KINEMATIC AND DYNAMIC MODEL OF THE HUMAN CENTRIFUGE

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Abstract. Human centrifuge is dynamic flight simulator used to provide motion and forces cues of modern combat aircraft. It is mainly intended for safe and reliable generation of high G onset rates and high levels of sustained G for pilot trainings and research. In this paper, modeling of human centrifuge as a three DoF robot manipulator with revolute joints is presented. Pilots seat is controlled as end-effector. Here, Rodriguez formula is proposed for modeling kinematics and dynamics of the human centrifuge. Algorithms of direct and inverse kinematics are developed. Velocities and accelerations of CM's (centers of masses) of centrifuge links are determined and the results are compared with results obtained from developed Jacobian where singular positions are particularly discussed. Inverse dynamics algorithm based on covariant form of Lagrange equations of the second kind is given. Developed kinematic and dynamic models are implemented into control unit and simulated in offline part if control system.

1. Introduction

Combat aircrafts of the last generation are characterized by the feature of super maneuverability. Pilots are exposed to dangerous effects of high G-forces, impetuous angular movements, spatial disorientation, etc. In such conditions their ability to control aircraft is reduced and they may suffer from the loss of consciousness induced by the high G loads, so-called G-LOC. Human centrifuge is dynamic flight simulator used to provide motion and forces cues of modern combat aircraft [1].

Accelerations simulated in the cabin of centrifuge must be authentic to those that pilot experience during the flight at the most rotating maneuvers. This device can be modeled as a three DoF robot manipulator with revolute joints [2]. Main motion is rotation of the centrifuge arm about vertical, planetary axis. Arm carries gondola which is able to rotate about two axes, pitch and roll, as shown in Fig. 1 [3].

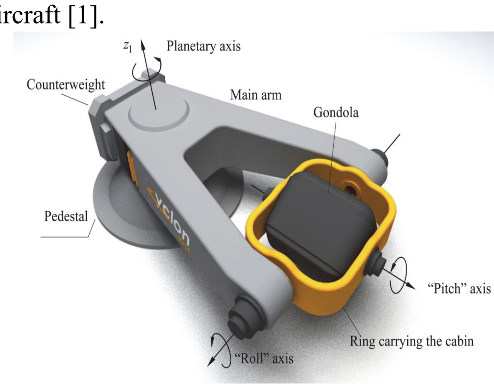

Figure 1. Human centrifuge-rotational axes

Control system intended for control of industrial robots have been previously developed in Lola Institute [4]. System is set up in modules: kinematical, dynamical, servo, path interpolator etc. [5]. This system is used as control system for human centrifuge by implementing kinematical and dynamical model and by adding functionalities specific for human centrifuge.

2. Kinematic model

In this section algorithms of forward and inverse kinematics, as well as development of Jacobian and analysis of singular positions are presented. Centrifuge is modelled as tree-like multibody system which moves in uniform gravitational field. Its motion is described by Lagrange generalized internal coordinates [6].

2.1. Forward kinematics

Local orthogonal Cartesian coordinate system $C_i\xi_i\eta_i\zeta_i$ which moves together with link i is joined to links. Here C_i is the center of mass of the link i. Based reference frame is denoted by $\xi_0\eta_0\zeta_0$. Rodriguez formula is used to obtain transformation matrices which define mutual position and orientation of centrifuge's links [6]:

$$\mathbf{A}^r = [I] + (1-\cos q)\left(\mathbf{e}^d\right)^2 + \sin q\,\mathbf{e}^d \tag{1}$$

where q is angle of rotation about axis determined by unit vector $\mathbf{e}$. $\mathbf{e}^d$ is skew symmetric matrix of vector $\mathbf{e}$ .Transformation matrices are given in (2a, 2b, 2c). Hereafter superscript 0 is used to denote that quantuty is given w.r.t. based reference frame.

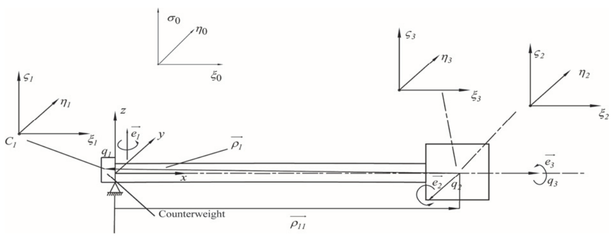

Figure 2. Coordinate frames in reference position

$$\boldsymbol{A}_{0,1} = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \boldsymbol{A}_{1,2} = \begin{bmatrix} c_2 & 0 & -s_2 \\ 0 & 1 & 0 \\ s_2 & 0 & c_2 \end{bmatrix}, \boldsymbol{A}_{2,3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_3 & -s_3 \\ 0 & s_3 & c_3 \end{bmatrix} \qquad \text{(2a, 2b, 2c)}$$

Pilot's seat is placed at the intersection of gondola axes. Its position w.r.t. to based frame is:

$$\boldsymbol{r}_E^0 = \sum_{k=1}^{n}\left[\boldsymbol{A}_{0,k}\right]\left\{\left(\boldsymbol{\rho}_{kk} + q_k\boldsymbol{e}_k\right)\right\} = \left[a_1c_1 \quad a_1s_1 \quad 0\right]^T \tag{3}$$

Here c_i, s_i are cosine and sine of the q_i angle, a_1 is the length of the centrifuge arm. c_{ij} is c_ic_j , s_{ij} is s_is_j , $s_{ijk}=s_is_js_k$ and $\boldsymbol{\rho}_{ii} = \overrightarrow{O_i'O_{i+1}}$. Vector $\overrightarrow{O_{i+1}C_i} = \boldsymbol{\rho}_i$ defines position of the $i+1$-th center of mass of the w.r.t. to frame $C_i\xi_i\eta_i\zeta_i$. These positions are obtained from mechanical design performed in 3CAD design software CATIA. Position of the *i*-th center of mass of the w.r.t. to based frame is given by equation:

$$\overrightarrow{OC_i} = \vec{r}_{Ci} = \sum_{k=1}^{i} \left(\boldsymbol{\rho}_{kk} + \xi_k \mathbf{e}_k q_k \right) + \boldsymbol{\rho}_i \tag{4}$$

Here $\mathbf{e}_k$ represents unit vector along joint axis. Parameter ξ_k has value 1 for rotation and 0 for translation.

Orientation of pilot seat is defined by classical Euler angles of precession ψ, nutation θ, and intrinsic rotation φ:

$$A_{0,3} = \begin{bmatrix} c_{12} & -s_1c_3 - c_1s_{23} & s_{13} - c_{13}s_2 \\ s_1c_2 & c_{13} - s_{123} & -c_1s_3 - s_{12}c_3 \\ s_2 & c_2s_3 & c_{23} \end{bmatrix} = \begin{bmatrix} c_\psi c_\varphi - s_\psi c_\theta s_\varphi & -c_\psi c_\varphi - s_\psi c_\theta c_\varphi & s_\psi s_\theta \\ c_\psi c_\varphi + c_\psi c_\theta s_\varphi & -s_\psi c_\varphi + c_\psi c_\theta c_\varphi & -c_\psi s_\theta \\ s_\theta s_\varphi & s_\theta c_\varphi & c_\theta \end{bmatrix} \tag{5}$$

$$\psi = arctg\left(-\frac{s_{13} - c_{13}s_2}{-c_1s_3 - s_{12}c_3} \right) + k\pi, \quad k = 0, \pm 1, \pm 2, \ldots \quad \varphi = arctg\left(-\frac{s_2}{c_2s_3} \right) + k_2\pi, \;\; k_2 = 0, \pm 1, \pm 2, \ldots \tag{6a, 6b}$$

$$\theta = arctg\left(-\sqrt{\left(s_{13} - c_{13}s_2\right)^2 + \left(-c_1s_3 - s_{12}c_3\right)^2} \; / \; c_{23} \right) + k_1\pi, \;\; k_1 = 0, \pm 1, \pm 2, \ldots \tag{7}$$

Angular velocities and accelerations of the link *i* w.r.t. based frame are obtained from following expressions:

$$\boldsymbol{\omega}_i^0 = \sum_{\alpha=1}^{i} \mathbf{A}_{0,i} \left\{ \mathbf{e}_\alpha \right\} \dot{q}_\alpha \, , \boldsymbol{\varepsilon}_i^0 = \sum_{\alpha=1}^{i} \mathbf{A}_{0,\alpha} \left\{ \mathbf{e}_\alpha \right\} \ddot{q}_\alpha + \sum_{\alpha=1}^{i} \sum_{\beta=1}^{\alpha} \mathbf{A}_{0,\beta} \left\{ \mathbf{e}_\beta^d \right\} \mathbf{A}_{\beta,\alpha} \left\{ \mathbf{e}_\alpha \right\} \dot{q}_\alpha \dot{q}_\beta \tag{8a, 8b}$$

Linear velocities and accelerations of the link's center of masses are obtained from:

$$\mathbf{v}_{C_i} = \sum_{\alpha=1}^{i} \vec{T}_{\alpha(i)} \dot{q}_\alpha \quad \mathbf{a}_{C_i} = \sum_{\alpha=1}^{i} \vec{T}_{\alpha(i)} \ddot{q}_\alpha + \sum_{\alpha=1}^{i} \frac{d\vec{T}_{\alpha(i)}}{dt} \dot{q}_\alpha \tag{9a, 9b}$$

where $\vec{T}_{\alpha(i)} = \partial \vec{r}_{Ci} / \partial q_\alpha$.

2.2. Inverse kinematics

Inverse kinematics problem is to find the values of joint angles (internal coordinates) required to obtain the desired values of position and orientation of end-effector (in this case pilot's seat). Input (external coordinates) can have different forms, depending on purpose. In this paper, Euler angles are chosen as input. Joint angles obtained from (10) and by multiplying (10) with $A_{1,0}$ are given in (11a, 11b, 11c):

$$A_{0,3} = \begin{bmatrix} c_\psi c_\varphi - s_\psi c_\theta s_\varphi & -c_\psi c_\varphi - s_\psi c_\theta c_\varphi & s_\psi s_\theta \\ c_\psi c_\varphi + c_\psi c_\theta s_\varphi & -s_\psi c_\varphi + c_\psi c_\theta c_\varphi & -c_\psi s_\theta \\ s_\theta s_\varphi & s_\theta c_\varphi & c_\theta \end{bmatrix} = \begin{bmatrix} n_{x3} & o_{x3} & a_{x3} \\ n_{y3} & o_{y3} & a_{y3} \\ n_{z3} & o_{z3} & a_{z3} \end{bmatrix} = \begin{bmatrix} c_{12} & -s_1c_3 - c_1s_{23} & s_{13} - c_{13}s_2 \\ s_1c_2 & c_{13} - s_{12}s_3 & -c_1s_3 - s_{12}c_3 \\ s_2 & c_2s_3 & c_{23} \end{bmatrix} \tag{10}$$

$$q_1 = arctg\left(\frac{n_{y3}}{n_{x3}} \right), q_2 = a\tan 2\left(n_{z3}, c_1n_{x3} + s_1n_{y3} \right), q_3 = arctg\left(\frac{o_{z3}}{a_{y3}} \right) \tag{11a, 11b, 11c}$$

2.3. Jacobian and singular positions

For robot manipulators, Jacobian is defined as the matrix that transforms the joint rates in the actuator space to velocity state in end-effector space:

$$\dot{\bar{\mathbf{q}}} = \mathbf{J}\dot{\mathbf{q}}, \mathbf{J} = \begin{bmatrix}\mathbf{J}_I & \mathbf{J}_{II}\end{bmatrix}^T \tag{12}$$

Here, $\bar{\mathbf{q}}$ is vector of external coordinates, and $\mathbf{q}$ is vector of internal coordinates. If external coordinates describing position and orientation of the end-effector are given in the form of (3) and (6a,6b,7) $\mathbf{J}_I$ and $\mathbf{J}_{II}$ are given in following equations:

$$\mathbf{J}_I = \begin{bmatrix} -s_1 a_1 & 0 & 0 \\ c_1 a_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{J}_{II} = \begin{bmatrix} 1 & -s_\psi c_\theta / s_\theta s_1 - c_\psi c_\theta c_1 & -s_\psi c_\theta / s_\theta c_{12} + c_\psi c_\theta s_1 c_2 + s_2 \\ 0 & c_\psi s_1 - s_\psi c_1 & c_\psi c_1 c_2 + s_\psi s_1 c_2 \\ 0 & s_\psi / s_\theta s_1 + c_\psi / s_\theta c_1 & s_\psi / s_\theta c_{12} - c_\psi / s_\theta s_1 c_2 \end{bmatrix} \tag{13}$$

If we place centers of masses into joints in (9a), velocities of joints are obtained and it has been shown that these terms coincide from those obtained from $\mathbf{J}_I$.

Determination of singular positions is of great importance in the design of control system. From $\det(\mathbf{J}_{II}) = c_2$ singular position of manipulator $q_2 = \pm\pi/2$ is obtained. In this position for definitely small values of end-effector velocities, indefinitely large values of joint velocities are required. This position is avoided by control algorithm.

3. Inverse dynamics problem

The inverse dynamics problem is to find the actuator torques and/or forces required to generate a desired trajectory of the manipulator. In this paper, this problem is solved in joint space by applying covariant form of Lagrange's equation of motion. This method is based on knowledge of kinetic and potential energy of the robot or its links. Kinetic energy of link *i* is calculated from following equations[6]:

$$E_{ki} = E_{ki}^{tr} + E_{ki}^{rot} = \frac{1}{2} m_i \boldsymbol{v}_{ci}^2 + \frac{1}{2} \boldsymbol{\omega}_i^i \boldsymbol{J}_{ci}^i \boldsymbol{\omega}_i^i, i = n,...,2,1, \tag{14a, 14b}$$

Generalized force of α link from gravity is:

$$Q_\alpha^G = \sum_{i=1}^{n} m_i \boldsymbol{g} \cdot \vec{T}_{\alpha(i)}, \alpha = n,...,2,1 \tag{15}$$

From covariant form of Lagrange's equation of motion:

$$\sum_{\alpha=1}^{n} a_{\alpha\gamma} \ddot{q}_\alpha + \sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} \Gamma_{\alpha\beta,\gamma} \dot{q}_\alpha \dot{q}_\beta = Q_{\gamma g} + Q_{\gamma a}, \gamma = 1,2,...,n \tag{16}$$

required actuator torque for q_γ generalized coordinate, $Q_{\gamma a}$ is obtained. Here, $\Gamma_{\alpha\beta,\gamma}$ are Christoffel symbols:

$$\Gamma_{\alpha\beta,\gamma} = \frac{1}{2}\left(\frac{\partial a_{\beta\gamma}}{\partial q_\alpha} + \frac{\partial a_{\gamma\alpha}}{\partial q_\beta} - \frac{\partial a_{\alpha\beta}}{\partial q_\gamma} \right). \tag{17}$$

$a_{\alpha\beta}$ are metric tensor coefficients :

$$a_{\alpha\beta} = \frac{\partial^2 E_k}{\partial \dot{q}_\alpha \partial \dot{q}_\beta}, \; E_k = \sum_{i=1}^{n} E_{ki} = \frac{1}{2} \sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} a_{\alpha\beta} \dot{q}_\alpha \dot{q}_\beta \tag{18}$$

4. Simulation in the off-line unit of control system

In robotics, during control design prior to the device realization, it is of great importance to test control algorithms for all possible conditions. The purpose of development of kinematic and dynamic models is possibility of analysis of controlled object behavior in simulated environment. As mentioned before in this paper, control system for this device is obtained by adding new functionalities into control unit for robot programming L-IRL (Lola Industrial Robot Language) and by implementing kinematic and dynamic model. This system has offline and online part. In the offline unit programming and testing of L-IRL code is done[7].

One of the possibilities for testing and analysis provided by offline system is an overview of all important kinematic and dynamic quantities obtained by implementing previously described model during different centrifuge operational scenarios. Graphs of some of these kinematic and dynamic quantities for planetary axis in the case of *open-loop* operational centrifuge mode produced by L-IRL offline unit are given *in Figs.* 4-8. Here, the predefined nonlinear profiles of the absolute acceleration in the center of gondola which is the input in inverse dynamics algorithm is given in Fig. 3. This profile is set by GMOVE instructions [8].

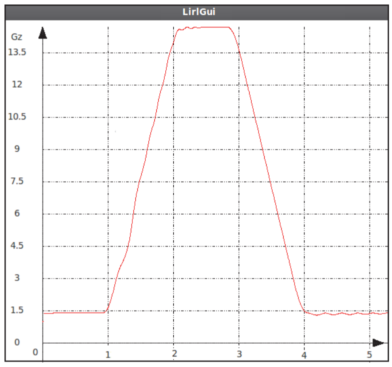

Figure 3. Abs. acceleration in the center of the gondola-input in inv. dyn. algorithm

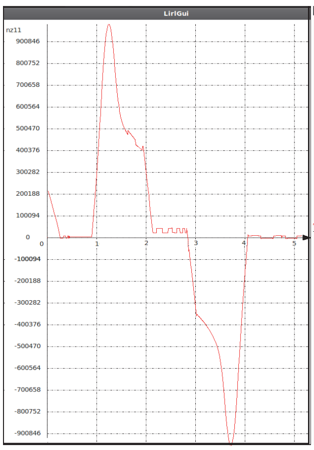

Figure 4. Actuator moment (Planetary axis)

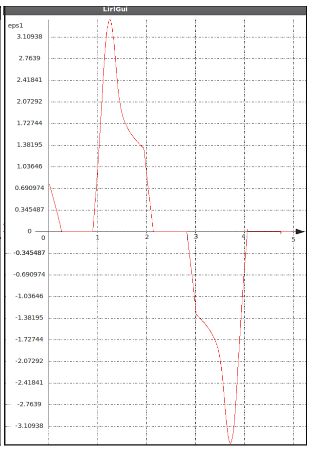

Figure 5. Angular acceleration of planetary axis

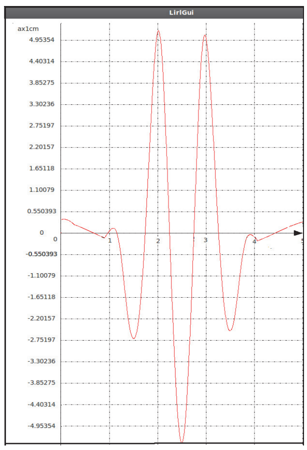

Figure 6. *x* component of linear acceleration of C w.r.t. based frame

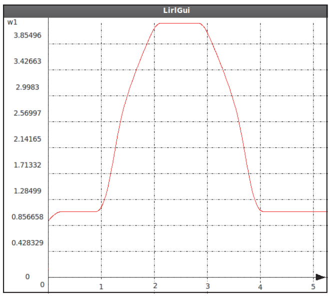

Figure 7. Angular velocity of first link

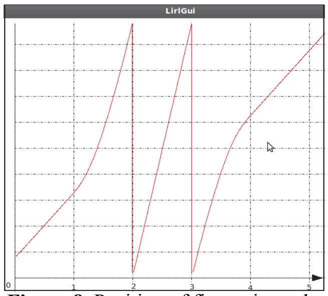

Figure 8. Position of first axis angle

5. Conclusion

In this paper modeling of human centrifuge as three DoF robot manipulator is presented. Algorithms of forward kinematics given by position of the pilot's seat w.r.t. to fixed frame and Euler angles, as well as the inverse kinematics algorithm are given. Velocities and accelerations of centers of masses as well of joints are determined. Development of manipulator Jacobian has shown that manipulator has singular position for $q_2 = \pm\pi/2$. Inverse dynamics algorithm based on covariant form of Lagrange equations of the second kind is given. These algorithms have been implemented into control unit and model has been simulated in offline part of the control unit. Simulation performed in offline system for open-loop centrifuge operational mode has given a fairly good insight of conduct of all important kinematical and dynamical quantities of this device.

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