Chapter

Extended Intuitionistic Fuzzy Line Graphs: Theory and Properties

Venkata Naga Srinivasa Rao Repalle, Keneni Abera Tola and Maamo Abebe Ashebo

Abstract

The introduction of fuzzy set theory was given by Zadeh. The introduction of fuzzy graph theory was given by Kauffman. Later the structure of fuzzy graph was developed Rosenfeld. The traditional fuzzy set cannot be used to completely describe all the evidence in problems where someone wants to know in how much degree of non-membership. Such a problem got the solution by Atanassov who introduced intuitionistic fuzzy set which described by a membership, a non-membership and a hesitation functions. An intuitionistic fuzzy set is used to solve problems involving uncertainty and imprecision that can't be handled by a traditional fuzzy set. This chapter introduced the interval-valued intuitionistic fuzzy line graphs (IVIFLG) and explored the results related to IVIFLG. As a result, many theorems and propositions related to IVIFLG are developed and supported by proof. Moreover, some remarkable isomorphic properties, strong IVIFLG, and complete IVIFLG have been investigated, and the proposed concepts are illustrated with the examples.

Keywords: fuzzy set, interval-valued intuitionistic fuzy graph, interval-valued intuitionistic fuzzy line graph, isomorphism

1. Introduction

Since Euler was presented with the impression of the Königsberg bridge problem, graph theory has received recognition in a variety of academic fields, including natural science, social science, engineering, and medical science. In the field of graph theory, some operations such as the Wiener index of graphs, line graphs, total graphs, cluster and corona operations of graphs, edge join of graphs, and semi-total line have been useful. In addition, some properties of boiling point, heat of evaporation, surface tension, vapor pressure, total electron energy of polymers, partition coefficients, ultrasonic sound velocity, and internal energy can be analyzed in chemical graph theory. These operations are not only useful in classical graphs but also in fuzzy graphs and generalizations of fuzzy graphs. Because real-world problems are frequently fraught with uncertainty and imprecision, Zadeh proposed fuzzy sets and membership degrees [1]. Accordingly, Kaufman presented the concept of fuzzy relations based on Zedeh's work in [2]. Rosenfeld [3] assembled both Zedeh's and Kaufman's work and then introduced fuzzy graphs.

Later on, Atanassov observed that fuzzy sets (FS) did not handle many problems with uncertainty and imprecision [4]. Based on these observations, he combined the membership degree with the falsehood degree and presented intuitionistic fuzzy sets (IFS) with relations and IFG, which is a generalization of FS [4–6]. It has many applications in fuzzy control, and defuzzification is the most computationally intensive part of fuzzy control. Mordeson investigated the concept of fuzzy line graphs (FLG) for the first time and explored both sufficient and necessary conditions for FLG to be a bijective homomorphism to its FG. He developed some theorems and propositions [7]. Firouzian et.al [8] introduced the notion of degree of an edge in fuzzy line graphs and congraphs.

Akram and Dudek discussed interval valued fuzzy graph (IVFG) and its properties in [9]. Later, different classes of IVIFGs such as regular, irregular, highly irregular, strongly irregular and neighbourly irregular IVIFGs were discussed [10]. Then, Akram drived IVFLG from IVFG [11]. Interval-valued intuitionistic $(S, T)-$fuzzy graphs were introduced by Rashmanlou and Borzooei [12]. Afterward, the idea of intuitionistic fuzzy line graph (IFLG) studied by Akram and Davvaz [13]. Furthermore, IFLG and its properties are investigated in [14].

Based on the defined concepts, we gave the definition of IVIFLG in this chapter. Our works are novel in the following ways: (1) IVIFLG is presented and illustrated with an example, (2) numerous theorems and propositions are developed and proved; (3) further, interval-valued intuitionistic weak line isomorphism and interval-valued intuitionistic weak vertex homomorphism are proposed. Readers should refer [5, 7, 11] for notations that are not declared in this chapter.

2. Body of the chapter

This section contains some basic definitions used to introduce IVIFLG. Throughout this chapter we considered only simple graph.

Definition 1.1. The graph $G = (V, E)$ is an intuitionistic fuzzy graph (IFG) if the following conditions are satisfied [15]

i. $\sigma_1 : V \to [0, 1]$ and $\gamma_1 : V \to [0, 1]$ are membership and nonmembership value of vertex set of G respectively and $0 \le \sigma_1(v) + \gamma_1(v) \le 1 \ \forall v \in V$,

ii. $\sigma_2 : V \times V \to [0, 1]$ and $\gamma_2 : V \times V \to [0, 1]$ are membership and nonmembership with $\sigma_2(v_i v_j) \le \sigma_1(v_i) \wedge \sigma_1(v_j)$ and $\gamma_2(v_i v_j) \le \gamma_1(v_j) \vee \gamma_1(v_j)$ and $0 \le \sigma_1 2(v_i v_j) + \gamma_2(v_i v_j) \le 1, \forall v_i v_j \in E$.

Definition 1.2. The line graph L(G) of graph G is defined as any node in $L(G)$ that corresponds to an edge in G, and pair of nodes in $L(G)$ are adjacent if and only if their correspondence edges $e_i, e_j \in G$ share a common node $v \in G$.

Definition 1.3. For the given graph $G = (V, E)$ with $n-$vertices and $S_i = \left\{v_i, e_{i_1}, \cdots, e_{i_p}\right\}$ such that $1 \le i \le n, 1 \le j \le p_i$ and $e_{ij} \in E$ has v_i as a vertex. Then (S, T) is called intersection graph where $S = \{S_i\}$ is the vertex set of (S, T) and $T = \left\{S_i S_j | S_i, S_j \in S; S_i \cap S_j \ne \emptyset, \text{for } i \ne j\right\}$ is an edge set of (S, T).

Definition 1.4. The line(edge) graph $L(G) = (H, J)$ is where $H = \{ \{e\} \cup \{u_e, v_e\} : e \in E, u_e, v_e \in V, e = u_e v_e$ and $J = \left\{S_e S_f : \ e, f \in E, e \ne f, S_e \cap S_f \ne \emptyset\right\}$ with $S_e = \{e\} \cup \{u_e, v_e, e \in E\}$ [11].

Definition 1.5. Let $G = (A_1, B_1)$ is an IFG with $A_1 = (\sigma_{A_1}, \gamma_{A_1})$ and $B_1 = (\sigma_{B_1}, \gamma_{B_1})$ be IFS on V and E respectively. Then $(S, T) = (A_2, B_2)$ is an intuitionistic fuzzy intersection graph of G whose membership and nonmembership functions are defined as [14]

i. $\sigma_{A_2}(S_i) = \sigma_{A_1}(v_i), \qquad \gamma_{A_2}(S_i) = \gamma_{A_1}(v_i), \forall S_i, S_j \in S$

ii. $\sigma_{B_2}(S_i\, S_j) = \sigma_{B_1}(v_i v_j), \qquad \gamma_{B_2}(S_i S_j) = \gamma_{B_1}(v_i v_j) \forall S_i S_j \in T.$

where $A_2 = (\sigma_{A_2}, \gamma_{A_2}), B_2 = (\sigma_{B_2}, \gamma_{B_2})$ on S and T respectively. So, IFG of the intersection graph (S, T) is isomorphic to G(means, $(S, T) \cong G$).

Definition 1.6. Consider $L(G^*) = (H, J)$ be line graph of $G^* = (V, E)$. Let $G = (A_1, B_1)$ be IFG of G^* with $A_1 = (\sigma_{A_1}, \gamma_{A_1})$ and $B_1 = (\sigma_{B_1}, \gamma_{B_1})$ be IFS on X and E receptively. Then we define the intuitionistic fuzzy line graph $L(G) = (A_2, B_2)$ of G as

i. $\sigma_{A_2}(S_e) = \sigma_{B_1}(e) = \sigma_{B_1}(u_e v_e),$

$\gamma_{A_2}(S_e) = \gamma_{B_1}(e) = \gamma_{B_1}(u_e v_e)$, for all $S_e, S_e \in H$

ii. $\sigma_{B_2}(S_e S_f) = \sigma_{B_1}(e) \wedge \sigma_{B_1}(f)$

$\gamma_{B_2}(S_e S_f) = \gamma_{B_1}(e) \vee \gamma_{B_1}(f), \forall S_e S_f \in J..$

where $A_2 = (\sigma_{A_2}, \gamma_{A_2})$ and $B_2 = (\sigma_{B_2}, \gamma_{B_2})$ are IFS on H and J respectively.

The $L(G) = (A_2, B_2)$ of IFG G is always IFG.

Definition 1.7. Let $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ be two IFGs. The homomorphism of $\psi : G_1 \to G_2$ is mapping $\psi : V_1 \to V_2$ such that [14].

i. $\sigma_{A_1}(v_i) \leq \sigma_{A_2}(\psi(v_i)), \quad \gamma_{A_1}(v_i) \leq \gamma_{A_2}(\psi(v_i))$

ii. $\sigma_{B_1}(v_i, v_j) \leq \sigma_{B_2}(\psi(v_i)\psi(v_j)),$

$\gamma_{B_1}(v_i, v_j) \leq \gamma_{B_2}(\psi(v_i)\psi(v_j)) \; \forall v_i \in V_1, v_i v_j \in E_1.$

Definition 1.8. The interval valued FS A is characterized by [9].

$$A = \{v_i, [\sigma_A^-(v_i), \sigma_A^+(v_i)] : v_i \in X\}.$$

Here, $\sigma_A^-(v_i)$ and $\sigma_A^+(v_i)$ are lower and upper interval of fuzzy subsets A of X respectively, such that $\sigma_A^-(v_i) \leq \sigma_A^+(v_i) \; \forall v_i \in V.$

For simplicity, we used IVFS for interval valued fuzzy set.

Definition 1.9. Let $A = \{[\sigma_A^-(v), \sigma_A^+(v)] : v \in X\}$ be IVFS. Then, the graph $G^* = (V, E)$ is called IVFG if the following conditions are satisfied;

$$\sigma_B^-(v_i v_j) \leq (\sigma_A^-(v_i) \wedge \sigma_A^-(v_j))$$
$$\sigma_B^+(v_i v_j) \leq \sigma_A^+(v_i) \wedge \sigma_A^+(v_j)$$

$\forall v_i, v_j \in V, \;\; \forall v_i v_j \in E$ and where $A = [\sigma_A^-, \sigma_A^+], B = [\sigma_B^-, \sigma_B^+]$ is IVFS on V and E respectively.

Definition 1.10. Let $G = (A_1, B_1)$ be simple IVFG. Then we define IVF intersection graph $(S, T) = (A_2, B_2)$ as follows:

1. A_2 and B_2 are IFS of S and T respectively,

2. $\sigma^-_{A_2}(S_i) = \sigma^-_{A_1}(v_i)$ and $\sigma^+_{A_2}(S_i) = \sigma^+_{A_1}(v_i), \forall S_i, S_j \in S$ and

3. $\sigma^-_{B_2}(S_iS_j) = \sigma^-_{B_1}(v_iv_j), \sigma^+_{B_2}(S_iS_j) = \sigma^+_{B_1}(v_iv_j), \ \forall S_iS_j \in T.$

Remark: The given IVFG G and its intersection graph (S, T) are always isomorphic to each other.

Definition 1.11. An interval valued fuzzy line graph (IVFLG) $L(G) = (A_2, B_2)$ of IVFG $G = (A_1, B_1)$ is defined as follows [11]:

- A_2 and B_2 are IVFS of H and J respectively, where $L(G^*) = (H, J)$
- $\sigma^-_{A_2}(S_i) = \sigma^-_{B_1}(e) = \sigma^-_{B_1}(u_ev_e), \ \sigma^+_{A_2}(S_i) = \sigma^+_{B_1}(e) = \sigma^+_{B_1}(u_ev_e),$
- $\sigma^-_{B_2}(S_eS_f) = \sigma^-_{B_1}(e) \wedge \sigma^-_{B_1}(f), \ \sigma^+_{B_2}(S_eS_f) = \sigma^+_{B_1}(e) \wedge \sigma^+_{B_1}(f)$ for all $S_e, S_f \in H, S_eS_f \in J.$

Definition 1.12. A graph $G = (A, B)$ with underlying fuzzy set V is IVIFG if

i. the mapping $\sigma_A : V \to [0, 1]$ and $\gamma_A : V \to [0, 1]$ where $\sigma_A(v_i) = [\sigma^-_A(v_i), \sigma^+_A(v_i)]$ and $\gamma_A(v_i) = [\gamma^-_A(v_i), \gamma^+_A(v_i)]$ denote a membership degree and non membership degree of vertex $v_i \in V$, receptively such that $\sigma^-_A(v_i) \leq \sigma^+_A(v_i)$, $\gamma^-_A(v_i) \leq \gamma^+_A(v_i)$ and $0 \leq \sigma^+_A(v_i) + \gamma^+_A(v_i) \leq 1 \ \forall v_i \in V$,

ii. the mapping $\sigma_B : V \times V \subseteq E \to [0, 1]$ and $\gamma_B : V \times V \subseteq E \to [0, 1]$ where $\sigma_B(v_iv_j) = [\sigma^-_B(v_iv_j), \sigma^+_B(v_iv_j)]$ and $\gamma_B(v_iv_j) = [\gamma^-_B(v_iv_j), \gamma^+_B(v_iv_j)]$ such that

$$\sigma^-_B(v_iv_j) \leq \sigma^-_A(v_i) \wedge \sigma^-_A(v_j, \qquad \sigma^+_B(v_iv_j) \leq \sigma^+_A(v_i) \wedge \sigma^+_A(v_j)$$
$$\gamma^-_B(v_iv_j) \leq \gamma^-_A(v_i) \vee \gamma^-_A(v_j), \qquad \gamma^+_B(v_iv_j) \leq \gamma^+_A(v_i) \vee \gamma^+_A(v_j)$$

where $0 \leq \sigma^+_B(v_iv_j) + \gamma_B + (v_iv_j) \leq 1$ and $\forall v_iv_j \in E.$

In the next section, we begin the main findings of this chapter by introducing and demonstrating examples of IVIFLG.

Definition 1.13. Consider $L(G) = (H, J)$ is IVIFLG of IVIFG $G = (A_1, B_1)$ and denoted by $L(G) = (A_2, B_2)$ whose membership and non membership function is defined as

i. A_2 and B_2 are IVIFS of H and J respectively, such that

$$\sigma^-_{A_2}(S_e) = \sigma^-_{B_1}(e) = \sigma^-_{B_1}(u_ev_e)$$
$$\sigma^+_{A_2}(S_e) = \sigma^+_{B_1}(e) = \sigma^+_{B_1}(u_ev_e)$$
$$\gamma^-_{A_2}(S_e) = \gamma^-_{B_1}(e) = \gamma^-_{B_1}(u_ev_e)$$
$$\gamma^+_{A_2}(S_e) = \gamma^+_{B_1}(e) = \gamma^+_{B_1}(u_ev_e) \ \ \forall S_e \in H.$$

ii. The edge set of L(G) is

$$\sigma^-_{B_2}(S_eS_f) = \sigma^-_{B_1}(e)\wedge\sigma^-_{B_1}(f), \qquad \sigma^+_{B_2}(S_eS_f) = \sigma^+_{B_1}(e)\wedge\sigma^+_{B_1}(f)$$

$$\gamma^-_{B_2}(S_eS_f) = \sigma^-_{B_1}(e)\vee\gamma^-_{B_1}(f), \qquad \gamma^+_{B_2}(S_eS_f) = \gamma^+_{B_2}(e)\vee\gamma^+_{B_1}(f) \text{ for all } ,S_eS_f \in J..$$

Example 1.14. Given IVIFG $G = (A_1, A_2)$ as shown in **Figure 1**.
From the given IVIFG we have

$$\sigma_{A_1}(v_1) = \left[\sigma^-_{A_1}(v_1), \sigma^+_{A_1}(v_1)\right] = [0.3{,}0.6]$$
$$\sigma_{A_1}(v_2) = \left[\sigma^-_{A_1}(v_2), \sigma^+_{A_1}(v_2)\right] = [0.2{,}0.7]$$
$$\sigma_{A_1}(v_3) = \left[\sigma^-_{A_1}(v_3), \sigma^+_{A_1}(v_3)\right] = [0.1{,}0.3]$$
$$\sigma_{A_1}(v_4) = \left[\sigma^-_{A_1}(v_4), \sigma^+_{A_1}(v_4)\right] = [0.3{,}0.4]$$
$$\gamma_{A_1}(v_1) = \left[\gamma^-_{A_1}(v_1), \gamma^+_{A_1}(v_1)\right] = [0.1{,}0.4]$$
$$\gamma_{A_1}(v_2) = \left[\gamma^-_{A_1}(v_2), \gamma^+_{A_1}(v_2)\right] = [0.1{,}0.2]$$
$$\gamma_{A_1}(v_3) = \left[\gamma^-_{A_1}(v_3), \gamma^+_{A_1}(v_3)\right] = [0.4{,}0.5]$$
$$\gamma_{A_1}(v_4) = \left[\gamma^-_{A_1}(v_4), \gamma^+_{A_1}(v_4)\right] = [0.4{,}0.5]$$
$$\sigma_{B_1}(v_1v_2) = \left[\sigma^-_{B_1}(v_1v_2), \sigma^+_{B_1}(v_1v_2)\right] = [0.2{,}0.5]$$
$$\sigma_{B_1}(v_2v_3) = \left[\sigma^-_{B_1}(v_2v_3), \sigma^+_{B_1}(v_2v_3)\right] = [0.1{,}0.2]$$
$$\sigma_{B_1}(v_3v_4) = \left[\sigma^-_{B_1}(v_3v_4), \sigma^+_{B_1}(v_3v_4)\right] = [0.1{,}0.1]$$
$$\sigma_{B_1}(v_4v_1) = \left[\sigma^-_{B_1}(v_4v_1), \sigma^+_{B_1}(v_4v_1)\right] = [0.2{,}0.4]$$

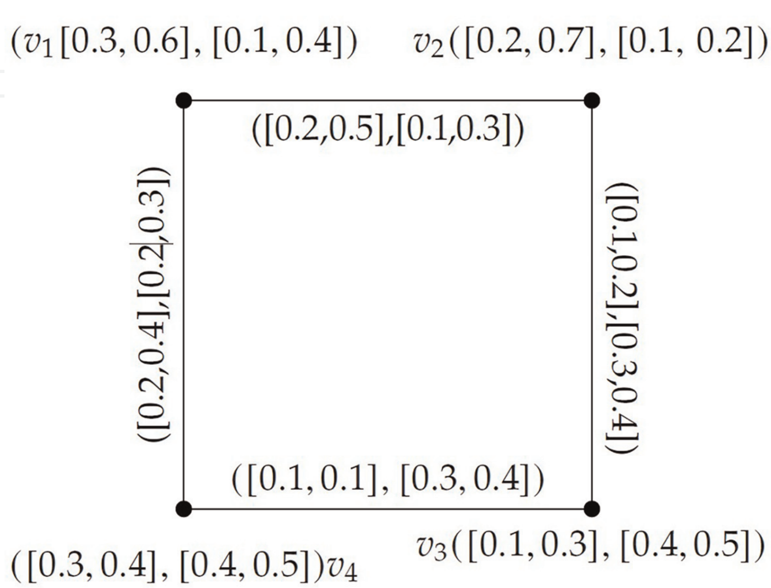

Figure 1.
IVIFG G.

$$\gamma_{B_1}(v_1v_2) = \left[\gamma^-_{B_1}(v_1v_2), \gamma^+_{B_1}(v_1v_2)\right] = [0.1,0.3]$$
$$\gamma_{B_1}(v_2v_3) = \left[\gamma^-_{B_1}(v_2v_3), \gamma^+_{B_1}(v_2v_3)\right] = [0.3,0.4]$$
$$\gamma_{B_1}(v_3v_4) = \left[\gamma^-_{B_1}(v_3v_4), \gamma^+_{B_1}(v_3v_4)\right] = [0.3,0.4]$$
$$\gamma_{B_1}(v_4v_1) = \left[\gamma^-_{B_1}(v_4v_1), \gamma^+_{B_1}(v_4v_1)\right] = [0.2,0.3]$$

To find IVIFLG $L(G) = (H, J)$ of I such that

$$H = \left\{v_1v_2 = S_{e_1}, v_2v_3 = S_{e_2}, v_3v_4 = S_{e_3}, v_4v_1 = S_{e_4}\right\} \text{ and}$$
$$J = \left\{S_{e_1}S_{e_2}, S_{e_2}S_{e_3}, S_{e_2 3}S_{e_4}, S_{e_4}S_{e_1}\right\}.$$

Now, consider $A_2 = \left[\sigma^-_{A_2}, \sigma^+_{A_2}\right]$ and $B_2 = \left[\sigma^-_{B_2}, \sigma^+_{B_2}\right]$ are IVFS of H and J respectively. Then we have

$$\sigma_{A_2}(S_{e_1}) = \left[\sigma^-_{B_1}(e_1), \sigma^+_{B_1}(e_1)\right] = [0.2,0.5]$$
$$\sigma_{A_2}(S_{e_2}) = \left[\sigma^-_{B_1}(e_2), \sigma^+_{B_1}(e_2)\right] = [0.1,0.2]$$
$$\sigma_{A_2}(S_{e_3}) = \left[\sigma^-_{B_1}(e_3), \sigma^+_{B_1}(e_3)\right] = [0.1,0.1]$$
$$\sigma_{A_2}(S_{e_4}) = \left[\sigma^-_{B_1}(e_4), \sigma^+_{B_1}(e_4)\right] = [0.2,0.4]$$

$$\gamma_{A_2}(S_{e_1}) = \left[\gamma^-_{B_1}(e_1), \gamma^+_{B_1}(e_1)\right] = [0.1,0.3]$$
$$\gamma_{A_2}(S_{e_2}) = \left[\gamma^-_{B_1}(e_2), \gamma^+_{B_1}(e_2)\right] = [0.3,0.4]$$
$$\gamma_{A_2}(S_{e_3}) = \left[\gamma^-_{B_1}(e_3), \gamma^+_{B_1}(e_3)\right] = [0.3,0.4]$$
$$\gamma_{A_2}(S_{e_4}) = \left[\gamma^-_{B_1}(e_4), \gamma^+_{B_1}(e_4)\right] = [0.2,0.3]$$

$$\sigma_{B_2}(S_{e_1}S_{e_2}) = \left[\sigma^-_{B_1}(e_1)\wedge\sigma^-_{B_1}(e_2), \sigma^+_{B_1}(e_1)\wedge\sigma^+_{B_1}(e_2)\right] = [0.1,0.2]$$
$$\sigma_{B_2}(S_{e_2}S_{e_3}) = \left[\sigma^-_{B_1}(e_2)\wedge\sigma^-_{B_1}(e_3), \sigma^+_{B_1}(e_2)\wedge\sigma^+_{B_1}(e_3)\right] = [0.1,0.1]$$
$$\sigma_{B_2}(S_{e_3}S_{e_4}) = \left[\sigma^-_{B_1}(e_3)\wedge\sigma^-_{B_1}(e_4), \sigma^+_{B_1}(e_3)\wedge\sigma^+_{B_1}(e_4)\right] = [0.1,0.1]$$
$$\sigma_{B_2}(S_{e_2}S_{e_3}) = \left[\sigma^-_{B_1}(e_4)\wedge\sigma^-_{B_1}(e_1), \sigma^+_{B_1}(e_4)\wedge\sigma^+_{B_1}(e_1)\right] = [0.2,0.4]$$

$$\gamma_{B_2}(S_{e_1}S_{e_2}) = \left[\gamma^-_{B_1}(e_1)\vee\gamma^-_{B_1}(e_2), \gamma^+_{B_1}(e_1)\vee\gamma^+_{B_1}(e_2)\right] = [0.3,0.4]$$
$$\gamma_{B_2}(S_{e_2}S_{e_3}) = \left[\gamma^-_{B_1}(e_2)\vee\gamma^-_{B_1}(e_3), \gamma^+_{B_1}(e_2)\vee\gamma^+_{B_1}(e_3)\right] = [0.3,0.4]$$
$$\gamma_{B_2}(S_{e_3}S_{e_4}) = \left[\gamma^-_{B_1}(e_3)\vee\gamma^-_{B_1}(e_4), \gamma^+_{B_1}(e_3)\vee\gamma^+_{B_1}(e_4)\right] = [0.3,0.4]$$
$$\gamma_{B_2}(S_{e_2}S_{e_3}) = \left[\gamma^-_{B_1}(e_4)\vee\gamma^-_{B_1}(e_1), \gamma^+_{B_1}(e_4)\vee\gamma^+_{B_1}(e_1)\right] = 0.2,0.3]$$

Then L(G) of IVIFG G is shown in **Figure 2**.
Proposition 1.15. $L(G) = (A_2, B_2)$ is IVIFLG corresponding to IVIFG $G = (A_1, B_1)$.

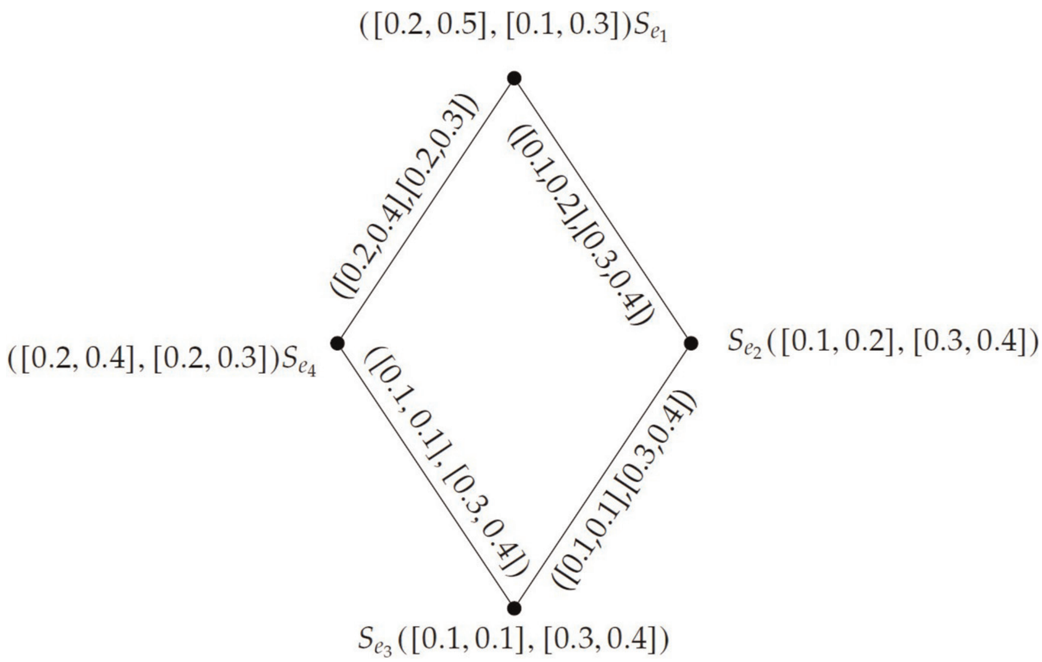

Figure 2.
IVIFLG of G.

Definition 1.16. A homomorphism mapping $\psi : G_1 \to G_2$ of two IVIFG $G_1 = (M_1, N_1)$ and $G_2 = (M_2, N_2)$ $\psi : V_1 \to V_2$ is defined as

i. $\sigma^-_{M_1}(v_i) \le \sigma^-_{M_2}(\psi(v_i)), \quad \sigma^+_{M_1}(v_i) \le \sigma^+_{M_2}(\psi(v_i))$
$\gamma^-_{M_1}(v_i) \le \gamma^-_{M_2}(\psi(v_i)), \quad \gamma^+_{M_1}(v_i) \le \gamma^+_{M_2}(\psi(v_i))$ for all $v_i \in V_1$.

ii. $\sigma^-_{N_1}(v_iv_j) \le \sigma^-_{N_2}(\psi(v_i)\psi(v_j)), \quad \sigma^+_{N_1}(v_iv_j) \le \sigma^+_{N_2}(\psi(v_i)\psi(v_j))$
$\gamma^-_{N_1}(v_iv_j) \le \gamma^-_{N_2}(\psi(v_i)\psi(v_j)), \quad \gamma^+_{N_1}(v_iv_j) \le \gamma^+_{N_2}(\psi(v_i)\psi(v_j))$ for all $v_iv_j \in E_1$.

Definition 1.17. A bijective homomorphism $\psi : G_1 \to G_2$ of IVIFG is said to be a weak vertex isomorphism, if

$$\sigma_{M_1}(v_i) = \left[\sigma^-_{M_1}(v_i), \sigma^+_{M_1}(v_i)\right] = \left[\sigma^-_{M_2}(\psi(v_i)), \sigma^+_{M_2}(\psi(v_i))\right]$$
$$\gamma_{N_1}(v_i) = \left[\gamma^-_{N_1}(v_i), \gamma^+_{N_1}(v_i)\right] = \left[\gamma^-_{N_2}(\psi(v_i)), \gamma^+_{N_2}(\psi(v_i))\right], \quad \forall v_i \in V_1.$$

A bijective homomorphism $\psi : G_1 \to G_2$ of IVIFG is said to be a weak line isomorphism if

$$\sigma_{B_1}(v_iv_j) = \left[\sigma^-_{B_1}(v_iv_j), \sigma^+_{B_1}(v_iv_j)\right] = \left[\sigma^-_{B_2}(\psi(v_i)\psi(v_j)), \sigma^+_{B_2}(\psi(v_i)\psi(v_j))\right],$$
$$\gamma_{B_1}(v_iv_j) = \left[\gamma^-_{B_1}(v_iv_j), \gamma^+_{B_1}(v_iv_j)\right] = \left[\gamma^-_{B_2}(\psi(v_i)\psi(v_j)), \gamma^+_{B_2}(\psi(v_i)\psi(v_j))\right] \quad \forall v_iv_j \in E_1.$$

If $\psi : G_1 \to G_2$ is an isomorphism that holds Definition 1.17, then ψ is called a weak isomorphism of IVIFGs G_1 and G_2.

Proposition 1.18. The IVIFLG $L(G)$ is connected graph if and only if its corresponding IVFG G is connected graph.

Proof: Assume that $L(G)$ is a connected IVIFLG of the IVIFG G. First, We want to show that necessary condition. Lets say G is disconnected IVIFG. Then there are at least two nodes of graph G which are not joined by path, say v_i and v_j. If we take one edge e in the first component of the edge set of G, then it doesn't have any edges which adjacent to edge e in other components. So that, the IVIFLG of graph G is disconnected and contradicts our assumption. Therefore, the IVIFG G must be connected. On the other hand, assume that IVIFG G is connected graph. Then, there is a path between each pair of nodes. This implies, edges which are adjacent in graph G are adjacent nodes in IVIFLG. As a result, every pair of nodes in IVIFLG of G are linked by a path. Therefore, the proof finished.

Proposition 1.19. An Interval valued line graph of star graph $K_{1,n}$ is a complete Interval valued graph K_n with $n-$vertices.

Proof: Consider the vertex $v \in V(K_{1,n})$ that adjacent to all other vertices $u_i \in V(K_{1,n})$ for $i = 1, 2\cdots, n$. Now, all the vertices in IVIFLG of $K_{1,n}$ are adjacent. This means, IVIFLG of $K_{1,n}$ is a complete graph.

Example 1.20. Suppose that the IVIFG $K_{1,3}$ with $V = \{v, v_1, v_2, v_3\}$ and $E = \{vv_1, vv_2, vv_3$ where

$$v = ([0.3,0.5], [0.1,0.4]),\ v_1 = ([0.3,0.4], [0.2,0.5])$$
$$v_2 = ([0.5,0.8], [0.1, 0.2]),\ v_3 = ([0.1,0.3], [0.5,0.7])$$
$$e_1 = vv_1 = ([0.2,0.3], [0.3,0.5]),\ e_2 = vv_2 = ([0.2,0.5], [0.0,0.3])$$
$$e_3 = vv_3 = ([0.1,0.2], [0.3,0.6]).$$

Then by definition of IVIFLG, the vertex sets of $L(K_{1,3})$ is $V = \{S_{e_1}, S_{e_2}, S_{e_3}\}$ and $\{S_{e_1}S_{e_2}, S_{e_1}S_{e_3}, S_{e_2}S_{e_3}$ edge sets where

$$S_{e_1} = ([0.2,0.3], [0.3,0.5]), \qquad S_{e_2} = ([0.2,0.5], [0.0,0.3]),$$
$$S_{e_3} = ([0.1,0.2], [0.2,0.6]), \qquad S_{e_1}S_{e_2} = ([0.2,0.3], [0.3,0.5]),$$
$$S_{e_1}S_{e_3} = ([0.2,0.3], [0.3,0.5]), \qquad S_{e_2}S_{e_3} = ([0.1,0.2], [0.2,0.6]).$$

Here $L(K_{1,3})$ is complete graph K_3 (**Figure 3**).

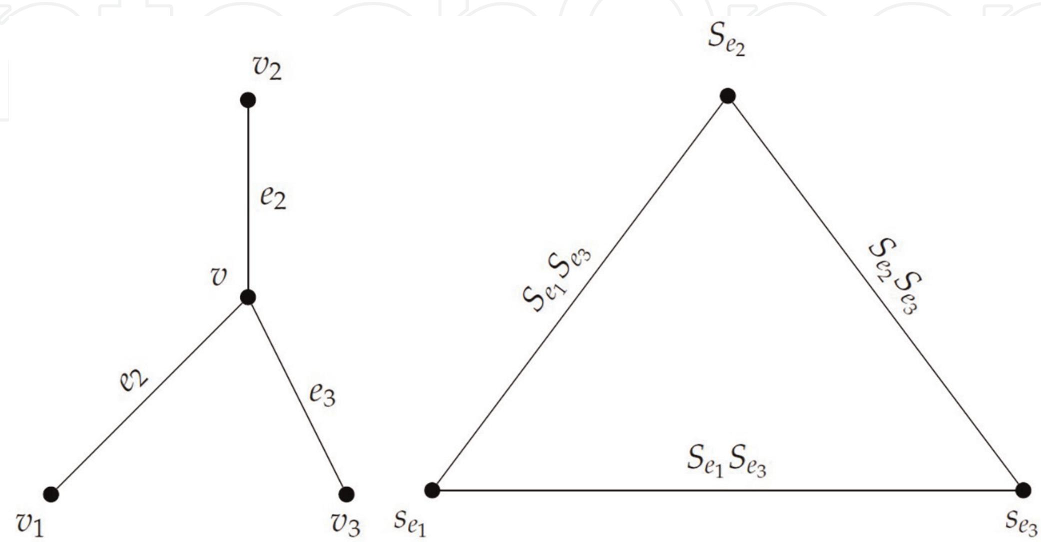

Figure 3.
Graphs of $K_{1,3}$ and $L(K_{1,3})$.

Proposition 1.21. Let $L(G)$ be IVIFLG of IVIFG of G. Then $L(G^*)$ is a line graph of G^* where $G^* = (V, E)$ with underlying set V.

Proof: Given $G = (A_1, B_1)$ is IVIFG of G^* and $L(G) = (A_2, B_2)$ is IVIFLG of $L(G^*)$. Then

$$\sigma_{A_2}(S_e) = \left[\sigma^-_{A_2}(S_e), \sigma^+_{A_2}(S_e)\right] = \left[\sigma^-_{B_1}(e), \sigma^+_{B_1}(e)\right],$$

$$\gamma_{A_2}(S_e) = \left[\gamma^-_{A_2}(S_e), \gamma^+_{A_2}(S_e)\right] = \left[\gamma^-_{B_1}(e), \gamma^+_{B_1}(e)\right] \ \forall e \in E.$$

This implies, $S_e \in H = \{\{e\} \cup \{u_e, v_e\} : e \in E, u_e, v_e \in V \& e = u_e v_e\}$ if and only if $e \in E$.

$$\sigma_{B_2}(S_e S_f) = \left[\sigma^-_{B_2}(S_e S_f), \sigma^+_{B_2}(S_e S_f)\right] = \left[\sigma^-_{B_1}(e) \wedge \sigma^-_{B_1}(f), \sigma^+_{B_1}(e) \wedge \sigma^+_{B_1}(f)\right]$$

$$\gamma_{B_2}(S_e S_f) = \left[\gamma^-_{B_2}(S_e S_f), \gamma^+_{B_2}(S_e S_f)\right] = \left[\gamma^-_{B_1}(e) \vee \gamma^-_{B_1}(f), \gamma^+_{B_1}(e) \vee \gamma^+_{B_1}(f)\right]$$

$$\forall S_e S_f \in J,$$

where $J = \{S_e S_f \mid S_e \cap S_f \notin \varnothing, e, f \in E \& e \notin f\}$. Hence, $L(G^*)$ is a line graph of G^*.

Proposition 1.22. Let $L(G) = (A_2, B_2)$ be IVIFLG of $L(G^*)$. Then $L(G)$ is also IVIFLG of some IVIFG $G = (A_1, B_1)$ iff

i. $\sigma_{B_2}(S_e S_f) = \left[\sigma^-_{B_2}(S_e S_f), \sigma^+_{B_2}(S_e S_f)\right] = \left[\sigma^-_{A_2}(S_e) \wedge \sigma^-_{A_2}(S_f), \sigma^+_{A_2}(S_e) \wedge \sigma^+_{A_2}(S_f)\right],$

ii. $\gamma_{B_2}(S_e S_f) = \left[\gamma^-_{B_2}(S_e S_f), \gamma^+_{B_2}(S_e S_f)\right] = \left[\gamma^-_{A_2}(S_e) \vee \gamma^-_{A_2}(S_f), \gamma^+_{A_2}(S_e) \vee \gamma^+_{A_2}(S_f)\right]$
$\forall S_e, S_f \in H, S_e S_f \in J.$

Proof: Suppose both conditions (i) and (ii) are satisfied. i.e., $\sigma^-_{B_2}(S_e S_f) = \sigma^-_{A_2}(S_e) \wedge \sigma^-_{A_2}(S_f)$, $\sigma^+_{B_2}(S_e S_f) = \sigma^+_{A_2}(S_e) \wedge \sigma^+_{A_2}(S_f)$, $\gamma^-_{B_2}(S_e S_f) = \gamma^-_{A_2}(S_e) \vee \gamma^-_{A_2}(S_f)$ and $\gamma^+_{B_2}(S_e S_f) = \gamma^+_{A_2}(S_e) \vee \gamma^+_{A_2}(S_f)$ for all $S_e S_f \in W$. For every $e \in E$ we define $\sigma^-_{A_2}(S_e) = \sigma^-_{A_1}(e)$, $\sigma^+_{A_2}(S_e) = \sigma^+_{A_1}(e)$, $\gamma^-_{A_2}(S_e) = \gamma^-_{A_1}(e)$ and $\gamma^+_{A_2}(S_e) = \gamma^+_{A_1}(e)$. Then

$$\begin{aligned}
\sigma^-_{B_2}(S_e S_f) &= \left[\sigma^-_{B_2}(S_e S_f), \sigma^+_{B_2}(S_e S_f)\right] \\
&= \left[\sigma^-_{A_2}(S_e) \wedge \sigma^-_{A_2}(S_f), \sigma^+_{A_2}(S_e) \wedge \sigma^+_{A_2}(S_f)\right] \\
&= \left[\sigma^-_{B_1}(e) \wedge \sigma^-_{B_1}(f), \sigma^+_{B_1}(e) \wedge \sigma^+_{B_1}(f)\right]. \\
\gamma^-_{B_2}(S_e S_f) &= \left[\gamma^-_{B_2}(S_e S_f), \gamma^+_{B_2}(S_e S_f)\right] \\
&= \left[\gamma^-_{A_2}(S_e) \vee \gamma^-_{A_2}(S_f), \gamma^+_{A_2}(S_e) \vee \gamma^+_{A_2}(S_f)\right] \\
&= \left[\gamma^-_{B_1}(e) \vee \gamma^-_{B_1}(f), \gamma^+_{B_1}(e) \vee \gamma^+_{B_1}(f)\right].
\end{aligned}$$

We know that IVIFS $A_1 = \left(\left[\sigma^-_{A_1}, \sigma^+_{A_1}\right], \left[\gamma^-_{A_1}, \gamma^+_{A_1}\right]\right)$ yields the properties

$$\sigma^-_{B_1}(v_iv_j) \le \sigma^-_{A_1}(v_i) \wedge \sigma^-_{A_1}(v_j)$$
$$\sigma^+_{B_1}(v_iv_j) \le \sigma^+_{A_1}(v_i) \wedge \sigma^+_{A_1}(v_j)$$
$$\gamma^-_{B_1}(v_iv_j) \le \gamma^-_{A_1}(v_i) \vee \gamma^-_{A_1}(v_j)$$
$$\gamma^+_{B_1}(v_iv_j) \le \gamma^+_{A_1}(v_i) \vee \gamma^+_{A_1}(v_j)$$

will suffice. From definition of IVIFLG the converse of this statement is well known.

Proposition 1.23. An IVIFLG is always a strong IVIFG.

Proof: It is straightforward from the definition, therefore it is omitted.

Proposition 1.24. Let G_1 and G_2 IVIFGs of G_1^* and G_2^* respectively. If the mapping $\psi : G_1 \to G_2$ is a weak isomorphism, then $\psi : G_1^* \to G_2^*$ is isomorphism map.

Proof: Suppose $\psi : G_1 \to G_2$ is a weak isomorphism. Then

$$v \in V_1 \Leftrightarrow \psi(v) \in V_2 \text{ and}$$

$$uv \in E_1 \Leftrightarrow \psi(u)\psi(v) \in E_2.$$

Hence the proof.

Theorem 1.25. Let $G^* = (V, E)$ is connected graph and consider that $L(G) = (A_2, B_2)$ is IVIFLG corresponding to IVIFG $G = (A_1, B_1)$. The,

1. there exists a map $\psi : G \to L(G)$ which is a weak isomorphism if and only if G^* is a cyclic graph with

   $$\sigma_{A_1}(v) = \left[\sigma^-_{A_1}(v), \sigma^+_{A_1}(v)\right] = \left[\sigma^-_{B_1}(e), \sigma^+_{B_1}(e)\right],$$

   $$\gamma_{A_1}(v) = \left[\gamma^-_{A_1}(v), \gamma^+_{A_1}(v)\right] = \left[\gamma^-_{B_1}(e), \gamma^+_{B_1}(e)\right],$$

   such that $A_1 = \left(\left[\sigma^-_{A_1}, \sigma^+_{A_1}\right], \left[\gamma^-_{A_1}, \gamma^+_{A_1}\right]\right) \& \ B_1 = \left(\left[\sigma^-_{B_1}, \sigma^+_{B_1}\right], \left[\gamma^-_{B_1}, \gamma^+_{B_1}\right]\right)$, $\forall v \in V, e \in E$.

2. The map ψ is isomorphism if $\psi : G \to L(G)$ is a weak isomorphism.

Proof: Consider $\psi : G \to L(G)$ is a weak isomorphism. Then we have

$$\sigma_{A_1}(v_i) = \left[\sigma^-_{A_1}(v_i), \sigma^+_{A_1}(v_i)\right] = \left[\sigma^-_{A_2}(\psi(v_i)), \sigma^+_{A_2}(\psi(v_i))\right]$$
$$\gamma_{B_1}(v_i) = \left[\gamma^-_{B_1}(v_i), \gamma^+_{B_1}(v_i)\right] = \left[\gamma^-_{B_2}(\psi(v_i)), \gamma^+_{B_2}(\psi(v_i))\right]$$
$$\forall v_i \in V.$$

$$\sigma_{B_1}(v_iv_j) = \left[\sigma^-_{B_1}(v_iv_j), \sigma^+_{B_1}(v_iv_j)\right] = \left[\sigma^-_{B_2}(\psi(v_i)\psi(v_j)), \sigma^+_{B_2}(\psi(v_i)\psi(v_j))\right]$$

$$\gamma_{B_1}(v_iv_j) = \left[\gamma^-_{B_1}(v_iv_j), \gamma^+_{B_1}(v_iv_j)\right] = \left[\gamma^-_{B_2}(\psi(v_i)\psi(v_j)), \gamma^+_{B_2}(\psi(v_i)\psi(v_j))\right] \ \forall v_iv_j \in E.$$

This follows that $G^* = (V, E)$ is a cyclic from Proposition 1.24.

Now let $v_1v_2v_3 \cdots v_nv_1$ be a cycle of G^* where vertices set $V = \{v_1, v_2, \cdots, v_n\}$ and edges set $E = \{v_1v_2, v_2v_3, \cdots, v_nv_1\}$. Then we have IVIFS

DOI: http://dx.doi.org/10.5772/intechopen.110182

$$\sigma_{A_1}(v_i) = \left[\sigma^-_{A_1}(v_i), \sigma^+_{A_1}(v_i)\right] = \left[t^-_i, t^+_i\right]$$
$$\gamma_{A_1}(v_i) = \left[\gamma^-_{A_1}(v_i), \gamma^+_{A_1}(v_i)\right] = \left[f^-_i, f^+_i\right]$$

and

$$\sigma_{B_1}(v_iv_{i+1}) = \left[\sigma^-_{B_1}(v_iv_{i+1}), \sigma^+_{B_1}(v_iv_{i+1})\right] = \left[\iota^-_i, \iota^+_i\right]$$
$$\gamma_{B_1}(v_iv_{i+1}) = \left[\gamma^-_{B_1}(v_iv_{i+1}), \gamma^+_{B_1}(v_iv_{i+1})\right] = \left[q^-_i, q^+_i\right],$$

where $i = 1, 2, \cdots, n$ and $v_{n+1} = v_1$. Thus, for $t^-_1 = t^-_{n+1}, t^+_1 = t^+_{n+1}, f^-_1 = f^-_{n+1}, f^+_1 = f^-_{n+1}$

$$\begin{aligned} &\iota^-_i \le t^-_i \wedge t^-_{i+1}, \\ &\iota^+_i \le t^+_i \wedge t^+_{i+1}, \\ &q^-_i \le f^-_i \vee f^-_{i+1} \\ &q^+_i \le f^+_i \vee f^+_{i+1}. \end{aligned} \quad (1)$$

Now

$$H = \{S_{e_i} : i = 1, 2, , \cdots, n\} \text{ and } J = \left\{S_{e_i}S_{e_{i+1}} : i = 1, 2, , \cdots, n-1\ \right\}.$$

And also,

$$\begin{aligned} \sigma_{A_2}(S_{e_i}) &= \left[\sigma^-_{A_2}(S_{e_i}), \sigma^+_{A_2}(S_{e_i})\right] \\ &= \left[\sigma^-_{B_1}(e_i), \sigma^+_{B_1}(e_i)\right] \\ &= \left[\sigma^-_{B_1}(v_iv_{i+1}), \sigma^+_{B_1}(v_iv_{i+1})\right] \\ &= \left[\iota^-_i, \iota^+_i\right] \\ \gamma_{A_2}(S_{e_i}) &= \left[\gamma^-_{A_2}(S_{e_i}), \gamma^+_{A_2}(S_{e_i})\right] \\ &= \left[\gamma^-_{B_1}(e_i), \gamma^+_{B_1}(e_i)\right] \\ &= \left[\gamma^-_{B_1}(v_iv_{i+1}), \gamma^+_{B_1}(v_iv_{i+1})\right] \\ &= \left[q^-_i, q^+_i\right] \\ \sigma^+_{B_2}\left(S_{e_i}S_{e_{i+1}}\right) &= min\left\{\sigma^+_{B_1}(e), \sigma^+_{B_1}(e_{i+1})\right\} \\ &= min\left\{\sigma^+_{B_1}(v_iv_{i+1}), \sigma^+_{B_1}(v_{i+1}v_{i+2})\right\} \\ &= min\left\{\iota^+_i, \iota^+_{i+1}\right\} \\ \sigma^-_{B_2}\left(S_{e_i}S_{e_{i+1}}\right) &= min\left\{\ \sigma^-_{B_1}(e), \sigma^-_{B_1}(e_{i+1})\right\} \\ &= min\left\{\ \sigma^-_{B_1}(v_iv_{i+1}), \sigma^-_{B_1}(v_{i+1}v_{i+2})\right\} \\ &= min\left\{\iota^-_i, \iota^-_{i+1}\right\} \end{aligned}$$

$$\begin{aligned}\gamma^+_{B_2}\left(S_{e_i}S_{e_{i+1}}\right) &= max\left\{\gamma^+_{B_1}(e),\gamma^+_{B_1}(e_{i+1})\right\}\\ &= max\left\{\gamma^+_{B_1}(v_iv_{i+1}),\gamma^+_{B_1}(v_{i+1}v_{i+2})\right\}\\ &= max\left\{q^+_i,q^+_{i+1}\right\}\\ \gamma^-_{B_2}\left(S_{e_i}S_{e_{i+1}}\right) &= max\left\{\gamma^-_{B_1}(e),\gamma^-_{B_1}(e_{i+1})\right\}\\ &= max\left\{\gamma^-_{B_1}(v_iv_{i+1}),\gamma^-_{B_1}(v_{i+1}v_{i+2})\right\}\\ &= max\left\{q^-_i,q^-_{i+1}\right\}\end{aligned}$$

where $v_{n+1}=v_1, v_{n+2}=v_2, \iota^+_1=\iota^+_{n+1}, \iota^-_1=\iota^-_{n+1}, q^+_{n+1}=\iota^+_1, , q^-_{n+1}=q^-_1$, and $i=1,2,\cdots,n.\psi:V\rightarrow H$ is bijective map since $\psi:G^*\rightarrow L(G^*)$ is isomorphism. And also, ψ preserves adjacency. So that ψ persuades an alternative τ of $\{1,2,\cdots,n\}$ which $\psi(v_i)=S_{e_{\tau(i)}}$ and for $e_i=v_iv_{i+1}$ then $\psi(v_i)\psi(v_{i+1})=S_{e_{\tau(i)}}S_{e_{\tau(i+1)}}, i=1,2,\cdots,n-1$. Now

$$\begin{aligned}t^-_i &= \sigma^-_{A_1}(v_i)\le\sigma^-_{A_2}(\psi(v_i))=\sigma^-_{A_2}\left(S_{e_{\tau(i)}}\right)=\iota^-_{\tau(i)},\\ t^+_i &= \sigma^+_{A_1}(v_i)\le\sigma^+_{A_2}(\psi(v_i))=\sigma^+_{A_2}\left(S_{e_{\tau(i)}}\right)=\iota^+_{\tau(i)},\\ f^-_i &= \gamma^-_{A_1}(v_i)\le\gamma^-_{A_2}(\psi(v_i))=\gamma^-_{A_2}\left(S_{e_{\tau(i)}}\right)=q^-_{\tau(i)},\\ f^+_i &= \gamma^+_{A_1}(v_i)\le\gamma^+_{A_2}(\psi(v_i))=\gamma^+_{A_2}\left(S_{e_{\tau(i)}}\right)=q^+_{\tau(i)}.\end{aligned}$$

And let $e_i=v_iv_{i+1}$,

$$\begin{aligned}\iota^-_i &= \sigma^-_{B_1}(v_iv_{i+1})\le\sigma^-_{B_2}\left(\psi(v_i)\psi(v_{i+1})=\sigma^-_{B_2}\left(S_{e_{\tau(i)}}S_{e_{\tau(i+1}}\right)\right)\\ &= min\left\{\sigma^-_{B_1}\left(e_{\tau(i)}\right),\sigma^-_{B_1}\left(e_{\tau(i+1)}\right)\right\}=min\left\{\iota^-_{\tau(i)},\iota^-_{\tau(i+1)}\right\}\\ \iota^+_i &= \sigma^+_{B_1}(v_iv_{i+1})\le\sigma^+_{B_2}\left(\psi(v_i)\psi(v_{i+1})=\sigma^+_{B_2}\left(S_{e_{\tau(i)}}S_{e_{\tau(i+1}}\right)\right)\\ &= min\left\{\sigma^+_{B_1}\left(e_{\tau(i)}\right),\sigma^+_{B_1}\left(e_{\tau(i+1)}\right)\right\}=min\left\{\iota^+_{\tau(i)},\iota^+_{\tau(i+1)}\right\}\\ q^-_i &= \gamma^-_{B_1}(v_iv_{i+1})\le\gamma^-_{B_2}\left(\psi(v_i)\psi(v_{i+1})=\gamma^-_{B_2}\left(S_{e_{\tau(i)}}S_{e_{\tau(i+1}}\right)\right)\\ &= max\left\{\gamma^-_{B_1}\left(e_{\tau(i)}\right),\gamma^-_{B_1}\left(e_{\tau(i+1)}\right)\right\}=max\left\{q^-_{\tau(i)},q^-_{\tau(i+1)}\right\}\\ q^+_i &= \gamma^+_{B_1}(v_iv_{i+1})\le\gamma^+_{B_2}\left(\psi(v_i)\psi(v_{i+1})=\gamma^+_{B_2}\left(S_{e_{\tau(i)}}S_{e_{\tau(i+1}}\right)\right)\\ &= max\left\{\gamma^+_{B_1}\left(e_{\tau(i)}\right),\gamma^+_{B_1}\left(e_{\tau(i+1)}\right)\right\}\\ &= max\left\{q^+_{\tau(i)},q^+_{\tau(i+1)}\right\}\ for\ i=1,2,\cdots,n.\end{aligned}$$

Which implies,

$$\begin{aligned}t^-_i\le\iota^-_{\tau(i)},\quad t^+_i\le\iota^+_{\tau(i)}\\ f^-_i\le q^-_{\tau(i)},\quad f^+_i\le q^+_{\tau(i)}\end{aligned}\tag{2}$$

and

$$\iota_i^- \le min\left\{\iota_{\tau(i)}^-, \iota_{\tau(i+1)}^-\right\}, \quad \iota_i^+ \le min\left\{\iota_{\tau(i)}^+, \iota_{\tau(i+1)}^+\right\}$$
$$q_i^- \le max\left\{q_{\tau(i)}^-, q_{\tau(i+1)}^-\right\}, \quad q_i^+ \le max\left\{q_{\tau(i)}^+, q_{\tau(i+1)}^+\right\}. \tag{3}$$

Thus from the above equations, we obtain $\iota_i^- \le \iota_{\tau(i)}^-, \iota_i^+ \le \iota_{\tau(i)}^+, q_i^- \le q_{\tau(i)}^-$ and $q_i^+ \le q_{\tau(i)}^+$. and also $\iota_{\tau(i)}^- \le \iota_{\tau(\tau(i))}^-, \iota_{\tau(i)}^+ \le \iota_{\tau(\tau(i))}^+, q_{\tau(i)}^- \le q_{\tau(\tau(i))}^-$ and $q_{\tau(i)}^+ \le q_{\tau(\tau(i))}^+$. By proceeding this process, we get

$$\iota_i^- \le \iota_{\tau(i)}^- \le \cdots \le \iota_{\tau^k(i)}^- \le \iota_i^-$$
$$\iota_i^+ \le \iota_{\tau(i)}^+ \le \cdots \le \iota_{\tau^k(i)}^+ \le \iota_i^+$$
$$q_i^- \le q_{\tau(i)}^- \le \cdots \le q_{\tau^k(i)}^- \le q_i^-$$
$$q_i^+ \le q_{\tau(i)}^+ \le \cdots \le q_{\tau^k(i)}^+ \le q_i^+$$

where τ^{k+1} is the identity function. It follows $\iota_{\tau(i)}^- = \iota_{\tau(\tau(i))}^-, \iota_{\tau(i)}^+ = \iota_{\tau(\tau(i))}^+, q_{\tau(i)}^- = q_{\tau(\tau(i))}^-$ and $q_{\tau(i)}^+ = q_{\tau(\tau(i))}^+$. Again, from Eq. (3), we get

$$\iota_i^- \le \iota_{\tau(i+1)}^- = \iota_{i+1}^-, \quad \iota_i^+ \le \iota_{\tau(i+1)}^+ = \iota_{i+1}^+$$
$$q_i^- \le q_{\tau(i+1)}^- = q_{i+1}^-, q_i^+ \le q_{\tau(i+1)}^+ = q_{i+1}^-.$$

This implies for all $i = 1, 2, \cdots, n$, $\iota_i^- = \iota_1^-, \iota_i^+ = \iota_1^+, q_i^- = q_1^-$ and $q_i^+ = q+_1$. Thus, from Eqs. (1) and (2) we obtain

$$\iota_1^- = \cdots = \iota_n^- = t_1^- = \cdots = t_n^-$$
$$\iota_1^+ = \cdots = \iota_n^+ = t_1^+ = \cdots = t_n^+$$
$$q_1^- = \cdots = q_n^- = f_1^- = \cdots = f_n^-$$
$$q_1^+ = \cdots = q_n^+ = f_1^+ = \cdots = f_n^+.$$

As a result, the proof.

Theorem 1.26. Let G be connected simple IVIFG, then IVIFLG of G is a path graph if and only if G is path graph.

Proof: Suppose that G is a path IVIFG with $|V(G)| = k$. Thus, G is a path P_k with length k and $|E(G)| = k - 1$. Since the vertices set of IVIFLG $L(G)$ is an edge sets of G, clearly $L(G)$ is a path with $|V(L(G))| = k - 1$ graph and $|E(L(G))| = k - 2$. Implies that $L(G)$ is a path graph. On the other hand, assume $L(G)$ is a path. Then every degree of vertex $v_i \in G$ is can't be greater than two. If there is a vertex $v_i \in G$ is greater than two, then an edge e which incident to $v_i \in G$ would form a complete sub-graph of IVIFLG $L(G)$ of more than two vertices. As a result, the IVIFG G must be either path graph or cyclic. But, G can't be the cyclic graph since a line graph of the cyclic graph is the cyclic graph. The proof is finished.

3. Conclusion

In this chapter, we introduced interval-valued intuitionistic fuzzy line graphs (IVIFLG) and investigated their results. In addition, we developed many theorems, and propositions related to IVIFLG with proof. Moreover, some remarkable properties of isomorphic properties, strong IVIFLG, and complete IVIFLG have been investigated, and the proposed concepts are illustrated with the examples.

Acknowledgements

The authors do thankful to the editor for giving an opportunity to submit our book chapter.

Competing interest

The authors declared that they have no competing interests.

Author details

Venkata Naga Srinivasa Rao Repalle[1*†], Keneni Abera Tola[2†] and Maamo Abebe Ashebo[1†]

1 Wollega University, Nekemte, Ethiopia

2 Dambi Dollo University, Dambi Dollo, Ethiopia

*Address all correspondence to: rvnrepalle@gmail.com

† These authors contributed equally.

References

[1] Zadeh AL. Information and control. Fuzzy Sets. 1965;**8**(3):338-353

[2] Kaufmann A. Introduction Theory of Fuzzy Sets. New York: Academic Press; 1975. p. 4

[3] Rosenfeld A. Fuzzy Graphs, Fuzzy Sets and Their Applications. Cambridge, United States: Academic Press; 1975. pp. 77-95

[4] Atanassov K. Review and new results on intuitionistic fuzzy sets. International Journal Bioautomation. 2016;**20**:17-26

[5] Atanassov K. Intuitionistic Fuzzy Sets. Theory and Applications. New York: Physica-Verlag; 1999. DOI: 10.1007/978-3-7908-1870-3

[6] Atanassov KT. On Intuitionistic Fuzzy Sets Theory. 2012. p. 283. DOI: 10.1007/978-3-642-29127-2

[7] Mordeson J. Fuzzy line graphs. Pattern Recognition Letters. 1993;**14**(5): 381-384. DOI: 10.1016/0167-8655(93) 90115-T

[8] Firouzian S, Sedghi S, Shobe N. On edge fuzzy line graphs and their fuzzy congraphs. Control and Optimization in Applied Mathematics. 2021;**6**(1):81-89

[9] Akram M, Dudek W. Interval-valued fuzzy graphs. Computers and Mathematics with Applications. 2011;**61**: 289-299. DOI: 10.1016/j. camwa.2010.11.004

[10] Naz S, Malik M, Rashmanlou H. Hypergraphs and transversals of hypergraphs in interval-valued intuitionistic fuzzy setting. Journal of Multiple-Valued Logic and Soft Computing. 2018;**30**:399-417

[11] Akram M. Interval-valued fuzzy line graphs. Neural Computing and Applications - NCA. 2012;**21**:1-6. DOI: 10.1007/s00521-011-0733-0

[12] Rashmanlou H, Borzooei R. New concepts of interval-valued intuitionistic (S, T)-fuzzy graphs. Journal of Intelligent and Fuzzy Systems. 2016; **30**(4):1893-1901

[13] Akram M, Davvaz B. Strong intuitionistic fuzzy graphs. Univerzitet u Nišu. 2012;**26**(1):177-196. DOI: 10.2298/ FIL1201177A

[14] Akram M, Parvathi R. Properties of intuitionistic fuzzy line graphs. Notes on Intuitionistic Fuzzy Sets. 2012;**18**(3): 52-60

[15] Parvathi R, Karunambigai MG, Atanassov KT. Operations on intuitionistic fuzzy graphs. In: 2009 IEEE International Conference on Fuzzy Systems. 20 August 2009. pp. 1396-1401. DOI:10.1109/FUZZY.2009.5277067