FRAGMENT SHAPE DISTRIBUTION IN EXPLOSIVELY DRIVEN FRAGMENTATION

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Abstract. The paper considers the shape distribution of fragments generated by detonation of a fragmentation warhead. Determination of the fragment shape distribution is very important for analysis of fragment ballistics and then for treatment of fragment interaction with a target. Morphology of metal fragments originating from casings of high-explosive projectiles is examined. Subsequently, the idealized fragment geometry and shape parameters are defined. Two theoretical models of fragment shape distribution are presented: (a) the model based on Mott's two-dimensional fragmentation approach, and (b) the Curran's "slot machine" model. The results of both models are transformed into previously defined shape parameters. Theoretical results are shown to be in good agreement with available experimental data.

1. Introduction

Dynamic fragmentation is a common phenomenon in nature and engineering systems that takes place in different size and time scales (asteroid impacts, explosively driven fragmentation, fragmentation induced by impact of nuclei, etc.).

The present research is focused on fragmentation of a metal cylinder caused by internal detonation of an explosive charge. Modeling of fragmentation process implies determination of following fragments' properties: (i) size (or mass) distribution of fragments, (ii) velocity of fragments, (iii) spatial distribution of fragments, and (iv) distribution of shape of generated fragments. In contrast to the first three problems, for which there are numerous models (e.g. [1-6]), significantly smaller number of studies is related to the problem of distribution of fragment shapes. Determination of fragment shape distribution is important for analysis of fragments' flight (fragment ballistics), as well as for modeling of interaction of a fragment and a target (penetration mechanics of fragments).

2. Morphology and shape parameters of fragments

The first study of the shape of fragments generated by explosive fragmentation of a metal cylinder was Mott's report [7]. Mott observed two dominant processes that lead to the fracture of the cylinder: (i) tension of the external part of the cylinder that causes a brittle or ductile fracture, and (ii) adiabatic shear bands at the internal part of the cylinder. A few characteristic shapes of large fragments, which have both the inner and outer surfaces of the cylinder, are shown in Fig. 1a (fragments 1–4). Small fragments (fragment 5) are result of the adiabatic shearing. More complex classification of fragments originating from a thick-wall cylinder fragmented by internal explosive detonation was proposed by Mock and Holt [8, 9]. According to this approach, there are four types of fragments (Fig 1b): (i) type 1 – has both the inner and outer surfaces of the original cylinder, (ii) type 2 – has only the outer cylindrical surface; further subdivision (2A, 2B) is related to the character of the other surfaces of the fragment, (iii) type 3 – has only the inner cylindrical surface, and (iv) type 4 – does not contain either the inner or outer original cylindrical surface.

Figures 1c and 1d show actual appearance of fragments originating from metal rings, as reported in experimental investigations [10, 11].

As previously mentioned, determination of the particle shape is relevant for modeling of the further motion of a fragment after the separation. An important parameter of the model is the reference fragment area *A* – the orthographic projection of the fragment on a plane perpendicular to direction of motion. The fragment has complex motion along the trajectory and the reference area *A* is continuously changed. Given that virtually all positions of the fragment during the flight are equally likely, the relevant value of reference area can be considered as the average area of fragment projection on an arbitrary plane. It can be proved that $A=S/4$. This relation which shows that the average projection area *A* of a convex body is equal to a quarter of its total surface area *S* is known as the Cauchy theorem [12].

Similar argument regarding the reference area can be applied to the penetration mechanics of the fragment. The comprehensive study of non-ideal penetration by Goldsmith [13], and the review of the relevant models of fragment penetration mechanics [14] suggest a possible way to treatment of the problem.

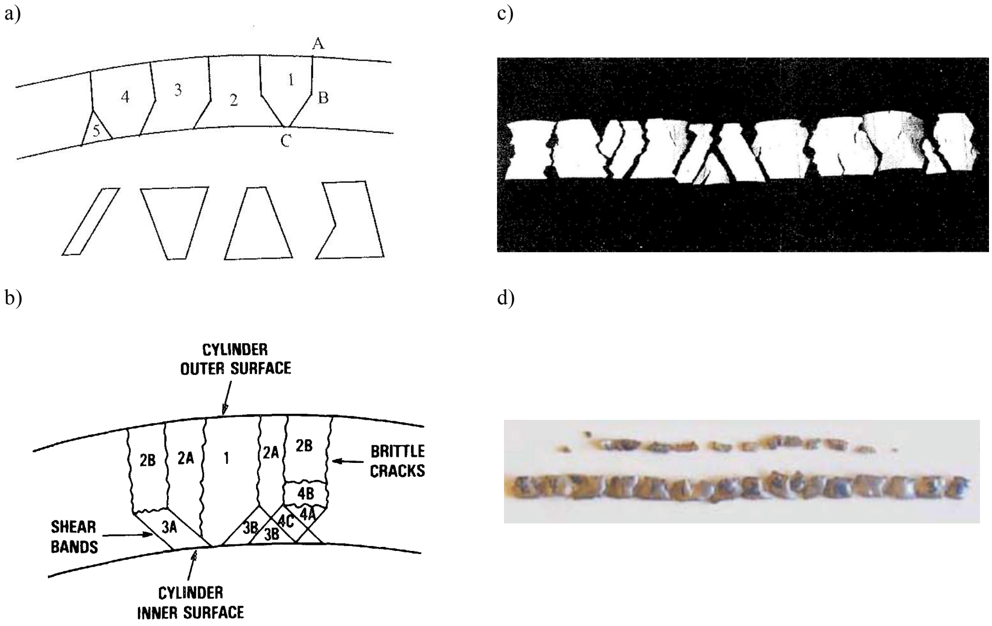

Figure 1. Morphology of fragments generated by internally detonated metal cylinder: a) Mott's classification of fragments from thin cylinder [7], b) complex classification of fragments originating from thick cylinders [8, 9], c) appearance of fragments from experiment [10], d) fragments from ring fragmentation [11]

Observing the diversity of possible fragment geometry, it is impossible to define a "universal" shape of the fragment. However, a triaxial ellipsoid has been adopted as a general approximation of fragment's shape, i.e. an idealization of the fragment convex envelope. An ellipsoid is defined by the length of semi-principal axes a, b and c ($a \geq b \geq c$), while its shape is determined by two parameters representing aspect ratios:

$$p = \frac{b}{a}, \quad q = \frac{c}{a}, \quad q \leq p \leq 1 . \tag{1}$$

An experimental approach to the fragment shape characterization implies measurement of fragment dimensions a, b and c, and calculation of corresponding shape parameters using Eq. (1). It should be noted that there is not a unique approach to determination of the characteristic dimensions of a fragment [15].

3. Models of fragment shape distribution

There are several analytical models that handle the shape distribution of fragments. Two of these basically two-dimensional (2D) models will be briefly analyzed.

2D model of fragment shape distribution. Investigation of biaxial 2D fragmentation, based on analytical treatment of Mott's model [16], leads to the distribution of area and aspect ratio of rectangular fragments in the form:

$$g(a,r) = \frac{n^2}{a_0}\left(\frac{a}{a_0}\right)^{n-1} \frac{1}{2r} \exp\left[-\left(\frac{a}{a_0}\right)^{n/2}\left[\left(\frac{r}{r_0}\right)^{n/2} + \left(\frac{r_0}{r}\right)^{n/2}\right]\right]. \tag{2}$$

In Eq. (2) a is the fragment area, $r=x/y$ is the aspect ratio of a fragment, n is the optimized shape parameter of the Weibull distribution, $a_0=x_0y_0$ where x_0 and y_0 are the scale parameters of the corresponding Weibull distributions. The nominal value of aspect ratio r_0 depends only on strain rates in two orthogonal directions:

$$r_0 = \frac{x_0}{y_0} = \left(\frac{\dot{\varepsilon}_y}{\dot{\varepsilon}_x}\right)^{2/3} . \tag{3}$$

Distribution of the aspect ratio of fragments is calculated by integration of eq. (2) over the whole domain of fragment area a:

$$h(r) = \int_0^\infty g(a,r)\mathrm{d}a = \frac{n}{r_0}\frac{\rho^{n-1}}{\left(1+\rho^n\right)^2} , \tag{4}$$

where $\rho=r/r_0$. The aspect ratio $r=x/y$, whose values are in the interval $(0,\infty)$, differs from previously defined shape parameter $p=b/a$, whose values belongs to the interval $(0, 1]$. Taking into account that $p=\min(r, 1/r)$, the probability density function of shape parameter p can be expressed in the form:

$$f_{2\mathrm{D}}(p) = \frac{2np^{n-1}(mp^{2n} + 2p^n + m)}{(p^{2n} + 2mp^n + 1)^2} \tag{5}$$

where the parameter m is defined by

$$m = \frac{r_0^{2n} + 1}{2 r_0^n}. \tag{6}$$

An analogous probability density function can also be derived for the shape parameter q.

"Slot machine" model. The second approach postulates that formation, growth and coalescence of thermoplastic shear instabilities (shear bands) is the main mechanism of fragmentation of the cylinder [17]. Various geometries and orientations of shear bands are possible, but experimental findings imply that half-penny shaped shear bands formed on internal surface of the cylinder are dominant. Consideration of overlapping of the crack's process zones and assumption of equal average fragment size in axial and circular direction yields the final form of the cumulative distribution of fragment shape:

$$F(n) = \frac{N(n)}{N_0} = \beta^{n-1}, \quad n \geq 1, \ \beta < 1. \tag{7}$$

In Eq. (7) $N(n)$ is the number of fragments whose length-to-width ratio is greater than n, N_0 is the total number of fragments and β is the parameter primarily dependent on the cylinder material properties and should be optimized for each experiment.
The shape parameter p is the reciprocal value of the fragment length-to-width ratio, $p=1/n$, which leads to the probability density function of the form:

$$f_{\mathrm{sm}}(p) = \ln\left(\frac{1}{\beta}\right) \frac{\beta^{\frac{1}{p}-1}}{p^2}. \tag{8}$$

4. Comparison with experimental results

Validation of analyzed theoretical models is performed by comparison with experimental results, which are very scarce. Marchand et al. [18] reported detailed test results for scaled models of two bombs: Mk 82 GP 500 lightly cased bomb and AN-Mk1 AP 1600 heavily cased bomb. Figures 2 and 3 show comparison of experimentally determined distribution of shape parameters p and q with results of two models considered. The model parameters m and β are optimized by the least squares method. As can be seen from the diagrams, probability density functions obtained by both models qualitatively follow the experimentally determined histograms for shape parameters p and q.
Results of fragmentation experiment of scaled BLU-109 munitions are presented in [17]. In this experiment with very thin cylinder wall (0.6 mm) only the fragment aspect ratio, and corresponding shape parameter p is determined. Theoretical distributions are in satisfactory agreement with the histogram obtained from the experiment (Fig. 4).
It is important to conclude that the 2D fragmentation model provides systematically better compatibility with the observation data than the "slot machine" model. Having in mind relative simplicity of the model and the fact that it involves only one adjustable parameter, the results of comparison can be considered promising. In order to fully validate, and possibly improve the model, it is necessary to use significantly larger experimental database.

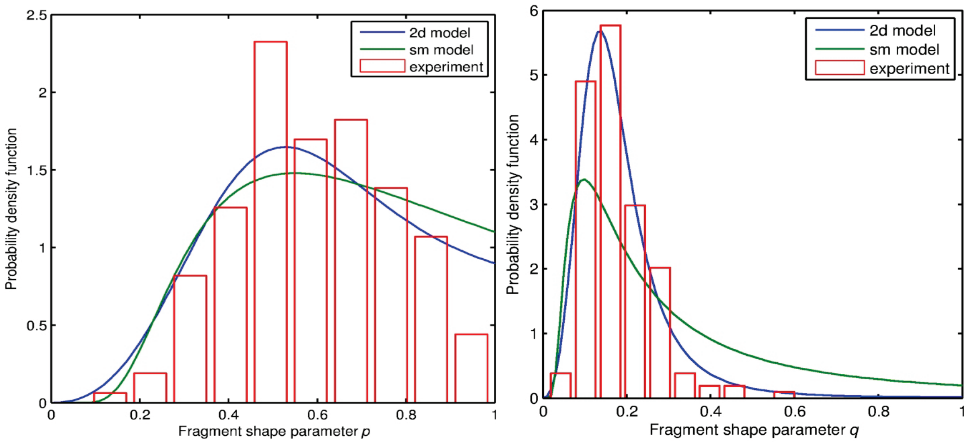

Figure 2. Comparison of experimentally determined histograms of shape parameters for a thin walled bomb [18] with probability density functions based on 2D fragmentation model and the “slot machine” model

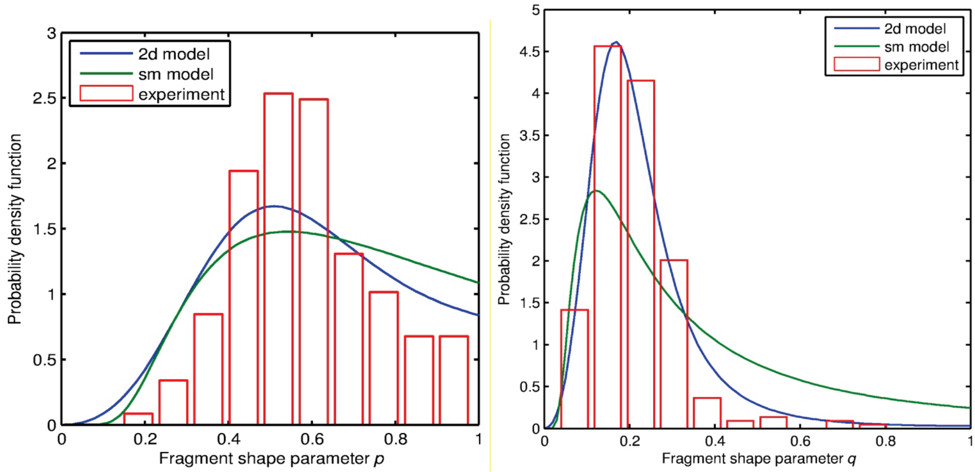

Figure 3. Comparison of experimentally determined histograms of shape parameters for a thick walled bomb [18] with probability density functions based on 2D fragmentation model and the “slot machine” model

4. Conclusion

The paper considers the shape distribution of fragments generated by detonation of a fragmentation warhead. Morphology of metal fragments originating from casings of high-explosive projectiles is examined and the fragment shape parameters are defined. Two theoretical models of fragment shape distribution are outlined: (a) the model based on Mott’s two-dimensional fragmentation approach, and (b) the Curran’s “slot machine” model. Theoretical results of 2D fragmentation model are shown to be in good agreement with limited experimental data.

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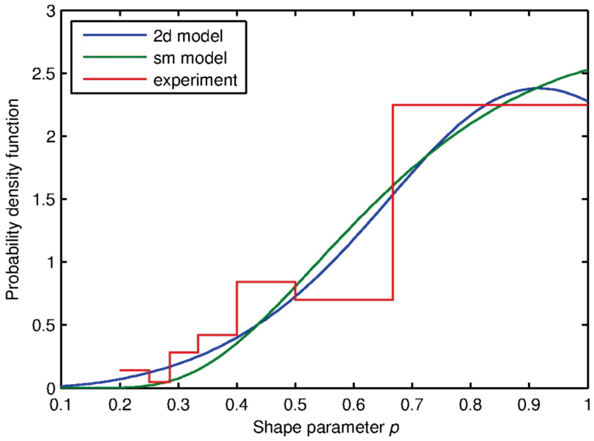

Figure 4. Comparison of experimental data for BLU-109 bomb [17] with probability density functions of 2D fragmentation model and “slot machine” model

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