation conditions for the modal operators, as **AV** makes them redundant by not permitting a truth-value t to be a member of both the valuation and anti-valuation sets of a proposition-world pair p_w.

Whether modal formulae have the valuation true or false is dependent only on whether the formulae in question have the valuation or anti-valuation relation to true at the relevant possible worlds. Under the assumption that for no world-proposition pair p_w and truth-value t is it the case that both $p_w\varepsilon^+t$ and $p_w\varepsilon^-t$, the truth and falsity of modal propositions become mutually exclusive, which invalidates the interdefinability rules for the modal operators and, consequently, blocks the occurrence of contradictions at the modal level.

While the formula $\Diamond(p \wedge {\sim}p)$ takes the valuation *true* at the actual world w_a in some interpretations, as the absolutist allows for a possible world *w'* accessible from w_a such that $(p \wedge {\sim}p)_{w'}\varepsilon^{+}1$, the formula ${\sim}\Box{\sim}(p \wedge {\sim}p)$ only takes the valuation *false* at w_a in every interpretation, as $\Box{\sim}(p \wedge {\sim}p)_{wa}\varepsilon^{+}1$ but *not* $\Box{\sim}(p \wedge {\sim}p)_{wa}\varepsilon^{+}0$. Given that for every possible world w accessible from w_a ${\sim}(p \wedge {\sim}p)_{w}\varepsilon^{+}1$, the occurrence of a possible world *w'* accessible from w_a such that ${\sim}(p \wedge {\sim}p)_{w'}\varepsilon^{-}1$ is precluded. Therefore, given **AV**'s semantics for the necessity operator and negation, we don't have $\Box{\sim}(p \wedge {\sim}p)_{wa}\varepsilon^{+}0$, or consequently ${\sim}\Box{\sim}(p \wedge {\sim}p)_{wa}\varepsilon^{+}1$, in any interpretation that $\Diamond(p \wedge {\sim}p)_{wa}\varepsilon^{+}1$. The interdefinability of $\Diamond A$ and ${\sim}\Box{\sim}A$ fails.

Similarly, while $\Box{\sim}(p \wedge {\sim}p)$ has the valuation *true* at w_a in every interpretation, for at every world w accessible from w_a ${\sim}(p \wedge {\sim}p)_{w}\varepsilon^{+}1$, ${\sim}\Diamond(p \wedge {\sim}p)$ only has the valuation *false* at the actual world w_a in some interpretations. Given that $\Diamond(p \wedge {\sim}p)_{wa}\varepsilon^{+}1$ in some interpretations, for the reasons already given, the occurrence of ${\sim}\Diamond(p \wedge {\sim}p)_{wa}\varepsilon^{+}1$ is precluded in those interpretations, as this would require both $\Diamond(p \wedge {\sim}p)_{wa}\varepsilon^{+}1$ and $\Diamond(p \wedge {\sim}p)_{wa}\varepsilon^{+}0$, which subsequently requires there to be a world *w'* accessible from w_a at which both $(p \wedge {\sim}p)_{w'}\varepsilon^{+}1$ and $(p \wedge {\sim}p)_{w'}\varepsilon^{-}1$, which is precluded by **AV**'s semantics. Therefore, there are interpretations of **AV** in which we don't have both $\Box{\sim}(p \wedge {\sim}p)_{wa}\varepsilon^{+}1$ and ${\sim}\Diamond(p \wedge {\sim}p)_{wa}\varepsilon^{+}1$. The interdefinability of $\Box{\sim}A$ and ${\sim}\Diamond A$ fails.

As we have stipulated that a truth-value t cannot be a member of both a proposition-world pair p_w's valuation and anti-valuation sets, we can easily show that there's no interpretation in which either $(\Box p \wedge {\sim}\Box p)_{w}\varepsilon^{+}1$ or $(\Diamond p \wedge {\sim}\Diamond p)_{w}\varepsilon^{+}1$, for any world w. Given the truth-conditions of conjunction and negation above, for the formulae to have the valuation true would require that $(\Box p)_{w}\varepsilon^{+}\{1,0\}$ and $(\Diamond p)_{w}\varepsilon^{+}\{1,0\}$, respectively. However, for either to occur in an interpretation would require that p contained the truth-value true in both its valuation and anti-valuation sets at some world *w'* accessible from w. Given that both $p_{w'}\varepsilon^{+}1$ and $p_{w'}\varepsilon^{-}1$ can't occur together in an interpretation in **AV**, no contradictions constituted by modal formulae have the *valuation* true in **AV**. That there is no interpretation in which $(\Diamond p \wedge {\sim}\Diamond p)_{w}\varepsilon^{+}1$ also demonstrates that there is no world w that is both a possible and impossible world. Possibility and impossibility cannot intersect in **AV**.

Given **AV**'s consequence relation,

> $\Sigma \vDash_{\mathrm{AV}} B$ iff for all interpretations $<W, w_a, R, \varepsilon^+, \varepsilon^->$ and $\forall w \in W$, if $\forall A \in \Sigma$, $A\varepsilon^+1$, then $B\varepsilon^+1$,

propositions of the form $\sim\Diamond(A \wedge \sim A)$ aren't logical truths in **AV**, as in some interpretations there's a possible world w' such that $\Diamond(A \wedge \sim A)_{w'}\varepsilon^+1$. Therefore, if for some contradiction C we have $C_{wa}\varepsilon^+1$, as the dialetheist theorises, we are *not* then committed to the actual world being an impossible world, as **AV**'s semantics don't permit $\sim\Diamond C_{wa}\varepsilon^+1$, given that $\Diamond C_{wa}\varepsilon^+1$ is ensured by R's reflexivity.

While invalidating the interdefinability rules for the modal operators, **AV**'s consequence relation does produce one result we wouldn't expect of a dialetheic logic. This is the validation of two forms of modal explosion:

> $\mathrm{ME_N}$) $\{\Box A \wedge \sim\Box A\} \vDash_{\mathrm{AV}} B$
>
> $\mathrm{ME_P}$) $\{\Diamond A \wedge \sim\Diamond A\} \vDash_{\mathrm{AV}} B$.

Given that neither $(\Box A \wedge \sim\Box A)_w\varepsilon^+1$ nor $(\Diamond A \wedge \sim\Diamond A)_w\varepsilon^+1$ are permitted by interpretations in **AV**, $\mathrm{ME_N}$) and $\mathrm{ME_P}$) follow from its consequence relation. This result may irk the paraconsistent logician, who believes that an inconsistent theory needn't be trivial regardless of whether that inconsistency is constituted of modal or non-modal propositions. However, for the absolutist and dialetheist, who need to deny the joint possibility of $p_w\varepsilon^+1$ and $p_w\varepsilon^-1$ to avoid the previous logic's unsavoury consequences, allowing for these particular forms of explosion seems a rather necessary and unimportant evil, especially given the potential theoretical cost of blocking $\mathrm{ME_N}$) and $\mathrm{ME_P}$).

AV delivers an intuitive semantics for the modal operators while ensuring that both of the interdefinability rules are invalidated. Consequently, both of the unsavoury consequences of the intuitive modal extension of **LP** we considered can be avoided. **AV** delivers everything we need it to, on the assumption, at least, that for no truth-value t and proposition-world pair p_w is there a permitted interpretation in which both $p_w\varepsilon^+t$ and $p_w\varepsilon^-t$.[134] It is this stipulated

[134] Additionally, if successful, **AV** could potentially provide the absolutist and dialetheist with another means with which to express *disagreement*. Given that the absolutist and dialetheist allow for both propositions to be true *and* false, and for contradictions to be true, neither can disagree with someone who states the truth of a proposition p by asserting either p's falsity or the truth of p's negation. After all, for the absolutist and dialetheist, neither the falsity of p, nor the truth of p's negation, preclude the truth of p. Until now, both parties have had to rely upon the communicative act of *denying p*, hypothesised as an act which precludes the *assertion* of p, to express disagreement; see Priest (2006a), pp. 103-111. However, with **AV**'s semantics, we can express disagreement without relying upon the communicative act of *denial*. If one party states that a proposition p is true, we can disagree with this statement by asserting that p has the *anti*-valuation true which, given the mutual exclusivity of the valuation and anti-valuation relations, *precludes p* having the valuation

partitioning of the set of truth-values that we now move on to, and the dilemma it poses for the dialetheist.

5.5 Stipulating Exclusivity and a Dilemma for the Dialetheist

The exclusivity of truth and falsity for modal formulae, and thus modal propositions, is ensured only by the hypothesised mutual exclusivity of the valuation and anti-valuation relations ε^{+} and ε^{-} for every proposition-world pair p_{w} and truth-value t. If this mutual exclusivity breaks down then so does the mutual exclusivity of truth and falsity for modal formulae, and the same problems would reappear for the absolutist. Thus, we need assurances that the absolutist can guarantee that $p_{w}\varepsilon^{+}t$ and $p_{w}\varepsilon^{-}t$ cannot occur together in an interpretation for any proposition-world pair p_{w} and true-value t. Yet, we know that simply stipulating mutual exclusivity isn't enough. Just as self-referential sentences threaten the mutual exclusivity of truth and falsity, although the classical logician stipulates their mutual exclusivity, so self-referential sentences can threaten the mutual exclusivity of $p_{w}\varepsilon^{+}t$ and $p_{w}\varepsilon^{-}t$. Consequently, if the absolutist is going to make **AV**'s semantics viable, ensuring that there's an available absolute logic to model her theory, she faces the same challenge that the classical logician does – to give a non-dialetheic response to certain self-referential sentences. Given that the absolutist wishes to stop her position from dissolving into dialetheism, however, this is hardly a new obligation. By allowing for true contradictions at non-actual possible worlds, while demurring on the question of true contradictions at the actual world, the absolutist was already required to avoid a dialetheic solution to the self-referential paradoxes. If the mutual exclusivity stipulated by **AV** requires something more of the absolutist, it's simply that she must double her efforts to give non-dialetheic solutions to the self-referential paradoxes. Otherwise, absolutism is destined to be a sub-species of dialetheism.

The situation for the dialetheist with regards to **AV** is far more interesting. The dialetheist herself requires **AV**'s semantics to block the consequence that the actual world is an impossible world, which subsequently requires her to maintain the mutual exclusivity of $p_{w}\varepsilon^{+}t$ and $p_{w}\varepsilon^{-}t$ for every proposition-world pair p_{w} and true-value t in **AV**. Yet, the dialetheist is famously uneasy with such stipulated mutual exclusivity, due to our ability in natural languages to form troublesome self-referential sentences such as (λ):

(λ) λ has the anti-valuation true [$\lambda\varepsilon^{-}1$].

---

true. This new expressive resource for the absolutist and dialetheist may help to answer some of the concerns that Shapiro (2004) has raised over the adequacy of *denial* as a *sole* means of expressing disagreement; a question that is beyond the scope of this chapter.

As with other self-referential sentences, the dialetheist would like to say that (λ) both has and doesn't have the anti-valuation true, which would entail that both $\lambda_w \varepsilon^+ 1$and $\lambda_w \varepsilon^- 1$, contrary to the stipulation of **AV**. This desire to give a dialetheic solution to (λ) poses a dilemma for the dialetheist.

The dialetheist needs to guarantee that her theory doesn't entail that the actual world is an impossible world. This requires, under the present proposal of **AV**, ensuring the mutual exclusivity of $p_w \varepsilon^+ t$ and $p_w \varepsilon^- t$ for every proposition-world pair p_w and truth-value t. Yet, the only way that the dialetheist can maintain this mutual exclusivity is by giving a non-dialetheic solution to (λ) above. Therefore, unless a new modal semantics can be introduced that removes this dilemma, the dialetheist must choose between endorsing the claim that the actual world is an impossible world and admitting that there is at least one self-referential sentence that cannot be given a dialetheic response.

5.6 One Man's Problem is Another's Opportunity

We have delivered a logic **AV** that has all the properties required of an absolute logic. However, the absolutist must still meet the challenge of self-referential sentences such as (λ) that put the mutual exclusivity of the valuation relations under threat. There is nothing we can do within our logic to preclude the relations' non-exclusivity; this is something dialetheism has taught us. If the absolutist is to prevent absolutism from dissolving into dialetheism, she must argue for the valuation relations' mutual exclusivity by dealing with self-referential sentences such as (λ) head on. Fortunately for the absolutist, unless the dialetheist has any other modal dialetheic logics available to her that don't entail the impossibility of the actual world, it's in the dialetheist's best interests to not dispute the stipulated mutual exclusivity of $p_w \varepsilon^+ t$ and $p_w \varepsilon^- t$. Therefore, in **AV** facilitating the dialetheist's avoidance of this unsavoury consequence, the absolutist has assurances that the dialetheist will be as determined as she is to maintain the mutual exclusivity of $p_w \varepsilon^+ t$ and $p_w \varepsilon^- t$. Those who would normally bring into question the mutual exclusivity of semantic categories have, on this occasion, reason to argue for their exclusivity. The dilemma that **AV** presents to the dialetheist, therefore, gives us reason to believe that the mutual exclusivity of $p_w \varepsilon^+ t$ and $p_w \varepsilon^- t$ can be maintained by the absolutist without much challenge. This would ensure that there is at least one absolute logic, **AV**, available to model the absolutist's theory. The sole possibility that could block the absolutist's construction of an absolute logic in the form of **AV** is the potential for the dialetheist to bite the bullet and admit the impossibility of the actual world.

In the previous and present chapters we have demonstrated that there are non-dialetheic, dialetheic, and absolute paraconsistent logics. In the next, we meet the challenge that, contrary to appearances, there are in fact no paraconsistent logics.

6. Slater on Contradictions and the Impossibility of 'Paraconsistent' Logics

> There may perhaps be principles so central to a concept that anyone who rejects them and nonetheless uses the usual name for the concept may be said to be using not the usual concept but another one of the same name.
>
> (Burgess in Boolos (1998) p. 142)

In the previous two chapters we putatively demonstrated that there are non-dialetheic, dialetheic, and absolute paraconsistent logics. Yet, in Slater's famous paper 'Paraconsistent Logics?' we find these results contradicted. Slater argues that, contrary to the prevailing view in the logical community, it's impossible for there to be paraconsistent logics. Given the theoretical advances that have been made in the field of paraconsistent logic, including inconsistency-adaptive logics that model scientific theory changes and paraconsistent theories of belief revision,[135] this argument constitutes quite a challenge to the status quo. It also constitutes an argument against the plausibility of the absolutist's position. If it's impossible for there to be any paraconsistent logics, then the absolutist's position dissolves into modal trivialism, *contra* the absolutist's hypothesis. Consequently, this is a challenge the absolutist must take seriously. In this chapter we consider the validity of both Slater's original argument against the possibility of paraconsistent logics, and a modified version of the argument against the possibility of *dialetheic* logics.

6.1 Slater's Argument

Slater (1995, p. 451) begins his ambitious article with the passage:

> If we called what is now 'red', 'blue' and vice versa, would that show that pillar boxes are blue, and the sea is red? Surely the facts wouldn't change, only the mode of expression of them. Likewise, if we called 'subcontraries', 'contradictories', would that show that 'it's not red' and 'it's not blue' were contradictories? Surely the same point holds. And that point shows that there is no 'paraconsistent' logic.

The message is clear; Slater thinks that putatively paraconsistent logics mistake subcontraries for contradictories. That is, they call subcontraries 'contradictories'. What is not quite so clear yet, however, is how this putative confusion demonstrates that there are no paraconsistent logics. Slater's argument proper begins with a description of **LP**'s semantics, concentrating on

[135] Good places to start for inconsistency-adaptive logics are Batens (1989, 1994 & 2000) and Batens & Meheus (2007). For an example of an adaptive logic being used to model scientific theories see Meheus's (1993 & 2002) use of an inconsistency-adaptive logic to model Clausius' derivation of Carnot's theorem. See Tanaka (2005) for a paraconsistent theory of belief revision. The research field of paraconsistent logics now even has its own mathematics subject classification: 03B53.

the fact that in **LP** propositional parameters can be assigned both truth-values and consequently that formulae of the form $A \wedge \sim A$ can be assigned both truth-values.[136] Given that $A \wedge \sim A$ is proposed as a formal representation of a contradiction in **LP**, and a contradiction is a conjunction of contradictories, Slater (1995, p. 451) rightly asserts that "it *looks like* the logic allows contradictories to be both true," (emphasis mine).

Slater (1995, p. 451) then proceeds to argue for the thesis that, given these semantics, "it is not the case that '$\sim A$' is the contradictory of 'A' in [**LP**]." Thus, contrary to first appearances, **LP** does not allow for contradictories to be true, for the $\sim A$ of $A \wedge \sim A$ is not the contradictory of A (and, given the symmetry of contradictories, A isn't the contradictory of $\sim A$). That is, **LP** may allow for formulae of the form $A \wedge \sim A$ to be assigned the truth-value true, which ensures that A and $\sim A$ can both be assigned the truth-value true, but given that A and $\sim A$ are *not* contradictories, **LP** hasn't shown that both contradictories can be true. Nor, parasitically, that contradictions can be true. Slater's justification for his main claim, that **LP**'s unary operator isn't a contradiction-forming operator, comes in two separate passages in the paper.

In the first, Slater (1995, p. 451) explicates the semantics of **LP** with the numerals 1, 0, and -1, representing the truth-values *true, both*, and *false*, respectively: for A to be true, the valuation of A need only be $v(A) \geq 0$, and for $\sim A$ to be true, the valuation of A need only be $v(A) \leq 0$. This is a rather roundabout way of showing that for the A of $A \wedge \sim A$ to be true only requires, using *just* the numerals 1 and 0 here for true and false respectively, $A\varepsilon 1$, which doesn't preclude $A\varepsilon 0$ *also*.[137] Whereas, for the $\sim A$ of $A \wedge \sim A$ to be true only requires that $A\varepsilon 0$, which doesn't preclude $A\varepsilon 1$ *also*.[138] Yet, according to Slater, what contradicts ($v(A) \geq 0$) is not ($v(A) \leq 0$), but rather ($v(A) < 0$). That is, two propositions are contradictories if and only if one of the propositions is either *true* or *both*, and the other is *false* (and cannot be *both*). Given that in **LP** both *true* and *both* are designated values (as for both, $A\varepsilon 1$), the point can be explained more clearly: two propositions are contradictories if and only if one of the propositions takes a designated value and the other doesn't. Given that the A and $\sim A$ of $A \wedge \sim A$ can both be assigned a designated value in a valuation of **LP**, according to Slater's criterion of contradictories, A and $\sim A$ do *not* symbolise contradictories in **LP**. Thus, "making '$\sim$' not a contradiction forming function, in [**LP**]," (Slater (1995), p. 451). Instead, A and $\sim A$ symbolise sub-contraries, as both can simultaneously be assigned the truth-value *true*, but cannot both be simultaneously assigned the truth-value *false* (although both can be false in taking the truth-value *both*). This condemns **LP**'s unary operator to the status of a sub-contrary operator, as

[136] See section 4.4.1 above.

[137] This is just Slater's '$v(A) \geq 0$'. The formula A is *at least* true, but it might be false also (and thus *both*).

[138] This is just Slater's '$v(A) \leq 0$'. The formula A is *at least* false, but it might be true also (and thus *both*).

Priest and Routley (1989, pp. 165-166) condemned da Costa's (1974) **C-systems**' unary operator.

The second passage is far more direct, and wears its source on its sleeve. It comes in the form of a definition, "no change of language can alter the facts, only the mode of expression of them… And one central fact is that *contradictories* cannot be true together – by definition," (Slater (1995), p. 453). Here then is the justification for Slater's claim that in **LP** $\sim A$ doesn't symbolise the contradictory of A. Contradictories just cannot be true together, *by definition.* Or, to talk in terms of designated values, 'By definition, contradictories cannot both have designated values'. Given that A and $\sim A$ *can* both be assigned designated values in a valuation of **LP**, the A and $\sim A$ of **LP** cannot be symbolising contradictories. From this it follows that the '~' of **LP** isn't a contradiction-forming operator. These two separate forms of Slater's justification don't differ in substance, except that Slater's first definition of 'contradictory' also ensures that one of the contradictories must be true, while the latter definition is silent on this issue. Slater's claim that what contradicts ($v(A) \geq 0$) is ($v(A) < 0$), is stated without any further elucidation or justification, which suggests it's a definition incognito. Both occurrences of Slater's justification for his claim, therefore, are definitional.

Slater has so far then explained the semantics of **LP**, argued that contradictories cannot both have designated values, and concluded that A and $\sim A$ don't symbolise contradictories in **LP**. We can systematise these argumentative steps a bit better to aid with analysis:[139]

P1) **LP** allows for formulae of the form $A \wedge \sim A$ to take a designated value. (Premise)
P2) In **LP** a conjunction takes a designated value if and only if both conjuncts take a designated value. (Premise)
P3) In **LP** both A and $\sim A$ can take a designated value. (From P1 & P2)
P4) Contradictories cannot both take a designated value. (Premise)
P5) The A and $\sim A$ of **LP** don't symbolise contradictories. (From P3 & P4)

This is the core of Slater's argument. From here it's easy to demonstrate that **LP** contains *no* formulae symbolising contradictories:

P6) If a logic contains formulae symbolising contradictories, then there must be an operator in the logic that always outputs a non-designated value when a designated value is inputted as an

[139] All premises P1-P5), except the obvious P2), are found in Slater (1995, p. 451).

argument and always outputs a designated value when a non-designated value is inputted as an argument. (Premise)
P7) **LP** contains *no* operator that always outputs a non-designated value when a designated value is inputted as an argument and always outputs a designated value when a non-designated value is inputted as an argument. (Premise)
P8) **LP** contains no formulae that symbolise contradictories. (From P6 & P7)

That Slater (1995, p. 452) believes he has demonstrated P8) is shown by his claim that "Priest might want to go on to argue, as a result of the present point about subcontraries, that there are *only* such subcontraries, since we must now realise there are no contradictories." Slater doesn't give us the necessary premises to reach this conclusion though, and so we are left to add the necessary steps ourselves. Thus, according to Slater, Priest's **LP** contains no contradiction-forming operator. As far as Slater is concerned, the best conclusion Priest can hope for is that there are no contradictories. This, after all, is what **LP**'s semantics imply. Yet, as Slater (1995, p. 452; cf. 2007, p. 464) recognises, Priest "wants there to be contradictions… *in reality*."[140] Slater thinks Priest wants something from his logic that its semantics simply don't deliver.

All the argument gives us so far is the conclusion that **LP** contains no formulae that symbolise contradictories. It neither demonstrates a fact about a *set* of logics, which the argument requires if it's going to show that there are no paraconsistent logics, nor indicates to us what relevance the non-occurrence of formulae symbolising contradictories in **LP** has to the existence of paraconsistent logics. For Slater's argument to be successful, both of these failures must be rectified. We can go some way to solving the first by revising Slater's argument so that it's applicable to all logics:

S1) A contradiction is a conjunction of contradictories. (Premise)
S2) If a conjunction is to be true, then its two conjuncts must be true. (Premise)
S3) If a contradiction is to be true, then its two constituent contradictories must be true. (From S1 & S2)

[140] Slater's talk of contradictions "*in reality*" is perhaps a bit misleading. If he means that the contradictions have to be somehow in the world then the dialetheist certainly isn't committed to this; see Mares (2004) discussion of *semantic* dialetheism. Indeed, it's unclear what sense can be made of contradictions "*in reality*" unless one is willing to admit some polarity into nature, perhaps with truthmakers. See Bobenrieth (2007) for the clearest statement of the incomprehensibility of objects in the world being contradictory (or consistent for that matter), and Beall (2000) for an account of how truthmakers could produce the polarity necessary to model contradictions "*in reality*".

S4) Two contradictories cannot both be true. (Premise)

S5) A contradiction cannot be true. (From S3 & S4)

S6) If a formula symbolises a proposition then it must preserve its truth-conditions. (Premise)

S7) If a formula can be assigned the truth-value true in a logic, then that formula does *not* symbolise a contradiction. (From S5 & S6)

S8) There is no formula such that it symbolises a contradiction and can be assigned the truth-value true. (From S7)

S9) If there's no formula such that it symbolises a contradiction and can be assigned the truth-value true, then there is no logic that contains a formula that symbolises a contradiction and can be assigned the truth-value true. (Premise)

S10) There's no logic that contains a formula that both symbolises a contradiction and can be assigned the truth-value true. (From S8 & S9)

This argument generalises from a conclusion regarding **LP** to all logics. Rather than questioning whether formulae of the form $A \wedge \sim A$ *actually* symbolise contradictions, as is proposed by proponents of **LP**, the argument attempts to establish that formulae with certain properties *cannot* be symbolisations of contradictions. In particular, it proposes that formulae assigned the truth-value true *cannot* be symbolising contradictions. The argument, in effect, takes the form: no contradictions can be true, therefore if a logic assigns the truth-value true to a formula, that formula cannot be symbolising a contradiction, whatever the logic purports. Its persuasive power rests on the proposition that contradictions cannot be true, which we deduced from the definition of contradictories in Slater's paper, plus the unproblematic premises that a contradiction is a conjunction of contradictories *and* that a conjunction is true if and only if both its conjuncts are true. If these premises are true, then we can validly infer there are no logics in which a formula symbolising a contradiction can be assigned the truth-value true.

Now that we have generalised Slater's argument, we can enquire whether it gives us good reason to conclude that "there is no 'paraconsistent' logic," (Slater (1995), pp. 451 & 453). That is, we need to ask whether the additional premise,

S11) If there are paraconsistent logics, then there are logics containing formulae that both symbolise contradictions and can be assigned the truth-value true,

is true. If it is, then paraconsistent logics could be in trouble.

6.2 Paraconsistent Logics

Paraconsistent logics are those logics that invalidate explosion. Paraconsistent logics, therefore, must somehow block the implication from an inconsistent premise set to an arbitrary proposition. Given the identity of paraconsistent logics, and his argument's reliance on showing S11) to be true, Slater must demonstrate that the only available mechanism by which logics can block the implication from an inconsistent set to an arbitrary proposition is by assigning the truth-value true to those formulae which putatively symbolise contradictions. If Slater fails to demonstrate this, then we will have no reason to believe that S11) is true, and consequently that Slater's argument establishes the impossibility of paraconsistent logics.

Unfortunately for Slater, there are paraconsistent logics, as we saw in section 4.3, that invalidate explosion without questioning the impossibility of contradictions being true. For example, while Jaśkowski's $\mathbf{D_2}$ invalidates explosion by giving a discursive interpretation of the elements of a logic, invalidating the rule of adjunction along the way, Jennings and Schotch's preservation logic invalidates explosion by altering the property to be preserved in the consequence relation.[141] Therefore, we can show that S11) is false by giving examples of paraconsistent logics in which the invalidation of explosion doesn't depend on the logics' postulating some formulae *F* that both symbolise contradictions and can be assigned the truth-value true.

Even if Slater places a stronger membership condition on paraconsistency, so that paraconsistent logics must invalidate conjoined explosion $\{A \wedge {\sim}A\} \vDash B$, the argument fails. While both $\mathbf{D_2}$ and Jennings and Schotch's preservationist logic fail to invalidate the conjoined form of explosion, Brown's (1999, 2000 & 2009) preservationist logic does invalidate it, and thus remains a counter-example.[142] Even strengthening the conditions on paraconsistency slightly, therefore, doesn't stop S11) from being false. Slater's argument fails; there are paraconsistent logics after all.

[141] Ironically, both Jaśkowski's $\mathbf{D_2}$ and Jennings and Schotch's preservationist logics contain contradiction-forming operators. In $\mathbf{D_2}$ there is a unary operator ~ such that both *A* and ~*A* cannot both be assigned the truth-value true at a possible world *w*, and a requirement for the underlying logic *L* of Jennings and Schotch's preservationist logic is that *L* contains an explosive operator. See sections 4.3.1 and 4.3.2 above.

[142] Brown (1999) himself has previously suggested that preservationist logics show that Slater's argument against paraconsistent logics is unsuccessful. He fails though to discuss Slater's argument in any detail, with the consequence that the essence of Slater's point is passed over. This, however, is understandable as Brown is far more interested in conveying the theoretical virtues of preservationist logics.

6.3 Dialetheic Logics

Although we have shown Slater's argument to be totally ineffective against paraconsistent logics, there is a category of paraconsistent logics for which Slater's argument could be more of a problem. These are *dialetheic* paraconsistent logics.[143] These logics are obviously very important for both the dialetheist and absolutist, as they allow for formulae symbolising contradictions to be assigned the truth-value true. Thus, if Slater's argument were successful against dialetheic logics, this would have serious philosophical repercussions. It would show that both the absolutist's and dialetheist's theses were logically incoherent.[144]

Let us reconsider Slater's argument, this time replacing 'paraconsistent logics' in S11) with 'dialetheic logics',

S1) A contradiction is a conjunction of contradictories. (Premise)
S2) If a conjunction is to be true, then its two conjuncts must be true. (Premise)
S3) If a contradiction is to be true, then its two constituent contradictories must be true. (From S1 & S2)
S4) Two contradictories cannot both be true. (Premise)
S5) A contradiction cannot be true. (From S3 & S4)
S6) If a formula symbolises a proposition then it must preserve its truth-conditions. (Premise)
S7) If a formula can be assigned the truth-value true in a logic, then that formula does *not* symbolise a contradiction. (From S5 & S6)
S8) There is no formula such that it both symbolises a contradiction and can be assigned the truth-value true. (From S7)
S9) If there's no formula such that it both symbolises a contradiction and can be assigned the truth-value true, then there's no logic that contains a formula that both symbolises a contradiction and can be assigned the truth-value true. (Premise)
S10) There's no logic that contains a formula that both symbolises a contradiction and can be assigned the truth-value true. (From S8 & S9)
S11') If there are dialetheic logics, then there are logics containing formulae that both symbolise contradictions and can be assigned the truth-value true. (Premise)
S12) There are no dialetheic logics. (From S10 & S11')

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[143] See section 4.4 above.
[144] A criticism similar to this interpretation of Slater's argument is made by Jennings and Schotch (2009, p. 31), directly at dialetheism: "[Dialetheism] avoids the classical consequences of contradiction by changing the meaning of 'contradiction'."

Given that none of the inferences in the argument are contentious, the only avenue for the advocate of dialetheic logics to resist this modified version of Slater's argument is to deny one of the premises. This leaves us with S1), S2), S4), S6), S9) and S11'). S11') is part of the definition of a dialetheic logic and therefore is immune to criticism. Similarly, the dialetheic logician has no motivation to reject S1) or S2), for they accept that a contradiction is a conjunction of contradictories and the standard semantics for conjunction. Similarly, there cannot be many doubts regarding S6) and S9). S6) is a fundamental principle of symbolisation, and S9) simply states that a consequence of there being no formulae of type T is that no logic can contain formulae of type T. So far so good for Slater's modified argument against dialetheic logics.

This leaves us with S4), Slater's (1995, p. 453; cf. 2007, p. 460) claim that "*contradictories* cannot be true – by definition." This is obviously a premise that the dialetheic logician will disagree with, for they assert that their logics contain formulae symbolising contradictories that can both be assigned the truth-value true in an interpretation. That S4) precludes their thesis, of course, isn't a good enough reason for rejecting the definition. They need to challenge it head on.

There are two possible ways to challenge S4). The first attempts to show that Slater's definition of contradictories doesn't in fact preclude the truth of contradictions, while the second questions the adequacy of Slater's definition of contradictories. We will be taking the latter option here, but it is worth mentioning the former challenge and why it may not be ultimately fulfilling.

Although Slater introduces a definition of contradictories that states contradictories *cannot* be true together, the dialetheic logician may respond that this definition only precludes the truth of both contradictories if we interpret the negated modal classically. By stating that contradictories *cannot* be true together, Slater was suggesting that two conditions,

A) If A and $\sim A$ are contradictories then, if $1 \in v(A)$ then $1 \notin v(\sim A)$

B) If A and $\sim A$ are contradictories then, if $1 \in v(\sim A)$ then $1 \notin v(A)$,

partly constitute the relation of contradictories. That is, two contradictories are never both assigned the truth-value true. However, this definition of contradictories only precludes the possibility of contradictions being true if it's not possible in one's logic to have both $1 \in v(A)$ and $1 \notin v(A)$. That is, if it's not possible for a proposition to be both true and *un*true.

Willingness to allow for this possibility in one's logic can be a consequence of either wanting to accommodate strengthened Liars such as 'This sentence is not true', or one's truth theory.[145]

Even if this reply to Slater's argument is coherent, and it makes sense to say of a proposition that it's both true and *un*true, it seems to miss the point of Slater's argument. The argument proposes that the definition of contradictories *does preclude* both contradictories being true. If this requires us to interpret 'cannot be true' classically, then so be it. Slater's challenge to dialetheic logics is that part of the concept of contradictories is that it precludes both contradictories from being true. One could only plausibly suggest that Slater's definition of contradictories is unproblematic for dialetheic logic if one refuses to accept that Slater's definition means what he wants it to mean. Yet, this would seem incredibly severe. Even those of us who are sympathetic towards the dialetheist's theories would be hesitant to say that we didn't understand what the classical logician means when she states that the truth of a proposition *p precludes* the truth of *p*'s contradictory. After all, we take the dialetheist to be disagreeing with the classical logician here, and that requires us to understand what she is saying.

One might think that Slater doesn't have the *right* to define contradictories such that the truth of one precludes the truth of the other, but this isn't to deny that we understand what he is stating with this definition. By admitting that the dialetheic logician understands what Slater means with his definition, she admits that she must meet the definition head on. The first type of challenge to Slater's argument seems destined to fail; by just refusing to understand what Slater is stating, we fail to appropriately interact with the argument. We do understand him, and our response to his argument shouldn't be to evade it by suggesting that we don't. If we are successful in showing that Slater's definition of contradictories is deficient, then the former reply to Slater's argument will be fundamentally fruitless. There is little point dialectically in demonstrating what a definition does and doesn't entail if it's a deficient definition.

6.4 The Definition

Slater's argument is tantamount to denying the dialetheist's thesis by replying that, by definition, contradictions cannot be true. This is dialectically acceptable unless the manoeuvre is shown to constitute begging the question against either party in the debate.[146]

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[145] If one were a dialetheist and endorsed a truth theory that included the axiom $Tr(\sim p) \rightarrow \sim Tr(p)$, as Beall (2009) does, then the conclusion that a proposition can be both true and *un*true is entailed by a proposition being both true and false, and the truth-reversing nature of negation. Thus, a true contradiction would ensure that both conjuncts of the contradiction were true and *un*true.

[146] Slater (1995, p. 459) actually thinks that "Priest's argument for the possibility that contradictories might be both true collapses immediately: *he has himself begged the question*, by presuming that

Pointing out that one has contravened the rules established via a definition, however, isn't begging the question. It is simply reminding one of the rules of the game. To show that one isn't obliged to stick to these rules, one must establish that they were not the rules of the game after all. That is, the dialetheic logician needs to demonstrate that Slater's definition of contradictories is *not* the philosophically operative definition of contradictories. She cannot simply complain that a particular definition D of contradictories is deficient *because* it precludes the truth of her thesis, for it's equally possible that Slater's definition of contradictories is indeed constitutive of what it is for two propositions to be contradictories. There is nothing wrong, after all, in showing that a position is incoherent because it's failed to follow the definitions established by the community. For example, given the definition of a circle as the set of all points in a plane that are equidistant from a given point, and the definition of a straight line as the shortest distance between two points, one would be justified in concluding that the statement that a circle is a straight line is incoherent because it contravenes these definitions. To challenge the incoherence of the statement, one must challenge the adequacy of the definition(s). If Slater's argument is successful, it will be because we can demonstrate the impossibility of dialetheic logics, and thus the incoherence of dialetheism and absolutism, by reading off a definition of contradictories or contradiction. If the dialetheic logician is going to resist Slater's argument, she will need to demonstrate that Slater's definition is deficient.

6.5 Contradictions Revisited

Thankfully for the dialetheist and the absolutist, we have already in essence accomplished this. In chapter 3 we considered the available definitions of 'contradiction' and found that one, the truth-value neutral account, was the most plausible. Slater's definition of contradictories is, parasitically, a version of the classically-assumed account of contradiction. While Slater only provides us with a definition of contradictories, any decent definition of contradictories must extend to a definition of contradictions (and vice versa). Yet, the two non-*ad hoc* extensions of Slater's definition of contradictories to a definition of contradictions,

CL_{Log}) Contradictions are a set of propositions Σ such that the conjunction of all $B \in \Sigma$ is logically false, but, necessarily, the occurrence of logical constants in the members of Σ ensure that one and only one member of Σ is true,

CL_{Conj}) Contradictions are a conjunction of two propositions A and B, such that the conjunction of A and B is logically false, but,

contradictories are defined otherwise," (emphasis mine). This quote demonstrates nicely that the kernel of the disagreement between Slater and the dialetheist is the definition of contradictories.

> the occurrence of logical constants in A and/or B ensure
> that one and only one of A and B is true,

are both philosophically implausible accounts of contradiction.[147] Given that an account of contradictories is implausible if the definition of contradiction it extends to is implausible, and vice versa, Slater's definition of contradictories is philosophically implausible.

By accepting that the truth-value neutral account is a more plausible definition of contradiction than the classically-assumed account, we can see that Slater's conclusion no longer follows from the definition of contradiction or contradictories. One cannot demonstrate that there are no dialetheic logics directly from the definition of contradictions as the 'conjunction of a proposition and its negation'. The truth-value neutral account is silent on whether contradictions can be true or not.

Slater's argument fails, as his definition of contradictories (and, parasitically, contradictions) is defective. The more adequate definition of contradictories and contradictions fails to entail that there are no true contradictions, and thus that there are no dialetheic logics. If Slater wants to demonstrate that there are no dialetheic logics, he will need to argue for it and not simply quote a definition that we have good reason to believe is faulty. Given that Slater's definition of contradictories would be an outright demonstration of the incoherence of dialetheism and absolutism, it's interesting that he formulated his argument in terms of logical systems. If Slater's definition of contradictories were adequate, he would need only to read out the definition to settle once and for all the debate over the philosophical plausibility of dialetheism. The definition would be conclusive evidence that the dialetheist's thesis is nonsense. That he doesn't do this is an indictment of his definition, for the definition will only show that there cannot be any dialetheic logics if it conclusively demonstrates that no contradiction can be true. By appreciating that the truth-value neutral account is the most plausible account of contradiction we have available, we can preclude the possibility of deriving the non-existence of dialetheic logics, and thus the absurdity of dialetheism and absolutism, from simply the definition of contradictories or contradictions.

6.6 Negation and All That

We have shown that the possibility of dialetheic logics isn't precluded by the definition of 'contradictories' itself. However, the truth-value neutral account, by defining contradictions as the conjunction of a proposition and its negation, brings negation into focus, and opens up the possibility of arguing from a definition of contradictories *and* negation to the conclusion that there cannot be any dialetheic logics. If we could achieve this then Slater's conclusion

[147] As we saw in section 3.1.1 above.

would be justified, although the justification would be constituted of definitions of both contradictories and negation.

We should be sceptical about the potential success of this move. There are two routes by which one could argue, via negation and the truth-value neutral account, for the conclusion that there are no dialetheic logics. The first is to give an implicit definition of negation, by giving the introduction and elimination rules of the operator. However, there are competing theories of negation available and we have no reason, without some independent justification, to consider a theory of negation T to be correct just because negation according to T ensures that a proposition and its negation cannot both be assigned the truth-value true. This would beg the question in favour of Slater's thesis. We would need then some independent motivation for considering such a theory of negation T to be correct. This may be forthcoming, but the debate over the correct theory of negation still seems very far from settled. No implicit definition of negation *by itself* then will demonstrate that there are no dialetheic logics.

Even if one is convinced of the accuracy of classical logic, and therefore the Boolean account of negation, merely giving the introduction and elimination rules of Boolean negation will not demonstrate that dialetheic logics are impossible. Not only may the dialetheic logician question the correctness of Boolean negation, which cannot be answered sufficiently by merely giving its introduction and elimination rules, but it has been part of the dialetheist's research project to demonstrate that Boolean negation doesn't ensure that a proposition and its negation cannot both be true.[148] Answering these challenges requires more than simply giving the implicit definition of the negation operator. Thus, giving the implicit definition of an operator, even if it is Boolean negation, cannot in conjunction with the truth-value neutral account of contradiction demonstrate that there are no dialetheic logics.

In addition, it seems that this prospective adaptation of Slater's argument misinterprets the original purpose of Slater's argument. Slater isn't attempting to demonstrate that classical negation is the correct theory of negation. After all, Slater is happy to call subcontrary operators 'negations' too. Instead, he is trying to tell us something about the concept of contradictories:

> [Priest] tries to show that Boolean negation likewise involves an operator for which the truth of $\sim p$ does not rule out that of p. But, even if this was true, it would merely show that Boolean negation was not a contradiction-forming operator… [T]he argument is a red herring.
>
> (Slater (2007) p. 458)

[148] Priest (1990 & 2007, pp. 469-471) has argued that one cannot demonstrate that Boolean negation ensures that a proposition and its negation cannot be true without begging the question against the dialetheic logician.

Priest's counter doesn't challenge Slater's argument because Slater isn't arguing for classical negation, or any other negation particle that putatively precludes the truth of a proposition and its negation. Rather, he is arguing for the incoherence of true 'contradictories'. Basing an argument for the non-existence of dialetheic logics on the implicit definition of a negation operator requires much more of Slater's argument than he initially intended, and thereby distorts it. Slater's argument doesn't rely on showing that there *is* a contradiction-forming operator, only that those operators that are found in putative dialetheic logics cannot be contradiction-forming operators, which entails that there are no dialetheic logics. Given, that is, that a defining property of a dialetheic logic is that it contains a contradiction-forming operator which allows for both contradictories to be true. By requiring that we both find a theory of negation T that ensures that a proposition and its negation cannot be true, and produce reasons to believe that T is the most plausible theory of negation, the amendment to Slater's argument completely changes the evidence it requires to be successful. The first route is not only unsuccessful, but distorts Slater's original argument.

The second route to the impossibility of dialetheic logics via the definition of negation is to make *being a contradiction-forming operator* one of the necessary conditions on an operator being negation. This route isn't available to Slater himself, as he allows for negations that are not contradiction-forming operators.[149] Yet, even if one didn't allow for non-contradiction-forming negations, our acceptable definition of contradiction doesn't entail the impossibility of dialetheic logics. All the truth-value neutral account tells us is that a contradiction is the conjunction of a proposition and its negation. Thus, by citing negation as a contradiction-forming operator one fails to show that dialetheic logics are impossible. It merely informs us of part of a contradiction's constituents. If an acceptable definition of contradiction had been a version of the classically-assumed account then this route might have been more successful, but in such a case we wouldn't have had to argue via the conditions on negation anyway. Instead, we could have argued directly from the definition of contradiction, as in Slater's original argument against dialetheic logics.

To successfully argue then from the truth-value neutral account of contradiction, and a definition of negation, to the conclusion that that there are no dialetheic logics, would require independent evidence that only operators O that ensure that a proposition and its negation cannot be true are deserving of the title 'negation'. Such an argument, however, isn't in keeping with the spirit of Slater's original argument. Slater thought that there could be other negations. What he protested against was calling an operator O a contradiction-forming operator when it allowed both a proposition p and Op to be true. Slater's argument is fundamentally about the

[149] Slater (2007) mentions on several occasions "dialetheic" negation, which he maintains are not contradiction-forming operators.

concept of contradictories, and not the concept of negation. If one has any doubts about this, then re-reading the beginning passage of Slater's paper should remove those doubts:

> If we called what is now 'red', 'blue' and vice versa, would that show that pillar boxes are blue, and the sea is red? Surely the facts wouldn't change, only the mode of expression of them. Likewise, if we called 'subcontraries', 'contradictories', would that show that 'it's not red' and 'it's not blue' were contradictories? Surely the same point holds.
>
> (Slater (1995) p. 451)

6.7 The Verdict

We have clarified and considered Slater's argument against the possibility of paraconsistent logics and found it to be unsuccessful. Modified as an argument against the possibility of dialetheic logics, it is as equally unsuccessful. The definition of contradictories, and parasitically contradictions, it relies upon is philosophically deficient. When presented with a more plausible account of contradictories and contradictions, namely the truth-value neutral account, we see that the definitions of contradictories and contradictions do not imply the impossibility of dialetheic logics. This is perhaps unsurprising given our findings in chapter 3 regarding the plausibility of the available definitions of 'contradiction'. After all, a demonstration of the impossibility of dialetheic logics from the definition of contradiction is tantamount to a demonstration of the absurdity of dialetheism and absolutism from the definition of contradiction. Contradictions, properly understood, neither preclude the truth of absolutism or dialetheism, nor the existence of dialetheic logics.

We have resisted one challenge to dialetheic logics and the absolutist's thesis. In the next chapter we consider another, a challenge from the putative *apriority* of the propositions of classical logic. This will lead us on to an important and insightful discussion of the empirical revisability of logic.

7. The Apriority of Logic

> No truth does have, and no truth could have, a true negation. Nothing is, and nothing could be, literally both true and false. This we know for certain, and *a priori*.
>
> (Lewis (1982) p. 434)

For those willing to admit that some propositions can be justified *a priori*, at least some logical propositions are considered to be among the most obvious cases.[150] Given that classical logic is by far the most widely endorsed logic in the philosophical community, it's no surprise that the hypothesis that some logical propositions are justifiable *a priori* extends to the view that some of the propositions of *classical* logic are justifiable *a priori*.[151] In addition, the history of philosophy is awash with claims that those propositions that can be justified *a priori* are necessarily true, analytic and immune to revision. Combined, these claims, if substantiated, would entail that some of the propositions of classical logic possess other philosophically important properties by virtue of being justifiable *a priori*.

The challenge presented in this chapter is to presume the *apriority* of certain propositions of classical logic that are incompatible with the absolutist's thesis, such as the mutual exclusivity of truth and falsity and the principle of explosion, and to consider whether the *apriority* of these propositions constitutes any obstacle to the absolutist's thesis. To state directly of these propositions that they are necessarily true or immune to revision would be to simply beg the question against the absolutist. However, if the classical logician could show that the propositions in question possessed these substantive properties as a consequence of being justifiable *a priori*, then she would no longer be begging the question against the absolutist. Instead, she would have a substantive criticism of absolutism.

7.1 Two Clarifications

Before we move on to consider the *a priori*/*a posteriori* distinction, two points of clarification are required. We need to be clearer on, firstly, what a proposition of logic is, and secondly, what form of challenge the putative *apriority* of these propositions of classical logic potentially raises against absolutism.

[150] Armstrong (2004, pp. 30 & 31), Boghossian (2001b, p. 640), Boghossian & Peacocke (2000, p. 8), BonJour (1998, p. 2 & 2005, p. 100), Field (2005b, p. 70), Harman (1996, p. 392), Horwich (2000, p. 169), Lewis (1987, p. 16), McGinn (1975-1976, pp. 199-200), Porteous (1936, pp. 119 & 137), Restall (2009, p. 37), Rey (1998, p. 28 & 2001, p. 653), and Russell (2001, p. 41). Historically, the thesis that our beliefs in logical propositions are justified *a priori* played an important role in Frege's (1981 & 1996) and Husserl's (2001) rejection of logical psychologism.

[151] Field (1996, p. 359).

7.1.1 Logical Propositions?

Logical systems are calculi, with a semantic theory and a metatheory.[152] Therefore, it may seem somewhat bizarre to speak directly of logical *propositions*. Even when we reason according to a logic we cannot be said to be accepting propositions, as *inferring according to a logical rule* is a distinct act from accepting a proposition.[153] Yet, there is a clear difference between *inferring according to a logical rule* contained in a logic *L*, and the outright endorsement of a particular logic *L* that some philosophers maintain.[154] We require, therefore, the means by which to refer to philosophers' endorsement of logics. Given that both belief and acceptance are commonly conceived as propositional attitudes, it seems only right to suggest that there are logical *propositions* for logicians to accept.[155] Logicians, by accepting a logic *L*, are committing themselves to the truth of propositions that are a consequence of *L*'s semantics or are included in *L*'s metatheory. For example, by endorsing a logic that contains both a consequence relation that preserves truth and *modus ponens*, one is committed to the proposition 'The rule of inference *modus ponens* is truth-preserving'.[156] It is the logician's belief in these propositions expressing the truth-preservation of the rules of inference, and the metatheory, of the logic that we are assuming can be justified *a priori*.

7.1.2 The Challenge to Absolutism

The absolutist requires an absolute logic to adequately model her logic. Given that classical logic both validates explosion and doesn't permit contradictions to be true, it fails to be an absolute logic. Therefore, classical logic is incompatible with the absolutist's thesis. Particularly, classical logic's validation of explosion and adherence to the mutual exclusivity of truth and falsity ensure that it's inadequate for the absolutist's purposes. Call those propositions of classical logic that are incompatible with absolutism the *troublesome* propositions. If any of these *troublesome* propositions of classical logic were found to have a

[152] Michael Resnik (1996, p. 491) considers logical *theories* to be a quadruple "consisting of a formal system, a semantics for it, the attendant metatheory, and a translation method for formalizing informal arguments." Unless Resnik intends some substantive distinction between logical *theories* and logical *systems* then we have good reason to disagree with him over the last element. It is undoubtedly possible to have a logic that has no application to any phenomena, especially informal arguments.

[153] A lesson learnt from the Lewis Carroll (1895) problem.

[154] This is demonstrated by the different forms of evidence required to be entitled to *infer according to a logical rule* in *L* and to be justified in one's philosophical endorsement of *L*. See Boghossian (2000) and Wright (2005).

[155] Although, as suggested by van Fraassen (1980) and Cohen (1989), there may be important epistemic distinctions between a belief *that p* and an acceptance *that p*, nothing we say here depends on making this distinction.

[156] There are at least two ways to understand the belief that 'The rule of inference *modus ponens* is truth-preserving'. Firstly, one could state that every argument of the form $\{A, A \rightarrow B\} \vdash B$ is truth-preserving. Secondly, using what Shapiro (2000, p. 337) calls *transfer principles*, we could translate the claim about logical inference into a claim about logical truth, 'Every proposition of the form $\ulcorner B$ is logically entailed by the combination of A and $A \rightarrow B \urcorner$ is true'.

truth-entailing, or other theoretically virtuous, property, then this could be problematic for the absolutist. For example, if we found that the proposition 'Truth and falsity are mutually exclusive' was rationally irrevisable, then the absolutist's thesis would entail a proposition that it is irrational to believe. Therefore, under the presumption that these *troublesome* propositions of classical logic are justifiable *a priori*, it is essential for the absolutist to ensure that a proposition *p* being justifiable *a priori* doesn't then entail that *p* possesses other properties which are theoretically damaging to absolutism.

It is important to note that the challenge raised by the *apriority* of these *troublesome* propositions cannot be adequately answered by merely endorsing a contradiction. While for the absolutist the necessary truth of a proposition *p* does not ensure *p*'s non-falsity, and the rational irrevisability of *p* doesn't entail that *p* is not also rationally revisable, there are good reasons for the absolutist to stay well away from this putative rebuttal of the challenge. Firstly, it would commit the absolutist to the *actual* truth of certain contradictions, a position that she wishes to distance herself from. Secondly, even if she could bring herself to endorse a true contradiction, the necessary truth of some of the *troublesome* propositions would still be devastating for the absolutist's thesis. For example, if the *troublesome* proposition 'Explosion is valid' were necessarily true, then being willing to endorse contradictions wouldn't save the absolutist. By accepting a contradiction constituted of a proposition expressing explosion's validity and another expressing its invalidity, the absolutist is committed to the *validity* of explosion. That is, she is not in a position to *reject* the validity of explosion. She must accept both its validity and invalidity. However, the validity of explosion is devastating for the absolutist, as it would reduce her thesis to modal trivialism. Simultaneously endorsing the invalidity of explosion wouldn't be of any help. Thus, in answering the challenge from *apriority* the absolutist must engage with the argument on its own terms, with a consideration of *a priori* justification, rather than relying on the truth of certain contradictions.[157]

In this chapter we will not argue, as Putnam's (1985c) Quine does, that there are no propositions justifiable *a priori*.[158] Nor will we suggest that the *a priori*/*a posteriori* distinction is of

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[157] Logical pluralism, *a la* Beall and Restall (2000 & 2006), could help the absolutist with both concerns. It would allow her to admit the necessary truth (or rational irrevisability) of the *troublesome* propositions without either precluding her own thesis or admitting any true contradictions. As the *troublesome* propositions would only be about the *classical* consequence relation, and more than one consequence relation can be correct, the necessary truth or rational irrevisability of the propositions would have no impact upon the truth or rationality of the propositions the absolutist holds in advocating an absolute logic. However, this response to the concerns would, firstly, require the absolutist to demonstrate how classical and absolute logics differ over their definitions of *cases*, the kernel of the logical pluralist's argument, when they seem not to, and secondly, require the absolutist to take on a completely unnecessary theoretical commitment.

[158] See also Devitt (1996, pp. 46-54; 1998; 2005), who argues against the existence of *a priori* justification on the basis that, firstly, the holistic nature of confirmation gives us good reason to think that logic and mathematics are not immune from empirical revision and, secondly, the concept of *a priori* justification is deeply obscure. Similar themes are found in Maddy (2000), where it's argued that *a priori* justification plays no part in the successful naturalistic model, or in explaining the success of scientific practice.

little epistemological importance, as Williamson (2007, Ch. 5 & 2013) does.[159] Instead, we will show that the putative *apriority* of these *troublesome* propositions of classical logic isn't a threat to the absolutist's research programme. The chapter offers the two conditionals that may accurately reflect the threat that the putative *apriority* of the *troublesome* propositions pose to absolutism,

A) If the *troublesome* propositions are justified *a priori*, then this *isn't* theoretically damaging to absolute logics,

B) If the *troublesome* propositions are justified *a priori*, then this *is* theoretically damaging to absolute logics,

and argues that whichever conditional is true, the absolutist has no reason to fear. If the correct account of *apriority* entails conditional A), then we can be content in this thesis to accept that the *troublesome* propositions of classical logic are justifiable *a priori* without any harm to the absolutist's thesis. Conversely, if the correct account of *apriority* entails conditional B), then we must take a different tack. Given that the only reasonable account of *apriority* that entails B) implies that propositions that are justifiable *a priori* are empirically indefeasible, B) entails that the *troublesome* propositions are empirically indefeasible. Yet, in the next chapter, we argue that logics can be rationally revised based on empirical evidence. This constitutes strong evidence that propositions of logic are *not* empirically indefeasible. Therefore, if the correct account of *apriority* entails conditional B), then the account also entails that logical propositions are not justifiable *a priori*, contrary to this chapter's hypothesis. Consequently, regardless of whether the correct account of *apriority* entails conditional A) or B), the putative *apriority* of the *troublesome* propositions is no obstacle to the absolutist's project.

7.2 The *a Priori/a Posteriori* Distinction

The *a priori/a posteriori* distinction is an epistemic distinction between ways humans come to be epistemically justified in believing a proposition: those that require experiential evidence and those that don't.[160] Plausibly, both types of coming to be justified are composed

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[159] Williamson's argument is criticised in Boghossian (2011); Williamson (2011) provides a reply in the same issue.

[160] This epistemic interpretation of the *a priori/a posteriori* distinction is in accordance with Kant's usage of the terms, who gave '*a priori*' the central place in the philosophical vocabulary that it still occupies. The pre-Kantian meaning of '*a priori*' is less clear and doesn't seem to correspond to our current epistemic understanding of the term:

> [W]hat was a priori and a posteriori in scholastic philosophy depended upon whether one was considering the order of being or the order of knowing. The terms literally mean 'from what comes before' and 'from what comes after', and the scholastic usage depends on the Aristotelian doctrine of the distinction between

of sub-types that differ in the ways we come to be justified in believing a proposition within the type's restraints. So, for example, *a posteriori* justification *may* contain the sub-types of direct perceptual justification and justification through testimony.[161] These sub-types that are members of the categories *a priori* and *a posteriori* justification will be called here *modes of justification*. An *a posteriori* mode of justification is simply an epistemic process that produces *a posteriori* justification for a proposition, and an *a priori* mode of justification is an epistemic process that produces *a priori* justification for a proposition. Scientific and philosophical inquiry may require that we become more fine-grained in our classification of modes of justification, due to relevant causal differences between, for example, instances of testimonial knowledge, however nothing in our discussion hangs on whether a more fine-grained classification is required or not.[162]

The two epistemic categories of *a priori* and *a posteriori* justification exhaust the possible modes of justification. There is no mode of justification M that isn't classifiable as either an *a priori* or *a posteriori* mode of justification. This exhaustiveness is a consequence of the excluded middle. Either a mode of justification requires experiential evidence or it doesn't. Equally, there is no mode of justification M that is classified as both an *a priori* and *a posteriori* mode of justification. The distinction between the two categories of justification hinges upon instances of *a posteriori* justification requiring experiential evidence in a way that instances of *a priori* justification do not. So, unless it's possible for a mode of justification to both require and *not* require experiential evidence in this particular way, a possibility not considered here, a mode of justification will be either an *a priori* mode of justification or an *a posteriori* mode of justification, but never both. The epistemic categories of *a priori* and *a posteriori* justification partition the set of modes of justification.

In contrast, while a proposition p can only be *a priori* or *a posteriori* justified, as these are the only available types of modes of justification, it does seem possible for p to be both *a*

> what is prior in nature and what is prior in knowledge. Something is prior in nature (*priora natura*) to something else if it could not exist without it: in this sense causes are prior to their effects. But something is prior in knowledge (*priora nobis*) to something else if we could not know the latter without knowing the former: in this sense effects may be prior to their causes.
>
> (Gaukroger (1989) pp. 101-102)

[161] Burge (1993) has argued, contrary to what is suggested here, that justified belief gained through testimony can be *a priori* justified, and thus testimonial justification is an *a priori* form of justification, due to the *a priori* entitlement we have to accept as true the testimony of others. See Christensen & Kornblith (1997) and Malmgren (2006) for criticisms of Burge's argument; Burge (1997) responds to Christensen & Kornblith (1997).

[162] While it is pretty clear that there are different *a posteriori* modes of justification, it may seem less clear that there are various *a priori* modes of justification. However, Peacocke (2000, pp. 284-285) has suggested that we might need different *a priori* modes of justification to explain all the putative cases of propositions we can justify *a priori*, due the propositions' distinctive features. While we may be *a priori* justified in believing the sentence 'I am here' due to a special connection between the indexicals it contains and its utterer, the *a priori* justification for 'Nothing can be red and green all over' will have nothing at all to do with indexicals.

priori and *a posteriori* justified. For example, one can be justified in believing the arithmetical proposition 'Two plus two equals four' by using a calculator or by counting one's fingers, and one can construct a proof that demonstrates two plus two equals four, constituting *a priori* justification for the proposition.[163] Therefore, while it's plausible that every mode of justification *M* is either an *a priori* or *a posteriori* mode of justification, but *not* both, some propositions can be both *a priori* and *a posteriori* justified.

7.3 The Minimal Definition of the *a Priori*

Given that the *a priori*/*a posteriori* distinction is an *epistemic* distinction between the possible *sources* of evidence for a proposition, it is a necessary condition on any adequate definition of *apriority* that it includes reference to this distinction between sources of evidence. Call this the *minimal* requirement for any definition of *apriority*. A *minimal* definition of *a priori* justification only includes this *minimal* epistemic requirement:

M-APrJ) Individual *I* is epistemically justified in believing a proposition *p* *a priori* iff *I* is epistemically justified in believing *p without experiential evidence*.[164]

As we are interested in the properties logical *propositions* possess in virtue of being propositions that are *a priori* justifiable, it will be useful to be able to refer directly to *a priori propositions*. Fortunately, as we shall see below, a proposition *p*'s status as *a priori* or *a posteriori* follows directly from the modes of justification by which *p* can be justified. Thus, by talking in terms of *a priori* propositions, we are neither making any theoretical concessions nor losing any explanatory power. Translating the minimal definition of *a priori* justification into a minimal definition of *a priori* propositions we achieve,

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[163] The case is even clearer if we consider disjunctive propositions. For example, 'Two plus two equals four or Paris is the capital of France' can be both justified *a priori* and *a posteriori*. While a proof can be produced for the first disjunct, constituting *a priori* justification for the disjunction, belief in the second disjunct can be justified by looking at a map, constituting *a posteriori* justification for the disjunction. For any proposition of the form $p \vee q$, where p can be justified *a priori* and q can be justified *a posteriori*, the proposition can be justified both *a priori* and *a posteriori*.

[164] There are alternative ways to formulate the minimal requirement for the *minimal* definition of *a priori* justification. Casullo (2003, pp. 29-30) distinguishes between two distinct forms of the minimal requirement: *Negative* formulations define *a priori* justification in terms of what *isn't required* for *a priori* justification, but *is* for *a posteriori* justification, and *positive* formulations define *a priori* justification in terms of particular *a priori* modes of justification. For positive formulations of *a priori* justification see Bealer (1996a, 1996b & 2000), BonJour (1995 & 1998, Ch. 1 & 4), and Butchvarov (1970, pp. 93-97). Negative formulations of *a priori* justification are favoured in Barnes (2007), Boghossian & Peacocke (2000), Edidin (1984), and Restall (2009). Our minimal account of *a priori* justification above is a *negative* formulation, favoured because many *positive* formulations are unnecessarily restrictive, and those that aren't seem equivalent to *negative* formulations; see Casullo (2003, pp. 30-32).

M-APrP) A proposition *p* is *a priori* iff *p can* be epistemically justified without experiential evidence.

It is a necessary requirement of any adequate theory of *apriority* that it distinguishes *a priori* from *a posteriori* justification by virtue of the differing roles experience plays in the two forms of justification. While *a priori* justification doesn't require experiential evidence, *a posteriori* justification does. Any account of *a priori* justification can add further restrictions to ensure that the distinction is epistemically enlightening, but any account that fails to meet this *minimal* requirement neglects the epistemic nature of the distinction between the forms of justification and is deficient.[165]

7.4 The Challenge of Default Reasonable Belief

Default reasonable beliefs are "reasonable in and of themselves, without any supporting justification."[166] While propositions and rules can similarly be default reasonable, with a default reasonable proposition being a proposition *p* that can be reasonably believed without any justification, and a default reasonable rule being a rule *R* that can be reasonably followed without any justification, our interest here is exclusively in default reasonable *beliefs*.[167] As our minimal definition of *apriority* stands, its adequacy is under threat from these default reasonable beliefs due to a conjunction of two factors.

Firstly, if we interpret *justification* widely enough to include externalist concepts of epistemic right such as warrant, M-APrJ) categorises default reasonable belief as *a priori* justified beliefs, given that the reasonableness of these beliefs doesn't depend on experiential evidence. Secondly, there seems to be an important epistemological difference between the epistemic property of *apriority*, which we've attempted to explicate in M-APrJ), and the epistemic properties that default reasonable beliefs possess in virtue of being default

[165] Definitions of *apriority* that fail to meet the minimal condition include: Putnam's (1985a, 1985b & 1985c) definition of *apriority* solely in terms of rational indefeasibility. While Putnam's (1985b, p. 99 & 1985c, p. 95) notion of the *contextual a priori* is an important idea, it's not an account of *apriority* epistemically understood; Frege's (1996, p. 4) definition of an *a priori* proposition as one which a proof of "can be derived exclusively from general laws, which themselves neither need nor admit of proof," which says nothing about the role of experience in justifying the "general laws". For more on the relation between Frege's account of *apriority* and Kant's traditional account, see De Pierris (1988) and Burge (2000); Reichenbach's (1965, p. 48) interpretation of Kant, which conflates *apriority* and necessity; Chisholm's (1987, pp. 121-123) account of *a priori* knowledge as acceptance of axioms or of propositions axiomatically entailed by an axiom, with *h* being an axiom if and only if necessarily "(i) it is true and (ii) for every *S*, if *S* accepts *h*, then *h* is certain for *S*." Chisholm, therefore, defines *apriority* in terms of necessary truth and certainty, neglecting the epistemic nature of the *a priori*/*a posteriori* distinction; Stalnaker's (1999, p. 83) notion of *a priori* truths, defined as statements that "expresses a truth in every context," which makes no mention of any epistemological conditions.

[166] Boghossian (2000, p. 238). Cf. Berry (2013, p. 3696).

[167] For more on default reasonable propositions and rules see Field (2000) and Harman (2003).

reasonable. While in the latter's case the entitlement isn't based on *any* evidence, and we simply have an epistemic right to the belief, in the case of *a priori* justified beliefs there is evidence for the truth of the belief, just not of the *experiential* type. In other words, the distinction between *a posteriori* justification and *a priori* justification isn't that the former requires experiential evidence while the latter requires an *absence* of evidence; it is that *a priori* justification requires evidence of a wholly different kind.

In combination, these factors cause a tension within our minimal definition of *a priori* justification. M-APrJ) classifies all default reasonable beliefs as *a priori* justified beliefs, when we have good reason to believe that they shouldn't be. To ignore the epistemic differences between the concepts, and run them together, would be theoretically unproductive. Something then needs to give, whether it's our current minimal definition or the wide interpretation of justification. There are three available options to meet the default reasonable beliefs' challenge to our *minimal* definition:

1) Formulate the definition of *a priori justification* explicitly in internalist language, thus precluding the possibility of any externalist account of *a priori* justification.

2) Reject the possibility of default reasonable beliefs.

3) Adapt M-APrJ) to ensure that *a priori* justified beliefs depend on *some* evidence for their justification.

Option 1) relies upon our ability to interpret the 'justification' of 'epistemic justification' in M-APrJ) such that it excludes cases in which we have the epistemic right to hold a belief *B*, but our epistemic right to believe *B* isn't dependent upon us *knowing* that we possess the epistemic right to believe *B*. As our warrant for holding default reasonable beliefs is of this kind, such beliefs would no longer be categorised as *a priori* justified beliefs. Unfortunately, any such interpretation of 'justification' would directly preclude the possibility of any externalist account of *apriority*, and this should be convincing evidence enough that it's an unviable solution. We can't reasonably conclude that an externalist framework cannot account for *a priori* justification simply because we were forced into a certain interpretation of 'justification' by the threat of default reasonable beliefs. Precluding externalist accounts of *apriority* should require far more substantial arguments.

Option 2) simply isn't a viable solution here. We simply don't have the time to explain away all the putative cases of default-reasonable beliefs, or deal with the concept of epistemic reasonableness that arises from such beliefs. Default reasonable beliefs may turn out to be explanatorily superfluous, or just downright implausible, yet we can neither argue for these conclusions here nor presume that they have been established elsewhere.[168]

This leaves us with option 3). To meet the challenge raised by default reasonable beliefs, we will need to adapt M-APrJ). This we can achieve by requiring that *a priori* justification is *constituted of non*-experiential evidence, rather than being simply justification *without* experiential evidence. Non-experiential evidence is any kind of evidence that isn't experiential, which ensures both that experiential and non-experiential in combination exhaust the possible types of evidence and that {experiential evidence} and {non-experiential evidence} don't intersect. With this change to the minimal definition, default reasonable beliefs are no longer categorised as *a priori* justified beliefs, as the entitlement agents possess for the beliefs isn't based on *any* evidence, experiential or non-experiential. In addition, the intuitive distinction between *a priori* and *a posteriori* justification based on different types of evidential sources is retained. The former depends on evidence that isn't experiential, while the latter depends on experiential evidence.

This change to the minimal definitions of *apriority* ensures that *a priori* and *a posteriori* modes of justification don't exhaust the possible modes of justification. If there are default-reasonable beliefs, then there will be beliefs that one can be entitled to hold without *any* evidence. For the same reason, it may be that the epistemic categories of *a priori* and *a posteriori* propositions don't partition {propositions that are justified at a possible world}. This will depend on whether there are any propositions that are both default-reasonable and incapable of being justified with either experiential or non-experiential evidence. Yet, these added complexities are a small price to pay for a precise definition of *apriority*.

Our new minimal definitions of *a priori* justification and propositions become, respectively:

M-APrJ') Individual *I* is epistemically justified in believing a proposition *p a priori* iff *I* is epistemically justified in believing *p* based sufficiently on non-experiential evidence.

M-APrP') A proposition *p* is *a priori* iff *p can* be epistemically justified based sufficiently on non-experiential evidence.

The 'sufficiently' qualifier included in these definitions doesn't ensure that an individual cannot be *a posteriori* justified in believing an *a priori* proposition. Instead, it only ensures that for some non-experiential evidence *e* to constitute *a priori* justification for a proposition *p*, *e*

[168] For arguments that our fundamental logical beliefs can be given a rule-circular justification, thus removing the need for these beliefs to be default reasonable as Field (2000) hypothesises, see Dummett (1978, ch. 17) and Boghossian (2000). For criticisms of the rule-circular justification of logic see Haack (1976 & 1982).

must be sufficient on its own, without any further contributing experiential evidence, to justify *p*. There might be this further contributing experiential evidence, but as long as the non-experiential evidence is *sufficient* for the justification of *p* without the experiential evidence, then the non-experiential evidence constitutes *a priori* justification. If an individual *I* possesses both *sufficient* non-experiential evidence for a proposition *p* and some experiential evidence for *p*, then the evidence available to *I* overdetermines her justification for believing *p*. However, this overdetermination doesn't damage her *a priori* justification for *p*, it simply ensures that she possesses both *a priori* and *a posteriori* justification for *p*.

With the added complication of default reasonable beliefs, it's also worth stating the minimal definitions of *a posteriori* justification and propositions, respectively:

> M-APoJ') Individual *I* is epistemically justified in believing a proposition *p a posteriori* iff *I* is epistemically justified in believing *p* based on experiential evidence.

> M-APoP') A proposition *p* is *a posteriori* iff *p can* be epistemically justified based on experiential evidence, but *cannot* be epistemically justified based sufficiently on non-experiential evidence.[169]

Note that we don't require the 'sufficiently' qualifier to characterise *a posteriori* justification. Any justification for a belief that requires experiential evidence, even if it also requires non-experiential evidence, constitutes *a posteriori* justification. This is a clear case of asymmetry between *a priori* and *a posteriori* justification.

Now that we have our minimal definitions of *apriority*, having revised them to deflect concerns raised by default reasonable beliefs, some aspects of the definitions require clarification.

7.5. Clarifying the Minimal Definition

There are four main aspects of the minimal definition that require clarification:

1) What counts as experience in 'experiential evidence'?

2) What distinguishes the role experience plays in the two types of justification?

[169] The latter condition in M-APoP') ensures that although a proposition can be both *a priori* and *a posteriori* justified, it *cannot* be both an *a priori* and *a posteriori* proposition.

3) Why does the minimal definition assign *apriority* to justification rather than knowledge?

4) What is the modality of the '*can* be epistemically justified' in M-APrP')?

7.5.1 What counts as Experience in 'Experiential Evidence'?

There are two available interpretations of experience in the context of 'experiential evidence'. The first interprets experience such that it includes both the experience of phenomena exterior to us through our sensory apparatus *and* experience of our own mental states, whereas the second includes only the former type of experience. Thus, according to the latter interpretation of experience, we could be justified in holding beliefs about our own mental states by inner awareness *without* any experiential evidence, as this awareness doesn't constitute experience.

By accepting the latter interpretation, and thus narrowing the domain of what counts as experiential evidence, we allow beliefs about our own mental states to be justified *a priori*. This doesn't commit us to saying that our beliefs about our own mental states are only justifiable *a priori*, for we could reasonably come to believe that we were angry earlier that afternoon from CCTV footage of our behaviour. Regardless, it does seem rather strong to commit oneself to the view that *every* proposition about our own mental states that we could become justified in believing, through introspection or internal sensation, can be justified *a priori*. Yet, by accepting this narrow account of experience we would have no option but to admit this. We commit ourselves to saying that propositions such as 'I'm frustrated because of the weather' or 'I'm unhappy because of the situation at work' are justifiable *a priori*. The reflection required of us to find evidence for these propositions, though, seems phenomenologically similar to our experiences of the external world, and *both* forms of evidence seem equally to be cases of experience in virtue of their shared phenomenological properties. That we are *forced* to accept that propositions about our mental states can be justified *a priori* seems an unfortunate consequence of choosing the narrow interpretation of experience.

Intuitively miscategorising certain propositions about our mental states as justifiable *a priori*, however, is not the most theoretically troubling aspect of the narrow interpretation of experience. After all, intuitive miscategorisations can always be accommodated if the theoretical considerations for doing so are strong enough. Rather, the most theoretically damaging aspect of the decision to interpret experience narrowly is the ease and unjustified nature of the victory it buys for the *a priori* advocate. Even if our beliefs about our own mental states are justifiable *a priori*, this isn't plausibly because we don't *experience* pain, frustration or unhappiness. Rather, it would be because such experience wasn't necessary as *evidence* for our belief, and thus wouldn't constitute our *justification* for the belief.

If we had good reason to think that we could be *a priori* justified in believing propositions expressing our mental states, it wouldn't be because we had stipulated that what we commonly consider to be cases of experience are *not actually* cases of experience. Instead, it would be because these experiences are *not* constitutive of our justification for our beliefs about our mental states. When we know that we are in pain, so the argument would go, we *experience* the pain, but such *experiences* don't constitute our epistemic justification for the belief that we are in pain. To attempt to establish that we have *a priori* justified beliefs about our mental states by this method is completely philosophically respectable, as it would be based on giving good reasons. To establish the same conclusion, by stipulating that what we commonly consider to be experience shouldn't be counted as such, for the sake of this particular discussion, is a cheap and totally unjustified victory for the *a priori* advocate.

By interpreting experience in the broad sense we leave open the debate on two counts. Firstly, we leave open the possibility that a scientifically respectable account of experience may find that there are particular mental phenomena, such as cases of imagining, that should *not* be considered experience. In which case, we could make substantive conclusions about whether a mental phenomenon M constitutes experience or not. Secondly, we leave open the debate over whether we can have *a priori* justified beliefs about some of our mental states, which will hinge upon whether we can have sufficient non-experiential evidence for beliefs about our mental states.

To remove an unnecessarily restrictive stipulation, and open the way for possibly constructive debate on whether our experience of our mental states is constitutive of our justification for our beliefs about our mental states, is undoubtedly a theoretical virtue. By accepting a wider interpretation of experience, propositions such as 'I'm hungry' and 'I'm in pain' could still count as *a priori*, for although both parties could agree that pangs of hunger and shooting pains are *experiences*, they can disagree over whether such experience constitutes the only evidence for the propositions. The disagreeing parties can argue over whether one can be justified in believing the propositions without citing the experiences as *evidence*. Thus, the possible retort from the *a priori* advocate that she *needs* to interpret experience narrowly to be able to reach her conclusion, or argue her case, simply isn't true. Both sides can share the same account of experience and still reasonably disagree. Excessive stipulation can cause our distinctions to lose their application to our actual practices and thus make them unimportant and uninteresting. By excessively stipulating, the *a priori* advocate may gain more entries into the category of *a priori* propositions, but at the cost of making boring and philosophically irrelevant what could have been an interesting epistemological distinction. Experience in our discussion of *apriority* will include both the experience of phenomena exterior to us through our sensory apparatus and experience of our own mental states through introspection or inner sensation, while remaining aware that there may be interesting cases of mental phenomena, such as imagining, which don't constitute experience.

7.5.2 What Distinguishes the Role Experience Plays in the Two Types of Justification?

Given that there is a distinction between experiential and non-experiential evidence, what differing roles does experience play in the distinct types of justification? After all, we seem to require experience to some extent to be justified in believing *any* proposition. Unless we are born with an innate knowledge of mathematical concepts, we will require some experience to understand what concepts the symbols '2', '4', '+' and '=' symbolise, if we are ever going to be justified in believing that '2 + 2 = 4'. Given that postulating an innate knowledge of mathematical concepts is hardly attractive, we must admit that experience is required for even paradigm cases of *a priori* justification. We need then a principled way to distinguish between the experiences required for *a priori* justification, and those required for *a posteriori* justification. Kant (2009, B1) both appreciated and indicated a possible answer to this problem in the *Critique*:

> [A]ll our cognition begins with experience; for how else should the cognitive faculty be awakened into exercise if not through objects that stimulate our senses and in part themselves produce representations, in part bring the activity of our understanding into motion…[N]o cognition in us precedes experience, and with experience every cognition begins.[170]

Even if there are propositions that can be justified by non-experiential evidence, for these propositions *p* to be *understood* by an agent, we must allow experience to play some role in the agent's coming to be justified in believing *p*. This experience that allows one to *understand* a proposition, by grasping its semantic or conceptual content, rarely simultaneously *justifies* the proposition. Understanding the constituent semantic or conceptual parts of a proposition is rarely, if ever, *evidence* for the truth of the proposition. Rather, such experience *enables* the evidential project to begin. One must understand a proposition before one can appreciate what constitutes evidence for its truth. Thus, even *a priori* justification requires the individual to have the experiences necessary to understand the semantic or conceptual content of the proposition.

While experience is required in an *enabling* role for both types of justification, aiding individuals to understand a proposition, experience plays an additional role in cases of *a posteriori* justification. In cases of *a posteriori* justification experience also provides the constitutive *evidence* for the truth of the proposition. So, with my *a priori* justified belief that

[170] Cf. Locke (1996), Bk. 1.

'2 + 2 = 4', the only role that experience plays in my belief is the role of *enabling* my learning of the content of the proposition. Whatever other resources I need, or intellectual processes I need to go through, to have an *a priori* justified belief in the proposition, they do not involve experience. Whereas, with *a posteriori* propositions such as 'Stockholm is the capital of Sweden', experience plays both the *enabling* role in my coming to understand the proposition, and an *evidential* role in my coming to be justified in believing it. With *a posteriori* propositions, I cannot come to be justified in believing them just by having *enabling* experiences. Whatever non-experiential resources or processes I call upon, they will be of no help in justifying the proposition. Experience, therefore, is necessary to give one this extra information that can act as a reason for believing the truth of the proposition.

Admittedly, not much has been said so far about what constitutes the *evidential* role of experience in cases of *a posteriori* justification. What is it for experience to give evidence for the truth of a proposition? Fortunately, I don't think we need to give an answer to that question here. We are generally aware of when experiences play an evidential role in justifying a proposition, and this is enough to draw the distinction between the roles experience plays with regards to *a priori* and *a posteriori* justification. *How* experience can be said to provide evidence for the truth of a proposition is a question we can leave to the philosophy of science and perception for a later time. It is part of the grander question of how any phenomena can be said to constitute evidence for the truth of a proposition, a question far too grand for us to speculate on here. We have our distinctive roles that experience plays, even if we don't fully understand how those roles are fulfilled.

To the extent that *a priori* and *a posteriori* justification can be differentiated by the *roles* experience plays in both, the distinction between the roles of experience, and their relation to *a priori* and *a posteriori* justification, is already appreciated in our minimal definitions of *apriority*. *A priori* justification is defined as justification that requires *non-experiential evidence*, and thus doesn't require experience in an *evidential* role, although experience *is* required in an *enabling* role. Our definitions of *apriority* are, therefore, already satisfactory on the count of appreciating the distinct roles experience plays in *a priori* and *a posteriori* justification.[171]

It is important to recognise that the advocate of *a priori* justification, by permitting justification that only requires experience in an *enabling* role, is not committing herself to the existence of analytic sentences, or to the thesis that analyticity is a necessary condition for *a priori* justification. To paraphrase *a priori* justification as *knowledge solely in virtue of meaning* or

[171] The twin roles that experience plays in justification is appreciated in Ayer (1987, pp. 99-100), BonJour (1995, p. 53; 1998, pp. 9-10; 2001b, p. 696; & 2005, p. 99), Burge (1993, p. 460), Chisholm (1987, pp. 118-119 & 126), Field (2005b, p. 70), Geirsson (1991, p. 204), Jenkins (2008, p. 444), Kitcher (1980a, p. 5 & 1983, p. 23), Manfredi (2000, p. 174), Plantinga (1993, pp. 103-104), Russell (2001, p. 41), and Swinburne (1975, pp. 239-240). The distinction has a strong connection with Frege's (1979c, p. 2) criticism of the empiricist tendency in philosophy to declare that all our knowledge is empirical "because our thinking takes its rise from experience."

similar, without argument, would be both extremely misleading in one's explanation of experience's *enabling* role in *a priori* justification, and very presumptive towards the possible sources of non-experiential evidence.

Although it was not an uncommon view throughout twentieth-century philosophy that all *a priori* propositions are expressed by analytic sentences,[172] there do seem to be other live options for the advocate of *a priori* justification. These include theories of *a priori* justification based on concept acquisition,[173] and theories that explicitly reject the link between *apriority* and analyticity by proposing non-experiential sources of justification, such as intuition or rational insight, that give us direct access to the nature of the world.[174] Of course, both of these theoretical possibilities may in the end be hopeless,[175] or they may carry a similar burden

[172] See Ambrose & Lazerowitz (1962, p. 17), Ayer (1987, Ch. 4), Hempel (1949, p. 241), Jackson (2000, p. 324), although he comes close to simply conflating *apriority* and analyticity by defining an *a priori* sentence as "one such that understanding it is sufficient for being able to see that it is true", Kneale & Kneale (1962, p. 636), Lewis (1946, p. 35), Pap (1958, p. 95), Salmon (1967, p. 39), Spicer (2007, p. 334), and Swinburne (1975, p. 241). Kripke (1981, p. 39) accepts the conditional '*if* something is analytically truth then it is *a priori*', but stops short of suggesting that all cases of *apriority* are cases of analyticity by stipulating that analytic truths are necessary truths, which ensures cases of the contingent *a priori* are not analytic (p. 122, fn. 63).

[173] See Eagle (2008, p. 81), Giaquinto (1996 & 1998), Peacocke (1993, 1998, & 2000), and Rey (1998). Dummett (1991, p. 1) implies he shares this view by stating that philosophy "[has] not returned to the belief that a priori reasoning can afford us substantive knowledge of fundamental features of the world. Philosophy can take us no further than enabling us to command a clear view of the conceptions by means of which we think about the world."

[174] See BonJour (1995; 1998, sections 1-4 & 6.7; 2001a; 2001b; & 2005), whose 'rational insight' theory of *apriority* is probably the most discussed in the literature – see Beebe (2008), Boghossian (2001b), Casullo (2001 & 2003 pp. 15-17, 100-110, 149-176, & 202-205), Devitt (2005), Gendler (2001), Harman (2001), Miscevic (1998), Rey (2001), and Thurow (2009); Bealer (1996a, 1996b & 2000), although Bealer is not adverse to switching between *a priori* justification constituted by concept possession (2000, p. 22) and a quasi-perceptual faculty that allows us to see the workings of nature (2000, p. 4); Ewing (1939-1940, pp. 217, 239 & 243); Gödel (1964), who speaks of our mathematical beliefs "represent[ing] an aspect of objective reality, but, as opposed to the sensations, their presence in us may be due to another kind of relationship between ourselves and reality," which he calls "intuition" (p. 272). Similarly to Bealer though, Gödel sometimes speaks as if mathematical knowledge is based on unpacking a concept, such as set theory being "supplemented without arbitrariness by new axioms which only unfold the content of the concept of *set*," (p. 264); Plantinga (1993, pp. 102-121), who explicitly rejects the equating of *apriority* and analyticity (p. 103), and talks of *a priori* knowledge as a type of *seeing* with a "peculiar sort of phenomenology" (p. 106), a product of the faculty of "*reason*" (p. 107); and Russell (2001, pp. 50 & 59-60), who commends Kant for recognising that not all *a priori* knowledge is analytic (p. 46), and suggests that *a priori* knowledge is knowledge of the relations between universals, which is strictly not knowledge of anything that exists but of entities that "*subsist* or *have being*" (p. 57). Russell (1975, p. 77) later suggests that he changed his mind on the existence of analytic *a priori* knowledge, "I no longer think that the laws of logic are laws of things; on the contrary, I now regard them as purely linguistic." Broad (1936) doesn't explicitly admit the existence of synthetic sentences that express *a priori* propositions, however he does state that "either there are no a priori truths at all, *or* there are such truths and they are all synthetic," (p. 105).

[175] Horwich (2000) has criticised the view that our *a priori* justification "is engendered by our understanding of words or by our grasp of concepts," (p. 150) concentrating particularly on our knowledge of logic and arithmetic. Horwich himself seems to prefer the view that *a priori* justification stems both from pragmatic decisions and the innateness (perhaps psychological indispensability) of certain beliefs (pp. 167-168). However, it's unclear how a pragmatic decision to hold a belief creates *epistemic* justification, or how our physical/psychological constitution forcing us to hold a belief entails that the belief is epistemically justified. Nietzsche (1968, §483 & 487;

to theories of analyticity.[176] However, to show that an advocate of *a priori* justification is committed to the analyticity of sentences expressing *a priori* propositions requires far more than demonstrating that *a priori* justification only requires experience in an *enabling* role.

7.5.3 Why does the Minimal Definition assign Apriority to Justification rather than Knowledge?

So far in our minimal definition we have assigned *apriority* to justification, yet one may wonder why we haven't been willing to assign *apriority* instead to knowledge. It's not unusual, after all, to find philosophers speaking of *a priori knowledge*. Our reason for assigning *apriority* to justification, rather than knowledge, is two-fold.

Firstly, it is the type of evidence that constitutes *justification* for a belief that dictates whether we have an instance of *apriority* or *aposteriority*. While we could speak primarily of either *a priori* justification or *a priori* knowledge, in assigning *apriority* to justification we single out the property that distinguishes *apriority* and *aposteriority*; this being the *source of justification* for a proposition. Speaking primarily of *a priori* justification, rather than knowledge, we make more transparent the distinguishing properties of *apriority* and *aposteriority*.

Secondly, assigning *apriority* primarily to knowledge could have the unfortunate consequence that we mistake properties of knowledge, which cases of *a priori* and *a posteriori* knowledge both possess, for properties unique to *apriority*. Given that *knowing* a proposition requires the fulfillment of further conditions than being *epistemically justified* in believing a proposition, there is a higher probability that requirements for knowledge may be mistaken for distinguishing properties of *apriority* than is the case with requirements for justification. This confusion may be particularly damaging if one maintains strong conditions on knowledge such as indefeasibility and certitude,

Indefeasibility:	An individual *I* knows *p* only if there is no possible defeating evidence for *p*.
Certitude:	An individual *I* knows *p* only if *I* possesses psychological certitude over the truth of *p*.[177]

2003, §11 & 39; & 2007, §424) observes persuasively on several occasions that there's no necessary link between what is true, or what one is justified in believing is true, and what is pragmatic or psychologically indispensable. Psychological indispensability seems no *epistemic* justification at all, and thus cannot constitute an account of *a priori* justification.

[176] The critique of analyticity is most famously found in Quine (1936, 1951, 1960a & 1960b), but is also found in Giaquinto (1996 & 2008), Harman (1996), and Williamson (2007, Ch. 3-4).

[177] Descartes seemingly endorsed both of these conditions on knowledge. The indefeasibility of knowledge is suggested by Descartes's (1991, p. 147) comparison of knowledge with conviction:

By both including these conditions in one's theory of knowledge, and insisting that we talk directly of *a priori* knowledge, one is at risk of confusing these hypothesised properties of *knowledge* for properties of *apriority*.[178] If these conditions are indeed conditions on knowledge then they will be conditions on *a posteriori* and *a priori* knowledge alike. By demanding that *a priori* knowledge fulfill these conditions because of one's theory of knowledge, one's demands of *a priori* knowledge stem not from *a priori* knowledge *qua* a case of *apriority*, but from *a priori* knowledge *qua* knowledge. One would have equal demands of *a posteriori* knowledge. In this chapter we are interested in the epistemic properties that distinguish *apriority* and *aposteriority*, not which properties *a priori* knowledge possesses in virtue of being knowledge. Thus, by assigning *apriority* to knowledge instead of justification we would be increasing our risk of introducing properties into the *apriority*/*aposteriori* distinction that don't constitute the distinction, but which are instead consequences of our theory of knowledge.

None of this entails that one cannot argue reasonably that indefeasibility or certitude are conditions on *apriority*, but we must be sure that these conditions derive from the *source of justification* particular to *a priori* justification, and not just from one's theory of knowledge. We are interested in properties that *apriority* possesses which *aposteriority* doesn't. Put simply, assigning *apriority* to knowledge rather than justification adds unnecessary complications. If we are going to suggest that indefeasibility and certitude are conditions on *apriority* then we want to be sure that the conditions are requirements of *apriority* particularly and not of knowledge. Any argument that suggests *a priori* propositions possess such properties should be derived as a consequence of the minimal definition of *apriority*, and not built into the minimal definition via one's theory of knowledge. For these reasons, we will be assigning *apriority* primarily to justification, rather than knowledge, in this chapter.

7.5.4 What is the Modality of the 'Can be Epistemically Justified' in M-APrP')?

In our definition of an *a priori* proposition, M-APrP'), we stated that an *a priori* proposition '*can* be epistemically justified based sufficiently on non-experiential evidence'.

> [With conviction] there remains some reason which might lead us to doubt, but knowledge is conviction based on a reason so strong that it can never be shaken by any stronger reason.

The association of psychological certitude with knowledge is suggested by Descartes's (1984, p. 101) comment that,

> [N]o act of awareness that can be rendered doubtful seems fit to be called knowledge.

[178] If one is in any doubt that this occurs, see Casullo's (2009) criticism of Kitcher's (2000) arguments for his account of *a priori* knowledge. The properties that Kitcher assigns to *a priori* knowledge depend on the beliefs' status as knowledge rather than their status as instances of *apriority*. Thus, the account fails to adequately distinguish *apriority* and *aposteriority*.

What is unclear, however, is the strict modal interpretation of this *can*. To clarify our intended interpretation of 'can' we will introduce a model for *apriority*.

Our model for *apriority* is the quintuple <*W*, *R*, *I*, *E*, *v*>. *W* is our set of possible worlds, *R* is the binary accessibility relation between sets of worlds, *I* is the set of possible individuals, *E* is the set of possible evidence, and *v* is our valuation function. Some properties of *R* and *E* require clarification here.

Our accessibility relation *R* in this instance should reflect *epistemic* possibility rather than metaphysical possibility. We are interested in those possible worlds *w* at which individuals have the same, or similar, cognitive abilities and sensory apparatus. There is little use in possessing a model for *apriority* if it distorts our epistemic situation. To include worlds at which some individuals can be justified in believing the colour of a flower through introspection or know the (in)validity of a logical proof because of distinct types of hunger pangs would distort the use of the epistemic distinction and thus limit the model's utility. For this reason, a world *w'* can only be accessible from a world *w* if *w'* is similar in certain epistemically important respects to *w*. What exactly these epistemically important respects are, and what restrictions we must place on *R* to ensure that it reflects *epistemic* possibility, are questions far beyond the scope of this chapter. However, we are clear that it is *epistemic* possibility that our model is attempting to capture.

With regards to *E*, we are assuming for the sake of our model that evidence consists of propositions of some kind. This ensures that we can identify the distinct members of *E* by propositional content. Whether there are any other conditions propositions must fulfill to be members of *E* isn't a matter we will settle here. We have a good enough understanding of paradigm cases of evidence to be able to model effectively *apriority* without knowing exactly what constitutes evidence. Two further points regarding *E* are necessary.

Firstly, any $e \in E$ may be a conjunction of propositions. We are interested here in evidence that *sufficiently* constitutes justification for a proposition, and sometimes such evidence will be a conjunction of propositions. Specifying in abstract the point at which a set of propositions, understood as evidence, sufficiently constitutes justification for a proposition is a seemingly impossible project to undertake. To ensure that this imprecision doesn't nullify our model's utility, we will idealise and assume that there is a matter of fact as to whether a set of evidence sufficiently constitutes justification for a proposition or not. Given that in modeling *apriority* we are interested in *sufficient* evidence, it is only sufficient evidence that we will model. Therefore, every $e \in E$ is sufficient evidence for some proposition *p*, and sometimes this will require *e* to be conjunction of propositions. We aren't interested in cases of insufficient evidence for justification, and thus we won't include those instances in our model.

Secondly, given that we are interested in two forms of evidence, experiential and non-experiential evidence, we need to denote the type of evidence that an individual i possesses for a proposition p. To achieve this, we will suffix the members of E with a plus or minus superscript: e^{-} denotes *non*-experiential evidence, e^{+} denotes experiential evidence, and e^{-+} denotes a combination of *non*-experiential and experiential evidence. At times we will also use e without a suffix, informing us that there is sufficient evidence for a proposition without committing us to the (non-)experiential nature of that evidence. In other words, it is the disjunction of the three forms of evidence.

With this structure in place, we can formulate the possible interpretations of '*can*' in our minimal definition of *a priori* propositions,

WorldWeak: Some proposition p is *a priori* at w iff for some $w' \in W$, such that wRw', there is some $i \in I$ that possesses some $e^{-} \in E$ for p at w'.

WorldStrong: Some proposition p is *a priori* at w iff for all $w' \in W$, such that wRw', there is some $i \in I$ that possesses some $e^{-} \in E$ for p at w'.

WorldJustified: Some proposition p is *a priori* at w iff for all $w' \in W$, such that wRw', if at w' there is some $i \in I$ that possesses some $e \in E$ for p, then at w' there is some $i \in I$ that possesses some $e^{-} \in E$ for p.

While WorldWeak requires that an *a priori* proposition p at w is sufficiently justified by non-experiential evidence at *some* possible world w' accessible from w, WorldStrong requires that an *a priori* proposition p at w is sufficiently justified by non-experiential evidence at *all* possible worlds accessible from w. In contrast, WorldJustified requires that an *a priori* proposition p at w is sufficiently justified by non-experiential evidence at *every* possible world w' accessible from w at which p is sufficiently justified by some evidence of whatever form.

Fortunately, our choice between the candidates is an easy one. The WorldStrong interpretation places far too strong requirements on *apriority*. Given that there are possible worlds at which the truth of the proposition p simply hasn't been broached yet, the WorldStrong interpretation would ensure that there are no *a priori* propositions. The WorldJustified interpretation removes this concern by ensuring that we are only interested in those possible worlds at which a proposition p is epistemically justified. Yet, if we consider paradigm cases of *a priori* propositions, such as arithmetical propositions, we also find good reasons to believe that the WorldJustified interpretation is too strong. Not only are there possible worlds at which an arithmetical proposition, such as '2 + 2 = 4', can be justifiably believed because of experiential evidence, but it seems there are worlds *w* at which the proposition is justifiably believed because of experiential evidence *and* no one at *w* is justified in believing the proposition because of non-experiential evidence. For example, imagine a world *w* at which there is only one child, and this child is justified in believing the proposition that '2 + 2 = 4' by counting the combination of two collections of two building blocks, or similar objects, on multiple occasions. This is a world at which '2 + 2 = 4' is justified *a posteriori* and not *a priori*. According to the WorldJustified interpretation of *a priori* propositions, this would entail that '2 + 2 = 4', and other simple arithmetical propositions, are not *a priori* propositions. Yet, these arithmetical propositions are paradigm cases of *a priori* propositions. Given that it seems the same point can be made with many other paradigm cases of *a priori* propositions, the WorldJustified interpretation miscategorises too many paradigm *a priori* propositions as not *a priori*. By not placing too strong a condition on *apriority*, and thus not miscategorising paradigm cases of *a priori* propositions, the WorldWeak interpretation is by far the most suitable interpretation of the modal verb in our definition of *a priori* propositions.[179]

7.5.5 The Minimal Definition Revisited

With these clarifications we can restate our minimal definitions of *apriority* and the contrasting definitions of *aposteriority* using our model:

Apriority

M-APrJ") Some $i \in I$ is *a priori* justified in believing a proposition p at a world w iff i possesses some $e^{-} \in E$ for p at w.

M-APrP") Some proposition p is *a priori* at w iff for some $w' \in W$, such that wRw', there is some $i \in I$ that possesses some $e^{-} \in E$ for p at w'.

Aposteriority

M-APoJ") Some $i \in I$ is *a posteriori* justified in believing a proposition p at a world w iff i possesses some $e^{+} \in E$ or $e^{-+} \in E$ for p at w.

[179] The same understanding of an *a priori* proposition, though not necessarily formulated in terms of possible worlds, can be found in Boghossian & Peacocke (2000, p. 2), Kitcher (1980a, p. 4), Kripke (1981, p. 34), Stang (2011, p. 466), and Swinburne (1975, p. 238).

M-APoP") Some proposition p is *a posteriori* at w iff for some $w' \in W$, such that wRw', there is some $i \in I$ that possesses some $e^{+} \in E$ or $e^{-+} \in E$ for p at w', and there is no $w'' \in W$, such that wRw'', at which some $i \in I$ possesses some $e^{-} \in E$ for p.[180]

7.6. The Properties of *Apriority*

Now that we have clarified the minimal definition of *apriority*, we can move on to consider those properties that *a priori* propositions putatively possess in virtue of fulfilling the minimal requirement. That is, those properties that are putatively *necessary* conditions for *apriority*. In concluding whether *apriority* entails some property P, we can deduce whether the *troublesome* propositions of classical logic possess property P in virtue of being *a priori* propositions. If we find that the *troublesome* propositions do possess some property P, the threat that P poses to the absolutist's theory will be dependent on the nature of P.

7.6.1 *Apriority* & Analyticity

The analytic/synthetic distinction is commonly conceived as a distinction between sentences, syntactic entities that can *have* meaning, rather than propositions which *are* meanings, and, particularly, as a distinction based on the *semantic* properties of sentences.[181] As with the *a priori*/*a posteriori* distinction, the analytic/synthetic distinction has had varying meaning and importance throughout the history of philosophy. To give a complete, or even thorough, historical account here of the distinction is impossible.[182] Fortunately, a lack of

[180] In both M-APoJ'') and M-APoP'') we have ignored the case in which a belief is *a posteriori* justified from a combination of experiential evidence and default reasonable beliefs. This is because, given that default reasonable beliefs are not beliefs dependent on evidence, they cannot be captured using E. Thus, to integrate default reasonable beliefs into our model would require the inclusion of another element. Given the complications that this would add to our model, and the unimportance of default reasonable beliefs for our purposes, default reasonable beliefs can be put aside for the time being.

[181] Gillian Russell (2008, pp. 21-22) gives two reasons for considering sentences, rather than non-syntactic truth-bearers such as propositions, to be the objects of the analyticity/syntheticity distinction. Firstly, "analytic truths are supposed to be true *in virtue of meaning*…requiring that it is only things which *have* meanings that are apt to be analytic…[Yet] propositions do not have meanings, rather they *are* the meanings of sentences." Secondly, *prima facie* cases of analytic sentences that include indexicals, such as Kaplan's (1989, pp. 508-509) 'I am here now', lose all appearance of analyticity if we consider the proposition expressed by the sentence; 'James is at Rochester cathedral at five o'clock on the 27th May 2010' has no appearance of analyticity.

[182] To note a few sources: Gasking (1972) discusses nine different definitions of analyticity, Swinburne (1975) discusses three types of account of analyticity, Quinton (1963-1964) compares four descriptions of analyticity, and BonJour (1998, Ch. 2) devotes a chapter to exploring possible accounts of analyticity. Additionally, in a symposium on synthetic *a priori* truths, which includes Broad (1936), Jackson (1936), and Porteous (1936), numerous formulations of analyticity are given, including Porteous's (1936, p. 126) equating analyticity with "possessing the character of (self-)evidence and certainty (incorrigibility)."

comprehensiveness is inconsequential, as we are only concerned with the most plausible interpretations of the distinction. We are interested in properties that *a priori* propositions possess in virtue of fulfilling the conditions in M-APrP"). Hence, if an account of a semantic distinction is implausible, then it's implausible that a proposition may have one of the properties defined by the distinction.[183] There are four accounts of the analyticity/syntheticity distinction that we will consider: the Kantian, Fregean, metaphysical, and epistemological accounts.

The Kantian (2009, B10) account is found in the *Critique*:

> In all judgments in which the relation of a subject to the predicate is thought…this relation is possible in two different ways. Either the predicate *B* belongs to the subject *A* as something that is (covertly) contained in this concept *A*; or *B* lies entirely outside the concept *A*, though to be sure it stands in connection with it. In the first case I call the judgment analytic, in the second synthetic. Analytic judgments (affirmative ones) are thus those in which the connection of the predicate is thought through identity, but those in which this connection is thought without identity are to be called synthetic judgments.[184]

Whatever the historical interest of the Kantian account of analyticity, it has little to no application to logical propositions and thus isn't relevant to our concerns. The account only applies to sentences of a universal subject-predicate form, 'All *A*'s are *B*'s', and as many logical propositions, such as 'Either a proposition or its negation is true' or 'A proposition is either true or false, but never both', do not fit this form, it cannot tell us much about properties that logical propositions possess in virtue of being *a priori* propositions. Indeed, if logical and arithmetical propositions are paradigm cases of *a priori* propositions, then Kant-analyticity cannot be a necessary condition on *a priori* propositions, as many of these propositions cannot be expressed by a Kant-analytic sentence. At most then, Kant-analyticity is a *sufficient* condition on *a priori* propositions, which doesn't admit us to infer that a proposition *p* is expressed by a Kant-analytic sentence *s* from *p* being an *a priori* proposition.

The Fregean (1996, p. 6) account is found in *The Foundations of Arithmetic*:

> If, in carrying out this process [of a proof], we come only on general logical laws and on definitions, then the truth is an analytic one, bearing in mind that we must take account also of all propositions upon which the admissibility of any of the definitions depends.

Given that logical propositions can be proved from themselves, assuming the reflexivity of our consequence relation, Frege's account of analyticity can apply to logical propositions.

[183] One sometimes finds a definition of analytic sentences as 'a sentence known by *a priori* means or which is necessarily true', as in Lewis (1946, p. 57) and Kripke (1981, p. 36). Given that these definitions ensure a correspondence between *apriority* and analyticity by either amalgamating concepts without good reason or stipulation, they aren't of interest to us here.
[184] Cf. Kant (1992, 4:266 & 2002b, §2).

Whether it can apply to all logical propositions, including the meta-logical and structural propositions of a logic, will depend on how broad Frege's conception of logical laws and definitions are. However, irrespective of whether Frege-analyticity can apply to these meta-logical and structural propositions, it's clear that any connection between *apriority* and Frege-analyticity will be rather trivial.

Firstly, for Frege-analyticity to be a necessary condition on *a priori* propositions would require all paradigm cases of *a priori* propositions to be reducible to logical laws or definitions. However, given the failure of the Logicist research programme, the prospect of achieving this, even for arithmetical propositions, seems grim. Thus, the strongest connection one could hope for between *apriority* and Frege-analyticity is that a proposition *p*'s being expressed by a Frege-analytic sentence *s* is *sufficient* for *p*'s being an *a priori* proposition, though not *necessary*.

Secondly, given that the theorems of any logic can be proved within *that* system, there are propositions of all logics that would be Frege-analytic. Thus, demonstrating that a sentence expressing a proposition was Frege-analytic wouldn't show that classical logic had any substantial advantage over other logics. For our purposes here, therefore, Frege-analyticity wouldn't demonstrate anything substantial about the *troublesome* propositions that could hinder the absolutist's thesis.

Thirdly, even if we allowed Frege-analyticity to deflect our second criticism by adapting Frege's definition to "we come only on general [classical] logical laws and on definitions," thereby begging the question against a non-classical logician, the utility of the account for our purposes is questionable. All the account tells us, after all, is that a sentence *s* is analytic if and only if it can be reduced to the general logical laws and definitions. Given that we are interested in some propositions of classical logic, we already know that they can be proved from classical logical laws and definitions. Demonstrating that a logical proposition is Frege-analytic, therefore, doesn't tell us anything beside the fact that the classical consequence relation is reflexive, which we knew already. Frege-analyticity tells us nothing interesting about propositions that are both logical and *a priori*, and thus doesn't constitute a challenge to absolutism.

This leaves us with the *metaphysical* and *epistemological* accounts of analyticity, outlined in Boghossian (1996):[185]

> **Metaphysical Analyticity (M-Analyticity)**: A sentence *s* is M-analytic iff *s*'s meaning that *p* alone entails the truth of the proposition *p*.

[185] Cf. Tappenden (1993) and Williamson (2007, Ch.'s 3 & 4).

> **Epistemological Analyticity (E-Analyticity)**: A sentence *s* is E-analytic iff grasping the meaning of *s* alone is sufficient for an individual to be justified in believing the proposition *p* that *s* expresses.

The respective accounts of syntheticity would be,

> **Metaphysical Syntheticity (M-Syntheticity)**: A sentence *s* is M-synthetic iff *s*'s meaning that *p* does *not* by itself entail the truth of the proposition *p* – other things must be so and so in the world for *p* to be true.

> **Epistemological Syntheticity (E-Syntheticity)**: A sentence *s* is E-synthetic iff grasping the meaning of *s isn't* sufficient for an individual to be justified in believing the proposition *p* that *s* expresses – one must be aware of other facts about the world to be so justified.

Both M-analyticity and E-analyticity make a distinction between kinds of meaningful declarative sentences based on semantic properties that one kind possesses and the other does not. The two accounts of analyticity differ over which semantic properties the distinction picks out. M-analyticity suggests that the distinction picks out those properties that *exhaustively dictate a sentence's truth-value*, such that analytic sentences possess these properties and synthetic sentences don't. E-analyticity suggests that the distinction picks out those properties that ensure *the justification criteria* of the propositions the sentences express are independent of non-meaning facts. Again, analytic sentences possess these properties and synthetic sentences don't. Given that the two accounts differ over the semantic properties of the sentences that the distinction hinges upon, they are bound to have differing relations to *apriority*, unless we find that M-analyticity and E-analyticity are necessarily co-extensive.

As E-analyticity concentrates on the distinction between those sentences *s* that ensure justified belief in the proposition *p* they express, through the grasping of *s*'s meaning alone, and those that don't, E-analyticity could reasonably have a correlation with *apriority*. If a sentence *s* is E-analytic, then the proposition *p* it expresses can be justifiably believed by grasping *s*'s meaning alone. The propositions that E-analytic sentences express, therefore, fulfill the minimal definition of an *a priori* proposition. The propositions would be epistemically justified based on non-experiential evidence. However, there are two reasons to be unimpressed by this result.

Firstly, we have assumed, generously, that there *are* some E-analytic sentences. Yet, there are at least as many dissenters as advocates of E-analyticity.[186] To establish any kind of correlation between *apriority* and E-analyticity would require a demonstration of the existence of E-analytic sentences. Without this demonstration it matters little that we can, assuming the existence of E-analytic sentences, show that a proposition *p*'s being expressed by an E-analytic sentence *s* is *sufficient* to establish that *p* is an *a priori* proposition. Nor can one successfully argue for the existence of E-analytic sentences from the presumption of the existence of *a priori* propositions. Firstly, there are other available candidates to account for the existence of *a priori* justification, such as conceptual knowledge and rationality, and secondly, we could have good reason to believe the evidence we possess for a proposition *p* is non-experiential without knowing the exact source of the justification. If the latter were not true, then every advocate of *a priori* justification would have, and would be expected to have, a fully-fledged theory of how we possess such justification; yet, clearly they don't, and they are not. Thus, we cannot establish the existence of E-analytic sentences (nor conceptual truths or rational insight) by simply appealing to the existence of *a priori* justification. To establish a correlation between E-analyticity and *apriority* we need good reasons to believe that there are E-analytic sentences, and these good reasons cannot be simply that we require an explanatory model for *a priori* justification.

Secondly, even if we possess these reasons to believe that there are E-analytic sentences, this would only show that E-analyticity is a *sufficient* condition on *apriority*, and not a *necessary* condition. Indeed, unless we can rule out the success of all other possible explanatory models for *a priori* justification, the prospects of demonstrating that E-analyticity is a *necessary* condition on *apriority* seem dim. However, given the numerous possible explanatory models of *apriority* this seems an almost impossible task. If one is going to establish that the *troublesome* propositions of classical logic are expressed by E-analytic sentences, it won't be by virtue of their putative *apriority*. Thus, although E-analyticity could reasonably be a *sufficient* condition on *apriority*, which is dependent on whether one can demonstrate the existence of E-analytic sentences, the possibility of establishing that E-analyticity is a *necessary* condition on *apriority* seems hopeless. The absolutist has nothing to fear from the putative relations between E-analyticity and *apriority*.

In contrast to E-analyticity, M-analyticity says nothing about the conditions under which propositions expressed by analytic sentences can be justifiably believed. Instead, M-analyticity picks out those sentences *s* whose meaning that *p* alone entails the truth of *p*. There is good reason to believe M-analyticity isn't a *necessary* condition on *apriority*.

[186] See, for example, Williamson's (2007, ch. 4) and Giaquinto's (2008) arguments against the applicability of E-analyticity to natural language sentences. Giaquinto (2008, pp. 102-108) also questions the applicability of E-analyticity to mathematical and logical definitions, building on Horwich's (2000) discussion of stipulation.

For M-analyticity to be a *necessary* condition on *apriority*, we must require that all *a priori* propositions are expressed by M-analytic sentences, which entails that all *a priori* propositions are expressed by sentences. Yet, this seems far too strong a condition on *a priori* propositions. We certainly don't tend to think that all propositions have been expressed by sentences, so why should we think that all *a priori* propositions have been? Indeed, if the logical truths of classical propositional logic are *a priori* propositions, then we know that not all *a priori* propositions are expressed by sentences, for there are an infinite number of logical truths. Therefore, some *a priori* propositions are not expressed by sentences and, consequently, some *a priori* propositions are not expressed by M-analytic sentences. M-analyticity, thus, isn't a necessary condition on *apriority*.[187] Additionally, given that some of these paradigm *a priori* propositions not expressed by sentences are true, such as some propositions of the form '$A \rightarrow A$' or '$\sim(A \wedge \sim A)$', there are some *a priori* propositions p, at least, whose truth is not solely dictated by the meaning of some sentence s; a consequence of p not being expressed by any sentence s.[188]

We have considered four accounts of analyticity and found that none are *necessary* conditions on *apriority*. Thus, none of the four accounts of analyticity we've considered have demonstrated that *a priori* propositions, *in virtue of* being *a priori* propositions, must be expressed by analytic sentences.

7.6.2 *Apriority* & Necessary Truth

The suggestion that there is some substantial connection between *apriority* and necessary truth goes back at least to Kant (2009, B4):

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[187] A relatively new theory of M-analyticity advanced by Gillian Russell (2008) in her book *Truth in Virtue of Meaning* is worth mentioning here. Russell (2008, pp. 45-46) distinguishes between four senses of the 'meaning' of a word:

Character: The thing speakers must know (perhaps tacitly) to count as understanding an expression.

Content: What the word contributes to what a sentence containing it says (the proposition it expresses).

Reference Determiner: A condition which an object must meet in order to be the referent of, or fall in the extension of, an expression.

Referent/Extension: The (set of) object(s) to which the term applies.

Unlike conventional interpretations of M-analyticity that conceive of analyticity as dependent on *content*, Russell (2008, pp. 52-66) argues instead that it's the *reference determiner* of an expression that's operative in M-analyticity. Russell's theory is both impressive and progressive, and we could not possibly hope to do full justice to it here. What is important to note for our purposes is that Russell's theory of M-analyticity, because it's defined in terms of *reference determiners* rather than *content*, does not even pretend to establish a close connection between M-analyticity and *apriority* (2008, p. 68).

[188] For further criticism of M-analyticity see Boghossian (1996), Sober (2000) and Williamson (2007, Ch. 3).

> Necessity and strict universality are therefore secure indications of an *a priori* cognition.[189]

There are three important interpretations of necessary truth in philosophy: *Logical*, *metaphysical*, and *physical* necessity. All three place different restraints on the propositions that can be true:

Logical:	A proposition p is logically (necessarily) true iff p is of a logical form F that is true under all interpretations.
Metaphysical:	A proposition p is metaphysically (necessarily) true iff p is true at every possible world w.
Physical:	A proposition p is physically necessarily true iff p is true at every possible world w that shares the same laws of nature as the actual world.

While it is physically necessary that the speed of light in a vacuum is 299,792,458 m/s, there are metaphysically possible worlds at which the speed of light in a vacuum is different. Thus, although the proposition 'the speed of light in a vacuum is 299,792,458 m/s' is physically necessarily true, it is *not* metaphysically necessarily true. Similarly, although the proposition that 'Hesperus is Phosphorus' is metaphysically necessarily true, it's not *logically* necessarily true, as the proposition is of the logical form a = b, which isn't true under all interpretations.

[189] Cf. Kant (1996a, 5:53; 1996b, 4:455; 2002a, 8:235; 2002b, 4:268, 4:277 & 4:294; and 2009, A1, B6 & A114). Two clarifications regarding Kant's hypothesised association of *apriority* with necessary truth need to be made. Firstly, Stang (2011) has recently argued that there are three distinct types of necessity noted in Kant's work (*formal*, *empirical* and *noumenal* necessity), and that it's only *formal* necessity which Kant considered to be co-extensive with *apriority*. Secondly, there is an alternative interpretation of Kant's quotes on the connection between *apriority* and necessary truth. This interpretation suggests that Kant maintained not that all justified beliefs in a necessary truth were justified *a priori*, but that any justified belief in the *modal status* (necessity or contingency) of a proposition's truth or falsity must be justified *a priori*. The most forceful evidence for this interpretation is from "On a Discovery":

> [T]hey are cognizable as truths *a priori*, which is completely identical with the proposition: they are cognizable as necessary truths.
> (Kant (2002a) 8:235)

For more on this interpretation of Kant's association of *apriority* and necessity see Stang (2011, pp. 463-465) and Casullo (2003, pp. 90-93 & 185-194).

Both Barnes (2007) and Horvath (2009) have argued persuasively for the conclusion that experiential evidence *cannot* produce knowledge of a proposition's metaphysical *necessity*, whether the proposition is metaphysically necessarily true or false. This leads Barnes (2007, p. 521) to conclude that "if we do have knowledge of necessity, then this gives us strong reason to believe that we have some *a priori* knowledge." While this putative relation between *apriority* and modal knowledge may have some plausibility, we haven't considered it in detail here because at most the property of *informing us of the metaphysical necessity of another proposition* is a *sufficient* condition on *a priori* propositions, and not a necessary condition.

Yet, under almost all interpretations of the three forms of necessity, the inverse isn't true. If a proposition is logically necessarily true, then it is metaphysically necessarily true, and if it is metaphysically necessarily true, then it is physically necessarily true.[190] Thus, using *Ln*, *Mn*, and *Pn*, to denote the set of logically, metaphysically, and physically necessary truths, respectively,

$$Ln \subset Mn \subset Pn.$$ [191]

Our interest now is to consider whether any of these forms of necessary truth are *necessary* conditions on *apriority*. Given that the set of physically necessary truths is a superset of both the set of metaphysically necessary truths and the set of logically necessary truths, if we can establish that there are some *a priori* propositions that are not physically necessary truths, then we will have done the same for metaphysical and logical necessary truth also.

Any suggestion that physical necessary truth is a *necessary* condition on *apriority* can be quickly dismissed. Given that any form of justification is fallible, *a priori* justification is fallible.[192] Thus, it is entirely possible for an individual to be *a priori* justified in believing a proposition *p*, thus ensuring that *p* is an *a priori* proposition, while being totally mistaken about the truth of *p*. Indeed, whenever two parties propose sufficiently strong non-experiential evidence for contradictory propositions, we can be assured that one of the *a priori* justified propositions isn't true.[193] Famous historical examples of this phenomenon include: the debate over whether we can know that we're not a brain in a vat, the disagreement between Leibniz and Clarke over the nature of space, the debate over whether there is a Russell-set, and non-experiential evidence for and against the existence of God (such as the ontological argument and arguments for the incompatibility of the divine attributes). As one of the contradictory propositions in each debate isn't true, at least some *a priori* justified propositions aren't physical necessary truths. Thus, it's possible for an individual to be *a priori* justified in believing

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[190] This isn't necessarily true under Tarski's (1983a) account of logical consequence in terms of satisfaction. If one admits the quantifiers and identity into one's set of logical constants, then propositions of the form $\exists x \exists y(x \neq y)$ will come out as either logically true or logically false in Tarski's account, as they contain no non-logical constants to be substituted for by variables (as detailed in Etchemendy (1990), Ch. 5). Consequently, as there *are* at least two objects at the actual world, propositions of the form $\exists x \exists y(x \neq y)$ come out as logically true in Tarski's account. Yet, 'There are at least two objects' doesn't seem to be a metaphysically necessary truth. It seems that there are metaphysically possible worlds at which there's only one object and, if one accepts Baldwin's (1996) *subtraction* argument, one might also have good reason to believe that there's a metaphysically possible *empty* world. Thus, Tarski's (1983a) account of logical consequence allows for logical truths that aren't metaphysically necessary truths. For a detailed examination of Tarski's theory of consequence see Etchemendy (1988 & 1990)

[191] Given that necessity and possibility are opposites, the inverse is true for sets of possible truths (using *p* for 'possible' in place of *n*),

$$Pp \subset Mp \subset Lp.$$

[192] In this section we are taking it as a given that *a priori* justification is fallible. For arguments that justify this assumption, see section 7.6.3 below.

[193] Unless, that is, we admit the possibility of true contradictions – which we *aren't* in this chapter.

a proposition that isn't a physically necessary truth and, consequently, it's possible for an *a priori* proposition p to not be a physically necessary truth. The fallibility of justification, whether *a priori* or *a posteriori*, always ensures that there can be *a priori* or *a posteriori* propositions that aren't true. Thus, none of the philosophically important interpretations of necessary truth are *necessary* conditions on *a priori* propositions.[194]

7.6.3 *Apriority* & Infallibility/Indefeasibility

There are three other main properties that have been historically considered to have a substantive relationship with *apriority*. These are infallibility, strong indefeasibility, and weak indefeasibility.[195] Given that the three properties have important connections to one another, it's prudent for us to consider them together. We can produce two versions of each property, one of which is a putative *necessary* condition on *a priori* justification (AP-J) and the other is a putative *necessary* condition on justification (of any kind) for an *a priori* proposition (AP-P):

> **AP-J Infallibility**: If some $i \in I$ possesses some $\bar{e} \in E$ for p at some $w \in W$, then p is true at w.
>
> **AP-P Infallibility**: If some $i \in I$ possesses some $e \in E$ for p at some $w \in W$, and there is some $i' \in I$ who possesses some $\bar{e} \in E$ for p at some $w' \in W$, such that wRw', then p is true at w.
>
> **AP-J Strong Indefeasibility**: If some $i \in I$ possesses some $\bar{e} \in E$ for p at some $w \in W$, then there is no $e \in E$ at w that defeats i's evidence $\bar{e}$ for p.

[194] In addition to the general appeal to the fallibility of justification, there may be some additional, more direct, reasons to consider logical and metaphysical necessity to not be necessary conditions on *a priori* propositions. With regards to logical necessity, there are paradigm examples of propositions that can be *a priori* justified, such as 'If a table is red then it is coloured' and 'No two numbers have the same successor', but which are *not* logical truths. Similarly, there are putative cases of metaphysically contingently true proposition that one can be *a priori* justified in believing. The most famous cases are in Kripke's (1981) *Naming and Necessity*, such as the examples of the metre stick (pp. 54-57) and Neptune (pp. 79 fn. 33 & 96 fn. 42), but others include Kaplan's (1989, pp. 508-509) 'I am here' and Kitcher's (1980b, pp. 92-95) 'I exist' and 'I am actual'.

[195] Kant (2009, B3-4) also suggested that judgements which are "thought in strict universality, i.e., in such a way that no exception at all is allowed to be possible," can only be *a priori* justified. However, the *universality* of such judgements (or propositions) is suggested at most to be a *sufficient* condition for their *apriority*, and not a *necessary* condition. Therefore, this putative property of (some) *a priori* propositions is irrelevant to our purposes. For more on the putative relationship between *universality* and *apriority* see Divers (1999).

> **AP-P Strong Indefeasibility**: If some $i \in I$ possesses some $e \in E$ for p at some $w \in W$, and there is some $i' \in I$ who possesses some $e^{-} \in E$ for p at some $w' \in W$, such that wRw', then there is no $e' \in E$ at w that defeats i's evidence e for p.

> **AP-J Weak Indefeasibility**: If some $i \in I$ possesses some $e^{-} \in E$ for p at some $w \in W$, then there is no $e^{+} \in E$ or $e^{-+} \in E$ at w that defeats i's evidence e^{-} for p.

> **AP-P Weak Indefeasibility**: If some $i \in I$ possesses some $e \in E$ for p at some $w \in W$, and there is some $i' \in I$ who possesses some $e^{-} \in E$ for p at some $w' \in W$, such that wRw', then there is no $e^{+} \in E$ or $e^{-+} \in E$ at w that defeats i's evidence e for p.

AP-J infallibility states that every proposition p that is *a priori* justified at a possible world w is true at w, while AP-P infallibility states that every *a priori* proposition that is (*a priori* or *a posteriori*) justified at a possible world w is true at w.

AP-J strong indefeasibility states that every proposition p that is *a priori* justified at a possible world w has no defeating evidence at w, while AP-P strong indefeasibility states that every *a priori* proposition that is (*a priori* or *a posteriori*) justified at a possible world w has no defeating evidence at w.

AP-J weak indefeasibility states that every proposition p that is *a priori* justified at a possible world w has no defeating experiential evidence at w, while AP-P weak indefeasibility states that every *a priori* proposition that is (*a priori* or *a posteriori*) justified at a possible world w has no defeating experiential evidence at w.

It's worth mentioning the pertinent logical relations between the corresponding *a priori* justification (AP-J) and *a priori* proposition (AP-P) versions of the properties. AP-J infallibility only entails AP-P infallibility if one assumes that the propositions under discussion have the same truth-value at all accessible possible worlds. Thus, one could establish that the *a priori* justification for an *a priori* proposition p was AP-J infallible without establishing that any justification for p was AP-infallible, as long as p didn't have the same truth-value at all accessible possible worlds. If AP-J infallibility were established, an *a priori* proposition p would be true at all those worlds w that it was *a priori* justified. Additionally, if p had the same truth-value across all possible worlds, then p would be true at all those words w at which it was epistemically justified. Yet, without the assumption that p has the same truth-value at every possible world, AP-J infallibility allows for the possibility that p is false at some worlds w at which it is *a posteriori* justified. Thus, the justification for an *a priori* proposition *p* could be AJ-P infallible, without being AP-P infallible, as long as *p* doesn't possess the same truth-value at all possible worlds.

Similarly, neither AP-J strong, nor AP-J weak, indefeasibility entail their respective AP-P properties. We can show this by just considering strong indefeasibility, as the result immediately extends to weak indefeasibility. It's possible for AP-J strong indefeasibility to be established without AP-P strong indefeasibility being established. Establishing AP-P strong indefeasibility requires one to show not only that *a priori* justification for an *a priori* proposition *p* at a world *w* is indefeasible at *w*, but that *any* justification for an *a priori* proposition *p* at a world *w* is indefeasible at *w*. Now, given that *a posteriori* justification is a form of justification, AP-P strong indefeasibility requires that *a posteriori* justification for an *a priori* proposition *p* at a world *w* is indefeasible at *w*. As AP-J strong indefeasibility only requires that *a priori* justification, and not *a posteriori* justification, is strongly indefeasible, AP-J strong indefeasibility doesn't entail AP-P strong indefeasibility. Consequently, the same is true for AP-J and AP-P weak indefeasibility.

Let's now consider the plausibility of including either version of the three properties into a definition of *apriority*. Beginning with infallibility, it seems pretty clear that both AP-J and AP-P infallibility are far too strong requirements on *any* justification for *a priori* propositions. Indeed, the only modern philosopher who seems to have taken the infallibility proposal seriously is Kitcher.[196] Condition 2c) of Kitcher's (1980, pp. 8-9) definition of *apriority* entails the infallibility of *a priori* justification (or, to use his preferred terminology, warrant):[197]

1) X knows *a priori* that *p* iff X knows that *p* and X's belief that *p* was produced by a process which is an *a prior* warrant for it.
2) α is an a priori warrant for X's belief that *p* iff α is a process such that, given any life *e*, sufficient for X for *p*, then
    a) some process of the same type could produce in X a belief that *p*,
    b) if a process of the same type were to produce in X a belief that *p* then it would warrant X in believing that *p*,

[196] We won't be including any discussion here of infallibility as a general condition on knowledge, given that it isn't relevant to our purposes. We are concerned solely with properties that *a priori* justification has in virtue of being *a priori* justification, and not in virtue of being a possible instance of knowledge *tout court*.

[197] Cf. Kitcher (1982, p. 219 & 1983, p. 24). Kitcher's account of *a priori* justification is so restrictive that it ensures there's no *a priori* justification of any mathematical proposition. By producing such restrictive criteria for *a priori* justification, Kitcher could be said to be ensuring the absence *a priori* justification by stipulation. It's unsurprising that those who take *a priori* justification seriously admit the fallibility of such justification: see Bealer (1996a, p. 123 & 2000, p. 3), BonJour (1995, pp. 48 & 56), Hale (1987, p. 129), Kripke (1981, p. 39), Peacocke (2000, p. 257), Plantinga (1993, pp. 106 & 109), and Pollock (1974, pp. 320-321).

c) if a process of the same type were to produce in X a belief that *p* then *p*.

To explain the somewhat idiosyncratic language of condition 2), X's "life" is the total sequence of experiences X has had, and a life being "sufficient for X for *p*" equates to that life being sufficient for X to gain the semantic information necessary to entertain the thought that *p*.[198] In other words, this is Kitcher's method of allowing the *enabling* role of experience into his definition.

Now, given that at the actual world there are false propositions for which paradigm instances of sufficient non-experiential evidence have been provided, AP-J infallibility is clearly too strong a condition on the *a priori* justification of an *a priori* proposition. Consider for example a very capable mathematician who constructs a putative proof for a proposition *p*, but the putative proof turns out to be faulty and *p* is, in fact, false. This is a clear case of an *a priori* justification for an *a priori* proposition being fallible, and such cases appear to actually occur.[199] Thus, including AP-J infallibility as a condition on *apriority* would entail that most, if not all, paradigm cases of *a priori* justification were not in fact *a priori*. Given that AP-J infallibility is too strong a condition on the justification for *a priori* proposition, AP-P infallibility is assured to be also. After all, if an *a priori* proposition *p* is false at some world *w* at which it is *a priori* justified, then *p* is also false at some world *w'* at which it is justified (*a priori* or *a posteriori*).

We can also establish that AP-P infallibility is too strong a condition on the justification of an *a priori* proposition *p* by considering solely worlds at which *p* is *a posteriori* justified. Imagine a possible world *w'* at which a talented and esteemed mathematician constructs a putative proof for 589038 + 238768 = 827805, checking her proof several times without finding error. Given that the mathematician has good reason to believe in her ability to construct arithmetical proofs, we would say that she is epistemically justified *a priori* in believing that 589038 + 238768 = 827805. The proposition is, however, false; the possibility of this has already been conceded to us with establishing the inadequacy of AP-J infallibility. Here then we have a false *a priori* proposition.

Now imagine a possible world *w*, which *w'* is accessible from, at which there's an individual who prides himself on being able to count large numbers of building blocks accurately. This individual has no independent mental arithmetical skills, such as being able to

[198] Kitcher (1980, pp. 5-6).
[199] If one's sceptical over the possibility of *faulty formal proofs* transmitting any justification to a proposition, then the same points in this section can be made with informal mathematical arguments, such as Antoine Arnauld's argument against negative numbers:

> Arnauld questioned that -1:1 = 1:-1 because, he said, -1 is less than +1; hence, how could a smaller be to a greater as a greater is to be a smaller?
>
> (Kline (1972) p. 252)

add large integers together. He has, however, in the past been congratulated by very capable mathematicians who assured him that his block counting has produced the correct answer when he counts the two or more collections of blocks separately *and then* counts the two or more collections of blocks continuously (the sum of the first countings equal the latter counting). The counter, therefore, has very good reasons to be confident in his counting skill and believe that his counting is accurate. Thus, on this occasion of counting these two particular collections of blocks, he is rationally justified in believing that if he counts carefully enough and recounts several times, the number of blocks he counts when he counts the two collections separately will sum the number of blocks he counts when he counts the two collections continuously. He proceeds to count the first collection of blocks, and counts 589038 blocks, recounting them another two times to ensure accuracy, and the second collection of blocks, counting 238768, again recounting to ensure accuracy. Both of his countings in this case are accurate, and he is justified independently of this accuracy in believing that the first collection is 589038 in number and the second is 238768. He then proceeds to count the two collections continuously. He counts 827805 blocks, recounting five times and always getting the same result. His counting in this case isn't accurate however, for there are actually 827806 blocks in total. Yet, his rational justification for believing that there are 827805 blocks isn't defeated by this one instance of inaccuracy. He is rationally justified in believing that 589038 + 238769 = 827805 as, firstly, it is the result of his almost universally accurate counting and, secondly, he's been provided with no evidence to believe that his counting is any less accurate on this occasion. Here then we have an instance of *a posteriori* justification for an *a priori* proposition *p* even, though *p* is false. Therefore, AP-P infallibility is too strong a condition to place on the justification of *a priori* propositions. Both *a priori* and *a posteriori* justification for an *a priori* proposition is fallible.

The reason for the inappropriateness of both versions of the infallibility condition is clear. Justification of any kind is fallible. The clear fallibility of *a posteriori* justification already ensures that AP-P infallibility is too strong a condition on the justification for *a priori* propositions. However, *a priori* justification is distinguished as a form of justification by the non-experiential source of the justification, and the nature of any evidence, experiential or non-experiential, is that it can be misleading. Consequently, the nature of any justification is that it can be mistaken.[200]

To reasonably conclude that *a priori* justification differs in both evidential *type* and *strength* from *a posteriori* justification, we would need goods reasons. Yet, firstly, there are no

[200] See Hoffmann (2011) for an interesting discussion of the putative infallibility of *a priori* justification, where it's concluded that, as "even the most diligent exercise of reason is not failsafe, pure reason cannot deliver any variety of infallible justification," (p. 17).

plausible arguments for the infallibility of *a priori* justification available in the literature and,[201] secondly, we have very strong evidence against the infallibility of *a priori* justification in the form of propositions *p* that are both false and paradigm cases of propositions that are, or have been, *a priori* justified. Thus, by including AP-J infallibility as a condition on *a priori* justification, one would likely ensure that there are no cases of *a priori* justification. When even *a priori* apologists argue that *a priori* justification is fallible, including an infallibility condition in a definition of *apriority* that ensures there are no cases of *a priori* justification would be an empty victory. It would disguise a genuinely philosophically interesting distinction between different sources of justification by nullifying the distinction. Recognising that there is both no argument for including AP-J infallibility as a condition on *a priori* justification, and that the inclusion of the condition would nullify a potentially interesting epistemic distinction, should be enough to demonstrate that the condition shouldn't be included in any definition of *apriority*.

The strong indefeasibility conditions seem somewhat more plausible candidates for conditions on *apriority*, given that (in)defeasibility and *apriority* are both epistemic properties.[202] Additionally, some justifications for beliefs are plausibly strongly indefeasible. For example, it's difficult to imagine any evidence that would rationally defeat one's justification for the belief that one is in pain, or that one exists. Indefeasibility, therefore, doesn't seem too strong a property to expect of certain rational justifications. The question is whether *a priori* justification, or all justification for *a priori* propositions, is among these indefeasible justifications.

To show that strong indefeasibility is too strong a condition on *apriority*, we only need to show that paradigm cases of *a priori* justification fail to be strongly indefeasible. This will demonstrate that both AP-J and AP-P strong indefeasibility are implausible conditions on *apriority*. If it's possible for *a priori* justification to be defeated, then it's possible for justification for an *a priori* proposition *tout court* to be defeated.

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[201] The only argument for the infallibility of *a priori* justification available is Kitcher's (1980a, pp. 17-18):

> [I]f a person is entitled to ignore empirical information about the type of world she inhabits then that must be because she has at her disposal a method of arriving at belief which guarantees *true* belief. (This intuition can be defended by pointing out that if a method which could produce false belief were allowed to override experience, then we might be blocked from obtaining knowledge which we might have otherwise gained.).

The argument depends explicitly on the weak indefeasibility of *a priori* justification. However, firstly, we haven't yet established the weak indefeasibility of *a priori* justification and, secondly, even if we did, it seems unlikely we would be willing to infer the infallibility of *a priori* justification from its weak indefeasibility. After all, even if *a priori* justification for a proposition *p* entitles us to ignore experiential evidence, this doesn't preclude the possibility of non-experiential evidence justifying the conclusion that *p* is false. Weak indefeasibility doesn't entail infallibility.

[202] The strong indefeasibility of *a priori* justification is endorsed in Levine (1993, p. 124).

Consider a very able mathematician who constructs a putative proof, with the expected standards of rigour. The mathematician, having checked her working out with enough care, is rationally justified *a priori* in believing the putative proof's conclusion. Yet, this rational justification for the putative proof's conclusion can be defeated in at least two distinct ways. Firstly, an eminent mathematician could convince the able mathematician that the supposed proof contains a mistake, and thus isn't indeed a proof, which, due to the eminence of the mathematician, could reasonably remove the able mathematician's rational justification for the proposition. Secondly, an eminent mathematician could construct a putative proof for the contradictory proposition. Unless the able mathematician is a dialetheist, and thus willing to entertain the possibility that both contradictory propositions are true, she will be reasonable in believing that her supposed proof must be faulty due to the eminence of the mathematician who has putatively proved the contradictory claim, even if she doesn't quite know how. Thus, again, the able mathematician's *a priori* rational justification for a proposition has been defeated, and perhaps replaced by an *a posteriori* rational justification for the contradictory proposition.

Again, mathematical proofs are paradigm cases of *a priori* justification. Showing that paradigm cases of *a priori* justification don't fulfill the condition is good evidence for believing that the condition isn't constitutive of *apriority*. If we want to allow for the possibility of *a priori* justification, and thus ensure that a potentially insightful epistemic distinction isn't nullified, we need to exclude both of the strong indefeasibility conditions from definitions of *apriority*.

This leaves us with weak indefeasibility, which is by far the most discussed addition to the minimal definition of *apriority* in the literature. To present the weak indefeasibility conditions in their most plausible light, we need to make a distinction between two ways in which evidence might defeat justification:[203]

U-defeaters: Evidence that shows we didn't have the justification for a proposition *p* that we thought we had. Such evidence *removes* our justification for *p*.

O-defeaters: Evidence that shows a proposition *p* we have justification for is false. Such evidence does *not* remove the justification we have for *p*, but instead gives a greater degree of justification to a competing proposition *q* that entails *p*'s falsity.

The two counter-examples from our discussion of strong indefeasibility are examples of the two types of defeaters. The case of the eminent mathematician spotting a supposed flaw in the

[203] Cf. Pollock (1974, pp. 42-43), Field (1996, p. 362), and Casullo (2003, pp. 44-48).

able mathematician's proof is an example of a U-defeater, in that it only shows that the able mathematician doesn't have the justification for the conclusion she originally thought, and not that the putative proof's conclusion is false. In contrast, the case of the eminent mathematician who constructs a putative proof for the contradictory proposition is an example of an O-defeater, in that it doesn't remove the justification for the original proposition *p* that the able mathematician gained through the putative proof of *p*. Instead, the justification produced by the eminent mathematician's putative proof for the contradictory of *p*, simply outweighs the justification produced by the able mathematician's original putative proof for *p*.

Field (1996 & 1998) draws this distinction between U-defeaters and O-defeaters with a definition of *apriority* including a weak indefeasibility condition in mind. Field (1996, p. 361) recognises that including a weak indefeasibility condition into a definition of *apriority* that excluded the possibility of U-defeaters, *as well as* O-defeaters, would ensures that definition excluded paradigm cases of *a priori* justification, such as mathematical proofs. For example, we can realise that we've made a mistake in our proofing due to sensory experience, and thus realise that we didn't have the *a priori* justification we thought we did for the putative proof's conclusion. For this reason, Field only excludes the possibility of experiential O-defeaters with his weak indefeasibility condition..

In our discussion of weak indefeasibility we will follow Field, and only exclude the possibility of experiential O-defeaters with our weak indefeasibility conditions. To require *a priori* justification to be immune to experiential U-defeaters would be far too restrictive, given that paradigm cases of *a priori* justification *can* be U-defeated by experiential evidence. Again, we don't want to add conditions to our minimal definition of *apriority* that would ensure there's no *a priori* justification, and, consequently, no *a priori* propositions. With this restriction placed on weak indefeasibility, we can reformulate AP-J and AP-P weak indefeasibility:

> **AP-J Weak Indefeasibility^{O}**: If some $i \in I$ possesses some $e^{-} \in E$ for p at some $w \in W$, then there is no $e^{+} \in E$ or $e^{-+} \in E$ at w that *O*-defeats *i*'s evidence e^{-} for *p*.
>
> **AP-P Weak Indefeasibility^{O}**: If some $i \in I$ possesses some $e \in E$ for p at some $w \in W$, and there is some $i' \in I$ who possesses some $e^{-} \in E$ for p at some $w' \in W$, such that wRw', then there is no $e^{+} \in E$ or $e^{-+} \in E$ at w that *O*-defeats *i*'s evidence e for *p*.

Justification for a proposition, therefore, can be weakly indefeasible^{O}, while still being susceptible to experiential U-defeaters.

The inappropriateness of including AP-P weak indefeasibility^{O} into any definition of the justification for *a priori* propositions is pretty clear. The condition requires that *any* justification for an *a priori* proposition cannot be O-defeated by experiential evidence. Yet, there are possible worlds at which *a priori* propositions are justified by *experiential* evidence, which is to say they are justified *a posteriori*, and no one doubts that experiential evidence can O-defeat *a posteriori* justification. Therefore, including AP-P weak indefeasibility^{O} as a condition on the justification for *a priori* propositions would ensure that many, if not all, paradigm cases of *a priori* propositions turned out not to be *a priori*. If any version of weak indefeasibility^{O} should be included in the definition of *apriority*, it must be AP-J weak indefeasibility^{O}.

There are at least two *prima facie* plausible reasons for introducing AP-J weak indefeasibility^{O} into the minimal definition of *apriority*. The first is that AP-J weak indefeasibility^{O} is entailed by the epistemic principle that evidence for a proposition *p* can only be O-defeated by evidence of the same epistemic type. The second is that the condition is entailed by the minimal definition of *apriority*. We will consider each in turn.

The first argument for including AP-J weak indefeasibility^{O} in the definition of *apriority* is the epistemic principle that contradictory propositions must possess a *parity of evidence types* when one of the pair is justified *a priori*. Thus, if one member of a contradictory pair *p* is justified *a priori*, then the other member *q* can *only* be justified *a priori*. Call this principle the *evidential-type parity* principle. The *evidential-type parity* principle ensures that experiential evidence cannot O-defeat *a priori* justification, as to O-defeat the justification for a proposition *p* would be to show that the contradictory proposition *q* has a greater degree of justification, and the *evidential-type parity* principle ensures that contradictories can *only* be justified by the same type of evidence.

Given that there are only two types of evidence, *a priori* and *a posteriori* evidence, evidential-type parity between contradictories with regards to *a priori* justification entails evidential-type parity between contradictories with regards to *a posteriori* justification. If we require that for every contradictory pair {*p*, *q*}, if one of the pair *p* is *a priori* justified then *q* can only be *a priori* justified, then we must require the same of *a posteriori* justification. If we allow for one of the contradictory pair *p* to be justified *a posteriori* and the other *q* to be justified *a priori*, then we will have contravened our original principle that contradictories have an evidential-type parity with regards to *a priori* justification. This is a simple consequence of there being only two relevant categories of evidence.

Thus, if one endorses an *evidential-type parity* principle for propositions that are justified *a priori*, one must also endorse an *evidential-type parity* principle for propositions that are justified *a posteriori*. This conclusion is based not only on the consideration that to restrict the *evidential-type parity* principle to cases of *a priori* justification would seem somewhat *ad hoc*, but that an *evidential-type parity* principle for propositions justified *a priori* actually *entails* the same principle for propositions justified *a posteriori*. Therefore, to establish AP-J weak indefeasibility^{O} as a condition on *apriority* through evidential-type parity, the credibility of a general epistemic principle must be established:

ETP Principle: If one member *p* of a contradictory pair {*p*, *q*} is epistemically justified by evidence of type *E*, then the other proposition of the contradictory pair *q*, if it is justified, can only be justified by evidence of type *E*.

Given that *a priori* and *a posteriori* evidence exhaust the possible types of evidence, *E* must either stand for *a priori* or *a posteriori* evidence. The principle entails that only evidence of the same epistemic type can O-defeat evidence for a proposition, as one O-defeats the justification for a proposition by showing that one is *more* justified in believing its contradictory.

Unfortunately for the advocate of AP-J weak indefeasibility^{O}, the ETP principle fails. There are numerous plausible cases of *a posteriori* justifications for propositions being O-defeated by non-experiential evidence. For example, an arithmetical proposition *p* could be justified by counting, but then this justification for *p* could be O-defeated by a proof for its contradictory. Thus, it's possible for one member of a contradictory pair to be justified *a posteriori*, and for the other to be justified *a priori*. The ETP principle has very clear counter-examples and, therefore, it provides no justification for the weak indefeasibility^{O} of *a priori* justification.

Consequently, the advocate of the weak indefeasibility^{O} condition must pin her hopes on the second argument, which relies upon the intuition that the weak indefeasibility^{O} of *a priori* justification is entailed by the minimal definition of *apriority*. The argument is Kitcher's (1983, pp. 88-89):[204]

> To make this concession [that *a priori* justification can be O-defeated by experiential evidence] is to abandon the fundamental idea that *a priori* knowledge is knowledge which is independent of experience. The *apriorist* would be saying that one can know *a priori* that *p* in a particular way, even though, given appropriate experiences, one would not be able to know that *p* in the same way. But if alternative experiences could undermine one's knowledge then there are features of one's current experience which are relevant to the knowledge, namely those features whose *absence* would change the current experience into the subversive experience. The idea of the support lent by kindly experience is the obverse of the idea of the defeat brought by uncooperative experience.

[204] Cf. Kitcher (1982), p. 222. Kitcher (2000, p. 73) no longer endorses the argument, although he does provide further, rather weak, arguments for the weak indefeasibility condition (pp. 74-80). For an insightful discussion of Kitcher's new arguments for the weak indefeasibility condition see Casullo (2009).

Given that *a priori* justification is supposed to be justification based on evidence that is *independent of experience*, to allow for O-defeating experiential evidence would ensure that, *contra* our minimal definition M-APrJ'), *a priori* justification does rely on some experiential evidence. Notably, the experiential evidence that there is an *absence of O-defeating experiential evidence*. Therefore, according to Kitcher's argument, the weak indefeasibility^{O} of *a priori* justification is entailed by the minimal definition of such justification.

The argument has been criticised by *modest apriorists*, who reject the inclusion of the weak indefeasibility^{O} condition in the definition of *apriority*.[205] These criticisms commonly claim that Kitcher fails to appreciate the distinct logical and causal roles that experience plays in *supporting* a proposition, by adding to the proposition's justification, and in *defeating* a proposition, by adding to the contradictory proposition's justification.[206]

The debate is complex and unsettled. Its resolution requires both an understanding of what constitutes evidence for and against a proposition, and how a type of evidence E may constitute evidence against a proposition p, but not for p (and similarly for the inverse). These are questions that are far beyond the scope of this thesis. Fortunately, for two reasons, they are also irrelevant to our purposes.

Firstly, our concern in this chapter has been to establish whether the putative *a priori* status of the *troublesome* propositions of classical logic is theoretically damaging to the absolutist's project. To this end, we have considered certain philosophically interesting properties that were plausibly *necessary* conditions on a proposition p's being justifiable *a priori*. While we haven't been able to establish the implausibility of Kitcher's argument for the AP-J weak indefeasibility^{O} condition, we can be pretty sure that a definition of *a priori* justification including this condition wouldn't be that damaging to the absolutist's thesis. The joint *apriority* of the *troublesome* propositions and the weak indefeasibility^{O} of *a priori* justification would only entail that, if some individual i was *a priori* justified in believing the *troublesome* propositions at a world w, the absolutist wouldn't be able to O-defeat *with experiential evidence* the justification for the *troublesome* propositions at w. It wouldn't entail that the *troublesome* propositions were necessarily true, or that they were indefeasible^{O}, only that they were *experientially* indefeasible^{O} at any world w at which they were justified *a priori*. Thus, the absolutist could still O-defeat the *a priori* justification for the *troublesome* propositions with non-experiential evidence, thereby justifying *a priori* their own theory's propositions. In the end, the inclusion of weak

205 Campos (2003, p. 706), Casullo (2003, Ch. 2), Edidin (1984, pp. 196-197), Giaquinto (1996, pp. 260-262), Manfredi (2000), Markie (2000), Peacocke (1993, p. 193), Plantinga (1993, pp. 112-113), and Rey (1998, pp. 27 & 32).

206 Casullo (1988, pp. 195-200) and Summerfield (1991, pp. 41-50). However, not all modest apriorists agree on how to respond to the possibility of introducing a weak indefeasibility^{O} condition; see Casullo's (2003, pp. 75-77) criticism of Summerfield's response. For a thoroughgoing critique of the weak indefeasibility^{O} condition, see Casullo (2003, pp. 39-48).

indefeasibility^{O} as a condition on *a priori* justification wouldn't be that damaging to the absolutist's theory, if at all.

Secondly, rather than taking a stand on the validity of Kitcher's argument for the weak indefeasibility^{O} of *a priori* justification, we can argue directly that logical propositions are *not* AP-J weakly indefeasible^{O}, by considering the empirical revisability of logic. Establishing that logic can be rationally revised based on empirical evidence would have at least two consequences for absolutism. Firstly, it would show that at least some logical propositions are weakly indefeasible^{O}, which would give us reason to believe that either logical propositions are *not* justifiable *a priori*, or that AP-J weak indefeasibility^{O} isn't a condition on *apriority*. Secondly, it would offer us an interesting potential route to establishing whether it's possible for contradictions to be true or not. Thus, establishing that we have reason to believe that the *troublesome* propositions of classical logic are not AP-J weakly indefeasible, by demonstrating that logic is empirically revisable, has the bonus of producing other appealing results relevant to this thesis's goals; a virtue that a comprehensive response to Kitcher's argument, in all probability, wouldn't possess.

7.7 The Choice: Minimal and Strong *Apriority*

We have formulated a minimal definition of both *a priori* justification and propositions, and considered whether there are any other philosophically interesting properties that an *a priori* proposition possesses by virtue of being *a priori*. We have considered the properties of being expressed by an analytic sentence, necessary truth, infallibility, and both strong and weak indefeasibility, and found that only one version of one of these properties, AP-J weak indefeasibility^{O}, may be a necessary condition on *apriority*. Due to the complex issues surrounding Kitcher's argument for the inclusion of AP-J weak indefeasibility^{O} in the definition of *a priori* justification, we have suspended judgement over the success of that argument and, consequently, over whether the condition should be included within the definitions. This leaves us with two possible definitions of *a priori* justification and propositions, the minimal and the strong:

Min-APrJ) Some $i \in I$ is *a priori* justified in believing some p at some $w \in W$ iff i possesses some $e^{-} \in E$ for p at w.

Str-APrJ) Some $i \in I$ is *a priori* justified in believing some p at some $w \in W$ iff i possesses some $e^{-} \in E$ for p at w and there is no $e^{+} \in E$ or $e^{-+} \in E$ that *O*-defeats *i*'s evidence e^{-} for p at w.

Min-APrP) Some p is *a priori* at some $w \in W$ iff for some $w' \in W$, such that wRw', there is some $i \in I$ that possesses some $e^- \in E$ for p at w'.

Str-APrP) Some p is *a priori* at some $w \in W$ iff for some $w' \in W$, such that wRw', there is some $i \in I$ that possesses some $e^- \in E$ for p at w', and there is no $w'' \in W$, such that wRw'', at which some $i' \in I$ possesses some $e^- \in E$ for p and some $e^+ \in E$ or $e^{-+} \in E$ O-defeats i'('s) evidence e^- for p at w''.

In the next chapter we argue that logical propositions are empirically revisable, which will give us good reason to believe that the *troublesome* propositions of classical logic are not AP-J weakly indefeasible^O.[207] This will allow us to conclude that whichever definition of *a priori* propositions, the minimal or the strong, is correct, the putative *apriority* of the *troublesome* propositions has no impact upon the absolutist's theory.[208] It may also provide us with a potential route to establishing the (im)possibility of true contradictions.

[207] Although we won't argue directly for the empirical defeasibility^O of the *troublesome* propositions in the next chapter, by establishing that logic is empirically revisable, and thus that *some* logical propositions are empirically defeasible^O, we create a defeasible reason to believe that the *troublesome* propositions are empirically defeasible^O. Perhaps some logical propositions are empirically defeasible^O and some aren't, and perhaps the *troublesome* propositions are examples of the latter. However, establishing that some logical propositions are empirically defeasible^O puts the onus on those proposing the empirical indefeasibility^O of the *troublesome* propositions to demonstrate what makes these *troublesome* propositions relevantly different from the empirically defeasible^O logical propositions.

[208] Either Min-APrP) is the correct definition of *a priori* propositions, in which case,

Min) The putative *apriority* of the *troublesome* propositions has no impact upon the absolutist's theory,

or, Str-APrP) is the correct definition of *a priori* propositions, in which case,

Str) Given that logical propositions are empirically defeasible, the *troublesome* propositions are not *a priori* propositions.

By establishing that either Min) or Str) is true, we can demonstrate that the putative *apriority* of the *troublesome* propositions is irrelevant to the plausibility of the absolutist's thesis.

8. Can Empirical Evidence Justify a Rational Revision of Logic?

> [There has] been very little attempt made to establish the thesis of the revisability, on empirical grounds, of logical laws by demonstrating, in any given case, that a revision of our logic on such grounds is both meritorious and properly described as occurring in response to experience.
>
> (Dummett (1976) p. 46)

8.1 Revising Logic?

The question of whether it can ever be rational to revise logic based on empirical evidence is steeped in ambiguity. What is it to *revise* logic? To this there seem to be three possible answers, two of which leave the topic without any interest at all. The first is that we could be revising the logical *facts*. That is, we could be changing what actually follows from what. Brief reflection on the analogous case of the facts of nature reveals, however, that this cannot be what one means by a revision of logic. We cannot revise the logical *facts*, just as we cannot revise the facts of nature, whether by empirical or non-empirical evidence. Facts simply are. Just as the laws of nature would hold regardless of whether anyone formulated them in a scientific theory, so the logical facts of what follows from what would persist without any theory of logic. These facts are beyond us, and we have no capability to revise them.

Our second option is revising the *rules of inference* that we infer according to, with inferring according to a rule being distinct from accepting a proposition expressing the rule. For example, one may infer by the inference rule *Modus Ponens* without having accepted the proposition expressing this rule ('If *A* and if *A* then *B*, then infer *B*'). The import of this distinction between *inferring according to a rule* and *accepting the propositional expression of the rule* is recognised in the Lewis Carroll problem, in which the Tortoise always requires the acceptance of a new conditional statement before he will admit that the conclusion follows from the premises.[209] By admitting that the practice of inference isn't the acceptance of propositions expressing rules of inference as additional premises, but rather *inferring according to rules*, the problem dissolves, for there are no further conditional statements to accept.

Admitting the importance of *rules of inference* as distinct from propositions, however, doesn't ensure that this interpretation of 'logic' is well fitted to our question. Can we really be interested in whether it's ever rational to revise the *rules of inference* one infers according to, based on empirical evidence? Meaning, I take it, that one changes in a systematic fashion the way one infers because of empirical evidence. It doesn't seem so. Firstly, by interpreting the question in this fashion, we ensure that the potential empirical (un)revisability of logic is

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[209] Carroll (1895). Cf. Smiley (1995).

unrelated to the historical distinction between areas of enquiry that admit revision justified by empirical evidence, and those that don't, contrary to the appearance that the question is embedded within this distinction. If the empirical sciences are paradigm examples of areas of study that admit revision justified by empirical evidence, and mathematics, let's presume, is an area of study that does not, it's hard to see where logic, interpreted as *rules of inference*, fits into this distinction. The empirical sciences and mathematics have nothing analogous to the *rules of inference* to refer to when we say that the empirical sciences admit revision based on empirical evidence and mathematics does not.[210] What we seem to be talking about in these cases are empirical scientific *theories* and mathematical *theories*. When we say that empirical science admits revisions based on empirical evidence, we mean that an empirical scientific *theory* may be revised by some propositions being excluded from the theory and others replacing them, and that this revision of the theory is justified by empirical evidence. By interpreting logic in this context as the *rules of inference* one infers according to, rather than as *theory*, we ensure that the empirical revisability of logic isn't relevant to the distinction between types of theory that admit revisions based on empirical evidence and those that don't.

While there is still the possibility of proposing that interpreting logic as *rules of inference* doesn't diminish the importance of our question, although it does separate the question of the empirical revisability of logic from that of the empirical sciences and mathematics, this response to our first criticism can be answered by a second. It just seems blatant that the *rules of inference* one infers according to can be rationally revised in a systematic fashion due to empirical evidence. For example, one may infer according to the rule of inference:

MH) When one is given n number of options O, and each option has the initial probability $\frac{1}{n}$ of being correct, if one then chooses an option O_1 and another party, who is aware of the correct option, offers one the opportunity to change one's decision while removing an *incorrect* option O_2 from one's possible choices, each remaining option O still has the probability $\frac{1}{n}$ of being correct and, therefore, it's as equally rational to stick with the initial choice of O_1 as it is to change one's option.

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[210] There are rules of inference in mathematics, such as the rule 'To obtain the successor of a natural number n calculate the sum of $n + 1$' in number theory. However, these rules of inference clearly don't cover all the putative examples of the empirical indefeasibility of mathematics. When we claim that mathematics is empirically indefeasible, we are also talking about the empirical indefeasibility of propositions that are clearly not rules of inference, such as 'Every natural number has a successor'.

Inferring according to this rule is at the root of the Monty Hall problem. With such a rule, however, it's perfectly possible to rationally revise one's inferential practices given empirical evidence. Concentrating on the simplest possible case, where an individual has three options: by running two separate series of the game, such that in one series one always switches one's option, and in the other series one always sticks with one's original choice, one comes to recognise that the probability of the options being correct *does* change. One should always switch and, therefore, one shouldn't infer according to MH). One has become justified in changing the way one infers in a systematic fashion. This running of the series of the game mentioned, however, is empirical evidence; whether it's running a computer model of the scenario, or actually playing the game a statistically significant number of occasions. Thus, the fact that one can have a justified solution to the Monty Hall problem through empirical evidence shows that it can be rational to revise the rules of inference one infers according to due to empirical evidence. Therefore, if we are interested in the question of whether it's possible for empirical evidence to justify a rational revision of the rules of inference one infers according to, our question has a simple answer – yes. Not only this, but, as we remarked, it places out of context the debate over the putative empirical indefeasibility of logic, which is embedded within the wider debate of whether historically *a priori* areas of research such as mathematics and logic are empirically defeasible.

We are left then with the third and final interpretation of the question, which interprets logic as logical *theory*. This interpretation brings the question back into line with our similar concerns regarding the empirical defeasibility of mathematical and empirical theories. Our question becomes the more challenging; *can a rational revision of logical theory ever be justified by empirical evidence*?

Even here ambiguity stifles us somewhat. Not necessarily over what a logical *theory* is, because we know on most occasions when we come across one, even if we can disagree on the necessary or sufficient conditions a theory must fulfill to be considered logical.[211] Instead, the ambiguity relates to whether we are interested in *all* or only *certain* revisions of a logical theory. Logical theories as mathematical structures can have many different purposes, both as being worthy of study in themselves or as having application to modeling different phenomena. In regard to this second purpose, there are undoubtedly cases where the phenomena one is attempting to model will influence the logic one chooses. Consequently, if the collection of data regarding the phenomena is achieved empirically then it seems beyond doubt that, given the function of the logic, it's totally rational to revise the logical theory based on empirical evidence. For example, if one is attempting to construct a logical theory that models the

[211] Again, for an account of logical theories that has merit see Resnik's (1996, p. 491) definition of logical theories as a quadruple. We have the same qualm here with Resnik's definition as in the previous chapter – logics don't need to contain a translation method for formalising informal arguments. That some do distinguishes them from others in their potential function, but that's all.

inferences scientists make when constructing theories, then new compelling data collected on how scientists infer, when constructing theories, is bound to sometimes justify a rational revision of one's logical theory if it conflicts with the theory. If our question is to have import, it cannot be that we are interested in whether it's possible for empirical evidence to justify a rational revision of logical theory including *these* cases, for then the answer would be yes. Instead, we must be interested in the rational revision of logical theory when that logical theory is being used for *particular purposes*.

It seems difficult to highlight one purpose of a logical theory and feel confident that it is *this* purpose that has led some in the philosophical community to believe that it's always irrational to revise our logic based on empirical evidence. However, there are two reasons to be relatively confident that it is logical theories attempting to model *what validly follows from what*, in other words logical consequence, that are pertinent to the debate over the empirical revisability of logic. Firstly, historically the debate over the empirical revisability of logic has been about logics as theories of truth-preservation. For example, Putnam's (1975b) insistence that empirical evidence from quantum mechanics gives us reason to believe that the distributive laws are not truth-preserving is a challenge to the classical theory of what validly follows from what. Secondly, the use of logics as theories of truth-preservation has been their primary use for the majority of the history of their study, which increases the probability that it is logic as theories of truth-preservation which is pertinent to the empirical revisability of logic.[212] Thus, our question of import is whether empirical evidence can ever justify a revision of logical theories that are attempting to model *truth-preservation*.[213]

Given that our concern is with the revision of logical theories that attempt to model *truth-preservation*, we must ensure that the revisions of these theories we are interested in are also of the right type. We are not interested in, for example, merely cosmetic changes to a logical theory. It must be revisions relevant to the question of which inferences are truth-preserving. Therefore, the revisions must be either the acceptance/rejection of a theorem, inference, or

[212] See, for example:

"[Logic's] central problem is the classification of arguments, so that all of those that are bad are thrown into one division, and those which are good into another," (Peirce (1902) pp. 20-21).

"The crucial notion [in logic], ultimately, is that of one sentence following logically from others," (Etchemendy (1990) p. 11).

"Logic is about consequence. Logical consequence is the heart of logic," (Beall & Restall (2006) p. 3).

[213] A third reason, particular to this thesis, for being concerned with the empirical revisability of logics modeling *truth-preservation* is that the disagreement between the absolutist/dialetheist and the classical logician is partly over logical consequence. Therefore, considering the empirical revisability of logics modeling logic consequence is the most relevant to this thesis's concerns.

metalogical principle (such as changing from a bivalent to a many-valued logic) that was previously rejected/accepted.[214]

The ambiguity of the given question hasn't only affected us mere mortals. Putnam's presentation of the question in 'The Logic of Quantum Mechanics' is full of ambiguity. At the beginning of the article Putnam (1975b, p. 174) asks, "Could some of the 'necessary truths' of logic ever turn out to be false *for empirical reasons*?" At least three points of potential confusion are present here, two of which are related to our previous conversation. Firstly, given Putnam's reference to the *putative* necessary truths of logic, implied by his mentioning, rather than using, the phrase 'necessary truths', it's clear that what's at stake are not the *facts* of logic but logical theory. The putative necessary truths of logic are those endorsed by a particular logical theory as necessarily true. It makes no sense to say of the logical *facts* that they turn out false. Thus, the issue is whether a logical theory accurately reflects the logical facts (if there are any such facts). Secondly, from Putnam's question it's implied that he is only concerned with cases of *putative* logical truths turning out false, but there's no need for this restriction. If we are interested in the revision of a logical theory, then this revision could as well come from a putative logical falsehood being considered neither logically false nor logically true, or an invalid form of inference being counted as valid (or the inverse). Simply concentrating on possible cases of putative necessary truths of logic turning out false is unnecessarily narrow. Finally, Putnam talks of a putative necessary truth turning out false "*for empirical reasons*". Talk of empirical *reasons* making a proposition false is odd. Perhaps it makes sense to speak of empirical *facts* making a proposition false (leaving aside for the moment what makes a fact *empirical*), but *reasons* don't make a proposition false (or true) unless the proposition is *about* reasons. Reasons constitute motivation for a belief or action, and don't (normally) alter a proposition's truth-value. Thus, to make sense of Putnam's question, we need to not say that the putative necessary truths *turn out false* for empirical *reasons*, but that we have reason to believe the putative necessary truths are false due to empirical reasons. Which is just to say that we have good reason to revise our logical theory based on empirical evidence. Our interpretation of the question that we have built in this chapter, therefore, is not only satisfactory due to the other interpretations being *un*satisfactory, but because it accords well with the most plausible interpretation of what Putnam was attempting to establish in his ground breaking paper.

Now that we are clearer on how our question should be interpreted in order to have substance, we are also clearer on how to seek a reasoned answer. To answer the question

[214] One could undoubtedly also be interested in whether empirical evidence could rationally justify an *extension* of a logical theory. However, our concern here is solely with the revisions of logical theories noted. We haven't offered a full theory of logical rivalry here as, firstly, there are already some fine explications of logical rivalry in the literature, see Haack (1996, pp. 1-7) and Aberdein & Read (2009), and secondly, adding to these explications would require committing more time to the topic than we can spare.

affirmatively, we need to have good reason to believe that empirical evidence can justify a revision of logical theory that includes a revision of what validly follows from what.

This reason could come from three sources. Firstly, one could advance a theory of epistemology, as Quine (1951) attempted, establishing that *all* propositions, and thus all logical propositions, are empirically defeasible. Secondly, one could argue by analogy, using a case of empirical defeasibility from an area of research putatively similar to logic. By contending that there's a case of empirical defeasibility in *this* area of research and *this* area of research doesn't differ in any relevant epistemological way from logic, one could argue that some logical propositions are also empirically defeasible. Using this method, Łukasiewicz (1967) argued for the defeasibility of classical logic via the existence of non-Euclidean geometries. Lastly, one could give direct examples in which it seems perfectly rational for some logical propositions to be revised based on empirical evidence. Currently, the only serious proposal of this kind is Putnam's. We will attempt in this chapter to add to these putative direct examples by taking the latter option, thereby meeting Dummett's complaint.

We will present two possible cases of empirical evidence justifying a revision of logical theory. The first is drawn from the theory choice between traditional syllogistic and classical first-order logics, concentrating on the disagreement over the existential import of universal conditions. The second is an adaptation of the first theory choice, with a disagreement between first-order classical logic and a new logic, first-order partial logic, over the existential import of the existential quantifier. In considering these cases it's important to note that they are not intended to reflect any historical rational revision of logic, only persuasive *possible* cases of empirical evidence justifying a rational revision of logic. This, after all, is all we require to show that there's good reason to believe that empirical evidence *can* justify a rational revision of logical theory (suitably interpreted).

8.2 Case One: Traditional Syllogistic Logic vs. Classical Logic

Traditional syllogistic logic came under attack in the twentieth-century for categorising as valid certain putatively invalid inferences. Particular cases of such putatively invalid reasoning were those syllogisms that allowed one to validly infer properties of a particular from only generalised premises.[215] For example, the syllogistic mood *Darapti*,[216]

All *B*'s are *C*'s,
All *B*'s are *A*'s,

[215] This criticism of traditional syllogistic logic originated in Russell (2010b, Ch. 5).
[216] In standard syllogistic notation, with the double-turnstile ('⊨') as a metalogical symbol for 'validly entails':

$$BaC, BaA \vDash AiC.$$

Therefore, some *A*'s are *C*s,

allows one to validly infer the existence of particulars with a particular property from the truth of two universal conditions ('All *A*'s are *B*'s'). In classical logic, the inference contained in *Darapti* is invalid because it produces arguments with true premises and false conclusions in cases where the property picked out by the conditionals' antecedents is uninstantiated, given that the two universal conditions are interpreted as conditionals in first-order classical logic.[217] The two logical theories, therefore, disagree over whether one can validly infer conclusions regarding particular properties that particulars possess from solely generalised premises. According to traditional syllogistic logic *Darapti* is a valid form of inference, as any argument fitting the schema is truth-preserving, whereas in first-order classical logic it's invalid.

In our example of logical revision here, we will concentrate solely on the tension between traditional syllogistic logic and classical logic, created by those putatively valid syllogistic moods that permit existential import from solely universal conditions. Nothing about the case hangs on this, except that it aids in clarity and economy of description. If needed, the considerations in the example could be expanded and accommodated into the overall competition between the relevant research programmes. This would be achieved by either giving an explanation of why it could be reasonable for a logician to allow her choice of logic to hang on the position she takes on this particular disagreement, or by admitting that other considerations were prevalent in her choice, but that her choice wouldn't have been as reasonable without her consideration of the conflict based on existential import. In both cases, it would still be reasonable to say that empirical evidence justified the rational revision of the logical theory. Anyhow, we don't intend with our example to suggest that this disagreement is the only tension between traditional syllogistic logic and classical logic, nor that it's the only aspect of the tension that an individual should concentrate on when making an informed decision between the logics. In giving an account of a logical revision, and particularly when our interest isn't with what historically occurred, but instead *could* occur, we are bound to emphasise conflicts pertinent to our argument at the cost of other interesting conflicts between the theories.

For this case, we are assuming that traditional syllogistic and first-order classical logic are the only two logical theories of truth-preservation available to choose from, whether this be due to a lack of the necessary formal machinery to construct other formal systems or a lack of philosophical insight into the need, or possibility, of logics with differing semantics. This assumption, again, isn't necessary for the argument to go through, but it makes the case

[217] There are further syllogistic moods, such as *Camestres* (AaB, $AeC \vDash BeC$) and *Felapton* (AeC, $BaC \vDash AoB$), which suffer from the same problem of (putatively unwarranted) existential import. Note that we are only interested in *traditional* syllogistic logic here, namely the syllogistic logic invented by Aristotle and added to by Scholastic commentators, and not modern syllogistic logics.

simpler to describe. Including in our example other postulated logics that differed, or agreed, with traditional syllogistic and classical logic on the subject of the existential import of universal conditions would merely facilitate a choice of more logics. It wouldn't block the possibility of empirical evidence justifying a rational revision of a logical theory.

Now, imagine a logician who is interested in using formal logic as a reliable method of assessing natural language arguments. She is, in other words, interested in using formal logic as a method to assess the truth-preservation of natural language arguments. The logic she wishes to choose is a logical system that models truth-preservation. The logician is aware that, given the role she intends for logic, she must be sensitive to the way individuals use grammatical devices, such as conditionals, negation, disjunction, etc. To ignore the way that these grammatical devices are commonly used would majorly increase the probability of her, and other logicians, mistranslating natural language arguments into formal arguments, and thus inaccurately representing the arguments as valid or invalid. In the logician's view, this would lead to the function of the logical theory she has in mind not being fulfilled, which isn't acceptable. If we are going to accurately assess natural language arguments using formal logic, then it must be a fundamental tenet of our theory that the truth-conditions of the natural language sentences are preserved in the symbolisation process. The semantics of our formal logic must match the pertinent semantic properties of the relevant natural language grammatical particles.

The logician doesn't assume either that she needs to consider what all members of the linguistic community believe is the correct use of a grammatical device, or that she needs to consider what members of the community consider to be *valid arguments* to be able to construct a plausible logical system. On the first count, our logician understands that there are always going to be members of the linguistic community who use a grammatical device idiosyncratically, or just blatantly incorrectly, and that these uses needn't be accommodated into her overall conclusions. Such idiosyncrasies will either be irrelevant to the function of her chosen logic, ironed out through social pressure, or become the social norm of how to understand that grammatical device (at which point the logician will have to re-evaluate which system best suits the intended function).

On the second count, the logician doesn't believe that the validity of arguments is a matter to be decided by convention. An argument is either valid or it isn't. To work out whether a particular argument is (in)valid, however, one needs to be aware of the meaning of the grammatical particles being symbolised, so that what these particles (perhaps in combination with other grammatical particles) entail can be worked out. Equally, our logician is *not* confusing the proper function of logic, as a study of what validly follows from what, for a study of what individuals tend to infer from what, or what individuals *think* validly follows from what. She is interested in studying which natural language arguments are truth-preserving using formal methods, and isn't willing to admit that the validity of an argument is dependent on an individual's *decision* that a particular proposition follows from another. There is a fact of the matter as to what follows from what; logical consequence is not established by convention. Our logician endorses neither logical conventionalism nor logical psychologism.

Our logician does, however, recognise the need to distinguish logical consequences and the meaning of the grammatical particles constituting our premise set, from which we draw the logical consequences. This latter information requires not only being clear on the role of verbs, nouns, and adjectives in the language, but those grammatical particles that logicians give a special status to because of the putatively special role they play in the (in)validity of inferences. These grammatical particles express what logicians call *logical constants*.[218] Whereas the consequence relation isn't a matter of convention, the logical constants a linguistic community express in their language *are* a matter of convention, which entails that the logical constants that a member of a community can include in their premise set of an argument is also a matter of convention. *Given* the meaning of the grammatical particles, the logician admits, what can be validly inferred from sentences including those grammatical particles *is* a factual matter, and not one of convention. Even if no one had formulated a logic or language containing a particle defined with the semantics of classical conjunction, it would still be the case that if a classical conjunction is true then both conjuncts are true. What is entailed by the truth of a molecular proposition constituted of two atomic propositions conjoined by classical conjunction is a factual matter, independent of convention.

Yet, what has been assumed so far is that the logical constants expressed by the grammatical particles in a language are given, that they are somehow transparent to us.[219] However, the logical constants expressed by these grammatical particles are not a given. We need to work for them, and one needs to know the logical constants that the grammatical particles express before one can infer the logical consequences of natural language premises containing these grammatical particles. To work out the logical consequences of a premise set, one needs to understand the relevant semantic properties of the members of the set, including the logical constants expressed by particles in the premise set. To make the point using an

[218] It's important to distinguish between a grammatical particle being *identical* to a logical constant and expressing a logical constant. Considering the classical extensional connective conjunction, for example, while the English 'and' isn't identical to the classical conjunction, given that 'and' can be used for purposes other than classical conjunction, it's also (arguably) used to *express* classical conjunction in certain contexts. It's the latter that the classical logician is proposing by sometimes symbolising natural language arguments containing 'and' as formulae containing classical conjunction. Thus, the idea is that certain grammatical particles express logical constants, in virtue of certain semantic properties those particles possess. This case is analogous to the relationship between sentences and propositions, as sentences express propositions so some grammatical particles (in certain contexts) express logical constants.

[219] This is a complaint that Priest (2006a, p. 171) has of Quine: "[Quine] *assumes* that the logical constructions of the vernacular *are* those of classical logic."

inferentialist theory of the meaning of logical constants, one needs to know the natural-deduction rules for the grammatical particles before one can work out what validly follows from premises containing these particles.[220]

Thus, what follows from sentences containing these grammatical devices isn't a conventional matter *given the meanings of the special grammatical particles*, which are just the logical constants they express. However, the meanings of these special grammatical devices, and thus the logical constants used by a linguistic community, are a matter of convention. Therefore, as the logician is interested in the validity of natural-language arguments, and wants a logical theory that accurately models truth-preservation, she is particularly interested in a logic that *accurately models the truth-preservation of inferences made with the grammatical particles found in the natural language.*

That the meaning of these special grammatical particles is a matter of convention doesn't entail that they can have just any meaning. These special grammatical particles, to be special, have to express a logical constant.[221] The point is that a particular grammatical particle *g* could have, before a convention was established, expressed any number of logical constants. Also, we don't merely mean that *g* could have meant classical conjunction rather than classical disjunction, for example, with another particle *g'* meaning classical disjunction when it could have meant classical conjunction. This would ensure that the set of logical constants *C* that the set of a natural language's special grammatical particles *G* expressed wouldn't be a matter of convention, although the matter of which member of *G* expressed which member of *C* would be. Instead, we mean that the set of logical constants *C* expressed in a natural language by the language's special grammatical particles is also a matter of convention. So, for example, a language's particles might express an exclusive disjunction but no inclusive disjunction, and the same is true of the inverse.

As an example of this distinction between the *un*conventionality of the consequence relation and the conventionality of the logical constants expressed in the natural language, imagine a society that has at least grammatical particles expressing classical conjunction, negation and the material conditional. In addition, imagine that the society has a grammatical particle, call it 'or', that expresses exclusive disjunction in some contexts but no grammatical

[220] If one finds Prior's (1960) *tonk* convincing evidence that there's something wrong with *inferential semantics* then one won't look favourably on the meanings of these special grammatical particles being fixed by natural-deduction rules (although see Boghossian (2003a) for an encouraging post-*tonk* treatment of meaning-constituting rules for the logical constants). One might prefer such meaning to be fixed by something like a truth-table. We're not arguing here that the meanings of these special grammatical particles *are* constituted by particular natural-deduction rules, only that this method of explicating their meanings nicely draws the distinction. Using truth-tables to draw the distinction would work just as well. We also won't be broaching here the question of how individuals actually grasp the meanings of these special grammatical particles; that's a matter for psychologists and linguists, not for philosophers.

[221] To fully explicate this point we would need to know what exactly a logical constant is, and we don't as of yet have a viable answer to that question. This isn't a weakness of our thesis though, as the lack of a viable answer is a problem for the whole philosophical community.

particle expressing classical inclusive disjunction in any context. Now, of course, members of the society could express a classical inclusive disjunction using an iteration of conjunctions and negations, but they don't have a particular grammatical particle that fits the truth table of classical disjunction. That they don't, and instead use only (at the propositional level) classical conjunction, negation, material conditional and exclusive disjunction *is* a matter of convention.

Given that their grammatical particle 'or' expresses exclusive disjunction, and their 'not' expresses classical negation, however, it's *not* then a matter of convention that inferences of the form,

p or *q*,
p,
Therefore, not *q*,

are valid. Given the fixed meaning of 'or', and the other particles expressing logical constants, a member of the community is committed to accepting the validity of this argument. To argue that the truth of a proposition *p* doesn't entail the classical negation of a proposition *q*, given the truth of '*p* or *q*', would be to demonstrate that one didn't mean an *exclusive* disjunction by 'or', but rather an *inclusive* disjunction. Language communities get to set the logical constants that their linguistic particles express, but they don't then get to decide what follows from propositions containing those logical constants.

Our logician's concern is that this conventionalist aspect of building a logical theory can be forgotten. That one can become so wrapped up in working out what follows from what, given the stipulated meaning of the logical constants in one's system, that one forgets it's important to recognise whether the natural language, whose arguments one is assessing, contains grammatical particles expressing the logical constants (or combinations of) given in the logic and not others. In modern logic "most results take some vocabulary for granted, and then develop various deductive and other systems on this basis. Most major results are about semantic consequence and computability given some language. The question how we chose its vocabulary in the first place does not arise. Modern logic has much more to say on derivability than on definability," (van Bentham (1999) p. 27).

While logicians are sometimes clear that their "formal language is designed to represent the logical behaviour of a select few natural-language words," (Sider (2010) p. 3) the prerequisite for achieving this goal, that the semantics of the logical constants in the logic accurately represent the pertinent semantic properties of the grammatical particles, is more often assumed than argued for:

> The structural traits of first-order sentences we have called 'logical constants' have correlates in vernacular sentences (or thought-vehicles)

> whose semantic significance is presumed to be *accurately* represented by the semantic significance of the corresponding traits in the first-order expressions that translate them.
>
> (Pérez-Otero & García-Carpintero (1999) p. 171. Emphasis mine)

Our logician is clear that merely presuming the adequacy of the logical constants in our logic for this role is inadequate; we have to discover their adequacy.

Returning to our logician's project, in order to assure herself of which logic most reliably assesses the (in)validity of natural language arguments, she collects data on the truth-conditions of universal conditions in her society's natural language. On collecting that data, she finds that universal conditions don't commonly have existential commitments built into them. If an individual makes a statement of objects of one type *A* that all objects of type *A* are also of type *B* (where *A* and *B* needn't be atomic natural language descriptions), they don't intend to imply that there exist any objects of type *A*. Unless, that is, the speaker is in a context where some members of the type *A* are before her, her statement would be inappropriate if interpreted without existential import (as in a history lecture on the British Parliamentary system), or particular implicative conditions are fulfilled. If an individual commonly wishes to communicate their existential commitment to an object with a universal condition then they will explicitly add it to the generalisation by prefacing the types of objects with an adverb such as 'actually' or 'really', or by adding 'and some exist' after the generalisation. The logician also finds that any member of the linguistic community who shirks these rules gets quickly misinterpreted, and thus concludes that the lack of existential import in a universal condition is a well accepted (if implicit) aspect of the natural language of her society.

Due to her theoretical commitments, these data lead the logician to conclude that she must accept a logic that has the logical machinery appropriate to deal with the universal conditions of her community accurately. If the logician uses traditional syllogistic logic to assess the (in)validity of natural language arguments, then she knows that given the existential import built into the logic's universal generalisations, she is going to judge as valid many invalid arguments that contain universal conditions (and, likewise, judge many arguments as entailing a contradiction when they don't). Given that the only available logic possessing the required logical machinery to accurately translate the natural language universal conditions is classical first-order logic, our logician comes to the conclusion that it's reasonable to adopt classical logic as a reliable formal guide to valid reasoning. Now, if we assume that our logician, previous to collecting this data, used traditional syllogistic logic to evaluate natural language arguments, which we are entitled to for the sake of our argument, then we seem to have an example of empirical evidence justifying a revision of logical theory. If the empirical data the logician collected had been different, then she certainly could have rationally chosen differently. Her justification for choosing classical logic over traditional syllogistic logic was constituted, in part, by the empirical evidence gathered on the (lack of) existential import built into universal conditions by competent users of language in her linguistic community.

We seem to have a case, therefore, in which the logician has rationally revised her logical theory from one that categorised a particular inference, *Darapti*, as valid, to a logic that categorised the same inference as invalid. Additionally, it seems to have been empirical evidence that justified this rational revision of the logical theory.

Now, given that traditional syllogistic logic may be considered evidently inferior to first-order classical logic in its expressibility, such as its lacking truth-functors to handle more complex propositions, one may have qualms over the possible rationality of empirical evidence justifying the revision of one's logic from traditional syllogistic to classical logic. Perhaps the inadequacy of traditional syllogistic logic is *so* evident as to preclude the possibility of empirical evidence justifying such a revision; we would always have recommended the revision before such evidence came in.[222] I think these concerns are unjustified; it certainly seems possible for one to not realise these other shortfalls of traditional syllogistic logic, and thus base one's decision, rationally, on the said empirical evidence. However, to ensure that any such concerns are fully answered, we can retell our story with a different theory choice, with a new logic being preferred to classical logic.

8.3 Case Two: Partial Logic vs. Classical Logic

This new system, first-order *partial* logic, is almost identical to first-order classical logic, in that it contains the same Boolean connectives, uses the same model-theoretic account of consequence, and uses functions for predicates. It differs from classical logic in only three respects.

Firstly, first-order *partial* logic doesn't contain an existential quantifier as conceived of in first-order classical logic. Instead, the logic contains a *partial* quantifier which differs from the existential quantifier in not carrying any existential import.[223] Thus, while the partial

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[222] Traditional syllogistic logic is, after all, inconsistent; see Goddard (1998 & 2000).
[223] The idea originally comes from McGinn (2000), as part of his argument that 'existence' is a first-order predicate. McGinn (2000, pp. 35-36) states that,

> In the orthodox notion expressed by '∃x' we have conflated two linguistic functions into the '∃x' symbol – the function of saying how many and the function of implying existence (and the name 'existential quantifier' only captures this latter aspect). But, as we have separated these functions for 'all' [with the universal quantifier], so we should for 'some'…[W]hat quantifier words do is abandon singularity; what 'exists' does is attribute the property of existence to objects that are either denoted or quantified over. It invites confusion to try and merge these two functions together into a single primitive symbol '∃x'.

Therefore,

quantifier requires only *at least one* member of the domain of quantification to fulfil the conditions within its scope, it doesn't require that the member of the domain *exists*. As with the universal quantifier, this partial quantifier simply specifies the amount of particulars being considered, not that those particulars exist. Therefore, when the partial quantifier symbolises propositions of the type 'There is some *x* such that…', it doesn't carry the commitment that any such *x*'s exist.

Secondly, to make sense of the partial quantifier quantifying over particulars that don't exist, the system's domain of quantification includes members that don't exist. The logic achieves this by having a domain ranging over 'actual objects or objects we talk about'. In doing so, first-order partial logic ensures that for every predicate *P* there's at least one member of the logic's domain that is a member of *P*'s extension. By assigning meaning to a predicate term *P* it gains an extension in partial logic, although the same isn't true for *P*'s anti-extension or for the extension of a conjunction of properties.[224]

Thirdly, with an expanded domain, the logic needs to be able to effectively communicate when a particular being considered is postulated as *existing* in an argument. After all, the existence of a particular entity is sometimes important. Therefore, the logic requires a mechanism to express existence. First-order partial logic can achieve this by introducing an existential predicate '**E**', which has an extension like other predicates. Thus, to communicate that a member of the domain exists, one only needs to assign the relevant variable to **E** as one would with any other predicate. So, take the example of 'Some witches are wicked'. Without the existential commitment that some wicked witches exist, the sentence is symbolised as (using '*W*' for witch and '*K*' for wicked),

$$\exists x(Wx \wedge Kx),$$

while the sentence with the existential commitment is symbolised as,

$$\exists x((Wx \wedge Kx) \wedge \mathbf{E}x).$$

First-order partial logic has all the expressive power of first-order classical logic and more.

> [W]e do better to call 'some' the *partial* quantifier, on analogy with the universal quantifier – neither logically implies existence.
>
> (McGinn (2000) p. 35)

[224] If one agrees with Mulder (1996) that the best candidate for justifying the validity of *Darapti*, and similar syllogistic moods, within a traditional syllogistic framework is to assume that every class and its complement is non-empty, then there emerge apparent similarities between the traditional syllogistic-classical logics disagreement and the partial-classical logics disagreement. On this justification for the validity of the traditional syllogistic moods, see also William Jacobs's (1979, p. 282) suggestion that "Aristotle did not accept the possible referential failure of the subject of an assertion," which elevates the potential justification to the status of an historical explanation for the syllogisms' perceived validity.

An interesting property of first-order partial logic is that it differentiates between two types of *Darapti*. *Atomic Darapti*,

$$\begin{array}{l} \forall x(Px \rightarrow Qx) \\ \underline{\forall x(Px \rightarrow Rx)} \\ \exists x(Qx \wedge Rx) \end{array}$$

is valid in first-order partial logic, as there's no interpretation in which the conditionals are true, and conclusion false, due to *P* having an empty extension. However, the *molecular Daraptis* are invalid (as they are in first-order classical logic):

Conjunctive *Darapti*

$$\begin{array}{l} \forall x((Px \wedge Sx) \rightarrow Qx) \\ \underline{\forall x((Px \wedge Sx) \rightarrow Rx)} \\ \exists x(Qx \wedge Rx) \end{array}$$

Conditional *Darapti*

$$\begin{array}{l} \forall x((Px \rightarrow Sx) \rightarrow Qx) \\ \underline{\forall x((Px \rightarrow Sx) \rightarrow Rx)} \\ \exists x(Qx \wedge Rx) \end{array}$$

Negative *Darapti*

$$\begin{array}{l} \forall x(\sim Px \rightarrow Qx) \\ \underline{\forall x(\sim Px \rightarrow Rx)} \\ \exists x(Qx \wedge Rx) \end{array}$$

Although every *n*-placed predicate *P* in first-order partial logic has a non-empty extension, there's no assurance that the extensions of every pair of *n*-placed predicates *P* and *S* intersect. Therefore, there will be some interpretations of the predicate-terms in the conjunctive and conditional forms of *Darapti* such that no member of the domain is in the extensions of both *P* and *S*, and no member is in the extensions of both *Q* and *R*. Likewise, there's no assurance in first-order partial logic that the anti-extensions of every *n*-placed predicate *P* will be non-empty. Therefore, there will be some interpretations of the predicate terms in the negative form of *Darapti* such that no member of the domain is in the anti-extension of *P*, and no member is in the extension of both *Q* and *R*. In other words, as soon as the antecedent of the conditionals in the *Darapti* are non-atomic formulae, classical first-order validity is recaptured, because there are no assurances that any member of the domain fulfils the conditions of the non-atomic formulae (unlike with atomic formulae).

Thus, while first-order partial logic agrees with classical logic on the invalidity of the *molecular* forms of *Darapti*, it disagrees with classical logic on the invalidity of the *atomic Darapti*.

Therefore, there appears to be a disagreement between the two logics over the validity of certain arguments, as there was between traditional syllogistic and classical first-order logic.

Let's return to our logician. In this scenario she has the choice between only first-order partial and classical logic to most adequately model the truth-preservation of natural language arguments, precluding the possibility of other theory choices for the simplicity of our argument. Again, understanding that she must ensure that the logical constants in her logic accurately reflect the semantics of certain important grammatical particles in the natural language, she collects data on the truth-conditions of claims that the extensions of two (or more) predicates intersect. Thus, assuming that she's collecting data on English speakers, she would be investigating the truth-conditions of claims of the form 'Some *A*'s are *B*'s'.

Now, on collecting this data our logician finds that individuals don't include existential import in statements that *some* objects of type *A* are of type *B*. For example, members of the community commonly state that 'Some witches are wicked' and 'Some superheroes can fly'. As before, we can imagine that individuals have extra communicative resources to indicate that the objects being quantified over exist, or that context is relied upon.[225] Due to her theoretical commitments, these data lead the logician to conclude that she must accept a logic that has the machinery appropriate to deal with these claims when they are embedded into an argument. If the logician uses first-order classical logic to assess the (in)validity of natural language arguments, then she's aware that the non-inclusion of a partial quantifier not carrying existential import in the logic will ensure the miscategorisation of certain valid arguments as invalid. As the only available logic containing a partial quantifier that doesn't carry existential import is first-order partial logic, it's reasonable for the logician to adopt this logic as a reliable formal guide to truth-preservation.

If we now assume that our logician, previous to collecting the data, had used first-order classical logic to model the truth-preservation of natural language arguments, we have a case of empirical evidence justifying a revision of logical theory. If the empirical data the logician collected had been different, then she could have rationally chosen differently. Her justification for choosing first-order partial logic over classical logic was constituted, in part, by the empirical evidence gathered on the (lack of) existential import built into the partial quantifier by competent users of language in her linguistic community.

Unlike with our logician's first theory choice between traditional syllogistic and classical logics, where there were numerous additional disagreements, the disagreements between the first-order partial and classical logics are minimal, if important. The logics disagree over whether the partial quantifier carries existential import. This disagreement is related to the logics' disagreement over the scope of the domain of quantification. By having

[225] Unlike McGinn (2000, p. 35), who holds that the partial quantifier ('some') *doesn't* contain existential import in natural languages, we are only relying on the weaker claim that it's *possible* for a natural language to contain a partial quantifier that doesn't carry existential import.

an extended domain of quantification, first-order partial logic requires that the extension of every *n*-placed predicate term is non-empty, whereas classical logic allows for the extensions of predicate terms to be empty. Lastly, because existential import isn't built into the partial quantifier in first-order partial logic, and the logic uses a predicate to pick out members of the domain that exist, the logic is committed to 'existence' being treated as a first-order predicate to adequately model the truth-preservation of natural language arguments.[226]

Now, unless the irrationality of any of these commitments of the first-order partial logic are so great as to outweigh its benefits of accurately modelling the partial quantifier in the natural-language, the empirical evidence postulated *would* make it rational for the logician to adopt first-order partial logic, given her theoretical commitments. This is particularly plausible once one remembers that first-order partial logic shares all the other powerful formal properties of classical logic. Here then we have an even more persuasive possible case of empirical evidence justifying a revision of logic, which includes a revision of whether certain inferences, such as those of the *atomic Darapti* form, are (in)valid.

In both of our examples, we have made assumptions regarding the possibility of logical revisions and the role that empirical evidence played in the theory choices. It will be helpful, therefore, to consider two pertinent objections to our putative cases of empirical evidence justifying a revision of logic. This may show that the cases are somewhat hardier than they may first appear. We begin with the objection that a change of doctrine is only a change of subject.

8.4 Objection One: A Change of Doctrine is a Change of Subject

The objection is Quine's. It states that if two logical theories disagree over whether a particular inference is truth-preserving, then really they are talking about *different* inferences. This ensures that there is no conflict or disagreement between the logical theories. In making this point, Quine uses as his example a deviant logician who proposes violations of the law of non-contradiction:

> My view of this dialogue is that neither party knows what he is talking about. They think they are talking about negation, '~', 'not'; but surely the notation ceased to be recognizable as negation when they took to regarding some conjunctions of the form '$p \wedge {\sim}p$' as true, and stopped regarding such sentences as implying all others. Here, evidently, is the deviant logician's predicament: when he tries to deny the doctrine he only changes the subject.
> (Quine (1986) p. 81)

[226] Whether this requires advocates of the logic to endorse the view that 'existence' is a first-order predicate is unclear. Perhaps one could insist that 'existence' must be treated as a first-order predicate for the purposes of modelling natural language arguments, while suspending judgement over whether it actually is.

While Quine concentrates on logical laws in most of his logical writings, the same *change of subject* principle can equally apply to inferences. Relating the objection to one of our cases, if first-order partial and classical logic disagree over the validity of *atomic Darapti*, then this must be because the logics are talking about different logical constants with their use of the partial quantifier in the argument form. One of the constants carries existential import while the other doesn't. Thus, although both the first-order partial and classical logicians have a translation manual for English sentences of the form 'Some *A's* are *B*'s', they don't assign sentences of this form the same truth-conditions in their respective symbolisations. Thus, when the classical logician states that you cannot validly infer according to the *atomic Darapti*, and the partial logician states that you can, they aren't disagreeing with one another because they assign different meanings to sentences of the form 'Some *A*'s are *B*'s'. In other words, both parties are talking about different propositions.

If the *change of subject* argument is successful then the revision of logic in our cases won't be a revision from a logical theory T_1 that accepts a particular inference as truth-preserving to another T_2 that denies the inference is truth-preserving (or vice versa), but will be rather a revision from a logic talking about one inference I_1 to another talking about a completely different inference I_2. Thus, our cases will no longer hold weight if the objection is sustained.

When confronting this objection, one must appreciate that it holds against the possibility of *any* form of disagreement between logics on the subject of (in)validity. There can be no revision from one logical theory to another based on a disagreement of (in)validity, whether it be rational, irrational, justified by empirical or non-empirical evidence. While one can introduce a new theory of validity, this new theory in no way disagrees with other theories. Thus, accepting such a new theory of validity doesn't constitute a rejection of other theories of validity. The objection isn't only of consequence to our discussion here, but also to the debate between the classical, intuitionist and dialetheic logician on the meaning of negation. According to the objection, none of the parties are arguing with one another; they are merely talking about different subjects.

Recognising the objection's scope, let us try and meet it. The predicament we are faced with is,

either

a) Traditional syllogistic and classical first-order logics do *not* disagree over the existential import of universal conditions, and first-order partial and classical logics do *not* disagree over the existential import of the partial quantifier, or

b) There are plausible possible cases of empirical evidence justifying a rational revision of logical theory.

For our cases to hold, we don't need to show that logics are never talking about different subjects when they appear to disagree over the truth-preservation of an inference. For example, two logics may appear to disagree over the falsity under all truth-value assignments of '$p \wedge {\sim}p$' and the truth under all truth-value assignments of '$p \vee {\sim}p$', while in reality they agree over the truth-conditions of sentences of either logical form. The appearance of disagreement can be explained by one of the logics using '$\wedge$' to symbolise classical conjunction and '$\vee$' to symbolise classical disjunction, and the other using '$\vee$' to symbolise classical conjunction and '$\wedge$' to symbolise classical disjunction. This is a simple case of, as yet unrecognised, divergence over symbol usage, and not a disagreement over logical consequence. To show that our putative cases are not of this kind, we need to demonstrate that the logics in our cases do disagree over the validity of certain inferences.

As Priest (2006a, pp. 169-170) has recognised, it's only when a set of logics are intended for the same purpose that they can disagree with one another.[227] Before a logic is interpreted for a particular function it is just a mathematical structure, and these can hardly disagree with one another on a matter. Just as different geometries are not rivals until they are applied to *model some phenomena,* such as the dimensions of a physical space, so logics aren't rivals until they attempt to model the same phenomena. If classical logic is never interpreted as a model for the way scientists reason when they are engaged in theory construction, and some paraconsistent logics are only ever used for this purpose, then the logics will never disagree or be rivals. Including explosion as a theorem when the logic intends by this that a premise set containing contradictory propositions entails any arbitrary proposition, and excluding explosion as a theorem when the logic intends by this that scientists don't infer the truth of arbitrary propositions from inconsistent premises, does *not* constitute disagreement.[228]

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[227] Cf. Beall & Restall (2006, p. 36).

[228] Woods (2003, pp. 149-153) has claimed that part of the apparent disagreement between (non-dialetheic) paraconsistent logicians and classical logicians can be explained away by both parties' conflation of inference and implication with regards to the validity of explosion. We seem to find just such a confusion of implication and inference in the work of some preservationist logicians:

> A more humanistic logic is required. Such a logic would accord with what we must frequently do, namely the best we can with data which, although inconsistent, are nevertheless, the best data we are able to command. We would like to be able to reflect but also judge the reasonings of ordinary doxastic agents.
>
> Even when our stock of beliefs is inconsistent we routinely draw a distinction between what follows from it and what does not, and regard certain inferences from

If two logics only disagree when they are attempting to model the same phenomena, we need evidence that in our cases both logics are doing just this. This isn't difficult, as it was a part of our stories that both logics were being used for the same purpose. That is, to act as a reliable formal method to judge the (in)validity of natural language arguments; to model truth-preservation. The fact that the two logics were given the same function, however, isn't sufficient to show that there's disagreement between the logics, although it is necessary. It's still possible that the theories' apparent divergence in opinion over the validity of instances of an argumentative schema is caused by their talking about different inferences, in which case the theories would be talking past one another. What we need to show is the substance of their disagreement, thereby demonstrating that their divergence of opinion is indeed over the validity of the same inferences.

Logics that profess candidacy for truth-preservation don't simply stipulate the meaning of their logical constants, give the logic's structural rules, and then derive the constants' consequences *given* these meanings. If this were all there was to logical systems modelling truth-preservation, then all logics that derived theorems, according to the meaning of the constants as stipulated, would be preserving truth. Yet, this isn't all we expect of a logic that purports to model truth-preservation. This is implied by our talk of *modelling* truth-preservation. There needs to be something initially to model.

Take as an example Putnam's argument for quantum logic, and its rejection of the distributive laws $(p \wedge (q \vee r)) \leftrightarrow ((p \wedge q) \vee (p \wedge r))$. Putnam wasn't arguing that the distributive laws didn't follow from classical conjunction and disjunction – they do. Rather, he was suggesting that classical conjunction and disjunction had got it wrong in some way, by denying that "there are *any* precise and meaningful operations on propositions which have the properties classically attributed to 'and' and 'or'," (Putnam (1975b) p. 189). Now, one might think Putnam's claim here is a little strong. It doesn't seem that classical conjunction or disjunction are lacking of any more precision, or meaning, than quantum logic's conjunction or disjunction. But, even softening Putnam's account, the point is that classical conjunction or disjunction got it *wrong* regarding the meaning of the English particles 'and' and 'or' in certain contexts (at least with regards to their use in scientific discourse).

In constructing a logic with the intent of modelling truth-preservation, one takes a stand on the identity of the privileged grammatical particles in natural language that play a special role in dictating the (in)validity of natural language arguments. It is a tenet of all attempts to construct a formal account of the (in)validity of arguments that there are these particular grammatical particles, expressing logical constants, which dictate the (in)validity of arguments. Now, of course, logics can disagree over which grammatical particles hold this

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> our beliefs as improper, in spite of the circumstances. If our procedures were genuinely classical we would not, indeed could not, make such a distinction.
> (Jennings & Schotch (2009) pp. 23 & 26)

special status, which may entail that they disagree over which inferences are valid. Let us assume for the moment, however, that two logics don't disagree over which grammatical particles are of logical import. Once these particular particles have been identified, however, there is still the task of giving them the correct content within one's logic. That is, accurately reflecting their meaning in the natural language by preserving those properties that constitute their meaning in the logic's symbolisation process.

Now, logical systems can disagree over the meaning of a certain grammatical particle *g*, which will be reflected by the logical constant *c* that the systems choose to symbolise *g* with. Thus, for example, while the classical logician will symbolise the 'not' in 'Emma's not walking up the stairs' as a classical Boolean negation, the dialetheic logician will symbolise it as a paraconsistent De Morgan negation. They agree that the individual is expressing a meaning historically associated with the logical constant negation, yet both parties disagree over some of the properties that this negation possesses.[229] Thus, the logics are arguing, and disagreeing, over the logical constant that a particular grammatical particle, perhaps in certain circumstances, expresses.

The claim that two logics are actually talking about different inferences when they appear to disagree over an inference's (in)validity is ambiguous. If a logical constant within a logic is defined by its introduction and elimination rules, and an inference is defined by the occurrence of logical constants in it then obviously, aside from proofing errors, the two logical systems must be talking about different constants if they disagree over the (in)validity of a rule of inference. However, none of this holds when the theories are attempting to model particular phenomena, that is, something external to the systems. In such cases, there can be disagreement without a change of subject. Reference to the phenomena that they are attempting to model can ensure that they are talking about the same subject, just as competing scientific theories attempting to accommodate data can ensure that they are talking about the same phenomena. Once there is some data external to the logics that they are attempting to accommodate, we have a reference point that defines the subject of their disagreement. Logics can disagree over the logical constant that a grammatical particle, in particular circumstances, expresses while both are attempting to answer the same question by accommodating the same data, which in this case would be the (in)validity of instances of an argument schema.

When two logics are applied to natural language arguments, so that both are being used as theories of truth-preservation in natural language arguments, those logics are attempting to model the same phenomena. There is no change of subject, guaranteed by the fact that they keep referring to the same natural language phenomena. They are arguing over

[229] We won't take a stand here on what makes Boolean negation and De Morgan negation types of *negation*, or similarly for the other categories of logical constants. Presumably, it will be dependent on the logical constants having certain similar properties, but what these properties are we won't speculate on.

which of their constants accurately reflects the meaning of the grammatical particles in the natural language that are pertinent to the (in)validity of arguments. There must be little doubt that logicians are engaged in this enterprise. Modal extensions of propositional logics have been constructed to openly model our modal, deontic, and doxastic concepts so as to evaluate natural language arguments containing them, and the success of a particular system at capturing our use of these concepts in natural language is evidence for that logic's preferability over others.

All that is left for us to show is that the logics in our examples are being used as theories to model the same phenomena, and thus that they are disagreeing over the correct interpretation of the grammatical particle's meaning. It is this *disagreement*, over the logical constant *c* that the grammatical particle *g* expresses, that can cause a disagreement over whether instances of an argumentative form containing *g* are valid. For the sake of brevity, we will consider only the apparent disagreement between the traditional syllogistic and classical first-order logics. If it is shown that there can be real disagreement between these logics then it will follow, given the similarities in the cases, that there can be real disagreement between partial and classical first-order logics also.

We can go some way to demonstrating that there can be real disagreement between traditional syllogistic and classical first-order logic by pointing to Russell's (2010b, p. 62) original criticism of traditional syllogistic logic:

> I want to say emphatically that general propositions are to be interpreted as not involving existence. When I say, for instance, 'All Greeks are men', I do not want you to suppose that that implies that there are Greeks. It is to be considered emphatically as not implying that. That would have to be added as a separate proposition.

Russell is explicit that by interpreting 'All *A*'s are *B*'s' as *necessarily* containing, by virtue of being a "general proposition", existential content, traditional syllogistic logic is *making a mistake*. Such general propositions don't always have existential import, and whether they do or not will depend on the content of the proposition, or the context in which it's spoken. Russell's argument for classical logic then is that the universal quantifier it contains more accurately reflects the meaning of general propositions in natural language.[230] The two theories disagree over the meaning of the general proposition and, according to Russell, classical logic reflects its meaning more accurately. Russell's interest here in reflecting the meaning of particular natural language grammatical particles in a logic, is the same as our logician's. Thus,

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[230] Similarly, Strawson (1950) argues for the validity of subalternation on the grounds that it better fits usage in natural language. In arguing that all of the four traditional syllogistic categorical forms – *AaB*, *AeB*, *AiB*, *AoB* – presupposed that *A* was non-empty, thereby validating subalternation, Strawson appealed to "this interpretation [being] far closer to the most common use of expressions beginning with 'all' and 'some' than is any Russellian alternative," (Strawson (1950) p. 344). Here then we have a case of empirical evidence, in the form of linguistic evidence, justifying a choice of logic.

given that those active in the actual debate considered the logics to be disagreeing over the meaning of the natural language particles, this constitutes strong evidence that the logics in our case can disagree over the meaning of certain natural language particles.

As the logics are being used as theories to model the truth-preservation of natural language arguments, and they are disagreeing over the meaning of a putatively logically important grammatical particle *g*, they come to disagree over the (in)validity of some natural language arguments containing *g*. The disagreement between the theories is a disagreement over what follows from what in natural language arguments. The traditional syllogistic and classical logicians don't disagree over what follows from what *within* a logical system; after all, intuitionists and dialetheists can construct derivations in classical logic as well as any classical logician. Instead, they differ over whether inferences modelled within the two logics accurately represent natural language arguments. That is, they disagree over truth-preservation, which is a property not of pairs of formulae within a logic, but of natural language arguments.[231] Consequently, the stance logicians take on this disagreement dictates whether they think natural language arguments such as,

> All chimeras are animals
> <u>All chimeras breathe flame</u>
> Therefore, some animals breathe flame,

are truth-preserving or not. By a logician changing her theory for modelling natural language arguments from traditional syllogistic to classical first-order logic, the logician changes her theory of what natural language inferences are valid, and thus which arguments are valid. Any traditional syllogistic logician stating that *Darapti* is a valid syllogistic mood is also stating that the natural language argument above is valid, and any classical logician stating arguments of the form,

$$\forall x(Px \rightarrow Qx)$$
$$\underline{\forall x(Px \rightarrow Rx)}$$
$$\exists x(Qx \wedge Rx)$$

are invalid, is also stating that the natural language argument above is invalid. They *are* disagreeing over an interpretation of the same phenomena, which ensures a disagreement over the (in)validity of certain natural language arguments; there is no change of subject.

There is one rejoinder available to the advocate of the *change of subject* thesis. While the logics *appear* to disagree over the validity of the argument,

[231] For an argument to be truth-preserving, the objects constituting the argument need to be capable of being true. Formulae within logics are not capable of being true, whereas natural language sentences are. Therefore, when a logic is modelling truth-preservation, it's modelling the relations between (sets of) natural language sentences.

All chimeras are animals
<u>All chimeras breathe flame</u>
Therefore, some animals breathe flame,

this is solely due to us our misconceiving of natural language arguments as constituted of *sentences*. If, instead, we correctly conceive of the natural language arguments being modelled as constituted of *propositions*, then its clear that the two logics are considering different arguments to one another. The appearance that the two logics are modelling the same phenomena, therefore, is misleading.

While the classical logician understands 'All chimeras are animals' to express 'All_C chimeras are animals', where 'All_C' doesn't carry existential import, the traditional syllogistic logician understands the sentence as 'All_A chimeras are animals', where 'All_A' does carry existential import. While propositions of the form 'All_A *A*'s are *B*'s' entail propositions of the form 'Some *A*'s are *B*'s', propositions of the form 'All_C *A*'s are *B*'s' don't entail propositions of the form 'Some *A*'s are *B*'s'. Thus, when we appreciate that the logics are symbolising the propositions that a natural language argument *expresses*, and not directly the natural language argument constituted of sentences, we come to realise that the two logics are not disagreeing over the validity of the same argument. After all, they disagree over the meaning of some of the grammatical particles in the natural language arguments, conceived of as sets of sentences. This ensures that the logics disagree over the set of propositions that the arguments express, with the consequence that the logics consider different propositions as their premise sets. In which case, we have a change of subject between the theories, in the form of two distinct arguments being evaluated; the appearance that they are modelling the same phenomena is illusionary. Thus, the disagreement between the logics isn't a disagreement over logical consequence, but *solely* a disagreement over the meaning of certain grammatical particles.

Although this rejoinder clarifies the essence of the *change of subject* objection with regards to logics that appear to disagree over the validity of arguments, it is ultimately unfulfilling. It is important, firstly, to recognise that both logics, when symbolising a natural language argument, are certainly symbolising the same natural language argument *as constituted of sentences*. Not only this, but the logics give different answers to the question of whether particular natural language arguments, constituted of sentences, are valid; notably in the case of arguments that are instances of the form *Darapti*. That these different answers were considered to constitute a disagreement between the logics can be seen in Russell's quote above. To the extent that the logics disagree over the validity of these natural language arguments, as they appear constituted of sentences, this represents *prima facie* evidence for the conclusion that the two logics do disagree over some natural language arguments' (in)validity. If the rejoinder is to convince us otherwise, the advocate of the *change of subject* thesis will need to explain away these appearances of disagreement that even the working logician is susceptible to. There are two reasons to think that the rejoinder has failed in explaining away any such appearances.

Firstly, even if we admit that the advocate of the *change of subject* thesis has shown that the two logics fail to disagree over validity, *on the assumption that* the arguments they are evaluating are constituted of propositions, we haven't yet been given reason to believe that arguments constituted of propositions should be considered the *sole* objects of logical evaluation. After all, our objector's conclusions certainly don't follow if we consider arguments constituted of sentences as the objects of logical formalisation and, ultimately, evaluation. If the arguments were constituted of sentences, any disagreement over the meaning of certain grammatical particles in the argument wouldn't entail that the logics were evaluating different arguments. Rather, the logics would be evaluating the same argument but disagreeing over its validity, due to a divergence in the logics' interpretations of the meaning of certain grammatical particles in the argument. Now, as the debate over the identity of the proper objects of logical evaluation is far from settled, and thus there are no assurances that propositions are the sole proper objects of logical evaluation, the advocate of the *change of subject* thesis has failed to explain away the apparent disagreement between the logics.[232] To achieve that, she will need to show that it's solely arguments constituted of propositions (that is, arguments individuated exclusively by their propositional content), that we are interested in modelling with our logical theories.[233] This would preclude the possibility of logics modelling the same natural language arguments while disagreeing over the propositional content of the arguments. As it stands, however, the appearance of disagreement is still apparent.

Secondly, the rejoinder's attempt to explain away the apparent disagreement between the two logics doesn't seem to hold water when we apply the explanation to other areas of apparent disagreement. Disagreement over the propositional content of a term doesn't then preclude disagreement over the extension of the term, or the truth of sentences that the term occurs in. Similarly, then, disagreement over the propositional content of an element of an argument shouldn't preclude disagreement over the validity of the argument.

Consider, firstly, two individuals' disagreement over whether Shane Warne is a great athlete. It's very possible that, during their disagreement, the two parties come to realise that they possess different criteria for applying the descriptor 'great athlete'. Yet, we wouldn't suggest that this disagreement over the criteria for applying the descriptor entails that their disagreement over whether Shane Warne is a great athlete is any less substantial. That is, their disagreement over the meaning of 'great athlete' doesn't entail that they aren't then substantially disagreeing over its extension. Indeed, on many occasions, such a disagreement

[232] For a recent evaluation of the candidates for the objects of logic, see Brun (2008).
[233] Achieving this will require showing that previous attempts to treat sentences (and any other truthbearers that aren't propositions) as the objects of logical study, such as Montague's (1974) use of disambiguated sentences, were somehow misguided.

over extension can be reasonably explained in terms of a divergence in opinion over the meaning of the predicate. Yet, this doesn't preclude the parties disagreeing, or having a meaningful debate, over the predicate's extension. The two parties are not somehow talking past one another when they give different answers to the question of whether Shane Warne is a great athlete.

The same point can be made using Putnam's (1975a, pp. 239-242) example from geometry. The disagreement between Euclidean and Riemannian geometry over the truth of the sentence,

> St) One cannot reach the place from which one came by travelling away from it in a straight line and continuing to move in a constant sense,

constituted by Riemannian geometry's rejection of the parallel postulate, is no less a disagreement because the rejection of the postulate represents a disagreement over the meaning of 'straight line'. When the two geometries are applied to actual space, the systems disagree over whether St) accurately models space or not, and this disagreement isn't mitigated by any divergence over the meaning of 'straight line'. Divergence in the propositional content assigned to a term t by two (or more) scientific theories doesn't entail that these theories containing diverging interpretations of t cannot be competing theories of the same phenomena. The disagreement between the theories is constituted by the divergence in the predictions they make about the same set of phenomena.

Similarly to these cases, any disagreement between logics over the propositional content of a natural language argument $\mathfrak{I}$, constituted of sentences, doesn't entail that there can be no substantive disagreement over $\mathfrak{I}$'s validity. By taking a stand on the validity of arguments constituted of propositions, one is in practice required to take a stand on the validity of natural language arguments as they appear in everyday life, in the form of sentences written on a page or spoken in a conversation. These arguments, as they appear in everyday life, can still act as phenomena that both logics are attempting to explain the (in)validity of, and indeed any logic attempting to model truth-preservation would be useless if it were not willing to take a stand on the (in)validity of these arguments. Thus, given that the logics can diverge in opinion on the validity of natural language arguments *constituted of sentences*, they can *disagree* on the validity of the arguments. Consequently, disagreement over a natural language argument $\mathfrak{I}$'s propositional content doesn't preclude disagreement over $\mathfrak{I}$'s validity.

Both of these reasons constitute good evidence that the rejoinder has failed to demonstrate that the apparent disagreement between the logics is illusionary. The disagreement seems as real as ever, as was suggested by Russell's proclamation that traditional syllogistic logic had miscategorised some invalid arguments as valid. Our cases are untouched by the objection that a change of doctrine constitutes a change of subject.

Before we move on to consider our second objection, it is worth briefly reminding ourselves of the importance of the *change of subject* argument to the debate over the empirical revisability of logic. Although we have spent time here attempting to show that the argument fails to demonstrate that there can be no disagreement between logics over truth-preservation, it's clear that any attempt to bury the argument for the empirical revisability of logic, via an appeal to the *change of subject* argument, would be ultimately unfulfilling. In this chapter we have been concerned with the question of whether empirical evidence could justify a revision of logic which includes the acceptance/rejection of a theorem, inferential rule or metalogical principle that was previously rejected/accepted. Yet, according to the *change of subject* argument no such revision of logic is possible. It is indiscriminate in the justification for adjudicating between rival logics that it precludes, because it precludes the possibility of rival logics. If two logics cannot disagree over validity, then *no* evidence, empirical or non-empirical, can justify a revision of one's theory of logical consequence.[234] Yet, in enquiring whether empirical evidence can justify a revision of one's theory of validity, we were assuming that non-empirical evidence *can* justify such a revision, and that this putative property of non-empirical evidence potentially distinguishes non-empirical from empirical evidence. Simply ruling out the possibility of empirical evidence justifying such a revision of one's theory of validity, with no reference to the particular nature of *empirical* evidence, is to downplay the importance of the question we are asking, rather than answering it appropriately in the negative. We want to know if non-empirical evidence can play a justificatory role in the revision of logical theory that empirical evidence cannot, and *not* whether any kind of revision of one's theory of validity is impossible. The *change of subject* challenge, therefore, though interesting, is ultimately beside the point when debating the empirical revisability of logic.

8.5 Objection Two: Logical Psychologism

Our second objection to the logician's cases presumes the evident absurdity of any theory entailing logical psychologism, a presumption prevalent since Frege and Husserl.[235] Given that our logician enquired into the meanings that the linguistic community assigned

[234] The *change of subject* argument does allow for revisions of logical theory, but no such revision constitutes disagreement with the previous logical theory. This ensures that there can be no revision of logical theory that includes the acceptance/rejection of a theorem, inferential rule or metalogical principle that was rejected/accepted in a previous theory. Therefore, the *change of subject* argument precludes *tout court* the type of revision that is of interest to us in this chapter.

[235] Although Bolzano and Herbart were the first critics of logical psychologism, Frege's (1981) and Husserl's (2001) critiques of logical psychologism are the more commonly noted and acclaimed. It's with some irony that we find in Husserl's *Prolegomena* the most detailed examination of psychologism given that Frege's own criticisms of psychologism had been squarely aimed at Husserl's (1970) *Philosophie der Arithmetik*.

particular grammatical particles, and these findings subsequently influenced her theory choice of a logic of truth-preservation, it might be supposed that our logician is committed in both of our cases to the doctrine of logical psychologism. This, anyhow, is the hypothesis of Godden and Griffin (2009, p. 184):

> Placing logic on a linguistic foundation seems to expose it to the contingencies of psychology, at least to the extent that psychology is involved in explaining linguistic meaning, whether through propositions or mental images.

This is a challenge that requires meeting, especially given the damning view of logical psychologism in most philosophical circles.[236]

Psychologism has two forms, a metaphysical and an epistemic form:[237]

Metaphysical Psychologism

Phenomena *P* supervene on mental phenomena *M*.[238]

Epistemic Psychologism

Theories explaining phenomena *P* are explanatorily reducible to empirical psychology.[239]

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[236] To note several cases:

> Both logic and psychology, if they are to exist at all, must remain each in principle independent. The undistinguished use of both at once must, even where instructive, remain in principle confusion. And the subordination of one to the other, whenever seriously attempted, will never, I think, fail to make manifest in its result the absurdity of its leading idea. (Bradley (1922) p. 613).

> [A]ll attempts to ground the fundamentals of logic on psychology are seen to be essentially shallow. (Peirce (1934) 5.28. Cf. Peirce (1982) p. 164).

> The laws of logic, while they are customarily called 'laws of Thought', are just as objective, and depend as little on the mind, as the law of gravity. (Russell (1992) p. 136).

> [Psychologism is] the tendency to confuse *logical* issues with *psychological* issues. (Pap (1958) p. 435).

> Twentieth-century analytic philosophy is distinguished in its origins by its non-psychological orientation. (Hacker (1996) p. 4).

> The very foundation of analytic philosophy…[is] the principle that logic and psychology are *categorically* divorced from one another. (Shanker (1998) p. 65).

[237] Many other definitions of psychologism have been advanced since the nineteenth-century, but most of these lack precision and theoretical interest. See Kusch (1995, pp. 118-120) for a survey of the definitions of psychologism found in early twentieth-century philosophical literature. Kusch quotes *twenty-one* non-equivalent definitions of psychologism in Moog (1913, 1917, 1918 & 1919).

[238] Found in Brockhaus (1991, p. 494), Godden & Griffin (2009, p. 172), Mohanty (2003, pp. 115-116), Pelletier *et al.* (2008, p. 4), Wheeler (2008, p. 137), and Willard (1989, p. 287). It's unclear whether it's *strong* or *weak* supervenience that's operative here, however the differences between the forms have no impact upon our discussion. For the distinction between *strong* and *weak* supervenience, see Kim (1987).

[239] Found in Aach (1990, p. 315), George (2003, p. 43), Hanna (2006, p. 1), Jacquette (2001, p. 261), Jerusalem (1905, p. 78; cited in Kusch (1995, p. 115)), Mohanty (1989, p. 2), Musgrave (1989, p.

These two forms of psychologism are related in so far as empirical psychology attempts to explain mental phenomena. If we assume that,

1) If phenomenon *P* supervene on phenomenon *Q*, then we can give a reductionist explanatory model for *P* in terms of *Q* (the subvenient domain),

2) Empirical psychology is the only research area that can adequately explain mental phenomena,

and

3) Empirical psychology is only engaged in explaining mental phenomena,

then the two forms of psychologism entail one another.[240]

The type of psychologism that one is interested in, whether it's metaphysical or epistemic, is determined by the type of phenomena that *P* stands for. So, logical psychologism is either of the theses that,

> **Logical Metaphysical Psychologism**
> Logic supervenes on mental phenomena,
>
> **Logical Epistemic Psychologism**
> Theories explaining logic are explanatorily reducible to empirical psychology,

or both. There are as many 'psychologisms' as there are phenomena that could allegedly supervene on mental phenomena, or be explained by empirical psychology.[241] For the sake of this section, we are concentrating solely on logical psychologism, for the reason that the subject of our logician's theorising is logic.

It isn't the goal of this section to deflect the accusation of logical psychologism by showing that the justifications for the absurdity of such psychologism are wayward.[242] Instead,

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315), Notturno (1989a, p. 162), Pandit (1971, p. 86), Richards (1980, pp. 19-20), Stumpf (1892, p. 468; cited in Kusch (1995, p. 103)), and Yagisawa (1989, p. 458).

240 Assumption 1) is particularly dubious. Cases of 'explanatory gaps', such as materialism's hypothesised inability to explain qualia in Levine (1983), give us good reason to doubt 1).

241 For discussions of different forms of psychologism see the Notturno (1989b) and Jacquette (2003) collections.

242 On the success of Frege's and Husserl's alleged refutations of logical psychologism, see Meiland (1976), Baker & Hacker (1989), Aach (1990), Cohen (1998), Mohanty (2003), and Hanna (2006, pp. 8-9). While rejecting Frege's and Husserl's criticisms of logical psychologism Hanna (2006, pp. 9-21) offers his own, arguing that logical psychologism is a version of scientific naturalism and that scientific naturalism is false.

we aim to demonstrate that our logician doesn't commit herself to either form of logical psychologism. This we can achieve by showing that neither empirical psychology, nor mental phenomena, play any important role in our logician's justification for her theory revision in the two cases.

Logic, as a model of truth-preservation, is concerned with the consequence relation between natural language sentences (or some other truth-bearer). Therefore, to be guilty of logical psychologism in this circumstance would require either that one's theory assumed that consequence relations were dependent upon mental states, or that one was committed to one's theory of logical consequence being fully reducible to a theory of empirical psychology. Our logician, however, seems to neither assume that consequence relations between sentences are dependent upon mental states, nor commits herself to theories of logical consequence being fully reducible to empirical psychological theories.

Beginning with logical *metaphysical* psychologism, in justifying her decision as to which logical theory best models the truth-preservation of natural language arguments, our logician didn't assume that logical consequence was dependent upon mental facts. In answering the question of whether a sentence *s* is a logical consequence of a set of sentences **Σ**, our logician wouldn't state that the answer is dictated by an individual's mental state, or even the mental states of a group of individuals. Instead, she would state that whether *s* is a logical consequence of a set of sentences **Σ**, or not, is dependent on the logical forms of the sentences, and that their logical form is dictated by the occurrence of grammatical particles in the sentences that express logical constants.[243] Here the logical consequence relation of the premise-conclusion pair is determined by the meaning of particular special grammatical particles contained in the sentences, without any reference to mental phenomena. Our logician appeals to meaning, rather than mental states, to explain facts of logical consequence.

Although our logician fails to appeal to mental states, there is still the possibility that she is committed to metaphysical psychologism. Given that logical consequence is dependent on logical form, logical form is dependent on the logical constants expressed in a sentence, and which logical constants are expressed within a sentence is dependent upon the *meaning* of the sentence, there is the additional unanswered question of the ontological basis of meaning. What if meaning, for example, is best understood as supervening on thoughts, a type of mental phenomena? In such a case, our logician would be committed to a form of *meaning* psychologism. Not only this, but if logical facts supervene on meaning facts, by transitivity, logical facts end up supervening on mental facts:

[243] We aren't ruling out the possibility here of sentences instantiating more than one logical form. It's pretty clear that sentences have different logical forms within different logical systems, such as the variation in logical form of 'All men are mortal' in propositional and first-order logics. Rather, the expression of members of the set of logical constants *C*, acknowledged in a logic *L*, by grammatical particles in a sentence *s* goes some way at least to constituting the logical form of *s* in *L*.

S1) Logical facts supervene on meaning facts
S2) Meaning facts supervene on mental facts
S3) Therefore, logical facts supervene on mental facts.

If the argument is sound, then our logician ends up being committed to *logical* metaphysical psychologism after all, contrary to appearances, given the supervenience relation between meanings and thoughts.

Now, granted both of the premises S1) and S2) above are dubious. The thesis that facts of meaning supervene on mental facts has been mentioned here as a suggestion at best, and it's unclear whether facts about logical consequence supervene on facts of meaning alone. The meaning of a sentence may not fully dictate its logical form. Indeed, if we understand the meaning of a sentence to be fully determined by its truth-conditions, then the failure of logical form to supervene on facts of meaning is pretty clear. We have noted on several occasions in this thesis that truth-conditional semantics fails miserably at accounting for logical form.[244]

However, the importance of highlighting the argument above isn't the present plausibility of the premises, but instead the general inference. If our logician accepts S1), and we find good reason to accept S2), then our logician will be committed to logical metaphysical psychologism. Thus, our logician must reject either S1) or S2) if she wishes to avoid what's being considered here an unfortunate commitment. Yet, in this regard, our logician is in no worse position than any other logician. She's no more committed to S1) or S2) than others. If it's possible to show that logical facts don't supervene on facts of meaning, perhaps because certain syntactic facts contribute to a sentence's logical form, then the rejection of S1) is as consistent with our logician's background assumptions as other logicians' theories. Our logician's commitment to investigating the meaning of certain grammatical particles in the natural language doesn't entail that she's committed to S1). Our logician is committed to facts of meaning contributing to the logical form of a sentence *s*, but no more so than other logicians. Similarly, if there are reasons to doubt the truth of S2), these reasons are as consistent with our logician's background assumptions as any other logicians'. There's nothing in our logician's theory that commits her to a particular ontology of meaning. Her theory is as consistent with a denial of S2) as it is with its acceptance. Our logician isn't committed to theses relevant to logical metaphysical psychologism any more than other logicians. Thus, the argument above is no more an argument against our logician than it is against any logician. If we find S1) and S2) to be true, then not only will we have to conclude that all logicians are committed to logical metaphysical psychologism, but we will have to reevaluate the putative absurdity of logical metaphysical psychologism.

[244] See sections 3.1.1 and 3.2.1 above.

Moving on to logical *epistemic* psychologism, there seems little reason to believe that our logician is committed to logical theories being reducible to empirical psychological theories. Even if we make the assumption that,

> S4) Linguistic data can be fully explained by psychological theories such that it becomes parasitically psychological data,[245]

so that our logician relies upon psychological data to establish the meaning of certain grammatical particles in natural language sentences, this psychological data only contributes to our logician's awareness of the logical form of sentences. A logical theory tells us much more than the logical constants that certain grammatical particles in sentences express. A logical theory demonstrates the logical consequences of a sentence *s* of a given logical form *F*.[246] It can achieve this either semantically, for example using truth tables, or syntactically with a proofing system. Thus, for our logician to be committed to the thesis that logical theories are theoretically reducible to empirical psychological theories, it needs to be demonstrated that,

> Psy) In virtue of psychological theories being able to give an account of the meaning of the grammatical particles expressing logical constants, which dictates the logical form of natural language sentences, psychological theories can give an account of the logical consequences of sentences.[247]

If we show that Psy) is false, we can establish that our logician isn't committed to logical epistemic psychologism. To undermine Psy), we need to demonstrate how our logician can use psychological data, under assumption S4), to determine the meaning of particular grammatical particles, without being committed to a psychological theory of logical consequence. The answer that our logician gives to Psy)'s challenge will be dependent on her theory of the meaning of logical constants, as this dictates what semantic properties a grammatical particle *g* must possess in order to express a logical constant *c*. We will consider our logician's response

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[245] We won't comment here on the plausibility of S4). It's enough that the assumption is necessary for the criticism to have any weight and that it's held by some, such as Godden and Griffin (2009), in the psychologism debate. If the assumption fails then so does the following argument.

[246] A logical theory needn't demonstrate all of the logical consequences of a sentence *s*, only those which *s* has in virtue of instantiating a particular logic form *F*. So, for example, classical propositional logic can demonstrate all of the logical consequences the sentence 'There are some black swans' has in virtue of instantiating the propositional form *p*, such as '*If* there are some black swans *then* there are some black swans'. However, propositional logic cannot demonstrate all of the logical consequences of this sentence *tout court*, for it has other logical consequences in virtue of instantiating other logical forms, such as the first-order form $\exists x(Bx \wedge Sx)$. Whether there is any logic that captures *all* of the logical consequences of every meaningful declarative sentence isn't important for our purposes here.

[247] Although the meaning of the grammatical particles in a sentence may not exclusively dictate the logical form of a natural language sentence, we will be assuming here that it does, as without this assumption the challenge from logical epistemic psychologism collapses.

to Psy) both from the presumption of a proof-theoretic account and a truth-conditional account of the logical constants.

Firstly, if our logician accepts a proof-theoretic account of logical constants, so that a grammatical particle *g* expresses a logical constant *c* in virtue of playing the same role in a set of inferences *I* as *c*, she could appeal to two different types of logical inferences; one of which psychological theories can capture, and the other they cannot. In drawing such a distinction, the logician can hope to explain why psychological theories can give an account of the meaning of grammatical particles expressing logical constants, while denying that psychological theories can fully explain the logical consequences of sentences. The distinction that our logician has in mind is that between meaning-constituting inferences and *non*-meaning-constituting inferences for a logical constant *c*, and thus potentially for a grammatical particle *g*. The meaning-constituting inferences for a grammatical particle *g* expressing some logical constant are a set of logical inferences *I* involving *g* so fundamental, that accepting the members of *I* constitutes the meaning of *g*. Therefore, in failing to reason according to these set of inferences *I* with *g*, one would be simply assigning *g* a different meaning, which is to say that *g* is being used to express a different logical constant (or none at all). This ensures that one cannot be making a *logical* mistake in failing to infer according to all the members of *I* with *g*, for failing to infer according to the members of *I* determines that one is assigning *g* a different meaning. Equally, "[f]undamental inferences that are meaning-constituting are not epistemically culpable, even if they are not supported by reflectively appreciable warrants. To demand more of a thinker is to demand the provably impossible," (Boghossian (2001a) p. 31). An individual is neither epistemically culpable for making these meaning-constituting inferences, nor susceptible to *logical* error, because she is setting up the rules for the meaning of *g*. The convention of *g*'s meaning is being established, and thus there's no rational justification available for the inference, nor is there the possibility of a *logical* mistake involving these inferences. Instead of making a *logical* error, an individual failing to infer according to the members of *I* with *g* is giving *g* a different meaning. It's possible that such an individual is making a *linguistic* mistake in failing to fall into line with the rest of the community in assigning *g* a particular meaning, constituted by the set of inference *I*, but a *linguistic* mistake isn't a *logical* mistake.

Now, given that individuals cannot make *logical* mistakes with such meaning-constituting inferences, we have every reason to think that psychological theories, on the assumption of S4), can account for the meaning of grammatical particles expressing logical constants. However, as long as the set of meaning-constituting inferences for a particle *g* is a proper subset of the set of inferences involving *g*, admitting that there can be a psychological account of the meaning of *g* doesn't entail that there can be a psychological theory of all the logical consequences of sentences involving *g*. It's open to our logician to suggest that individuals often make many logical errors when it comes to these more complex, non-meaning-constituting, inferences, which undoubtedly they do.[248] Thus, our logician can recognise that we cannot rely on the psychological data of how people *tend* to infer with these non-meaning-constituting inferences to dictate our logical theory of consequence. Instead, we can only logically rely upon the meaning-constituting inferences for a particle *g* they reason according to. The distinction between meaning-constituting and non-meaning-constituting inferences gives our logician the justification to reject Psy). One can rely upon a psychological theory, under the assumption of S4), to give the meaning of the grammatical particles, without being committed to the view that psychological theories can give an account of the logical consequences of sentences.

Psy) envisages two roles for psychological theories,

R1) Psychological theories can give an account of the meaning of grammatical particles that express logical constants,

R2) Psychological theories can give an account of the logical consequence of sentences.

The distinction between meaning-constituting and non-meaning-constituting inferences allows us to demonstrate how psychological theories can fulfill role R1) without fulfilling role R2). This justification for the rejection of Psy) relies upon three assumptions:

A1) The meaning of grammatical particles expressing logical constants are given by proof-theoretic semantics.

A2) There's a distinction between meaning-constituting and non-meaning-constituting inferences for those grammatical particles *g* that express logical constants.

A3) We have access to the meaning-constituting inferences for these grammatical particles.

Although we have separated these assumptions, A2) and A3) are assumptions made by any plausible version of proof-theoretic semantics. Given that we do have access to the meaning of grammatical particles, if the meaning of a grammatical particle *g* is dictated by a set of inferences involving *g*, we must have access to these inferences. Additionally, identifying the set of inferences *I* that constitute the identity of a logical constant *c*, and parasitically the meaning of a

[248] Famous cases of individual's making logical errors are Wason's selection task (see Wason (1966 & 1968) and Wason & Shapiro (1971)) and the conjunction fallacy (see Tversky & Kahneman (1983)). Findings of studies on other logical errors are noted in Stanovich & West (2000) and Oaksford (2005).

grammatical particle *g*, is a genuine problem. Yet, this is a problem for the proof-theoretic account of logical constants in general, and not particularly for our logician. The assumptions that our logician has made in this denial of Psy), are assumptions that any plausible proof-theoretical semantics for the logical constants must make. If one of A1-A3) is unacceptable, then this will only require our logician to endorse a different account of the meaning of the logical constants to adequately answer Psy)'s challenge.

This brings us onto our logician's second option for refuting Psy), on this occasion assuming a truth-conditional account of the logical constants. On this account, a grammatical particle *g* expresses a logical constant *c* in virtue of sharing certain truth-conditional properties with *c*. In this case our logician can explain how psychological theories can fulfill role R1), but not R2), by appealing to the difference between a psychological theory giving us access to certain meanings and the theory explaining the logical consequences of sentences containing these meanings.

We are assuming that psychological theories can fulfill the role of R1). After all, we do seem to understand the meaning of grammatical particles expressing logical constants, and this can only be because linguistic data give us access to the meaning of a grammatical particle *g*. Given our assumption S4), it's entailed that psychological theories can account for the meaning of the grammatical particles expressing logical constants. The question is whether psychological theories must fulfill role R2) by virtue of fulfilling R1).

For a psychological theory to explain the logical consequences of sentences as a logical theory does, the theory must have access to more than the meaning of grammatical particles that express logical constants. The psychological theory must explain, using solely mental phenomena, how a sentence *s* is a logical consequence of a set of sentences Σ, which cannot be achieved by solely giving the meaning of certain grammatical particles in terms of truth-conditions. Demonstrating the logical consequences of a set of sentences Σ relies upon showing how these sentences jointly *entail* another set of sentences Γ, in virtue of the sentences containing (combinations of) grammatical particles that express logical constants. In logical theories, we are able to explain these logical consequences in terms of interpretations. If logical theories are to be fully reducible to psychological theories, however, then psychological theories cannot rely upon these logical devices to account for truth-preservation. Instead, they must appeal to some mental phenomena to account for the entailment of sentences.

Unfortunately for any attempt to reduce logical theories of consequence to psychological theories, the closest that a psychological theory can get to accounting for the entailment relation solely in terms of mental phenomena, is by appealing to data on how individuals actually *infer* some sentences from others. Yet, no study of inference is ever going to adequately capture logical consequence. The subject of logical theories of truth-preservation is not inference but implication. By attempting to account for logical consequence with inferences, a psychological theory would both overgenerate and undergenerate validity. The psychological theory will overgenerate validity because individuals often make logical mistakes in their inferences, which ensures that the theory will categorise as valid certain arguments which the logical theory it's attempting to accommodate classifies as invalid.[249] Similarly, the psychological theory will undergenerate validity because, in classical propositional and first-order logics at least, every meaningful declarative sentence has an infinite number of logical consequences.[250] As it isn't possible for any individual, or group of individuals, to perform an infinite number of inferences, no psychological theory can fully model the logical consequences of sentences in terms of inferences. This distinction between gaining access to the meaning of grammatical particles expressing logical constants, and *generating* logical consequences from sentences containing these particles, demonstrates that a theory can fulfill the role of R1) without fulfilling R2). This, again, gives us good reason to reject Psy).

We have given two good reasons to reject Psy), one from the presumption of a proof-theoretic account of the logical constants, and another from the presumption of a truth-conditional account. This demonstrates that, unless there are any plausible novel accounts of the meaning of the logical constants, there's nothing particular about our logician's background assumptions that commit her to logical epistemic psychologism. In conjunction with our demonstration that she isn't any more committed to logical metaphysical psychologism than other logicians, this shows that our logician has nothing to fear from accusations of logical psychologism.

8.6 Maybe Logic is Empirically Revisable…

In this chapter we have clarified the question of whether empirical evidence can ever justify a revision of logic. Having settled on the question of whether empirical evidence can justify a rational revision of a logical theory of truth-preservation, we then considered two putative cases of empirical evidence justifying just such a revision. The first argued that empirical evidence could justify a revision from traditional syllogistic to first-order classical logic, and the second argued that empirical evidence could justify a revision from first-order

[249] "[The] laws of reasoning are, properly speaking, the laws of right reasoning only, and their actual transgression is a perpetually recurring phenomenon… We must accept this as one of those ultimate facts," (Boole (1854) p. 408).
[250] To give two logical rules of inference in classical propositional logic which demonstrate this:

L1) For any proposition A derive $\sim(A \wedge \sim A)$.

L2) For any proposition A derive $A \rightarrow A$.

Every iteration of L1) and L2) produces new logical consequences of the original proposition A, *ad infinitum*.

classical to partial logic. In both cases, the disagreement between the logics focused around the validity of *Darapti*. We then considered two objections to our cases: that in neither case was there disagreement between the logics, as a change of doctrine is tantamount to a change of subject, and that both cases included an unacceptable commitment to logical psychologism. Both objections were found to be toothless. It seems then that we possess two reasonable possible cases of empirical evidence justifying a revision of a logical theory of truth-preservation. If we are going to continue to endorse the empirical indefeasibility of logical theories, we will need to find good reason to discount the force of the two cases presented in this chapter.

8.7 Consequences for Absolutism

While the case for the empirical defeasibility of logical theories presented in this chapter could have a general influence upon how we conceive of the relation between linguistic data and our logical theories, it could also have an important impact upon the debate between the absolutist and classical logician. In chapter 3 we found that contradictions are most plausibly conceived as the conjunction of a proposition and its negation. This entailed that the debate between the absolutist and the classical logician hinges upon whether conjunctions of propositions and their negations can be true or not.

Consequently, whether a contradiction can be true will be partly dependent, at least, upon the semantics of conjunction and propositional negation. Yet, the relevant question isn't whether the *classical* conjunction of a proposition and its *classical* negation can be true, or whether the *dialetheic* conjunction of a proposition and its *dialetheic* negation can be true; after all, we know the answer to both these questions. Instead, by enquiring into the semantics of conjunction and propositional negation, the logical community is questioning which logical theory, if any, has successfully modelled the conjunctions and negations of propositions as they occur in natural language. We express contradictions in our natural language, and thus those conjunctions and negations of propositions that we are interested in are those that are expressed in our natural language. Consequently, the debate between the classical logician and absolutist is over whether conjunctions of a proposition and its negation, as conjunctions and negations are expressed in our natural languages, can be true.

We know that 'London is the capital of England and two plus two equals four' is the *conjunction* of 'London is the capital of England' and 'Two plus two equals four', and we know that 'All men are mortal' is the *negation* of 'Some men are immortal' and that 'Emma isn't at the beach' is the *negation* of 'Emma is at the beach'. These are paradigm examples of what we mean by conjunction and propositional negation. The questions for the logician are,

1) In virtue of what properties is the proposition 'London is the capital of England and two plus two equals four' the *conjunction* of 'London is the capital of England' and 'Two plus two equals four', and similarly for other paradigm cases of conjunction?

and

2) In virtue of what properties are the propositions 'All men are mortal' and 'Emma isn't at the beach' the negations of 'Some men are immortal' and 'Emma is at the beach', respectively, and similarly for other paradigm cases of propositional negation?

Our findings in this chapter suggest that we can look to natural language for answers to 1) and 2) without any concern. We can look to natural language for an answer to what semantic properties 'All men are mortal' possesses in virtue of being the negation of 'Some men are immortal', and similarly for conjunction. In doing so, we have demonstrated that there is nothing absurd with natural language data informing the semantics of a logical theory without the theories collapsing into logics of inference.

In debating whether contradictions can be true, the parties are questioning whether there is anything intrinsic about the meaning of a conjunction, or the relation between a proposition and its negation, which ensures that the conjunction of a proposition and its negation cannot be true. By looking at the meaning of conjunctions and the negations of propositions in our natural language, we may gain access to just the data we need to answer this question. After all, if contradictions are conjunctions of a proposition and its negation, and we express contradictions with some natural language sentences, then we must express both conjunctions and negations of propositions in our natural language. By studying the meaning of these sentences, we may gain some insight into the meanings of the conjunctions and negations of propositions.[251]

By giving us access to the meaning of conjunctions and propositional negations, this natural language data may indicate whether there is any characteristic of the meaning of either conjunctions, or the negations of propositions, which ensures that the conjunction of a proposition and its negation cannot be true. To this extent, gathering natural language data on the meaning of conjunctions and propositional negations may be a potentially fruitful area of research for both parties in the debate. The previous reasons for trepidation that a logician may have felt, in allowing for linguistic data to influence her logical theory, have hopefully

[251] We aren't suggesting here that there's been a paucity of research conducted on the meaning of conjunction and propositional negation in natural languages. Horn (1989) and Miestamo (2007), for example, detail the vast, and important, research that has been carried out studying negation. Instead, we are suggesting that it's often only once that research is carried out with a theoretical disagreement in mind, that we come to recognise the relevance of certain data to the disagreement.

been dissolved by our findings in this chapter. Unless any new considerations can be brought against the cases presented, it *can* be rational for empirical evidence to justify a revision of one's logic, and allowing empirical evidence to influence one's logical theory isn't to commit oneself to a form of logical psychologism.

Of course, gathering this linguistic data, even if it does offer insights into the properties of conjunctions and negations of propositions, may not settle conclusively the debate between the absolutist and the classical logician. Even if we found no linguistic evidence that suggested the semantic properties of conjunctions and propositional negation preclude the possibility of a contradiction being true, this will not conclusively demonstrate that absolutism is true. For example, there may be certain metaphysical considerations that preclude the truth of a proposition and its negation, although the semantic properties of conjunctions and propositional negations themselves don't. None of these considerations, however, nullify the utility of gathering the relevant linguistic data. If we find no evidence that the semantic properties of conjunction and propositional negation preclude the truth of contradictions, this will only demonstrate to the classical logician that she needs to explore other theoretical avenues if she's to suitably meet the absolutist's challenge.

With this potential route of research, we can see a distinction between the challenges that the dialetheist and absolutist present the classical logician. The dialetheist commonly suggests that the truth of particular contradictions are the unforeseen consequences of certain properties of natural languages, such as their being semantically closed.[252] In contrast, the absolutist may suggest that the possibility of true contradictions is a simple consequence of conjunction and propositional negation, as they are expressed in natural languages, failing to preclude the truth of contradictions, properly understood as the conjunction of a proposition and its negation.

Regardless of whether this linguistic data ultimately provides evidence for one of the party's positions in the debate or not, the findings from both chapter 3 and the present chapter demonstrate that one cannot preclude the possibility of linguistic data playing some meaningful role in the debate over the possible truth of contradictions.

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[252] To use Beall's (2009, p. 5) analogy with architecture and evolutionary biology, true contradictions are "spandrels" of language.

9. Conclusion: The Future of Absolutism

> [W]e cannot firmly establish in the present state of our research, that the unintelligibility of contradictory states of affairs, that is for us and for a vast majority of people a very strong *feeling*, is not a mere product of a millennium of tradition, originated in contingent facts.
>
> (Miró Quesada (1989) p. 650, fn. 58)

In our introduction, we set ourselves four goals:

1) To make clearer the absolutist's challenge by clarifying the notion of a true contradiction.
2) To tackle the question of whether the absolutist's position can be stated in a philosophically and logically coherent way.
3) To remove from the field of debate some arguments against the possibility of true contradictions, clearing the way for better ones.
4) To point the way towards future research that may produce evidence for either the possibility or impossibility of true contradictions.

Our findings in the previous chapters suggest that we have achieved all these goals, at least to some extent.

In chapter 2 we proposed absolutism as the thesis that it's metaphysically possible for at least some contradictions to be true, and formulated the thesis as Ab). The following chapter considered the numerous available non-equivalent definitions of contradiction to embed within Ab), and we found that one definition of contradiction, the truth-value neutral account, was far more plausible than its competitors. Appreciating the plausibility of the truth-value neutral account of contradiction allowed us to give full expression to absolutism, the thesis that it's metaphysically possible for at least some conjunctions of a proposition and its negation to be true, formulated as Ab_C). Additionally, we found that the truth-value neutral account of contradiction could be embedded within Ab) without precluding the truth of the absolutist's thesis; a point we revisited in chapter 6. By evaluating the available definitions of contradiction, and embedding the most plausible within the absolutist's thesis, we both fulfilled our first goal of clarifying the notion of a true contradiction, and went some way to establishing that absolutism can be stated in a philosophically and logically coherent manner.

The following two chapters went even further in demonstrating that absolutism is a philosophically coherent position, by giving the thesis a suitable logic. We gave details of both non-dialetheic and dialetheic paraconsistent logics, and proposed an absolute paraconsistent logic **AV** to model absolutism. By demonstrating that the most plausible definition of contradiction available in the literature doesn't preclude the truth of the absolutist's thesis when embedded within Ab), and that there is an absolute logic available to model absolutism, we have subsequently demonstrated that absolutism can be given a philosophically and logically coherent expression. This ensures that the classical logician cannot simply reject the absolutist's thesis as nonsense.

In our consideration of Slater's argument against the possibility of paraconsistent logics in chapter 6, we found that the argument was unsuccessful both when its target was paraconsistent logics as a whole, and when its target was refined to only include dialetheic logics. Slater's argument against these logics is that they presume that formulae symbolising contradictions can be assigned the truth-value true, which they can't in virtue of the definition of *contradictories*. It was enough to revisit our conclusions from chapter 3 to demonstrate that neither a plausible definition of contradictories, nor contradictions, entail by themselves that formulae symbolising contradictions cannot be assigned the truth-value true. Considering in depth Slater's argument should hopefully put to bed once and for all the argument that dialetheism is absurd because contradictions can't be true *by definition*.

Our consideration of Slater's argument and the potential argument from the *apriority* of classical logic in chapters 6 and 7, respectively, and our subsequent conclusion that neither are damaging to absolutism, constitutes a partial fulfillment, at least, of our third goal. We can quite confidently conclude that neither the definition of contradiction, nor the putative *apriority* of classical logic, are viable threats to the truth of absolutism.

While we have failed to highlight more promising arguments against absolutism, this is for a good reason; at present, it's difficult to conjure any up with sincerity.[253] We simply don't know enough yet about contradictions. If our conclusions regarding the available accounts of contradiction in chapter 3 are accurate, then contradictions are a conjunction of a proposition and its negation. Whether contradictions can be true or not, therefore, will depend heavily upon the meaning of conjunction and the relation of propositional negation. However, it's currently unclear what data either side could propose as evidence for the truth of their respective thesis.[254] How does one establish whether the meaning of conjunction or propositional negation permits contradictions to be true or not? It is in the context of this potential stalemate, that the prospective importance of the previous chapter's findings comes into focus.

With logic's historical status as an *a priori* area of enquiry, logicians have in general distanced themselves from the possibility of using empirical evidence to motivate logical theory choice. By appreciating that logical theories of truth-preservation can be rationally

[253] Priest (2004 & 2006a) has made a point of demonstrating how poor many historical arguments against the possibility of true contradictions are.
[254] This stalemate obviously doesn't occur in the debate between the dialetheist and classical logician over the *actual* truth of contradictions. Both parties in that debate have the putative instances of true contradictions to use as data.

revised based on empirical evidence, however, we come to recognise potential sources of data that could lead to one of the parties in our debate gaining evidence for their position. As we noted in the previous chapter, by gathering data on the meaning of conjunctions and propositional negations that are expressed in our natural language, we may gain evidence that could both motivate our logical theory choice, and give us reason to believe that contradiction can or cannot be true. In opening up the possibility of empirical evidence justifying our choice of logic, we have subsequently drawn attention to an area of research that may produce evidence for either the possibility or impossibility of true contradictions. If our conclusions in the previous chapter are correct, then we have no reason to fear linguistic data influencing our logical theories.

Our goal in this thesis was never to establish absolutism's truth. Instead, it was to show in the strongest possible light the absolutist's challenge to the classical logician. Dialetheism has gone some way in the last forty years to call into question the consistentist principles of classical logic. All too often, however, philosophers are content to answer the dialetheist's claims either by putatively demonstrating that hypothesised true contradictions aren't true, or by asserting that 'Contradictions just can't be true, *by definition*'. Yet, while a demonstration that the dialetheist's putative examples of true contradictions are unsuccessful is in no sense a demonstration of the *impossibility* of true contradictions, we have shown in this thesis that the classical logician certainly hasn't established that contradictions can't be true *by definition*.

Given contemporary philosophy's exposure to dialetheism, one cannot help but be surprised by the paucity of reasonable arguments, found in the literature, against the *possibility* of true contradictions. The hope for this thesis is that it refocuses the classical logician's attention on to the principles that she endorses, just as threats of scepticism can refocus the epistemologist's attention. While demonstrating that dialetheism has failed to provide any actual cases of true contradictions is undoubtedly important for the classical logician, so is justifying her claim that contradictions *can't* be true. No consistent solution to the self-referential paradoxes will entail that contradictions *can't* be true.

Some necessary philosophical and logical groundwork has been done in this thesis to clarify the absolutist's challenge, and to present that challenge in its most coherent form. It is now up to the classical logician to meet the absolutist's challenge, and to recognise that she's under an intellectual obligation to show that contradictions *can't* be true, and not just that they *aren't* true.

Appendix A: A Survey of the Definitions of Dialetheism

Below is a survey of the definitions of dialetheism found in the literature. Each definition is categorised as defining dialetheism as one of:

TF: The thesis that some propositions are both true and false.
C: The thesis that some contradictions are true.
B: Both the thesis that some propositions are true and false, and the thesis that some contradictions are true.
O: Some other thesis.

Armour-Garb (2004, p. 114) **<u>B</u>**

Dialetheism is the view that some propositions (or, generally, truth bearers) are both true and false…Dialetheism is the view that there are true contradictions.

Armour-Garb & Beall (2001, p. 593) **<u>O</u>**

Dialetheists maintain that some sentences are equivalent to their own negations.

Armour-Garb & Beall (2002, p. 217) **<u>O</u>**

Dialetheists…believe that some truths have true negations.△

Armout-Garb & Priest (2005, p. 167) **<u>TF</u>**

According to dialetheism, paradoxical sentences are both true and false (i.e. have a true negation).◇

Armour-Garb & Woodbridge (2006, p. 395) **<u>TF</u>**

[*D*]*ialetheism*, the view that certain sentences are properly characterized as both true and false.

Batens (1999, p. 270) **<u>TF</u>**

The position defended there…is called dialetheic: some sentences are both true and false.

Beall (2004b, p. 197) **<u>O</u>**

Dialetheism is the view that some truths have true negations.△

Beall (2009, p. vii) **TF**

[T]he basic dialetheic claim: there are some true falsehoods.

Beall (2012, p. 517 fn.) **TF**

[D]ialetheism – the view that there are 'gluts' or 'true falsehoods', that is, true sentences of the form $A \wedge \sim A$.$^{\triangledown}$

Bremer (2008, p. 208) **C**

Dialetheism is the claim that some contradictions are true.

Brown (2002, p. 628) **O**

[Dialetheists] hold that the world is inconsistent.

da Costa & French (1990, p. 187) **C**

Dialetheism: the view that there are *true* contradictions

Everett (1996, p. 657) **C**

Dialethism, the thesis that our world contains true contradictions.

Kroon (2004, p. 245) **C**

Dialetheists – those who think there are true contradictions.

Littmann & Simmons (2004, p. 314) **B**

According to the dialetheist, there are sentences that are both true and false, and the LNC fails.

Mares (2004, p. 266) **C**

Dialetheism holds that there are true contradictions.

McGee (2004, p. 280) **TF**

The dialetheist – advocate of the thesis that there are statements that are both true and false.

Parsons (1990, p. 336) **TF**

[Dialetheism's] basic claim is that some unambiguous sentences are both true and false.

Priest (1995, p. 4) **C**

Dialetheism (the view that there are true contradictions).

Priest (2004, p. 29) **C**

The view that the LNC fails, that some contradictions are true, is called *dialetheism*.

Priest (2006a, p. 1) **B**

Dialetheism is the view that some contradictions are true: there are sentences… *p*, such that both *p* and ~*p* are true, that is, such that *p* is both true and false.

Restall (2002, p. 428 fn.) **C**

I am not a *dialetheist* who thinks that some contradictions are true.

Shapiro (2004, p. 336) **O**

Dialetheism is the view that there are propositions *p* such that both *p* and ~*p* are true.[○]

Tennant (2004, p. 355) **C**

Dialetheism – the view that there are true contradictions.

Weir (2004, p. 385) **C**

[T]he dialetheist's claim that there are true contradictions is provably false.

Zalta (2004, p. 419) **B**

[Dialetheism] is the doctrine that, in some of these really hard cases, there are indeed true contradictions. Dialetheists argue that some sentences are both true *and* false.

Notes

△ = If one conceives of a contradiction as the conjunction of a proposition and its negation, then this definition is equivalent to the definition of dialetheism as the thesis that there are true contradictions. In contrast, if one considers 'false' to simply mean 'has a true negation', then this definition is equivalent to the definition of dialetheism as the thesis that some propositions are both true and false.

▽ = If one conceives of contradictions as propositions of the form $A \wedge {\sim}A$, then this definition defines dialetheism as *both* the thesis that some propositions are both true and false, and the thesis that come contradictions are true.

◇ = If one conceives of a contradiction as the conjunction of a proposition and its negation, and conjunction has its normal semantics, then this definition defines dialetheism as *both* the thesis that some propositions are both true and false, and the thesis that come contradictions are true.

○ = If tilde symbolises negation in the definition, and one conceives of a contradiction as the conjunction of a proposition and its negation, then this definition is equivalent to the definition of dialetheism as the thesis that there are true contradictions.

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