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Statistical Process Control Applied at Level Crossing Incidents

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Abstract

Given the increased importance of accidents at level crossing (LC) and the enormous damage caused by these accidents, it will be useful to question the safety measures at LC. Different scenarios are studied to compare the performance of control charts (CC) for detecting upward shifts of the quotient between the magnitude X and the time between events (TBE) which corresponds to a process's deterioration: TBE distribution is exponential and for X two situations were retained (normal and Gamma distribution). The database covering the period 2009-2018 is obtained from the Tunisian railway company and the objective is to specify the best CC to monitor the situation at LC's. The T distribution CC alone as well as that relative to X distribution alone does not make it possible to detect overruns and therefore does not lead to effective control of the situation at the LC. Unlike some previous research, the CC corresponding to the quotient X & T also does not allow rapid detection of overshoots if T is considered exponential and X is assumed to be normal. The objective of setting up a CC that makes it possible to control the situation at the concerned LC is achieved in the case where T is exponential and X follows a gamma distribution. In this case, the CC corresponding to the quotient (Q) makes it possible to detect overshoots quickly.

Keywords: Control chart; time between events; statistical process control; control limits; level crossing safety; magnitude.

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1. Introduction

Considering that the LC accident rate in major countries is at the level of 1 per 10 LCs, a simple scan of the data obtained from the Tunisian railway's company shows that the majority of LCs have recorded an acceptable number of incidents (within standards) but others show quite large numbers. Since different LCs do not present the same safety situation during the reference period (2009-2018), it will be necessary to find a method that will allow the situation to be controlled in the LCs deemed critical.

A railway LC is an intersection where a road passes through a railway track. Safety levels in the railway interfaces continue to be very inter-related despite the increasing emphasis on improving design and enforcement practices. Despite the various safety measures undertaken, the frequency of accidents, many of which lead to deaths, remains high and the corresponding economic losses are remarkable. The majority of accidents are observed in public passengers, where active warning devices (barriers, lights, bells, etc.) are in place and functioning properly. This directly reflects the need to review the safety measures in place, safety assessment methods, and design practices at grade crossings.

Over the selected period (2009-2018), there were 777 accidents at LC in Tunisia. The global loss caused by incidents is estimated to 16669533$. Women committed only nine accidents, a percentage of 1.16%. Men committed the largest proportion, 98.84% of these accidents. 140 accidents were in autumn with a percentage of 18.02%, 27.67% (215) in summer, 28.57% (222) in winter, and 25.74% in spring (200). The majority of accidents were observed in broad daylight with a percentage of 70.91%, i.e. 551 accidents, and 138 accidents were observed in the middle of the night (17.76%). For the rest of the accidents, i.e. 91 accidents (11.71%), the time is not available. The climatic conditions are divided into clear climate and rainy climate. In the first case, there were 634 accidents, or 81.6%, throughout the study period which caused 170 victims. The total economic loss recorded at this level is equal to $ 14236850. The remaining percentage (18.4%) represents accidents in a rainy climate.

When consulting the works dealing with safety at LC, the majority of research has been concentrated on the causes of accidents at LC or on the search for an incident prediction model at a LC. Dixon (2007) [1] found that eight human factors can lead to violations or error behavior through literature examinations, visits crossings, visits to the signal box, and interviews. These human factors include competence, distraction, inadequate design, perceived control, risk compensation, familiarity, complacency, and mental models. Borowsky and his colleagues in [2] tested the reaction of 20 drivers to practical signs. Experience has shown that drivers are less likely to identify traffic signs that are located in unexpected places. The authors in [3] concluded that the information should be simplified for inexperienced drivers based on the results of the eye tracking method. Bohua and his colleagues in [4] also used an eye tracker to study attitudes towards visual signs and concluded that the amount of traffic guide signs must not exceed 5 to avoid increasing the fixation duration. The authors in [5] presented the road vehicle test to collect vehicle data before approaching crossings. It was concluded that the locations of the signs, the design of the floor, traffic, and pedestrian entourage are important for the safety of LC. Boros and his colleagues in [6] seek the impacts of explanatory factors on the safety of railway crossings. They found that annual daily road traffic and annual daily train traffic are significant predictors but some other predictors such as crossing angle, track alignment, number of tracks and sight distances turned out to be not significant. Djordjević and his colleagues in [7] proposed a new approach for evaluating safety at railways level crossing (RLC) using the non-radial Data Envelopment Analysis (DEA) model. The introduced model is employed for the evaluation of railway efficiency of European countries regarding the level of safety at RLC through considering desirable and undesirable inputs, as well as desirable and undesirable outputs for the period from 2010 to 2012 and the year 2014. Through empirical study, the improved non-radial DEA model can provide an evaluation of the efficiency of the degree of safety at RLC at both the macro and micro levels, as well as benchmarking for different levels. Evaluation of other factors and their impacts on accidents at RLC is also possible with the model. Pacha and his colleagues in [8] established a new hazard prediction model (Florida Priority Index Formula) which they recommend to rank the LC in the State of Florida. The Florida Priority Index Formula assesses the potential hazard of a given highway-rail grade crossing based on the average daily traffic volume, average daily train volume, train speed, existing traffic control devices, accident history, and crossing upgrade records.

The authors in [9] evaluate the effects of Highway-rail grade crossings (HRGC) geometric parameters on crash occurrence and severity likelihoods. They take four parameters namely the distance between crossings and their nearest roadway intersections, the crossing angle that affects sight distance at a crossing, the number of traffic lanes, and the number of main tracks. Keramati and his colleagues selected the competing risks model (CRM) to identify contributing geometric factors and quantify their effects on HRGC crash occurrence and severity probabilities. For crash analysis, CRM estimates the probability of a crash occurring at a crossing over time; the crossing can experience more than one crash event, including property damage only crash, injury crash, and fatal crash. On average, the distance between a crossing and the nearest highway intersection and the acute crossing angle have negative impacts on property damage only (PDO), injury, and crash occurrence probability. However, they both have positive impacts on fatal crash probability. The finding indicates that fatal crash rates could increase with longer roadway distance between crossing and its nearest roadway intersections or with 90° crossing. This could result from aggressive drivers and a high likelihood to have T-bone collision suggesting there may be a need to implement additional safety interventions to reduce aggressive driving behavior and to provide additional protection to highway users at HRGCs. In this research, the authors determined the long-term effects of the number of main tracks while holding all other contributors, including rail/roadway traffic, at their mode level. The finding indicated the cumulative crash probability is 7% with three main tracks in 30 years of service. The cumulative probability increased to 20 % when the track number increased to four, resulting in a nearby 200% hazard increase. The potential rationale may be that higher track numbers indicate a wider crossing for highway users to traverse, which in turn could increase the crash likelihood.

Zhou and his colleagues in [10] considered Matthews correlation coefficient (MCC) and accuracy as general overall performance indicators. Their comparison case study indicates that the overall performance of the random forest (RF) model is better than the Decision Tree (DT) model because both accuracy and MCC are taking all four values in the confusion table into consideration. The RF model performs better than the DT model in terms of the percentage of correct positive predictions compared to the total positive predictions provided by the model (precision). However, the DT model performs better than the RF model in terms of the percentage of correct positive predictions compared to the total observed positive outcomes (sensitivity). They concluded that the RF model could improve both crash and non-crash forecasting power and effectively reduce false alarm rates for unbalanced data.

The authors in [11] conducted a case study using the data for the 6,109 public level crossings and 2,896 private level crossings in the State of Florida. The proposed approach estimates the overall delay per day at a given level crossing before implementation of any additional countermeasures. A set of 10 candidate countermeasures were evaluated for implementation at the considered level crossings. Results from the case study revealed that traffic delays were higher at the public level crossings in Florida as compared to the private level crossings, both before and after implementation of the candidate countermeasures. It was also discovered that low-cost countermeasures could be more advantageous in terms of safety effectiveness and traffic delays as compared to many high-cost conventional countermeasures that have been commonly used by practitioners over the years. Since the duration between two accidents can be measured by the number of days and the value of an accident can be estimated in monetary value, it will therefore be possible to apply the CC method to control the situation at a LC by using CCs for TBE or for the magnitude. To monitor a given process, it was necessary to present a CC. The study was focused on the control charts. In LC situation, it is known that a Poisson model has always modelled the number of accidents. Instead of dealing with the number of accidents, it will be more interesting to focus on the TBE that can be modelled by an exponential distribution. The objective is to choose the best control chart that will detect as soon as possible an out of control situation.

The story began in 1931 with Walter A. Shewart [12] who is considered the father of CCs (one of the basic tools used for statistical process control (SPC)). According to him if the process is stable, 99.7% of the values will exist between the upper and lower control limits (UCL and LCL), and therefore the probability that a point exceeds one of these two limits is 0.3%. A count of the number of defects that follow the Poisson distribution is often used to monitor the quality of a production process. The c-chart was useful in this situation but when TBEs are studied, this chart did not give good results. While working with TBE, Rudolf and his colleagues in [13] tried to transform the exponential distribution of TBE to obtain a normal distribution then he proposes to present an exponentially weighted moving average (EWMA) and Cumulative Sum (CUSUM) CCs. He wants to take the fourth root of the exponential distribution. Almost similar work, according to him was done in 1969 by the authors in [14], in 1985 by the author in [15], and by Nelson [16] in 1994. Jones and his colleagues in [17], ensure that if defects occur according to the homogenous Poisson process, the TBE is then independent and identically distributed according to an exponential distribution. A comparison among the Gamma and the exponential CUSUM charts was made by Zhang and his colleagues in [18] shows that the Gamma chart is more sensitive than the exponential chart and the performance of a Gamma chart with $r = 4$ is comparable with that of an exponential CUSUM optimally designed. Given that an overshoot of UCL while working with TBE means a possible decrease in defect rate that means a process improvement and an increase of the occurrence rate (system deterioration) will be translated by an overshoot of the LCL, then it will be necessary to calculate the UCL and LCL in the case of a Gamma distribution. The objective of this work is to focus on the LC with the highest number of incidents during the study period (LC50) and to look for the best CC that will be used to monitor the situation at this LC. Other LCs with the same characteristics (protection type, crossing angle, area, number of lanes, and lane trajectory) recorded an acceptable number of incidents in the study period. Therefore, the problem at this level is to find a method that makes it possible to control the situation at this LC. Does the establishment of a CC of the distribution T or that of X make it possible to detect overshoots? Alternatively, should we work with the X & T quotient?

At this level, it will be necessary to find a control chart that allows monitoring the situation at a LC. The study will be carried out in two ways: a study of control chart which allows detecting overruns based on TBE (T) and the use of control chart for the magnitude of the incident (X) and for the quotient Q = X&T. Finally, the control chart that will make it possible to quickly detect overruns will be used to control the situation at LC50. To achieve this goal, it will be necessary to make a comparison between the work carried out by Rudolf and his colleagues (1999), Jones and his colleagues (2002), and Zhang and his colleagues (2007) to retain the best method for monitoring the situation at the LC50. This comparison will allow determining the best CC that will ensure instantaneous control of the situation at LC50 given that the economic losses and the number of incidents registered at this passage are quite significant and finding the right control chart to monitor this situation will be very interesting.

2. Methodology

In what follows, a CC for the LC with the highest number of incidents will be presented. Since the number of accidents can be likened to several defects and therefore can be modeled by a homogeneous Poisson distribution, then the TBE is exponentially distributed and so it can be written as follows:

$$f(x) = \lambda e^{-\lambda x} \qquad (1)$$

Where λ is the failure rate.

First, it will be necessary to check if the TBEs are exponentially distributed, then CCs for LC will be established and finally, the best CC to use for monitoring LC will be selected. The accident number per LC in Tunisia varies from 0 to 21 incidents over the period 2009-2018. For LC with a low number of incidents, the number of observations will be insufficient to make a satisfactory analysis. The LC concerned by this study (LC50) is the one that recorded the highest number of incidents during the study period. This LC has the characteristics listed in table1 (LC50 characteristics).

Table1: LC50 characteristics.

Influencing factors	characteristics
Lane number	1
Lane trajectory	totally straight
Crossing angle (with road)	greater than 45 degrees but less than 90 degrees
Area	urban zone
Protection type	protection with a Saint Andrew's cross

LCs with the same characteristics as well as the number of accidents recorded for each of them during the study period are grouped in table 2 (Accident number per LC).

Table 2: Accident number per LC.

LC_number	Accident_number
17	1
18	1
19	1
27	7
33	1
36	1
50	21
51	1
100	1
101	1
102	1
192	2
193	1
220	1
222	4
247	1
254	2
351	1
360	1
361	1
362	1
393	1

LC50 recorded the highest number of incidents throughout the study period while several other almost similar LC did not have a number quite high that justifies the importance of studying and monitoring the situation of this LC. The LC50 recorded 21 accidents during the study period that means an average of more than 2 accidents per year with an estimated loss value equal to 75400$. The data corresponding to this LC containing TBE and magnitude for incidents collected from Tunisian Railway Company is presented in Table 3 (Time between events and incident's magnitude for LC50).

Table 3: Time between events and incident's magnitude for LC50.

Acc_Nr	TBE	X($)	Acc_Nr	TBE	X($)	Acc_Nr	TBE	X($)
1	52	333,333	8	238	333,333	15	195	333,333
2	157	14633,333	9	26	333,333	16	596	11666,667
3	57	9633,333	10	153	4000,000	17	103	15300,000
4	144	333,333	11	11	333,333	18	132	6666,667
5	278	3333,333	12	98	1000,000	19	220	333,333
6	175	333,333	13	223	500,000	20	95	333,333
7	80	333,333	14	297	5000,000	21	128	333,333

Before starting the presentation of the CCs, it will be necessary to have a phase of reference (phase I) where the process is considered in control with an estimated occurrence rate of incidents. It is easy to see in Fig. 1 (Probability plot for TBE, LC50) that the distribution of TBE is not normal since the P-Value is less than 5%.

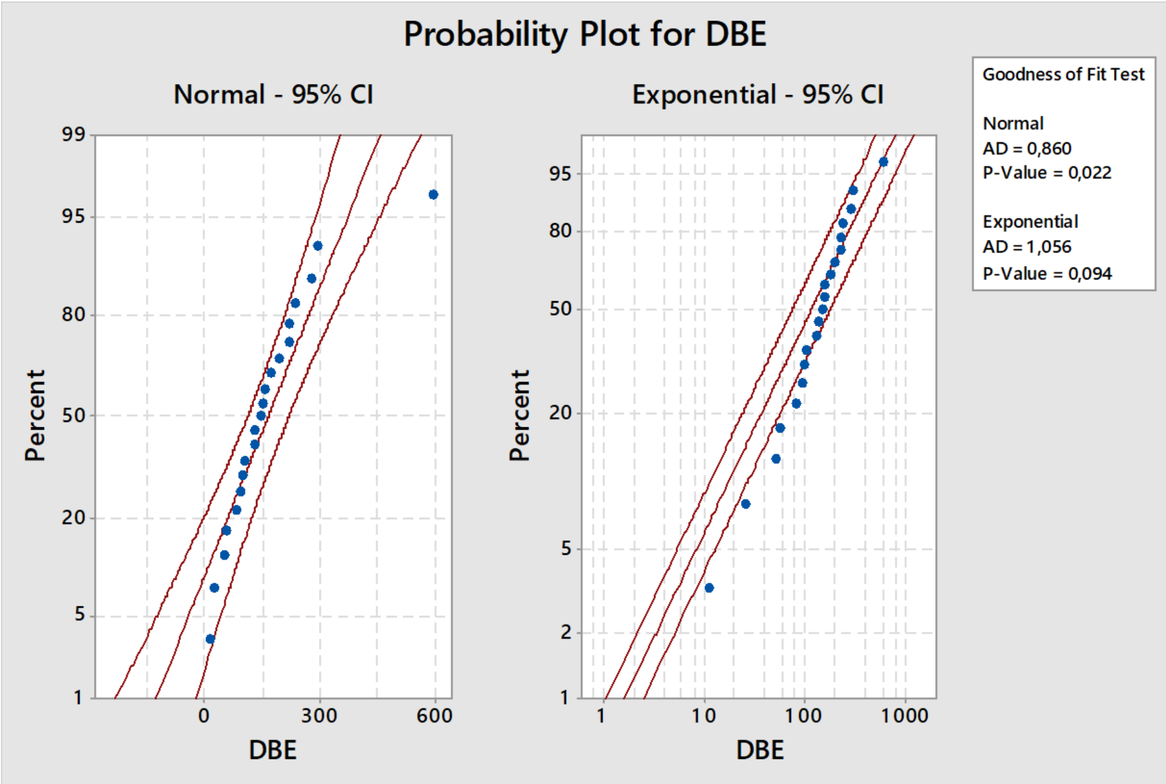

Figure1: Probability plot for TBE, LC50.

The P-Value for the exponential distribution is greater than 5% (0.094> 0.05), which means that it is impossible to reject the null hypothesis H0 and therefore it can be considered that the data follow an exponential distribution. In what follows the CC that corresponds to phase I will be first built and then will be used to monitor new events. The transformation made by Rudolf will be now performed.

For the original data, the UCL and the LCL can be calculated as:

$$UCL = -\frac{1}{\lambda}\ln\left(\frac{\alpha}{2}\right) \tag{2}$$

$$LCL = -\frac{1}{\lambda}\ln\left(1-\frac{\alpha}{2}\right) \tag{3}$$

Where α is the individual false alarm rate.

After the transformation of the data, Rudolf calculates the limits for the new data as follows:

$$\text{UCL} = \left[-\frac{1}{\lambda} \ln\left(\frac{\alpha}{2}\right) \right]^{1/4} \quad (4)$$

$$\text{LCL} = \left[-\frac{1}{\lambda} \ln\left(1 - \frac{\alpha}{2}\right) \right]^{1/4} \quad (5)$$

$$\text{Central Line}\left(\text{CL}\right) = \left[-\frac{1}{\lambda} \ln(0.5) \right]^{1/4} \quad (6)$$

The fourth root of the data has a mean of 3,401 that is almost equal to the median (3,464) and a standard deviation of 0,6856. The Skewness is nearby zero, the Kurtosis is less than 3 and the difference between mean and median is only 6%. The probability plot of this distribution appears in **Fig. 2** (Probability plot & EWMA chart of fourth root):

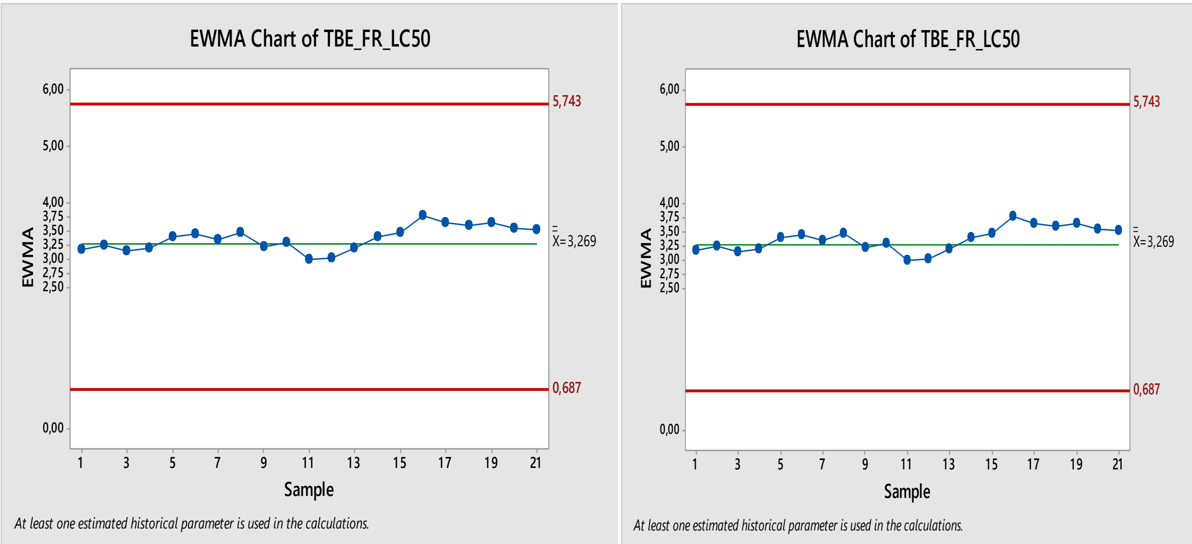

Figure 2: Probability plot & EWMA chart of fourth root.

The P-Value is greater than 5% that mean the normality of the distribution. Now limits will be calculated. As done by Rudolf and his colleagues (1999) the maximum likelihood estimate of $1/\lambda$ is $\overline{X}$, which is the sample mean of the original data over the base period. As a result, the estimated LCL will $\left[-\overline{X}\ln(1-\alpha/2)\right]^{1/4}$, the estimated UCL will $\left[-\overline{X}\ln(\alpha/2)\right]^{1/4}$ and the estimated central line will $\left[-\overline{X}\ln(0,5)\right]^{1/4}$. Alternatively, it is known that $\overline{X}$ is equal to 164.667 (phase I) which will give this (with a false alarm rate equal to 0,27%):

- UCL = 5.743.
- LCL= 0.687.

- CL = 3,269.

The EWMA chart for the fourth root data with a weighting factor of 0.2 presented in **Fig. 2** (Probability plot & EWMA chart of fourth root) doesn't signal and so the process is in control and during last year's there is an improvement. Three new observations are chosen so that they are very close to each other to see if this CC can detect deterioration:

- Event 1: after 5 days of the last event.
- Event 2: after 8 days.
- Event 3: after 10 days.

Thus, the CC in **Fig. 3** (EWMA chart of fourth root) will be obtained:

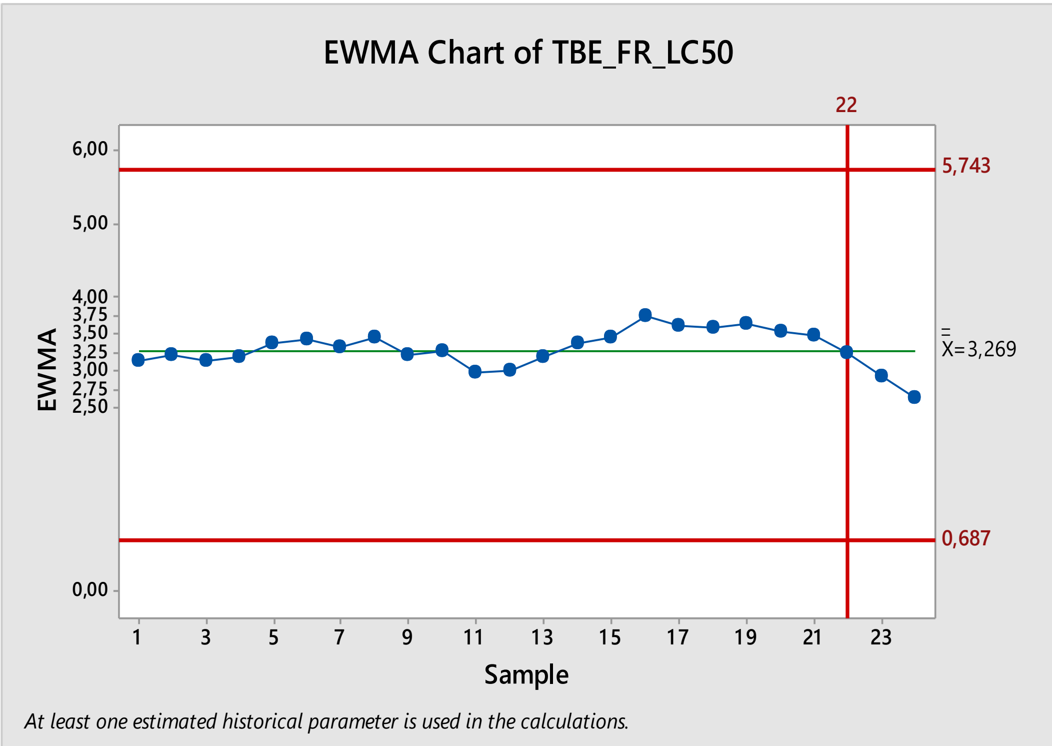

Figure 3: EWMA chart of fourth root.

No overtaking was observed even after adding new observations that are very close to each other, so this CC cannot be used to control the situation at this LC. Now, the calculation of the control limits will be done according to the method used by Jones and his colleagues (2002). They calculate a phase I CC based on TBE data CL and control limits as cited below:

$$UCL = \frac{m\bar{x}}{1 + (m-1)F_{2(m-1),2,1-\alpha/2m}} \quad (7)$$

$$CL = \bar{x} \tag{8}$$

$$LCL = \frac{m\bar{x}}{1 + (m-1)F_{2(m-1),2,\alpha/2m}} \tag{9}$$

Where:

m: number of initial independent observations from an in control process.

α : The false alarm rate.

Fi: an F distribution with numerator and denominator degrees of freedom respectively equal to 2(m-1) and 2.

In this case, the initial sample contains 21 independent observations and the false alarm rate will be fixed at 0.05. Thus, the calculations give the followings:

$$UCL = \frac{m\bar{x}}{1 + (m-1)F_{2(m-1),2,1-\alpha/2m}} = \frac{21*164.667}{1+(20)F_{40,2,0.0012}} = \frac{3458.007}{1+20*0.12} = \frac{3458.007}{3.4} = 1017.061$$

$$CL = \bar{x} = 164.667 .$$

$$LCL = \frac{21*164.667}{1+(20)F_{40,2,0.9988}} = \frac{3458.007}{1+20*830} = \frac{3458.007}{16601} = 0.208$$

The phase I CC for TBE is presented in **Fig. 4** (Phase I & LC50 CC):

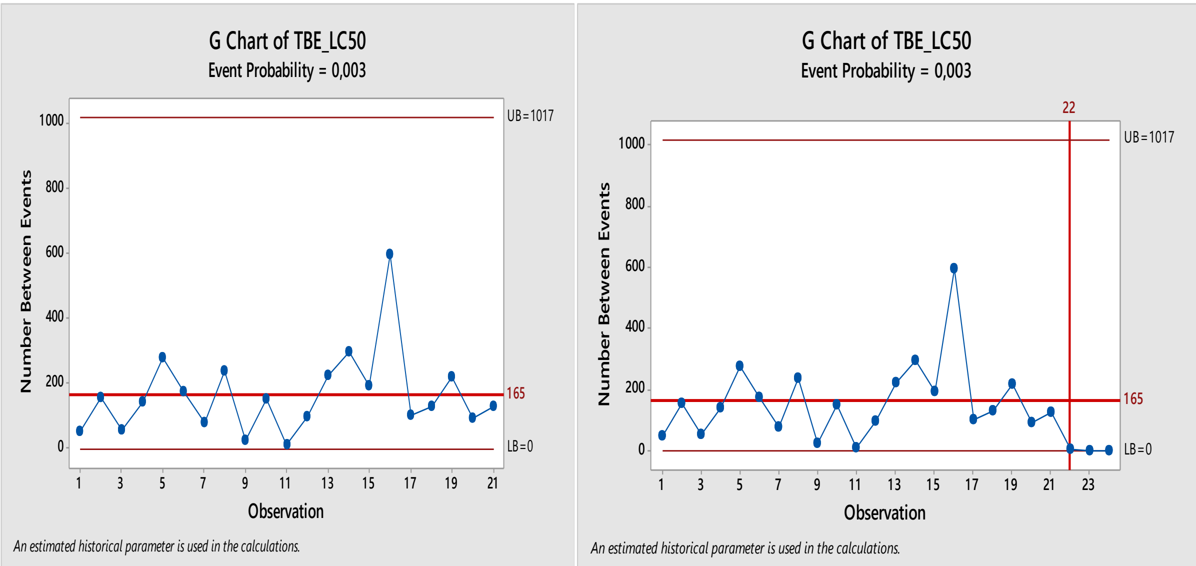

Figure 4: Phase I & LC50 CC.

It can be seen in this graph that there are no values plotting outside the control limits, and then the phase II CC for the LC50 with the three new observations can be established (**Fig. 4**: Phase I & LC50 CC). As it can be seen, there are no values plotting outside the control limits. From the first years, the majority of observations are close to the LCL but the process is still under control. This process needs more attention given that the last observations are nearby zero and it seems that there are no measure taken that is why, thereafter no improvement can be detected over the rest of the observations.

Now the method applied by Zhang and his colleagues will be followed. A gamma chart with r = 2 for LC incident over the period 2009-2018 will be presented. To determine the UCL and LCL in the case of a Gamma distribution with parameters (r, λ_0) it will be necessary to solve two equations:

$$F_{G_0}(L) = 1 - e^{-\lambda_0 L} \sum_{j=0}^{r-1} \frac{(\lambda_0 L)^j}{j!} = \alpha \qquad (10)$$

$$F_{G_0}(U) = 1 - e^{-\lambda_0 U} \sum_{j=0}^{r-1} \frac{(\lambda_0 U)^j}{j!} = 1 - \alpha \qquad (11)$$

Where:

α : The design false alarm rate.

U: UCL.

L: LCL.

As they demonstrated, the unbiased estimate of λ , in case this parameter is unknown in advance, is given by:

$$\bar{\lambda} = \frac{n - 1}{\sum_{i=1}^{n} X_i} \qquad (12)$$

Where n is the size of the preliminary sample and Xi are the individual TBE observations that follow an exponential distribution with parameter λ .

In the studied example, the process was considered in control with an estimated occurrence rate of incidents (the unbiased estimate of λ) $\lambda_0 = 0,00578$ relative to phase I.

In order to present the Gamma chart with r = 2, and a false alarm rate $\alpha = 0.0027$, the Table 4 (Tabulation of the Gamma chart for LC incident) is drawn.

Table 4: Tabulation of the Gamma chart for LC incident.

Index of TBE	X	G(r = 2)	Index of TBE	X	G(r = 2)	Index of TBE	X	G(r = 2)
1	52		8	238	318	15	195	
2	157	209	9	26		16	596	791
3	57		10	153	179	17	103	
4	144	201	11	11		18	132	235
5	278		12	98	109	19	220	
6	175	453	13	223		20	95	315
7	80		14	297	520	21	128	

Zhang and his colleagues (2007) retained a false alarm rate $\alpha = 0,004$. If this value is retained, these control limits will be obtained:

LCL=26.81.

UCL= 2233,37.

The gamma chart is presented in **Fig. 5** (Gamma chart with r=2 for LC50 incident data):

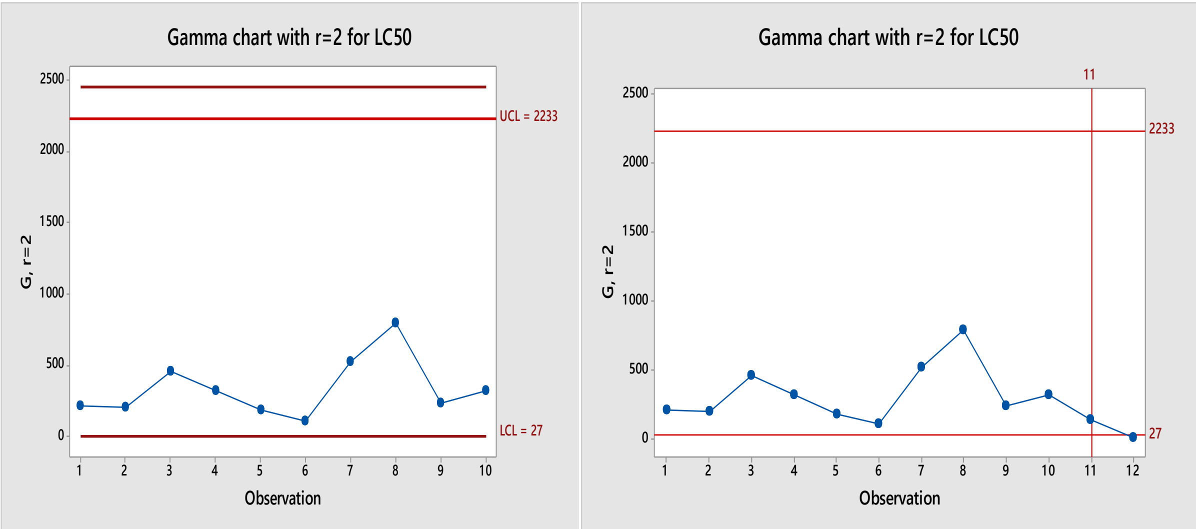

Figure 5: Gamma chart with r=2 for LC50 incident data.

This is the phase I chart which does not signal any overtaking and then the process is under control since no values are plotting outside the control limits. The CC with 3 new events (**Fig. 5** Gamma chart with r=2 for LC50 incident data) shows an out of control point for the observation 12: The situation is risky since the new value is very close to the LCL that's why this LC50 must undergo an intervention to improve this situation. If a false alarm rate $\alpha = 0,0027$ is used instead of 0.004, the gamma chart for LC50 incident data is established (**Fig. 6**:

Gamma chart with r=2 for LC50 incident data):

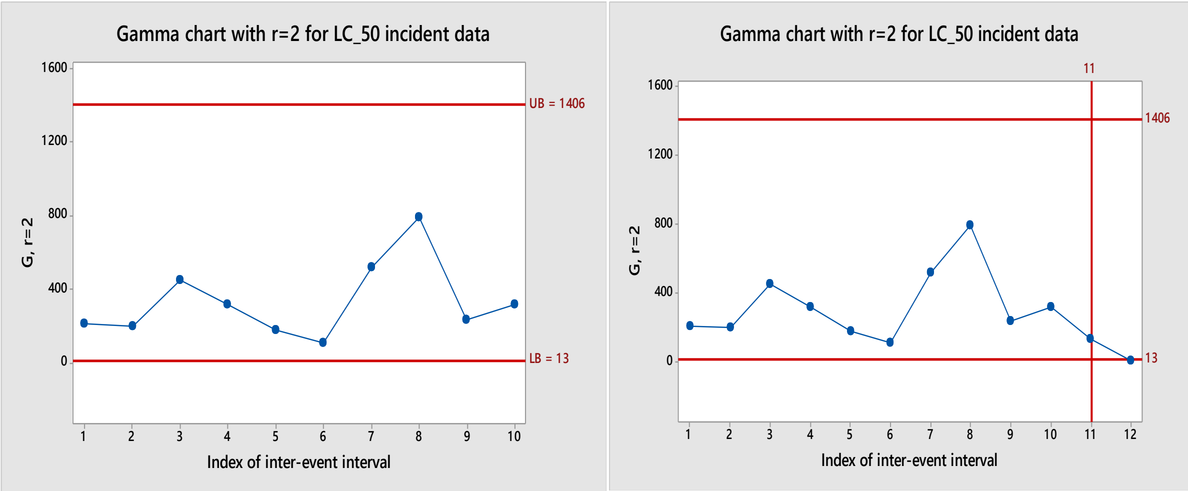

Figure 6: Gamma chart with r=2 for LC50 incident data.

There are no values plotting outside the control limits and then the process is under control. Then the same thing is done for the chart with 3 new values (**Fig. 6**: Gamma chart with r=2 for LC50 incident data): There is no big difference with the previous chart: when changing the design false alarm rate from 0,004 to 0,0027, the Gamma control chart gives same results.

2.1 Magnitude CC

In the previous paragraph, a CC that allows detecting overruns based on TBE has been studied. In this section, a CC for the X and for the quotient Q will be established. It is assumed that the T and X are independent of each other. T is exponential as it is demonstrated previously and for X, it is suggested to study two situations namely: normal and Gamma distribution.

Wu and his colleagues in [19] demonstrated the potential of quotient chart not only for manufacturing systems, but also for non-manufacturing sectors. The Q chart makes use of information about both T and X, and thus is more effective than an individual T chart or an individual X chart. There are two charting parameters associated with a quotient chart. If it is designed to detect the upward quotient shifts (decreasing T and/or increasing X), the two parameters are the LCLt of the T chart and the UCLx of the X chart. If the Q chart is designed to detect the downward quotient shifts (increasing T and/or decreasing X), the two parameters are the UCLt of the T chart and the LCLx of the X chart.

Qu and his colleagues in [20] presumed that X is usually assumed to follow a normal distribution with mean μ and standard deviation σ. By using the Average Time to Signal (ATS) as criteria, Qu and his colleagues (2014) demonstrated that the overall performance of the Q chart is much better than that of the individual T chart and X chart. The Q chart is always significantly more effective than the T chart except when T shift prevails, and it substantially outperforms the X chart except when X shift is dominant.

Rahali and his colleagues in [21] evaluated the Shewhart TBE and amplitude CCs for several distributions. Three statistics Z1, Z2 and Z3 have been established. Rahali and his colleagues (2019) concluded that if the shift is due to mainly a change in X or if it is due to a simultaneous change in X and T, the overall best choice is statistic Z1. If the shift is mainly due to a change in T, the overall best choice is clearly the statistic Z3.

In what follows the work will aim to research for an upward shift of the quotient Q given that if T increases or X decreases, the quotient Q increases (improvement) and that if T decreases or X increases, the quotient Q increases (deterioration).

For each chart, control limits will be calculated and CC will be presented to see if it detects any overruns, the obtained results will be compared in order to select the best combination that will be the one with the smallest ATS.

2.2 TBE Chart

As determined in the previous section, the TBE distribution is exponential for the LC50. For phase I data, the values characterizing this distribution have to be determinate from the historical values (21 observations) which will be considered as the in control value.

The control limits for the T CC are determined as follows:

$$LCL_T = \frac{-\ln(1-\alpha)}{\lambda_0} \tag{13}$$

And

$$UCL_T = \frac{-\ln(\alpha)}{\lambda_0} \tag{14}$$

Where:

α : type I error value.

λ_0 : The reciprocal of the mean value of T: The event (accident) takes place every $1/\lambda_0$ time units. The in-control value λ_0 can be estimated from the sample mean of T that is usually obtained from historical data.

The ATS relating to this distribution is determined as follows:

$$ATS = \frac{P_k}{\lambda} + (1 - P_k)(\frac{1}{\lambda} + ATS_{L_t}) \tag{15}$$

Where:

$$P_{Lt} = \Pr(t < LCL_T) = 1 - e^{-\lambda \times LCL_T} \quad (16)$$

P_k is the probability that the Q shift is signaled at the k^{th} occurrence of the event:

$$P_k = \Pr(t_0 + t_1 < LCL_t) = F_{t_0+t_1}(LCL_t) = \int_0^T \Pr(t_0 + y < T \mid y = t_1).f_{t_1}(t_1)dt_1$$

$$P_k = \begin{cases} \dfrac{\lambda_0 e^{-\lambda LCL_t} + \lambda(e^{-\lambda_0 LCL_t} - 1) + \lambda_0}{\lambda_0 - \lambda}, \text{if} \lambda \neq \lambda_0 \\ 1 - e^{-\lambda LCL_t}, \text{if} \lambda = \lambda_0 \end{cases} \quad (17)$$

$$ATS_{L_t} = \frac{1}{\lambda.P_{L_t}} \quad (18)$$

2.3 Magnitude Chart (X)

For this distribution, two situations will be studied: a normal then a Gamma distribution. If the normal distribution is retained as done by Qu and his colleagues (2014), this X chart has only an UCLX for detecting increasing X shifts. It is determined by:

$$UCLX = \mu0+\sigma0\Phi-1\ (1-1/\lambda0\tau) \quad (19)$$

Where Φ–1() is the inverse standard normal cumulative distribution function.

If the Gamma distribution will be retained for the X chart, the UCL will be equal to:

$$UCL_X = F_X^{-1}(1 - \alpha) \quad (20)$$

Where $F_X^{-1}(X)$ is the inverse function of the cumulative probability function Fx(X)

The ATS relating to the magnitude distribution X will be equal to:

$$ATS = (P_k \times 1 / \lambda) + (1 - P_k) \times ((1 / \lambda) + ATS_{UCL_X} \quad (21)$$

Where :

$$P_{UCL_X} = \Pr(X > UCL_X) = 1 - F(UCL_X) \quad (22)$$

$$P_k = 1 - F(UCL_x) \quad (23)$$

$$ATS_{UCL_x} = \frac{1}{\lambda \times P_{UCL_x}} \quad (24)$$

2.4 Quotient Chart (Q=X&T)

Two combinations will be studied namely:

- T exponential and X normal.
- T exponential and X follows a Gamma distribution.

The control limits of the Q chart will be determined as demonstrated by Wu and his colleagues (2009) in the case where the objective is to detect an upward shift. Thus, these limits will be calculated:

$$LCL_t = \frac{-\ln(1-(1-\omega)f)}{\lambda_0} \quad (25)$$

$$UCL_x = F_x^{-1}(1-\omega f) \quad (26)$$

Where:

$$f = \frac{1-\sqrt{1-4\times\alpha\times A}}{2\times A} \quad (27)$$

$$A = \omega(1-\omega)(1-2\xi+2\xi^2) \quad (28)$$

Since an upward shift is looked for, in what follows it will be considered that A=0,25 ($\omega = 0,5$ and $\xi = 0$).

In all cases, the determination of the ATS relating to the quotient Q will be carried out as follows:

$$ATS = P_k \times (1/\lambda) + (1-P_k) \times ((1/\lambda) + ATS_q) \quad (29)$$

Where:

$$P_k = F_{t_0+t_1}(LCL_t) + 1 - F(UCL_x) - F_{t_0+t_1}(LCL_t) \times (1 - F(UCL_x)) \quad (30)$$

$$ATS_q = 1/\lambda \times P_q \quad (31)$$

$$P_q = (1 - e^{-\lambda \times LCL_T}) + (1 - F(UCL_X)) - (1 - e^{-\lambda \times LCL_T}) \times (1 - F(UCL_X)) \qquad (32)$$

The objective is to calculate the ATS under the conditions mentioned above and see for which distribution the lowest ATS is obtained in order to determine the exceeding as quickly as possible. Since the calculation of the ATS carried out by several researchers resulted in the fact that the Q CC is more efficient in detecting overruns during phase II, the following work will be interested in determining the control limits for each of these CCs to see which of them will be the best to monitor the LC50.

2.5 Detection of Increasing Event Ratio Q

As retained by Wu and his colleagues (2009) and for detecting an upward shift these values will be fixed:

$\xi = 0$; $\omega = 0.5$; $\tau = 100/\lambda_0$

From the historical data concerning the LC50, the available information is drawn in Table 5 (Historical values).

Table 5: Historical values.

Distribution	X normal	X Gamma
Mean	3590.48	0.56
SD	5027.14	6407.27

For T distribution, the event take place once every 165 days ($1/\lambda_0$ times units) which means that λ_0 is equal to 1/165 = 0.006.

It will be supposed that the false alarm is produced every 100 occurrences of the event which means that τ is supposed to be equal to 16500.

If the overall type I error of a chart is α and ATS0 is made equal to τ the following values will be obtained:

$\alpha = \frac{1}{\lambda_0 . \tau}$ Since

$$ATS_0 = \frac{1}{\lambda_0} . ARL_0 = \frac{1}{\lambda_0} . \frac{1}{\alpha} \qquad (33)$$

So $\alpha = \frac{1}{0.006.16500} = 0.01$

For the normal distribution X the three charts (T, X and Q) are designed and the charting parameters for the three situations are listed below:

• T chart: LCLT = 1,692

• X chart: UCLX = 15266,39

• Q chart: LCLT = 0,846, UCLX = 772.056

These three charts will be presented for phase I and phase II under the hypothesis presented in Table 6 (Hypothesis: values to study).

Table 6: Hypothesis: values to study.

Observations	T	X
1	15 days	2000 $
2	30 days	15000 $
3	20 days	7000 $
4	10 days	12000 $

In **Fig. 7** (TBE chart & LCLt) the T chart is presented:

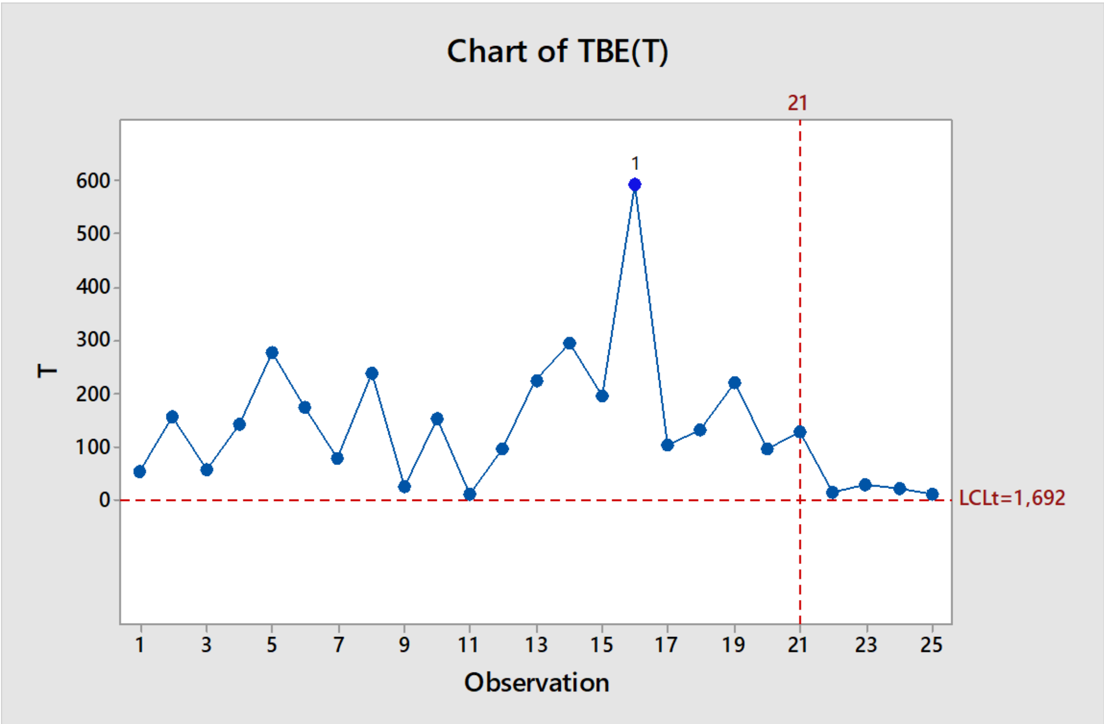

Figure 7: TBE chart & LCLt.

There is no overtaking observed. Thus, the T chart does not allow detection of any overruns during phase II.

The same observation can be made for the X chart (**Fig. 8**: X chart & UCLx):

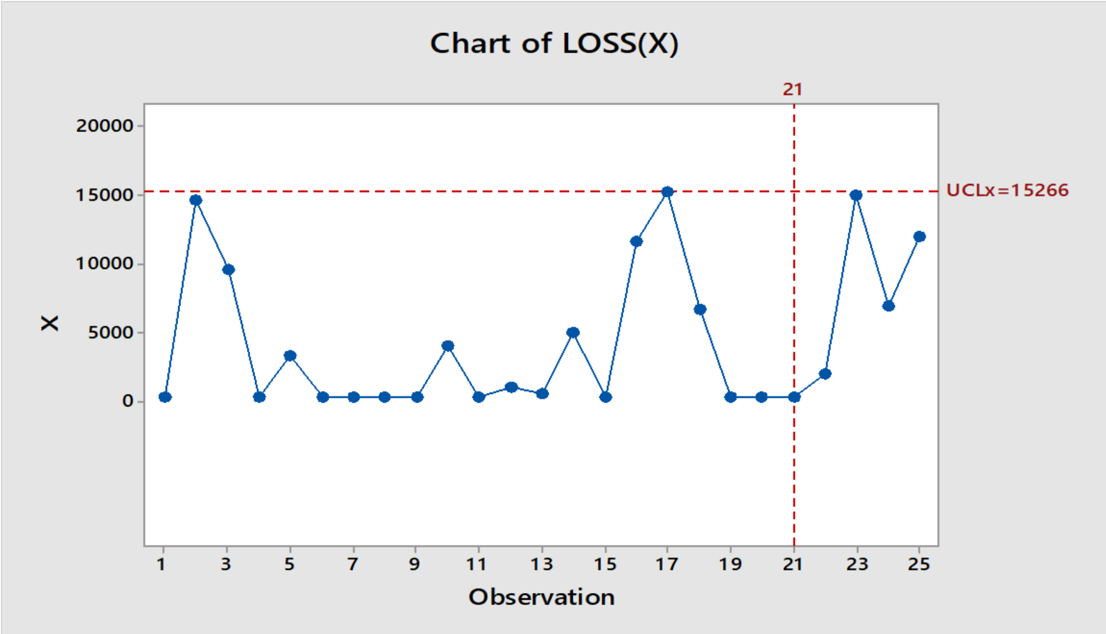

Figure 8: X chart & UCLx.

All observations are below the UCL and no overruns can be reported during phase II.

For the Q chart (**Fig. 9**: Q chart with control limits), the exceeding is noted at the last observation where an overruns of the UCL is seen.

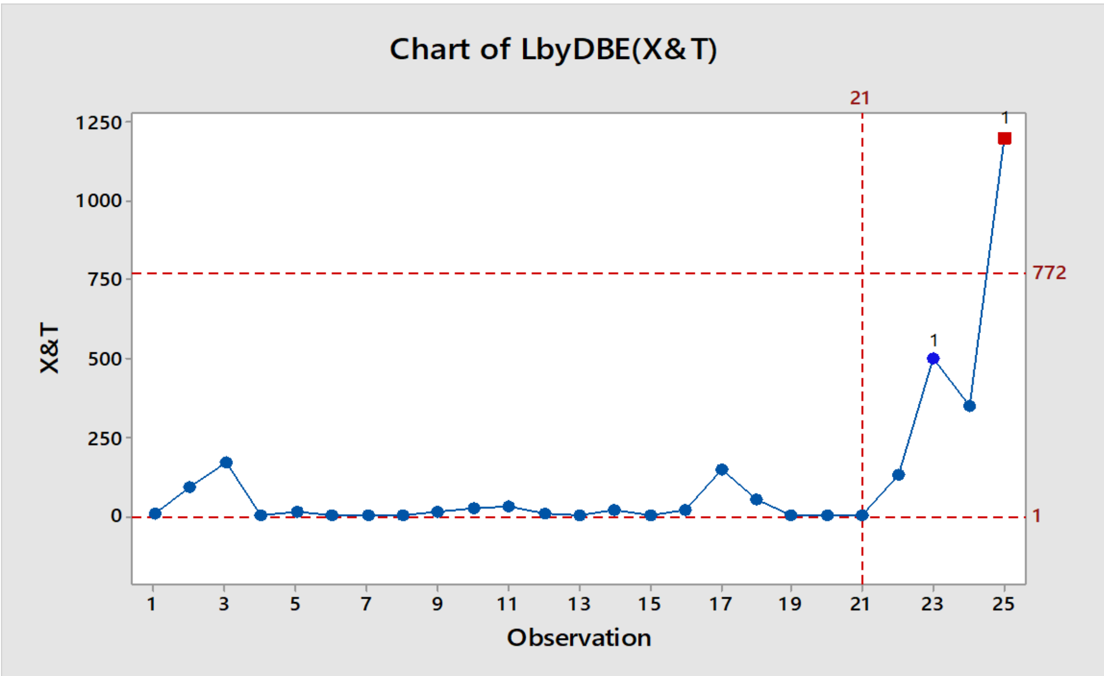

Figure 9: Q chart with control limits.

This finding confirms what has been put forward by Wu et and his colleagues (2009), Qu and his colleagues

(2014) and Rahali and his colleagues (2019) regarding the performance of this chart in terms of detecting exceeding. Thus, the T and the X charts did not allow detecting overshoots while the Q CC did.

For the Gamma distribution, the values a_0 and b_0 can be estimated from the in-control values $\bar{x}$ of sample mean and sample variance s^2 due to the following,

$$a_0 = \bar{x}^2/s^2 \text{ and } b_0 = s^2/\bar{x} \qquad (34)$$

Based on the above specifications, the three charts (T, X and Q) are designed and the charting parameters for the three situations are listed below:

- T chart: LCLT = 1,692
- X chart: UCLX = 22326.68
- Q chart: LCLT = 0,846, UCLX = 218.074

The three charts for phase I and phase II (Table 4) are presented as follows:

- T chart: The same chart as precedent (**Fig. 7**).
- X chart (**Fig. 10**: X chart & UCLx (Gamma distribution)): This CC does not allow discovering overruns.

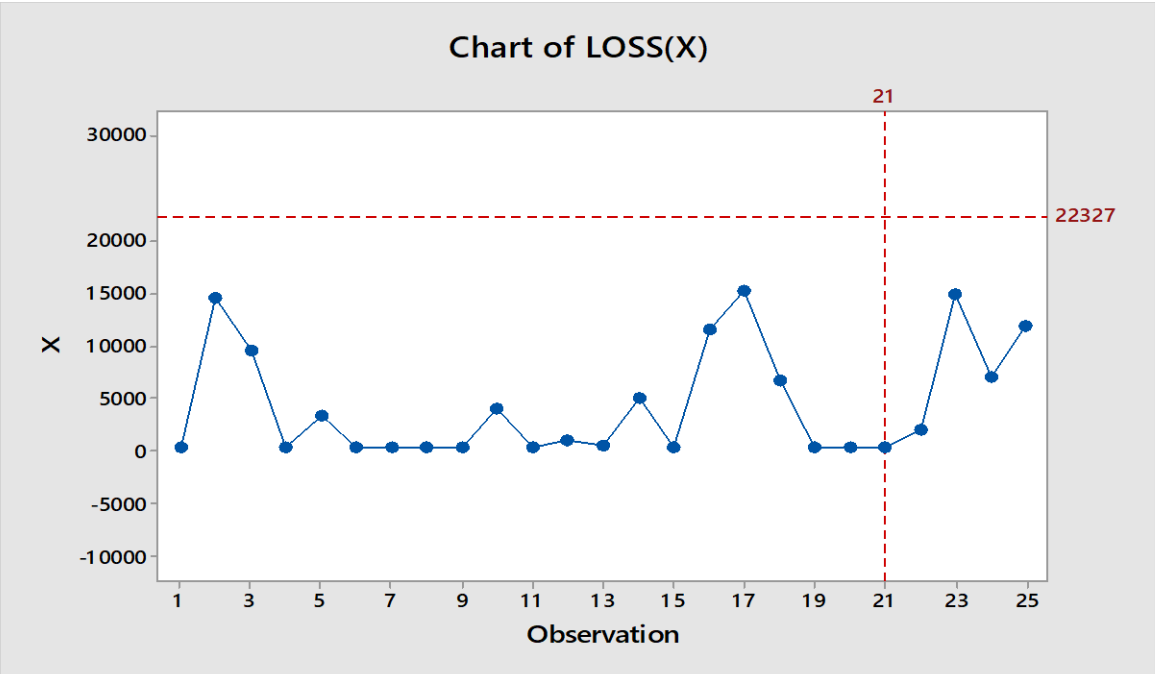

Figure 10: X chart & UCLx (Gamma distribution).

The situation during phase I is under control and it is the same for phase II.

Q chart (**Fig. 11**: Q chart with control limits (Gamma distribution)):

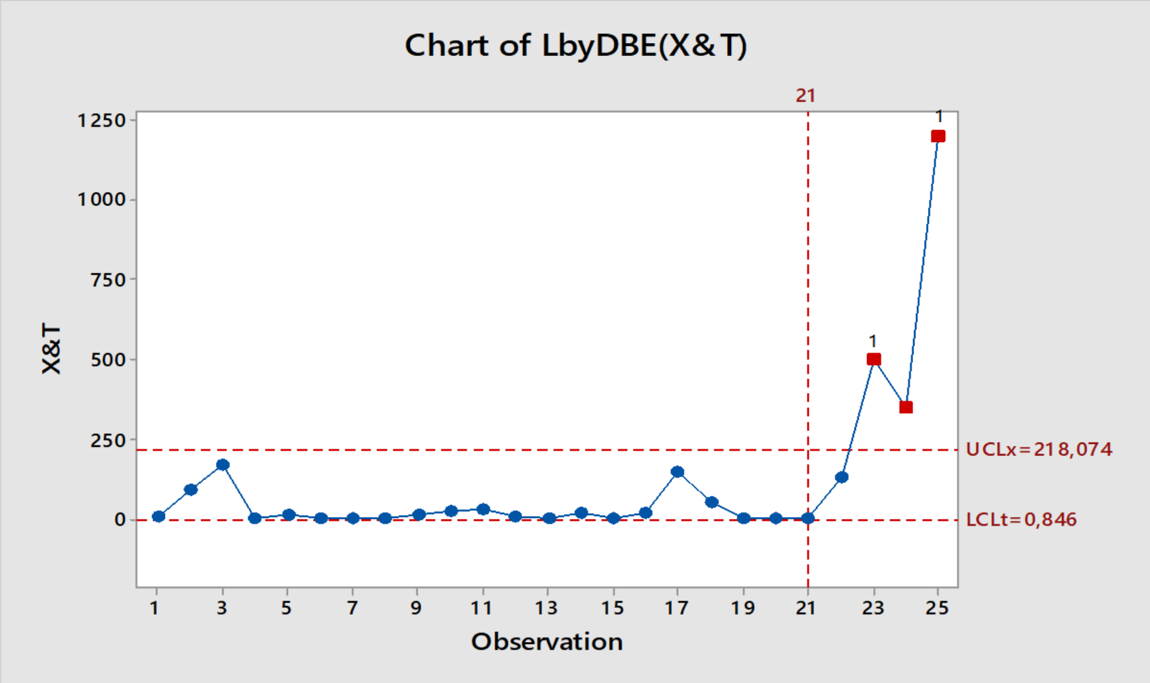

Figure 11: Q chart with control limits (Gamma distribution).

During phase I the process is under control while during phase II the existence of overruns can be noticed (observation 23, 24 and 25).

So in the hypothesis where X follows a Gamma distribution and T is exponential, the CC corresponding to the X&T combination is the most efficient in terms of detecting an out of control situation for the LC50.

3. Analysis and Discussion

In this paragraph, a presentation of several control charts is advanced: these charts are relating to the data corresponding to the TBE (T), to the magnitude (X) of the estimated economic losses (EEL) and to the quotient Q (X & T) between X and T. It is assumed that T and X are independent of each other. T is exponential as is demonstrated previously and for X, it is suggested to study two situations namely: normal and Gamma distribution. Wu and his colleagues (2009) [18] demonstrated the potential of the quotient chart not only for manufacturing systems but also for non-manufacturing sectors. The Q chart makes use of information about both T and X and thus is more effective than an individual T chart or an individual X chart. There are two charting parameters associated with a quotient chart. If it is designed to detect the upward quotient shifts (decreasing T and/or increasing X), the two parameters are the LCLt of the T chart and the UCLx of the X chart. If the Q chart is designed to detect the downward quotient shifts (increasing T and/or decreasing X), the two parameters are the UCLt of the T chart and the LCLx of the X chart. Qu and his colleagues (2014) [19] presumed that X is usually assumed to follow a normal distribution with mean μ and standard deviation σ. By using the Average Time to Signal (ATS) as criteria, Qu and his colleagues (2014) demonstrated that the overall performance of the Q chart is much better than that of the individual T chart and X chart. The Q chart is always significantly more effective than the T chart except when T shift prevails, and it also substantially outperforms the X chart except when X shift is dominant. Rahali and his colleagues in [20] evaluated the Shewhart TBE and amplitude CCs for several distributions. Three statistics Z1, Z2, and Z3 have been established. Rahali and his colleagues (2019) concluded that if the shift is due to mainly To control the situation at LC and see if it is necessary to intervene to take certain corrective measures to improve the situation, the CCs were used. The problem at this level is to find the best CC to control the situation at the LC. The LC50 presents a high number of accidents during the reference period and the fact of not detecting any overrun with the assumption of a very short time between incidents does not seem logical. The CC of distribution T as well as that of distribution X used above does not detect any overshoot; the process is therefore in this case under control, despite the LC50 recorded 21 incidents throughout the study period while several others almost similar LCs did not have this number quite high. So finding that this process is under control seems strange.

This finding was confirmed by the CC established by following the work done by Zhang and his colleagues (2007). This CC detects overruns which proves the process concerned sometimes goes into an out of control state, which obliges the managers to intervene to take corrective measures to make improvements to the process and make it under control. Previous studies carried out by various researchers have demonstrated by calculating the ATS that the CC corresponding to Q is the most efficient in terms of detection of overruns. It has been assumed that the distribution T is exponential. For the distribution X, two situations were studied, namely a normal and a Gamma distribution. The parameters of each distribution for phase I (under control phase) were estimated from historical data. Subsequently, the control limits were determined and the CCs were presented.

Following the results found by various researchers, it turned out that the CCs relating to the TBE and magnitude distributions did not allow the detection of exceeding during phase II. Similarly, the use of a CC for the quotient assuming that T is exponential and that X is normal does not make it possible to quickly detect overruns. On the contrary, the CC corresponding to Q makes it possible to detect several overshoots when X is assumed to follow a Gamma distribution. This last CC is, therefore, faster in detecting overruns.

In conclusion, for the LC50, it will be proposed to retain the CC relating to the quotient Q with the hypothesis that X follows a Gamma distribution. It will then be proposed to retain the control chart relating to the quotient Q (X&T) with the hypothesis that the magnitude X follows a Gamma distribution. This control chart brings together 21 observations in phase I and will be a means of control that will allow monitoring the situation for the LC50.

For the implementation of the above, it will be necessary to monitor the situation of the LC50 and proceed as follows:

- Establishment of an intervention policy to improve the situation of the LC50.
- Any intervention must be the subject of an agreement between the various stakeholders and take into account the economic cost, the application time, and the added value in terms of security.

- Based on the design of the experiment method to reveal the factors harming safety at the LC50, the CC selected for this work will be used at this level as a control tool.

4. Conclusion

Despite the various safety measures undertaken at LCs, the frequency of accidents remains high, and the corresponding economic losses are remarkable. The situation at the majority of LCs seems satisfactory in Tunisia but some LCs record a fairly large number of incidents during the study period (2009-2018). The objective of this work is to focus on the LC with the highest number of incidents during the cited period and to look for the best CC that will be used to monitor the situation at this LC. LC50 recorded the highest number of incidents throughout the study period while several other almost similar LCs did not have this number quite high which justifies the importance of studying and monitoring the situation at this LC. Other LCs with the same characteristics (protection type, crossing angle, area, number of lanes, and lane trajectory) recorded an acceptable number of incidents in the study period. So, the problem at this level is to find a method that makes it possible to control the situation at this LC (LC50).

In the beginning, a CC that allows detecting overruns based on TBE and magnitude X has been studied. These CCs failed to control the situation at the concerned LC. Then a CC for the quotient Q was established, it is assumed at this stage that the T and X are independent of each other. T is exponential and for X, it was suggested to study two situations namely: normal and Gamma distribution. This work aimed to research for an upward shift of the quotient Q given that if T increases or X decreases, the quotient Q increases (improvement) and that if T decreases or X increases, the quotient Q increases (deterioration). For each chart, control limits are calculated and CCs are presented to see if any overruns is detected.

By the results found by various researchers, it turned out that the CCs relating to the TBE and magnitude distributions did not allow the detection of exceeding during phase II and the same is true for CC corresponding to Q when the magnitude is assumed to be normal and T is exponential. On the contrary, the CC detects several overshoots when X is assumed to follow a Gamma distribution. This last CC is, therefore, faster in detecting overruns. In conclusion, for the LC50, it will be proposed to retain the CC relating to the quotient Q with the hypothesis that X follows a Gamma distribution.

In this study which deals with the Tunisian case, a comparison was made between Rudolf and his colleagues (1999) transformation, Jones and his colleagues (2002) method to calculate the control limits of their control chart, and the Gamma chart proposed by Zhang and his colleagues (2007).

In order to improve the situation of the LC50, which recorded a high number of accidents and a fairly significant economic loss, contact was made with the various stakeholders to apply certain measures to improve safety at the LC50 level. Some corrective measures have been suggested to reduce the EEL caused by incidents at LC.

After the implementation of these safety measures, it will be necessary to record future accidents at this LC for a well-defined period to establish an experimental design that will allow us to have an idea of the economic improvements brought by these measures.

The control of the improvement of the situation at LC must be the subject of a future study since it is necessary to wait a more or less long period to apply these corrective measures on the one hand and to record the future losses to see their trends on the other hand. This study provides a CC which makes it possible to control the situation at a LC. This CC is necessary but it must be completed by another work (research in progress) which will allow:

- The establishment of a model to explain the number of accidents at a LC by explanatory variables (to be determined).

- Calculate a security index that alerts any risky situation.

- Exploit the CC already studied in this paper to control the situation and intervene in case of overrun...

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