Describing law of motion of flexible inextensional shell in gravity force field

Descripción de ley de movimiento de cascarilla plegable no expansible en campo de fuerzas de gravedad

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ABSTRACT

This work makes use of Navier-Stokes equations to describe an analytical method of finding the motion speed of a flexible inextensional shell falling down to the ground from a preset height and determines the duration of this fall. The soft shell in question is a fabric body of aerodynamic shape or an item of clothes, an airborne vehicle element, etc. Analytical relations are presented for the speed at which the shell moves in the air, taking account of the air resistance and the shell fall duration. The boundary problem of the soft shell vertically falling in the air is solved.

Keywords: Analytical calculation method; Flexible inextensional (soft) shell; Motion resistance forces; Navier-Stokes equations; Shell gravity force.

RESUMEN

Aquí se usan las ecuaciones de Navier-Stokes para describir un método analítico de calcular la velocidad del movimiento de una cascarilla plegable no expansible que cae en tierra de una altura predeterminada y calcular la duración de la caída. La cascarilla plegable en cuestión es un objeto de tejido de configuración aerodinámica o una prenda de vestir, un elemento de un vehículo aéreo, etc. Se presentan las relaciones analíticas para calcular la velocidad con que la cascarilla cae en el aire. Se tienen en cuenta en las fórmulas la resistencia del aire y la duración de la caída. Se resuelve el problema de límites para la cascarilla que cae verticalmente en el aire.

Palabras claves: Cálculo analítico; Cascarilla plegable no expansible; Fuerzas de resistencia al movimiento; Ecuaciones de Navier-Stokes; Fuerzas de gravedad de la cascarilla.

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1. INTRODUCTION

In modern mechanics of continua an important place belongs to the actively evolving section of soft aircraft dynamics; this section is dedicated to solving aircraft motion control problems. Soft airborne vehicles include aerodynamic shapes, ram-air parachutes, hand gliders, parachutes, and other devices that can be modeled in the form of flexible inextensional shell. To solve of the soft shell freely moving in the air is necessary for studying the motion of various airborne vehicles with a density exceeding the air density; as a result, a soft shell freely moving in the air falls down to the ground by gliding from a preset height.

A major application of the falling shell problem is the intrusion of an airborne vehicle onto the air strip (Perl, 2006). The summary showing how severe this issue is can be found in the hearings held by the US Subcommittee on Aviation in 2001. While landing an airborne vehicle has to enter into a gliding fall. Another topical problem is the safe landing control after a long supposed fall of humanoid robots (Bingham *et al*., 2014). Humanoid robots face a high risk of fall, when walking or working in an undefined medium, and for which it is necessary to ensure their safe controllable contact with technical object surface (Zhang, *et al.,* 2017). The autonomous control of microsize airborne vehicles imitating flying insects in unknown media is a complex problem because these artificial subjects have small dimensions and are, therefore, exposed to wind influences. Similarly to insects, microsize airborne vehicles shall have highly efficient visual motor control systems (Kerhuel, Viollet & Franceschini, 2007). The issues related to controlling microsize airborne vehicles and humanoid robots led to the creation of hybrid routing models with semi-autonomous control during robot-assisted operations in unknown remote, dangerous, or otherwise unavailable areas (Lin *et al*., 2014).

The motion of technical objects in the air is described by Navier-Stokes equations, taking account of air viscosity and turbulent airflow stresses (Indenbirken *et al*, 2000; Rocha *et al.,* 2000). That said, the forces with which the airflow contacts with the moving body surface are also taken into account (Paivanas & Hassan, 1981). However, the applied methods of calculating moving body speeds and pressures are defined either numerically or by experiment.

This work proposes an analytical method of finding the motion speed of a soft inextensional shell falling down to the ground from a preset height and determining the duration of this fall. The shell's fall down to the ground is modeled, taking into account the airflow motion resistance forces but without taking into account the forces with which the wind affects the shell.

The task of describing the soft shell motion law involves such major aspects as designing the shell and its forming in the field of gravity and elasticity forces (Byrdina, Bekmurzaev & Mitsik, 2017a; 2017b). Another topical issue is the vertical stability of the shell with an airflow discharged inside it (Bekmurzaev, Mitsik & Byrdina, 2018). It should be noted that, similarly to the flow of fluid, the airflow can be described not only by Navier-Stokes equations but also by other equations, e.g., by Euler equations when the air viscosity is not taken into account (Kocharenko, Mitsik & Aleynikova, 2016). This work is an important part of describing dynamical behavior of flexible inextensional shells used in various environments.

2. SOFT SHELL MOTION EQUATIONS

Let us consider the common patterns typical of the considered physical model that are described in (Loytsyanskiy, 1978).
The flow of viscous gas will be analyzed proceeding from the following assumptions:

1.Gas is perfect, i.e., its pressure p, density ρ, and absolute temperature T meet the Clayperon-Mendeleev equation known as the ideal gas law

$$p = \frac{\rho \cdot R_0 \cdot T}{m},$$
(1)

where R_0 – is the universal gas constant and m – is the gas molecular weight.

2. Dynamic viscosity coefficient μ is the function of T only. Various laws of this relation can be used hereafter. In the Sutherland formula recorded as

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{\frac{3}{2}} \cdot \frac{T_0 + T_s}{T + T_s},$$
(2)

T_s is the Sutherland constant the value of which for air is around 122 K;
T_o and μ_0 are the absolute temperature and the viscosity coefficient correspondent to some initial gas state.

3. Specific heat capacitance coefficient c does not depend on the absolute gas temperature and is a physical gas constant.
4. Gaseous heat conduction coefficient λ is proportionate to μ, i.e., the Prandtl number is attained and recorded as

$$\frac{\mu \cdot c}{\lambda} = \sigma = const.$$
(3)

5. The above described equations are supplemented by the motion continuity equation recorded as

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0.$$
(4)

6. Gas is a Newtonian medium subject to the known Newton's generalized law of the linear relation between the stress tensor and the strain speed tensor
The main Navier-Stokes viscous gas dynamics equations are

$$
\begin{cases}
\rho \dfrac{du}{dt} = \rho F_x - \dfrac{\partial p}{\partial x} + \mu\left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2}\right) + \mu \dfrac{\partial}{\partial x}\left(\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z}\right) - \\
-\dfrac{2}{3}\dfrac{\partial}{\partial x}\left(\mu \cdot div\overline{V}\right); \\[2mm]
\rho \dfrac{dv}{dt} = \rho F_y - \dfrac{\partial p}{\partial y} + \mu\left(\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2} + \dfrac{\partial^2 v}{\partial z^2}\right) + \mu \dfrac{\partial}{\partial y}\left(\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z}\right) - \\
-\dfrac{2}{3}\dfrac{\partial}{\partial y}\left(\mu \cdot div\overline{V}\right); \\[2mm]
\rho \dfrac{dw}{dt} = \rho F_z - \dfrac{\partial p}{\partial z} + \mu\left(\dfrac{\partial^2 w}{\partial x^2} + \dfrac{\partial^2 w}{\partial y^2} + \dfrac{\partial^2 w}{\partial z^2}\right) + \mu \dfrac{\partial}{\partial z}\left(\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z}\right) - \\
-\dfrac{2}{3}\dfrac{\partial}{\partial z}\left(\mu \cdot div\overline{V}\right);
\end{cases}
\tag{5}
$$

where $u, v, w -$ are the projections of the $\overline{V}$ wind speed vector to the OX, OУ, OZ coordinate axes; $\overline{F} = \left(F_x; F_y; F_z\right) -$ is the vector of the external bulk forces affecting the gas in each point in space; $\mu -$ is the dynamic viscosity coefficient;

$$
div\overline{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}.
$$

The full derivatives $\dfrac{du}{dt}, \dfrac{dv}{dt}, \dfrac{dw}{dt}$ are recorded in detail as

$$
\begin{aligned}
\frac{du}{dt} &= \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}u + \frac{\partial u}{\partial y}v + \frac{\partial u}{\partial z}w; \\
\frac{dv}{dt} &= \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x}u + \frac{\partial v}{\partial y}v + \frac{\partial v}{\partial z}w; \\
\frac{dw}{dt} &= \frac{\partial w}{\partial t} + \frac{\partial w}{\partial x}u + \frac{\partial w}{\partial y}v + \frac{\partial w}{\partial z}w.
\end{aligned}
\tag{6}
$$

The assumption that can be considered for this model is

$$
F_x = 0; F_y = 0; F_z = -mg,
\tag{7}
$$

where $g -$ is the gravity acceleration.

The physical meaning of this assumption is that it presupposes the absence of any external force action on the shell freely falling down through the air and moving only because of its higher density.
Boundary conditions of the problem:

1. At the initial time instant the speed in each point of the shell is zero.
2. At the initial time instant of free fall, the speeds of the airflow outside the shell are zero in each point in space.

3. SIMPLIFYING THE FREE FALLING SHELL PROBLEM

In the first phase of simplifying the freely falling shell problem let us additionally assume that the process is isothermal, i.e., the airflow temperature is constant and equal to 15^0C in each point of the airflow. Actually, this condition means that heat losses on friction are excluded. Let us also suppose that in this problem the air flow is incompressible. Proceeding from these assumptions, the airflow temperature will be constant and the air density in each point of the airflow will be constant as well.

These circumstances lead to constant pressures in each point of the airflow according to the Clayperon-Mendeleev equation. i.e.,

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = \frac{\partial p}{\partial z} \equiv 0 \tag{8}$$

Since the airflow is supposed incompressible, then

$$div\overline{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \equiv 0 \tag{9}$$

By virtue of condition (9), the latter two summands in each equation of system (5) are zero; taking account of formulas (1) and (6) and assumption (7), system (5) will be changed to a simpler form recorded as

$$\begin{cases} \rho\left(\dfrac{\partial u}{\partial t} + \dfrac{\partial u}{\partial x}u + \dfrac{\partial u}{\partial y}v + \dfrac{\partial u}{\partial z}w\right) = \mu\left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2}\right); \\[3mm] \rho\left(\dfrac{\partial v}{\partial t} + \dfrac{\partial v}{\partial x}u + \dfrac{\partial v}{\partial y}v + \dfrac{\partial v}{\partial z}w\right) = \mu\left(\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2} + \dfrac{\partial^2 v}{\partial z^2}\right); \\[3mm] \rho\left(\dfrac{\partial w}{\partial t} + \dfrac{\partial w}{\partial x}u + \dfrac{\partial w}{\partial y}v + \dfrac{\partial w}{\partial z}w\right) = -\rho mg + \mu\left(\dfrac{\partial^2 w}{\partial x^2} + \dfrac{\partial^2 w}{\partial y^2} + \dfrac{\partial^2 w}{\partial z^2}\right); \end{cases} \tag{10}$$

After the admitted assumptions, system (10) is a system of three nonlinear equations in partial derivatives relative to three unknown functions, i.e., (10) is a closed system.

Let us make system (10) even simpler by assuming that the shell freely falls without moving horizontally, i.e., the fabric points motion paths have constant x and y coordinates and the shell configuration changes only along the OZ axis (Fig. 1).
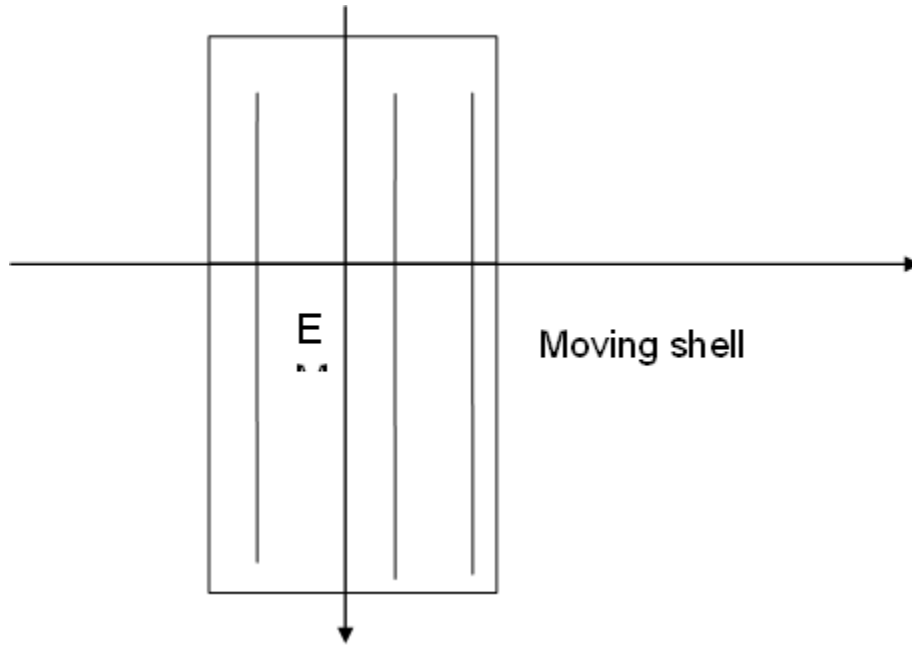
Figure 1. Behavior of the shell freely falling in the airflow in the XOZ plane

The supposition that the airflow will not change along the OY axis means that functions $v = v(x, y, z) = 0, u = u(x, y, z) = 0$ have zero values and function $w = w(z)$, i.e., it does not depend on variables x and y. Thus the actual 3D flow is replaced with the 1D flow model.

In these conditions system (10) will be simplified as

$$\rho\left(\frac{\partial w}{\partial t} + \frac{\partial w}{\partial z} w\right) = \rho mg + \mu \frac{\partial^2 w}{\partial z^2} \tag{11}$$

Equation (11) will be solved by approximate methods given that above exposed problem restrictions 1) – 5) are met.

To solve equation (11), it will be considered that the shell falling process is almost stationary because of the low shell weight and the inhibitory effect of the air; thus

$$\frac{\partial w}{\partial t} = 0 \tag{12}$$

and equation (11) will be simplified to

$$\frac{\partial w}{\partial z} w = mg + \frac{\mu}{\rho} \frac{\partial^2 w}{\partial z^2} \tag{13}$$

Equation $\dfrac{\mu}{\rho} \dfrac{\partial^2 w}{\partial z^2}$ in the right part of equation (13) determines the aerodynamic resistance forces of the air medium inhibiting the shell.

Equation (13) will be transformed on the assumption that, according to (Targ, 1986), the air resistance force in engineering aerodynamics in the square resistance zone is approximately found as

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$$\frac{\mu}{\rho}\frac{\partial^2 w}{\partial z^2} \approx -\frac{\mu k}{\rho}w^2, \tag{14}$$

where $k -$ is the coefficient found by experiment.

The minus sign in the right parts of (14) means that the second partial derivative of the speed vector projection is negative due to the inhibitory influence of the viscous medium.

By virtue of (14) equation (13) is converted to

$$\frac{\partial w}{\partial z}w = mg - \frac{\mu k}{\rho}w^2. \tag{15}$$

Equation (15) is basic for finding the speed of the shell freely falling in stationary mode.

4. FINDING FREELY FALLING SHELL DIMENSIONS

Let us consider the geometric properties of the freely falling shell. The vertical projection of the speed vector in the initial phase of motion is zero, i.e.,

$$w(z;0) = 0. \tag{16}$$

The vertical component of the shell speed vector will first increase due to the gravity force affecting the shell. However, with the increase in vertical component $w = w(z)$ the viscous medium resistance force increases as well; consequently, after some time the airflow gravity force directed vertically downwards will be balanced by the medium resistance force. Consequently, the vertical component of the flow will tend to some constant value. That said, $w = w(z)$ shall be monotonously increasing against variable z .

Let us solve equation (15) to meet initial condition (16). The general solution of (15) is

$$w(z) = \sqrt{\frac{mg\rho}{k\mu} + C \cdot e^{-\frac{\mu k}{\rho}z}}, \tag{17}$$

where $C -$ is the arbitrary constant.

The value of C is found from initial condition (16) as

$$C = -\frac{mg\rho}{k\mu}. \tag{18}$$

Taking into account equation (18), the shell speed distribution is recorded as

$$w(z) = \sqrt{\frac{mg\rho}{k\mu}\left(1 - e^{-\frac{\mu k}{\rho}z}\right)}, \tag{19}$$

Assume that the shell freely falls down from height H . In this case the shell motion speed is found as

$$w_{cp} = \frac{1}{H}\int\limits_0^H \sqrt{\frac{mg\rho}{k\mu}\left(1 - e^{-\frac{\mu k}{\rho}z}\right)}\,dz. \tag{20}$$

Since the shell fall duration is

$$t = \frac{H}{w_{cp}}, \tag{21}$$

the shell fall duration found from (21), taking (20) into account, is

$$t = \frac{H^2}{\int\limits_0^H \sqrt{\frac{mg\rho}{k\mu}\left(1 - e^{-\frac{\mu k}{\rho}z}\right)}\,dz}. \tag{22}$$

5. CONCLUSIONS

1. The actual 3D airflow streamlining the falling shell can be treated in practice as unidimensional to a sufficient degree of adequacy.
2. It is convenient to analyze 1D flows using the Navier-Stokes equations on the assumption that the shell moves vertically and by approximating the medium friction forces according to the square resistance law.
3. The theoretical relations for the distribution of the speeds and duration of the shell falling down to the ground from a preset height have been found.
4. The study results can be used for modeling the fall of various technical objects in the atmosphere without taking account of wind influences.

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