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Game between the third party payment service provider and bank in mobile payment market

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Abstract: With the innovation and integration of the Internet and the financial industry, the third-party payment market has developed greatly and has great potential. This paper discusses the duopoly game between third-party payment service providers and banks, which are the main participants in the mobile payment market. By constructing Nash game model, the conditions of equilibrium point, stability and bifurcation are analyzed. The effects of adjusting parameters and cooperation coefficient on business volume and profit are discussed. The conclusions are as follows: excessive investment will lead to unpredictable fluctuations in the market and fall into chaos; By strengthening cooperation, all participants in the mobile payment industry chain can improve business volume and profits while curbing chaos in the mobile payment market.

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Keywords: Market cooperation, mobile payment market, duopoly game model, bifurcation.

1. INTRODUCTION

With the popularization and promotion of mobile network and financial payment, mobile phones have realized the function of mobile payment. Based on such a shopping settlement system, mobile payment and “no paper money payment” have brought great convenience to enterprises and consumers (Dahlberg et al. (2008)).

Mobile payment generally conducts transactions with merchants or individuals through mobile terminals such as mobile phones and tablet computers, which is safer and faster than cash payment (Kanniainen (2010)). The mainstream mobile payment industry chain we are currently seeing includes third-party payment service providers (such as Alipay, PayPal, etc.), banks, mobile operators, and equipment suppliers (chip manufacturers, mobile phone manufacturers, equipment terminal suppliers, etc.), merchants, consumers, etc.

In 2020, China's mobile payment users will reach 790 million¹. The advantages of electronic payment, such as security and convenience of mobile payment market, enable such a life changing model to gain a huge market. It is often difficult for consumers to notice when using. The rapid development of the mobile payment market mainly benefits from the extensive cooperation and technical docking between third-party payment service providers and banks. At the same time, there is an inevitable competitive relationship between third-party service providers and banks. Many scholars study the open mobile payment market (Dahlberg et al. (2015); Weir et al. (2006); Pal et al.

(2015)), mainly from the market cooperation framework (Hedman and Henningson (2015)). At the same time, they can conduct qualitative research on consumers' adoption of mobile payment (Mallat (2007)).

Consumer psychology can improve the consumption efficiency of consumers and the business efficiency of operators by studying consumer behavior (Park et al. (2019); Kim et al. (2019)). With the rise of mobile payment, it is particularly important to explore the relationship between payment system and user behavior mode from the perspective of consumer psychology (Meyl and Walter (2019)). Scholars generally study the positive impact of consumer emotion and expected regret on the behavioral intention of mobile payment through the emotional response model of consumer psychology (Verkijika (2020)).

All participants in the mobile payment industry chain create value for the mobile payment market through division of labor and technical cooperation (Jocovski et al. (2020)). Some scholars discussed the business model of cooperation between mobile operators and financial institutions to analyze the industrial chain selection and profit distribution of mobile payment (ZiFu et al. (2013); Hsiao (2019)), Yang Lu (Yang et al. (2012)) established a model reflecting the characteristics and use environment of mobile payment services, and analyzed the important determinants that mobile payment services should adopt. Kauffman (Kauffman et al. (2015)) studies the infrastructure investment of mobile payment system through a new option based stochastic Valuation Modeling Method of uncertain IT investment. The research on mobile payment mainly adopts the combination of empirical research and

¹ <https://www.iresearch.com.cn>.

qualitative research. There is little research on the cooperation and game between third-party payment service providers and banks. From the perspective of game theory (Ma et al. (2018); Lou and Ma (2018); Guckenheimer and Holmes (2013); Ma and Xie (2019)), this paper studies the competition and cooperation between two important participants in the mobile payment industry chain, third-party payment service providers and banks.

2. MODEL DESCRIPTION

The explosive growth of mobile payment has made mobile payment gradually cover every aspect of consumers' daily consumption. At the same time, the popularity of mobile payment has also made banks, as the monopoly of traditional payment, challenged by third-party payment service providers. Third-party payment service providers have gained increasing support from consumers through technological innovation and payment convenience. Therefore, in the market share of mobile payment, third-party payment service providers can occupy a larger market share than banks.

In order to maximize profits in the emerging mobile payment market, third-party payment service providers and banks are competing fiercely for market share. Banks representing traditional forces and third-party payment service providers representing emerging forces actually monopolize the mobile payment transaction market, so it can be considered that China's mobile payment market is an oligopolistic market.

Based on the understanding of the development status and trend of China's mobile payment market, this paper constructs the operation model of the duopoly market of third-party payment service providers and banks in the mobile payment market. We believe that third-party payment services are homogeneous. Both banks and third-party payment service providers follow the principle of maximizing their own interests. At the same time, participants in the third-party payment market can obtain complete information of the market. We mainly consider bank and the third party payment service provider in mobile payment market. q_1 is the business volume of mobile payment services provided by the third party payment service provider, q_2 is the business volume of mobile payment services provided by bank, and the total business volume of mobile payment services in the market is $Q = q_1 + q_2$. The reverse demand function of mobile payment service in mobile payment market is $P = \alpha - \beta(q_1 + q_2)$. The marginal costs of the third party payment service provider and bank are c_1 and c_2 , respectively. The third party payment service provider costs more than bank, that is, $c_1 > c_2$. Therefore, we can obtain that the profit functions of the third-party payment service provider and the bank are

$$\begin{cases} \pi_1 = (\alpha - \beta(q_1 + q_2))q_1 - c_1q_1, \\ \pi_2 = (\alpha - \beta(q_1 + q_2))q_2 - c_2q_2. \end{cases} \quad (1)$$

Considering the need to ensure positive profits for the players in the game, conditions $\alpha > c_1$ and $\alpha > c_2$ holds. Find the first-order partial derivative of eq.(2) to get the marginal profit as

$$\frac{\partial \pi_1}{\partial q_1} = \alpha - c_1 - \beta q_2 - 2\beta q_1, \quad \frac{\partial \pi_2}{\partial q_2} = \alpha - c_2 - \beta q_1 - 2\beta q_2.$$

Players in the game often use a variety of decision-making modes to adjust the game strategy. It is assumed that the participants adopt the bounded rational expectation adjustment strategy, and adjust the strategy of the period $t+1$ according to the level of the period t . Both sides of the game control the business volume adjustment coefficient $\rho_i, i = 1, 2$ to control the adjustment rate of the forecasting strategy.

$$q_1(t+1) = q_1(t) + \rho_1 q_1(t) \frac{\partial \pi_1}{\partial q_1}, \quad q_2(t+1) = q_2(t) + \rho_2 q_2(t) \frac{\partial \pi_2}{\partial q_2}.$$

The mobile payment discrete mapping duopoly model based on bounded rationality prediction is established as follows

$$\begin{cases} q_1(t+1) = q_1(t) + \rho_1 q_1(t)(\alpha - c_1 - 2\beta q_1(t) - \beta q_2(t)), \\ q_2(t+1) = q_2(t) + \rho_2 q_2(t)(\alpha - c_2 - \beta q_1(t) - 2\beta q_2(t)). \end{cases} \quad (2)$$

3. EQUILIBRIUM ANALYSIS

It is generally believed that neither side of the game will use the strategy of zero and negative expected profit, so the zero solution and negative solution in the game are not allowed by both sides of the system. In the solving model, the non negative equilibrium solution has practical guiding significance. We assume that $q_1(t+1) = q_1(t) = q_{10}$ and $q_2(t+1) = q_2(t) = q_{20}$ in (2). It follows that

$$\begin{cases} q_{10} = q_{10} + \rho_1 q_{10}(\alpha - c_1 - 2\beta q_{10} - \beta q_{20}), \\ q_{20} = q_{20} + \rho_2 q_{20}(\alpha - c_2 - \beta q_{10} - 2\beta q_{20}). \end{cases}$$

Based on the solution of the equation, the equilibrium solutions of the equation are $E_1 \left(\frac{\alpha - c_1}{2\beta}, 0 \right)$, $E_2 \left(0, \frac{\alpha - c_2}{2\beta} \right)$, and $E_3 \left(\frac{\alpha - 2c_1 + c_2}{3\beta}, \frac{\alpha + c_1 - 2c_2}{3\beta} \right)$, respectively Since there will be no player in the game whose return is 0, we exclude the equilibrium solution such as E_1 , and E_2 . Equilibrium E_3 is positive under the conditions

$$\alpha > c_1, \alpha > c_2, c_1 > c_2, \alpha + c_2 - 2c_1 > 0. \quad (3)$$

The following is the Jacobian matrix E_3 :

$$J_3 = \begin{pmatrix} 1 - 2\beta\rho_1q_1^* & -\beta\rho_1q_1^* \\ -\beta\rho_2q_2^* & 1 - 2\beta\rho_2q_2^* \end{pmatrix}, \quad (4)$$

By calculating the trace and determinant of E_3 matrix, we obtain that the necessary and sufficient condition for $E_3(q_1^*, q_2^*)$ stability are $1 - Tr + \Delta > 0$, $1 + Tr + \Delta > 0$, and $1 - \Delta > 0$.

$$\begin{cases} 1 - (2 - 2\beta(\rho_1q_1^* + \rho_2q_2^*)) + 1 - 2\beta(\rho_1q_1^* + \rho_2q_2^*) \\ + 3\beta^2\rho_1\rho_2q_1^*q_2^* > 0, \\ 1 + 2 - 2\beta(\rho_1q_1^* + \rho_2q_2^*) + 1 - 2\beta(\rho_1q_1^* + \rho_2q_2^*) \\ + 3\beta^2\rho_1\rho_2q_1^*q_2^* > 0, \\ 1 - (1 - 2\beta(\rho_1q_1^* + \rho_2q_2^*) + 3\beta^2\rho_1\rho_2q_1^*q_2^*) > 0. \end{cases} \quad (5)$$

The (6) can be obtained by solving (5):

$$\rho_2 < \min \left\{ \frac{2 - \beta\rho_1q_1^*}{\beta q_2^*}, \frac{4 - 4\beta\rho_1q_1^*}{4\beta q_2^* - 3\beta^2\rho_1q_1^*q_2^*} \right\}. \quad (6)$$

Theorem 1. The system has three equilibrium points E_1, E_2 and E_3 , among which E_3 is the only positive equilibrium point. E_3 is stable under condition (6).

Here, for the convenience of calculation and display, we assume

$$\alpha = 5, \beta = 0.5, c_1 = 0.4, c_2 = 0.2, \rho_1 = 0.6. \quad (7)$$

then

$$\frac{2 - \beta\rho_1q_1^*}{\beta q_2^*} \approx 0.6721, \quad \frac{4 - 4\beta\rho_1q_1^*}{4\beta q_2^* - 3\beta^2\rho_1q_1^*q_2^*} \approx 0.2118.$$

and $\min\left\{\frac{2 - \beta\rho_1q_1^*}{\beta q_2^*}, \frac{4 - 4\beta\rho_1q_1^*}{4\beta q_2^* - 3\beta^2\rho_1q_1^*q_2^*}\right\} = 0.2118$. We get the values of the three equilibrium points as $E_1(4.6, 0)$, $E_2(0, 4.8)$, and $E_3\left(\frac{44}{15}, \frac{10}{3}\right)$.

The Fig. 1 shows that the system start from the initial points near E_1 and E_2 , all tend to equilibrium E_3 . This also proves that E_3 is a stable equilibrium solution.

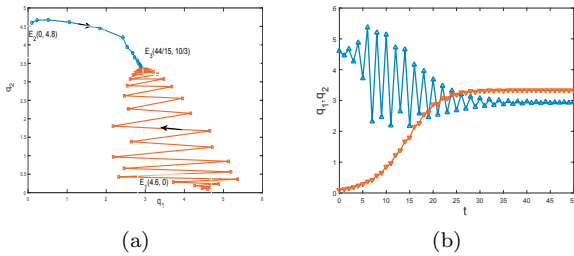


Fig. 1. For system (2) with $\rho_2 = 0.1$ and (6), (a) two solutions with the initial points $(0.1, 4.6)$ and $(4.6, 0.1)$; (b) time series of $q_1(t)$ and $q_2(t)$ of the solution with the initial point $(4.6, 0.1)$.

The eigenvalues of positive equilibrium E_3 are

$$\lambda_{31} = 1 - \beta(\rho_1q_1^* + \rho_2q_2^*) + \beta\sqrt{(\rho_1q_1^* - \rho_2q_2^*)^2 + \rho_1\rho_2q_1^*q_2^*},$$

$$\lambda_{32} = 1 - \beta(\rho_1q_1^* + \rho_2q_2^*) - \beta\sqrt{(\rho_1q_1^* - \rho_2q_2^*)^2 + \rho_1\rho_2q_1^*q_2^*}.$$

Now set

$$\bar{\rho}_2 = \frac{4 - 4\beta\rho_1q_1^*}{4\beta q_2^* - 3\beta^2\rho_1q_1^*q_2^*}. \quad (8)$$

Base on the eigenvalue $\lambda_{32} < -1$ under the condition $\rho_2 > \bar{\rho}_2$, the equilibrium E_3 is unstable. For $\rho_2 = \bar{\rho}_2$, the eigenvalue $\lambda_{32} = -1$, and may be a period doubling bifurcation occurs (Guckenheimer and Holmes (2013)). The model has a stable period-2 solution for

$$\rho_2 \in \left(\frac{4 - 4\beta\rho_1q_1^*}{3\beta^2\rho_1q_1^*q_2^* - 4\beta q_2^*}, \frac{4 - 4\beta\rho_1q_1^*}{3\beta^2\rho_1q_1^*q_2^* - 4\beta q_2^*} + \epsilon\right), \quad (9)$$

There is a point (q_{10}^*, q_{20}^*) , which will return to the original point after two iterations, so $E_{41}(q_{10}^*, q_{20}^*) \rightarrow E_{42}(q_{11}^*, q_{21}^*) \rightarrow E_{41}(q_{10}^*, q_{20}^*)$.

$$\begin{cases} q_1 = \bar{q}_1 + \rho_1\bar{q}_1(\alpha - c_1 - 2\beta\bar{q}_1 - \beta\bar{q}_2), \\ q_2 = \bar{q}_2 + \rho_2\bar{q}_2(\alpha - c_2 - \beta\bar{q}_1 - 2\beta\bar{q}_2), \\ \bar{q}_1 = q_1 + \rho_1q_1(\alpha - c_1 - 2\beta q_1 - \beta q_2), \\ \bar{q}_2 = q_2 + \rho_2q_2(\alpha - c_2 - \beta q_1 - 2\beta q_2). \end{cases} \quad (10)$$

The following result is obtained.

Theorem 2. The equilibrium point E_3 of the system is unstable under condition (8). The system will have period doubling bifurcation under condition (9), and two stable fixed points of period 2 will appear at the same time, which are E_{41} and E_{42} respectively.

We can get that $\bar{\rho}_2 = \frac{4 - 4\beta\rho_1q_1^*}{3\beta^2\rho_1q_1^*q_2^* - 4\beta q_2^*} \approx 0.2118$.

In Fig. 2, we can see that the system has a stable equilibrium point E_3 . When $\rho_2 = 0.2118$, the system

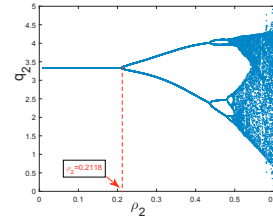


Fig. 2. Bifurcation diagram of system (2) with (7) and $\rho_2 \in (0, 0.6)$

enters period doubling bifurcation and two stable fixed points E_{41} and E_{42} will appear at the same time. For the players in the mobile payment industry chain, there are certain requirements for investment stability and profit expectation in the game process. Players will try their best to ensure the stable growth of profits while expanding the market scale, and abnormal and huge investment will make the system enter a very unstable growth and fall into chaos, which is not conducive to the long-term and stable development of the industry.

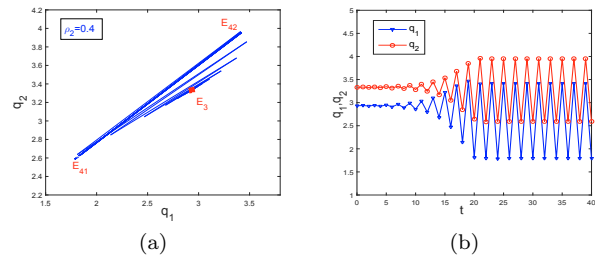


Fig. 3. The solution of system (2) with $\rho_2 = 0.4$ and (7), (a) the portrait; (b) the time series of $q_1(t)$ and $q_2(t)$.

As can be seen from Fig. 3, $E_3\left(\frac{44}{15}, \frac{10}{3}\right)$ tends to two equilibrium points $E_{41}(1.7907, 2.5912)$ and $E_{42}(3.4170, 3.9526)$ from the initial point $(2.93, 3.33)$, and the two period 2 equilibrium points E_{41} and E_{42} are stable under condition (7).

4. PROFIT ANALYSIS

We will adjust the business volume by ρ_2 as the main adjustment parameter to discuss its impact on the system.

The unit period profit of third-party payment service providers and banks are

$$\bar{\pi}_1 = (\alpha - \beta(q_1^* + q_2^*))q_1^* - c_1q_1^* = \frac{(\alpha - 2c_1 + c_2)^2}{9\beta},$$

$$\bar{\pi}_2 = (\alpha - \beta(q_1^* + q_2^*))q_2^* - c_2q_2^* = \frac{(\alpha + c_1 - 2c_2)^2}{9\beta}.$$

When doubling occurs in the system, $E_{41}(q_{10}^*, q_{20}^*) \rightarrow E_{42}(q_{11}^*, q_{21}^*) \rightarrow E_{41}(q_{10}^*, q_{20}^*)$. The unit period profit of third-party payment service providers and banks are

$$\bar{\pi}_1(\rho_2) = [(\alpha - \beta(q_{10}^* + q_{20}^*))q_{10}^* - c_1q_{10}^* + (\alpha - \beta(q_{11}^* + q_{21}^*))q_{11}^* - c_1q_{11}^*]/2,$$

$$\bar{\pi}_2(\rho_2) = [(\alpha - \beta(q_{10}^* + q_{20}^*))q_{20}^* - c_2q_{20}^* + (\alpha - \beta(q_{11}^* + q_{21}^*))q_{21}^* - c_2q_{21}^*]/2.$$

Based on hypothesis (7), we can get $\frac{\alpha + c_1 - 2c_2}{3\beta} = \frac{10}{3}$ and $\bar{\pi}_2 = (\alpha - \beta(q_1^* + q_2^*))q_2^* - c_2q_2^* = \frac{(\alpha + c_1 - 2c_2)^2}{9\beta} = \frac{50}{9}$. Fig. 4(a) shows that the unit business volume of bank is $\frac{10}{3}$ for $\rho_2 \in (0, 0.2118)$, increases first and then decreases for $\rho_2 \in (0.2118, 0.5)$, takes the maximum value at

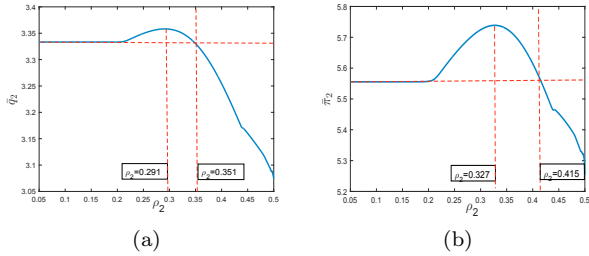


Fig. 4. For system (2) with $\rho_2 \in (0.05, 0.5)$ and (7), (a) the unit business volume of bank; (b) the unit profit of bank.

$\rho_2 = 0.291$, and is less than $\frac{10}{3}$ for $\rho_2 \in (0.351, 0.5)$. It's seen from Fig. 4(b) that the unit profit of bank is $\frac{55}{9}$ for $\rho_2 \in (0, 0.2118)$, increases for $\rho_2 \in (0.2118, 0.327)$, decreases for $\rho_2 \in (0.327, 0.5)$, and is large than $\frac{50}{9}$ for $\rho_2 \in (0.2118, 0.415)$. The change of ρ_2 will not affect the business volume and unit profit of the bank. The bank's maximum unit profit in the game at $\rho_2 = 0.327$.

5. COOPERATIVE ANALYSIS

The participants in the game adopt the decision-making mode of limited rational expectation, so the business volume in the period of $t + 1$ is determined by the marginal profit in the period of t . Therefore, the profit model in multi-cycle game are

$$q_1(t+1) = q_1(t) + \rho_1 q_1(t) \frac{\partial \pi_1}{\partial q_1}, \quad q_2(t+1) = q_2(t) + \rho_2 q_2(t) \frac{\partial \pi_2}{\partial q_2}.$$

With the upgrading and innovation of the payment industry in the Internet and financial industry, banks and third-party payment service providers will increase business volume by strengthening cooperation. Here, it is assumed that both parties in the game increase the business volume by $H_i q_1(t) q_2(t)$ ($i = 1, 2$) at the same time. And $\rho_1 h_1 q_1(t) q_2(t)$ and $\rho_2 h_2 q_1(t) q_2(t)$ are the value of increased business volume of both parties. Here, a multi-period game model considering cooperation and competition can be obtained

$$\begin{cases} q_1(t+1) = q_1(t) + \rho_1 q_1(t) \\ (\alpha - c_1 - 2\beta q_1(t) - \beta q_2(t)) + \rho_1 h_1 q_1(t) q_2(t), \\ q_2(t+1) = q_2(t) + \rho_2 q_2(t) \\ (\alpha - c_2 - \beta q_1(t) - 2\beta q_2(t)) + \rho_2 h_2 q_1(t) q_2(t). \end{cases}$$

where $c_1 > c_2$, $h_1 \geq 0$, $h_2 \geq 0$. h_1 and h_2 are cooperation coefficients of the third party payment service provider and bank, respectively. Then

$$\begin{cases} q_1(t+1) = q_1(t) + \rho_1 q_1(t) \\ (\alpha - c_1 - 2\beta q_1(t) - (\beta - h_1) q_2(t)), \\ q_2(t+1) = q_2(t) + \rho_2 q_2(t) \\ (\alpha - c_2 - (\beta - h_2) q_1(t) - 2\beta q_2(t)). \end{cases} \quad (11)$$

The equilibrium solution $\tilde{E}_3(\tilde{q}_1^*, \tilde{q}_2^*)$ of system (11) is

$$\begin{cases} \tilde{q}_1^* = \frac{2\beta(\alpha - c_1) - (\beta - h_1)(\alpha - c_2)}{4\beta^2 - (\beta - h_1)(\beta - h_2)}, \\ \tilde{q}_2^* = \frac{2\beta(\alpha - c_2) - (\beta - h_2)(\alpha - c_1)}{4\beta^2 - (\beta - h_1)(\beta - h_2)}. \end{cases} \quad (12)$$

The condition that makes the equilibrium solution stable is

$$\begin{cases} 4 - 4\beta(\rho_1 \tilde{q}_1^* + \rho_2 \tilde{q}_2^*) + (4\beta^2 - (\beta - h_1) - \\ (\beta - h_2))\rho_1 \rho_2 \tilde{q}_1^* \tilde{q}_2^* > 0, \\ 4 - 2\beta(\rho_1 \tilde{q}_1^* + \rho_2 \tilde{q}_2^*) > 0. \end{cases} \quad (13)$$

One eigenvalue of positive equilibrium \tilde{E}_3 is -1 at $\rho_2 = \tilde{\rho}_2$, where

$$\tilde{\rho}_2 = \frac{4 - 4\beta\rho_1\tilde{q}_1^*}{4\beta\tilde{q}_2^* - [4\beta^2 - (\beta - h_1)(\beta - h_2)]\rho_1\tilde{q}_1^*\tilde{q}_2^*}, \quad (14)$$

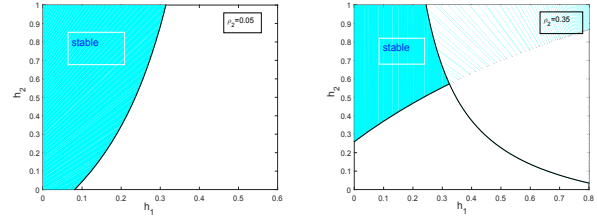


Fig. 5. (a) The stable regions of equilibrium \tilde{E}_3 of system (11) with (7) and (a) $\rho_2 = 0.05$; (b) $\rho_2 = 0.35$,

The previous condition (7) still holds in system (11). Fig. 5 shows the stability region of the equilibrium solution of system (11) when $\rho_2 = 0.05$ and $\rho_2 = 0.35$.

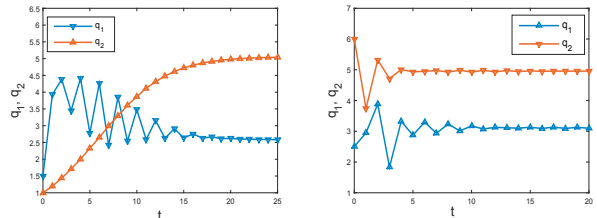


Fig. 6. The time series of $q_1(t)$ and $q_2(t)$ of system (11) with (7) and (a) $\rho_2 = 0.05$, $h_1 = 0.1$, $h_2 = 0.6$; (b) $\rho_2 = 0.35$, $h_1 = 0.2$, $h_2 = 0.55$.

Based on hypothesis (7), the unit business volume of the third-party payment platform and the bank when $h_1 = 0.1$ are

$$\tilde{q}_1^*(h_2) = \frac{2.68}{0.8 + 0.4h_2}, \quad \tilde{q}_2^*(h_2) = \frac{2.5 + 4.6h_2}{0.8 + 0.4h_2}. \quad (15)$$

and the unit profit of bank are

$$\tilde{\rho}_2(h_2) = \frac{1.6h_2 - 0.016}{0.98 + 1.8032h_2}, \quad \tilde{\rho}'_2(h_2) = \frac{1.5969}{(0.98 + 1.8032h_2)^2}. \quad (16)$$

Fig. 7 shows the business volume of the payment market in the case of complete non cooperation and limited cooperation, where $h_2 = 0.25$. As can be seen in Fig. 7(b), the sum of business in the case of limited cooperation is higher than that in the case of non cooperation. At the same time, the addition of cooperation coefficient delays the time when the system enters the period doubling bifurcation. Therefore, we can delay the chaos of the mobile payment market by increasing the degree of cooperation between the two sides.

In order to better explore the complex characteristics of system (11), we assume

$$\alpha = 5, \beta = 0.3, c_1 = 0.3, c_2 = 0.2, \rho_1 = 0.62, \rho_2 = 0.31. \quad (17)$$

Fig. 8 shows the bifurcation diagram and portals of the cooperative mobile payment system respectively. We can see that the system keep stable at $h_2 \in (0.356, 0.418)$, and enter the bifurcation at $h_2 = 0.356$ and $h_2 = 0.418$. Fig. 9 shows the of the system (11) at H_2 bifurcation diagrams and portraits with specific values. This fully demonstrates the unique complex characteristics of system (11).

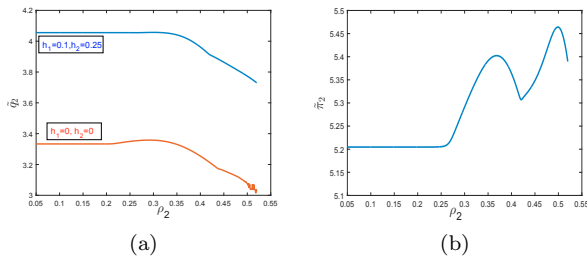


Fig. 7. $\rho_2 \in (0.05, 0.52)$, (a) the unit business volumes of financial institution for $h_1 = 0, h_2 = 0$ and $h_1 = 0.1, h_2 = 0.25$; (b) the unit profit of financial institution for $h_1 = 0.1, h_2 = 0.25$.

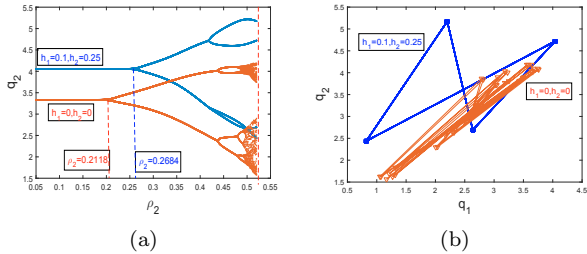


Fig. 8. (a) bifurcation diagrams of stable equilibria for $\rho_2 \in (0.05, 0.52)$, $h_1 = 0, h_2 = 0$, and $h_1 = 0.1, h_2 = 0.25$; (b) the portraits for $\rho_2 = 0.52$, $h_1 = 0, h_2 = 0$, and $h_1 = 0.1, h_2 = 0.25$.

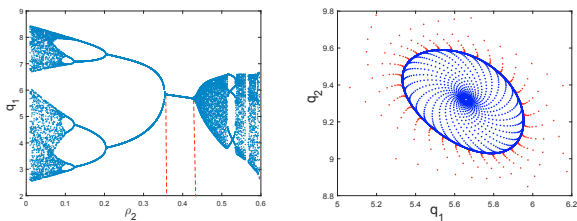


Fig. 9. For system (11) with (17), (a) bifurcation diagram of stable equilibria for $h_2 \in (0, 0.6)$; (b) the portraits for $h_2 = 0.44$.

6. CONCLUSIONS

With the rapid development of Internet and financial technology, mobile payment technology, as a product of Internet financial integration, has received more attention. As important players and players in the mobile payment industry chain, banks and third-party payment service providers have been in the state of cooperation and game in the process of industry development. Duopoly game model is built based on the marginal profit and the adjusting parameter. The unit profit of bank may decrease with the adjustment coefficient increasing in a certain range and there are some defects model. In addition, considering to meet the rapid development of the market, cooperation is introduced to build duopoly multi-period game model. Theoretical analysis and numerical simulation show that increasing business volume through cooperation can suppress chaos in mobile payment market.

This paper constructs a duopoly game model of the mobile payment service market under complete information. Through analysis and simulation, we believe that the mobile payment market can still continue to be deeply developed and developed to avoid the business shrinkage of any participant leading to the fracture of the industrial chain. When the strategy of providing the above given business volumes is adopted, neither the third party pay-

ment service provider nor bank has enough reason to break the balance in order to achieve the expected maximum income. The overall mobile payment industry chain is also affected by the business. If the adjustment coefficient is too large, the industry chain will enter a chaotic state, and the business volume of participants in the mobile payment market will become unpredictable, resulting in the collapse or unpredictable changes of the mobile payment market. Therefore, it is necessary to establish a mobile payment supervision system with the participation of all supervision subjects. At the same time, it should also establish a communication and coordination mechanism for daily supervision, reasonably allocate supervision resources, and realize the all-round supervision of the mobile payment industry, so as to promote the healthy and orderly development of the mobile payment market.

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