Estimating the size of Earth's umbral shadow using sky brightness light curves during a lunar eclipse

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We present a simple method to estimate the size of Earth's umbral shadow in a classroom setting. The method uses the published sky brightness curves obtained during a total lunar eclipse and requires only a conceptual understanding of lunar eclipses and simple geometric considerations. It is suitable for use in introductory and upper level astronomy courses.
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Aristarchus of Samos was the first astronomer to estimate the relative sizes of the Earth and Moon and the distance to the Moon.[1] His method involved observing the time between the Moon's entrance to and exit from the Earth's umbral shadow during a lunar eclipse. By comparing this transit time with the time for the Moon to traverse a distance equal to its own diameter, Aristarchus determined the size of Earth's shadow in terms of the lunar diameter. From a single lunar eclipse observation, he found that the umbral shadow at the distance to the Moon was 2 times the lunar diameter. Once Aristarchus determined the distance to the Moon, he used simple geometry to determine the size of Earth relative to the Moon using the diameter of the umbral shadow and the Earth-Moon distance.[1,2] Today, we know that for a central lunar eclipse, in which the Moon passes through the center of Earth's shadow, the diameter of the umbral shadow averages 2.65 lunar diameters. The diameter of the umbral shadow can be as small as 2.578 lunar diameters if an eclipse occurs at lunar perigee or as large 2.735 lunar diameters if the eclipse occurs at apogee.

Cowley describes a classroom activity using lunar eclipse photographs to determine the Earth-Moon distance.[3] His method relies on the measured curvature of Earth's shadow, as first done by Hipparchus, using photographs found in textbooks. Bruning used Cowley's method on a single photograph containing several phases of the July 5–6, 1987 lunar eclipse.[4] The size of Earth's shadow can also be determined using observations of craters during a lunar eclipse: this technique uses the time it takes for the umbral shadow to traverse a crater of known diameter.[5] We propose a new eclipse classroom activity, using published light curves of night sky brightness taken during lunar eclipses to estimate the size of Earth's umbral shadow at the location of the Moon, employing the method of Aristarchus. This method is accessible to non-science majors and provides them with the experience of using real data to reproduce an important scientific experiment.

Figure 1 shows a schematic representation of the Moon's motion through the penumbral and umbral shadows during a total, central eclipse. Figure 2 shows a plot of night sky brightness during the umbral phase of a total lunar eclipse, measured at zenith on the night of January 20–21, 2019 in Morehead, KY. The specific details of the instrumentation and observing conditions are available in another paper and are not particularly important for the activity.[6] On the night of a lunar eclipse, the Full Moon will rise and the sky will begin to brighten. As the Moon enters Earth's umbral shadow (U1), the sky brightness will decrease. Once the entire Moon has entered the umbral shadow (U2), the sky will generally be about as bright as a New Moon night and will remain so until the Moon begins to exit the umbral shadow (U3).

The difference in the time between U2 and U1 and, likewise, between U4 and U3 represents the time it takes for the Moon to move one full diameter in the sky, assuming a central eclipse. The difference between U3 and U1 and, likewise, between U4 and U2 represents the total time for the Moon to travel through the full umbral diameter. To determine the size of the umbral shadow, you divide the average time for the Moon to traverse the umbral shadow by the average time it takes for the Moon to move one diameter,

$$D_{\text{umbra}} = \frac{(t_{U3} - t_{U1}) + (t_{U4} - t_{U2})}{(t_{U2} - t_{U1}) + (t_{U4} - t_{U3})} D_{\text{Moon}}. \quad (1)$$

Using the data from Fig. 2, we find an umbral shadow diameter of 1.9 ± 0.3 lunar diameters. Why is this value less than the average (and even the minimum)? The most likely interpretation is that the January 20–21, 2019 eclipse was not a central eclipse! Students can verify this by examining the eclipse data sheets available on NASA's Eclipse website.[7]

Our method assumes that the time elapsed between U1 and U2 and U3 and U4 actually represents the time for the Moon to move a distance equal to one full lunar diameter. However, this is only true for a central eclipse. It turns out that for non-central eclipses, this is an over-estimate of the time as the Moon will have to move further in the sky to be completely in the umbral shadow, as shown in Fig. 3. We can compare the average of these two times with the actual rate of motion in the sky. Recall that the synodic period of the Moon is 29.5 days. In one hour, the Moon moves

$$t = \frac{360^\circ}{29.5\ \text{days}} \times \frac{1\ \text{day}}{24\ \text{h}} = 0.508^\circ/\text{h}. \quad (2)$$

So in 60 min, the Moon moves a distance equal to one full lunar diameter. If the average of the two time differences, $t_{U2} - t_{U1}$ and $t_{U4} - t_{U3}$, is more than 60 min, can we argue that the eclipse is not a central eclipse? (This is understood by comparing positions U1 and U2 (or U3 and U4) in Figs. 1 and 3.

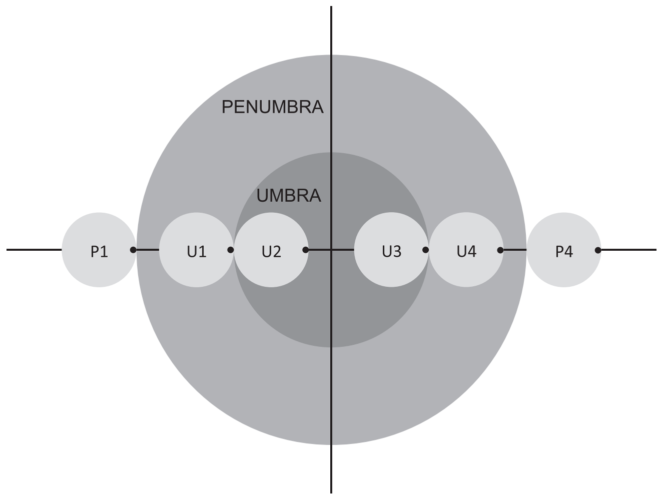

Fig. 1. A schematic of the progression of the Moon through the penumbral and umbral shadows during a central lunar eclipse. In this diagram, the diameter of the umbral shadow is 2.65 lunar diameters.

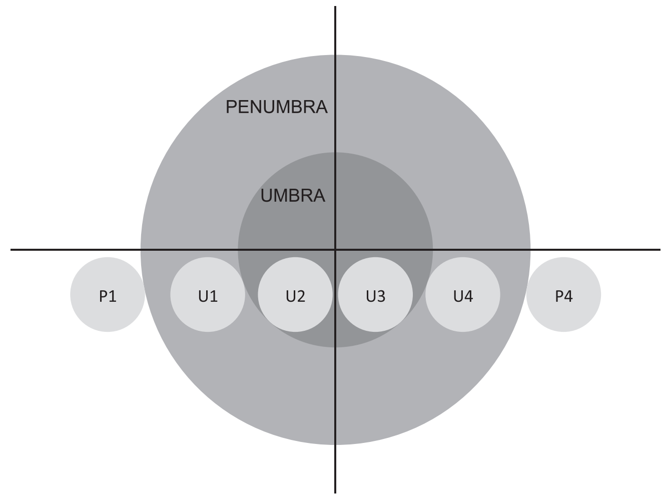

Fig. 3. In a non-central eclipse, the distance that the Moon must traverse between the U1 and U2 stages of the eclipse is greater than that in the central eclipse in Fig. 1. Thus, the time between U1 and U2 stages in a non-central eclipse overestimates the time to move one full lunar diameter in the sky.

In Fig. 1, the Moon must travel exactly one diameter to fully enter or exit the umbra, while in Fig. 3, it must travel more than one diameter. Since the Moon travels one diameter in every 60 min, it will take longer than this to enter or exit the umbra in a non-central eclipse.) If the time in the denominator is over-estimated, then the quotient in Eq. (1) is underestimated, and hence, our method will tend to underestimate the size of the umbral shadow in a non-central eclipse. If the eclipse is central, the night sky brightness curve should be symmetric. Note the asymmetry of the light curve of our non-central eclipse: would asymmetry be a characteristic of a non-central eclipse light curve? (If students examine the "Eclipse Wise"[8] data sheets for these three eclipses, they will find that the 1982 eclipse is very nearly a central eclipse, while those of 2004 and 2019 are definitely non-central. This eliminates non-centrality as a possible explanation for asymmetry in light curves. We posit that the observed asymmetry in the 2019 eclipse light curve is the result of changes in the lunar altitude, relative to the zenith observation point, during the eclipse. Without further data, we cannot test this hypothesis. This is a good lesson for students: more observations are often required to refute an explanation.) These questions should be accessible to general astronomy students in both a service course and physics majors. A question that you might ask physics majors: why do we not have to take into account Earth's motion around the Sun during the lunar eclipse? (The answer lies in Eq. (1): the diameter of the umbra is a ratio of two times, and the Earth's motion around the Sun affects both the numerator and the denominator in the same way.) This should stimulate discussions regarding the motion of the center of mass of the Earth-Moon system. Other ways to challenge upper level physics students include having them calculate the diameter of the penumbral shadow during the eclipse based on the known lunar distance[10] and some simple geometry.[11,12] More advanced physics students can consider whether stating a measurement uncertainty makes sense given that this method will

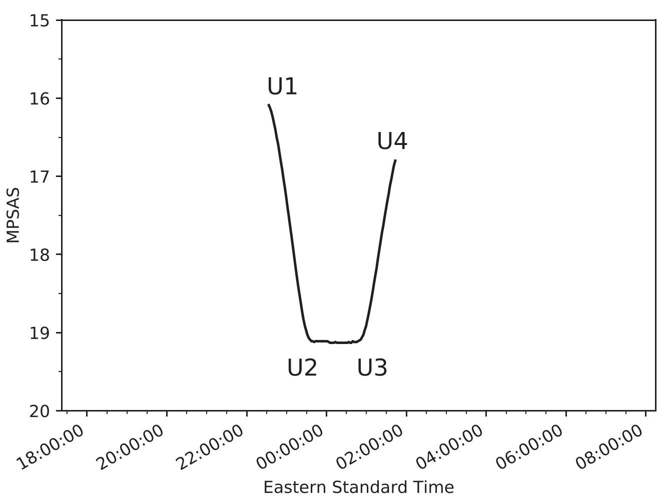

Fig. 2. Sky brightness at zenith during the January 20–21, 2019 total lunar eclipse as a function of local time. The units of sky brightness are in magnitudes per square arc second: recall that the magnitude is an inverse scale so that larger numbers indicate a darker sky. These data were obtained by the authors using a Unihedron Sky Quality Light Meter (SQM) located in Morehead, KY. This plot was created with the Matplotlib library in PYTHON (Ref. 9).

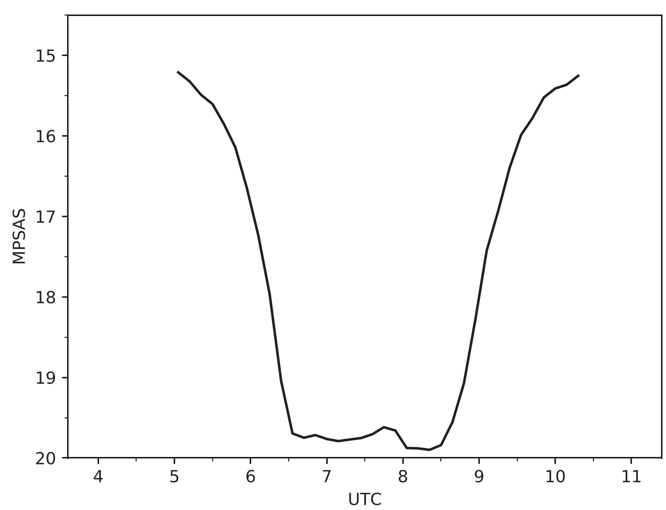

Fig. 4. Sky brightness curve from the July 6, 1982 total lunar eclipse. These data were obtained by Morton (Ref. 13) using the 31-in. refractor at Lowell Observatory. The telescope tracked a patch of sky 20° north of the Moon, tracking at lunar speed. This graph is reproduced using the WebPlotDigitizer (Ref. 15) online software using the automatic extraction feature.

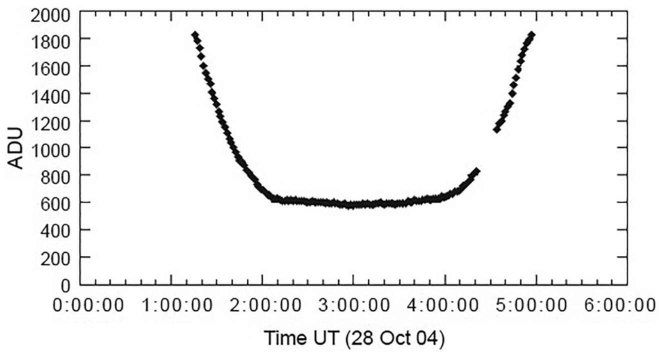

Fig. 5. Sky brightness on the night of the October 28, 2004 total lunar eclipse. These data were obtained by Shawn Dvorak using a telescope equipped with a CCD while observing the variable star QQ Cas (Ref. 14).

tend to underestimate umbral diameters. You can really make this activity challenging for upper level physics majors: provide minimal guidance on the analysis technique and have them read the three articles with sky brightness data, applying this method to all three sets of data.

Sky brightness curves for total lunar eclipses are rare. We have found only two other papers with night sky brightness light curves: one obtained during the July 6, 1982 total lunar eclipse, see Fig. 4, and another from the October 27–28, 2004 lunar eclipse, see Fig. 5.[13,14] Although these light curves were obtained using very different instrumentation, the umbral portions of the light curves share the same characteristics as our data. Neither of these authors use their data to estimate the size of the umbral shadow, and so students can use these data to estimate the size of the umbral shadow under different lunar eclipse conditions using our method. Our method is unique in that it employs the same basic ideas as Aristarchus' original method but uses light curves.

Given that our method tends to underestimate the size of the umbral shadow for non-central eclipses, why would you use this method? Well, nearly sixty percent of all lunar eclipses between 1999 and 3000 A.D. are central, total eclipses.[16] Our observation method,[6] using a Unihedron Sky Quality Meter pointed at zenith,[6] is simple and relatively inexpensive (about $250 US) and requires only a meter, a mount, and a clear eclipse night. If there is a total lunar eclipse in your future, a class or a single student could take similar data and use this method.

Also, note that the night sky brightness curve during a total lunar eclipse bears a striking resemblance to exoplanet transit curves. Exoplanet transits can be central or non-central, and this affects the length of the flat portion of their light curve similar to a lunar eclipse. The radius of an exoplanet affects the depth of the light curve. On the other hand, the depth of the sky brightness light curve during an eclipse should only depend on the lunar altitude and local night sky brightness.[17] The duration of exoplanet ingress and egress times coupled with the exoplanet transit time can be used to determine the radius of the exoplanet.[18] These parallels make our method a useful educational precursor to the astrophysical study of exoplanets!

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