Reflecting Surface Assisted Energy Harvesting with Optimized NOMA Downlink Transmissions

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***Abstract*—This paper studies an intelligent reflecting surface (IRS) assisted wireless communication system with multiple downlink data and energy harvesting users. We assume that base station uses non-orthogonal multiple access (NOMA) for transmission and downlink users employ successive interference cancellation to decode their information from the received signal. With this setting, our goal is to maximize the harvested energy at the energy harvesting users while guaranteeing the minimum rate requirements of the individual data users. We propose an alternating optimization based algorithm, where semidefinite relaxation is used to obtain the optimal beamforming design at the base station and the IRS. Specifically, an iterative rank minimization approach is used to obtain the optimal reflection phase vector at the IRS. The convergence of the proposed algorithm is also proved. Finally, the efficacy of the proposed algorithm is demonstrated with the help of simulation results.**

***Index Terms*—Intelligent reflecting surface, non-orthogonal multiple access, successive interference cancellation, wireless energy harvesting.**

I. Introduction

Wireless energy harvesting technology can provide a viable solution to the problem of wirelessly powering the unapproachable small size devices/sensors [1]. On one hand, it is an attractive solution while, on the other hand, the limitations imposed by strong propagation losses hinder the application of this technology in practical situations. Due to the ability of intelligent reflecting surfaces (IRSs) to reconfigure the wireless environment, the losses owing to propagation environment can be mitigated to some extent. Essentially, IRS is composed of a large number of reflecting elements which can change the direction of coherently reflected signals.

Due to the low-cost solution offered by an IRS, many recent studies [2]–[9] have focused on assessing the performance benefits realized by using IRSs in modern wireless communication systems. General challenges specific to the IRS applications in wireless communication systems are highlighted in [2]. In [3], theoretical performance limits of IRS-based wireless communication systems are explored. Mu *et. al.* in [4], [5] have provided algorithms for maximizing the sum rate while optimizing the active and passive beamforming as well as location of the IRS. Specifically, in [4], the authors use successive convex approximation techniques to maximize the weighted sum rate achieved at a single-antenna user by adjusting the transmit power of a single-antenna base station, IRS phases and IRS location. This work is extended in [5] by incorporating a multi-antenna base station. A signal-to-interference-plus-noise ratio (SINR) fairness problem is considered in [8], where IRS is divided into multiple modules of reflecting elements; then an efficient scheme is devised to achieve fairness among SINR by triggering/untriggering the individual modules.

This research work was supported in part by the Academy of Finland.

Another important consideration in wireless communication systems is the energy consumption/utilization. In wireless communication research community, the research problems related to energy are categorized into two types: energy minimization to address energy consumption issues and energy efficiency maximization to address energy utilization issues.

An energy minimization approach is proposed in [7]. Particularly, a multi-IRS assisted wireless communication system is considered in [7] and efficient algorithms with provable convergence are devised to minimize the energy consumption at the base station while guaranteeing the SINR requirements of the downlink users. An energy efficiency maximization algorithm is proposed in [6] by using the deep reinforcement learning techniques. Particularly, machine learning approaches are adopted in two steps. The first step predicts users' teletraffic demand with the help of a real dataset and then in the second step a decaying double deep Q-network based position-acquisition phase-control algorithm is proposed to solve the joint problem of deployment and design of the IRS.

The above works considered conventional communication systems without focus on energy harvesting based wireless communication systems. An IRS, due to its ability to function without requiring extra circuitry, provides a low cost solution to the long-distance propagation loss problem [9]. Therefore, the work of [1] was extended in [9] by incorporating an IRS into the system, after which an alternating optimization based algorithm was proposed to maximize the harvested energy while guaranteeing the individual rate constraints of downlink users. Particularly, semidefinite relaxation based active and passive beamforming was proposed and Gaussian randomization technique was used to obtain the reflection phases.

In this paper, we study an IRS-based energy harvesting wireless communication system, where downlink users can employ successive interference cancellation to decode non-orthogonal multiple access transmissions from a base station. We propose an efficient alternating optimization algorithm to maximize the harvested energy with convergence guarantee and demonstrate the performance thereof. Especially, it avoids Gaussian randomization to obtain the rank-one solution, hence the feasibility of the obtained solution is also guaranteed.

II. System Model and Problem Formulation

The system model consists of a base station (BS) equipped with N antennas, an intelligent reflecting surface (IRS) with L reflecting elements, M single-antenna energy harvesting receivers and K single-antenna downlink data users. The BS uses non-orthogonal multiple access (NOMA) to transmit data to the downlink users. Further, we assume that the downlink data users employ successive interference cancellation (SIC) to decode their desired information from the received signal.

By denoting the precoding vector and transmitted symbol for the k-th data user by $\mathbf{w}_k$, and x_k, respectively, the overall transmitted signal from the BS can be written as $\mathbf{x} = \sum_{k=1}^{K} \mathbf{w}_k x_k$, with $E(|x_k|^2) = 1$. We denote the direct (resp. reflected) channel between the BS (resp. IRS) and k-th data user by $\mathbf{h}_{d,k}^H$ (resp. $\mathbf{h}_{r,k}^H$). Similarly, we use $\mathbf{s}_{d,m}^H$ (resp. $\mathbf{s}_{r,m}^H$) to denote the channel between the BS (resp. IRS) and m-th energy harvesting receiver. Moreover, the channel between the BS and the IRS is denoted by $\mathbf{H}$. Denoting by θ_l the reflecting phase shift caused by the IRS on the incident wave at the l-th reflecting element of IRS, the reflection-coefficient matrix at the IRS can be written as $\boldsymbol{\Theta} = \text{diag}([e^{j\theta_1}, ..., e^{j\theta_L}])$, where $\text{diag}(\mathbf{v})$ converts vector $\mathbf{v}$ into a diagonal matrix.

We can write the signal received at the k-th data user as

$$r_k = (\mathbf{h}_{d,k}^H + \mathbf{h}_{r,k}^H \boldsymbol{\Theta} \mathbf{H})\mathbf{x} + n_k = \mathbf{h}_k^H \mathbf{x} + n_k, \qquad (1)$$

where $\mathbf{h}_k^H = \mathbf{h}_{d,k}^H + \mathbf{h}_{r,k}^H \boldsymbol{\Theta} \mathbf{H}$ and n_k is the additive Gaussian noise term at the k-th data receiver with variance σ^2. Since we assume SIC at the data users, the SINR may differ for a particular user if the decoding order in SIC is altered. Let $\pi(k)$ denote the decoding order of k-th data user. Hence, k-th data user first successively decodes the signal of each j-th user with $\pi(j) < \pi(k)$ before decoding its own signal, while the signals of each i-th data user with $\pi(i) > \pi(k)$ are treated as interference. Then, the SINR at j-th data user to decode the signal of k-th data user, with $\pi(k) \leq \pi(j)$, is given as

$$SINR_{k\to j} = \frac{|\mathbf{h}_j^H \mathbf{w}_k|^2}{\sum_{\pi(i)>\pi(k)} |\mathbf{h}_j^H \mathbf{w}_i|^2 + \sigma^2}. \qquad (2)$$

Correspondingly, the achievable data rate for decoding j-th data user at k-th data user is given as

$$R_{k\to j} = \log(1 + SINR_{k\to j}). \qquad (3)$$

From the same transmission, the energy received at the m-th energy harvesting receiver is given as

$$S_m = \sum_{k=1}^{K} |(\mathbf{s}_{d,m}^H + \mathbf{s}_{r,m}^H \Theta \mathbf{H})\mathbf{w}_k|^2 = \sum_{k=1}^{K} |\mathbf{s}_m^H \mathbf{w}_k|^2, \qquad (4)$$

where $\mathbf{s}_m^H = \mathbf{s}_{d,m}^H + \mathbf{s}_{r,m}^H \Theta \mathbf{H}$.

With the above setting, we formulate an optimization problem where the objective is to maximize the sum of harvested energies over all the energy harvesting users while satisfying the minimum data rate requirements of the data users with a total power budget P.

Mathematically, the optimization problem **P1** can be written as follows:

P1

$$\begin{aligned}
\underset{\mathbf{w}_k,\, \theta_l}{\text{maximize}} \quad & \sum_{m=1}^{M} S_m \\
\text{subject to} \quad & C1: \; R_{k\to j} \geq \gamma_k, \quad \forall\, \pi(k) \leq \pi(j), \\
& C2: \; \sum_{k=1}^{K} ||\mathbf{w}_k||^2 \leq P, \\
& C3: \; 0 \leq \theta_l \leq 2\pi, \; \forall\, l \in \{1, \cdots, L\},
\end{aligned} \qquad (5)$$

where γ_k is the minimum required rate for k-th data user. Due to the coupling between $\mathbf{w}_k$ and θ_l, the optimization problem **P1** is non-convex and difficult to solve. In the next section, we provide an alternating optimization based approach for **P1**.

III. Proposed Solution for P1

In this section, first we provide alternating optimization based solution methodology for **P1**. Particularly, first for fixed values of θ_l, we solve **P1** and then we solve **P1** over θ_l.

A. Optimization for Fixed Value of $\boldsymbol{\Theta}$

Even with a fixed value of $\boldsymbol{\Theta}$, problem **P1** is non-convex due to the data rate constraints. To address this non-convexity, we introduce several auxiliary variables (Y_{kj} and Z_{kj}). Then, we introduce a new optimization problem **P1.1** as follows:

P1.1

$$\begin{aligned}
\underset{\mathbf{w}_k,\, Y_{kj},\, Z_{kj}}{\text{maximize}} \quad & \sum_{m=1}^{M} \sum_{k=1}^{K} |\mathbf{s}_m^H \mathbf{w}_k|^2 \\
\text{subject to} \quad & C1: \; \log\left(1 + \frac{Y_{kj}}{Z_{kj}}\right) \geq \gamma_k, \\
& C2: \; \sum_{k=1}^{K} ||\mathbf{w}_k||^2 \leq P, \\
& C5: \; Y_{kj} \leq |\mathbf{h}_j^H \mathbf{w}_k|^2, \\
& C6: \; Z_{kj} \geq \sum_{\pi(i)>\pi(k)} |\mathbf{h}_j^H \mathbf{w}_i|^2 + \sigma^2.
\end{aligned} \qquad (6)$$

We have the following lemma for problems **P1** and **P1.1**.

Lemma 1. *With any fixed value of* $\boldsymbol{\Theta}$*, the problems **P1** and **P1.1** are equivalent.*

Proof. It is clear that if constraints $C5$ and $C6$ are met with equality, then both problems are equivalent. Suppose for any j, k constraint $C5$ is not strict, then we can increase the value of Y_{kj} so that the constraint is met with equality. By doing so the objective value of the feasible solution has not changed and also constraint $C1$ is not violated. Now assume that for j, k, constraint $C6$ is not strict, then we can reduce the value of Z_{kj} to meet the constraint with equality. By doing so the objective value of any feasible solution has not changed and constraint $C1$ is not violated. □

Due to constraints $C1$ and $C5$, the problem **P1.1** is still non-convex. To tackle this issue, we introduce new variables as $\mathbf{W_k} = \mathbf{w}_k \mathbf{w}_k^H$. Then, problem **P1.1** can be written as follows:

$$\mathbb{L}(\Omega, \alpha_{kj}, \zeta_{ki}^{j}, \mathbf{M}_k, \mathbf{W}_k, Y_{kj}, Z_{kj}) = \sum_{m=1}^{M}\sum_{k=1}^{K} \mathrm{Tr}\left(\mathbf{W}_k \tilde{\mathbf{H}}_m^H \phi\phi^H \tilde{\mathbf{H}}_m\right) + \sum_{j=1}^{K}\sum_{k\geq j}^{K} \alpha_{kj}\left(\mathrm{Tr}\left(\mathbf{W}_k \tilde{\mathbf{H}}_j^H \phi\phi^H \tilde{\mathbf{H}}_j\right) - Y_{kj}\right) + \Omega\left(P - \sum_{k=1}^{K}\mathrm{Tr}\left(\mathbf{W}_k\right)\right) + \sum_{k=1}^{K}\sum_{i<k} \zeta_{ki}^{j}\left(\mathrm{Tr}\left(\mathbf{W}_i \tilde{\mathbf{H}}_j^H \phi\phi^H \tilde{\mathbf{H}}_j\right) + \sigma^2 - Z_{kj}\right) + \sum_{k=1}^{K}\mathbf{W}_k\mathbf{M}_k + C \quad (8)$$

$$\begin{aligned} \textbf{P1.2} \quad & \underset{\mathbf{W}_k, Y_{kj}, Z_{kj}}{\text{maximize}} \quad \sum_{m=1}^{M}\sum_{k=1}^{K} \mathrm{Tr}\left(\mathbf{W}_k \tilde{\mathbf{H}}_m^H \phi\phi^H \tilde{\mathbf{H}}_m\right) \\ & \text{subject to} \quad \tilde{C1}: \ Y_{kj} \geq Z_{kj}\hat{\gamma}_k, \quad \forall\ \pi(k) \leq \pi(j), \\ & C2: \ \sum_{k=1}^{K}\mathrm{Tr}\left(\mathbf{W}_k\right) \leq P, \\ & C5: \ Y_{kj} \leq \mathrm{Tr}\left(\mathbf{W}_j \tilde{\mathbf{H}}_k^H \phi\phi^H \tilde{\mathbf{H}}_k\right), \\ & C6: \ Z_{kj} \geq \sum_{\pi(i)>\pi(j)} \mathrm{Tr}\left(\mathbf{W}_i \tilde{\mathbf{H}}_k^H \phi\phi^H \tilde{\mathbf{H}}_k\right) + \sigma^2, \\ & C7: \ \mathrm{rank}\left(\mathbf{W}_k\right) = 1, \\ & C8: \ \mathbf{W}_k \succeq \mathbf{0}, \end{aligned} \quad (7)$$

where $\tilde{\mathbf{H}}_k = \begin{bmatrix}\mathrm{diag}(\mathbf{h}_{r,k}^H)\mathbf{H} \\ \mathbf{h}_{d,k}^H\end{bmatrix}, \tilde{\mathbf{H}}_m = \begin{bmatrix}\mathrm{diag}(\mathbf{s}_{r,m}^H)\mathbf{H} \\ \mathbf{s}_{d,m}^H\end{bmatrix}, \phi = [e^{j\theta_1}, \cdots, e^{j\theta_L}, 1]^H$ and $\hat{\gamma}_k = 2^{\gamma_k} - 1$.

It is clear that problems **P1.1** and **P1.2** are equivalent. Hence, we will solve problem **P1.2** instead. Then after having obtained the value of $\mathbf{W}_k^*$ satisfying rank$(\mathbf{W}_k^*) = 1$, we can obtain $\mathbf{w}_k^*$ through Choleskey decomposition. However, problem **P1.2** is still non-convex due to the rank-one constraint. If constraint $C7$ is relaxed, then problem **P1.2** is a convex optimization problem. In this regard, we have the following lemma for problem **P1.2**.

Lemma 2. *The optimal solution $\mathbf{W}_k^*$ of problem **P1.2** satisfies rank$(\mathbf{W}_k^*) = 1$.*

Proof. First, we must have rank$(\mathbf{W}_k) \geq 1$ to satisfy the minimum rate requirements of the data users. In the following, we show that rank$(\mathbf{W}_k) \leq 1$ in order to complete the proof that rank$(\mathbf{W}_k) = 1$. As problem **P1.2** is a convex optimization problem if rank constraint is removed, we use Karush–Kuhn–Tucker (KKT) conditions to find its solution.

The Lagrangian for problem **P1.2** can be written as shown at the top of this page, where C is a constant term independent of $\mathbf{W}_k$'s and $\Omega, \alpha_{kj}, \zeta_{ki}^{j}, \mathbf{M}_k$ are the Lagrange multipliers for constraints $C2, C5, C6, C8$, respectively. Based on the KKT conditions, the gradient of $\mathbb{L}(\Omega, \alpha_{kj}, \zeta_{ki}^{j}, \mathbf{M}_k, \mathbf{W}_k, Y_{kj}, Z_{kj})$ with respect to $\mathbf{W}_k^*$ must be equal to zero for optimality, i.e.,

$$\Omega^*\mathbf{I} = \mathbf{M}_k^* + \tilde{\mathbf{S}}^H\phi\phi^H\tilde{\mathbf{S}}, \quad (9)$$

where $\tilde{\mathbf{S}} = \sum_{m=1}^{M}\tilde{\mathbf{H}}_m + \sum_{j\leq k}\alpha_{kj}^*\tilde{\mathbf{H}}_j$. Furthermore, at optimality, we must have $\mathbf{W}_k^*\mathbf{M}_k^* = \mathbf{0}$. After multiplying both sides of (9) with $\mathbf{W}_k^*$ and using $\mathbf{W}_k^*\mathbf{M}_k^* = \mathbf{0}$, we have

$$\Omega^*\mathbf{W}_k^* = \tilde{\mathbf{S}}^H\phi\phi^H\tilde{\mathbf{S}}\mathbf{W}_k^*, \quad (10)$$

which ultimately implies rank$(\mathbf{W}_k^*) \leq 1$. Combining this with the fact that rank$(\mathbf{W}_k^*) \geq 1$ completes the proof. □

Thus, we can solve problem **P1.2** by relaxing the rank constraint through standard convex solvers, e.g., CVX.

B. Optimization over Reflection Phases

By introducing a new variable $\mathbf{V} = \phi\phi^H$, the problem **P1** for given values of $\mathbf{w}_k = \mathbf{w}_k^*, Y_{kj} = Y_{kj}^*, Z_{kj} = Z_{kj}^*$ can be written as follows:

$$\begin{aligned} \textbf{P1.3} \quad & \underset{\mathbf{V}, \phi}{\text{maximize}} \quad \sum_{m=1}^{M}\sum_{k=1}^{K} \mathrm{Tr}\left(\tilde{\mathbf{H}}_m \mathbf{W}_k \tilde{\mathbf{H}}_m^H \mathbf{V}\right) \\ & \text{subject to} \quad \hat{C5}: \ Y_{kj} \leq \mathrm{Tr}\left(\tilde{\mathbf{H}}_k \mathbf{W}_j \tilde{\mathbf{H}}_k^H \mathbf{V}\right), \\ & \hat{C6}: \ Z_{kj} \geq \sum_{\pi(i)>\pi(j)} \mathrm{Tr}\left(\tilde{\mathbf{H}}_k \mathbf{W}_i \tilde{\mathbf{H}}_k^H \mathbf{V}\right) + \sigma^2, \\ & C9: \ \mathrm{Tr}(\mathbf{1}_l\mathbf{V}) \geq 1, \\ & C10: \ \mathrm{Tr}(\mathbf{1}_l\mathbf{V}) \leq 1, \\ & C11: \ \mathbf{V} = \phi\phi^H, \end{aligned} \quad (11)$$

where $\mathbf{1}_l$ is a square matrix of size L with the only non-zero entry being the (l,l)-th entry which is equal to one. Thus, constraints $C9, C10$ ensure that (l,l)-th entry of $\mathbf{V}$ is 1. If we remove the rank constraint, $C11$, the above problem is a quadratic constrained quadratic program (QCQP). However, the solution obtained by relaxing the rank constraint may not be optimal and/or feasible. Therefore, in the following, we describe an iterative rank minimization (IRM) approach that guarantees the rank-one solution and convergence to the local optimal solution. The convergence to the local optimal solution is important since otherwise it is not possible to prove the overall convergence of the alternating optimization approach.

To proceed further, we introduce the following relations:

$$\mathbf{Q}_0 = -\sum_{m=1}^{M}\sum_{k=1}^{K}\tilde{\mathbf{H}}_m\mathbf{W}_k\tilde{\mathbf{H}}_m^H, \hat{\mathbf{Q}}_{kj} = -\tilde{\mathbf{H}}_k\mathbf{W}_j\tilde{\mathbf{H}}_k^H, \quad (12)$$

$$\tilde{\mathbf{Q}}_{kj} = \sum_{\pi(i)>\pi(j)}\tilde{\mathbf{H}}_k\mathbf{W}_i\tilde{\mathbf{H}}_k^H, \mathbf{Q}_l = \mathbf{1}_l, \bar{\mathbf{Q}}_l = -\mathbf{Q}_l, \quad (13)$$

$$\tilde{Z}_{kj} = Z_{kj} - \sigma^2, \tilde{Y}_{kj} = -Y_{kj}. \quad (14)$$

Using the relations in (12)–(14), we can rewrite **P1.3** as

$$\begin{aligned} \textbf{P1.4} \quad & \underset{\mathbf{V}, \phi}{\text{minimize}} \quad \mathrm{Tr}\left(\mathbf{Q}_0\mathbf{V}\right) \\ & \text{subject to} \quad \tilde{C5}: \ \mathrm{Tr}\left(\hat{\mathbf{Q}}_{kj}\mathbf{V}\right) \leq \tilde{Y}_{kj}, \\ & \tilde{C6}: \ \mathrm{Tr}\left(\tilde{\mathbf{Q}}_{kj}\mathbf{V}\right) \leq \tilde{Z}_{kj}, \\ & \tilde{C9}: \ \mathrm{Tr}(\bar{\mathbf{Q}}_l\mathbf{V}) \leq -1, \\ & \tilde{C10}: \ \mathrm{Tr}(\mathbf{Q}_l\mathbf{V}) \leq 1, \\ & C11: \ \mathbf{V} = \phi\phi^H. \end{aligned} \quad (15)$$

As a first step in solving problem **P1.4** efficiently, we convert the rank-one constraint, $\mathbf{V} = \phi\phi^H$, into multiple quadratic equality constraints. Let $\mathbf{V}_{\mu_t}$ be a principle submatrix of $\mathbf{V}$ with entries taken from the rows and columns indexed by μ_t, and ϕ_{μ_t} consists of entries taken from vector ϕ indexed by μ_t. Here, μ_t is defined as the complete decomposition of the set $\{1, 2, \ldots, L\}$. Then, problem **P1.4** can be written [10] to

$$\textbf{P1.5} \quad \underset{\mathbf{V}, \phi}{\text{minimize}} \quad \text{Tr}\left(\mathbf{Q}_0\mathbf{V}\right) \quad \text{subject to} \quad \tilde{C5}, \tilde{C6}, \tilde{C9}, \tilde{C10}, \quad \tilde{C11}: \ \mathbf{V}_{\mu_t} = \phi_{\mu_t}\phi_{\mu_t}^H. \tag{16}$$

However, after all rank-one solutions for $\mathbf{V}_{\mu_t}$ are obtained, it is not guaranteed that all the common elements among different $\mathbf{V}_{\mu_t}$ are same [10]. To address this issue, the following rank-one constraints for extended submatrices are also considered:

$$C12 : \text{rank}\left(\begin{bmatrix}\mathbf{V}_{\mu_t} & \phi_{\mu_t} \\ \phi_{\mu_t}^H & 1\end{bmatrix}\right) = 1. \tag{17}$$

Here, the idea is to minimize a linear function while satisfying linear constraints and the rank constraints. To address problem **P1.5**, we resort to an iterative approach which satisfies all the rank-one constraints. In this direction, we make use of the following result:

Lemma 3. *For a non-zero positive semidefinite matrix* $\tilde{\mathbf{V}}_{\mu_t} = \begin{bmatrix}\mathbf{V}_{\mu_t} & \phi_{\mu_t} \\ \phi_{\mu_t}^H & 1\end{bmatrix}$, *we have rank* $\left(\tilde{\mathbf{V}}_{\mu_t}\right) = 1$ *if and only if*

$$\tilde{\mathbf{V}}_{\mu_t} \succeq 0 \wedge \mathbf{P}_{\mu_t}^H\tilde{\mathbf{V}}_{\mu_t}\mathbf{P}_{\mu_t} \preceq 0, \tag{18}$$

where $\mathbf{P}_{\mu_t}$ *is a matrix comprising of the* μ_t *smallest eigenvalues of matrix* $\tilde{\mathbf{V}}_{\mu_t}$.

Proof. The result can be easily proved by noting that the only non-zero eigenvalue of $\tilde{\mathbf{V}}_{\mu_t}$ is its largest eigenvalue and the rest of the eigenvalues are zero. □

Hence, problem **P1.5** is equivalently reformulated to

$$\textbf{P1.6} \quad \underset{\mathbf{V}, \phi}{\text{minimize}} \quad \text{Tr}\left(\mathbf{Q}_0\mathbf{V}\right) \quad \text{subject to} \quad \tilde{C5}, \tilde{C6}, \tilde{C9}, \tilde{C10}, \quad C13 : \tilde{\mathbf{V}}_{\mu_t} \succeq 0, \quad C14 : \mathbf{P}_{\mu_t}^H\tilde{\mathbf{V}}_{\mu_t}\mathbf{P}_{\mu_t} \preceq 0. \tag{19}$$

Since it is not possible to know $\mathbf{P}_{\mu_t}$ before solving for $\tilde{\mathbf{V}}_{\mu_t}$, we use an iterative approach to solve problem **P1.6**. During the z-th iteration we solve the following problem:

$$\textbf{P1.7} \quad \underset{\mathbf{V^z}, \phi^{\mathbf{z}}, \nu^z}{\text{minimize}} \quad \text{Tr}\left(\mathbf{Q}_0\mathbf{V}^z\right) + \xi_z|\nu^z|_1 \quad \text{subject to} \quad \tilde{C5}, \tilde{C6}, \tilde{C9}, \tilde{C10}, C13, \quad \tilde{C14} : \nu_{\mu_t}^z\mathbf{I} \succeq \left(\mathbf{P}_{\mu_t}^{z-1}\right)^H\tilde{\mathbf{V}}_{\mu_t}^z\mathbf{P}_{\mu_t}^{z-1}, \tag{20}$$

where $\nu^z = [\nu_1^z, \cdots, \nu_T^z]$ and ξ^z is the increasing weight of the norm for the z-th iteration. In each iteration, the aim is to minimize the objective function while simultaneously reducing $|\nu^z|_1$. As a result, when $\nu_{\mu_t}^z = 0$ for all μ_t, the rank one constraint will be met.

The overall proposed alternating optimization based algorithm for **P1** is summarized below.

Algorithm 1 Alternating optimization based algorithm for solving optimization problem **P1**

1: set $p = 0, \epsilon_1, \epsilon_2, p_{max}, z_{max}, \xi_0$
2: While $p \leq p_{max} \wedge |f(\mathbf{\Theta}^p, \mathbf{w}_k^p) - f(\mathbf{\Theta}^{p-1}, \mathbf{w}_k^{p-1})| \geq \epsilon_1$:
2.1: solve problem **P1.2**
2.2: set $z = 0$ and solve **P1.4** through SDR to find $\tilde{\mathbf{V}}_{\mu_t}^z$ and then find $\mathbf{P}_{\mu_t}^z$
2.3: $z = 1$
2.4: While $z \leq z_{max} \wedge |\nu^z|_1 \geq \epsilon_2$:
2.4.1: solve problem **P1.7**
2.4.2: update the values of $\mathbf{P}_{\mu_t}^z$
2.4.3: $z = z + 1$
2.4.4: set $\xi_z \geq \xi_{z-1}$
2.4.5: end while
2.5: $p = p + 1$
2.6: end while

C. Convergence of the Alternating Optimization Algorithm

Since we have used alternating optimization algorithm it is important to show that the proposed algorithm has non-decreasing objective value after every iteration of the alternating optimization algorithm. Although this is not always true for all the alternating optimization based algorithm, the following lemma shows that our proposed algorithm results in non-decreasing values of objective value — thus proving the convergence of the overall optimization algorithm.

Lemma 4. *Each iteration of the proposed algorithm results in non-decreasing values of objective function of problem **P1**.*

Proof. Without loss of generality, let us consider the l-th iteration. Further assume that the optimal solution of the algorithm during the $(l-1)$-th iteration is given by $\mathbf{\Theta}^{l-1}, \mathbf{w}_k^{l-1}$. Then we need to show that

$$f(\mathbf{\Theta}^{l-1}, \mathbf{w}_k^{l-1}) \leq f(\mathbf{\Theta}^{l}, \mathbf{w}_k^{l}). \tag{21}$$

where $f(\cdot, \cdot)$ denotes the objective value of the problem **P1**. First, we note that

$$f(\mathbf{\Theta}^{l-1}, \mathbf{w}_k^{l-1}) \leq f(\mathbf{\Theta}^{l-1}, \mathbf{w}_k^{l}) \tag{22}$$

is valid since $\mathbf{w}_k^l$ are the optimal solution of problem **P1.1** for a fixed value of $\mathbf{\Theta}$. Next we show that

$$f(\mathbf{\Theta}^{l-1}, \mathbf{w}_k^{l}) \leq f(\mathbf{\Theta}^{l}, \mathbf{w}_k^{l}). \tag{23}$$

It can be proved that the KKT conditions of problem **P1.7** and problem **P1.5** are the same when $\lim_{l\rightarrow\infty}|\nu^l|_1 = 0$ [10]. Hence, (23) is true. By combining (23) with (22), we conclude that (21) is valid for the proposed alternating optimization algorithm. This completes the proof. □

Combining the above lemma with the fact that the objective value of problem **P1** is bounded proves the convergence of the proposed alternating optimization algorithm.

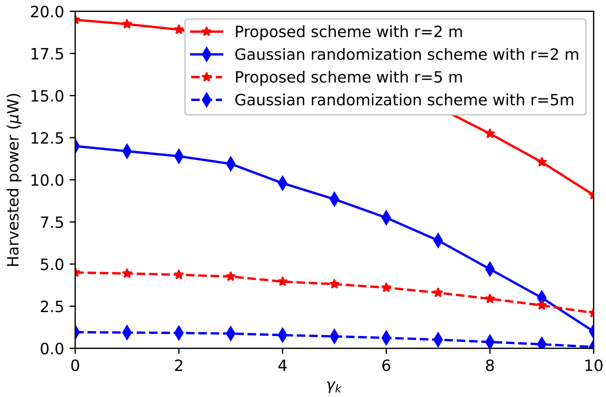

Fig. 1. Harvested power with respect to SINR threshold when $R = 20$ m.

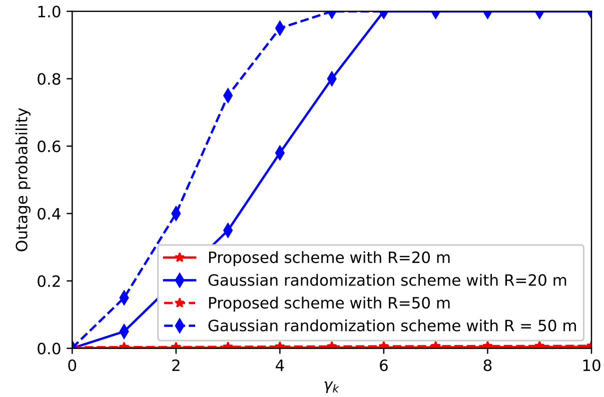

Fig. 2. Outage probability with respect to the SINR threshold.

IV. Simulation Results

For simulations, we assume $N = 8, M = 4, K = 8, L = 100, \sigma^2 = -90$ dBm and $\gamma_k = [0 - 10]$ dB $\forall k$. Also, we assume that the data receivers are distributed randomly in a semi-circle of 20 m radius around the IRS while the energy harvesting receivers are located in a semi-circle of $(2, 5)$ m radius around the IRS. For SIC, we assume farthest-to-nearest user decoding, where the farthest user is decoded first and then second farthest and so on. The distance between the IRS and the BS is set to 50 m. For convenience, we use r to denote the radius of the semi-circle between the IRS and the energy harvesting receivers and R to denote the radius of the semi-circle between the IRS and the data users.

First, we present the harvested energy results in Fig. 1. We compare the proposed algorithm with the Gaussian randomization scheme, where the optimal values of the reflecting elements are obtained by first solving the SDR problem related to the reflection phase optimization and then Gaussian randomization [9] is used to obtain the rank one solution. It is clear that the proposed algorithm performs better than the Gaussian randomization scheme. It is also noted that, as SINR threshold of the data receivers increases, the harvested energy at the energy harvesting receivers decreases. This is due to the fact that the increased performance requirement of the data receivers results in strong beamforming toward the data receivers. This causes reduced transmit power intensity in the direction of energy harvesting receivers. Hence, a reduction in the harvested energy performance is observed with the higher communication performance requirements.

Next, in Fig. 2, we present the outage probability result for the Gaussian randomization scheme to illustrate the effect of ignoring the rate constraint while performing the Gaussian randomization to obtain the rank-one solution. Since the rate constraints are considered in the proposed scheme, the outage probability of the proposed scheme is miniscule and arises only due to occurrence of extremely weak channel conditions which render the problem infeasible. On the other hand, it can be observed that due to the neglecting of the rate constraints while performing Gaussian randomization the outage probability rises quite steeply with increase in SINR threshold.

V. Conclusion

We presented an alternating optimization based algorithm for maximizing the harvested power in an IRS-assisted wireless communication system. Iterative rank minimization is used for finding the optimal reflection phases at IRS elements. The convergence proof of the proposed alternating optimization algorithm is provided. It is shown that the proposed algorithm outperforms a Gaussian randomization based algorithm in terms of harvested power and outage probability.

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