GREY WOLF OPTIMIZATION FOR POSITION CONTROL OF A DIRECT CURRENT MOTOR DRIVEN BY FEEDBACK LINEARIZATION METHOD

Mitra Vesović*,
Radiša Jovanović

Faculty of Mechanical Engineering,
Belgrade,Serbia

Abstract:

Several studies dealing with position control of the DC motor have reported issues concerning friction force. This article demonstrates a nonlinear control and optimization strategy for position control of a series servo motor. Once it is empirically verified that the linear model does not adequately reflect the system, the model is upgraded from linear to nonlinear. In the course of the research, the nonlinear feedback linearizing the controller's behavior is examined. A grey wolf metaheuristic optimization algorithm is used to find the coefficients of the controller's gains. In this way, modern methods are applied to take a fresh look at the existing problem. Furthermore, performance for various targeted output signals is compared to show the approach proposed in the study. Also, a comparative analysis with whale optimization algorithm is performed. The experimental results acquired on the stated system are shown, and they validate the usage of the nonlinear control, demonstrating the effectiveness of using optimum feedback linearization in electrical machines.

Keywords:

Nonlinear Model and Control, Grey Wolf Optimization, Feedback Linearization Approach.

INTRODUCTION

The position of the output series DC motor shaft may be controlled using a variety of approaches. Traditional feedback control systems, such as proportional-integral-derivative (PID-like) controllers, are very widely utilized. They are inexpensive (in comparison to more complex control systems), simple, and variations of these manage to maintain the system's output within error limitations. They, on the other hand, suffer from a lack of resilience [1]. There are various nonlinear controllers in addition to standard ones. Some of them employ adaptive control techniques [2], while others are constructed using Artificial Neural Networks (ANN) [3]. The significant nonlinear features of the system make control challenging in general. Another approach, such as Fuzzy Logic Controller (FLC), can be designed to avoid this challenge [4]. Because of their high nonlinearity and various local optima, global optimization issues are difficult to solve efficiently. Finding the optimal minimum error function is a fundamental and difficult topic. For researchers in this field, nature has been an important source of inspiration [5].

Correspondence:
Mitra Vesović

e-mail:
mvesovic@mas.bg.ac.rs

DOI: 10.15308/Sinteza-2022-36-43

The genetic algorithm (GA), particle swarm optimization (PSO), whale optimization algorithm (WOA), grey wolf optimization (GWO) [6], and others are examples of these algorithms. For instance, nonlinear evolution and genetic algorithms have been utilized to optimize the design of a phase controller for tracking the trajectory of moving robots [7]. Other strategies can be used in conjunction with other nonlinear control systems. The GWO approach demonstrates its superiority for step and stochastic load disturbances in a wide range of situations. On the other hand, there is the nonlinear method whose basic idea is to algebraically convert a nonlinear system's dynamics into a (fully or partially) linear one, allowing linear control techniques to be used. Feedback linearization (FBL) is a strong nonlinear strategy that works by cancelling nonlinearities. This strategy has been effectively utilized in a variety of control tasks, including robotic systems, high-performance aeroplanes, helicopters, biomedical devices, and industry in general [8].

To a large extent articles dealing with comparable themes built the nonlinear model by flux and motor current nonlinearities [9], [10] a [11]. To control the position of the DC motor, the FBL was carried out in this work utilizing a mathematical model that takes into consideration friction-induced nonlinearity (Tustin model). Furthermore, a unique model was developed in which the discontinuous nonlinearity was approximated by a differentiable nonlinearity of the hyperbolic tangent, guaranteeing that the FBL application requirements were satisfied. After the feedback linearization strategy was successfully applied to algebraically change the nonlinear states of the system to their linear forms, a conventional linear system technique was adopted. To solve the problem of finding controller gains, the GWO and others optimization approaches were applied.

The experimental evidence of the efficiency of nonlinear system control is the paper's last contribution.

The rest of the paper is organized as follows: in Section 2, modeling and a schematic diagram of the object is provided. Contrary to many articles where nonlinearities are based on flux or motor current, modeling was performed using function that is suitable for the FBL and which takes into account the nonlinearity resulting from friction. Then, linear and nonlinear models are verified and compared. Sections 3 and 4 are overview of the theoretical derivations of the FBL and the GWO. In Section 5 we obtain the control signal based on the FBL and optimize its coefficients using the GWO. Finally, coa mparative analysis with another nature-inspired algorithm (whale optimization algorithm) is presented.

2. OBJECT DESCRIPTION AND MODELING

The creation of a mathematical model is among the first stages in the development of a control system. This saves time and profit in the long run. Contradictorily, accurate mathematical models are difficult to come by. Figure 1 shoof ws a schemamotorsepresentation of series wounded DC motor. Choosing motor voltage V_m as input variable $V_m=u$, and position of the load shaft θ_l as the output variable, $\theta_l=y$ the system's linear model is:

$$J_{eq}\ddot{y}(t) + B_{eq,v}\dot{y}(t) = A_m u(t)$$

Equation 1 – A linear model of the DC motor

In Eq. (1) J_{eq}, $B_{eq,v}$, and A_m are a total moment of inertia, equivalent damping term, and actuator gain.

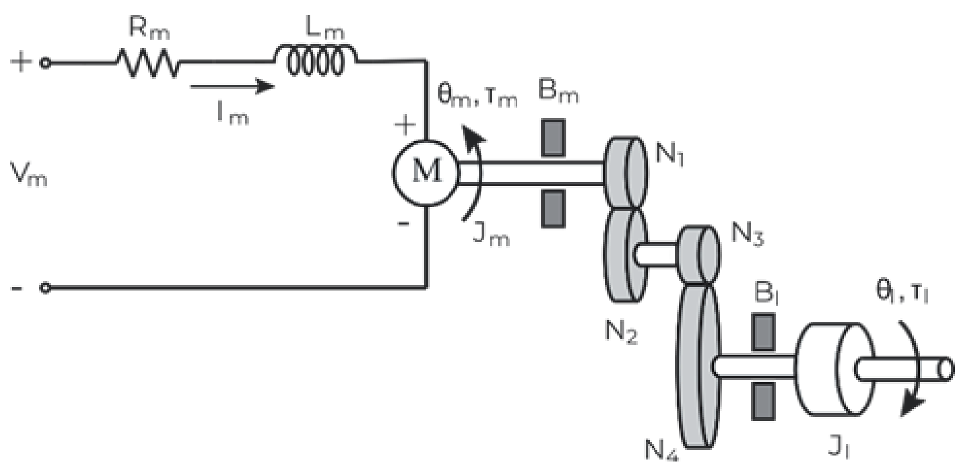

Figure 1 – A schematic representation of this series wounded DC motor.

The DC motor's nonlinear mathematical model was created using the speed-dependent friction nonlinearity. Many friction models, which have been extensively researched in the literature, differ primarily in how they describe the moment of friction. The friction torque is described as a static and/or dynamic function of rotational velocity in these models [12]. First, the friction model Tustin was used in this study:

$$J_{eq}\ddot{y}(t) + T_{st}(\dot{y}(t)) + B_{eq,n} = A_m u(t)$$

Equation 2 – A nonlinear model of the DC motor

where $T_{st} = T_{st}(\theta l) = T_{st}(y) = 0.0174 sgn(\dot{y}) + 0.0087e - \frac{\dot{y}}{0.064} sgn(\dot{y})$ is the nonlinear part of the Tustin friction model. To avoid the jump discontinuity of the suggested friction model and because the FBL approach demands differentiable functions (as it will be apparent from the supplied definitions in the following section), the approximation is achieved using the tangent hyperbolic function. Only Coulomb and viscous friction are modeled in this manner, and the exponential section of the Stribeck curve (static friction) is ignored [13].

$$f(\theta_l) = f(y) = \lambda_1\left(\frac{2}{1+e^{-\lambda_2\dot{\theta}_l}} - 1\right)$$

Equation 3 – Approximation of the part of the friction function

The state equation of the system was produced by choosing to designate nonlinearity as f(*x*).

$$\dot{x} = \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & \frac{-B_{eq,n}}{J_{eq}}\end{bmatrix}x + \begin{bmatrix}0\\ -1\end{bmatrix}f(x) + \begin{bmatrix}\dot{x}_1\\ \frac{A_m}{J_{eq}}\end{bmatrix}u \quad (1)$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}x \quad (2)$$

Equation 4 – State equation of the system

In Eq. (4) state variables are given as $x_1=\theta_l$ and $x_2=\dot{\theta}_l$ and $B_{eq,n}$ is an equivalent damping term with linear viscous friction already comprehended.

2.1. VERIFICATION OF THE MATHEMATICAL MODELS

The object's real operation is demonstrated during the experiment, Figure 3.

For step and sinusoidal inputs, comparisons were done with the responses obtained from the linear and nonlinear models. The real object's and linear model's reactions to step and sinusoidal excitations do not match well. The model does not reflect the system's actual behavior for the step input. The sine wave also exhibits nonlinearity in the form of the dead zone. The effect of friction is represented by this nonlinearity. It's particularly important when stated in low-frequency sinusoidal functions (and when the rotation direction changes), because the friction impact is amplified.

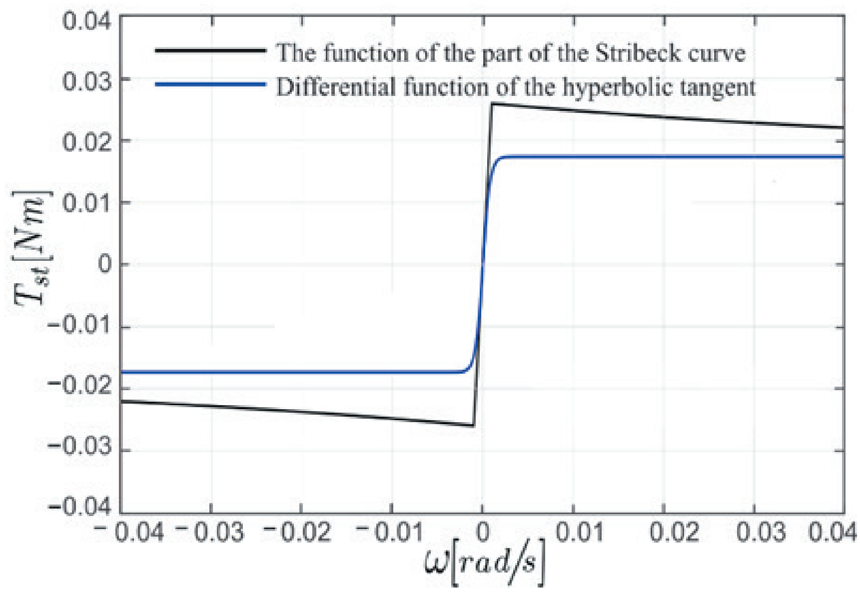

Figure 1 – A schematic representation of this series wounded DC motor.

Parameters	Values and Units	Parameters	Values and Units
J_{eq}	0.0021 kgm^2	$B_{eq,n}$	0.0721 Nm/(rad/s)
$B_{eq,v}$	0.0840 Nm/(rad/s)	First coefficient from Eq. (3) – Approximation of the part of the friction function λ_1	0.0173607
A_m	0.1284 Nm/V	Second coefficient from Eq. (3), λ_2	2500

Table 1 – Numerical values.

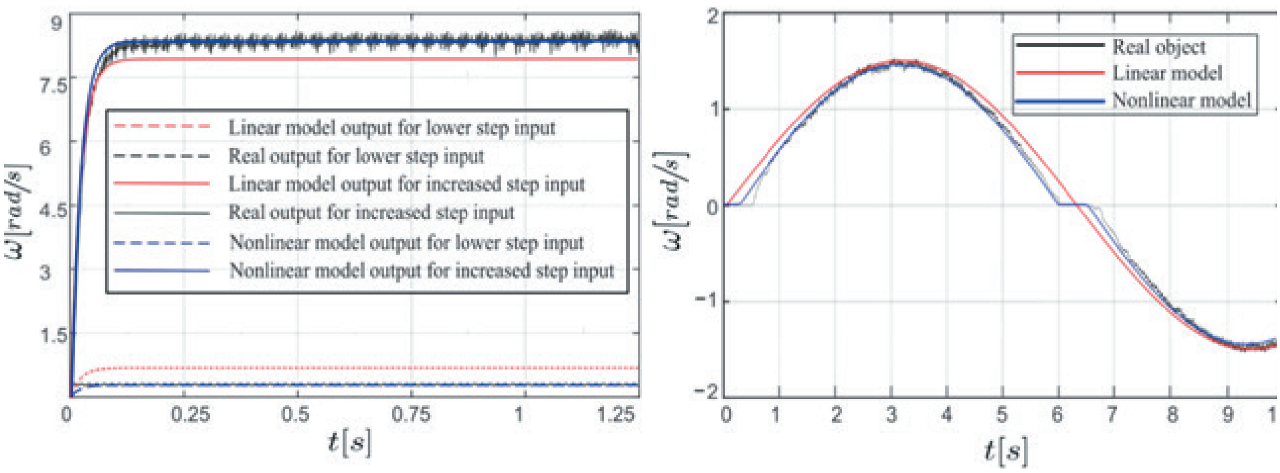

Figure 3 – Comparison between real behaviour and two models.

The nonlinearity of friction must be considered to produce the most realistic model of the DC motor and to permit a decent synthesis of the control system afterward. From Figure 3 a significant conclusion may be derived: the plant's simulated linear model does not match the reaction of the real system. The mathematical model of a series DC motor is nonlinear.

3. FEEDBACK LINEARIZATION LIMITATIONS

In order to eliminate nonlinearities from the system, the theoretical foundation for implementing the recommended FBL method will be presented in this section. The theoretical derivation is based on [14]. Designing the control signal with the feedback linearization rule, which cancels the nonlinearity, will be very important. This method does not rely on approximation in any way but, without a doubt, generalization of this concept is not always possible - there must be a unique set of systemic characteristics that allows cancellation. To reach this level of control, four restrictions must be met.

1. State equation of the system requires the following form Eq. (5).

$$\dot{x} = A_x + B_y(x)\left[u - a(x)\right],$$

Equation 5 – The appropriate form for applying the FBL

where A is $n \times n$ matrix, while B is $n \times p$ matrix. The functions: α: $R^n \longrightarrow R^p$, γ: $R^n \longrightarrow R^{p \times p}$ are defined on the domain that contains the origin and reflect possible nonlinearities in the system. Sometimes, when system is not in the form of the Eq. (5) it may be adjusted, because state space model of system is not unique and depends on the choice of state variables.

2. Differentiability is required for all functions;

3. It's easy to see that to cancel a nonlinear component by subtraction $\alpha(x)$, the control signal u and the nonlinearity must appear as the sum. To reverse the nonlinear member $\gamma(x)$ by division, on the other hand, control and nonlinearity must appear as the product. So, the third condition is that $\gamma(x)$ must be nonsingular for all $x \in D$; and

4. Pair (A,B) has to be controllable.

With these requirements met, the following control law might be generated:

$$u = a(x) + \frac{1}{\gamma(x)} v \;,$$

Equation 6 – Control law for the FBL

with a new control signal v.

4. GREY WOLF OPTIMIZER - OVERVIEW

Due to its great qualities, the GWO has been widely customized for a broad variety of optimization problems: it has extremely few parameters, and no derivation information is necessary for the first search. It mimics the hunting technique, as well as the grey wolves highly ordered pecking order and social scale in the wild [6]. In a group, there are four different wolf ranks: α, β, δ, and ω. The α is the pack's leader, and the other members of the pack obey him. Furthermore, all wolves participate in the major activity of prey hunting, which is divided into two steps: seeking for the prey and attacking. The following Eq. (7) of the distance vector **D** and the vector for position updating **X** (t+1):

$$\mathbf{D} = \left|\mathbf{C}\mathbf{X}_p(t) - \mathbf{X}(t)\right|, \;\; \mathbf{X}(t+1) = \mathbf{X}_p(t) - \mathbf{A}\mathbf{D} \,,$$

Equation 7 – Encircling the prey

are used to create a mathematical model of prey encirclement [6]. The coefficient vectors **A** and **C** may be computed as follows: A=2ar_1-a and C=2r_2. r_1 and r_2 are the random vectors in the range [0, 1]. Component a decreases from 2 to 0. Finally, t is the current iteration, X_p is the prey's location, and X is the agent's position vector. A mathematical simulation of hunting behavior is given with:

$$\mathbf{D}_\alpha = \left|C_1\mathbf{X}_\alpha - \mathbf{X}\right|, \;\; \mathbf{X}_1 = \left|\mathbf{X}_\alpha - A_1\mathbf{D}_\alpha\right| \qquad (1)$$

$$\mathbf{D}_\beta = \left|C_2\mathbf{X}_\beta - \mathbf{X}\right|, \;\; \mathbf{X}_2 = \left|\mathbf{X}_\beta - A_2\mathbf{D}_\beta\right| \qquad (2)$$

$$\mathbf{D}_\delta = \left|C_3\mathbf{X}_\delta - \mathbf{X}\right|, \;\; \mathbf{X}_3 = \left|\mathbf{X}_\delta - A_3\mathbf{D}_\delta\right|, \qquad (3)$$

Equation 8 - Hunting

and

$$\mathbf{X}(t+1) = \frac{\mathbf{X}_1 + \mathbf{X}_1 + \mathbf{X}_3}{3} \;.$$

Equation 9 – Position update

X_α, X_β, X_δ denote position vectors of the α, β, and δ wolves respectively, and A_1, A_2, A_3, C_1, C_2, C_3 are the elements expressed in the column vector. To put it another way, the agents separate to look for the prey, then converge to assault the prey. This is what encourages exploration and allows the GWO algorithm to search worldwide, or in other words, to have a broad search [6]. It's also simple, straightforward to use, adaptable, and scalable, with a unique capacity to find the correct balance between exploration and exploitation during the search, resulting in favorable convergence.

5. EXPERIMENTAL RESULTS

In order to meet all the conditions for the application of the method, from Eq. (4) and Eq. (5) follows:

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ 0 & \frac{-B_{eq,n}}{J_{eq}} \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 0 \\ \frac{A_m}{J_{eq}} \end{bmatrix}, a(x) = \frac{J_{eq}}{A_m} f(x), \gamma(x) = 1,$$

Equation 10 – FBL model of the system

so the system has the required form. As function f(x) is hyperbolic tangent and γ(x)=1, conditions 2 (differentiability) and 3 (nonsingularity) are also met. It remains only to check the controllability matrix:

$$U = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 0 & \frac{A_m}{J_{eq}} \\ \frac{A_m}{J_{eq}} & -\frac{B_{eq,n}}{J_{eq}^2} \end{bmatrix}, \;\; rank U = n = 2.$$

Equation 11 – Checking the controllability condition for the FBL

Eq. (11) shows the fulfillment of the fourth condition. The relative degree of the system is r=2 so Input-Output Feedback Linearization is feasible and full state linearization can be performed without fear of the internal dynamics. The control signal is chosen to be in the form:

$$u == \frac{J_{eq}}{A_m}\left[\frac{J_{eq,n}}{A_m} x_2 + f(x) + v\right], \;\; v = -K_0 x_1 - K_1 x_2 + K_r x_{ref} \,.$$

Equation 12 - Proposed control law

In Eq. (12) x_{ref} represents desired output. Traditional control is can be ineffective in dealing with a variety of issues such as steady-state error, rapid position and velocity changes. In this study, the controller gains must be established and optimized for the optimal operation to provide good dynamic behaviour. To compensate the effects of backlash and friction, FBL control approach with gains optimized with the GWO algorithm is used. Furthermore, the aforementioned parameters are all programmed into a single wolf, i.e. a single agent, who is supplied with a vector containing three parameters in our scenario. The integral of absolute errors (IAE) is used for the objective function performance criteria, as $IAE=\int|\varepsilon(\tau)|d\tau$. The number of search agents in the proposed GWO algorithm is fixed at 30, with a maximum number of iterations of 500. Furthermore, one agent represents a single possible optimum controller. The following are the scaling factor parameters acquired after optimization: $K_0=K_r=450$; $K_1=30.8387$. In the experimental part, we compared GWO with another modern optimization algorithm WOA [5]. The parameters of the WOA algorithm are taken from the paper [5]. The objective function, numbers of iterations and search agents are the same for both algorithms, due to a fair comparison. Both algorithms give similar results, with mean absolute error (MAE) shown in Table 5, with GWO being slightly better. With minor variations, the output and intended trajectory signals are essentially similar. On the Figure 4 and Figure 5 results are shown only for GWO. From Figure 4 (left) it is clear that system responds quite quickly. Both the rising and settling times are under 0.35 seconds with overshoot less than 3% and steady state error 0.0170. Sinusoidal signals in which the direction of rotation of the output shaft varies throughout operation are also very essential references for testing the performances of a nonlinear control system. Therefore, sinusoidal reference with amplitude 1 and frequency 0.5Hz is shown on Figure 4 (right). Position tracking for value: π (left) and for arbitrary signal with rapid changes (right) are depicted in the following Figure 5.

Reference	GWO	WOA
Unit step	0.0256	0.0242
Sinusoidal signal	0.0330	0.0324
Constant	0.0824	0.0975
Arbitrary signal with rapid changes	0.0755	0.0766

Table 2 – Comparison of MAE for GWO and WOA.

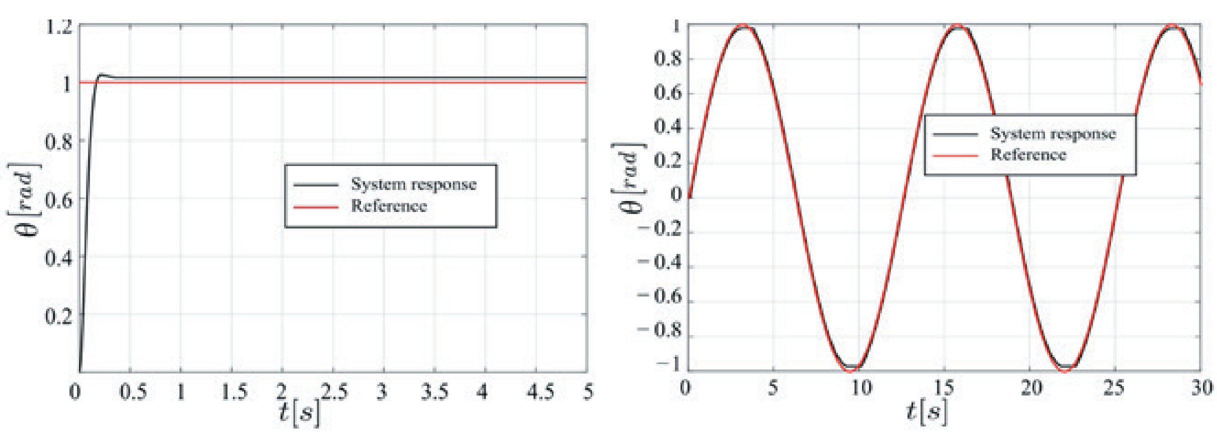

Figure 4 – Position tracking for unit step and sinusoidal signal as reference signals.

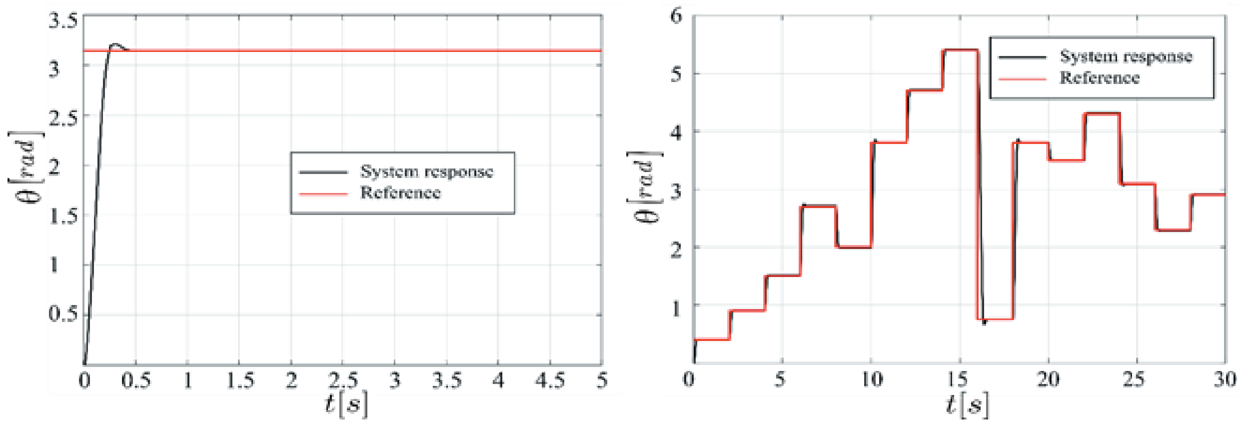

Figure 5 – Position tracking for constant and arbitrary signal with rapid changes as reference signals.

6. CONCLUSION

This paper is a continuation of the research [15], where the velocity of the load shaft of the DC motor is controlled by the FBL method, whose gains were optimized with the GWO algorithm. In this research, a nonlinear control technique was also utilized, but in order to control the load shaft's position of the DC motor. The introduction of Coulomb friction led to the development of a nonlinear mathematical model. Hyperbolic tangent was discovered as an approximation of the portion of the Stribeck friction curve and it was used as the function that represents nonlinearity. Afterward, the requirements for successfully implementing FBL were investigated and the theory of the GWO technique was provided. The fulfillment of the prerequisites for the synthesis of the control law with this technique has been summarized and supplied. Finally, the GWO optimization technique was employed to generate gains of the proposed FBL controller in the Matlab and Simulink environments, according to the IAE performance criterion. The results revealed that the proposed controller was capable of coping with the DC motor's nonlinearities. The desired output was followed by the plant response. Because the major purpose of this research was to get the DC motor to follow a particular position, it's crucial to note that this technique works for a variety of outputs. Provided control method might also be used to operate certain more complicated systems that employ this sort of engine. One interesting topic of future research may be optimization using alternative metaheuristic methods and comparing them.

7. ACKNOWLEDGEMENTS

The paper is a part of the research done within the project supported by the Science Fund of the Republic of Serbia, Grant No. 6523109, AI- MISSION4.0, 2020-2022, and by the Ministry of Education, Science and Technological Development of the Serbian Government, MPNTR RS under contract 451-03-68/2022-14/200105 from date 4.2.2022, Grant TR35004.

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