

The Reflection Strategies on Students' Thinking Structures in the Mathematical Problem Solving Steps

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Abstract

Thinking is a process of understanding various things encountered in everyday life, finding certain opinions or ideas, making judgments, and solving problems. One of the tasks teachers must do in learning mathematics is to help students convey the cognitive processes experienced when solving issues. The thinking errors are and the algorithms that are not yet complete in solving student problems. But in reality, students still have difficulties that cause errors in solving mathematical problems. This study describes the reflection strategy on thinking to correct students' misconceptions in solving mathematical problems. This type of research is descriptive research with a qualitative approach. Research data consists of student answers, results think aloud, and the results of semi-structured interviews. The results showed that students with low problem-solving abilities experienced assimilation at the stage of understanding the problem only. In contrast, at the scene of developing strategies and implementing plans, accommodation occurred. The students will attack because students can decompose problems into simpler ones but require reflection to connect mathematical material to solve problems. In addition, students also need thinking to correct errors and complete incomplete algorithms when carrying out plans for solving mathematical problems.

Keywords: Reflection strategy; Thinking structure; Mathematical problem solving

INTRODUCTION

Problem solving is the activities as a series of actions that allow students to find solutions to problems (Tambunan, 2019; Tippmann et al., 2017). Problem solving is also a process that students do to overcome or solve a problem through stages, including defining the problem, finding out the main factors causing the pain, finding solutions, and applying these solutions so that existing problems can solve (Gog et al., 2020). Problem solving is very important to master because students will encounter various learning process types and everyday life (Mathew et al., 2019). Problem solving is a competency that must prioritize for students to apply and adapt strategies to solve other problems in different contexts (Halpern, 2014).

In solving mathematical problems, students' thinking processes occur when processing data or information to solve problems (Hamdani et al., 2021). The thinking process includes receiving information, managing, storing, and recalling that information through students' memories (Demirel et al., 2015; Rochana, 2018).

It is clear that students certainly do a process to make decisions and resolve internal thinking (Dorko, 2019). The thought or thinking process involves understanding the things encountered in everyday life, finding certain opinions or ideas, making judgments, and resolving the problem early, one of the tasks that must perform a mathematics teacher. I need to help students convey cognitive processes experienced during situation solving to know where the mistake of thinking is. The algorithm is not yet complete on solving the problem of students (Bormanaki, 2017).

But in reality, students still have difficulty in solving mathematical problem solving problems. This refers to evidence by the results of an initial study conducted by researchers in class XI MA Daruttauhid Malang MA Islam Sabilurrosyad showing that many students make mistakes. Based on the interviews, mathematics teachers at the school informed that errors cause because students had difficulty understanding the problem, creating a settlement plan, lack of knowledge of the prerequisite material, and not describing the reasons in detail regarding the solutions found. Thus, this problem is a factor causing students' thinking process errors. Based on the above issues, treatment needs can stimulate students to correct thinking process errors in solving mathematical problems. One possible solution to correct the mistakes and complete students' thinking processes is through reflection.

Reflection is an activity that aims to thoroughly review students' problem solving processes (Muhtadin, 2020). Reflection provides feedback in questions that stimulate students to study problem solving (Muhtadin, 2020; Ainurrohmah, 2016). The student can also reflect by giving students individual assignments in the form of questions that provoke students to find appropriate and accurate thinking steps (Wahyudi, 2020). Therefore, reflection is one of the solutions to develop students' problem-solving abilities because there is a critical effort by looking back at the mental processes that have been passed (Muhtadin, 2020).

Reflection can positively impact students' thinking stages to be structured and systematic (Muhtadin, 2020; Ainurrohmah, 2016). Through the review, students can also apply the theory gained from learning and practice it according to the problem's context (Muhtadin, 2020). In addition, reflection makes students understand the issue entirely and make plans well to get the correct answer (Mailani, 2017; Muhtadin, 2020; Stronge, 2009).

Several studies related to reflection have been carried out, including (Muhtadin 2020) research on defragmenting students' thinking structures through thinking to correct students' mistakes in solving story problems. Listiyani's research 2018. conducts reflections to measure the effectiveness of inquiry learning on student learning outcomes and activities. Ismayanti and Muhammad Arsyad's research worked reviews to improve students' creative thinking processes. Nuryana's research 2018 conducts reflection to stimulate students to improve the stages of problem solving.

The reflection in this study aims to reflect the structure of students' thinking to correct errors in solving mathematical problems by providing feedback that stimulates students to improve and complete the stages of thinking to be more optimal. There is no research on giving reflection to students' thinking structure to correct errors in solving math problems.

RESEARCH METHOD

This research method is a descriptive qualitative approach. Subjects were students of class Senior High School. The purposive sampling technique makes the subject selection. Research data in the form of student answers, think aloud, and the results of semi-structured interviews. Prospective subjects are 98 students who have obtained linear programming material. All students do a problem-solving test related to linear programming. Then the researchers grouped students based on problem-solving abilities, which referred to the problem-solving categorization of Samo (2017).

The research subjects taken were two students who had low problem-solving abilities coded with S1 and S2. Then the researcher gave a test in the form of questions adopted from PISA linear programming material, validation tests carried out to material experts and learning experts, and readability tests. The test contains questions used are as follows.

1. Dua media massa koran di Jakarta sedang membutuhkan orang untuk bekerja sebagai penjual koran. Iklan yang menunjukkan bagaimana kedua media massa membayar gaji penjual koran disajikan dalam Gambar 1 dan 2.



Gambar 1. Iklan Indopos

Gambar 2. Iklan Kompas

Melihat kedua iklan tersebut, Budi tertarik dan memutuskan untuk melamar menjadi penjual koran. Oleh karena itu, ia perlu mempertimbangkan antara bekerja di Indopos atau Kompas. Buatlah grafik yang menggambarkan bagaimana pendapatan pekerja kedua media massa koran!

Picture 1. PISA Problems for Linear Program Material

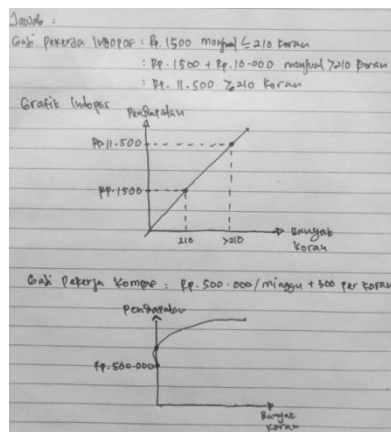
Data analysis aims to determine the shortcomings of students' thinking processes. It was known through errors and incomplete algorithms to become a guideline for researchers to do reflection. Furthermore, the data of students' thinking processes before and after reflecting on mathematical problem solving were reanalyzed through Piaget's (1959) theory of change in thinking schemes, namely accommodation and assimilation. The analysis technique carries through stages which include data reduction, data presentation, and concluding.

RESULTS AND DISCUSSION

A. Student Thinking Structure in Problem Solving and Mathematical Reflection

S1 is a subject that has common problem solving abilities. S1 can describe the issue more straightforwardly, but S1 cannot relate the problem to mathematical

material to solve the problem. The S1 thinking structure does not match the given situation. More details can see in Picture 2.

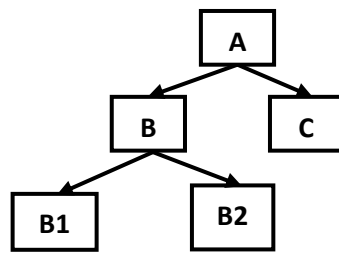


Picture 2. Subject 1 Work Before Reflection

Based on Picture 2, at the stage of understanding the problem, S1 does not describe the information in the answer, but through the results of think-aloud S1 illustrates that the income of Indopos workers is Rp. 1.500.00, a bonus of Rp. 10.000.00 per newspaper, the rest is sold from 210 newspapers. In addition, S1 also explained that the income of Kompas workers was Rp. 500,000.00 and a bonus of Rp. 300.00 per newspaper sold. By comparing the salaries of workers in the two newspapers. Then S1 also explained the information that asks, namely a graph depicting the income of Indopos and Kompas workers.

Furthermore, in formulating the strategy, S1 did not make an objective function first, but S1 immediately made a mathematical model and graph that described the income of Indopos and Kompas workers. However, S1 made a mistake where the income model of Indopos workers obtained 1,500 selling 210 newspapers and 11,500 selling >210 newspapers. In contrast, the income of Kompas modeling workers was Rp. 500,000.00 per week plus a bonus of Rp. 300.00 per newspaper. It explains that S1 cannot connect the problem with mathematical material obtain previously to solve the problem.

At the stage of implementing the plan, S1 also made an error, namely the shape of the income graph of Indopos and Kompas workers, which was incorrect because the cut line formed was linear. The error is, of course, caused by previous students' mistakes in formulating strategies. In addition, S1 does not conclude the answer regarding a graph that describes workers' income from the two mass media newspapers. The following scheme describes the undergraduate thinking process in solving mathematical problems:



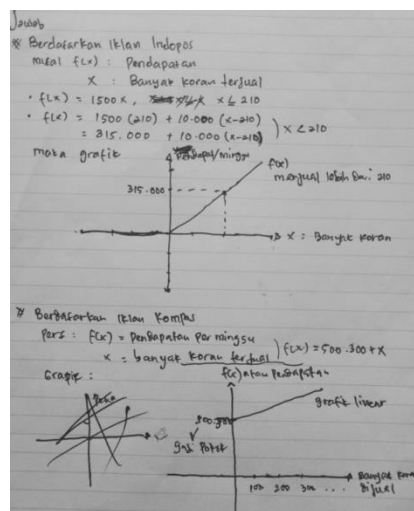
Information:

- A: Problem
- B: Information is known
- B1: Information known about Indopos
- B2: Information know regarding Kompas
- C: Information in question

Picture 3. Thinking Structure of Undergraduate Mathematics Problem Solving Before Reflection

Based Picture 3 informs that S1 can describe the problem but cannot connect the problem with mathematical material or concepts to solve the problem. It impacts S1's error in determining the graph cut line and the revenue function of the two newspapers. Therefore, the researcher provided a reflection that stimulated S1 to correct the mistakes and complete the thinking structure described as follows.

The researcher started by reflecting mathematical connections to stimulate S1 to recognize patterns in the problem so that S1 could recall mathematical concepts to build solutions. Researchers provide reflections related to S1 knowledge of linear programming and solve problems related to linear programming. Through this reflection, S1 can remember that the function must be objective in making a graph in a linear program. Then the researcher asked S1 to solve the problem by looking for the objective function following S1's knowledge of the understood linear programming material. The results of student work after being given a reflection are as follows.



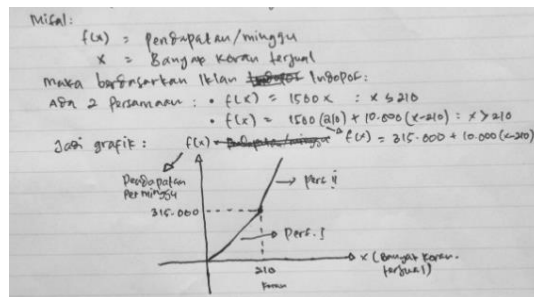
Picture 4. S1 Answers After Reflection Through Mathematical Connections

Based on Picture 4, S1 can connect the concepts studied with the problem after being given reflection. S1 realized that to create a graph, the first thing to find was the objective function of the worker's income. The hypothesis supported by the interview results that S1 assumes $f(x)$ is the income per week, with x as the number

of newspapers sold. In addition, S1 realizes that before finding a graph, the first is to determine the income objective function of each Indopos and Kompas worker.

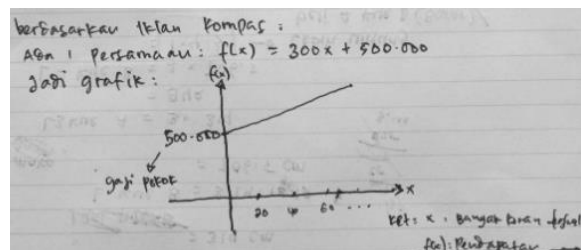
S1 creates an Indopos worker's income function, including $f(x) = 1,500x$; $x \leq 210$, and $f(x) = 315,000 + 10,000(x - 210)$; $x > 210$. S1 also creates a function for the income of Kompas workers, which is $f(x) = 500,000 + 300x$. However, in applying the S1 pattern, an error occurred in determining the intersection point of the second Indopos income graph. S1 only drew one straight line so that the graph cut line formed from the two Indopos income functions was linear. S1 also made an error in making the intersection point of the Kompas graph, namely the income graph of Kompas workers that S1 found did not match the example made.

As for the mistakes made by S1 in graphing the incomes of Indopos and Kompas workers, the researcher reflects through cognitive conflicts regarding the difference in workers' pay when selling 210 newspapers and 211 newspapers. Through this reflection, S1 can stimulate S1 to find out the difference in the intersection points of the graph formed so that S1 can conclude that the diagram depicting the income of Indopos workers has two straight lines with different slopes. The results of the repair of S1 can see in the image below.



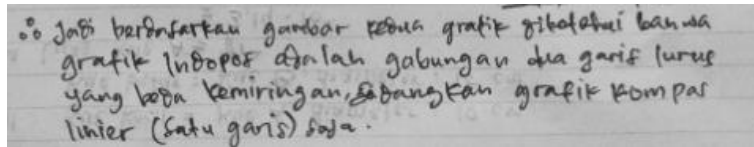
Picture 5. S1 Answers Graphing the Functions and Income of Indopos Workers

Next, the researcher gives a reflection, namely how to determine the Kompas worker's income graph based on the information contained in the advertisement. The thought given is in the form of questions related to how S1 did the previous example to create a function that describes workers' income. Through the reflection given can stimulate S1 to correct the error shown in the image below.



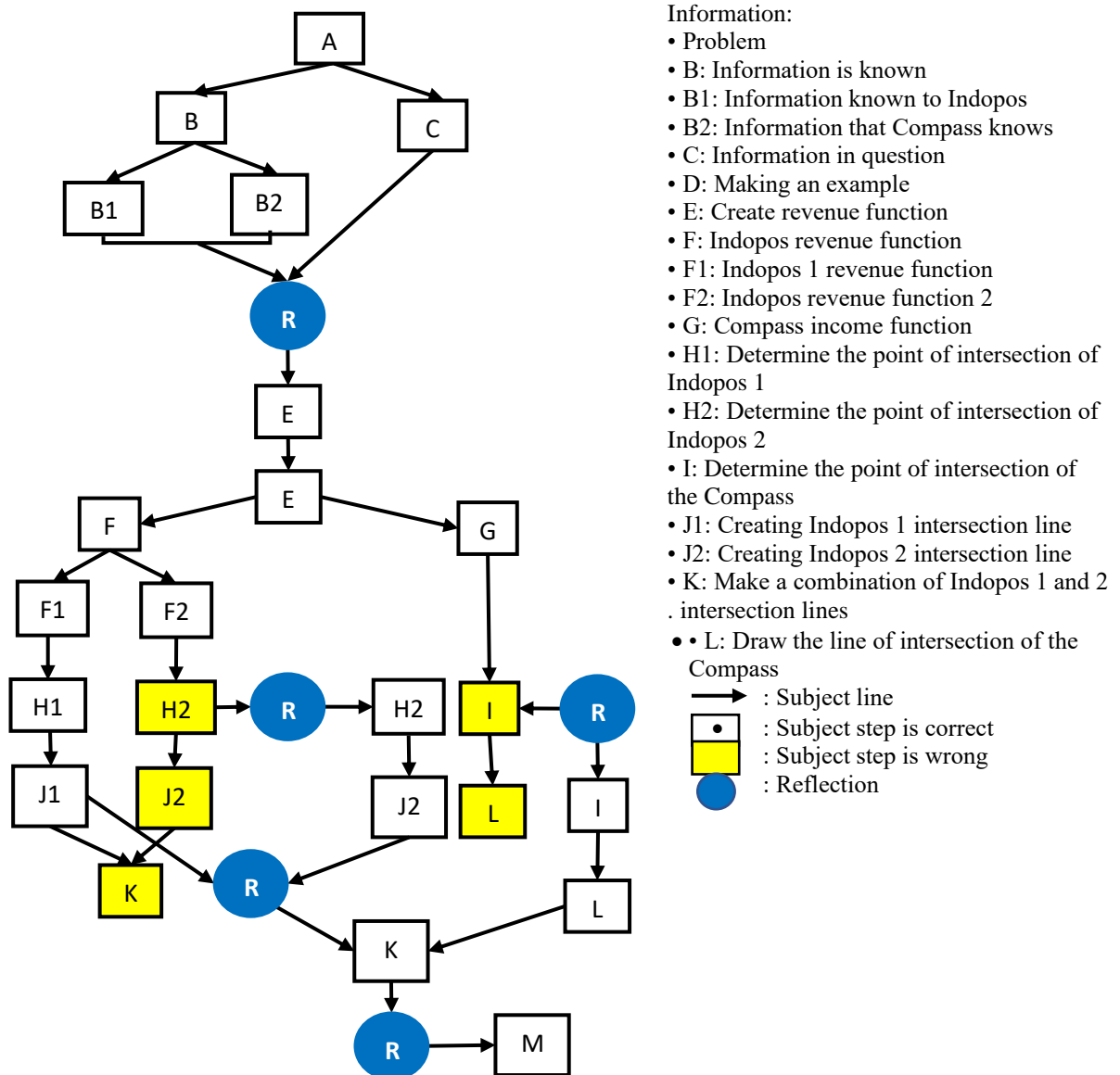
Picture 6. S1 Answers Create Functions and Income Graphs for Indopos Workers

After correcting S1's errors, the researcher gave a reflection aimed at S1 making conclusions about the solutions found. Through these questions, S1 can briefly describe that the income graph of Indopos workers has two different slopes, while the graph of workers' income is linear. The improvement of S1's answer show in Picture 5.



Picture 7. S1 answer pieces make conclusions

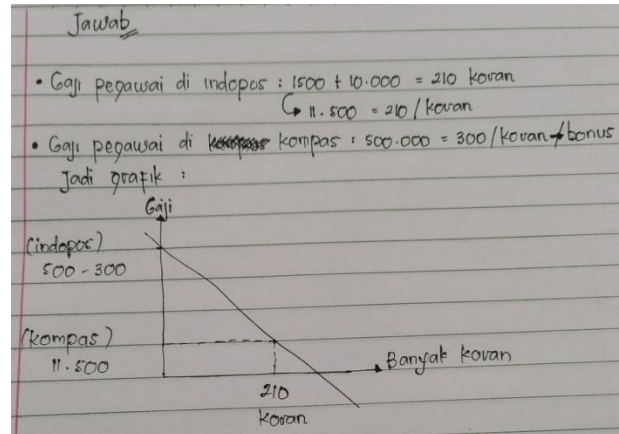
Based on the reflection process that has been described, the following scheme describes the S1 thinking process during the reflection process:



Picture 8. S1 Thinking Scheme on Solving Mathematical Problems During Reflection

B. S2 Thinking Structures in Problem Solving and Mathematical Reflection

S2 is a subject that has common problem solving abilities. S2 can describe the issue more simply but cannot relate the problem to mathematical material to solve the problem because the thinking structure possessed by S2 is not following the given situation. More details can see in Picture 8.

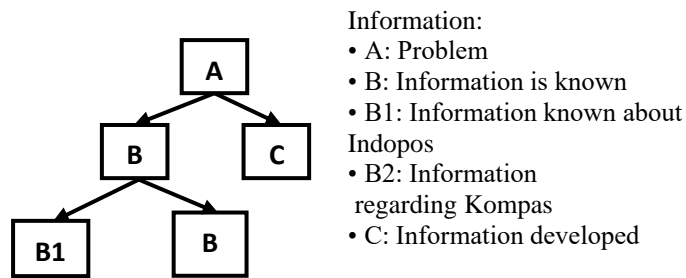


Picture 9. S2 Answers Before Reflection

Based on the picture above, it can see that when understanding Master's problem, She described the problem by comparing the salaries and bonuses of the two mass media newspapers. S2 explained that workers' income at Indopos is Rp. 1.500.00 if they sell 210 newspapers and a bounty of Rp. 10,0000.00 per newspaper for the remainder sold. S2 also found that the weekly income of Kompas workers was Rp. 500.000,00, and there is a bonus of Rp. 300.00 per newspaper. In addition, the results of postgraduate interviews describe the information asked, namely a graph depicting the income of Indopos and Kompas workers.

Furthermore, in formulating the Master's strategy, he immediately made mathematical models and graphs describing workers' income in Indopos and Kompas. S2 determines the points and intersections on the chart by simply matching the workers' pay and the number of newspapers sold in one week without creating an objective function. It proves that S2 cannot recognize patterns in the questions because S2 cannot relate the questions to the mathematical concepts or materials studied.

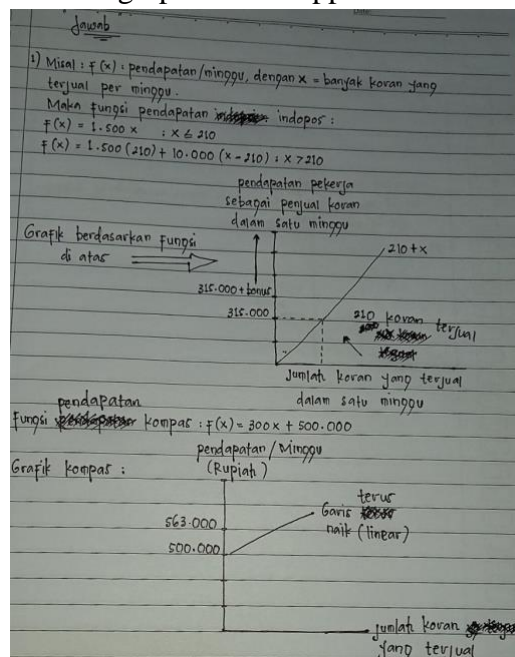
The teacher's inability to relate the problem to mathematical concepts or material to solve the problem ensures that the teacher makes mistakes in carrying out the plan. S2 determines the intersection point of the graph by performing the addition operation between the salary and bonus of the worker. In addition, the graph formed has parallel lines that depict the wages of Indopos and Kompas workers remain the same. The following is a schematic that describes the S2 thinking process in solving math problems:



Picture 10. S2 Thinking Structure on Mathematical Problem Solving Before Reflection

Based on Picture 9, it knows that S2 cannot connect problems with mathematical material or concepts to solve problems. It also impacts S2's error in determining the cut line of the graph and the revenue function of the two newspapers. Therefore, the researcher provides stimulating reflections on the Master's degree to correct errors and complete the thought structure described below.

The researcher started by reflecting on S2 to do pattern recognition. Reflection is through mathematical connections. The researcher demonstrates a Master's knowledge about the steps of solving linear programming problems. Through these reflections, S2 can recall that in solving linear programming problems, what must do is to determine the sample variables followed by creating an objective function to draw graphs. The snippet of the answer below evidences it.



Picture 11. S1 Answers After Reflection Through Mathematical Connections

Based on Picture 10, S2 assumes weekly revenue as $f(x)$ and the number of newspapers sold as x . Then S2 determines the objective function and then draws a graph of the income of Indopos and Kompas workers. S2 makes the income function of Indopos workers, including $f(x) = 1,500x; x \leq 210$, and $f(x) =$

$315,000 + 10,000(x - 210)$; $x > 210$. The compass income function found by S2 is $f(x) = 300x + 500,000$. S2 can find characteristic equations and connect the problems found with the material obtained previously, namely a linear program, after being given reflection.

Furthermore, in implementing the plan, S2 makes a graph that describes the income of Indopos workers per week. S2 replaces the value of $x = 210$ with $f(x) = 1500x$ to make the intersection between the worker's income and the number of newspapers sold 210 newspapers. Then S2 is substituted again to determine the cut-off point on the second Indopos income function to know the change in the cut-off point if selling newspapers more than 210, but S2 makes an error so that the intersection line of the graph formed from the two Indopos income functions is linear. In addition, after finding two forms of diagrams that describe the incomes of Indopos and Kompas workers, S2 does not conclude the answers obtained.

Therefore, researchers reflect cognitive conflicts by asking questions in the form of examples that lead to mental conflicts with teachers. The researcher reflects on the difference in workers' income if they sell 210 newspapers and 215 newspapers. Through these questions, S2 can find out that the difference in income earned by workers also impacts the difference in the cut line that form, so that S2 can correct the error that the graph depicting the income of Indopos workers has two straight lines with different slopes. The results of the increase in S2 can see in the following snippet of answers.

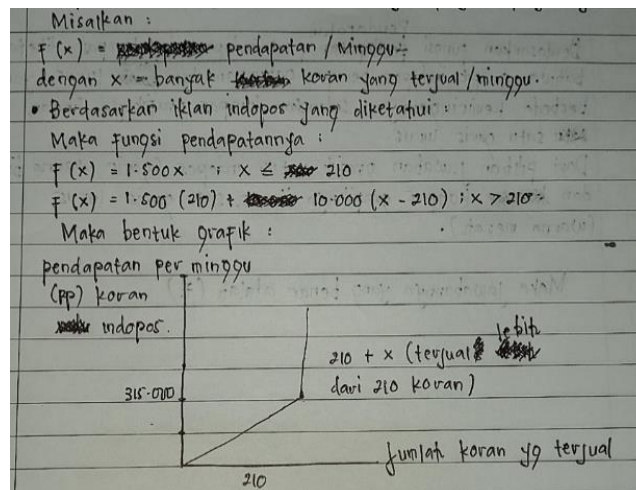


Figure 12. Answers to Master's Degree in Graphing the Functions and Income of Indopos Workers After Giving Reflections

S2's error can correct; the researcher reflects scaffolding to stimulate S2 to make conclusions about the solution so that S2 can explain that the income graph of Indopos workers has a different slope. In contrast, the income graph of Kompas workers is linear. For more details, see the following snippet of S2's answer.

Pendapatan
Berdasarkan fungsi ~~pendapatan~~ $f(x)$ indopos dan Kompas, diketahui
bahwa grafik indopos berupa gabungan dua garis lurus yang
berbeda kemiringannya; sedangkan Kompas grafiknya linear,
~~dan~~ satu garis lurus.

Figure 13. Master's Answers Making Conclusions on Answers After Giving Reflections

Based on the reflection process that has been described, the following schematic illustrates the master's thought process during reflection:

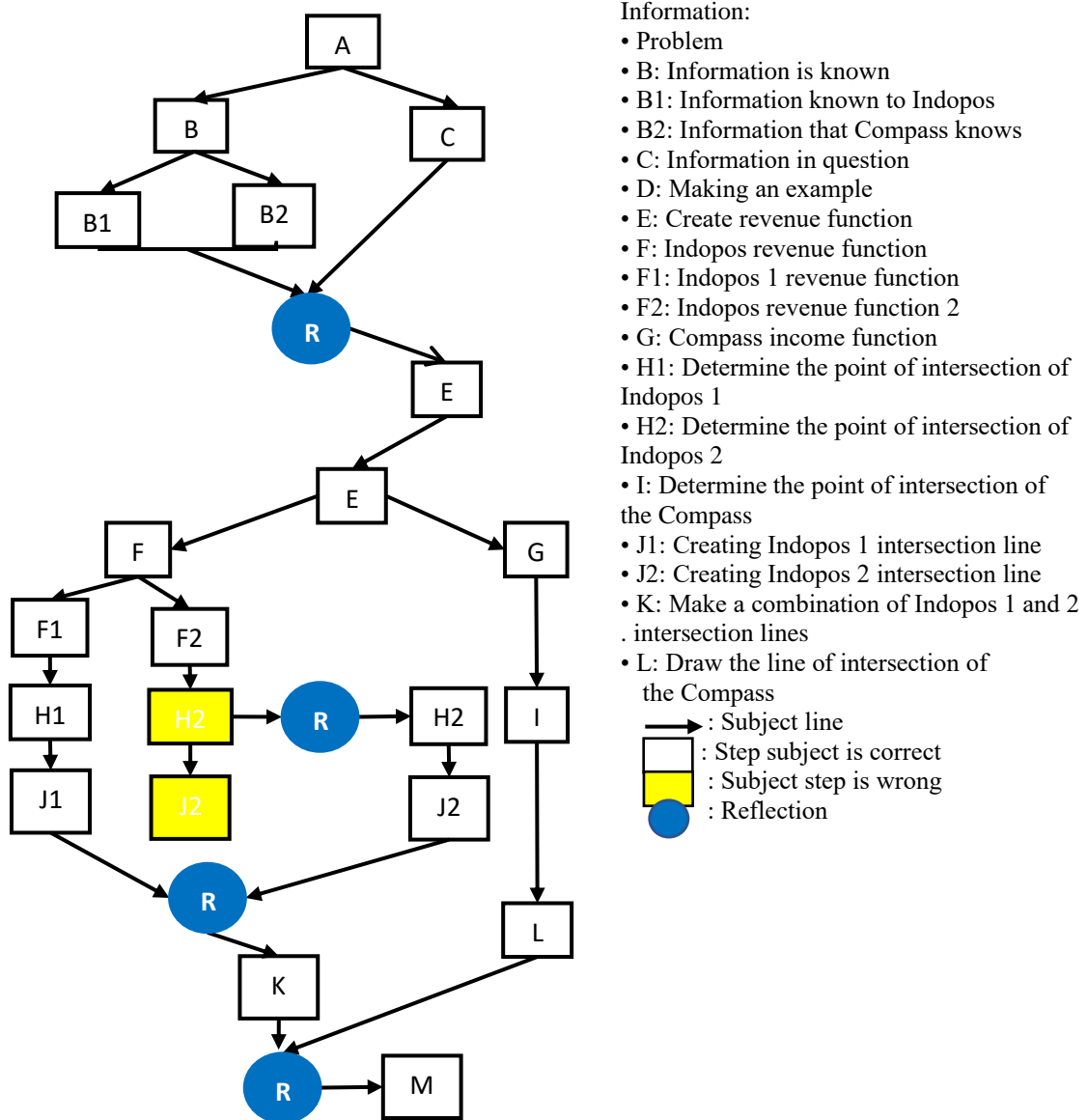


Figure 14. S2 Thinking Scheme of Solving Mathematical Problems During Reflection

Based on the study results, it can seem that students who have standard problem-solving abilities represented by S1 and S2 experience changes in thinking schemes with the same tendency to solve mathematical problems. There is an assimilation of students' thinking processes to understand the problem because their system follows the situation. According to Bormanaki (2017), when assimilated, students do not replace the existing schema because the structure of the problem found is following the available schema. Students can describe the issue directly but not wholly, both in answers and verbal expressions conveyed when problem-solving student's description of the problem is related to know during interviews. As Gog et al. (2020) and Halpern (2014) revealed, the stage of understanding the problem includes the ability to describe essential elements related to information that is known and asked in the situation.

Furthermore, at the strategy formulation stage, students experience accommodation. This is because students need reflection to connect material or mathematical concepts to build solutions to problems. Zhiqing (2015) and Dorko (2019) state that accommodation is a cognitive structure adapted from new experiences, resulting in new schemes or changing old systems based on the given stimulus. Researchers reflect mathematical connections to stimulate students to recall knowledge related to solving linear programming problems. It is in line with the theory of connection knitting (Subanji, 2015), which creates a relationship between thinking structures to develop strategies. Haseski et al. (2018) and King (2019) say that formulating a process is a step to interpret procedures based on concepts or materials that are mastered and allow them to be used to solve problems.

At the stage of implementing the plan, there is accommodation to the thinking process of students. Students' cognitive schemes are not following the planned strategy, so students need reflection. Hamdani et al. (2021) and Bormanaki (2017) state that accommodation changes students' cognitive structures caused by the treatment given, giving rise to new schemes. Students initially experienced an error, namely misapplying a procedure because it was not according to their strategy, so the researcher reflected through cognitive conflict to correct the mistake. According to Mathew et al. (2019), when students carry out plans, they will be significantly influenced by the strategies or techniques that they have prepared previously. After correcting students' thinking errors, the researcher reflected scaffolding on what conclusions could be drawn from the solutions found. Through these reflections can stimulate students to transform their thinking by making conclusions about answers.

As for the students' problem-solving steps, there is accommodation to students' thinking processes. Compromise occurs because, at first, students cannot build solutions, and there are unstructured steps due to mistakes made, so it requires reflection from the researcher. Hamdani (2021) refers to accommodation as a cognitive structure adapted through new experiences resulting from the given stimuli. Meanwhile, after review, students can perform transformations by correcting errors and completing steps reasonably and structured

CONCLUSION

Based on the results and discussion, reflection can stimulate students to correct errors in solving mathematical problems evidenced by changing the problem-solving steps taken by students to be more logical and systematic. Students with low problem-solving abilities experience assimilation to understand the situation while strategizing and implementing accommodation plans. Students can describe the problem more straightforwardly but require reflection to connect mathematical material to solve problems. In addition, students also need to review to correct errors and solve incomplete algorithms when carrying out mathematical problem solving plans.

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