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THE IMPACT OF THE PROPOSED MATHEMATICS ENRICHMENT PROGRAM ON THE UAE STUDENTS' MATHEMATICAL LITERACY IN LIGHT OF THE PISA FRAMEWORK

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Under the Supervision of Dr. Adeeb Jarrah

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Declaration of Original Work

I, Hanan Shaher Almarashdi, the undersigned, a graduate student at the United Arab Emirates University (UAEU), and the author of this dissertation entitled "*The Impact of the Proposed Mathematics Enrichment Program on the UAE Students' Mathematical Literacy in Light of the PISA Framework*", hereby, solemnly declare that this dissertation is my own original research work that has been done and prepared by me under the supervision Dr. Adeeb Jarrah, in the College of Education at UAEU. This work has not previously been presented or published, or formed the basis for the award of any academic degree, diploma or a similar title at this or any other university. Any materials borrowed from other sources (whether published or unpublished) and relied upon or included in my dissertation have been properly cited and acknowledged in accordance with appropriate academic conventions. I further declare that there is no potential conflict of interest with respect to the research, data collection, authorship, presentation and/or publication of this dissertation.

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Abstract

Mathematical literacy is an important skill that students must possess, as it helps students understand and use mathematics in the real world. The main aim of this dissertation is to investigate the impact of a proposed mathematical enrichment program developed based on the Programme for International Student Assessment (PISA) framework. This assessment is particularly important because it is on the National Agenda as the UAE aspires to be among the top 20 countries in the world in PISA by 2021. An explanatory mixed method design was adopted to achieve the purpose of this study. Subjects included were 102 grade 10 students taken from two schools. In the first phase, a nonequivalent pre–posttest quasi-experimental research method was conducted. Mathematical literacy test and motivation scale were used to collect data. There was a statistically significant difference between the experimental and control groups with an effect size above the mean as indicated by the covariate analysis. Female students recorded greater improvement than male students in all areas of mathematical literacy except for reasoning. However, there was no significant difference in motivation to learn mathematics between the experimental and control groups, nor between male and female students. The second phase included qualitative data collected by a perceptions survey. The students showed positive feelings towards their participation in the mathematical enrichment program as revealed by the thematic analysis. The study suggested that to improve students' mathematical literacy, schools should carefully implement the mathematics enrichment program. Findings from this study are expected to serve a larger goal of informing mathematics leaders in the UAE on how to improve the students' mathematical literacy. This study was one step up to fill in some of the gaps in the literature, and in the long term, much remains to be investigated and learned about mathematical literacy.

Keywords: Enrichment program, Mathematical Literacy, Problem-solving, Reasoning, Motivation, Contextual Teaching and Learning.

Title and Abstract (in Arabic)

أثر برنامج مقترح في الرياضيات لإثراء مهارات الرياضيات لطلاب دولة الإمارات العربية المتحدة في ضوء إطار برنامج تقييم الطلاب الدولي (PISA)

الملخص

تعد مهارات الرياضيات من أهم المهارات التي يجب أن يمتلكها الطلاب، لأنها تساعد الطلاب على فهم واستخدام الرياضيات في العالم الحقيقي. الهدف الرئيسي من هذه الرسالة هو البحث في تأثير برنامج إثرائي مقترح في الرياضيات والذي اعتمد في بناءه على البرنامج الدولي لتقييم الطلبة (PISA). هذا التقييم مهم بشكل خاص لأنه مدرج في الأجندة الوطنية حيث تطمح دولة الإمارات العربية المتحدة إلى أن تكون من بين أفضل 20 دولة في العالم في PISA بحلول عام 2021. وقد اعتمدت الدراسة على استخدام المنهج المختلط لتحقيق الغرض من هذه الدراسة. اشتملت عينة الدراسة على 102 طالبا من الصف العاشر من مدرستين. في المرحلة الأولى، تم إجراء طريقة بحث شبه تجريبية غير متكافئة قبل الاختبار البعدي. تم استخدام اختبار مهارات الرياضيات ومقياس الدافعية لجمع البيانات. كان هناك فرق ذو دلالة إحصائية بين المجموعتين التجريبية والضابطة بحجم تأثير أعلى من المتوسط كما يتضح من تحليل المتغير المشترك. سجلت الطالبات تحسناً أكبر من الطلاب الذكور في جميع مجالات مهارات الرياضيات باستثناء الاستدلال. ومع ذلك، لم يكن هناك فرق معنوي في الدافع لتعلم الرياضيات بين المجموعة التجريبية والضابطة، ولا بين الطلاب والطالبات. وتضمنت المرحلة الثانية بيانات نوعية تم جمعها من خلال مسح التصورات. أظهر الطلاب مشاعر إيجابية تجاه مشاركتهم في برنامج الإثراء الرياضي كما يتضح من التحليل الموضوعي. اقترحت الدراسة أنه لتحسين معرفة الطلاب بالرياضيات، يجب على المدارس تنفيذ برنامج الإثراء الرياضي بعناية. من المتوقع أن تخدم نتائج هذه الدراسة هدفًا أكبر يتمثل في إطلاع قادة الرياضيات في الإمارات العربية المتحدة على كيفية تحسين مهارات الرياضيات للطلبة. كانت هذه الدراسة خطوة واحدة لسد بعض الثغرات في الأدبيات، وعلى المدى الطويل، لا يزال هناك الكثير الذي يتعين التحقيق فيه وتعلمه حول مهارات الرياضيات.

مفاهيم البحث الرئيسية: برنامج إثرائي، مهارات الرياضيات، حل المشكلات، الاستدلال، الدافعية، التعليم والتعلم السياقي.

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Without you all, my dream would not have been achieved. Thank you.

Dedication

To my beloved parents, husband, and children
My family and friends

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List of Abbreviations

ANCOVA	Analysis of Covariance
CTL	Contextual Teaching and Learning
HOTS	Higher Order Thinking Skills
MEP	Mathematics Enrichment Program
ML	Mathematical Literacy
MLT	Mathematical Literacy Test
MSLQ	The Motivated Strategies for Learning Questionnaire
NCTM	National Council of Teachers of Mathematics
OECD	Organization for Economic Cooperation and Development
PISA	Programme for International Student Assessment
PBL	Problem Based Learning
RME	Realistic Mathematics Education
STEM	Science, Technology, Engineering and Mathematics
UAE	United Arab Emirates

Chapter 1: Introduction

1.1 Overview

How many students think they study mathematics that is rooted in everyday issues? What if students were able to learn about mathematics outside of the classroom and were confident to think independently, ask smart questions, and be motivated to change things for the better? What if students were involved in mathematical thinking where school mathematics could be used in real-life situations? The mathematics reforms hope to help students acquire lasting, useful, and meaningful knowledge. Reformers believe that the traditions of school mathematics are the reason why students have difficulty understanding mathematics and its use in their lives (Cobb, Perlwitz & Underwood-Gregg, 1998). Advocates of reform are pushing for classrooms that require the student to act like a mathematician because "knowing" in mathematics is "doing mathematics" (National Council of Teachers of Mathematics (NCTM), 1989, p. 7).

This chapter provides a comprehensive overview of this study. This study aims to contribute to the reform efforts in the United Arab Emirates (UAE) in prioritizing education by proposing an enrichment program aimed at improving students' Mathematical Literacy (ML). This reflects how well students use school mathematics in their lives and contribute to their readiness for future work effectively. Basically, this introductory chapter provides background information about the related aspects of mathematical literacy. The chapter intends to shed the light on the study's rationale by discussing the background of the study and the statement of the problem in addition to the purpose and significance of the study. Additionally, other technical issues like the definition of key terms, limitations, and delimitations of the study are also covered.

1.2 Background of the Study

"*It is our right to dream that our country will be one of the best countries in the world* " Sheikh Mohammed bin Rashid Al Maktoum, the Vice President and Prime Minister of the UAE and Ruler of the Emirate of Dubai.

The global economy will witness many important economic changes in the future. The National Agenda for the UAE Vision 2021 aims to place the UAE at its core as a major player at the international level. The government of the UAE has set a national agenda that aims to reach the ultimate and comprehensive goal of achieving the UAE's position as one of the best countries in the world by 2021, the year of the country's golden jubilee. The UAE National Agenda includes a set of national indicators in several sectors. These indicators are long-term and measure performance outcomes in each of the national priorities, and generally compare the UAE with global standards (The Official Portal of the UAE Government, 2020).

The UAE is an ambitious country that focuses on developing an equipped generation capable of facing life's challenges at the international level. In statements to him, His Highness Sheikh Mohammed bin Zayed Al Nahyan stressed the importance of developing education following comprehensive strategic plans to serve as a gateway to the future of the UAE and to enhance educational outcomes to keep pace with the comprehensive development in the country. He stated that "the UAE will continue to elevate the education system to the international benchmark to produce ambitious generations who are capable of writing new success stories in the nation's renaissance, launched since the founding of the Union by the late Shaikh Zayed bin Sultan Al Nahyan” (Zaman, 2017).

Education in the UAE is considered an essential element for the development of the nation and the best investment in its youth as it aims to invest in the knowledge economy instead of relying on oil and gas. Hussain bin Ibrahim Al Hammadi, UAE Minister of Education told participants at the Global Manufacturing and Industrialisation Summit, GMIS” that “A country cannot flourish if it does not have the right human capital, and education is the main pillar by which a society can move forward. The pace of transformation driven by the Fourth Industrial Revolution is so fast that future skilling is a must” (Khaleej Times, 2019). For this reason, the National Agenda for UAE Vision 2021 emphasizes the development of a first-rate education system, which will require a complete transformation of the current education system and teaching methods to better meet the country's future needs.

The National Agenda has set a goal of eight major educational goals, one of which is for UAE students to be among the best in the world in reading, mathematics, and science international tests. In the context of the continuous efforts to be one of the best countries in the field of education, students of the UAE have participated in some international studies aimed at helping governments regularly monitor the outcomes of education systems in terms of student performance. These tests include PISA (Programme for International Student Assessment), TIMSS (Trends in International Mathematics and Science Study), and PIRLS (Progress in International Reading Literacy Study). This allows the UAE to compare its students' achievements with those in other countries. In this way, this not only helps governments understand the practices of other countries but also learns from them to improve the effectiveness of their educational systems.

1.2.1 Mathematical Literacy (ML) in the UAE

Since 2008, the UAE has started its journey to participate in international tests to examine and benchmark the performance levels of its education system. It aims to achieve performance among the top 15 countries in TIMMS and the top 20 countries in PISA by 2021 as a goal of the UAE National Agenda (The Official Portal of the UAE Government, 2019).

PISA, the international assessment organized by the Organization for Economic Cooperation and Development (OECD) is of particular importance as it seeks to assess students' achievement in reading, mathematics, and scientific literacy. PISA is implemented every three years for students at the age of 15, thus approaching the end of compulsory education in most participating education systems. Moreover, the PISA assessment is not curricular oriented and goes beyond the school curriculum to measure the use of knowledge in daily tasks and challenges. For this reason, it measures young people's success in acquiring knowledge and their ability to use it in specific areas to meet real-life challenges. Although all three subjects are evaluated in each cycle, one major topic is assessed in depth, while the other two subjects are minor areas of that cycle. The results of the UAE in previous cycles of PISA, since the beginning of its journey in 2009, are as follows in Table 1 (Source: Ministry of Education, 2020; Sanderson, 2019). The highlighted parts of rows from Table 1 indicates the major subject of assessment in each cycle:

Table 1: The UAE Results in PISA in previous cycles

	PISA 2009		PISA 2012		PISA 2015		PISA 2018	
	Score	Rank	Score	Rank	Score	Rank	Score	Rank
Reading	431	42	442	44	434	47	432	46
Mathematics	421	41	434	48	427	47	435	50
Science	438	41	448	46	437	46	434	49

According to Sanderson (2019), PISA 2018 results show the UAE ranks highest in the Arab world on all three subjects. However, all Arab countries performed below the OECD average, including the UAE, in all three subjects. Internationally and more specifically in mathematics, the UAE dropped from 47th to 50th in mathematics out of nearly 80 countries in PISA 2018 despite an increase of its score by about 8 points from the 2015 cycle. “In mathematics, we are seeing the continuation of a positive trend, here the UAE is broadly on track of achieving its ambitious performance targets,” said Andreas Schleicher, director for education and skills at the OECD (Sanderson, 2019). However, this ranking achieved shows that the UAE still needs to do more to achieve its goal of being among the top 20 countries by 2021.

In PISA, there are six levels of proficiency with the lowest being Level 1 and the highest being Level 6. The OECD considers students at or above proficiency Level 2 as having the necessary skills to succeed in the knowledge economy. Looking deeply at the percentages of low achievers and top achievers in mathematics indicates that the general path of the UAE is "stable". The percentages of low achievers (below level 2) and high achievers (level 5 and 6) are represented in Table 2, from 2009 through 2018 (OECD, 2019a)

Table 2: Percentage of low achievers and high achievers in mathematics

PISA 2009		PISA 2012		PISA 2015		PISA 2018	
Below Level 2	Level 5 and 6	Below Level 2	Level 5 and 6	Below Level 2	Level 5 and 6	Below Level 2	Level and 6
51.3%	2.9%	46.3%	3.5%	48.7%	3.7%	45.5%	5.4%

The results of PISA 2018 indicate that about 54% of students in the UAE attained Level 2 or higher in mathematics (OECD average: 76%), compared to 98% in Beijing, Shanghai, Jiangsu, and Zhejiang (China), which means that about half (46%) of the UAE students perform below level 2. Those students could interpret and recognize how to represent mathematically a simple situation without direct instructions (OECD, 2019b).

Moreover, only 5% of the UAE students scored at Level 5 or level 6 in mathematics (OECD average: 11%) compared to 44% in Beijing, Shanghai, Jiangsu, and Zhejiang (China), and 37% in Singapore. Those students who are considered as gifted and most able students in any country can model mathematically complex situations, and they can apply problem-solving effectively and appropriately (OECD, 2019b).

Mathematical literacy measured in PISA indicates how ready the UAE students are for life by applying their knowledge and skills to real-life challenges. But student results in the UAE show a small percentage of outstanding performance in mathematical literacy compared to outstanding performance in other countries. Therefore, building students' mathematical literacy is critical to preparing them for their lives and contributing to nation-building as well.

Achieving the dream of making the UAE one of the best countries needs building a strong education especially in mathematics due to the increasing importance of the problems and situations we face in daily life. Thus, the emphasis of building students as problem solvers is a focal point in teaching mathematics as the students need to understand and apply what they have learned in schools to a variety of contexts in real life.

Heaton (2000), similar to what is needed to improve student performance reflected in PISA and other international assessments, described the current perspective of school mathematics as "dynamic, constructed, and reconstructed through an ongoing process of sense making by the learner" (p. 4).

1.2.2 High Achievers (and Gifted) in Mathematics

Farkas, Duffett and Loveless (2008) in their report about the teachers' views on how schools are serving high achievers, concluded that there's some confusion over the definition of equity in U.S. schools, particularly when it comes to high-achieving students. Their report revealed that sixty percent of teachers indicated that struggling students—not high achievers—are their top priority, while only 23 percent indicated that “academically advanced” students are a top priority. "The Standards propose that all students be guaranteed equal access to the same curricular topics; it does not suggest that all students should explore the content to the same depth or at the same level of formalism" (NCTM, 1989, p. 131).

Depending on the educational system in the UAE, those excelling in mathematics and other subjects mainly choose the advanced stream for their studies at the secondary level. Therefore, it stands to reason that these high-achieving and talented students would be primarily concentrated in these classes, even though not all students performed well. This means that extensive measures should be taken to raise the performance of these students.

Irtiqa'a, an inspection program launched in 2012 by the Abu Dhabi Department of Education (ADEK) to monitor and measure the quality of performance in schools, reported that critical thinking, mathematical thinking, and problem-solving were not sufficiently developed in many schools of cycle three, and they need guidance to apply real-world applications. Moreover, students do not always face a sufficient challenge to think critically for themselves. It is not always adequately adapted to meet the needs of high achiever students. In this context, Griffin (2012) revealed that while students at lower proficiency levels improve very quickly, students at the higher end of the scale hardly do better. These high-achieving students often unchallenged in the classroom, they are frequently asked to repeat what they have mastered before. This may lead these students not making adequate progress, which ultimately leads to poor academic achievement and stop learning due to boredom.

AlGhawi (2017) was the first researcher to investigate the implementation of gifted education programs in public schools in Dubai, UAE. AlGhawi pointed out that several gifted education programs have been monitored in schools. However, these were delivered to the entire class. No programs specifically designed to meet the unique needs of gifted students have been implemented, nor is there a special curriculum for gifted students. Although the UAE Ministry of Education (MoE) has recommended many types of gifted education programs, most teachers tend to rely solely on student registration in national and international competitions (AlGhawi, 2017).

1.3 Statement of the Problem

The UAE has invested heavily in educating its next generation as it seeks to reduce its dependence on oil and gas. In January 2017, Hussain Ibrahim Al Hammadi, UAE Minister of Education said: “We want to move from an economy based on oil to a new economy based on the human knowledge of both nationals and expatriates alike who will use knowledge as a tool to compete and move the country forward” (Zaatari, 2017). The UAE is making great strides towards education reform and developing the first educational system to achieve Vision 2021 and the indicators of the National Agenda to be among the best countries in the world. Educational reforms aim to ensure that all students can maximize their potential in their schools (Donerlson, 2008). Those students who resulted from this educational reform must use mathematics in their lives because helping students grow to be successful people outside of the classroom is just as important as teaching the curriculum.

Looking at the PISA 2018 results, UAE students showed generally low performance and ranked 50th in mathematics out of nearly 80 countries (OECD, 2019a). This indicates that students' ability to use mathematics to think about their lives, make plans for their future, and think about important problems and issues in their lives is insufficient. However, competing with global nations in mathematics means that students must be mathematically qualified, well-equipped, engaged, and reflective citizens of the 21st century.

Moreover, PISA 2018 results revealed that only about 5% of the UAE students can perform in the fifth and sixth levels (OECD, 2019b). These results indicated that the general path of the UAE is "stable" (OECD, 2019a). These alarming results require an educational intervention to increase the proportion of outperformers in the United Arab Emirates to perform at higher levels of mathematical literacy as they represent the primary strength of nation-building in many fields. Having said that, satisfying the needs of all learners is equally important for improving students' mathematical literacy. "The Standards propose that all students be guaranteed equal access to the same curricular topics; it does not suggest that all students should explore the content to the same depth or at the same level of formalism" (NCTM, 1989, p. 131).

NCTM (1995) suggested using the term promising in place of gifted, intentionally expanding the definition to include a much larger range of students and opening up the possibility of forming students with distinct mathematical aptitudes rather than simply identifying students with pre-existing mathematical experience and passion (Sheffield, 1999). Those who prefer this definition do not agree with other researchers (e.g. Gagné, 2004) who state “gifts" as a prerequisite for talent development. The Report of the Task Force on the Mathematically Promising recognizes that there are special issues relating to the education of the mathematically promising student (Sheffield, 1999) and has made recommendations that include the development of new curricular standards, programs, and materials that encourage and challenge the mathematically promising.

Thus, these high achieving students need challenging problem solving to activate their potential. When mathematical thinking (reasoning) and problem-solving are encouraged and enriched, it often leads to the development of cognitive processes usually associated with higher levels of chronological education (Piggott, 2004). Thus, providing students with excellent mathematics with an enrichment program to ensure promising students have the opportunity to engage in a high level of mathematical problem solving and reasoning can help improve their level of mathematical literacy.

Accordingly, this research aspires to meet these aforementioned calls and the growing need for high-quality programs and curricula in the field of mathematics teaching and learning for mathematical excellence for high achievers (and gifted students as well) by developing an enrichment program to improve mathematics knowledge.

1.4 Purpose of the Study

Creating an ideal, high-quality educational foundation is one of the main pillars of the UAE National Agenda (The Official Portal of the UAE Government, 2019), enhancing students' competencies and abilities in various literacy skills in reading, mathematics, and science. This requires creating educational environments that meet the needs of all learners, including the gifted and high achievers. Thus, the purpose of the current study had three objectives: First, to develop a proposed enrichment program in mathematics in the light of the PISA framework to improve students' mathematical literacy. Second, the main objective was to study the effectiveness of this program by testing it with tenth-grade students, as mathematical literacy is an indicator of students' readiness for the future and their ability to use what they learned effectively. Besides, it investigates the effect of the mathematics enrichment program on students' motivation. It also examines the emerging change, if any, that students' gender may have on their mathematical literacy and motivation to learn mathematics based on the application of the mathematics enrichment program. Finally, this study aims to explore 10th graders' perception of the mathematics enrichment program and how it can be improved.

By providing an enrichment program, this study aims to build a bridge between the school and the community by bringing real, relevant, and realistic contexts to the classroom to prepare learners to meet future requirements.

1.5 Research Questions and Hypotheses

To examine the effectiveness of the proposed Mathematics Enrichment Program (MEP), the researcher sought to identify the main and sub-questions that must be answered, and the hypotheses in this study as follows:

RQ1: What is the impact of the Mathematics Enrichment Program on the mathematical literacy of tenth grade students?

The first question was answered by comparing the performance of the experimental group students with the control group for both male and female students separately since they study in different learning environments, then comparing male and female students to identify any statistically significant differences after controlling the pretest scores by answering the following three sub-questions:

RQ1a: What is the impact of the Mathematics Enrichment Program on the mathematical literacy of tenth grade female students?

H0: There is no statistically significant difference between the experimental group and the control group on tenth grade female students' mathematical literacy.

RQ1b: What is the impact of the Mathematics Enrichment Program on the mathematical literacy of tenth grade male students?

H0: There is no statistically significant difference between the experimental group and the control group on tenth grade male students' mathematical literacy.

RQ1c: Are there any gender-based significant differences in mathematical literacy in response to the Mathematics Enrichment Program?

H0: There is no statistically significant influence of gender on students' mathematical literacy in response to the Mathematics Enrichment Program.

<u>RQ2: What is the impact of the Mathematics Enrichment Program on tenth grade students' motivation to learn mathematics?</u>

To answer this question, similar to the first question, three sub-questions are sought to be answered as follows:

RQ2a: What is the impact of the Mathematics Enrichment Program on tenth grade female students' motivation to learn mathematics?

H0: There is no statistically significant difference between the experimental group and the control group on tenth grade female students' motivation to learn mathematics.

RQ2b: What is the impact of the Mathematics Enrichment Program on tenth grade male students' motivation to learn mathematics?

H0: There is no statistically significant difference between the experimental group and the control group on tenth grade female students' motivation to learn mathematics.

RQ2c: Are there any gender-based significant differences in motivation to learn mathematics in response to the Mathematics Enrichment Program?

H0: There is no statistically significant influence of gender on students' motivation to learn mathematics in response to the Mathematics Enrichment Program.

<u>RQ3: What are the students' perceptions towards the Mathematics Enrichment Program after its implementation?</u>

It was sought to collect quantitative and qualitative data to learn about how the students perceive their experience of participating in the MEP then analyze it to answer this question.

1.6 Significance of the Study

PISA is an exceptional international comparative study because it focuses on applying skills, knowledge, and presenting problems in real-world contexts. Its purpose is to provide students with a measure of their degree of preparation for the future, not just their academic achievement. Mathematical literacy is important to be highly obtained by students as it is set as a goal of mathematics teaching for the first time at a broad level by the NCTM (Kaiser & Willander, 2005). The OECD has claimed that mathematical literacy refers to the ability to put mathematical knowledge and skills to functional use rather than merely mastering them in the school curriculum (OECD, 1999). As such, this enrichment program is expected to meet students' needs to improve their level of mathematical literacy to be successful not only in the classroom but to be functional citizens and make positive contributions to society outside the school environment.

The PISA was first conducted in 2000, where it rotates the focus of assessment among reading, mathematics, and scientific literacy in each cycle, one of which is the major domain and the other two are secondary domains. This study has a particular significance as PISA 2021 is assessing again the ML as the major domain as it was in 2003 and 2012. Although mathematics was assessed all PISA cycles that began in 2000, it was the main domain assessed only in the 2003 and 2012 cycles. The return of mathematics as the main domain in PISA 2021 is an important opportunity to continue comparing student performance. Moreover, this will highlight what to evaluate taking into account the changes that have occurred in the world to enlighten the educational field, educational policies, and practices in the UAE. The results of this study are expected to help the UAE to re-examine its vision of mathematical capability and shape its education to reach the goal of its National Agenda of being among the best 20 countries in the world in PISA 2021. Improving PISA scores requires a paradigm shift in teaching and assessment. The last-minute test preparation of the students is not going to work and has been proven to be ineffective over time. Consequently, the results of this study will provide the stakeholders with time to make the changes required to start early and focus on the skills needed not on the preparation for the mathematics PISA test.

Moreover, education systems are trying to prepare the students for success in a fast transforming, present and future, Innovation Age. Nowadays, Science, Technology, Engineering and Mathematics (STEM) has received a lot of attention worldwide as a critical element of the curriculum because STEM professions are perceived as a key driver of growth through innovation (OECD, 2014b). Mathematics fundamentally requires urgent attention because it is the foundation of STEM that is critical for the development of innovators.

The research findings from this study are expected to serve the larger goal of informing the education community on how educators can best design advanced instruction and programming appropriate to develop mathematical literacy for the high achiever students. In the enrichment program, students will use high order of thinking and integrate content from science, technology, literacy, and mathematics disciplines as appropriate to answer complex questions, and to develop solutions for challenges and real-world problems. Mathematically promising students need to be challenged in order not to turn off their talent.

Students who are gifted in mathematics are very important and considered one of the main resources with which they can be leaders in many fields. An investigation of the current state of mathematics education for the gifted, supported by research and experience, reveals the lack of a special mathematics curriculum. The opinions of mathematics teachers in the UAE were also negative regarding the effectiveness of the gifted programs, if any, in their schools (Jarrah & Almarashdi, 2019). Whereas, the general curriculum that is delivered to other students is not enough for the gifted because they need a deeper, wider, and faster curriculum (Johnson, 2000). Hence, this enrichment program can also be applied to students gifted in mathematics as they are kept in their regular classes.

1.7 Limitations of the Study

Although contributing to the literature in several ways, the current study is not without limitations. The current study focused only on public high schools because they apply the same curriculum and follow the same rules. Two high schools were included in this research; one for male students and the other for female students. Private schools are not included as they apply multiple curricula. This limitation of a particular research setting is restricted as this makes it difficult to replicate it elsewhere with different samples (Creswell, 2009).

The research focused on providing the mathematics enrichment program for two experimental groups consisted of 51 tenth-grade high achiever students in mathematics from the advanced stream (27 males and 24 females). The impact of this mathematics enrichment program was measured by comparing student achievement in the experimental groups with two control groups consisted of another 51 tenth-grade high achiever students in mathematics from the advanced stream (26 males and 25 females). This enrichment program relied on contextual problem-solving in problem-based learning (PBL) for two periods weekly with an independent study as needed.

Since the sample of students was not large enough, the results might not be generalized to all mathematically high achievers in the UAE. Moreover, the results are limited to this kind of enrichment.

1.8 Definition of Terms

Enrichment Programs: These are programs that provide richer content through strategies that supplement normal grade level work, for example, learning centers, providing extra work, field trips, Saturday programs, and independent study (Child, 2004: p. 268).

Mathematical Literacy (ML): “Mathematical literacy is an individual’s capacity to reason mathematically and to formulate, employ, and interpret mathematics to solve problems in a variety of real-world contexts. It includes concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to know the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective 21st century citizens” (OECD, 2018b, p. 7).

Programme for International Student Assessment (PISA): Is an international study that was launched by the OECD in 1997, first administered in 2000 and now covers over 80 countries. Every 3 years the PISA survey provides comparative data on 15-year-olds’ performance in reading, mathematics and science. (OECD, 2020, “PISA for Schools – faqs”, para. 2).

Problem Solving: “Problem-solving means engaging in a task for which the solution method is not known in advance. In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understandings” (NCTM, 2000, p. 52).

1.9 Organization of the Study

This research study is presented in five subsequent chapters. This chapter included the background of the problem, the statement of the problem, the purpose of the study, the research questions, the importance of the study, definitions of key terms, and the organization of the study. Chapter 2 is devoted to the theoretical framework of the study besides the reviewed related recent research and literature that of relevance to the subject of this research.

The research methodology and the design used in the study were outlined in Chapter 3. It also describes the procedures and instruments for conducting the study and provides information regarding the participants as well. Chapter 4 summarized the most significant findings of the study. Finally, Chapter 5 provides the discussion and interpretations of the results reached in Chapter four. It also includes the conclusions, recommendations for further research, and implications for practice.

Chapter 2: Literature Review

2.1 Introduction

This chapter summarizes the theories and studies published to give a comprehensive background to enrich the purpose of this study. It is extremely important to investigate the willingness of the UAE students to use mathematics in all parts of their daily life. This focus stems from the importance of mathematics and reflects the meaning of being mathematically competent. In this chapter, previous studies related to the current study were reviewed. This study aimed to improve students' mathematical literacy. Thus, this literature review began by shedding light on the goals of mathematics education, then introducing the concept of mathematical literacy and its components; Problem-solving and Mathematical reasoning. Then details about (MEP) were presented as a proposed solution to improve the students’ mathematical literacy. Finally, it includes a discussion of constructivism and the motivational theories that support this framework.

2.2 Goals of Mathematics Education

Mathematics is not only the language of science, it directly contributes to many essential domains like finance, business, and health. It opens career doors for students; it enables informed decisions for citizens; provides nations with the knowledge to compete in the technological community. The power of mathematics is needed to participate fully in the future (National Research Council, 1998). These statements were written before about 30 years; however, they are still true.

What is education for? What is the purpose of mathematics education? These questions are at the core of the educational process. Biesta (2009) claimed that one way to answer these questions is to think about the aims and ends of education starting from the actual functions educational systems perform. From Biesta's (2009; 2015) perspective of the goal of education, it could be classified into three different (but related) functions: qualification, socialization, and subjectivation (personal development). Qualification is a major function of education for any educational institution as it denotes the purpose of being qualified for the future (Biesta, 2015). Learners are provided with the knowledge, skills, and understanding and also with the dispositions that allow them to be qualified in "doing something". This qualification could be understood in a narrow sense (like being qualified in a certain task or job) or in a much wider sense like well preparing people to succeed in modern culture and complex society (Biesta, 2009). Thus, with no doubt, the qualification function of education has a critical role in preparing the workforce and contribute to economic development and growth. Moreover, the qualification function plays a more general role by providing the learners with the knowledge and skills needed to be well prepared to become responsible and reflexive citizens. The socialization function of education is about directing children and youth to traditions, cultures, and ways to do things. Although socialization is not the explicit goal of the educational process, it will still function implicitly as shown by research on the hidden curriculum (Biesta, 2009), while subjectification reflects the impact of education on the individualization and ways of being that hint at independence from others. This represents how children "come to exist as a person, as opposed to being an object" (Biesta, 2015, p.77). To make it clearer, Biesta (2015) referred to the previous three functions as the three aspects related to content, tradition, and person.

For mathematics education, this wide range of goals could be reflected as follows: qualifications for the use of mathematics in the study of next classes or university and jobs and more broadly how well could the students use their school math in their daily life; socialization also occurs unconsciously through the interaction culture between teacher and students that may result into liking mathematics and realizing its importance in becoming a responsible and reflective citizen who can understand and deal with the quantitative aspects of society (OECD, 2014a); and personal development of individuals' attitude towards using mathematics with confidence and independence in quantitative problems in daily life that may require mathematical knowledge and skills to solve them. When the teacher chooses to focus on mathematical goals and learning, they give the priority to the qualification of children. However, appropriate use of individuals' mathematical knowledge and skills is the common aspect of the three previous strands of goals. Therefore, mathematics education must deal with the complex relationship between mathematical knowledge and its applications in real-life situations (Golding, 2018).

The educational systems globally have been responded to the Industrial Age that was required to the 19^{th} century, and now it should put more emphasis on the depth of understanding the needs of the global society in the 21^{st} century. Issues that create personal and societal challenges in the twenty-first century, such as personal decisions and the global economy, that require mathematical thinking to understand its quantitative component. Nevertheless, Mahajan, Marciniak, Schmidt, and Fadel (2016) stated that mathematics remains the basis of economic growth through science, technology, and engineering as a basis for innovation and is critically essential to understanding the world and citizenship.

The importance of mathematics over several decades have been shifted toward new branches and topics (OECD, 2008). This indicates that what is relevant today is different than yesterday. For example, mathematics is an essential component of STEM learning. The STEM professions are a key driver of growth through innovation (OECD, 2014b). This has been clearly shown by the results of the OECD investigation about how much emphasis is put by the current systems on STEM from the students' total time of learning. The results showed that 30% of the countries put emphasis on STEM, and to be more specific, approximately 45% of the STEM total time was directed to mathematics that represented about 11% of the total teaching time (OECD, 2014a).

The literature of the goals of mathematics education reflects the increasing use of practical mathematical knowledge of quantitative problems in everyday life. It can be seen that it is now easier than before to visually represent a big collection of data or information using technology such as spreadsheets. This plays an essential role in political decision-making on economic and political issues as well. Gellert and Jablonka (2007) argue that "mathematisation of society" calls more than ever to equip students with the suitable mathematical knowledge and skills that allow them to participate effectively in this "mathematised world" and to understand the quantitative problems they face in their lives. Consequently, a focus on appropriate and relevant goals of mathematics education has been added to the political agendas of many countries.

Shepardson (1993) emphasized the importance of cognitive engagement in making effective classroom activities that can be linked to higher-order thinking skills (HOTS). According to this viewpoint, the most important criteria for promoting HOTS that the students should be involved in transforming knowledge and understanding. Teachers should create a communication environment for the effective interaction of students, and encourage them to verify, question, criticize and evaluate others' arguments, engage in knowledge building through various processes, and generate new knowledge and must realize that they have to be active learners who take initiatives and responsibilities of their learning.

The focus of mathematics education has been shifted to more emphasis on mathematical reasoning and problem solving and understanding of quantitative situations. To teach mathematics effectively, it is required to understand what the students know and need to know in order to challenge them and support them to learn it well (NCTM, 2000). Nowadays, technological tools help in performing mathematical calculations, however, mathematical reasoning and mathematical skills are beneficial for people in their daily life. There seems to be much international agreement on mathematics education to achieve goals that take into account the importance of preparing students to solve future problems in real-life situations. (OECD, 1999; Toner, 2011).

The UAE has made efforts in its educational reform to improve educational systems, including mathematics education in schools. For that reason, the UAE Vision 2021 National Agenda emphasizes the development of a first-rate education system. The goal is also to prepare students for college and careers. In this preparation, students need strong literacy skills in every discipline, including mathematics. This was clearly reflected in the strategic plan of the MoE in the UAE 2017-2021, where the mission statement stipulated the following: “Develop an innovative Education System for a knowledge and global competitive society, that includes all age groups to meet future labor market demand, by ensuring quality of the MoE outputs, and provision of best services for internal and external customers” (Ministry of Education, 2017).

Therefore, it is very important to participate in comparative international studies such as TIMSS and PISA as they provide a great deal of knowledge about how the UAE students perform in mathematics in the context of the institutions of the educational world. This comparison of our students' performance in relation to other countries helps the government policymakers to understand and learn how mathematics is taught by teachers and how it is learned and performed by students in different countries. National governments use the results of international assessments to guide educational policy, often under the slogan of "raising standards". Stigler, Gallimore, and Hiebert (2000), based on their experience as researchers conducting international studies, clarified the value of this kind of research, as an intercultural comparison is a powerful way to discover practices that are not observed everywhere and to reflect them on our own. This will give researchers and educators alternative ways to improve mathematics teaching (Stigler & Hiebert, 2004).

Both PISA and TIMSS have many overlapping features in their design and administration, however, there is a major difference in what the two international tests aim to measure (Olsen, 2005) and they reflect a different opinion about what school mathematics is or what it should be. The focus of TIMSS is on formal mathematical knowledge as most of its test items are either pure mathematical items without context or items with simple and artificial contexts that reflect the traditional mathematical problems in school, while PISA puts emphasis on the "mathematical literacy" as it tests the students in the type of applied mathematics required in their real lives as citizens of modern society (Grønmo & Olsen, 2006). It is assumed that the students have the necessary knowledge in pure mathematics to be able to find a correct mathematical solution then transfer it to the applied mathematics. This shows that applied mathematics however is more complex than pure mathematics (Grønmo & Olsen, 2006).

Problems related to the real-world context have an irregular structure such as the problems in PISA that is oriented towards students' HOTS in mathematics such as mathematical thinking (Ambarita, Asri, Agustina, Octavianty & Zulkards, 2018). Thomas and Thorne (2009) explained that "higher-level thinking" requires that we do something with the facts. We must understand them, infer from them, connect them to other facts and concepts, categorize them, manipulate them, put them together in new or novel ways, and apply them as we seek new solutions to new problems". Collins (2014) stated that HOTS cover three categories, including transfer (or the ability to apply what we learn and know), critical thinking (involving reflective thinking, reasoning, investigating, exploring viewpoints, comprehending, synthesizing, evaluating, comparing, and connecting) as well as problem-solving.

Stacey (2007) noted that, based on the framework used by PISA, mathematical literacy involves many components of mathematical thinking, including reasoning, modelling, and making connections between ideas. Thus, mathematics education in schools should aim to support the increasing integration of authentic mathematics (Golding, 2018). This can be found greatly in applications in everyday life or the workplace it can contribute to the curriculum. Therefore, today the world is calling for an increased link between mathematics and its applications, i.e. mathematical literacy. Moreover, it also provides compelling examples of mathematics in everyday life and the workplace. These examples can help build mathematics education programs to prepare students for the world (National Research Council, 1998).

2.3 What is Mathematical Literacy (ML)?

There is no clear definition of "Mathematical literacy" term, while there are many different concepts that refer to it. There are several international perspectives on ML too. ML is gaining more focus in the curriculum reform as it is measured in some international comparative assessments. This will be discussed in the following sub-sections.

2.3.1 Mathematical Literacy in Curriculum Reform

Since the early eighties, the conversation about being "mathematically literate" has started and gained more importance until today (Ekmekci & Carmona, 2014). Literacy is seen beyond just the ability to read and write, it also includes mathematics that is considered of equal importance in the definition of literacy (Jablonka, 2003; Moses & Cobb, 2001; Watson, 2002). Undoubtedly, the ability to use numbers and interpret quantitative information in today's society is an important component of literacy in addition to speaking, writing, and reading. The use of the term "literacy" may refer to a certain level as it does in other compound phrases, such as "statistical literacy" or "computer literacy" (Jablonka & Nice, 2014).

NCTM (1989) Standards talked about ML and being mathematically literate without giving an explicit definition. However, NCTM has set five broad goals seeking ML for all students: "(1) That they learn to value mathematics, (2) that they become confident with their ability to do mathematics, (3) that they become mathematical problem solvers, (4) that they learn to communicate mathematically, and (5) that they learn to reason mathematically" (NCTM, 1989, p. 5). Internationally, people who can apply mathematics in real-life situations and who can arrive at a solution can be considered mathematically literate (Jablonka, 2003).

The motivation for introducing ML arose from mathematics educators who opposed the idea of learning basic mathematics fundamentals by rote learning only because it ignores the importance of understanding mathematics and under what conditions it can apply and activate the acquired knowledge flexibly in new mathematical contexts and situations (Jablonka & Niss, 2014). The importance of mathematical literacy is emphasized by the NCTM document (1980) that was written in part in response to the so-called "back to basics" movement in the United States in the 1970s (Jablonka & Nice, 2014). The document focused on including those essentials for “meaningful, productive, immediate and future citizenship”, not just the need to make basic skills an essential part of every child's education. These “essentials” include among them problem solving, applying mathematics in everyday situations, alertness to the reasonableness of results, emphasizing the higher-order mental processes of logical reasoning, information processing, and decision making that should be considered basic to the application of mathematics (NCTM, 1980). For the first time at a very wide level, the notion of students attaining a high level of ML was claimed as a goal of teaching mathematics by NCTM at the end of the 1990s (Kaiser & Willander, 2005).

2.3.2 International Perspectives on Mathematical Literacy

Jablonka (2003) in her comprehensive study referred to various international views on ML and revealed that the views differed fundamentally based on the fundamental principles and values of stakeholders. This explains why it is difficult to refer to the distinct meaning of ML because “it varies according to the culture and context of the stakeholders who promote it" (Jablonka, 2003, p. 76). She outlines, through a literature review, five agendas underpinning ML concepts. These agendas are as follows: "developing human capital" (entailed in OECD-PISA), "cultural identity", "social change", "environmental awareness", and evaluating mathematical applications (Jablonka, 2003).

For this research, the focus will be on developing human capital. Jablonka (2003) views the human capital perspective as "a conception of ML in terms of the ability to analyze, reason and communicate ideas and results by posing and solving mathematical problems. This comprises a mathematisation and modelling perspective" (p. 80). According to this perspective, ML develops an individual's human capital by enhancing students 'mathematical skills, improving students' economic prospects, and thus making them better citizens (Jablonka, 2003).

Jablonka (2003) stated that the literature has revealed various concepts of ML related to certain relationships and factors such as the relationship "between mathematics and the surrounding culture and curriculum" (p. 80). In addition, another relationship exists between school mathematics and mathematics outside of school where ML is related to "an individual's ability to use mathematics that is supposed to be taught in school" (p. 97).

Although some concepts of ML are different, however, there are some similarities between them. The researchers referred to ML as Realistic Mathematics Education (RME) (Hope, 2007), Mathematisation (Freudenthal, 1991; Hope, 2007), mathematical modeling (Gellert , Jablonka & Keitel, 2001), as well as mathematics in action (McCrone & Dossey, 2007; Skovsmose, 2007). Realistic Mathematics Education (RME) "uses a theoretical framework that relies on real-world applications and modelling, a didactical belief propagated by Hans Freudenthal" (Gates & Vistro-Yu, 2003, p. 67). The theory of RME relies on five components that use a real-world context (Hope, 2007).

The above concepts are formal and involve high-level mathematical skills. However, on the other hand, researchers who advocate 'mathematics in action' believe that ML is a fundamental requirement for all people as it is an important part of their daily lives (McCrone & Dossey, 2007; Skovsmose, 2007). They argue that ML is not about "studying higher levels of formal mathematics, but about making mathematics relevant and empowering for everyone" (McCrone & Dossey, 2007, p. 32).

Taking into account the above discussion of the different concepts, skills alone are not sufficient to describe ML appropriately as it involves mathematical problems in situations that require features such as a conceptual understanding of mathematics in addition to problem-solving skills (Gellert et al., 2001). Moreover, it is believed that the differences between different concepts of ML "consist of the problems to which mathematics is applied" (p. 61). Mathematical literacy has a role in narrowing the gap between abstract and applied mathematics, as it differs in terms of complexity and context. Therefore, it is important to allow all students to apply what they have learned in abstract mathematics to situations in their daily lives.

2.3.3 International Studies about Mathematical Literacy

Several international studies measure students' knowledge and skills in mathematics. Many countries participate in international studies such as PISA and TIMSS because they provide policymakers with information about the education system and the country's relative ranking among other participating countries.

PISA is applied periodically every three years to assess reading, mathematical and scientific literacy of 15-year-olds in the participating countries and it aims to measure students' ability to use their knowledge and skills in solving real-life problems in different contexts. The focus of PISA is to represent the "yield" of learning at age 15, rather than the abstract knowledge in the acquired curriculum. (National Center for Education Statistics, 2008b), while TIMSS measures the mathematics and science performance of fourth and eighth graders of the participating countries periodically every four years. TIMSS's main goal is to “measure the mathematics and science knowledge and skills broadly aligned with curricula of the participating countries” (National Center for Education Statistics, 2008a, p. 5).

TIMSS measures achievement based on the common mathematical content as elaborated with specific objectives of the curriculum, while PISA intended to measure the ML level of 15 years old students; how well they can use and apply their school mathematics within real-world contexts (Grønmo & Olsen, 2006). In other words, the two studies differ in their focus as TIMSS “seeks to find out how well students have mastered curriculum-based scientific and mathematical knowledge and skills”, while the purpose of PISA is to “assess students’ scientific and mathematical literacy, that is, their ability to apply scientific and mathematical concepts and thinking skills to every day, non-school situations” (Nohara, 2001, p. 11). In this case, Mathematical literacy goes beyond the types of situations and problems usually encountered in the classroom to place greater emphasis on real-world problems. In addition, what counts in this perspective are “not the situations themselves, which are of interest, but only their mathematical descriptions” (Jablonka, 2003, p. 81).

OECD (1999) claims that ML implies the ability of functional use of mathematical knowledge and skills rather than just mastering them as a school curriculum. Consequently, the students need to possess a high level of ML to be successful as functional citizens that have a positive contribution to society not only successful inside classrooms. In this sense, ML goes beyond curricular mathematics. ML assessment is inseparable from current curricula and teaching methods because students' knowledge and skills depend, to a large extent, on what and how they learned them in school and how that learning was assessed (OECD, 2017).

The assessment literature usually refers to non-specific terms, such as "real world", "everyday life", "personal life" and "community" to indicate the contexts in which students are expected to participate. Jablonka and Niss (2014) indicate that efforts to classify contexts often lack a theoretical basis and that determining the knowledge basis of mathematically literate behavior remains in need of further research.

2.3.4 Mathematical Literacy Definition

There is a problem facing efforts to define “mathematical literacy” as it cannot be described exclusively in terms of mathematical knowledge because it is related to an individual's ability to use and apply that knowledge. Thus, it must be seen as functionally as applicable to situations in which this knowledge will be used (Jablonka, 2003). Much of the literature has not referred directly to ML but has dealt with related concepts because its focus is on topics such as the goal of mathematics education, and the role of mathematical knowledge in many areas (Jabolnka, 2003). Mathematics education literature deals with a range of related concepts such as mathematical literacy (Jablonka, 2003, 2015; Jablonka & Niss, 2014), numeracy (Rosa & Orey, 2015; Tout & Gal, 2015), critical mathematical numeracy (Frankenstein, 2010), quantitative literacy (Steen, 2001), mathemacy and statistical literacy (Jablonka & Niss, 2014; Rosa & Orey, 2015). There is a lot of research that uses the term “numeracy”, “quantitative literacy”, and “mathematical literacy” synonymously, while others distinguish between them. The terms “mathematical literacy” and “quantitative literacy” appear to be of American origin, while the term “numeracy” was created in the United Kingdom by Cockcroft (Jablonka & Niss, 2014).

Numeracy definitions usually include 'number sense' and 'symbol sense', which emphasized the mediating role between the symbolic, in both natures as numerical or algebraic, representations, and their interpretations. The numerical sense refers to the informal aspects of quantitative reasoning (McIntosh, Reys & Reys, 1992), while the symbol sense includes a sense of comfort when using and interpreting algebraic expressions (Arcavi, 1994). The term "quantitative literacy" is preferred by Steen (2001) as it emphasizes the standing of numeracy meaning in a society that continues to increase using the numbers and quantitative information, while Jablonka selects the term mathematical literacy “to focus attention on its connection to mathematics and to being literate, in other words to a mathematically educated and well-informed individual” (Jablonka, 2003, p. 77).

Although some researchers may interpret these notions differently, they all emphasize the recognition of the usefulness and ability to use mathematics in many aspects of life as an important goal of mathematics education (Jablonka & Niss, 2014). What all have in common is the individual's attempt to understand and understand real-life situations, thus using some forms of thinking, knowledge, and mathematical skills.

The focus was placed differently when defining ML: knowledge and skills were described by Ojose (2011), while on the other side Kilpatrick, Swafford, and Findell (2001) focus on proficiencies or competencies. Moreover, ML is seen as a context related to real-life situations (Steen, 2001). Internationally, it refers to competence (Christiansen, 2007). Gellert et al. (2001) and Jablonka (2003) perceive ML in terms of higher-order mathematical skills. Despite the diverse approaches that different mathematics educators follow, there is a consensus that there are many dimensions or competencies that create ML.

According to Jablonka (2003), the first attempt to define mathematical literacy within the initial OECD framework for PISA was very broad and demanding and reflects the human capital perspective as follows:

> "Mathematical literacy is the capacity to identify, to understand and to engage in mathematics and make well-founded judgements about the role that mathematics plays, as needed for an individual's current and future life, occupational life, social life with peers and relatives, and life as a constructive, concerned and reflective citizen." (OECD, 1999, p. 50).

Three roots could be tracked back in this definition: a tradition of pragmatic education (Bybee, 1997), Freudenthal’s conception of the term “the world” where the individual lives that “mathematical concepts, structures, and ideas have been invented as tools to organise the phenomena of the physical, social and mental world” (Freudenthal, 1983), and mathematics competencies (Niss, 2003). From there the PISA framework was developed so that PISA aims to test students' ability “to put their mathematical knowledge to functional use in a multitude of different situations” (OECD, 2003). However, with the same objective, this definition was revised several times in PISA 2012 and very recently for PISA 2021 framework as will be explained in the next section on the ML framework.

Kilpatrick, Swafford and Findell (2001) stated that the idea of "mathematical competence" refers to the meaning of successful learning of mathematics and defines it indirectly based on five strands: “conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition”. Then the notion of “mathematical competence” was the focus of the Danish KOM project about “competencies and mathematics learning” in Danish and elsewhere (Niss & Højgaard, 2011). Competencies in ML include problem-solving abilities, reasoning, connections, communication, and mathematical representation (OECD, 2017). ML is also seen to have four prior components to solve a problem that are: exploring, connecting, and reasoning as well as using diverse mathematical methods (Stacey & Turner, 2015, p. 12).

According to OECD (2009) when using the term "literacy", the focus of the PISA is on the total of the mathematical knowledge that a 15-year-old can use functionally in many contexts. Problems often require reflexive methods that include insight and some creativity. Thus, mathematics literacy is related to the mathematics function learned in school. Based on this definition, PISA aims to assess functional mathematical knowledge and skills in terms of students' ability to interpret a mathematical problem, analyze it, think about the process, solve it, and communicate the solution effectively. In other words, by assessing the ML level in PISA, the focus is on students' ability to use what they learn in situations they are expected to encounter in their daily lives rather than being limited to the content of the curriculum they have learned (OECD, 2003).

The different perspectives on ML clearly illustrate how different concepts differ in the degree of complexity concerning the required mathematical knowledge and skills as in some concepts requiring advanced mathematical knowledge, expert, and higher cognitive skills. However, the PISA definition and evaluation criteria are the best descriptions of the requirements for this study as PISA focus is on what extent the students could use their acquired knowledge and skills when they are faced with situations and challenges related to their skills. PISA adopts a "Real-life literacy" perspective rather than a curriculum-driven one (OECD, 2018b). More information is provided in the next sub-section about the framework of ML in PISA as it is relevant to this study.

2.3.5 Mathematical Literacy Framework

In PISA 2012, the major subject was mathematics literacy with a major change in its first framework. A new focus on the problem-solving processes where the OECD (2013) incorporated the mathematical modeling processes "formulate, employ, and interpret" by Lesh and Fennewald (2013), as follows:

> An individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged, and reflective citizens. (OECD, 2013, p. 25)

According to this definition, students are seen as actively problem solvers of real-world challenges and problems. Students are not only required to reproduce knowledge of mathematics but also to apply it to different contextual situations in real life. PISA 2012 mathematical literacy assessment framework consists of three components: "(1) real-world contexts; (2) mathematical contents; and (3) mathematical processes" (OECD, 2013, p. 26). The mathematical literacy model that illustrates the key constructs of this framework and shows how they rely on each other (OECD, 2013, p. 26) is shown in Figure 1.

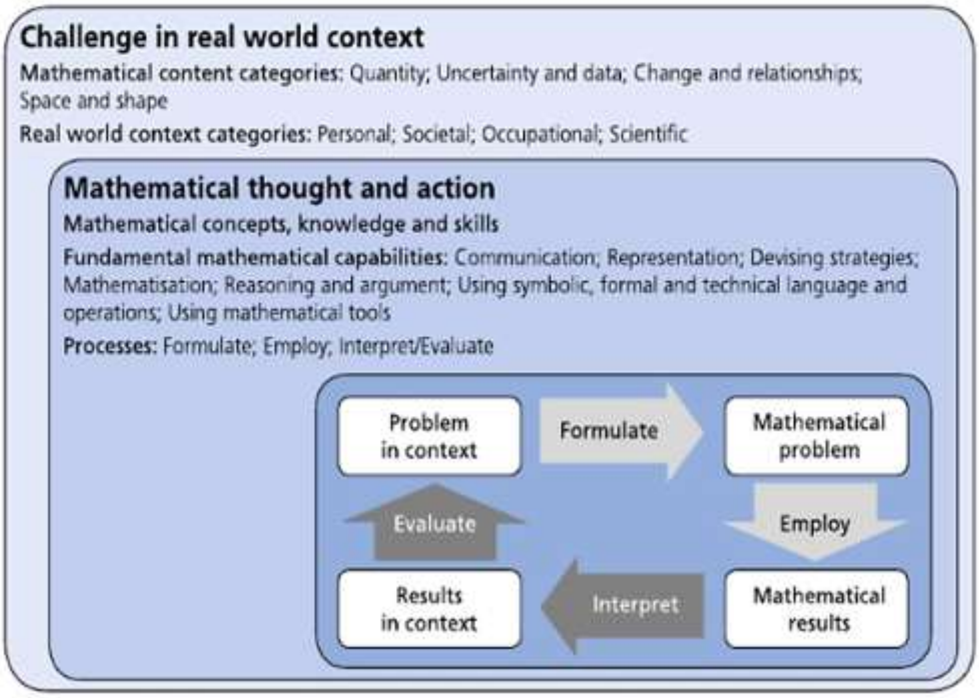

Figure 1: The PISA 2012 mathematical literacy framework

In Figure 1, the outer box depicts the mathematical content categories in addition as well as the real-world context categories. The middlebox represents the mathematical thoughts needed to solve these challenges, such as mathematical concepts, knowledge, and skills besides eight fundamental mathematical capabilities and the three mathematical processes. Finally, the inner box shows how these three processes are used to find a solution to the problem.

Real-world challenge context problems presented in the outer box can be categorized concerning their context or their content of mathematics. The context of problems can be classified into four categories which can be of a personal nature related to the challenges that an individual may face; a societal context that focuses on community whether local, national, or global, in which an individual lives; an occupational context centered around work situations; a scientific context related to how mathematics is applied in the world (OECD, 2013).

In addition to the contextual nature of the problem, it can also be characterized by the nature of the mathematical phenomenon which is based on four mathematical content categories called “overarching ideas” (OECD, 2013). This, to some extent, differs from the content approach that might be familiar from the perspective of mathematics education and school curriculum. However, the overarching ideas together generally include a set of mathematical topics that students are likely to learn (OECD, 2003). Mathematical content categories are (OECD, 2013, pp. 33-35): “change and relationship” where the students can model change and relationships with the suitable functions and equations; “Space and shape” in which students understand perspective, create and read maps, and manipulate 3D objects; “Quantity” in which 15-year-olds can understand multiple representations of numbers, participate in mental arithmetic, use estimation, and assess the reasonableness of results; “Uncertainty and data” where students use probability and statistics and other techniques of data representation and description to mathematically describe, model, and interpret uncertainty.

Students need mathematical thinking to be applied to the challenge to solve contextual problems. The framework of ML characterized in three different ways as shown in the middlebox. First, students need to build on many mathematical concepts, knowledge, and skills when they trying to solve a challenge. Second, the individual relies on this mathematical knowledge that is distinguished in the framework based on seven basic mathematical competencies such as representing and communicating mathematics and so forth. Third, as the student works on the problem, through the processes of problem-solving, the fundamental capabilities of the students are activated sequentially and simultaneously to create a solution drawing on mathematical content from appropriate topics.

The mathematical modelling cycle that is represented in the inner-most box of Figure 1 denotes the stages that the problem solver goes through when demonstrating ML that takes place with the problem in context. Working on a problem might require problem formulation, employing mathematical concepts or procedures, or interpreting and evaluating a mathematical solution. These three processes are important for both ML and the modeling course that builds on basic mathematical abilities that also build on an individual's mathematical knowledge about specific content. (OECD, 2013, 2017). Mathematical process categories are (OECD, 2013, pp. 28-30):

• Formulate: It refers to the ability of individuals to recognize and identify chances of using mathematics to provide mathematical structure to solve a problem presented in some contextualized form.

• Employ: It refers to the students' ability to apply "mathematical concepts, facts, procedures, and reasoning" to solve mathematically the formulated problems and get mathematical decisions.

• Interpret: It refers to the ability of individuals to "interpret, apply, and evaluate mathematical results". This includes translating mathematical solutions or thinking back in the context of the problem to assess plausible outcomes and understand the context of the problem.

The definition of ML continued to have the same focus with slight changes each cycle until its definition for PISA 2021 where the major subject will be ML, that defined as follows by (OECD, 2018a):

> Mathematical literacy is an individual's capacity to reason mathematically and to formulate, employ, and interpret mathematics to solve problems in a variety of real-world contexts. It includes concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to know the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective 21st century citizens. (OECD, 2018a, p. 8)

OECD (2018a, p. 10) presented an overview of the major constructs of this framework and indicates the relationship among these constructs shown in Figure 2.

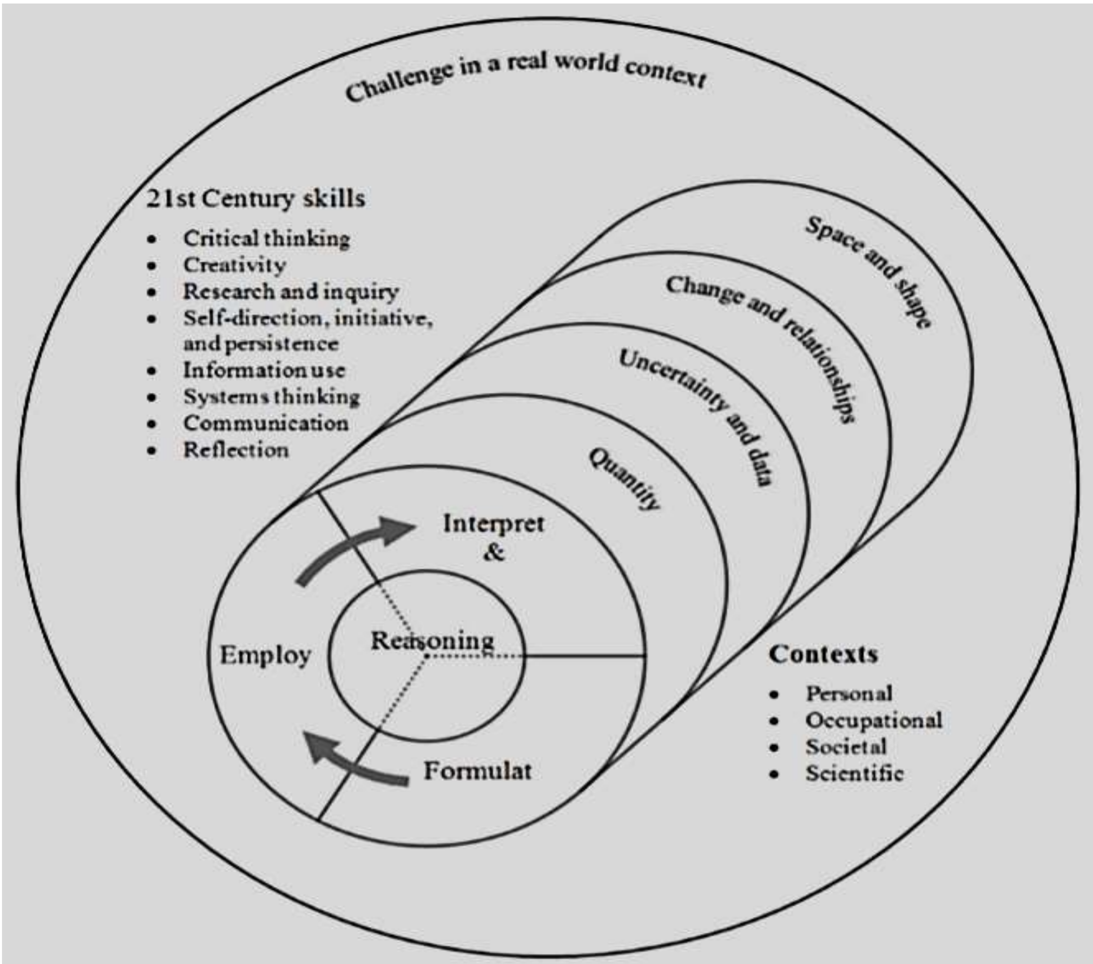

Figure 2: Mathematical literacy framework for PISA 2021

This definition clarifies, for assessment considerations, that mathematical literacy occurs in real-world contexts. Additionally, ML “assists individuals to know the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective 21st century citizens” (OECD, 2018a).

It is worth noting that the definition of ML focuses on mathematical thinking as well as on the use of mathematics to solve real problems (OECD, 2018a). Mathematical reasoning has been placed at the center of both the problem-solving cycle and ML as a new addition to the framework of PISA 2021. Therefore, ML consist of two main parts that cannot be separated, namely mathematical reasoning and problem-solving (OECD, 2018a, 2018b), while ML plays a vital role represented in the ability to use mathematics to solve real-life problems, mathematical reasoning goes beyond problem-solving in its traditional sense to include making judgments about societal problems that can be solved using mathematics. The assessment focus has been shifted, the trend is to move away from performing basic calculations to use new technologies because of the fast change of the world (OECD, 2018a, 2018b).

Jablonka (2003) stated that “ML is connected to learning how to think, but not to learning what to think about”. The primary implications of an emphasis on ML for mathematics teachers are clear. Mathematics must be logical for students to understand, and it must be based on their experiences, and any math education must be based on their previous experiences. (O’Shea, 2009).

2.3.6 Summary

The literature reveals different conceptions of ML. However, the most common descriptions were realistic mathematics education (Hope, 2007); mathematising (Gellert et al., 2001; Hope, 2007); and mathematics in action (McCrone & Dossey, 2007). Some researchers emphasize the formal application of mathematics to real-world contexts requiring a high level of mathematical knowledge and the competence to use and apply it (Gellert et al., 2001; Hope, 2007; Jablonka, 2003), while other researchers perceive ML as some basic level of literacy that is needed for all people to empower them to make well-informed decisions in their daily lives (McCrone & Dossey, 2007; Skovsmose, 2007).

All the above mentioned, make it a hard task to define ML. However, the PISA definition is the best fit with the purpose of this study as it aims to measure how well the students can use their mathematical knowledge in real-world contexts that they face in their lives. So, it can be concluded that mathematical literacy is one's ability to formulate, use, and interpret mathematics in situations or contexts of life. ML involves bridging the gap between abstract and applied mathematics as contexts and complexity degree vary. In the following two sections, problem-solving, and reasoning skills will be discussed as ML consists of these two parts.

2.4 Mathematical Problem Solving

In this section, the importance of mathematical problem solving will be discussed, in addition to the definition of the problem itself, its types, and the difficulties of using it in mathematics classrooms. Then the definition of mathematical problem solving will be discussed to lead to problem-solving mechanisms including the modeling process necessary for mathematical literacy as defined by the OECD and this research.

2.4.1 Importance of Mathematical Problem Solving

Problem-solving has gained a high position as a primary goal of mathematics learning over the past decades (Schoenfeld, 2014). It plays an essential role in mathematics education for K-12 students. Mathematics educators have designed problem-solving tasks for teaching and assessment purposes worldwide and specifically in the field of numeracy and mathematical literacy (Geiger, Goos & Forgasz, 2015). The National Council of Teachers of Mathematics (NCTM, 1980) stated that problem-solving should be at the core of mathematics teaching because it includes skills and functions as an important part of everyday life. Moreover, it can help people in their careers and other aspects of their lives to adapt changes and unexpected problems. Moreover, NCTM (1989) endorsed this recommendation by stating that problem-solving should be included in all aspects of mathematics teaching to give students the chance to experience the power of mathematics in the world around them.

The focus of mathematics education has been shifted from focusing on the procedures of mathematics to the process of problem-solving and creating problem solvers. Pólya (1980) stated that “If education fails to contribute to the development of the intelligence, it is obviously incomplete. Yet intelligence is essentially the ability to solve problems: everyday problems, personal problems...” (Pólya, 1980, p. 1). Additionally, recent definitions of intelligence discussed the practical intelligence that enables “the individual to resolve genuine problems or difficulties that he or she encounters” (Gardner, 1985, p. 60).

The Common Core State Standards Initiative (CCSSI) (2010) identified eight mathematical practices that were written to focus on the practices through solving problems. The first mathematical principle “Make sense of problems and persevere in solving them” focuses on problem-solving—understanding problems and persevere in solving them. This principle aligns with each of the five NCTM Process Standards that characterize "doing" mathematics. These practices include “problem solving, communication, reasoning and proof, representation, and connections” (NCTM, 2000).

Although problem-solving is just one of the Process Standards in NCTM (2000), it is very fundamental to learning mathematics by understanding. These Standards were the reason behind the Math Wars in the USA in the 1990s because of the different views of what is mathematics (Niss, Bruder, Planas, Turner & Villa-Ochoa, 2016). In addition, Cobb et al. (1991) suggested that engaging in problem-solving is not just to solve the problem itself, but to “encourage the interiorization and reorganization of the involved schemes as a result of the activity” (p. 187). This is because “Knowing” and “being able to do” are two different things.

When the students are allowed to use and build on their knowledge while solving problems, then they can reinforce and add to their previous knowledge and develop new mathematical understanding as well. Carpenter and colleagues (1999) stated that this is true even for young children as “children may actually understand the concepts that we are trying to teach but be unable to make sense of specific procedures that we are asking them to use” (Carpenter et al., 1999, p. xiv). The children can solve problems depending on their informal mathematics knowledge without direct teaching.

Problem-solving will not only result in developing the students' confidence in their ability to think mathematically (Schifter & Fosnot, 1993), but it is a "vehicle" for students to construct, evaluate and reflect on their theories about mathematics (NCTM, 1989). However, problem-solving is more than a vehicle that is used to teach and emphasize mathematical knowledge aiming to meet everyday challenges, it can provide the students with a context for learning mathematical knowledge. Problem-solving is seen as a very important skill for promoting logical thinking because individuals need more than knowing the rules to follow to get just the right answer, they need the ability to make decisions along the process. Therefore, problem-solving can be developed as a valuable way of thinking and can help people to transfer into new work environments with the skill to face several career changes during a working lifetime and everyday challenges (NCTM, 1989).

2.4.2 Definition of the Problem

There are many definitions of the "problem" and educators are far from agreement on its meaning. Additionally, some researchers like Schoenfeld (1992) believes that "problem" has several meanings that sometimes are contradictory. Mathematics educators have several terms to refer to the mathematical problems such as routine, non-routine; single-step, multi-step, and real-world; textbook, and non-textbook. Many scholars have focused more on the "problem", such as Shulman (1985) who called it his "favorite epigram." In some cases, it is clear what is meant by some of these terms; in other cases, it is not (Hoosain, 2004).

The problem situation is defined by Kantowski (1977) as "An individual is faced with a problem when he encounters a question he cannot answer or a situation he is unable to resolve using the knowledge immediately available to him. He must then think of a way to use the information at his disposal to arrive at the goal, the solution of the problem" (p. 163). In the same vein, McLeod (1988) defined the problem as the task that its goal or solution cannot be reached immediately and there is no clear algorithm to use to solve the problem. This distinguishes between a problem and an exercise. In an exercise, the solution algorithm is previously known while in the problem case the solution algorithm is unavailable.

Blum and Niss (1991) see the problem situation as to “challenge somebody intellectually who is not in immediate possession of direct methods/procedures/algorithms, etc. sufficient to answer the question" (p. 37). Later, Holth (2008) also defined the problem as a task that an individual does not know (immediately) what to do to get the solution. The common elements among the previous definitions that the algorithm to solve the problem are unknown and the problem solver must design a method to solve it. Having said this, regarding the framing of this research, the mathematical problem presents a goal without an immediate or clear solution (Pólya, 1981; Blum & Niss, 1991; Holth, 2008).

Depending on this definition, a key characteristic of the mathematical problem does not lie within the problem itself, since the problem is seen as relative to the individual involved. The complexity of the problem is a function of knowledge, experience, and dispositions of the problem solver, i.e. what is perceived as a problem for a person can be considered an exercise for another (Kilpatrick, 1985; Schoenfeld, 1985; Blum & Niss, 1991).

2.4.3 Types of Problems

The foremost form of representing the problem situations is the "word problems" (Verschaffel, Greer & De Corte, 2000; Verschaffel, Greer, Van Dooren & Mukhopadhyay, 2009). The word problems defined by Verschaffel, Depaepe, and Van Dooren (2014) as: "verbal descriptions of problem situations wherein one or more questions have raised the answer to which can be obtained by the application of mathematical operations to numerical data available in the problem statement" (p. 641).

Many researchers identified different categories of problems. For example, Pólya the father of problem-solving, identified two categories of problems: "problems to find", where the principal parts are unknown; and "problems to prove" which consists of a hypothesis and a conclusion (Pólya, 1985). Another classification of problem category is made by Blum and Niss (1991), they identified two categories of mathematical problems: the applied mathematical problems that refer to questions about situations real world and outside of mathematics; and pure mathematical problems which are rooted entirely in mathematics.

Niss et al. (2007) also classified types of problems by stating that mathematical problems consist of three types: modeling problems, word problems, and intra-mathematical problems where the degree to which they relate to real life is the difference between these types. This classification is very close to the three categories of mathematical problems in Schukajlow et al. (2012). They call the first category "intra-mathematical problems" which are direct problems with no connection to the real world. The second category is the "dressed-up problems" where the mathematical topic is camouflaged in a real-world contextual problem. The third is called "modeling problems" that "transfer processes between reality and mathematics" (p. 220). The focus of this research is on contextual mathematical problems that lend themselves to modeling problems type.

Contextual mathematical problems with an emphasis on problem-solving play an essential role in mathematics education. The concept of "word problems", in mathematics education in the twentieth century, became the title of this type of contextual mathematical problem (Hoogland, Pepin, De Koning, Bakker & Gravemeijer, 2018). Simply put, a contextual mathematical problem, the focus of this research, has two components: a representation of a real problem situation and a question. In these contextual mathematical problems, the problem situation is described from real life, while thinking, knowledge, and mathematical tools must be activated to answer the question posed.

Developing the skills that allow the student to solve the problem is more motivating than teaching the skills without a context. This notion emphasizes the importance of engaging students in problem-solving and not just giving them exercises to do. This places problem solving particularly important as a "vehicle " to learn new concepts and skills or to enhance previously acquired skills (NCTM, 1989). This will allow students to think about the problem in different ways, use strategies that make sense for them, and ultimately develop a deep mathematical understanding.

2.4.4 Difficulties about Contextual Problems

Word problems, including mathematical contextual problems, encounter many difficulties when used in the classroom of mathematics (Dewolf, Van Dooren, Ev Cimen & Verschaffel, 2014; Gellert & Jablonka, 2009). Students develop an "answer-getting” mindset as they continue to think of word problems as procedural exercise, selecting numbers from the text, and without understanding the problem or thinking about the authentic constraints, they just perform an operation on them (Depaepe, De Corte & Verschaffel, 2010). Sensemaking is an essential component in problem-solving processes, the answer-getting mindset called “suspension of sense-making” is considered a very important challenge in mathematics education (Schoenfeld, 1991, 1992; Verschaffel et al., 2000).

Teaching problem solving is a challenge that can cause difficulties for a mathematics teacher. Many mathematics curricula stated that the aim of integrating problem-solving into the curriculum is to improve higher-order skills and cognitive flexibility. However, much of the instructions focus on basic mathematical skills that only focus on developing automatic skills by assigning more problems is based on computation skills every day (Schoenfeld, 2004). Most teachers show students the procedure for doing mathematics without giving them the chance to get the students to understand something new on their own (Burns & Lash, 1988).

Van Dooren, Verschaffel, Greer and De Bock (2006) stress that it is important to pose problems to the student and to listen to the way the students explain how they are going to solve these problems. However, most of the teachers prefer safety as they stuck with providing just the workbook examples, computation skill worksheets, and drill during the instruction because they themselves are the product of such a similar approach (Fosnot, 1989; Wilburne, 2006). Educators should encourage students to gain an understanding of mathematical operations rather than just performing them.

Non-routine mathematical problems, which do not have a clear solution, should be incorporated into the mathematics content courses to provide students with rich and meaningful mathematical experiences. This type of problems trigger students' interest and encourage them to be engaged in the problem-solving process by applying different strategies. It also requires students to think logically, enhance their conceptual understanding, and to develop problem-solving approaches that can be used to solve other problems (Wilburne, 2006).

2.4.5 Definition of Mathematical Problem Solving

A clear distinction must be made between general problem solving and mathematical problem-solving. General problem solving has been defined by OECD (2013) as a cognitive process that transforms a certain situation into a goal situation if a clear solution is not present (Robertson, 2001). The assessment of general problem-solving skills by PISA differ from assessing tasks in areas of reading, mathematics, and science in that general problem-solving skills avoid relying on knowledge with specific knowledge as much as possible but rely on the cognitive process and information in a novel situation to solve the problem (OECD, 2013). However, several researchers believe that problem-solving skill is a domain-specific by nature where students need to retrieve schemas from their organized number of schemas stored in their long-term memory to use it in a novel situation (Sweller, Clark & Kirschner, 2010).

Although PISA (e.g. OECD, 2018a) considered problem-solving as an essential component of students' learning, the agreement on the importance of problem-solving does not say much about what the term might mean. Literature indicates that the dominant perceptions of problem-solving are seen as a process, as a curriculum, and as an instructional method (Xenofontos & Andrews, 2014).

Österman and Bråting (2019) pointed out that mathematical problem solving is not only a topic but a process underlying the entire mathematics that motivates the contextual learning of mathematical concepts and skills. Many researchers who view problem-solving as a process, have established frameworks that describe the components of the process that occur when an individual engages in solving a mathematical problem. Among these examples was the famous Pólya Four-Stage Model (1945) which provided the basis for other developed models. Later models emphasized the solver "needs to transform the situation or find new perspectives on it so that her/his mathematical knowledge can be applied to it" (Nunokawa, 2005, p. 327). This requires using the notions of heuristics introduced by Pólya (1945) such as trial-and-error, analogy, generalization, working backward, and draw-a-figure. These high-level processes provide guidance for obtaining a solution to a problem, enabling problem solvers to choose from a limited set of alternatives and order the solution process in a series of steps (Xenofontos & Andrews, 2014).

Problem-solving plays an important role in the intended curriculum of a country. In this regard, NCTM (2000) identified five fundamental processes that are a major objective in the curricula of many countries. Problem-solving is listed as one of the five Process Standards (NCTM, 2000). This designates that "Problem-solving is an integral part of all mathematics learning, and so it should not be an isolated part of the mathematics program" (NCTM, 2000, p. 52).

The third problem-solving perspective relates to how teachers use mathematical problems in their teaching. Teachers may use problem-solving in their classes in three ways: "teaching for problem-solving" where teachers may focus on students application of their mathematical knowledge to students to solve routine and non-routine problems; “teaching via problem-solving” in which problems are used to facilitate students' learning that is related to how to solve problems and manage the solution processes, and “teaching about problem-solving” when students present their thinking (Nunokawa, 2005).

For this study, the operational definition for mathematical problem-solving means “engaging in a task for which the solution method is not known in advance. In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understandings” (NCTM, 2000, p. 52). Similarly stated in another research, problem-solving in math means “becoming involved in a task for which the solution method is not known in advance. To find a solution, students must use the previously acquired knowledge and, through this process, gain new mathematical understandings” (Bahr & Garcia, 2010).

In addition, the definition of mathematical literacy (OECD, 2018a, 2018b) indicates the student’s ability to “formulate, employ, and interpret mathematics. These three words, formulate, employ and interpret”. The modeling problem-solving cycle provides a beneficial and meaningful structure to the mathematical problem-solving process that describes what individuals do to relate problem context to mathematics and problem-solving. Having said that, problem-solving is seen in this study as a process in which the modeling problem-solving cycle indicates the problem-solving process, that will be discussed deeply in the following sub-section.

2.4.6 Mechanisms of Mathematical Problem Solving

There are many ways to do problem-solving. Unlike an exercise, there is no single strategy that works for problems every time. Many researchers have created different procedures to solve problems. One of the most significant strategies was established by John Dewey. For example, in his earlier studies, Dewey (1910) highlighted five steps on how a problem is solved. These steps are (1) difficulty level, (2) location and definition, (3) suggestion(s) of possible solutions, (4) development of a suggested solution, and (5) accept or reject the suggested solution

Problem-solving procedure stages emphasize the importance to engage in follow or evaluation to examine whether the results satisfy the initial conditions (Dewey, 1933). In 1945, Pólya the father of problem-solving published his short book "*How to Solve It"* which provided a similar model to John Dewey in problem-solving. This model is a general outline of a problem-solving framework as a four-step method for solving mathematical problems: understanding the problem, creating a plan, implementing the plan, and looking back (Pólya, 1945). Wilburne (2006) stated that this model is used to enhance mathematical reasoning and advance students' ability to solve mathematical problems. This helps students become aware of their way of thinking when solving a problem and create connections with problems that they need to solve in the future. Moreover, Pólya's model is very useful in the problem-solving process to the students and teachers in the teaching process. For example, teachers can ask probing questions to make students understand a problem easier or give them clues without directing students to a solution. Problem-solving is not a linear process but rather a complex, interactive, and cyclic process. Students move forward and backward and across Pólya's model. This process is described by the Common Core State Standards that represent the first standard of mathematical practice "Make sense of problems and persevere in solving them" as follows:

> Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze

> givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. (New York State Education Department, 2017).

Pólya (1945) stated that there are many strategies to solve the problems that Pólya refers to as “heuristic” and are basic rules for making progress on difficult problems. There are, for example, use trial and error, draw a diagram, look for patterns, or working backwards, etc. The learners need to have many strategies to use in solving problems and choosing an appropriate strategy that is the result of solving many problems (Pólya, 1945). The problem-solving process is important because it is just as important, if not more, than getting the answer.

More recently, mathematical modeling is one of the basic standards of mathematics that is included in the Common Core State Standards, which is primarily intended to provide chances for students to use mathematics in solving daily, societal, and workplace situations (CCSSI, 2010). Thus, the role of context in supporting student learning by linking daily activities to school mathematics activities has been emphasized more by curriculum developers. Hence, understanding the mathematical modeling process will give us a better understanding of the mechanism involved when students solve mathematical contextual problems.

2.4.7 Mathematical Modeling Process

The problem-solving and modeling process has been extensively studied by many researchers (Blum, 2002, 2015; Schoenfeld, 1992, 2007, 2014). These studies focus on the processes and mechanisms that problem-solvers perform to solve problems of a quantitative nature. Despite many definitions of modeling, fundamental thinkers have simplified the definition to indicate the relationship that could be formed between the real world and mathematics.

Mathematical modelling is seen as “a matter of constructing an idealised, abstract model which may then be compared for its degree of similarity with a real system” (Giere, 1999, p. 50). Mathematical modeling is similar to ML in that both require applying the four main steps of Polya's model in problem-solving. Mathematical modeling is the complete process that starts with a real-life situation that is translated into a mathematical model, and then this model must be applied to obtain the mathematical results that must be translated and validated in the original position. (Blum & Niss, 2010).

Mathematical modeling is ‘linking classroom mathematics to something from everyday life that is not inherently mathematical’ (Cirillo, Pelesko, Felton-Koestler & Rubel, 2016, p. 3). This means that contextual problems should be used to “elicit student thinking—to reveal bases of understanding that can be built upon” (Schoenfeld, 1983, p. 407). Blum (2002) points to two other aspects to the definition of modeling that it has direction as it moves from reality to mathematics; and it is a "process leading from a problem situation to a mathematical model" (p. 153).

Contextual mathematical problems are more similar to modeling problems than to problems of application (Niss, Blum & Galbraith, 2007). When it comes to using mathematics in "real-world" problems, modeling and application are sometimes used similarly. However, there is a difference between them according to Niss et al. (2007) as follows:

> The term "modelling", on the one hand, tends to focus on the direction "reality $\longrightarrow$ mathematics" and, on the other hand and more generally,

> emphasises the processes involved. Simply put, with modelling we are standing outside mathematics looking in: "Where can I find some mathematics to help me with this problem?" In contrast, the term "application", on the one hand, tends to focus on the opposite direction "mathematics reality" and, more generally, emphasises the objects involved—in particular those parts of the real world which are (made) accessible to a mathematical treatment and to which corresponding mathematical models already exist. Again simply put, with applications we are standing inside mathematics looking out: Where can I use this particular piece of mathematical knowledge? (pp. 10-11).

What is expected of students when engaging in mathematical modeling is not only limited to dealing with one particular task, but they have to apply it to different situations that can be modeled by using a specific mathematical concept, relationships, or formula, developing a routine and fluency in mapping problem data to the basic mathematical model and in working through this model to reach a solution (Van Dooren, Verschaffel, Greer & De Bock, 2006). The modeling process has a cyclic nature where students do not move in succession through the different steps of the modeling process. The students, when modeling, go through many modeling cycles as they need to revisit their work many times, and gradually refine their model or sometimes reject it.

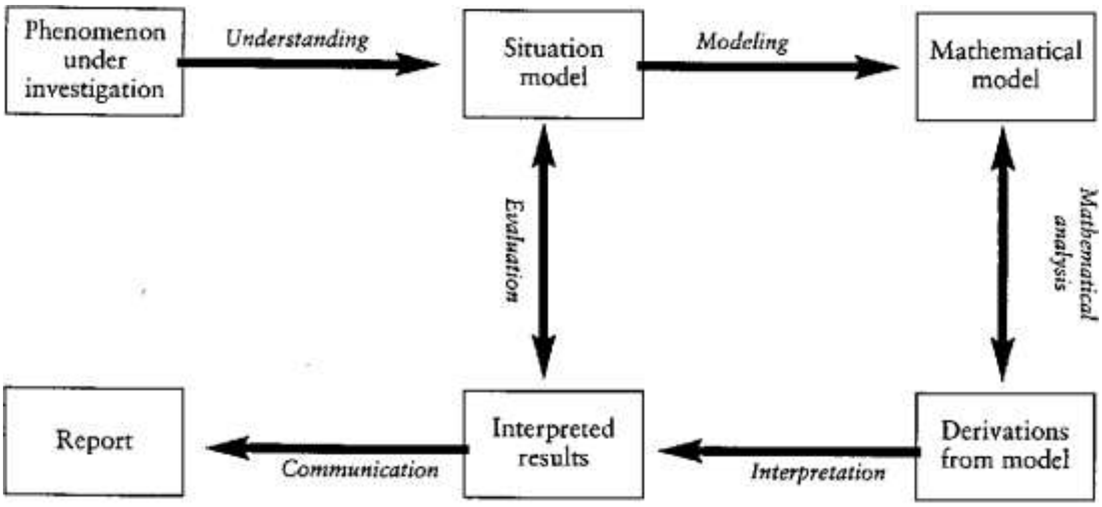

Figure 3: Schematic diagram of the process of modeling

In the literature, many researchers in their work on problem solving and modeling have presented diagrams for visualizing contextual mathematical problem-solving processes. Mathematical modeling of problem-solving is a complex procedure consisting of different stages (Van Dooren, Verschaffel, Greer & De Bock, 2006). Verschaffel et al. (2000, p.xii) presented the Schematic diagram of the modeling process to represent the stages of the problem-solving procedure as shown in Figure 3.

At the first stage, students need to understand the phenomenon under investigation to create a model of the relevant elements, and relations rooted in the situation. Students need to identify the key and less important elements that should be included in the situation model. The second stage is building a mathematical model of the related items by mathematising the situation model. The situation model is mathematised by translating it into a mathematical equation involving the key quantities and relations. Then students obtain the solution by manipulating the model. It is not enough just to get the answer, students need to evaluate their results against a situation model in which the students check their results are reasonable and appropriate to the original situation. At the final step, students are supposed to communicate the interpreted results considering the circumstances of the problem (Verschaffel et al., 2000).

Many Mathematical competencies, such as reading and communication, designing and applying problem solving strategies, or working mathematically (reasoning, calculating...) are closely related to modeling (Niss, 2010). By modeling, mathematics becomes more meaningful for learners and useful for cognitive analyses. Blum and Leiß (2007) presented a modeling cycle for solving these tasks in the seven-step model. Mathematical modeling is seen as a cognitively challenging activity due to the many competencies involved (Blum, 2015). This model presented by Blum (2015, p. 76) is shown in Figure 4 below.

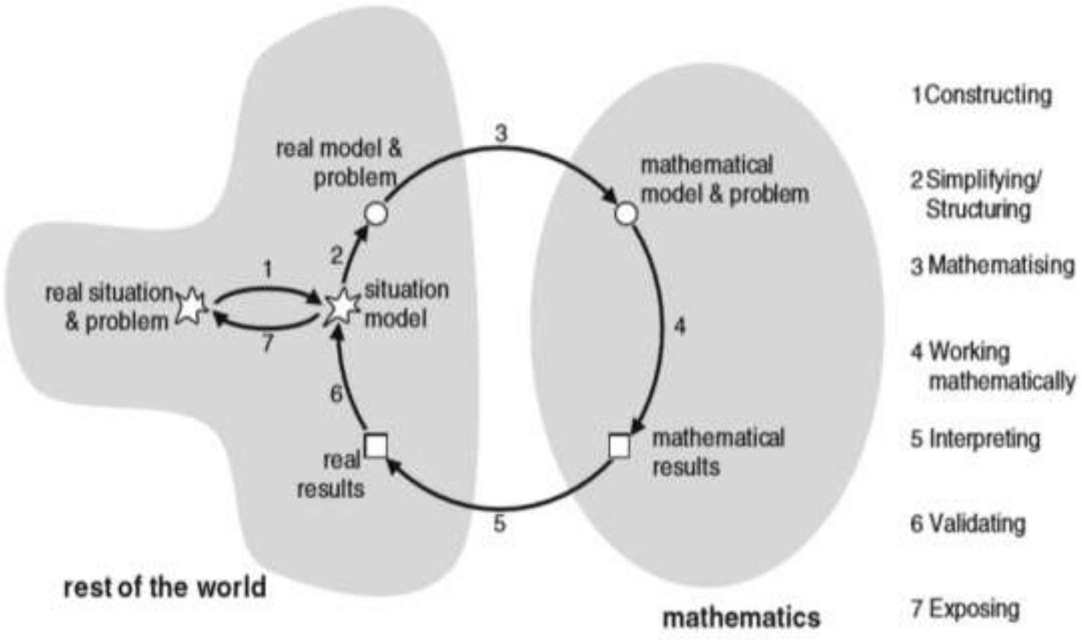

Figure 4: The seven-step modelling Schema

Many students remain stuck in the first step which is understanding the situation and constructing the situation model. The modeling process begins with a problem from a real-life context and then ends again in a real model of the original situation after simplifying the real situation. Understanding the situation model in the cycle is a very important phase during the modeling process. This is because it describes the transition between the real situation and situation model as a phase of understanding the problem. Then during the process of mathematisation, this real model is transformed into a mathematical model. Then working mathematically leads to the mathematical results to finally interpret it in the real world as real results (Blum, 2015; Blum & Niss, 2010).

There are two types of mathematisation, horizontal and vertical arithmetic formulated by Treffers (1978). Vertical mathematics appears to mean “formal” mathematics whereas horizontal mathematisation denotes the “informal” ML. The horizontal mathematisation is explained by (Freudenthal, 1991) as “going from the world of life into the world of symbols, while vertical mathematisation means moving within the world of symbols” (p. 24). Mathematisation is a term used by OECD which involves five elements describing how to solve a problem with roots in reality (Hope, 2007).

It is evident from the above discussion that the distinction between different concepts such as mathematisation and mathematical modeling is unclear. However, Blum and Niss (2010) clarified the difference as mathematisation is a one-way process that translates the real model into a mathematical model, whereas mathematical modeling is the process of translation between the real world and mathematics in both directions that involves the entire process.

Mathematical modeling has been considered as a cornerstone of the PISA framework for mathematics by OECD where it is incorporated into the definition of mathematical literacy that investigates the ability to deal with real-life contexts. Students implement mathematics and use mathematics tools to solve contextual problems through a series of stages. The definition of mathematical literacy mentioned three processes describing what individuals do to relate the context of a problem to mathematics and thus solve the problem (OECD, 2013).

According to Stacey (2011), mathematical modeling consists of three processes: formulating, solving, and interpreting. Likewise, Brown and Schäfer (2006) describe the same cyclical processes using the terms formulation, analysis, interpretation, and consolidation. These processes will be found in the central work of the teacher leading students from real-life situations to the application of appropriate mathematics. Moreover, these processes are key components of mathematical modeling and mathematical literacy as defined for cycle 2012 as well (OECD, 2013). This modeling cycle is presented by OECD (2013, p. 26) is shown in Figure 5.

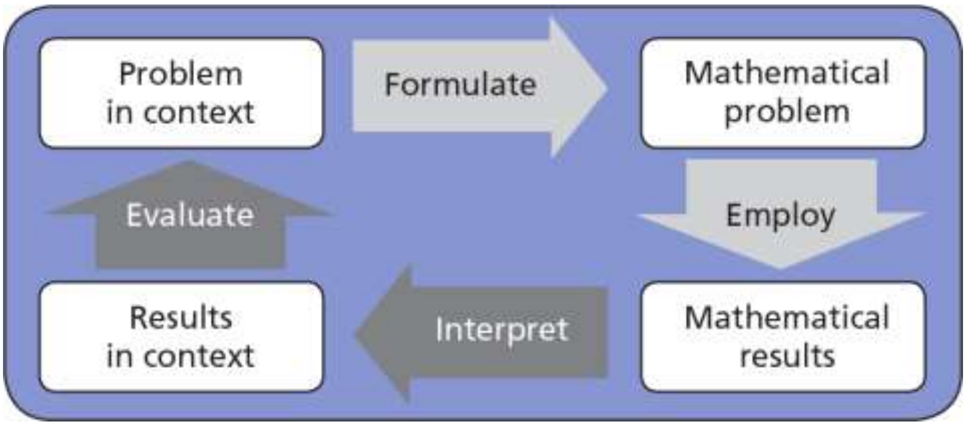

Figure 5: The mathematical modeling cycle of PISA 2012 framework

The mathematical modeling cycle takes place with a “problem in context”. To begin solving the contextual problem, the individual attempts to formulate the situation mathematically based on the relevant mathematics identified in the problem situation. In this stage, the problem solver transforms the “problem in context” into a “mathematical problem” to apply the mathematical treatment. Then, mathematical concepts, procedures, facts, and tools are employed to find “mathematical results”. This stage where mathematical reasoning, manipulation, transformation, and computation take place. In the next stage, the “mathematical results” need to be interpreted in terms of the original problem as “results in context”. The problem solver needs to “interpret, apply, and evaluate” the mathematical solution in the real-world context of the problem (OECD, 2013).

In a modeling cycle, it is often not necessary to go through all its stages despite it being an essential aspect of the PISA conception of students as active problem solvers, especially in the context of an assessment (Niss et al., 2007). Students' scores in the PISA 2012 indicated overall student achievement in the three processes based on the definition of mathematical literacy (OECD, 2013). However, the definition of mathematical literacy as "an individual's capacity to reason mathematically and to formulate, employ, and interpret mathematics to solve problems in a variety of real-world contexts" (OECD, 2018) does not focus solely on problem-solving to be mathematically literate, but also put reasoning at the center of the problem-solving cycle. Figure 6 illustrates the relationship that links problem solving and reasoning in the modeling cycle of mathematics in the PISA 2021framework.

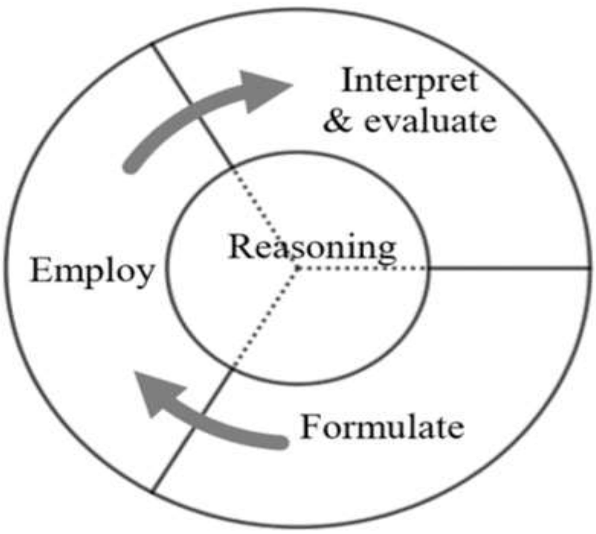

Figure 6: The mathematical modeling cycle of PISA 2021 framework

OECD (2018a, 2018b) stated that mathematically literate students could apply their mathematical knowledge to extract the abstract mathematical of the problem (more specifically contextual real-life problems) and then formulate it mathematically using appropriate terminology. This transformation process entails mathematical reasoning. Next, they need to use mathematical concepts, algorithms and procedures taught in schools to solve the resulting mathematical problem. However, making the appropriate selection of those tools may require making a strategic decision that also demonstrates mathematical reasoning. Mathematical reasoning is also embedded in the process of evaluating and interpreting a solution within the original real-world situation (OECD, 2018a, 2018b).

There is an intersection between mathematical reasoning and solving real-world problems. In addition, mathematical reasoning goes beyond solving practical problems as it is also a way of evaluating and interpreting the quantitative nature of problem-solution that is best understood mathematically. Thus, mathematical literacy is seen to be a composite of two connected aspects that are problem solving and mathematical reasoning (OECD, 2018b). The PISA 2021 framework places mathematical reasoning at the heart of the problem-solving process. Based on the importance of mathematical reasoning for mathematical literacy, the next section will be devoted to studying mathematical literacy (OECD, 2018a, 2018b).

2.4.8 Summary

Problem-solving received a lot of attention recently. It is extensively supported by NCTM documents to improve the quality of teaching as it was mentioned among the five mathematical processes standards (NCTM, 2000). Many researchers agree that the main characteristic of any problem that it has no clear solution (Blum & Niss 1991; Holth, 2008; McLeod, 1988; Pólya, 1981). Many types of problems were discussed since the work of Pólya the father of problem-solving. Niss et al. (2007) identified three types of problems: modeling problems, word problems, and intra-mathematical problems and stated that the difference between them is the degree of their connection to real-life (Niss et al., 2007). This research focus is on contextual mathematical problems that lend themselves to modeling problems that are the core of PISA's mathematical literacy assessment.

Problem-solving, through literature, is perceived in three ways: as a process, as a curriculum, and as an instructional approach (Xenofontos & Andrews, 2014). Those researchers who perceive it as a process developed frameworks and models to describe the components of the process and to analyze how it works starting from Dewey's (1933) problem-solving procedure stages and Pólya's model (1945) until the modeling cycle of OECD for both cycles 2012 and 2021 (OECD, 2013, 2018). The updated PISA 2021 framework, which is adopted for this study, puts reasoning at the center of the problem-solving cycle that will be discussed in the next section.

2.5 Mathematical Reasoning

The role of using problem-solving in teaching mathematics is not only as a means of developing knowledge and skills but also helping students to understand and make sense of mathematics and develop their reasoning abilities (NCTM, 1989). When the students develop the ability to confront a mathematical problem, persevere in its solution, and evaluate and justify their results, they became confident, self-reliant mathematical thinkers (Kanmani & Nagarathinam, 2018). Hoogland (2016) has claimed that there has been a shift in the interest in mathematics education in secondary and vocational education in particular, by giving more attention to mathematical reasoning, problem-solving, and understanding of quantitative situations. Mathematical reasoning is one of the most important activities that are used in many areas such as education, science, environmental issues, and real-life situations.

The reasoning is important for all disciplines, for example, the students in literature classes of their high school need to analyze, interpret, or think critically about what they are reading. However, it does play a more special and important role in mathematics. Part of mathematical thinking is making logical conclusions based on assumptions and definitions. It is often understood to involve formal reasoning or proof where evidence is required to arrive at a conclusion.

The standard “Reasoning and proof” is important to teaching and learning mathematics because it is one of the five processes standards (NCTM, 2000). Mathematical reasoning is a critical skill, as it develops, students realize that mathematics is logical and can be understood. For example, students learn how to choose a problem-solving strategy, draw logical conclusions, and learn to apply these solutions. In addition to considering a solution to determine if it makes sense (NCTM, 2000). They understand the importance of reasoning as an important part of mathematics (Kanmani & Nagarathinam, 2018). This skill could enable the students to make use of all other mathematical skills. The reasoning is very important to make decisions in many aspects of life such as choosing between possible options or thinking about how to solve a problem and much more (Kanmani & Nagarathinam, 2018).

Building on the “Principles and Standards for School Mathematics” (NCTM, 2000), the NCTM (2009) released another important document as the next step in its continuous efforts to promote high standards in mathematics education entitled “Focus in High School Mathematics: Reasoning and Sense Making”. This document demonstrates a clear focus on the process standards, mainly reasoning and sense-making. In this document, NCTM claims that if the emphasis is placed on reasoning and sense-making, students will be better prepared for future success.

Reasoning helps students to connect their ideas, develop connections between new learning and their existing knowledge, enjoy learning mathematics by gaining deeper conceptual understanding, which increases the likelihood that they will understand and retain new information. Learning to use mathematics in a meaningful way requires being curious, asking a lot of questions, and reasoning. In short, students learn that mathematics makes sense through reasoning (NCTM, 2009).

2.5.1 Mathematical Reasoning Definition

There are several terms used to refer to "reasoning": critical thinking, higher-order thinking, or logical reasoning. In general, reasoning can be defined as the process of drawing conclusions based on evidence or stated assumption (NCTM, 2009). Kanmani and Nagarathinam, (2018) stated that “reason is the capacity for consciously making sense of things, establishing and verifying facts, applying logic, and changing or justifying practices, institutions and beliefs based on new or existing information”. Students can deduce logical and objective conclusions and decisions that they can trust by applying the appropriate reasoning that they learned through mathematics classes (OECD, 2018b).

There are two important aspects of mathematical reasoning in today's life and to the PISA framework. The first is deductive reasoning where students can infer from explicit assumptions that are the hallmark of a mathematical process (OECD, 2018b). It involves making a logical argument, drawing conclusions, and applying generalizations to specific situations. This type of reasoning may involve eliminating unreasonable possibilities and justifying answers. The ability to use deductive reasoning improves with age. More complex reasoning skills are appropriate at the secondary level such as recognizing incorrect arguments (Kanmani & Nagarathinam, 2018).

The second aspect is very important which is statistical and probabilistic (inductive) reasoning. On a logical basis, it is important to help remove the confusion in individuals' minds between what is possible and what is possible so that they do not fall victim to false news. Moreover, from a technical perspective, making sense of the big data generated by an increasingly complex world and its multiple dimensions is one of the biggest challenges people face later in life. Students must make informed decisions in the context of real-life by being formalized with the nature of such data. (OECD, 2018b)

OECD (2018a) stated that mathematical reasoning is the core of mathematical literacy in which it was empowered by some of the basic concepts that support school mathematics. Some of these basic concepts are:

- “Understanding quantity, number systems, and their algebraic properties”;
- “Appreciating the power of abstraction and symbolic representation”;
- “Seeing mathematical structures and their regularities”;
- “Recognising functional relationships between quantities”;
- “Using mathematical modelling as a lens onto the real world (e.g. those arising in the physical, biological, social, economic, and behavioral sciences)”; and
- “Understanding variation as the heart of statistics”. (OECD, 2018a, p. 15).

The OECD (2018a, 2018b) indicated that mathematical reasoning (both deductive and inductive) can be manifested in contexts such as “evaluating situations, selecting strategies, drawing logical conclusions, developing and describing solutions, and recognising how those solutions can be applied". Moreover, OECD (2018b) added that students' reason mathematically when they:

- "Identify, recognise, organise, connect, and represent",
- "Construct, abstract, evaluate, deduce, justify, explain, and defend"; and
- "Interpret, make judgements, critique, refute, and qualify". (OECD, 2018b, p 14-15).

Francisco and Maher (2005), in their longitudinal study, examined the conditions for enhancing reasoning in problem-solving, stating that "providing students with the opportunity to work on complex tasks as opposed to simple tasks is crucial for stimulating their mathematical reasoning" (p. 731). Tasks should be selected carefully to promote reasoning skills. Students should be given an opportunity to work on complex tasks rather than simple tasks because such tasks are necessary to develop mathematical thinking (Francisco & Maher, 2005). According to Wilburne (2006), the best mathematical problem to use in the classroom is non-routine mathematical problems that promote rich and meaningful mathematical discussions. This type of problem does not show any clear solutions and those that require a student to use different strategies to solve them. Likewise, O'Shea (2009) stated that students succeed in solving mathematical problems when faced with difficult problems that fall within their current level of understanding. It could be surprising to a teacher when students engage in reflection, it might reveal that what students can do is far more than what the teacher might have given them credit for (Leavy & O'Shea, 2011).

2.5.2 Reasoning Habits and Modeling Cycle

It is emphasized that students should be given the chance to think mathematically that equip them with a conceptual understanding by using mathematical reasoning in conjunction with a small set of fundamental mathematical concepts that are manifested and reinforced through students' experience in learning mathematics. Focusing on reasoning and sense-making indicates that it is not enough for high school mathematics programs to only focus on “covering” mathematical topics, it should continuously give more attention to developing reasoning habits as an integral part of the curriculum not teaching it as new topics (Pólya, 1952, 1957; Schoenfeld, 1983; Harel & Sowder, 2005).

The curriculum of high school mathematics is very crowded, which leaves little room for introducing reasoning habits that should become routine in all mathematics classes. Thus, teaching reasoning habits as new topics might not be effective as desired. Alternatively, reasoning habits could be integrated into the existing curriculum and give more attention to ensure that students both understand and can use what they have been taught. NCTM (2009) describes “reasoning habits” as “productive ways of thinking that should become customary in the processes of mathematical inquiry and sense-making. It also has included a list of reasoning habits that demonstrate what type of thinking should become expected routine to exist in every mathematical class of the high school. The reasoning habits involve analyzing a problem; Implementing a strategy; seeking and using connections; Reflecting on solution (NCTM, 2009, p. 9-10).

Kilpatrick, Swafford and Findell (2001) have argued that a focus on reasoning and sense-making is central to achieving mathematical proficiency including conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Reasoning habits are directly helping in addressing strategic competence and adaptive reasoning. Similarly, these reasoning habits are strongly related to the problem-solving processes - formulate, employ, and interpret- based on OECD work. The current definition of mathematical literacy by OECD (2018) explains how mathematical reasoning relates to problem-solving in PISA 2021 framework. Mathematical reasoning is central to both of problem-solving cycle and mathematical literacy in general (OECD, 2018a, 2018b). “Without reasoning there is no mathematics” (NCTM, 2009, p. 3).

According to OECD (2018a, 2018b), mathematical literacy consists of two related and overlapped features: mathematical reasoning and problem solving. However, mathematical reasoning plays a more vital role than solving applied problems, and it is also important for assessing and providing important explanations and conclusions for public policies that can be better understood due to their quantitative nature.

The OECD (2018a, 2018b) asserted that for students to be mathematically literate, they need to first use their knowledge of mathematics to recognize the mathematics of any context including real-world problems, and then formulate it mathematically using appropriate concepts. Mathematical reasoning is a critical component to being mathematically literate as it is required for the transformation from a real-world problem to a well-defined mathematical problem. In addition, mathematical reasoning is also needed to make a strategic decision for selecting mathematical concepts, algorithms, and procedures that are taught in school to solve the resulting mathematical problem. Moreover, the definition of PISA places more emphasis on students' need to assess the relevance of mathematical solution by interpreting the solution of the problem with a return to the real situation.

2.5.3 Summary

Individuals try to make sense of situations they encounter around the world which means that they use some mathematical reasoning, knowledge, and skills due to the strong tendency towards the practical use of mathematical knowledge implied by many concepts like numeracy and mathematical literacy.

In high school mathematics, students study mathematics that prepares them for life, the workplace, the scientific, and the technical community (NCTM, 2000). It is very important to emphasis reasoning and infuses it into all these areas to enable the students to experience mathematics as a powerful way of making sense of the world to best preparing students for future success (NCTM, 2009).

The reasoning is identified as "the capacity for consciously making sense of things, establishing and verifying facts, applying logic, and changing or justifying practices, institutions and beliefs based on new or existing information" (Kanmani & Nagarathinam, 2018).

Students should learn to make sense of mathematics through reasoning. Hiebert (2003) explains that students are more likely to understand and retain new information when connected with their previous knowledge. Sensemaking and reasoning are complementary. Students in their reasoning depend on understanding that is derived from sense-making of a situation, and their attempt to justify why something is true improves that understanding of a situation (NCTM, 2009).

It is essential to provide students with the chance to work on challenging problems rather than simple problems to develop their mathematical thinking (Francisco & Maher, 2005). The best type of problem to be employed in a classroom is non-routine problems that have no clear solution and require the students to use a variety of strategies to solve them (Wilburne, 2006).

Reasoning habits are described as “productive ways of thinking that should become customary in the processes of mathematical inquiry and sense-making” (NCTM, 2009). They are not only strongly related to the problem-solving processes, but they go beyond solving practical problems in the traditional sense. The reasoning is the core of both problem solving processes and mathematical literacy (OECD, 2018a, 2018b). To sum up, “Without reasoning there is no Mathematics” (NCTM, 2009, p. 3).

2.6 Mathematics Enrichment Program (MEP)

The focus of educational policy has recently begun to shift from completing compulsory education to ensuring quality education that will develop students to be ready for international competition. Hence, improving students’ performance on international tests such as PISA or TIMSS is crucial as it is reflecting the quality of education.

2.6.1 Background

PISA results for the UAE students in mathematics indicate that they have difficulty in solving problems linking mathematical concepts to everyday life. The UAE students ranked 50th position in mathematics out of nearly 80 countries in PISA 2018 (Sanderson, 2019). The percentage of the students below level 2 is 46%, while the OECD considers that students performing proficiency level 2 or higher possess the skills necessary to succeed in the knowledge economy. This indicates that these students are able only to solve simple mathematical situations (OECD, 2019b). Whereas only about 5% of the UAE students, who perform at level 5 or level 6, can model mathematically complex situations and apply problem-solving effectively (OECD, 2019b). These are outstanding students including students who are gifted in mathematics in the UAE. Due to the contextual nature of problems that are presented in PISA, which is essentially at the heart of word problems, UAE students have less ability and are unable to solve high order thinking (HOT) problems, especially those at levels 5 and 6, compared to students from other countries.

Based on the literature, some reasons have been suggested to answer the question of why students are not very successful in solving word problems: first, students have limited experience with word problems (Bailey, 2002), second, lack of motivation to solve word problems (Hart, 1996), third, word problems were irrelevant to students' lives (Ensign, 1997). These three reasons can be addressed by providing students with word problems to improve the students' performance. In particular, contextual problems presented in real-life situations can address the two factors of motivation and relevancy, which may lead to the first being more experience with word problems.

However, one of the main reasons for students' poor performance and inability to solve problems outside of the classroom is that they lack the appropriate knowledge of problem-solving in real-life contexts. Most of the problem solving provided in schools consists primarily of structured conceptual problem solving not ill-structured problem solving (Dixon & Brown, 2012; Johnson, Dixon, Daugherty & Lawanto, 2011). Ill-structured problems, as indicated by Hong and Kim (2016) are framed with respect to real problems that are contextualized, which are similar to PISA problems, that require students to identify the information and skills needed to solve them. So "studying more to get more points" will definitely not lead to a successful solution to PISA's math problems. These problems can be considered as one of the measures that meet current social needs which emphasizes students' ability to solve real-life problems facing modern society (Hong & Kim, 2016). This suggests that it is imperative to provide students with opportunities to become real-life problem solvers by exposing them to the type of problem that develops their problem-solving abilities (Hong & Kim, 2016).

Most students have experienced "doing mathematics" that involves studying materials and working through abstract tasks. Nevertheless, the enrichment curriculum will provide students with the opportunity to experience "the joy of confronting a novel situation and trying to make sense of it - the joy of banging your head against a mathematical wall, and then discovering that there may be ways of either going around or over that wall" (Schoenfeld, 1994, p. 43). To achieve this goal, it is necessary to clarify the content, skills, knowledge, and classroom experience and to define the methodology for implementation (Piggott, 2004).

2.6.2 What is Enrichment?

The Cambridge Dictionary (2020) defines "Enrichment" as "the act or process of improving the quality or power of something by adding something else" (para. 1). Enrichment programs are described as activities designed to expand and develop learner's experience (Bragett, 1994). Eyre and Marjoram (1990) defined enrichment as any type of activity or learning that falls outside the core of the learning that most children do and described the goal of enrichment is related to improving the quality of life in the classroom and increasing sensitivity. Clendening and Davies (1983) defined “enrichment of content” as:

> any learning experience that replaces, supplements, or extends instruction beyond the restrictive bonds and boundaries of course content, textbook, and classroom and that includes depth of understanding, breadth of understanding, and relevance to the student and to the world in which he or she lives.

In the same vein, Piggott (2004) focuses on depth, breadth, and relevance as major components of enrichment. However, enrichment is considered a relative concept as all definitions refer to normal practices that are not standardized in schools and classes (Feng, 2006).

For the mathematics education field, enrichment is defined as “broadening students’ mathematical experiences by examining mathematics outside of the prescribed curriculum and also making connections with other curriculum areas” (Bicknell, 2009, p. 35). Additionally, enrichment in mathematics means allowing the learner to learn mathematics in more depth to expand the learner’s knowledge (Koshy, 2002).

Enrichment encourages mathematical thinking and problem solving which leads to the development of cognitive processes. Nevertheless, the word “enrichment” is mostly used in the context of provision for the mathematically gifted, with just a few exceptions such as Wallace (1986). Thus, if it is beneficial to provide gifted students with problem-solving and mathematical thinking linked to stimulating mathematical contexts, then it is surely worth doing for everyone for an extended period of time.

Enrichment should not only be available to the fastest and brightest students, it should be integrated into the curriculum as a whole. Moreover, enrichment is not only seen as a means for more capable students, but all students will also benefit from this experience, at least it can offer most students a more realistic option for classroom management (Piggott, 2004). Because, as Feng (2006) points out, enrichment is a way to introduce accessible aspects of mathematics not covered by the curriculum, promote mathematical reasoning, encourage extended problem solving, provide alternative approaches to curricular topics, and highlight links between aspects of mathematics presented separately in the curriculum.

2.6.3 Why is Enrichment Needed? And for Whom?

PISA 2018 test results indicated the poor performance of Emirati students in general, especially outstanding students, as evidenced by the performance of only about 5% of students in Level 5 and Level 6 in Mathematics (OECD, 2019b). Although intervention measures must be taken to help all students improve their learning levels, high achievers receive little attention although many researchers have indicated the crucial role of gifted and talented groups in developing and transforming societies (Meisenberg & Lynn, 2011). Several studies have shown that the levels of cognitive ability of societies are required to develop the aspect of positive value in developing countries (Rindermann & Thompson, 2011).

In 1980, An Agenda for Action: Recommendations for School Mathematics stated that “Outstanding mathematical ability is a precious societal resource, sorely needed to maintain leadership in a technological world” (NCTM 1980, p. 18). Unfortunately, most programs are geared towards meeting the needs of “at-risk” students so that their needs can be fully developed (Galloway, Armstrong &

Tomlinson, 2013), but not to support gifted students (Clark, 2008; Kokot & Kruger, 2005) as well as the high achievers. There is a tendency to neglect gifted students as well as high achievers due to the belief that they can take care of themselves. Child (2004) noted that "some teachers believe that the bright can look after themselves".

For a long time, mathematics education discussed equity and mathematics enrichment for high potential students separately. The discourse on equity focused primarily on providing access to a minimum of basic mathematics but ignored the high potential among disadvantaged students (Schnell & Prediger, 2017). According to DIME (2007), many countries indicate the equity and opportunities necessary for learning mainly in relation to students with low achievement and their chances of having some access to basic mathematics. However, the role of mathematics as a gatekeeper to higher education calls for an additional "measure of equity and access" (Pateman & Lim, 2013). Only recently, research and development have focused on potential among underprivileged students those who are not immediately identified as high potentials (Schnell & Prediger, 2017; Suh & Fulginiti, 2011).

There is a call for a wider conceptualization of mathematical potential due to the economical demands raised by the huge need for STEM academics in a technical civilization. The "mathematical potential" construct is used for students "who can achieve a high level of mathematical performance when their potential is realized to the greatest extent" (Leikin, 2009, p. 388) and characterized to have analytical and creative abilities, affective factors, commitment, and multiple opportunities. This concept can be carried over from the top 2% to a wider group to about 20% of all students, and thus they are less exclusive than the usual "talented" or "gifted" (Schnell & Prediger, 2017). Moreover, Leikin (2011) links the building of mathematical potential with learning situations because if students are subjected to a learning situation rich in mathematics, then the student can demonstrate certain potential.

Child (2004) stated that the most common methods used in nurturing gifted students mostly in combination are acceleration, segregation, and enrichment. Whatever the three methods, the curriculum needs to be differentiated for the gifted students from the mainstream curriculum. Moreover, Heward (2014) indicated that the enrichment method has been the most advocated method since the progressive movement in the 1920s. This method involves more in-depth instruction and ability grouping for gifted students. Koshy, Ernest and Casey (2009) suggested that enrichment is an alternative strategy for acceleration and differentiation. Moreover, this type of provision continued over years in different countries (Smith, Polloway, Patton & Dowoy, 2004).

Enrichment is mostly related to gifted provision models such as curriculum acceleration or compaction (Piggott, 2004). Acceleration gives students access to “standard curriculum material” earlier and encourages students to move faster through subject content leading to early entry to university. Renzulli and Reis (1997) stated that curriculum compacting restructures curriculum to enable students to cover portions of the standard curriculum "more efficiently". However, acceleration and compacting themselves are not what make difference to the gifted students, but the issue is to free up time that can be used for extracurricular activities because "doing more of the same" is not enriching. However, the model of enrichment is still seen as an add on to the standard curriculum.

Schnell and Prediger (2017) stated that enrichment means exposing the students to rich learning processes to expand their experiences and skills. There are two types of enrichment; either by broadening or deepening. Enrichment by broadening represents learning additional topics rather than what is normally studied at school as courses out of school, while enrichment by deepening enhances the depth and complexity of the subject being studied in the school (Schnell & Prediger, 2017). Enrichment by deepening the tasks and topics are mostly chosen because they are in line with the regular curriculum unlike broadening by extracurricular activities (Sheffield, 2003). For this study, enrichment by deepening is chosen because it suits the needs of advanced students, including those gifted in mathematics, in regular classes by deepening what they are already studying through an emphasis on problem-solving and mathematical reasoning (Piggott, 2004). When all students engage in this type of task, those with potential are expected to expand their expertise, skills as well as the rest of the students in the class depending on the level of each student.

Teachers should raise the ceiling of expectations when interacting with gifted students so that students can compete with their potential rather than with the norm. To maximize the potential of gifted students, teachers need to differentiate the materials, assignments, and products in the level of complexity, abstraction, and depth (Rief & Heimburge, 2006). The enrichment method for treating the gifted students is considered perfect for the high achievers, especially that some of these high achievers are also gifted too. This research aims to study the impact of enrichment on the mathematical literacy of tenth grade students in the UAE.

2.6.4 Paradigmatic Positions of Mathematics Enrichment

The enrichment activities aim to provide students with a stimulating mathematical experience, promote positive attitudes, raise the level of achievement, and contribute to efforts to enhance, generalize, and increase the general understanding of mathematics. From the enrichment literature, four paradigmatic positions can be identified to reflect their educational views and priorities. Feng (2006) listed enrichment positions as follows:

- development of exceptional mathematical talent;
- popular contextualisation of mathematics;
- enhancement of mathematics learning processes, and
- outreach to the mathematically underprivileged.

All of these mathematics enrichment positions are motivated to provide high-quality mathematics learning experiences. However, opposing views arose from differing perceptions of how best to achieve this and on whom it should be applied to achieve the most benefit. Nevertheless, the following three positions are directed at all students with a different focus for each of them.

According to Feng (2006), the first position is directed to few students, only gifted, as it aims to identify and develop (mathematical) talent and views enrichment as a method to meet their academic needs, and to cultivate an elite group to become leaders in civic, commercial and industrial contexts. This position was supported by many researchers, for example, Clendening and Davies (1983).

The second position applies to all students where its focus is on the application of mathematics as a means of engaging students in mathematics. This will make students appreciate the applications of mathematics to life, not just as an academic discipline. This is expected to break the negative stereotypes of mathematics by deepening students' understanding of mathematics and its applications.

The Third position of enrichment is best described as student-and experience-centered (Feng, 2006). This type of enrichment is an approach of the ongoing process that should infuse all aspects of teaching and learning as an integral part of education for all students, whether in regular classrooms or beyond. According to Feng (2006) “using this interpretation of enrichment, the engagement of all students in meaningful mathematical practices is an essential and worthwhile part of education; this also forms the main goal of mathematics enrichment”. This conceptualization promotes the linking of mathematical content presented separately in the curriculum with mathematical content and other fields of study. By providing students with a stimulating experience in mathematics, enrichment promotes mathematical thinking and problem-solving. It is important to note that depending on the different levels of students in the classroom, students will need different levels of support to take advantage of enrichment opportunities. Thus, enrichment in this sense emphasizes appropriate scaffolds and content differentiation: enrichment tasks are often designed to use mathematical concepts and techniques at various levels of difficulty and may lead to qualitatively different endpoints (Feng, 2006; Piggott, 2004).

The fourth position calls for social justice and equity, educators that support this view not only believe that enrichment should be open to all students, but also make proactive efforts to ensure mathematics enrichment for students who have not traditionally benefited from such provisions (Feng, 2006).

The focus of this study is mainly on the third position to enhance the mathematics learning process while using contextual mathematics which will lead also to the satisfaction of the second position of enrichment as popular contextualization of mathematics. If mathematics enrichment includes "mathematical problem solving and mathematical logic linked to mathematical contexts" (Piggott, 2004), enrichment should be the basis for many, if not all, aspects of the curriculum, and all students should be able to benefit from this experience (Feng, 2006).

In the next sub-section, the basic concepts including problems, problem-solving, and thinking will be put together in a meaningful enrichment framework based on mathematical literacy and its major components, mathematical problem solving and reasoning.

2.6.5 Enrichment Framework

In this study, enrichment by deepening is based on problem solving and mathematical reasoning as suggested by Piggott (2004). The literature on enrichment, problem-solving, and mathematical thinking lacks clarity as it fails to provide consistent interpretations of each term. This lack of clarity means that the concepts are closely related and thus difficult to separate (Piggott, 2004).

This enrichment program integrates PISA contextualized problems into its content consisting of HOT which requires thinking at a higher level such as mathematical thinking (Ambarita, Asri, Agustina, Octavianty & Zulkards, 2018). Collins (2014) stated that HOTS covers three categories, including transferring (or the ability to apply what we learn and know), critical thinking (involving reflective thinking, reasoning, investigating, exploring viewpoints, comprehending, synthesizing, evaluating, comparing, and connecting) as well as problem-solving. Looking at the PISA framework, many components of mathematical literacy involve mathematical thinking such as reasoning, modelling, and making connections between ideas (Stacey, 2007).

In this research, enrichment is provided for all students, therefore, the diversity of students' levels should be considered as providing problems of different levels of mathematical expertise to students. Moreover, Foster (2015) added that an appropriate "ramp" to the task allows the students to immediately think of something to do to solve the problem. Even though the solution must not be obvious to the learners, it's a difficulty that also should not be experienced as threatening. If the start point of a problem is beyond the student's zone of proximal development (Vygotsky, 1978), then the student cannot engage with the problem and it will not result in a learning gain no matter the amount of support was. Furthermore, a relatively easy beginning to the solution is not the end itself, it only helps students to get "into" the problem and a step towards appreciating and confronting the larger task (Foster, 2015). Moreover, this allows students to achieve some early success, which means positive engagement that motivates learners (Foster, 2015).

Although the provision of enrichment consists mainly of extra-curricular, it is still related to the mathematical skills that are currently being studied or previously studied. Lesson time may be used to enable all students to participate in this type of provision or any other available time. This enrichment perspective that focuses on enhancing the teaching and learning process is supported by Piggott (2004) and many other researchers (Feng, 2006). Thus, in agreement with Piggott (2004), the enrichment framework consists of two components: content and teaching. The next section will discuss these components as follows: 1) mathematical enrichment content and 2) contextual teaching and learning.

2.6.5.1 Mathematical Enrichment Content

Similar to Piggott (2004), as mentioned earlier, enrichment content in this study is based on mathematical literacy that consists of problem-solving and mathematical thinking (reasoning). The problem-solving content entails the general scope of skills that can be applied both inside and outside of mathematics curricula that describe the main elements of a problem-solving process (Piggott, 2004). Therefore, real-life applications can be incorporated into problem-solving to engage students in mathematics. Mathematical thinking is associated with specific mathematical skills that are needed to draw on for effective problem-solving. Looking at the PISA 2021 framework, the mathematics content is referred to as mathematical literacy that also covers problem-solving and reasoning (OECD, 2018a, 2018b). Moreover, Stacey (2007) states that the components of mathematical literacy include mathematical thinking such as reasoning and modeling. These elements then work together and interact with both teachers and students as shown in Figure 7 (Piggott, 2004).

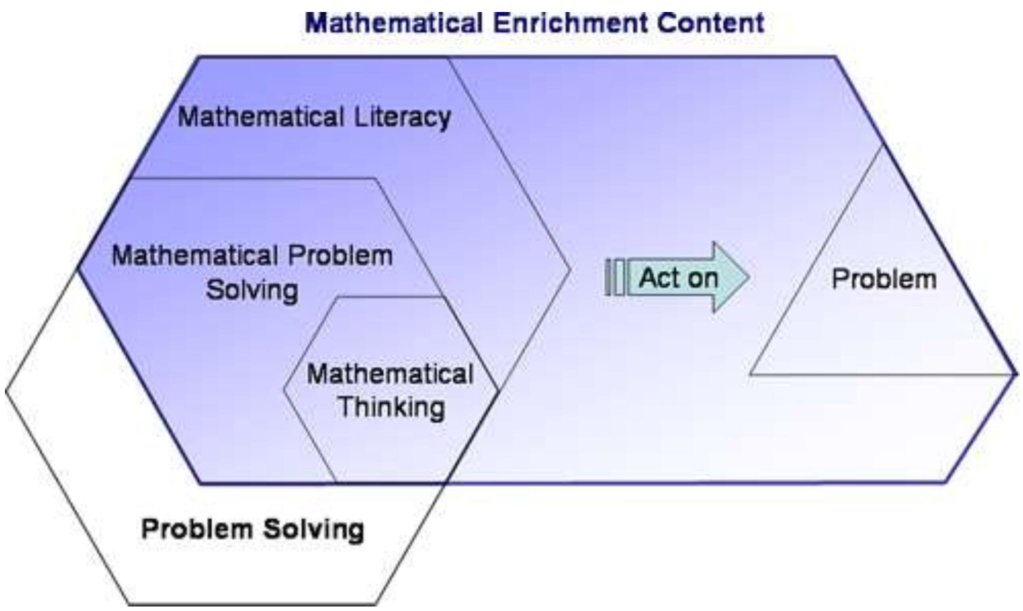

Figure 7: Mathematical Enrichment Content Framework

PISA 2021 (OECD, 2018a, 2018b) and Piggott (2004) agree that learning general problem-solving skills will not be sufficient when teaching about problem-solving, but students also need mathematical reasoning skills. Because without it they would not have the skills to apply it to the problem-solving process. The reasoning is central to problem solving (modeling processes) based on PISA 2021 framework.

PISA aims to measure the students' mathematical literacy that focuses on real-world problems as the students encounter situations and problems that go beyond what was learned in the school's classroom (OECD, 2009b). Students are required to use the skills and competencies they acquired through their school learning to solve these contextual problems (OECD, 2009b). However, school mathematics curricula are usually structured into topics that focus on procedures and formulas. Because of this organization, students may not be able to see or experience existing mathematics in new fields and applications (Brown, Collins & Duguid, 1989). Mathematics is presented to students as a set of disjointed pieces of factual knowledge, not as overarching concepts and relationships (OECD, 2009b).

Students need to deepen their understanding of mathematics, which they are learning in the classroom. It is not expected from students to just practice and memorizes mathematical knowledge that teachers provide, education today is "student-centered" where students favor discovering new ideas for themselves (Draper, 2002). The student is not 'an empty vessel' to be filled with knowledge (NCTM, 2000). Students are no longer required to only memorize formulas, but they must understand its applications and the meaning beyond its parts. Learning aims to provide learners with learning situations to assimilate new learning together with prior knowledge to construct their unique cognition (Ertmer & Newby, 2013).

For this content to make sense, learning and teaching environments need to encourage the effective use of resources so that students can develop the skills, strategies, and competence needed to effectively address problems and use basic thinking skills (Piggott, 2004). A specific view of teaching and learning supports engaging problems that develop and use problem-solving strategies and encourages mathematical thinking. This has put some implications on the teaching approach adopted as the second component of this Mathematical Enrichment Program.

2.6.5.2 Contextual Teaching and Learning (CTL)

For the success of this enriched content, a specific view of teaching and learning supports engaging problems that develop and use problem-solving strategies and encourages mathematical thinking. Pape, Bell and Yetkin, (2003) stated that for students to be successful problem solvers, they need to be creative, confident, and autonomous. This means that teaching should move away from teacher-centered practice. Based on the literature, when teacher-centered the classroom, the students' expectations of mathematics focus on activities that are concerned with procedures, accuracy, and lead by the teacher without social activity involvement. This limited interaction is likely to result in little learning gains compared to learning opportunities as students feel autonomous and independent where the teacher's role is a facilitator.

This curriculum addition of the contextual problems to the students' learning is in line with Correl's (1978) definition of "enrichment" as any experience that substitutes, supplements, or extends instruction beyond that normally offered. NCTM (2014) asserted that effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow many entry points and varied solution strategies.

Additionally, the effective use of enrichment opportunities means that some students require more support than others. Therefore, an appropriate scaffolding and differentiation of content are emphasized. Though, Swan (2008) stressed the crucial role of students' collaboration, building on the knowledge that students previously studied, and creating tension and cognitive conflict to be resolved drawing on collective knowledge and discussion for multiple solution pathways.

Thus, as the second component of the mathematical enrichment, the classroom environment and teaching should facilitate constructive approaches through social interaction between students and teachers. The teacher's role is to provide the appropriate tasks, create an atmosphere in which students are not passive and use interventions that do not perform but extract mathematics from students by making mathematical connections and helping them bridge the knowledge gaps (Piggott, 2004). Constructivism emphasized, as a learning theory, the role of students rather than that of the teacher. In constructivism, students can use their prior knowledge and experience in testing ideas and apply these ideas to a new situation (Berns & Ericson, 2001).

Students' achievement in mathematics is based on building skills on top of one another. Mathematics skills are important across the school year as with other basic subjects, such as reading and writing, because performance depends on what the student learned previously and apply it to new concepts and applications. Unfortunately, the UAE students failed to apply their school learning of mathematics to new contexts from real life as revealed by the PISA results. To bridge this gap, teachers need to present mathematics in a real-world context. Thus, teachers can apply Contextual Teaching and Learning (CTL) which is a method that helps the teachers to relate subject content to real-world application and motivate students to make connections (Berns & Ericson, 2001; Hudson & Whistler, 2007).

This CTL method could help the students to improve the students' mathematical literacy because “among mathematical problems, those which have some applications in other branches of science and technology or the ones which have been essentially derived from real-life problems might be more attractive for students, since they bring life to the abstract concepts of mathematics which they learn, and make the concepts more tangible” (Adams, 2003, p. 794). Students in classes are taught the basic knowledge of mathematics but as abstract concepts. To give more meaning to what they have learned, students need to apply these concepts to real-life problems. This CTL provides the means for reaching learning goals that require higher-order thinking skills (Satriani, Emilia & Gunawan, 2012).

There are five strategies suggested by Crawford (2002) that could be used in contextual learning are; 1) relating; 2) experiencing; 3) applying; 4) cooperating or study group; and 5) transferring (REACT) (Satriani, Emilia, & Gunawan, 2012). These strategies are relevant to the skills of mathematics literacy. Moreover, Crawford stated that the REACT strategy affected student’s motivation and their learning outcomes in both mathematics and science (Maryani & Widjajanti, 2020). There are several connections between the steps and components of learning with indicators of mathematical literacy abilities (Maryani & Widjajanti, 2020). Hence, Mathematical literacy could be improved through the application of contextual learning. The connections between the steps and components of learning with indicators of mathematical literacy abilities are illustrated in Figure 8 (Maryani & Widjajanti, 2020, p. 7) below.

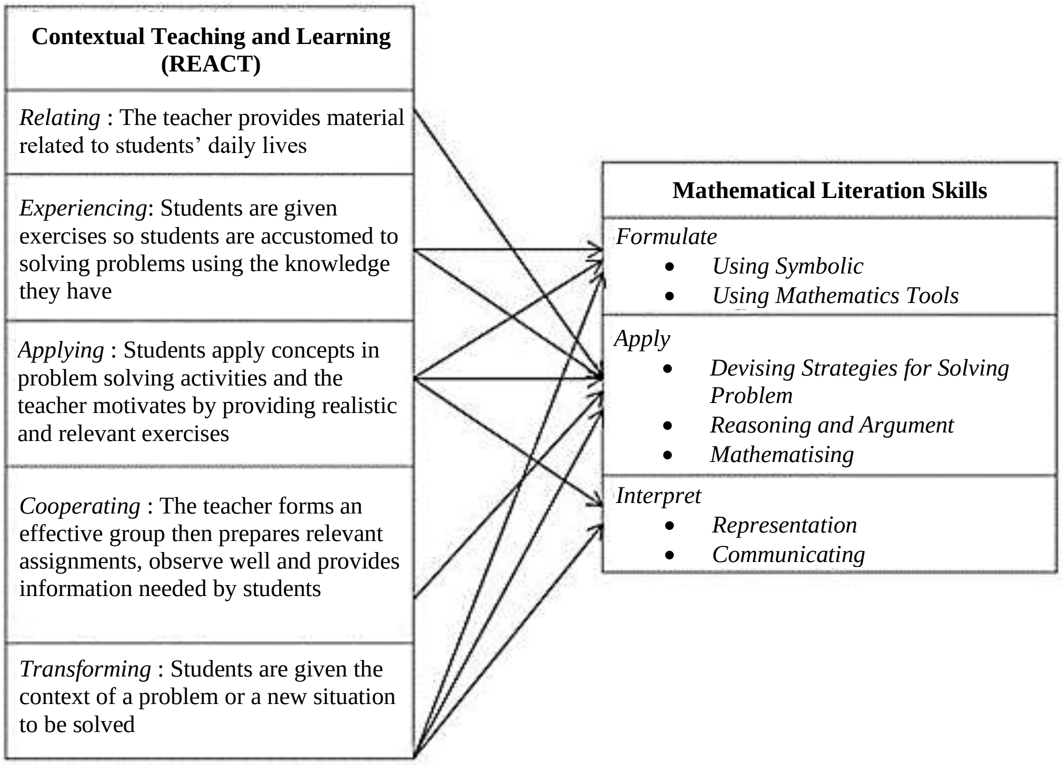

Figure 8: CTL and mathematical literacy

To apply CTL, there are five teaching approaches have emerged where context is its critical component to engage students in an active learning process. These approaches could be used individually or in combination with one or more others. These approaches are; Problem Based Learning, Cooperative Learning, Project-based learning, Service Learning, and Work-based learning (Berns & Ericson, 2001).

In this study, students were provided with extracurricular mathematical problems that give the students the chance to be exposed to contextual problems in a Problem Based Learning (PBL) as one of the CTL approaches. PBL is believed to promote the use of deep processing that means connecting different subjects together, and self-regulation and thus aims to stimulate high-quality learning (Wijnen, Loyens, Smeets, Kroeze & Van der Molen, 2017). This is because the learning strategies of students can be influenced by the instructional educational method applied in the study program (Vermunt, 2007). In addition, based on research that is supporting the use of PBL in education classrooms (Capon & Kuhn, 2004) and its correspondence with NCTM's Process Standards that make PBL a natural fit for the curriculum used in this study as an “enrichment” content.

Each of the NCTM standards can be found within the PBL goals. Hmelo-Silver (2004) stated that "PBL is designed to help students: (1) construct an extensive and flexible knowledge base, (2) develop effective problem-solving skills, (3) develop self-directed lifelong learning skills (4) become effective collaborators, (5) become intrinsically motivated to learn" (p. 240). The first four goals of PBL are also considered as core components to Process Standards (Communications, Reasoning & Proof, Problem Solving, and Connections) stated by the NCTM (2000) as described in their Principles and Standards. These Process Standards are important to describe ways to understand and apply mathematical content knowledge.

This approach reflects the constructive perspectives of learning through social interaction (Confrey, 1990). This knowledge building implies that learning builds on a student's prior knowledge, his interaction with resources, and interaction with members of their practice community. Additionally, the structure of a “good” problem itself requires the students to interact and build solution plans, revisit ideas, closely link with building on prior knowledge, and building mental patterns associated with a rational view of knowledge (Piggott, 2004). Each lesson in the “enrichment unit” was designed as such where the students are active in their learning with the teacher's guidance. In PBL, ill-structured and nonroutine problems let the students think of more than one solution using more than one strategy, while the students work together and use their prior experience to gain new information in the process of problem-solving, the teacher's role is to facilitate this collaboration.

2.7 Constructivism and Mathematical Problem Solving

This section is devoted to describing the theoretical propositions or concepts presented to support the importance of finding novel situations to stimulate and enrich students' problem-solving. This section addresses the theoretical perspectives on the key structures that frame this study, constructivism, and mathematical problem-solving.

The notion of constructivism is not new. It actually gained more focus in the 1980s as a new interpretation of learning shifted from the old notion that learners were seen as “disconnected knowledge processing agents” to be seen as “active knowledge makers or constructors” who bring a wide range of social and cultural experiences to their learning (James & Bloomer, 2001). Constructivism is “an approach to learning that holds that people actively construct or make their own knowledge and that reality is determined by the experiences of the learner” (Elliott et al., 2000, p. 256). Constructivism is widely known as an approach that helps the teacher in propping the level of the students' understanding and to show that this understanding can grow and change to higher-level thinking (Mvududu & Thiel-Burgess, 2012). Moreover, Arends (1998) states that constructivism believes in the personal construction of meaning by the learner through experience, and this meaning is affected by the interaction of previous knowledge and new events.

In this section, the focus is on constructivism from different perspectives that have influenced and continue to influence curricular developments. More specifically, the theoretical framework for this study is based on Constructivist approaches build upon Piaget and Vygotsky's ideas of using higher-order thinking and questioning techniques. Thus, what follows is a description of cognitive constructivism and social constructivism that comprise the basis of the emergent constructivism that is underpinning the theoretical framework of this study.

2.7.1 Cognitive Constructivism

The aim of cognitive constructivism as a teaching method is to support students in assimilating new knowledge to the existing knowledge. Constructivists claim that knowledge is not absorbed by students from extrinsic resources, rather they create their own meaning in their minds based on their prior knowledge and experience (Ertmer & Newby, 2013). Learning is seen as a process of active discovery as the knowledge is actively constructed. The teacher's role is to facilitate this discovery by guiding the students in their attempt to assimilate new knowledge to the old and to make modifications to the old to accommodate the new (GSI Teaching and Resource Center, 2016). According to Piaget, learning is an internal process that requires cognitive conflict to occur in the mind of the individual. The most important principle of Piaget's theory is the principle of equilibration that takes place in the process of adaptation (Powell & Kalina, 2009). Cognitive teaching methods aim to help students assimilate new information to existing knowledge and enable them to make appropriate adjustments to their existing intellectual framework to absorb that information (GSI Teaching and Resource Center, 2016).

Teachers and curriculum developers need to understand the critical transition in the student's cognitive development, how the students think and learn. Cognitive development is an active process when the students assimilate the new knowledge to the old, they will be able to make suitable adjustments to their existing intellectual framework to accommodate the new knowledge. Consequently, learning is relative to the students' stage of cognitive development (GSI Teaching and Resource Center, 2016).

According to Piaget, there are four main stages in the cognitive development of the child (Powell & Kalina, 2009). The "sensorimotor" stage during the first two years. The second stage is the "preoperational" and lasts until around the age of seven years. Then the next stage of "concrete operational" from seven to eleven years where the child begins to develop logic but only on concrete objects. The fourth and final stage is the formal operational stage, which is lasting from eleven years to the rest of the child's life. The child at this stage can approach mathematical problems intellectually in an organized way (Piaget, 1957).

Students at the age of 15 are generally in the fourth and final stage of Piaget's four stages of cognitive development that is called the formal operational stage. PISA is devoted to 15 years, old students. Thus, the released items of PISA are suitable for the students' stage of cognitive development. At this period, students are supposed to show improvement in their ability to think abstractly, use advanced reasoning skills, make hypotheses and inferences, and draw logical conclusions. Students at this stage of their life should be provided with new opportunities to adopt good thinking habits and mathematical practices. This stage is the focus of this research as it is important to understand what the abilities of the 15 years old students are. In this stage, the children could perform abstract intellectual operations, learn to formulate, and make abstract hypotheses. The children learn to appreciate others' opinion as well as their own.

In light of Piaget's theory, Gredler (2001) stressed the importance of the teacher role in choosing and providing the students with problems that allow the students to provide different ways to their solution, and all the students should be engaged in a problem-solving process that has an impact in encouraging the students and forcing them to think.

2.7.2 Social Constructivism

Hyslop-Margison and Strobel (2008) stated that Social Constructivism is a learning theory with roots in cognitive constructivism (Piaget, 1957) and sociocultural theory (Vygotsky, 1987). Even that Vygotsky agrees with Piaget's claim that learners respond to their interpretation of stimuli, not to external stimuli, however, he claimed that Piaget had ignored the social component of learning. Social Constructivism emphasizes that learning, like all cognitive functions, is dependent on interaction with others such as teachers, students, and parents (Vygotsky, 1999).

The process of learning mathematics requires the students to actively construct the meaning of concepts through individual re-construction of knowledge and social interaction with other students, and teachers (Belbase, 2014). Learning is seen as a contextual process that depends critically on its collaborative nature as the students learn from each other or with their teacher guidance (Schunk, 2012). Based on social constructivism, learning never comes from scratch, the successful learner embeds new learning within old to expand understanding to incorporate the new experience. According to social constructivism, learning is neither connected to the external world nor the learner's mind, but it exists as the outcome of mental contradictions that result from group interactions with the environment (Schunk, 2012).

Moreover, learning as seen in social constructivism is based on real-life adaptive problem solving which results from social interaction through shared experience and discussion with others where the learner adapts rules to make sense of the world based on matching the new ideas against existing knowledge. Kukla (2000) stated that societies together create the properties of the world as the reality-based on social constructivism is seen as constructed not discovered through human activity.

Vygotsky stressed the social learning leads to cognitive development. An important concept that is essential to this socially mediated cognitive development is the “Zone of Proximal Development” (ZPD) that has been introduced as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem-solving under adult guidance, or in collaboration with more capable peers" (Vygotsky, 1978, p. 86). In the ZPD there are two levels: the actual development level that indicates where the student has already reached, and to what level is the student capable of solving problems independently. The level of potential development is the ZPD level where the students can reach under the guidance of the teacher or by collaboration with other students. In the ZPD, the students will be able to solve problems and understand topics that they are not able to solve or understand at their level of actual development. This means that the level of potential development is the level at which learning happens.

The teachers should be aware that on one hand unless students are challenged and active in their learning, they will lose attention, while on the other hand, if the students face a great challenge, they might just give up. Vygotsky (1978) explained that “learning which is oriented toward developmental levels that have already been reached is ineffective from the viewpoint of the child’s overall development” (p. 89).

Therefore, according to Vygotsky, the intellectual development theory implies that teachers should organize learning to be just above the level of the actual developmental level of the individual student through the interaction of peer tutoring, collaboration, and small groups.

Vygotsky believes that the education role is to provide the students with suitable exercises within their ZPD to encourage and improve their learning. The teacher's role is facilitating the students learning as they share knowledge through social interaction that is considered an effective way of developing skills (Dixon-Dixon-Krauss, 1996). Vygotsky's theories imply the importance of collaborative learning as the group members have different levels of ability where the more advanced peers can help less advanced members function within their zone of proximal development (McLeod, 2019).

The ZPD and the term "scaffolding" are mentioned together like two faces for the same coin in literature as ZPD is defined as the area where a child can solve a problem with the help (scaffolding) of an adult or more competent peer (McLeod, 2019), while Wood, Bruner, and Ross (1976) defined scaffolding as the process to help and lead a child or novice through the ZPD to solve a task or achieve a goal that is not reachable without others' help. In the classroom, scaffolding strategies could be modeling a skill, providing hints or cues, and adapting material or activity (Copple & Bredekamp, 2009).

2.7.3 Emergent Constructivism

In response to educational reforms, the constructivist approach to teaching and learning is supported by many math educators (Draper, 2002). This research is framed by the emergent perspective of constructivism that draws on both Piaget and Vygotsky. For examining the mathematical growth of students, the emergent perspective is suitable "as it occurs in the social context of the classroom" (Cobb & Yackel, 1996, p. 176). The knowledge, in the emergent perspective, is personally constructed in the medium of social interaction and that individual and social dimensions of learning to complement each other (Tobin & Tippins, 1993; Murray, 1992; Cobb & Yackel, 1996). Students have an opportunity for mathematical learning when they try to understand other people's interpretations and when they try to compare their own solutions with those of others (Cobb & Yackel, 1996). From an emerging perspective, mathematics is a social activity as well as an individual activity. When negotiating criteria in the classroom, the teacher plays a pivotal role in initiating and guiding these standards, but the individual student has an active role in this formation as well (Cobb & Yackel, 1996).

Constructivists depend on teaching practices that are rich in conversation and constructivists understand that experience, environment play important roles in learning where learners create their own knowledge based on interaction with other people (Draper, 2002). Classroom discourse is one of the seven NCTM Standards for teaching and learning mathematics (NCTM, 2007). The NCTM states, "In practice, students' actual opportunities for learning depend, to a considerable degree, on the kind of discourse that the teacher orchestrates" (p. 32). Findell (1996) stated that for student-centered math classrooms, the teacher should play the role of questioner and problem poser, so students make sense of their learning.

2.8 Motivation

The majority of secondary students' achievement and engagement in mathematics decline; thus, the teachers are challenged to engage the students to avoid school failure. Despite that students' attention is drawn in many directions, their participation is crucial to solving problems and participating in meaningful class discussions. In fact, teaching mathematics is a much broader effort than simply helping students to acquire skills and problem-solving strategies. Therefore, teachers also attempt to develop motivation and positive dispositions toward studying mathematics. This will have long-term consequences on students' confidence to do mathematics and on their career choices.

George Pólya wrote in his book "How to solve it" on mathematical problem solving as follows:

> Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery. …If he [the teacher] fills his allotted time with drilling his students in routine operations, he kills their interest… (Pólya, 1945, p. v)

This quotation reflects the importance of motivation and dispositions for learning and teaching mathematics and the development of mathematics education. Additionally, The National Research Council's Adding It Up: Helping Children Learn Mathematics describes mathematical proficiency as five interconnected strands (Kilpatrick, Swafford & Findell, 2001): 1. Conceptual understanding 2. Procedural fluency 3. Strategic competence 4. Adaptive reasoning 5. Productive disposition. The first four strands are also components of mathematical literacy as mentioned before. Concerning the fifth strand, students with a productive disposition see that mathematics makes sense and both useful, and worthwhile, believing that continuous effort pays off, and see themselves as effective learners and practitioners of mathematics (Kilpatrick, Swafford & Findell, 2001). These issues of disposition received more attention, two books were published by NCTM for teachers about mathematics-related motivation issues included engagement, and dispositions (Brahier & Speer, 2011; Middleton & Jansen, 2011). For example, NCTM's 73rd annual book, Motivation, and Disposition: Pathways to Learning Mathematics, explores a variety of perspectives on motivation and disposition as they relate to mathematics teaching and learning (Brahier & Speer, 2011).

Many themes were discussed in the literature on students’ motivation to learn mathematics. One of the important themes is that students’ motivation could be affected by instructional practices and tasks despite that the motivation to learn is acquired early and remains almost the same over time (Middleton & Jansen, 2011). Teaching methods have the potential to encourage student participation and motivation through specific activities (Middleton & Jansen, 2011; Turner & Meyer, 2009). Thus, students’ level of motivation could be influenced by the students’ experiences in their lives out of school.

Contextual tasks are an important long-term part of school mathematics (NCTM, 2000, 2014), and under certain circumstances, they may be an appropriate source for enhancing student engagement (Hernández, Levy, Felton- Koestler & Zbiek, 2016). Research indicates that teachers tend to consider contextual problems mainly in terms of their potential to motivate their students (Lee, 2012; Pierce & Stacey, 2006). Students' understanding of formal mathematical concepts can arise from problem-solving experience or contextual problems that rely on contexts outside of mathematics (Freudenthal, 1991). Thus, curriculum materials have been created in line with this recommendation (Robinson, Robinson & Maceli, 2000). The contextual problems, for this study, are the main component of an enrichment program that is developed to improve the students' mathematical literacy level; therefore, this part of the study aims to examine the effect of the contextual problems in mathematics on the students' motivation to learn mathematics. This can inform curriculum development and instructional decisions. In this enrichment program, a variety of contextual tasks were a regular part of instruction and the students worked on contextual tasks on their own, in small groups, or as a class multiple times per week. Arguably, for almost all students, when they see the importance of their school mathematics in real life, they become more enthusiastic about learning mathematics (OECD, 2013, 2018).

2.8.1 The Significance of Studying Motivation

Based on the literature, students' motivation to learn mathematics is an important outcome to study for many reasons: first, Motivation has been empirically linked to student achievement in mathematics. Despite the complex relationship between them, the quality of student engagement is linked with positive gains in learning (Fredericks, Blumenfeld & Paris, 2004). Second, some claim that a student's positive mathematical behavior should be considered as a primary intended outcome of mathematics education (Kilpatrick, Swafford & Findell, 2001; Brophy, 2008). Third, engagement and motivation to learn mathematics is meeting the reform calls that require students to be actively involved in their learning (NCTM, 1989, 2000; CCSSI, 2010). Hence, it is worth considering the features of educational curricula and practices that may influence student engagement and motivation. The relevant contextual problem is a strong place to start in increasing students' motivation due to its feature of presence in nearly all mathematics curricula.

However, the claim that contextual tasks have the potential to enhance student motivation appears to contradict the notion in a school culture that students fear story problems. This issue was evident in the description of a typical mathematics classroom, Wilson (2003) wrote, “Ample time is usually left for practicing problems, and an audible sigh of relief is heard whenever word problems are not assigned” (p. 4). This indicates the need for further investigation into the relationship between student motivation and contextual tasks and more specifically for the UAE students.

2.8.2 Definition of Contextual Problems and Motivation

The term "contextual problems" refers to any problem that includes some kind of realistic or fictional scenario that is described at least partially through a non-mathematical language or a non-mathematical picture representation (Li, 2000). This term covers many terms found in literature such as word problems, story problems, real-world or applied problems, and real-world connections. Additionally, contextual problems are defined as the problems that involve setting that exist outside of pure mathematics (Reinke, 2019)

In this study’s framework, motivation is assumed not to be static traits of the learner but rather that “motivation is dynamic and contextually bound” (Duncan & McKeachie, 2005, p. 117). This means that the students' motivations change from course to course based on the nature of the course and the students’ interest in the course. Pintrich and Schunk (2002) also clarified motivation as the process where the students initiate and sustain goals that direct their activities. Motivation includes the underlying causes of people's behavior (Middleton & Jansen, 2011).

Motivation is a comprehensive construct that is related but not equivalent to many constructs like values, beliefs, and attitudes. Motivation is used to describe a person’s choice, persistence, and performance when engaging in an activity, especially goal-oriented activity (Brophy, 2004). This research aims to consider the effects of contextual tasks on motivation. Consistent with this aim, Brophy (2004) gave more attention to students’ “motivation to learn” meaning as a “tendency to find academic activities meaningful and worthwhile and to try to get the intended learning benefits from them” (p. 16). The focus of this definition is on learning which makes it different than extrinsic motivation which relates to performance-based rewards like grades.

In addition, motivation to learn is also a different construct than intrinsic motivation, which relates to the enjoyment of an activity. However, motivation to learn can coexist and be supported by extrinsic and intrinsic motivation. The motivation to learn, in essence, differs from the motivation to do other things or activities because of the nature of the school and its constraints (Brophy, 2004). Although the term engagement is not equivalent to motivation, they are sometimes used synonymously (Fredericks & McColskey, 2012). Motivation is the generally unnoticeable mechanism underlying people's behavior, while engagement is the observable appearance of motivation (Skinner, Kindermann & Furrer, 2008).

2.8.3 Contextual Problems and the Motivation to Learn Mathematics

Researchers have suggested several ways in which contextual problem solving can enhance and complicate mathematics education (Blum & Niss, 1991). Contextual problems are believed to provide motivation, enhance student participation and engagement, and develop students' abilities to apply mathematics to extra-mathematical situations in the future (Reinke, 2019). Moreover, Walkington, Sherman and Petrosino (2012) point out that students, in some cases, are more successful in solving contextual problems than non-contextual problems. Researchers also note that some contextual problems cause students to take advantage of their individual and family experiences outside of school in understanding mathematical ideas, which support mathematics learning and confirm their cultural identities (Turner et al., 2012). Problem-based learning is expected to improve the student's mathematical literacy as students will be extremely motivated to learn and develop high-level thinking skills, teamwork, and communication (Tan, 2007).

In fact, it is believed that adding the contexts to mathematics problems have the potential to promote student motivation in addition to other purposes (Middleton & Jansen, 2011). On one hand, the contextual tasks were the core of many curricula and instructional research programs in the past 30 years in alignment with recommendations from the research community (Lloyd, Herbel-Eisenmann & Star, 2011). Due to the NCTM recommendations to put more emphasis on real-world applications or to connect students' everyday mathematics and school mathematics, several countries including the US and UK have shifted their curriculum to develop mathematical ideas in context (Robinson, Robinson & Maceli, 2000).

From the developers' point of view, these tasks can result in catching the students' interest and engaging them in mathematics as well as supporting their learning (Lappan & Phillips, 2009). However, Thomas and Gerofsky (1997) indicated that many people think of contextual tasks negatively, they call it "the hated word problems". Thus, educators call for a reconsideration of the contextual tasks' role in the curriculum; they have argued that believing contextual tasks will motivate students is too simplistic (Gerofsky, 2004; Verschaffel, Greer & De Corte, 2000).

2.8.4 Theoretical Framework for Motivation

The modern motivational paradigm is dominated by cognitive theories that claim that individuals’ ideas, beliefs, and emotions together influence motivation (Wigfield & Eccles, 2002). According to the social-cognitive perspective, students’ motivation is relatively situation or context specific (Pintrich, Marx & Boyle, 1993). This approach emphasizes the important role of students' beliefs and interpretations of actual events, as well as the role of the achievement context for motivational dynamics (Pintrich et al., 1993; Wigfield & Cambria, 2010). Based on social cognitive theory, students who have or appreciate positive feelings about mathematics or have higher expectations for success tend to do more, learn more, and show higher mathematics performance (Pintrich & Schunk, 2002). In addition, students' emotional interactions with the task and the performance of their task affect their effort, perseverance, and performance (Pintrich & Schunk, 2002).

Motivation is not directly noticeable, and therefore the conceptual framework setting is important for its measurement. Motivation theories are concerned with understanding what motivates people to act in a specific way, and what makes an individual choose the direction and intensity of actions. Thus, some influential theories about motivation to learn mathematics will be reviewed. Motivational theories in education study cognitive, social, behavioral, and self-regulation perspectives as well as perspectives from self-determination theory (Ryan, 2012). Three theoretical approaches are particularly important for motivational processes in the context of (mathematics) education and more specifically suggest the theoretical potential of contextual tasks to motivate students to learn mathematics: self-determination theory, expectancy-value theories, and achievement goal theory.

2.8.4.1 Self-Determination Theory (SDT)

It is one of the most powerful motivational theories. SDT is a general theoretical framework used to study human motivation (Ryan & Deci, 2002). The SDT focuses on the degree to which an individual's behavior is self-motivated and self-determined. The SDT distinguishes different types of motivation based on different types of causes or goals that produce motivation. Intrinsic and different types of extrinsic motivation are distinguished to explain motivated behavior (Ryan & Deci, 2000). Among the issues, a central assumption of self-determination theory is that humans have innate psychosocial needs (i.e. competence, independence, and social relatedness) that develop in interaction with the surrounding social context and help to understand the process of goal chasing (Ryan & Deci, 2002, 2017). It is believed that meeting these needs is accompanied by positive emotional experiences (Ryan & Deci, 2002) and it permits individuals to develop intrinsic motivation and achieve a more in-depth understanding of the learning content, which in turn can contribute to positive achievement.

Intrinsic motivation: It is when the work is done "for its own sake," without expectation of external rewards. It entails personal development, enjoyment, or exploring that leads to feelings of "internal rewards,' or enjoyment of the moment. The focus is on the process more than the result itself (Deci & Ryan, 1985).

Extrinsic motivation: It indicates goals or reasons for reaching an external reward or avoiding negative consequences. Extrinsic motivation is often described as opposing intrinsic motivation; however, it can be intrinsic to the self to aspire to external rewards (Deci & Ryan, 1985).

Both internal and external factors guide the motivating behavior in practice. The difference between individuals lies in the balance between internal and external motivation.

2.8.4.2 Expectancy-Value Theory (EVT)

The most widely used expectancy-value model of achievement motivation derives from the more recent work of Wigfield and Eccles (2000). It is one of the social cognitive models of achievement motivation. This model shows that beliefs about value and expectation of success are related to the effort in learning mathematics. The focus of this model is on the role of students' expectations for academic success and how they perceive the value for academic assignments; it is based on personal, social, and developmental psychology (Pintrich & Schunk, 2002).

The EVT comprise a variety of motivation constructs that can be organized into two broad categories (Pintrich et al., 1993) : an expectancy of success that is reflected by the question “Can I do this task?” and value components that correspond to the question “Do I want to do this task and why?”. The EVT is believed to influence the engagement in a subject, educational choices, and ultimately achievement. The expectation component of the model refers to one's beliefs and judgments about his or her abilities to do and succeed in a task. Expectancy beliefs, including self-concept, ability perceptions, and expectancy for success. Both components are important predictors of achievement behavior (Wigfield & Eccles, 1992).

The value component of the model indicates the different reasons individuals have to engage in a task or not, and the strength of these values. The use of contextual tasks to motivate students is particularly supported by the value aspect of theory because it addresses the belief in the usefulness and personal relevance of the content (Wigfield & Eccles, 2000) and “awareness of its role in improving the quality of our lives” (Brophy, 2004, p. 133). In this model, value is consisting of four components: importance, interest, utility, and cost (Wigfield & Eccles, 1992). The attainment value is about the importance of doing well on a task. The second element is intrinsic value or interest, which is about the degree to which a person enjoys doing a task (Wigfield & Eccles, 1992). The third element, utility value indicates the usefulness in terms of the individual’s future goals (Pintrich & Schunk, 2002). The last component is the cost, which is perceived as the negative aspects associated with the task, such as the effort involved or the loss of opportunities to perform other tasks. Intrinsic value is conceptually similar to intrinsic interest in the SDT (Deci & Ryan 1985), while utility value resembles the extrinsic motivation component in the SDT (Michaelides et al., 2019).

2.8.4.3 Achievement Goal Theory

Students’ goal orientations are broader cognitive orientations that students have toward their learning and they reflect the reasons for doing a task (Dweck & Leggett, 1988). This theory is consistent with the previous two theories and assumes that students have different reasons to either engage or not in their learning (Pintrich, 2000). According to Patrick et al. (2011), these reasons affect what, how, and why students learn and perform. Two types of achievement goals are identified (Dweck & Leggett, 1988); (1) the mastery goal in which a mastery-oriented person is learning for the task's own sake (similar to intrinsic motivation in the SDT and intrinsic value in the EVT). (2) the performance goal where it reflects the desire to compare performance relative to others. The goal here is to do well and get rewards associated with high performance (Similar to extrinsic motivation in the SDT and utility value in the EVT). Each of these two goals has different consequences in the context of achievement, with mastery being associated with higher performance than performance orientation (Michaelides et al., 2019).

Taking into consideration that PISA also is studying the association between mathematics motivation and mathematics achievement in high school-aged students. This study looked at two different types of motivation as described by OECD (2013): intrinsic motivation, which described if students enjoyed and were interested in math, and instrumental (extrinsic) motivation, which described if students valued math for its role in their education or career goals—and found both correlated to math achievement (OECD, 2013). Chiu, Pong, Mori, and Chow (2012) believe that the way students engage in learning and doing mathematics is due to their beliefs about the value of mathematics. For example, Deci and Ryan (2000) indicated that students who love to learn mathematics (intrinsic motivation) often show higher achievement in mathematics, as well as students who see mathematics as a useful tool for other goals (instrumental motivation) (Pintrich & Schunk, 2002).

This program was mainly designed to connect mathematics to students' life based on the mathematical literacy framework because it can be said that for almost all students, the motivation to learn mathematics increases when they see the importance of what they have learned to the world outside of the classroom and other subjects (OECD, 2013, 2018). To study motivation to learn mathematics, the expectancy-value model was adopted that reflects the social cognitive theory. Expectancy-value theory stresses appreciating the worth of learning specific topics (Brophy, 2008). It is suggested that making the content meaningful and relevant, and connecting it to other important aspects of students' lives, will support their motivation to learn. The focus of this study is on two main scales namely-- intrinsic goal orientation and extrinsic goal orientation. The items were selected and modified from the scales of intrinsic and extrinsic motivation in the MSLQ questionnaire (Pintrich et al., 1991).

2.9 Related Studies

2.9.1 Related Studies on MEP

Jarrah and Almarashdi (2019) designed a survey of 19 statements to learn about mathematics teachers' perceptions regarding the gifted education implemented in their schools. The sample consisted of 66 mathematics teachers from Al Ain city in the UAE. The results showed that teachers were generally positive about gifted education and their proficiency in teaching gifted students. However, they mostly have negative perceptions regarding the effectiveness of gifted programs Another study (El-Demerdash, 2010) emphasizes the importance of having appropriate enrichment programs where students can learn and develop their creative potential if appropriate programs are used that successfully teach them the necessary creative skills and processes. The study used interactive geometry software to design an enrichment program then examined its effect on students' creativity in geometry. This study was applied to one group experimental design. The results showed improvement in students' creativity in geometry due to the enrichment program.

Nuurjannah and Sayoga (2019) conducted a quasi-experimental research design with a nonequivalent control group. Their research aimed to study the achievement and improvement of mathematical literacy ability of junior high school students in Indonesia. The contextual approach was applied to the experimental group of 33 students in the eighth grade, while 27 students in the eighth grade received conventional learning in the control group. The results showed that the acquisition of the experimental class posttest was higher than the control class. Similarly, Laurens, Batlolona, Batlolona and Leasa (2018) applied quasi-experimental research to investigate the difference in students' mathematics cognitive achievement after implementing Realistic Mathematics Education (RME) on a group of students, while another group receives conventional learning. The results confirmed the previous research by Nuurjannah and Sayoga (2019), the students who were taught with RME achieved better than the students who were involved in conventional learning.

Contextual teaching and learning can be applied through problem-based learning (PBL). To study the effectiveness of PBL versus traditional learning, various studies were performed to compare the two. For example, Wardono, Waluya, Mariani, and Candra (2016) investigated the effect of the PBL model using the Indonesian Realistic Mathematics Education (PMRI) approach that is in line with PISA mathematical literacy. Their research aimed to verify whether their model could improve the mathematical literacy ability of 7th graders in "Change and Relationship" content. The research applied a mixed-method study. Random sampling was applied to choose two experimental classes where the PBL with PMRI approach assisted E-learning was used for the first class, while the second used PBL only with the PMRI approach and the third class was the control class that used the expository method. They used documentation, tests, and interviews to collect data. The results of this study showed that the average mathematical literacy ability for both experimental classes was better than the control group. Moreover, the class that assisted in e-learning achieved the best improvement in mathematical literacy ability.

In another study by Firdaus, Wahyudin and Herman (2017), PBL was used to improve the mathematical literacy of fifth-grade students. The research approach used was a quantitative approach using a quasi-experimental method nonequivalent groups design pretest-posttests. The results of this study indicated that the PBL was more effective in improving students' mathematical literacy model than direct instruction. Moreover, no effect was found for the school's location on students' mathematical literacy.

Dolmans, Loyens, Marcq and Gijbels (2016) reviewed twenty-one studies dealing with PBL and students’ approaches to learning to investigate the effect of PBL on students’ deep and surface approaches to learning. The results indicate that deep learning was improved by PBL with a small positive average effect size of 0.11 and the positive effect was found in eleven studies, no effect on six studies, and a negative effect of four studies on deep learning. The review showed that no effect for PBL on surface learning with a very small effect size of 0.08 where eleven studies show no effect, six with a negative effect, and four show an increase in the surface approaches.

To improve the students’ mathematical literacy, Dewantara, Zulkardi, and Darmawijoyo (2015) conducted a study that aimed to produce PISA like mathematics tasks that are valid and practical. Then they focused on the activation of students' mathematical abilities underlying mathematical processes related to mathematical literacy as the main potential effect of the developed PISA-like tasks. Data were collected from a study sample of 28 students from seven students using student tests and interviews. The findings of this study indicated that the 10 developed items of PISA- like are most likely to enhance students' mathematical literacy within three mathematical processes. Moreover, the highest percentage of students' achievement in interpreting tasks was more than employ and formulate. In the same vein, Nizar, Putri, and Zulkardi (2018) also aimed to produce valid and practical PISA-like mathematical problems on the content of uncertainty and data using football and table tennis contexts in the 2018 Asian Games. Another aim of this study was to find the effect of these problems on the mathematical literacy of tenth-grade students. Data were collected using a walkthrough, document, observation, interview, and test methods. The results of this study indicated that these problems had the potential effect, showing the capability of communication and representation as revealed by a sample of 33 students of field test responses. Other similar studies on uncertainty and data by Efriani, Putri, and Hapizah. (2019) and Putri and Zulkardi (2020) also supported the same results. Another research by Ahyan, Zulkardi and Darmawijoyo (2014) produced PISA like problems that are valid and practical in the content of change and relationships and has potential effect for Junior High School students. The researchers developed and implemented 13 problems. The results showed that 12 of the developed mathematical problems were valid, practical, and had potential effects for Junior High School students.

2.9.2 Related Studies on Motivation

Few studies were found after reviewing the literature on mathematics education literature where they directly investigated the effects of contextual tasks on student motivation. The first study was found by Cordova and Lepper (1996) to investigate the effect of contextualizing and personalizing mathematics content as well as how the provision of choice on students’ intrinsic motivation, achievement, and other factors. The sample consisted of 70 students from grades four and five who played a specially designed computer game intended to teach order of arithmetic operations for 30-

minute sessions. The results showed that most students experienced greater motivation to learn and engaged in more challenging forms of play when the content was put in imaginative contexts, and the context scenario was personalized. Thus, the study supported the use of contextual mathematics tasks (the personal contexts) to promote student engagement and enjoyment of specific tasks. However, in this study, the intrinsic motivation to learn was problematic as its source might have resulted from the use of computers rather than the engagement of the content itself.

The second study by Ku and Sullivan (2000) approve Cordova and Lepper's (1996) findings related to the personalization of contexts in a traditional classroom environment. Ku and Sullivan (2000) study the effect of personalizing mathematics word problems for 5^{th} grade Taiwanese students. The researchers stated that students in the treatment group indicated significantly more positive attitudes and higher motivation related to the lessons and tasks than the control group. This study followed the personalization technique where textbook word problems were modified according to the most common interests and experiences. In addition, it has another strong point as it was completed in the students' classrooms rather than the computer environment used by Cordova and Lepper (1996).

In more recent research, a study conducted by Wijnia, Loyens, and Derous (2011) that compared the effects of PBL versus lecture-based environments on undergraduates' study motivation. The data was collected using a survey. The results revealed that PBL students scored higher on competence but did not differ from lecture-based students on autonomous motivation. The focus group students indicated that the collaboration was motivating, while other controlled factors such as mandatory presence were perceived as detrimental for students' motivation. The researchers concluded that PBL does not always seem to lead to higher intrinsic motivation. Thus, even in learning environments that are intended to be motivating for students, it is vital to balance controlling elements versus autonomy and to build the right amount of structure in the learning environment.

Moreover, a research paper written by Widjaja (2013) addresses the use of contextual problems to support mathematical learning based on current classroom practices in Indonesia. Examples of using contextual problems from the elementary classes were presented in this paper. Although the use of contextual problems can provide students with the potential to engage and motivate students in learning mathematics, it also presents some challenges for students in the classroom. The study found that contextual problems are not suitable for students' meaningful learning. Educators need to engage students in interpreting context to explore key mathematical ideas. In conclusion, establishing clear links between context and mathematical ideas is very important to support students' progress in their mathematical thinking.

Mulyono and Lestari (2016) suggested that students could develop their mathematical skills if they had high self-efficacy toward mathematics. Hence, their research aims to analyze students' mathematical literacy and self-efficacy using Search, Solve, Create, and Share (SSCS) learning with a contextual approach. The study showed that the mathematical literacy of students who were taught using SSCS learning with a contextual approach was better than students who were taught using traditional learning. Additionally, the results show that learning SSCS with a contextual approach can improve a student's self-efficacy.

2.9.3 Related Studies on Gender Differences

The existing literature on gender and academic achievement have different perspectives and findings. For example, the trend in OECD countries has been that male student achievement in mathematics in PISA outperforms females with males scoring five points higher than females (OECD, 2019b). However, student results in the UAE were inconsistent with the OECD trend where females demonstrated better mathematical literacy than males. The results showed girls outperforming boys in mathematics by nine points (OECD, 2019b).

Some of the study results were consistent with the trend in OECD countries. For example, Reis and Park (2001) focused their research on studying gender differences in students who excelled in mathematics and science concerning five factors including their achievement and self-concept. The study sample consisted of two sub-samples of distinguished students from two national studies. Their research results revealed that students excelling in mathematics and science outnumbered females in both subsamples. In addition, high-achieving males had higher self-concept and higher scores on standardized mathematics tests than high-achieving females. Whereas, contradictory results for the OECD trend were found by Hyde, Fennema, and Lamon (1990) who conducted a meta-analysis of 100 studies published from 1963 through 1988 of gender differences in mathematics performance. The results showed that females slightly outperformed males in computation and understanding mathematical concepts, while males were better in complex problem-solving. Notably, there were no gender differences in problem-solving in elementary or middle school, while females performed lower in problem-solving in high school. The gender differences in mathematics performance are small and the magnitude of the difference between the genders decreased over the years. Similar results were found by Stage, Kreinberg, Eccles, and Becker (1985) in their review of studies related to gender and achievement found that males perform better on reasoning tasks, while females perform better on computational tasks. Moreover, Fennema (2000) also found that there are males who were better than females concerning problem-solving. It has also been found that females tend to use more standard algorithms than boys, while males tend to innovate or use untaught problem-solving strategies more than females. Additionally, Innabi and Dodeen (2018) studied the gender differences in mathematics achievement of Jordanian students using TIMSS data for 2015. The results showed a marked superiority of females over male students. In more detail, the results showed that males were better at answering more difficult, unfamiliar, and life-related mathematical problems than females, while females were better at answering familiar, less difficult, and non-life-related problems.

However, some studies have found that does not support this trend. Ghasemi, Burley and Safadel (2019) examined data of fourth- and eighth-grade girls and boys from two international databases, namely IEA’s TIMSS 2015, and the World Economic Forum’s Global Gender Gap Report 2017 to study the gender difference in general achievement in mathematics. The results of their study revealed that no statistically significant large differences were observed comparing the performance of girls and boys in mathematics achievement and the number of high achievers. Moreover, Ajai and Imoko (2015) designed a pre-posttest quasi-experimental study to assess gender differences in mathematics achievement and retention using PBL. The study sample consisted of 260 male and 167 female students. The researchers developed the algebra achievement test as the main tool for gathering data and then analyzing these data using the t-test. The results of the study showed the female students’ performance was slightly better than male students, however, there is no significant difference between male and female students’ achievement and retention scores as well.

In a study conducted by Garduño (2001), the gender differences in self-efficacy, attitudes toward mathematics, and achievement of 48 gifted seventh- and eighth-grade students were investigated after participating in a two-week course on probability and statistics that was taught with a mathematical problem-solving approach. The students were randomly assigned to one of two experimental groups or to a control group during a summer enrichment program. The research findings indicated that there were no statistical differences in achievement or self-efficacy among the groups. However, the difference was statistically significant in attitudes toward mathematics favoring students in the whole-group instruction, competitive setting.

Research has confirmed gender differences in mathematics motivation indicating that males generally have better motivational profiles in mathematics than females (Kurtz-Costes et al., 2008). For example, Rodríguez, Regueiro, Piñeiro, Estévez and Valle (2020) conducted a study in Spain with a sample consisted of 450 males and 447 females students from 5th and 6th grades. Their study aimed to verify gender differences and to examine the explanatory potential of males' and females’ attitudes toward mathematics on their performance. The results supported the previous research that females’ attitude was less positive than males’ in particular lower motivation, worse perception of competence, and higher rates of anxiety with small effect size in all cases.

Various results emerged from a subsequent meta-analysis of 100 studies of gender differences in mathematics as well as 70 studies on gender differences in attitudes and affect associated with mathematics (Frost, Hyde & Fennems, 1994). The results revealed that males are slightly outnumbered females in mathematics performance, while the attitudes and affects of females were more negative concerning mathematics. Generally, gender differences in math performance and attitudes and affect appeared to be small to moderate only. A more recent meta-analysis analyzed the gender differences of mathematics performance in 242 studies published from 1990 to 2007 showed that males and females perform similarly in mathematics (Lindberg, Hyde, Petersen & Linn, 2010).

2.10 Concept Map of Theoretical Framework

The theoretical framework of this study could be summarized by the following concept map as shown in Figure 9 below.

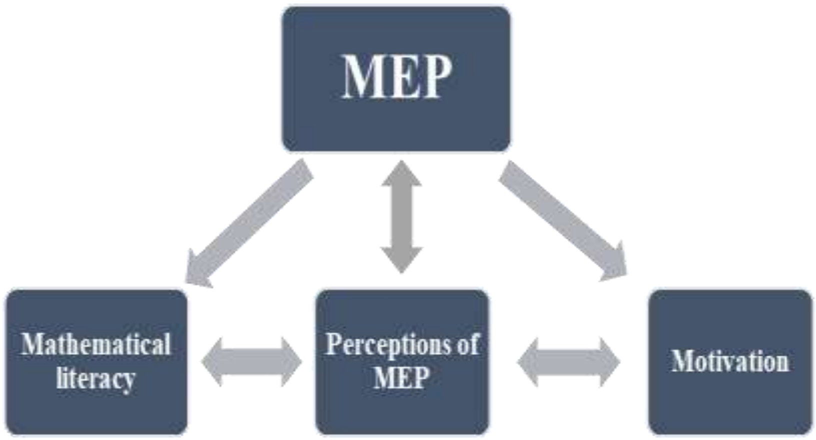

Figure 9: The concept map of the framework

Figure 9 presents the concept map of the theoretical framework of this study where the enrichment activities that consisted the MEP aim to provide students with a stimulating mathematical experience, promote positive attitudes, raise the level of achievement, and contribute to efforts to enhance, generalize, and increase the general understanding of mathematics.

According to Feng (2006) "using this interpretation of enrichment, the engagement of all students in meaningful mathematical practices is an essential and worthwhile part of education; this also forms the main goal of mathematics enrichment". This conceptualization promotes the linking of mathematical content presented separately in the curriculum with mathematical content and other fields of study. By providing students with a stimulating experience in mathematics, enrichment promotes mathematical thinking and problem-solving. It is important to note that depending on the different levels of students in the classroom, students will need different levels of support to take advantage of enrichment opportunities. Thus, enrichment in this sense emphasizes appropriate scaffolds and content differentiation: enrichment tasks are often designed to use mathematical concepts and techniques at various levels of difficulty and may lead to qualitatively different endpoints (Feng, 2006; Piggott, 2004).

Researchers have suggested several ways in which contextual problem solving can enhance and complicate mathematics education (Blum & Niss, 1991). Contextual problems are believed to provide motivation, enhance student participation and engagement, and develop students' abilities to apply mathematics to extra-mathematical situations in the future (Reinke, 2019). Based on social cognitive theory, students who have or appreciate positive feelings about mathematics or have higher expectations for success tend to do more, learn more, and show higher mathematics performance (Pintrich & Schunk, 2002).

Chapter 3: Methodology

This chapter aims to provide a detailed description of the study methods, data collection procedures, and tools used in data collection. It describes in detail the proposed enrichment program, the mathematical literacy test, and the motivation survey and how the students evaluate and perceive their MEP experience. In addition, this chapter describes the method of selecting participants and the context in which the study took place. The validity and reliability of the instruments used in this study.

3.1 Methods

This research applied the mixed methods design. Mixed-methods research is defined by Teddlie and Tashakkori (2011) as an inquiry that is conducted by "selecting and then synergistically integrating the most appropriate techniques from a myriad of QUAL, QUAN, and mixed methods" (p. 286). The method used in this study was based on a quantitative and qualitative (QUAN-Qual) model that is also known as an explanatory mixed methods design (Gay, Mills & Airasian, 2012; Mills & Gay, 2019). More specifically, it is represented using the symbol as "QUAN ⟶ qual" (deductive-sequential design, where the core component is quantitative and the supplemental component is qualitative) (Schoonenboom & Johnson, 2017). Kansteiner (2020) indicated that the Schoonenboom and Johnson (2017) approaches have been widely accepted because it is said to have many advantages ranging from philosophical positions to the question of method. This type of mixed-methods research is quantitatively dominated in which one relies on a quantitative, post-positivist view of the research process while recognizing at the same time that adding qualitative data and approaches is likely to be useful as it can expand understanding the quantitative data (Kansteiner, 2020; Schoonenboom & Johnson, 2017). Green (2007) lists five main purposes for mixed methods research; triangulation, complementarity, development, initiation, and expansion. This research seeks to achieve the complementarity purpose as the qualitative data clarify and elaborate on the data of the quantitative portion and seek to enhance the MEP.

The main method used in this research was the quasi-experimental design. This quantitative method is used to fulfill the intention of this research to learn about the effect of the enrichment program on the students' mathematical literacy. A quasi-experimental study was conducted based on groups of nonequivalent pretest-posttest design. A quasi-experimental design is chosen to fulfill the purpose of this study as it seeks to establish a cause-effect relationship between two or more variables (Gay, Mills & Airasian, 2012; Mills & Gay, 2019). The experimental groups were identified and enrolled in the math enrichment program. Results were compared with results from control groups who were not enrolled in the MEP.

Additionally, this study applied the survey design method to study the effect of the enrichment program on the students' motivation. The qualitative part of this mixed-methods study came from the perceptions survey to understand the students' perceptions of the math enrichment program. The use of the student survey to gain insight into students' perceptions was an additional design strength. Allowing students to complete an anonymous survey provided valuable data from those who enrolled directly in the math enrichment program to improve their mathematical literacy. This data was collected in a non-threatening, inexpensive way, and could be used to compare students' perceptions with actual results.

Internal validity is the degree to which the observed differences in the dependent variable are direct results of the treatment of the independent variable, not some other variables (Gay, Mills & Airasian, 2012; Mills & Gay, 2019). This study did not control many of the confounding variables that have been shown to affect student achievements such as differences in teachers’ pedagogy, language barriers, individual learning styles, and the instructional methods. Thus, the complexity of teaching and learning styles and inability to manipulate different aspects of the program addressed the various aspects of the program to distinguish differential effects were threat the internal validity. However, the nature of the MEP focused on communication and sharing of ideas during solving problems together. This nature was agreed upon it with teachers who implemented the MEP with their students as part of the PBL and CTL.

Random selection of group participants is argued to be the best way to control several external variables simultaneously and to ensure equivalency in groups (Gay, Mills & Airasian, 2012; Mills & Gay, 2019). Therefore, the lack of randomization of the participants was one of this study’s weaknesses. However, in an attempt to limit the effect of the lack of randomization of the participants, the researcher randomly assigned the experimental and the control groups as whole classes in each of male and female schools. Moreover, both schools were chosen from Al Ain, the same city, besides MEP was implemented only for grade 10 Advanced as a selected criterion. Thus, since there was no control over the previously mentioned variables, the events must be interpreted carefully because the cause-and-effect relationship might not be the ones that it appeared to be (Gay, Mills & Airasian, 2012; Mills & Gay, 2019).

The external validity or generalizability of the study was limited by the possible effect of pre-testing. Pretest-posttest designs are widely used in behavioral research, primarily to compare groups and/or measuring change resulting from experimental treatments (Gay, Mills & Airasian, 2012; Mills & Gay, 2019). To increase the generalizability of the study by limiting the researcher's influence on participants, the experimenter's influence was controlled by selecting both experimental and control female groups that were taught by the same teacher in addition to selecting experimental and control male groups that were taught by one teacher as well. The researcher was only involved in the pilot study application. The testing effect was minimized because two months were separating the administration of pretest and posttest.

The essential strength of mixed-methods design was that it would not be possible without the synergy between quantitative and qualitative research. The mixed-methods design allowed the researcher to use more than one approach to analyze research questions. Quantitative statistical data could be described by the insight of personal survey questions. Another positive feature of this research revolved around action research with the professional implications of the learning community and educational decisions. Action research in education is based on a systematic survey in the teaching and learning environment that will influence a positive change in the school environment (Gay, Mills & Airasian, 2012; Mills & Gay, 2019). This will allow stakeholders to make informed decisions to improve students’ mathematical literacy.

The General and Advanced streams are dominant in most secondary schools. Depending on the students’ performance, they can choose either to remain in the general stream or to join the advanced stream. In both, general and advanced streams students will continue Grades 10, 11 and 12. The key difference between the general stream and the advanced stream is the range of scientific subjects. Students in the advanced track will receive more in-depth instruction in mathematics and sciences than those in the general track. Thus, most high achiever students choose the advanced stream.

3.2 The Study Participants

The population of this study was male and female students from 10th grade in the United Arab Emirates because most of the 15-years old were predominantly in 10th grade. The population that could be reached was advanced grade 10 in Al Ain. This study was conducted in public schools for boys and girls. Public schools were purposefully selected for this study to reduce differences between participants because students in public schools studied the same curriculum and are exposed to the same assessment methods. In addition, the teaching in public schools was standardized by the MoE, while students enrolled in private schools follow different curricula that might affect their knowledge of mathematics differently. Moreover, the advanced stream was chosen for this study because it aimed to broaden the definition of gifted to include a much larger group of students and to open the possibility of having students with outstanding mathematical abilities and not simply identified students with mathematical pre-existing expertise and passion (Sheffield, 1999). Thus, the choice of the advanced stream was appropriate for this study as it normally includes gifted students.

It was very difficult to sample individuals within a group because it was very unlikely to obtain administrative approval for random selection and removing a small number of students from the classes for this study. A random sampling technique was used for sample selection whereby intact groups [not individuals] were randomly selected. Two schools were selected, one for males and one for females. In each school, two groups were randomly assigned as a control and experimental group. The same participants were involved in both stages of this study: the quasi-experimental and the survey research. Three female students and five male students were excluded due to missing more than three lessons from the program. The final number of participants was 102 of whom 9 (8.8%), 73 (71.6%), and 20 (19.6%) were students of 14, 15, and 16 years of age, respectively. Of the 102 participants, 53 (51.96%) were males, and 49 (48.03%) were females. This descriptive data is presented in Table 3.

Table 3: The gender of the participants

Gender	Number	Percentage
Male	53	51.96%
Female	49	48.03%
Total	102	100%

The participants in the two experimental groups were distributed as 27 (50.9%) males compared to 24 (48.9%) females. Similarly, the participants in the two control groups were distributed between males 26 (49.1%) and females 25 (51.0%). The distribution of the participants is shown in Table 4.

Table 4: Distribution of participants

Groups	No of students		Total	
	Male	Female	Subtotal	Overall total
Experimental	27(50.9%)	24(48.9%)	51	102
Control	26(49.1%)	25(51.0%)	51	
Total	53 (100%)	49(100%)	102	

3.3 Study Instruments

There were four instruments for this study; the Mathematical Enrichment Program (MEP) to fulfill the first purpose of this study; the Mathematical Literacy

Test (MLT) to measure the impact of the enrichment program on students achievement in mathematical literacy; the motivation survey to measure the effect of MEP; and the perceptions survey that aimed to measure the effect of the enrichment program from the students perspective. Both MEP and MLT were based on PISA mathematical literacy framework. The rationale of developing MEP and MLT based on PISA have been discussed in the following paragraphs before moving to the study's instruments.

PISA is an internationally standardized assessment jointly developed by participating countries and administered to 15-year-olds in educational programs (e.g. OECD, 2009b, 2013, 2019a, 2019b). Thus, it fits the same age period as most students who study in grade 10 in the UAE. PISA is unique for its worldwide scope and its regularity allowing countries to track their progress in meeting key learning goals. "PISA is the only international education survey to measure the knowledge and skills of 15-year-olds" (OECD, 2020). The main focus of PISA is on literacy and looks at the students' ability to apply knowledge and skills rather than examining the mastery of any specific country curriculum that makes it appropriate to measure the ML level of the students in all countries (OECD, 2020).

Moreover, hundreds of experts, academics, and researchers from participating countries and economies in the PISA study are involved in PISA's development, analysis, and reporting in addition to OECD staff and contractors (OECD, 2020). This involvement guarantees the international validity of PISA assessment instruments and considers the cultural and curricular context of OECD countries. "They also have strong measurement properties and emphasize authenticity and educational validity" (OECD, 2009b, 2013).

In fact, countries and economies participating in the PISA program are invited to submit questions that are then added to elements developed by OECD experts and entrepreneurs. Carefully reviewed questions that do not include any cultural bias are used in PISA. Moreover, before the main test is taken, a pilot test is conducted in all participating countries and economies. If any test questions proved to be too easy or too difficult in some countries and economies, for reasons not related to the general level of proficiency of students, they are excluded from the main test in all countries and economies (OECD, 2020). Consequently, the results of PISA have a high degree of validity and reliability (OECD, 2009b, 2013). In order to allow countries to track their performance over time, several questions from the PISA Survey were used more than once. These questions cannot be posted publicly as long as they are in use (OECD, 2020). Thus, the development of the MEP and MEPT was based on the released items that were available to the general public by OECD as this would allow assessing the students on a strong basis.

3.3.1 The Proposed Mathematics Enrichment Program (MEP)

The proposed MEP was an intervention program that was designed to help the students improve their mathematical literacy to fulfill the purpose of this study.

3.3.1.1 The Proposed MEP Development Principles

Based on the previous literature and the related studies, the following principles were identified to underline the development of the proposed enrichment program to improve the students' mathematical literacy.

- The proposed MEP was designed to build on students' prior knowledge, so it should demonstrate the most important prior knowledge related to the studied topic.
- The proposed MEP was based on contextual problems that address the modeling problems that would improve the high order thinking skills (problem-solving and reasoning).
- The proposed problems consisted of PISA problems of different levels. However, there are few problems with low levels like one and two, as such were the upper levels of 5 and 6 based on the PISA problem classification. Most of the problems lied in levels 3 and 4 to be fulfilled by students. Adopting PISA problems was to ensure that levels of problems are at the appropriate cognitive level for the students.
- The teacher's role was to facilitate students' construction of mathematical knowledge; support and expand student thinking by fostering discussions and encouraging students to develop their own problem-solving strategies and use informal or prior knowledge to help develop their conceptual understanding and use of alternative solution methods. The problem-based learning was a perfect fit for this role of the teacher in implementing the enrichment program that was based on constructivism and social constructivism where the students could interact and help each other.

3.3.1.2 The Content of the Proposed Enrichment Program

This proposed MEP aimed to improve the students' mathematical literacy. Nevertheless, mathematical literacy is a very broad and cumulative area. So, the scope of content for this enrichment program was identified to be restricted to the comprehensive framework of mathematical literacy in PISA. The MEP can be found in Appendix A. Mathematical literacy was the major topic in 2012 for PISA assessment that focuses mainly on the processes of problem-solving (modelling cycle) and it will return to be in 2021 with extra focus on reasoning as it is the core of the problem-solving processes (OECD, 2018a). Considering the framework of mathematical literacy by PISA, the main components of mathematical literacy involve mathematical thinking such as reasoning, modelling, and making connections between ideas (Stacey, 2007). According to Piaget's theory of constructivism, students of this age are cognitively capable of reasoning and solving problems that support the relevance of the MEP program because students at the age of 15 are in the operational stage (Piaget, 1957).

The development of the Enrichment Program design relied mainly on two components, which were a review of the basics of prior knowledge required for each lesson as well as relevant PISA elements released. Using released items from PISA was appropriate to the students’ cognitive level as it was designed to test the 15 years age students. This proposed enrichment program consisted of the four PISA mathematical literacy content areas which were change and relationship, space and shape, quantity, and uncertainty. In addition, addressing reasoning was embedded in these four content areas as the processes of solving these problems might be through formulating, employing, and interpreting in which reasoning was essential to all these processes.

Two lessons were developed for each of the four “overarching ideas”: quantity, space and shape, change and relationships, and uncertainty. The developed eight lessons and allocated time range for each lesson of the program were determined as shown in Table 5.

Table 5: Enrichment program content and time-range for lessons

Content Area	Lessons	No. of sessions
Change and relationship	1- Functions and variations	2
	2- Numerical trends and patterns	2
Space and shape	1- Geometric approximation	2
	2- The visual and physical world	2
Quantity	1- Percentages	2
	2- Quantification	2
Uncertainty	1- Probability	2
	2- Statistics	2
Total	8 lessons	16 sessions

This enrichment program, as shown in Table 5, consisted of eight lessons where two lessons were assigned to each of the content areas, each lesson was designed based on the most addressed topics from the released PISA items of different levels in addition, these released items were collected to build on students' prior knowledge. The time allotted for each lesson was two periods of 45 minutes. Only one lesson was discussed each week. As a result, eight weeks of time was required to implement the enrichment programs.

3.3.1.3 Appropriateness of the Proposed Enrichment Program

In light of what has been reached in the theoretical framework and previous studies, the development of the enrichment program took several steps to reach its final form. After the initial development of the enrichment program, it was presented to a group of experts, who are experienced in teaching and learning mathematics. The group of experts consisted of one professor in mathematics education, one professor in mathematics, and five expert mathematics teachers.

For judging the appropriateness of the proposed MEP, these experts were asked to decide to what extent it was appropriate to the level of students in the advanced tenth grade. Experts indicated that the proposed enrichment program was suitable for advanced tenth grade students. They remarked that released items for PISA were suitable to the group age of the students as they were specially developed for this age group and reviewed by several international experts all around the world. They also mentioned that this enrichment program included problems that might challenge most levels of students. In addition to the most important prior knowledge was necessary for every lesson. However, they suggested that more than two periods might be required to discuss each lesson. Unfortunately, due to the intensive curriculum of mathematics and time constraints, it was difficult to provide more than two periods for each lesson. Therefore, after discussing this point with the experts, we agreed that not all problems should be discussed with students in the classroom as the teacher's role as a facilitator of student learning. Therefore, students were encouraged to self-learn the rest of the problems independently.

Another point that was mentioned by the experts about the language level used in the problems. They stated that it involved difficult words that the students might not understand and might affect their ability to solve problems. After discussing this point, we agreed not to modify the language of the problems as the students needed to be exposed to the same level of problems that PISA provided. Additionally, this could be considered another challenge for students to use their skills to just anticipate the meaning of the difficult words from the context of the problem as using words was an essential part of the mathematical contextual problems.

The last point was about the order of the lessons, the experts suggested teaching the lessons about “quantity” just after “change and relationship” not after “space and shape”. This was because “quantity” might include concepts that are considered basics to other content areas. The experts’ suggestion was met, and the order of lessons was changed as they said.

3.3.2 The Mathematics Literacy Test (MLT)

Test items of PISA are a mixture of multiple-choice items and questions requiring students to construct their own responses. The items are organised in groups based on a passage setting out a real-life situation (OECD, 2009b, 2013). The multiple-choice test is a key feature of PISA assessment because it is reliable, effective, and supports robust and scientific analyses (OECD, 2020). These MLT items involve multiple options and structured answer items. Additionally, up to a third of the questions in the PISA evaluation are usually open that requires the students to construct a response either an extended-response or a short answer (OECD, 2009a). Moreover, there are two types of constructed response items: extended constructed response and short constructed response. The extended constructed response elements require students to solve a problem in addition to explaining their solution, while the short-constructed response items require only an answer.

This test served as a pretest and posttest for this study and it aimed to measure the students’ mathematical literacy levels based on content, context, and processes. This test consisted of 34 problems that were drawn from released PISA materials published on the OECD website. MLT can be found in Appendix B.

3.3.2.1 The Mathematics Literacy Test (MLT) Components

Table 6: A map for selected mathematics items in MLT

	Part 1: Q1-Q26			Problem Solving										
				Process			**Content**				**Context**			
Item number	**Item name**	**Item difficulty**	**Level of Proficiency**	**Formulate**	**Employ**	**Interpret**	**Change and relationships**	**Quantity**	**Space and shape**	**Uncertainty and data**	**Personal**	**Occupational**	**Scientific**	**Societal**
1	Charts Q1 *	347.7	BL1											
2	Charts Q2	415.0	L1											
3	Charts Q5	428.2	L2											
4	Which Car? Q1 *	327.8	BL1											
5	Which Car? Q2	490.9	L3											
6	Which Car? Q3	552.6	L4											
7	Garage Q1	419.6	L1											
8	Garage Q2.1	663.2	L5											
9	Apartment Purchase Q1	576.2	L4											
10	Drip Rate Q1	610.0	L5											
11	Drip Rate Q3	631.7	L5											
12	REVOLVING DOOR Q1	512.3	L3											
13	REVOLVING DOOR Q2	840.3	L6											
14	REVOLVING DOOR Q3	561.3	L4											
15	Sauce Q2	489.1	L3											
16	Sailing Ships Q1	511.7	L3											
17	Sailing Ships Q3	538.5	L3											
18	Sailing Ships Q4	702.1	L6											
19	Climbing Mount Fuji Q1	464.0	L2											
20	Climbing Mount Fuji Q2	641.6	L5											
21	Climbing Mount Fuji Q3	591.3	L4											
22	Helen the Cyclist Q1	440.5	L2											
23	Helen the Cyclist Q2	510.6	L3											
24	Helen the Cyclist (E) Q3	696.6	L6											
25	FERRIS WHEEL Q1	592.3	L4											
26	FERRIS WHEEL Q2	481.0	L3											
	Total		**26**	**8**	**14**	**4**	**7**	**7**	**8**	**4**	**8**	**4**	**6**	**8**
	Percentage %		**100**	**31**	**54**	**15**	**27**	**27**	**31**	**15**	**31**	**15**	**23**	**31**
	Part 2: Q27-Q34		8	Reasoning										

*BL1 means below level one
Source: OECD (2014), PISA 2012 Technical Report, OECD, Paris.

The above Table 6 demonstrated the map for selected mathematics items used in this study. This MLT was a PISA-style test where test items released from PISA 2012 were used to better compare with previous results. Those chosen items were spread across all six proficiency levels identified by PISA 2012 of three types of response format (multiple-choice, closed constructed, and open constructed response). Furthermore, the items were spread across the four content subdomains (quantity, space & shape, change & relationships, and uncertainty & data), and all four PISA contexts (personal, public, educational/public, and scientific). Finally, the three processes (formulate, employ, and interpret) that they had to be activated in order to connect the real world were included.

The test problems primarily attempted to assess the students' problem-solving in six proficiency levels that were presented in 26 of the test problems in addition to 8 problems that measured their reasoning skills. Moreover, Table 7 describes the distribution of test items based on the dimensions of the PISA framework for the assessment of mathematics

Table 7: Distribution of MLT items by dimensions of the PISA framework

Processes	#	Contents	#	Contexts	#
Formulate	8	Quantity	7	Personal	8
Employ	14	Space and shape	8	Occupational	4
Interpret	4	Change and relationship	7	Scientific	6
Reasoning *	8	Uncertainty	4	Societal	8
Total	26 + 8Reasoning	Total	26	Total	26

*Reasoning is the new addition to the PISA 2021 framework of mathematical literacy

In PISA, each question was assigned a difficulty level. Using Item Response theory and these difficulty levels the raw scores of the students were converted to a score on the PISA scale. The PISA scale in mathematics was also divided into six mathematical literacy levels to represent degrees of proficiency where level six was the highest. The mathematics proficiency levels are detailed in Figure 10 below.

Level	Lower score limit	What students can typically do
6	669	At Level 6 students can conceptualise, generalise and utilise information based on their investigations and modelling of complex problem situations. They can link different information sources and representations and flexibly translate between them. Students at this level are capable of advanced mathematical thinking and reasoning. These students can apply this insight and understanding along with a mastery of symbolic and formal mathematical operations and relationships to develop new approaches and strategies for attacking novel situations. Students at this level can formulate and precisely communicate their actions and reflections regarding their findings, interpretations, arguments, and the appropriateness of these to the original situations.
5	607	At Level 5 students can develop and work with models for complex situations, identifying constraints and specifying assumptions. They can select, compare, and evaluate appropriate problem-solving strategies for dealing with complex problems related to these models. Students at this level can work strategically using broad, well-developed thinking and reasoning skills, appropriately linked representations, symbolic and formal characterisations, and insight pertaining to these situations. They can reflect on their actions and formulate and communicate their interpretations and reasoning.
4	545	At Level 4 students can work effectively with explicit models for complex concrete situations that may involve constraints or call for making assumptions. They can select and integrate different representations, including symbolic representations, linking them directly to aspects of real-world situations. Students at this level can utilise well-developed skills and reason flexibly, with some insight, in these contexts. They can construct and communicate explanations and arguments based on their interpretations, arguments and actions.
3	482	At Level 3 students can execute clearly described procedures, including those that require sequential decisions. They can select and apply simple problem-solving strategies. Students at this level can interpret and use representations based on different information sources and reason directly from them. They can develop short communications reporting their interpretations, results and reasoning.
2	420	At Level 2 students can interpret and recognise situations in contexts that require no more than direct inference. They can extract relevant information from a single source and make use of a single representational mode. Students at this level can employ basic algorithms, formulae, procedures, or conventions. They are capable of direct reasoning and literal interpretations of the results.
1	358	At Level 1 students can answer questions involving familiar contexts where all relevant information is present and the questions are clearly defined. They are able to identify information and to carry out routine procedures according to direct instructions in explicit situations. They can perform actions that are obvious and follow immediately from the given stimuli.

Source: OECD (2014), PISA 2012 Technical Report, OECD, Paris.

Figure 10: Descriptions for the six levels of proficiency in mathematics

The results of PISA 2018 indicated that 45.5% of the students performed below level 2, and only about 5.4% performed above level (OECD, 2019b). This indicated that if the students’ performance improved in levels 2, 3, and 4 was crucial to the overall improvement in ML. In addition, this would be a small starting step towards the goal of being among the best 20 countries in the world and fulfilling the gifted needs too.

The items of the MLT were distributed to cover all the six proficiency levels of mathematics problems as presented in Table 8 as follows:

Table 8: Distribution of MLT items by levels of proficiency

Level of proficiency	# of items	Percentage %
Level 1 and Below	4	15
Level 2	3	12
Level 3	7	27
Level 4	5	19
Level 5	4	15
Level 6	3	12
Total	26	100

These “proficiency levels” described what students at given levels of proficiency typically know and can do. Where students were distributed between the first level, in which students succeeded only in basic tasks, and the sixth level, in which students could solve complex problems and had advanced thinking skills.

It is noteworthy that the Cronbach alpha test was performed to compute to find the reliability of the administered mathematical literacy test and found to be 0.85.

3.3.2.2 The Mathematics Literacy Test Appropriateness

As mentioned earlier, PISA questions are carefully reviewed and only those that have no cultural bias are used. “They also have strong measurement properties, and place an emphasis on authenticity and educational validity” (OECD, 2009b, 2013). This is a result of the PISA procedure to select test items. Thus, the results of PISA have a high degree of validity and reliability (OECD, 2009b, 2013). Moreover, for this study, the questions were presented to the same committee that reviewed the MEP to decide about the test appropriateness to measure the students’ level of ML. They reported that the test was comprehensive and is appropriate to measure ML that was based on both the MEP and PISA framework. They suggested reordering the questions so that the test starts from easy to hard questions. The researcher reordered the beginning of the test but did not order it fully from level 1 to level 6 because the questions were on the unit form that means two or more questions might be related to measuring the same context. Another reason, that the students might just skip doing all the test if they recognized that the rest questions were of high level.

Experts initially indicated that students would need more than 90 minutes for 34 PISA problems. However, although all math questions in PISA involved some real-world mathematical problem solving, not all students were required to take a complete problem-solving course due to the limited time students had to answer the questions: “the average allowable response time for each question is around two minutes, which is too short a period of time for students to go through the whole problem-solving cycle.” (OECD, 2009a, p. 160). Thus, bearing in mind that students should, as far as possible, undergo conditions similar to the original PISA test conditions, the committee decided that the test time was appropriate.

3.4.3 Motivation to Learn Mathematics Survey

This survey consisted of two sections. The survey can be found in Appendix C; it mainly consists of the seven-scale multiple-choice Likert. The first section collected personal information about the gender and age of the participant, while the second section of the survey measured the students’ motivation to learn mathematics using a 7-point Likert scale ranging from disagree strongly (1) to agree strongly (7). The survey was anonymous so that students would be more likely to give honest answers and was administered to students from both experimental and control groups before and after conducting the MEP.

3.4.3.1 Validity and Reliability of the Motivation Survey

To ensure the validity and reliability of the survey, a preliminary pilot field test of the survey was conducted to solve any problems before the survey was distributed to the research participants. Thus, the survey was administered to 51 students who met the demographic criteria of the study's population (26 males: 25 females) from grade 10 advanced from public high schools. Additionally, before administering the survey, it was a very important step to establish the validity to ensure that this survey would help in fulfilling the purpose of this study. "Validity refers to the degree to which a test measures what it is supposed to measure and, consequently, permits appropriate interpretation of scores" (Gay, Mills & Airasian, 2012, p. 160). The survey items were adapted from the original Motivated Strategies for Learning Questionnaire (MSLQ) that was developed by Pintrich et al. (1991). This motivation survey was about intrinsic goal-oriented and extrinsic goal-oriented. These items were administered to students in English with slight changes were made to be applicable to mathematics subject. Factor analysis was applied to the original MSLQ over three years when the survey was developed. Additionally, the MSLQ was applied and validated many times in research included the high school level (Montalvo & Torres, 2004).

Moreover, the content validity of the whole survey was established by asking three experts who were professors working in the College of Education at the United Arab Emirates University and five mathematics teachers to check for validity. The feedback provided by the teachers helped in rebuilding and modifying some items to ensure that it was understood before the advisor's approval.

Reliability "is the degree to which a test consistently measures whatever it is measuring" (Gay, Mills & Airasian, 2012, p. 165). Cronbach's alpha was the most common measure of scale reliability. It was measured for the sample of 51 students of the pilot testing. The reliability of the whole scale was found to be 0.72 which is acceptable reliability. Fraenkel and Wallen (1996) stated that the reliability of items is acceptable if the alpha is between 0.70 and 0.99.

3.4.4 The Perceptions Survey

This survey was designed by the researcher specifically for this study to gain insight into students' perceptions about their experience in the MEP. The survey can be found in Appendix D; it primarily consists of a multiple-choice seven-scale Likert and open-ended questions designed in three sections. The first section collected demographic data about the gender and age of the participant. The second section was dedicated to evaluating the program with 8 items in addition to one item of a yes/no response to indicate whether the students would recommend the program that was applied to other students to improve their mathematical literacy. Finally, the third section was of open-ended questions type that represented the qualitative part of the survey. It aimed to provide students with the opportunity to express their opinions about their personal experiences and to highlight and clarify their ideas. Another purpose of the qualitative part was to gain insights into the possible improvements for the program. Open questions were used to help validate and strengthen quantitative research by identifying patterns that emerged during data collection. The purpose of this post-program student survey was to gather valuable perception data from those who were enrolled in the MEP. The survey was created to discover if the students felt that the contextual problems in the program could help in improving their mathematical literacy.

After collecting and analyzing data about the student’s perceptions, the reliability of the perceptions survey was calculated. Cronbach’s Alpha, the most common measure of scale reliability, was used to calculate reliability and was found to be 0.94, indicating very strong reliability (Gay, Mills & Airasian, 2012; Mills & Gay, 2019).

3.5 Ethical Considerations

This study includes students as human subjects, therefore adherence to research ethics requirements such as privacy, honesty, fairness, confidentiality, and other ethical issues related to these students was essential (Walliman, 2015). Therefore, within the ethical procedures, the researcher requested approval of MOE, which was issued in a form of emails sent directly to the chosen schools (Appendix E) after providing full information about the research and the researcher, including a request to facilitate the researcher's task in the two selected schools (Appendix F). Then, the researcher provided the schools with information about the purpose, significance of the timeline, and procedures.

Approval for carrying out this study was requested from the MoE. Full information about the research and the researcher, including a request to facilitate the researcher's task in the two selected schools (Appendix E) were provided to MoE. Then, the MoE granted its approval in a form of emails sent directly to the chosen schools (Appendix F) after scheduling a researcher's meeting with an expert in PISA from the Department of International Examinations to validate MEP and MLT. After that, permission was obtained from the two high schools in which the MEP program would be implemented. Finally, a consent letter was sent to the students' parents to obtain their consent for their children to participate in the study (Appendix G). The consent letter guarantees that the data collected will be used for research purposes only and will not, under any circumstances, be shared with anyone. It also confirmed that students' participation is voluntary, and they have the right to withdraw at any time during the intervention without any consequences. Moreover, for further ethical consideration to assure the anonymity and confidentiality of the participants, the names of the students or any personal information were not required for this study.

3.6 The Research Procedure

After completing the preparation of the proposed MEP and MLT and determining their appropriateness, the researcher attempted to conduct a pilot experiment for the program aimed at determining the appropriate time range for each lesson and ensuring the experimental appropriateness of educational treatment as well as the MLT. In light of the pilot study, the proposed MEP was clear, appropriate to the students and the time range for each lesson of the program was found to be appropriate for implementation. Similarly, the MLT time was suitable for the students.

A list of schools that included advanced grade 10 was formed to select the schools that included two sections of advanced grade 10 or more to form the control group and experimental group from both male and female schools. The two sections from each school were chosen to be taught by the same teacher to eliminate the teacher factor in implementing the MEP. In addition, the results of the mock exam from the year 2018 were used to ensure that their level of performance was equivalent.

After getting the required approvals and meeting with the schools' administrations of the two selected schools, two meetings were scheduled with the involved teachers before implementing the MEP. The first meeting was to introduce the program and the second was for discussing their role in and the procedure of implementation. The teachers were provided by MEP and the answer key to the presented problems. Then weekly meetings were scheduled to meet the teachers to clarify anything related to the implementation process. However, these meetings were flexible based on the teacher's need and ongoing discussions using the WhatsApp application that would replace the actual meeting when needed.

The implementation of the program took a full semester over a period of 10 weeks, starting from 8-9-2019 to 14-11-2019. Where during the first week, both a pre-test of mathematical literacy and the survey of motivation were applied for both the experimental and control groups. The program was then applied to the experimental groups over 16 sessions of 45 minutes each. The teachers were given the freedom to choose when to conduct the two sessions in a manner consistent with the teachers 'plan to provide the flexibility to complete the core curriculum as well. In the tenth and final week, the post-test of mathematical literacy and the survey of motivation were applied to both the experimental and control groups. Additionally, students in the experimental groups were asked to express their opinion about the applied program through a survey to evaluate the program in its quantitative and qualitative sections.

3.7 Data Analysis

Descriptive and inferential statistics were used in order to address the research questions being posed in the study. An analysis of covariance (ANCOVA) was conducted to analyze the quantitative achievement data being collected and to determine if there were any differences in the mean scores of the two groups (Gay, Mills & Airasian, 2012; Mills & Gay, 2019), while controlling for the effects of the students' mathematical literacy level by setting the pretest as a covariate. ANCOVA was replicated for male and female students as well for both mathematical literacy and motivation. Additionally, an independent t-test was conducted to compare male and female perceptions of the MEP of tenth-grade students. Moreover, the thematic analysis was applied to analyze the qualitative part of students' perceptions regarding the MEP and their suggestions to improve the program.

Chapter 4: Results

This study aims to investigate the effect of the proposed MEP to improve students' mathematical literacy and their motivation to learn mathematics. The study used a mixed-methods design to fulfill this purpose. This chapter focused on data presentation and analysis. The main research questions to be answered in this study included the following:

RQ1: What is the effect of the mathematics enrichment program on the mathematical literacy of tenth grade students?

RQ2: What is the effect of the mathematics enrichment program on the motivation to learn mathematics of tenth grade students?

RQ3: What are the perceptions of the tenth-grade students regarding the mathematics enrichment program?

In the UAE schools, male and female students attend separate schools. Thus, this study was applied in the female school and repeated at the same time in a male school to study the effect of MEP on both genders. To answer the research questions, quantitative and qualitative data were collected. Therefore, the descriptive statistics of the means and standard deviations were calculated. Moreover, the inferential statistics that used were: a one-way ANCOVA that was performed to determine the effect of MEP on students' mathematical literacy controlling their prior knowledge by setting the pretest as a covariate; a t-test was required for analyzing quantitative data comparing tenth grade male and female perceptions of MEP. In addition, thematic analysis was applied to analyze the qualitative data. The answers to the research questions were presented as follows.

4.1 First Research Question

RQ1: What is the impact of the Mathematics Enrichment Program on the mathematical literacy of tenth grade students?

To answer this question, the descriptive statistics of means and standard deviations were performed for both the pretest and posttest of mathematical literacy for both male and female tenth-grade students, as shown in Table 9.

Table 9: Descriptive statistics for the 10th grade students' ML

Mathematical literacy test	Experimental group					Control group				
		Pretest		Posttest			Pretest		Posttest	
	N	M	SD	M	SD	N	M	SD	M	SD
Female students	24	11.79	3.87	18.17	5.00	25	9.20	3.58	10.80	3.89
Male students	27	5.37	2.39	12.81	4.39	26	5.69	2.31	8.85	1.71

In general, Table 9 showed the superiority of female students over male students in the experimental groups to which MEP was applied. The results of the post-test for both male and female students also showed an increase in mathematical literacy of the experimental and control groups, and that the increase achieved by the experimental group was much better for both genders. The female students got an average score of 18.17 for the experimental group and an average score of 10.80 for the control group, while male students scored an average score of 12.81 for the experimental group and an average score of 8.85 for the control group. This was a good starting point for inferring the impact of MEP implementation. Hence, if the experimental group scored higher than the control group in the post-test, it was expected that this could be due to treatment, provided that other confounding variables are controlled. Consequently, to ensure that this post-test difference is indeed a result of treatment and not the result of random variation in pretest between groups, one-way

ANCOVA should be used to examine the effectiveness of MEP in controlling the pretest of mathematical literacy. To minimize the chances of making a Type I error (when the true null hypothesis is rejected), the significance level of all the inferential statistics in this study was set to $\alpha = 0.05$ because this level would minimize the chances of making a Type I error. As this study was applied to separate schools for male and female students, the researcher sought to study the effect of the MEP separately on male and female students' mathematical literacy by controlling the pre-test, and then compare the difference in the effect on them using ANCOVA with the pretest as a covariate to control students' previous levels.

Before performing the statistical analysis, the researcher examined nine assumptions for using ANCOVA analysis (Laerd Statistics, 2020b). Four of these assumptions were mentioned by Green and Salkind (2011). The first three assumptions are related to the choice of research design and do not need statistical tests to verify (Laerd Statistics, 2020b). The research design for this study was a quasi-experimental, non-equivalent control group design that supports the first four assumptions. This applies to both the first and second questions of this research. These assumptions include: dependent variable (ML posttest for the first research question and the posttest motivation score for the second research question) and covariate variable (ML pretest for the first research question and the pretest motivation score for the second research question) measured at continuous level, and an independent variable consisting of two (or more) categorical, independent groups (experimental group and control group) and the third assumption is the independence of observations (participants in experimental groups cannot be involved in the control groups for both genders).

Additionally, another six assumptions must be met to apply the ANCOVA test (Field, 2013; Laerd Statistics, 2020b). These assumptions consisted of the covariate linearity, homogeneity of the regression slopes, normally distributed residuals, homoscedasticity, homogeneity of variance, and no outliers. The analyses tested each assumption by research sub-questions for each associated dependent variable. The first question was answered by comparing the performance of the experimental group students with the control group for both male and female students separately since they studied in different learning environments, then comparing male and female students after controlling the pretest scores to identify any statistically significant differences. Thus, the answer to the first research question obtained by answering the following three sub-questions:

4.1.1 First Sub-question

RQ1a: What is the impact of the Mathematics Enrichment Program on the mathematical literacy of tenth grade female students?

H0: There is no statistically significant difference between the experimental group and the control group on tenth grade female students' mathematical literacy.

Firstly, the researcher examined the six additional assumptions for using ANCOVA analysis (Field, 2013; Laerd Statistics, 2020b). For this research question, the dependent variable was the mathematical literacy as measured by posttest of female tenth grade students and the independent variable was the availability of the MEP. There were no outliers in the data, as they were assessed by no cases with standardized residuals greater than ±3 standard deviations as revealed by Figure 11 that represents the whisker plot graph of ML post-test for both experimental and control female student groups below.

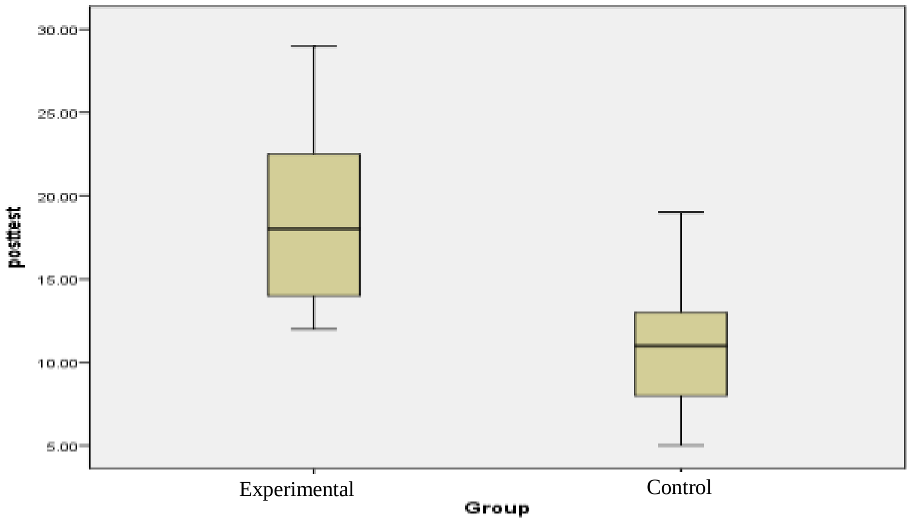

Figure 11: The whisker plot graph of ML of the female groups

There was a linear relationship between mathematical literacy in the pretest and posttest of the experimental and control groups, as assessed by visual inspection of the scatter plot as represented in Figure 12 below.

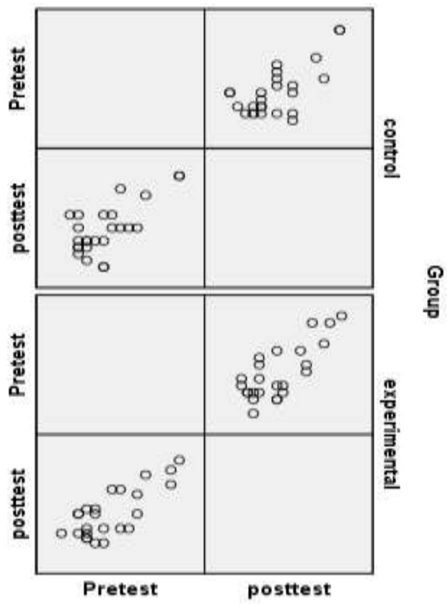

Figure 12: Scatter plot of female student’s ML pretest and posttest

There was homogeneity of regression slopes as the interaction term was not statistically significant F (1,47) = 1.238, p = 0.272. Data for the control group were normally distributed according to Shapiro-Wilk's test (p > 0.05) but not for the experimental group (p = 0.041) as revealed in Table 10 below.

Table 10: Tests of Normality of ML of the female groups

		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Group	Statistic	df	Sig.	Statistic	Df	Sig.
posttest	experimental	0.195	24	0.019	0.913	24	0.041
	control	0.160	25	0.100	0.933	25	0.100

Nevertheless, despite looking at a histogram distribution that to some extent reflects a normal distribution as represented in Figure 13 below, ANCOVA can still be operated because it is robust to the violation of normality (Laerd Statistics, 2020b). Additionally, Levy (1980) stated, "That ANCOVA is robust with respect to dual violations of the assumptions of equal regression and normality of distribution" (p. 835).

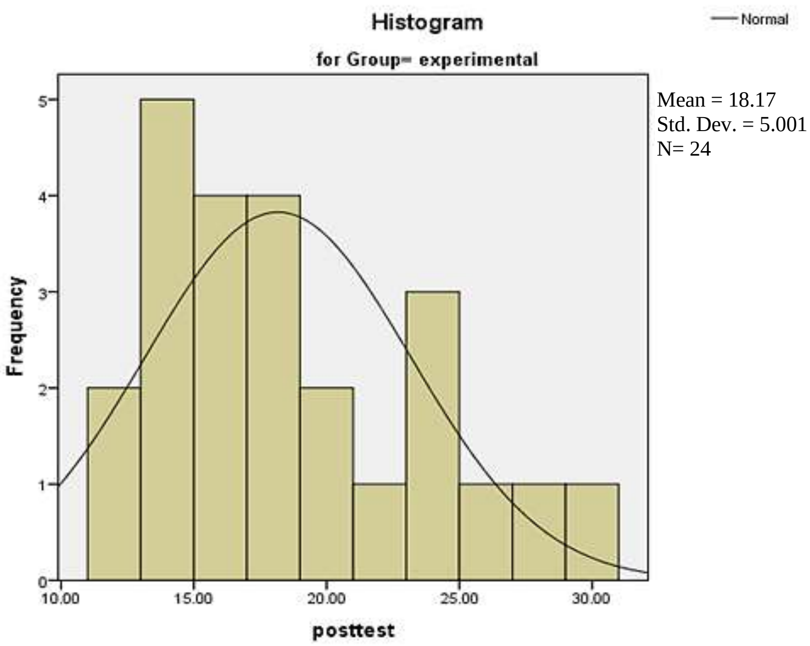

Figure 13: Histogram of ML of the female experimental group

The homogeneity of variance was assumed as shown by Levene's test of homogeneity of variance ($p = 0.114$). Moreover, the homoscedasticity was assumed as assessed by visual inspection of a scatter plot as represented by Figure 14 below.

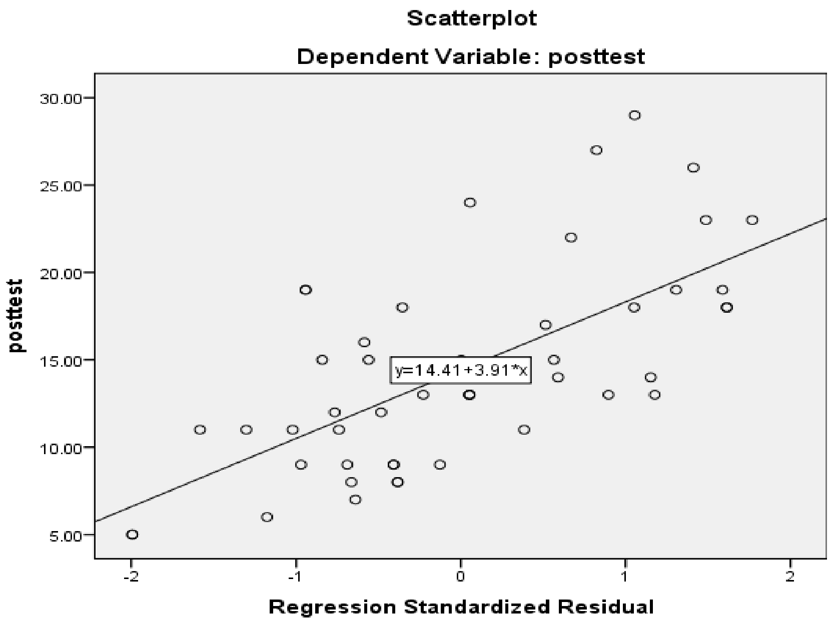

Figure 14: Scatter plot of ML of the female students

The effect of MEP on the adjusted mathematical literacy posttest of female students was examined using one-way ANCOVA. The results showed a statistically significant difference at the $p < 0.05$ level in the ML posttest between the control group and the experimental group when adjusted for ML pretest results as Table 11 shows.

Table 11: ANCOVA results for the female 10th grade students' ML

Source	df	Mean Square	F	P	η^2
Pretest	1	503.311	53.099	0.000	0.536
Group	1	281.653	29.714	0.000	0.392
Error	46	9.479			
Total	49				

Table 11 shows that the ANCOVA test was significant $F(1, 46) = 29.714$, $p < 0.0005$, partial $\eta^2 = 0.392$. The effect size eta-squared is interpreted as small, medium, and large if it possesses the values 0.01, 0.06, and 0.14, respectively (Stevens, 2009). Hence, for this study, the effect size is large and means that 39.2% of the ML posttest results are due to the MEP. A post-hoc comparison was performed using the Bonferroni method that showed a statistically significant difference between the experimental group and the control group ($p < 0.0005$) indicating that the implementation of MEP had a positive effect on the mathematical literacy of female tenth grade students' mathematical literacy. This can be seen from Table 12 where the experimental group outperformed the control group when comparing the adjusted mean scores with pretest as a covariate.

Table 12: Adjusted and unadjusted means for ML of female students

		Unadjusted		Adjusted	
Female Groups	No.	M	SD	M	SE
Experimental Group	24	18.17	5.00	17.00	0.648
Control group	25	10.80	3.89	11.92	0.634

Based on the results of the one-way ANCOVA test on mathematical literacy of tenth grade females, the null hypothesis was rejected. This demonstrates a difference across the means of the experimental and control group, and the results of the experimental group in the post-test of mathematical literacy were better than the results of the control group. In other words, the mathematical literacy of the female tenth graders in the experimental group was improved as a result of the application of MEP.

Similarly, this study was duplicated in a male school too to examine the impact of MEP on the mathematical literacy of male students, the same statistical test was followed to answer the second sub-question that stated as:

4.1.2 Second Sub-question

RQ1b: What is the impact of the Mathematics Enrichment Program on the mathematical literacy of tenth grade male students?

H0: There is no statistically significant difference between the experimental group and the control group on tenth grade male students' mathematical literacy.

For this research question, six additional assumptions for using ANCOVA analysis were examined. The dependent variable was the mathematical literacy as measured by posttest of 10th grade male students and the independent variable was the availability of the MEP. There were no outliers in the data, as assessed by no cases with standardized residuals greater than ±3 standard deviations. as revealed by Figure 15 that represents the whisker plot graph for both experimental and control male student groups below.

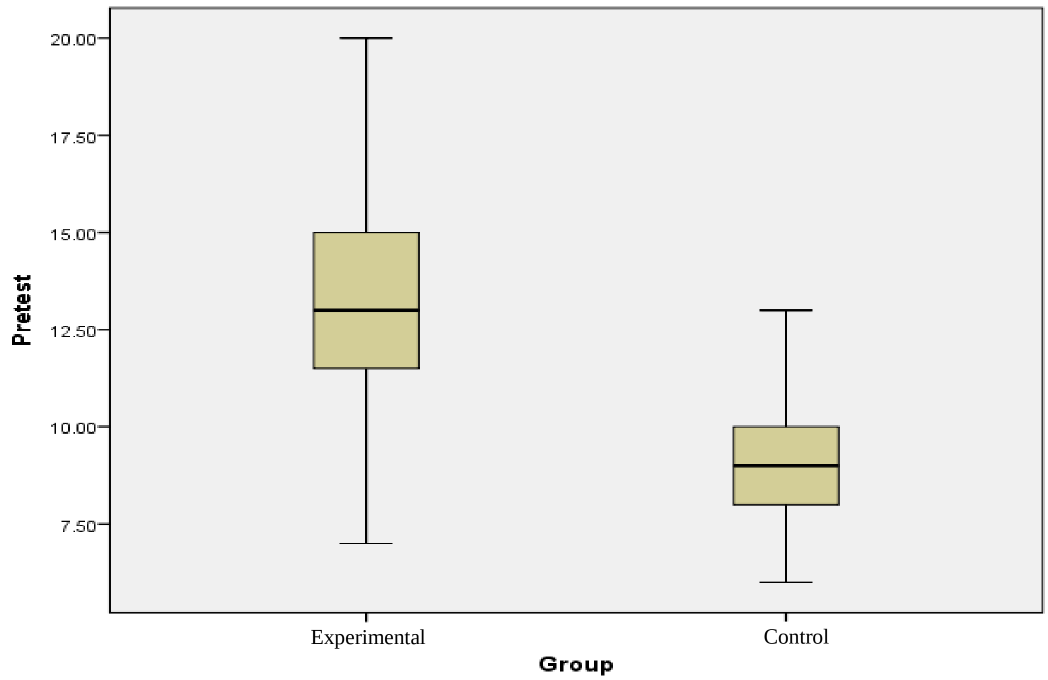

Figure 15: The whisker plot graph of ML of the male groups

There was a linear relationship between pretest and posttest mathematical literacy for experimental and control groups, as assessed by visual inspection of a scatter plot as represented in Figure 16 below.

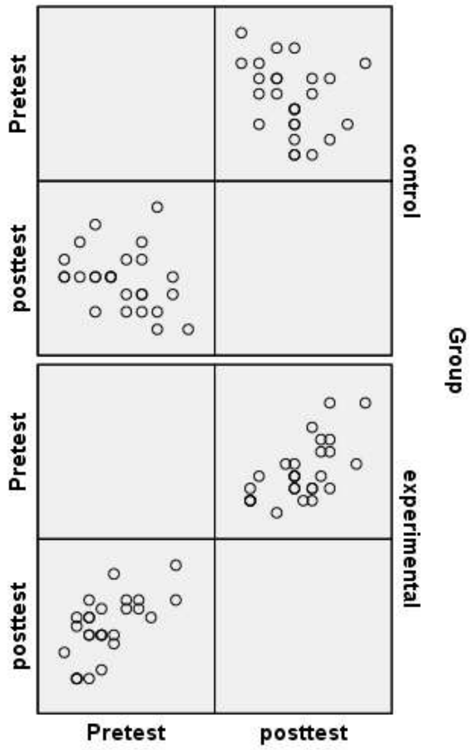

Figure 16: Scatter plot of male student's ML pretest and posttest

There was homogeneity of regression slopes as the interaction term was not statistically significant, F (1, 51) = 5.503, p = 0.053. The data of both experimental (p = 0.132) and control group (p = 0.197) were normally distributed as assessed by Shapiro-Wilk's test (p > 0.05) as revealed in Table 13 below.

Table 13: Tests of Normality of ML of the male groups

		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Group	Statistic	df	Sig.	Statistic	Df	Sig.
posttest	Experimental	0.149	27	0.125	0.941	27	0.132
	Control	0.195	26	0.012	0.947	26	0.197

The homogeneity of variance was assumed as shown by Levene's test of homogeneity of variance (p = 0.174). Moreover, the homoscedasticity was assumed as assessed by visual inspection of a scatter plot as represented by Figure 17 below.

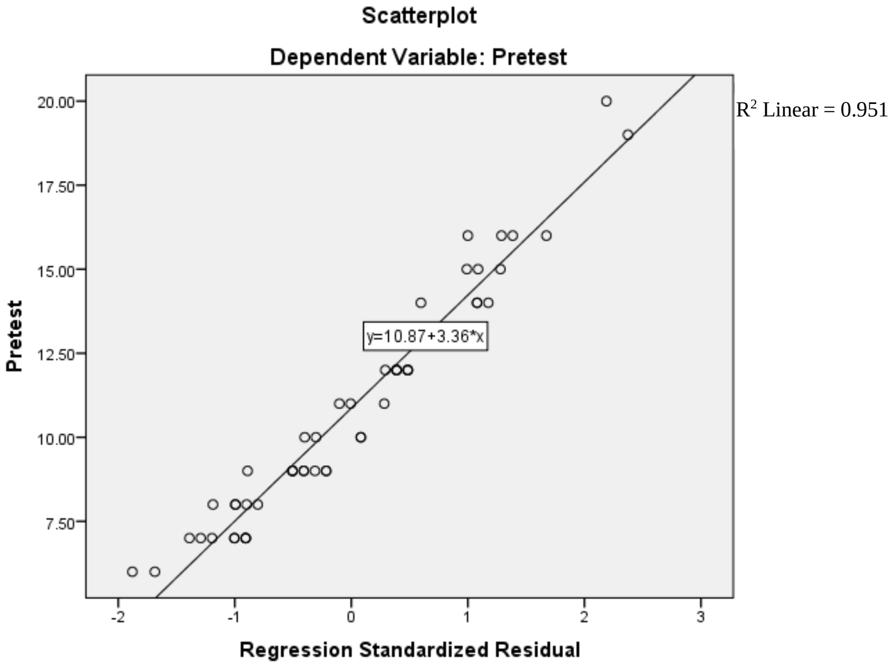

Figure 17: Scatter plot of ML of the male students

The effect of the MEP on the adjusted mathematical literacy posttest of male students was examined using ANCOVA. The results of one-way ANCOVA showed that there was a statistically significant difference at the $p < 0.05$ level in the posttest of ML results between the control group and the experimental group when adjusted for ML pretest results as shown in Table 14 below.

Table 14: ANCOVA results for the male 10th grade students' ML

Source	Df	Mean Square	F	P	η^2
Pretest	1	41.863	5.886	0.019	0.105
Group	1	220.790	31.045	0.000	0.383
Error	50	7.112			
Total	53				

Table 14 illustrates that the ANCOVA test was significant $F(1, 50) = 31.045$, $p < 0.0005$, partial $\eta^2 = .383$. Thus, for this study, the effect size is large and means that 38.3% of the ML posttest results were due to the MEP. A post-hoc comparison was performed using the Bonferroni method that showed a statistically significant difference between the experimental group and the control group ($p < 0.0005$) indicating that the implementation of MEP on the experimental group had a positive effect on the mathematical literacy of the tenth-grade male students. This can also be seen clearly from Table 15 as the experimental group outperformed the control group when comparing the adjusted mean scores of the experimental and control groups with pre-test as a covariate.

Table 15: Adjusted and unadjusted means for ML of male students

		Unadjusted		Adjusted	
Male Groups	No.	M	SD	M	SE
Experimental Group	27	12.81	4.39	12.88	0.514
Control group	26	8.85	1.71	8.78	0.524

The results of the one-way ANCOVA revealed that the mathematical literacy of the male tenth graders in the experimental group was improved as a result of the application of MEP. Thus, the null hypothesis was rejected to demonstrate a statistically significant difference across the means of the experimental and control group, and the experimental group's results in the post-test of mathematical literacy were better than that of the control group.

In light of the results for the sub-questions RQ1a and RQ1b, implementation of MEP improved mathematical literacy for both male and female tenth grade students. Moreover, although the settings are different in the male school and the female school in which this study was conducted, this study sought to compare the effect of MEP on males and females in the experimental group. This allowed further comparison with the trend of the OECD. This can be done by answering the following question:

4.1.3 Third Sub-question

RQ1c: Are there any gender-based significant differences in mathematical literacy in response to the Mathematics Enrichment Program?

H0: There is no statistically significant influence of gender on students' mathematical literacy in response to the Mathematics Enrichment Program.

For this research question, all six additional assumptions for the use of an ANCOVA analysis were met previously in RQ1a and RQ1b.

The effect of the MEP on the adjusted mathematical literacy posttest of male students was examined using ANCOVA. The results of one-way ANCOVA showed that there was no statistically significant difference at the $p < 0.05$ level in the posttest of ML results between the control group and the experimental group when adjusted for ML pretest results as shown in Table 16.

Table 16: ANCOVA results for the male 10th grade students' ML

Source	Df	Mean Square	F	P	η^2
Pretest	1	469.388	52.394	0.000	0.522
Group	1	5.193	0.580	0.450	0.012
Error	48	8.959			
Total	51				

Table 16 illustrates that the ANCOVA test was not significant $F(1, 48) = 0.580$, $p = 0.45$, partial $\eta^2 = 0.012$ indicating that there was no statistically significant difference in the posttest of mathematical literacy results between the female and male groups when adjusted for pretest results. As such, there was no need to perform post hoc analyses. Table 17 shows the adjusted mean scores for mathematical literacy for the tenth-grade female and male students in the experimental and control groups using pretest as a covariate.

Table 17: Adjusted and unadjusted means for ML for all students

		Unadjusted		Adjusted	
Groups	No.	M	SD	M	SE
Female students	24	18.17	5.00	15.77	0.764
Male students	27	12.81	4.39	14.85	0.706

Based on the results, one-way ANCOVA failed to reject the null hypothesis to demonstrate that there is no statistically significant difference between the means of the female and male groups regarding their mathematical literacy results due to MEP. This indicated that male and female students gained a nearly similar increase in their level of mathematical literacy. However, the adjusted means showed that females outperformed males, but the difference was very small, so it was not significant.

Generally, based on the answers to the previous three sub-questions, it was found that MEP has a positive effect on the students of the experimental groups of the tenth grade, whether they were male or female. Although there was no statistically significant difference between males and females' mathematical literacy, it was necessary to identify their performance in the test according to the six levels, four content areas, four contexts, and three processes and reasoning as addressed in the MLT based on the PISA mathematical literacy framework provided by OECD as follows:

Firstly, to study students' performance levels for MLT, the percentages of student performance at each level for males, females, and all students for the experimental and control group for both the pretest and the posttest were performed. The PISA performance levels are from level 1 to level 6 where level 6 is the highest. Table 18 displays the frequencies and percentages of students who answered the problems at each level below.

Table 18: The percentages of students' performance levels of MLT

MLT Levels	Students	Experimental group					Control group				
			Pretest		Posttest			Pretest		Posttest	
		N	F	%	F	%	N	F	%	F	%
Level 1 (n=4)	All	51	138	68	194	95	51	132	65	157	77
	Female	24	87	91	95	99	25	78	78	88	88
	Male	27	51	47	99	92	26	54	52	69	66
Level 2 (n=3)	All	51	83	54	110	72	51	78	51	69	45
	Female	24	52	72	66	92	25	41	55	42	56
	Male	27	31	40	44	56	26	37	47	27	35
Level 3 (n=7)	All	51	105	29	181	51	51	83	23	133	37
	Female	24	61	36	105	63	25	46	26	68	39
	Male	27	44	23	76	40	26	37	20	65	36
Level 4 (n=5)	All	51	26	10	122	48	51	21	8	35	14
	Female	24	22	18	75	63	25	18	14	22	18
	Male	27	4	3	47	35	26	3	2	13	10
Level 5 (n=4)	All	51	12	6	39	19	51	9	4	8	4
	Female	24	12	13	34	35	25	9	9	6	6
	Male	27	0	0	5	5	26	0	0	2	2
Level 6 (n=3)	All	51	3	2	4	3	51	3	2	0	0
	Female	24	3	4	3	4	25	3	4	0	0
	Male	27	0	0	1	1	26	0	0	0	0

As indicated in Table 18, mathematical literacy test results showed that performance at all six levels improved overall. The experimental group in both male and female students showed an increase at all levels compared to the control groups. Female students recorded greater improvement than male students at all levels. The most noticeable improvement was at the lower levels, where performance at level 1 improved the most and then gradually decreased to Level Six. At higher levels of the MLT test, the males in the experimental group failed to answer any of the questions at levels 5 and 6 of the pretest, while they showed little improvement on the post-test as they answered only 5% and 1% of levels 5 and 6, respectively. Female students showed better performance than males. They showed a marked improvement from 13% to 35% at level 5 but their performance at level 6 remained stable with 4% indicating no improvement. This can be clarified by Figure 18 below.

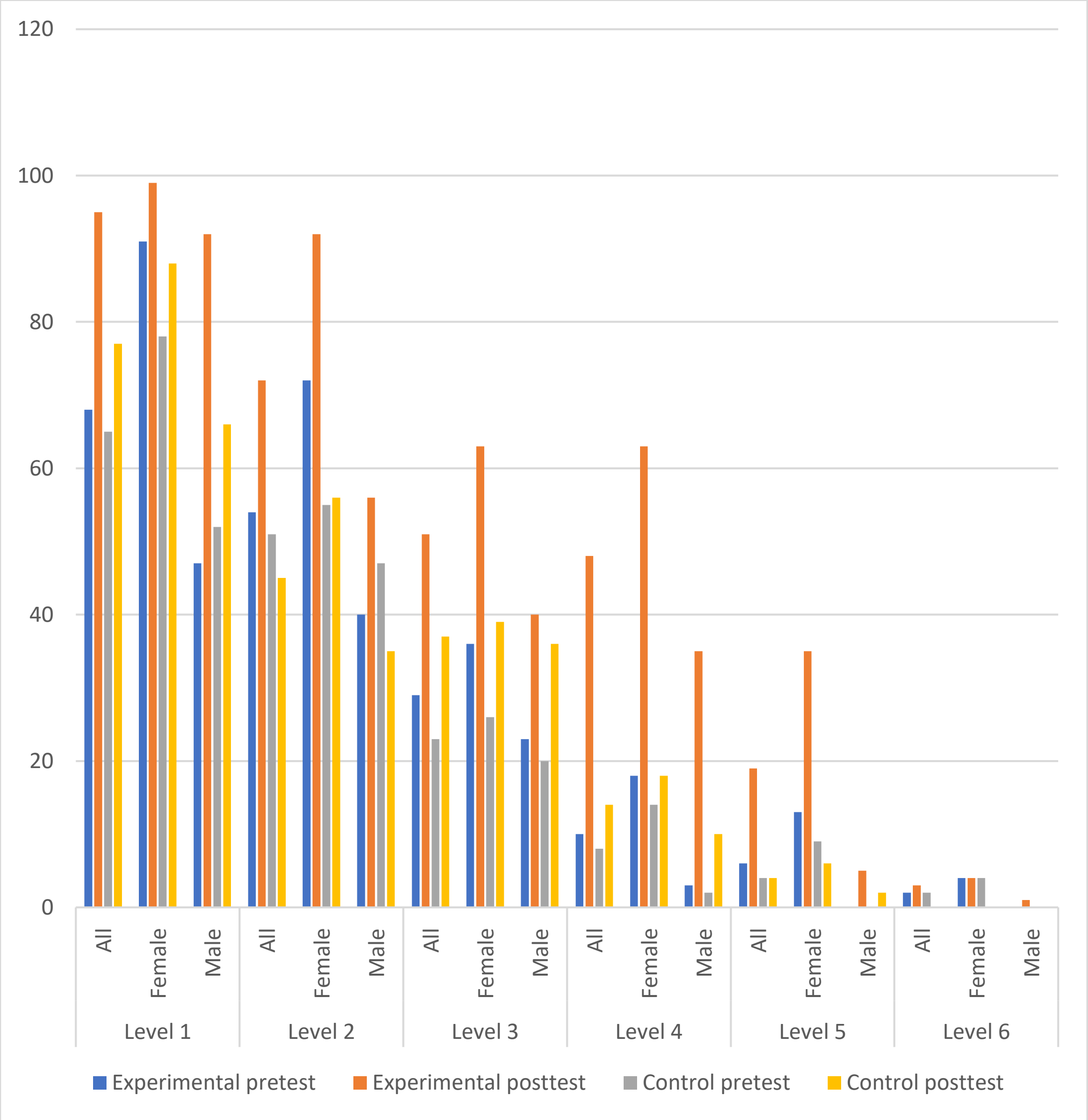

Figure 18: The percentages of students' performance levels of MLT

Likewise, Table 19 shows the frequencies for processes on MLT.

Table 19: The percentages of students' performance in processes of MLT

MLT processes	Students	Experimental group					Control group				
			Pretest		Posttest			Pretest		Posttest	
		N	F	%	F	%	N	F	%	F	%
Formulate (n=8)	All	51	68	21	143	33	51	60	15	88	22
	Female	24	50	26	81	42	25	36	18	44	22
	Male	27	18	8	62	29	26	24	12	44	21
Employ (n=14)	All	51	161	23	313	44	51	134	19	157	22
	Female	24	100	30	202	60	25	81	23	94	27
	Male	27	61	16	111	29	26	53	15	63	17
Interpret (n=4)	All	51	140	67	199	98	51	132	65	146	72
	Female	24	85	89	95	99	25	79	79	86	86
	Male	27	55	51	104	96	26	53	51	60	58
Reasoning (n=8)	All	51	61	15	132	32	51	52	13	98	24
	Female	24	46	24	58	30	25	35	18	44	22
	Male	27	15	7	74	34	26	17	8	54	26

As demonstrated in Table 19, the results also showed that females outperformed males in each of the modeling processes (formulate, employ, and interpret). The "Formulate process" scored the lowest score, while both genders scored the highest score in the "interpret" process. However, males scored higher than females in problems that require reasoning as presented in Figure 19 below.

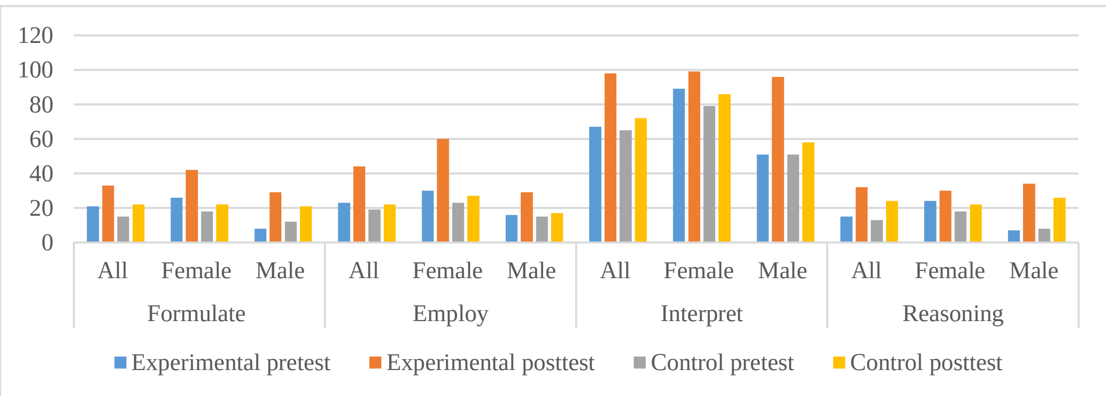

Figure 19: The percentages of students' performance in processes of MLT

Moreover, students' performance was analyzed in four major components of the Mathematical Literacy Test namely: change and relationship, quantity, space and shape, and uncertainty and data. The results are presented in Table 20 below:

Table 20: The percentages of students' performance in content of MLT

MLT Content	Students	Experimental group					Control group				
			Pretest		Posttest			Pretest		Posttest	
		N	F	%	F	%	N	F	%	F	%
Change and Relationship (n=7)	All	51	57	16	83	23	51	47	13	41	11
	Female	24	34	20	69	41	25	30	17	25	14
	Male	27	23	12	14	7	26	17	9	16	9
Quantity (n=7)	All	51	100	28	201	56	51	83	23	139	39
	Female	24	64	38	101	60	25	55	31	75	43
	Male	27	36	19	100	53	26	30	16	70	38
Space and shape (n=8)	All	51	68	17	172	42	51	65	16	76	19
	Female	24	52	27	118	61	25	38	19	46	23
	Male	27	16	7	54	25	26	27	13	30	14
Uncertainty and Data (n=4)	All	51	140	69	199	98	51	132	65	146	72
	Female	24	85	89	95	99	25	79	79	86	86
	Male	27	55	51	104	96	26	53	51	60	58

The results as revealed in Table 20 showed improvement in all four content areas. Female students scored higher than males in all four content areas. The highest percentage of male and female students scored in the "uncertainty and data" content area, where students answered almost all questions of this type of question content, while the lowest performance was in the "change and relationship" content area for both male and female students. This can be represented in Figure 20.

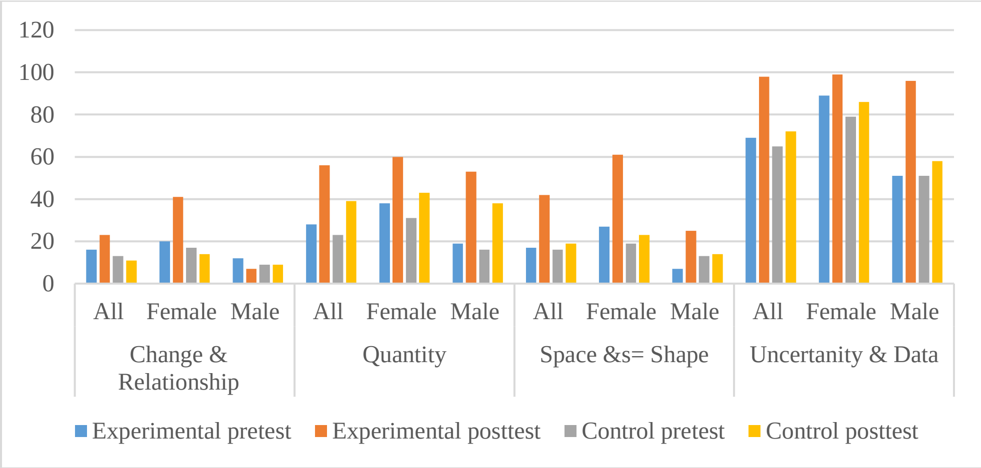

Figure 20: The percentages of students' performance in content of MLT

Furthermore, student performance was analyzed in four mathematical literacy contexts. Contexts included: personal, occupational, scientific, and societal, based on the PISA framework. The results of the frequencies and percentages are presented in Table 21 below:

Table 21: The percentages of students' performance in contexts of MLT.

MLT Context		Experimental group					Control group				
			Pretest		Posttest			Pretest		Posttest	
	Students	N	F	%	F	%	N	F	%	F	%
Personal (n=8)	All	51	150	37	228	56	51	109	27	153	38
	Female	24	86	45	124	65	25	67	34	88	44
	Male	27	64	30	104	48	26	42	20	65	31
Occupational (n=4)	All	51	37	18	78	38	51	37	18	39	19
	Female	24	31	32	52	54	25	25	25	23	23
	Male	27	6	6	26	24	26	12	12	16	15
Scientific (n=6)	All	51	36	12	100	33	51	36	12	47	15
	Female	24	27	19	58	40	25	22	15	29	19
	Male	27	9	6	42	26	26	14	9	18	12
Societal (n=8)	All	51	144	35	244	60	51	144	35	163	40
	Female	24	93	48	144	75	25	81	41	86	43
	Male	27	51	24	100	46	26	63	30	77	37

Table 21 demonstrates that the improvement was evident in all four contexts where females outperformed males. The highest percentage of male and female students scored in "societal" contexts, while the performance was lowest in “scientific” contexts but very close to occupational contexts as presented in Figure 21 below.

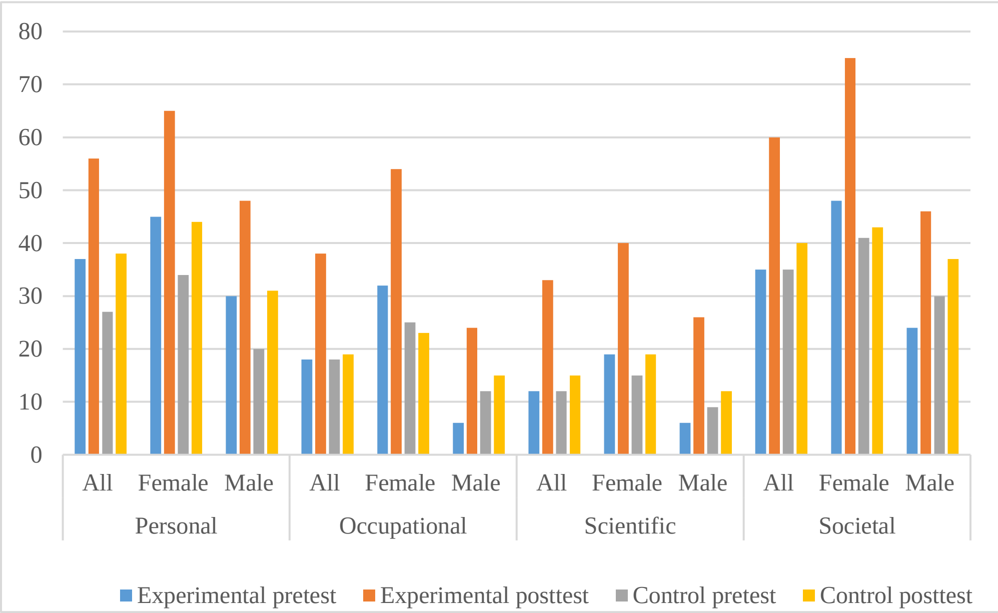

Figure 21: The percentages of students’ performance in contexts of MLT

As a result of all the previous analysis, it can be concluded that MEP could improve the students’ mathematical literacy performance. The experimental group in both male and female students showed better improvement at all levels, processes, content, and contexts compared to the control groups. Female students recorded greater improvement than male students in all areas except for reasoning.

Similarly, the statistical analysis of ANCOVA used separately in female and male schools to answer the first question was followed to answer the second question after controlling the motivation pretest score as follows:

4.2 Second Research Question

RQ2: What is the impact of the Mathematics Enrichment Program on tenth grade students’ motivation to learn mathematics?

To answer this question, similar to the first research question, the descriptive statistics of means and standard deviations were performed of both the pre and post-test for mathematical literacy for the female tenth-grade students, as shown in Table 20 below.

Table 22: Descriptive statistics of 10th-grade students’ motivation

Motivation to learn mathematics	Experimental group					Control group				
		Pretest		Posttest			Pretest		Posttest	
	N	M	SD	M	SD	N	M	SD	M	SD
Female students	24	43.75	3.55	47.08	4.51	25	43.36	3.83	46.80	5.16
Male students	27	46.19	6.24	48.78	6.10	26	44.23	7.07	47.31	5.44

In general, Table 22 shows that female students' motivation to learn mathematics was slightly less than that of male students in the experimental groups to which MEP was applied. The results of the post-test for both male and female students also showed that the experimental and control group increased and that the increase in motivation by the experimental group was slightly better for both genders; The female students got an average score of 47.08 for the experimental group and an average score of 46.80 for the control group, while the male students scored an average score of 48.78 for the experimental group and an average score of 47.31 for the control group. These means indicate that both groups of male and female students have roughly the same motivation score for learning math levels in both pretest and posttest. However, ANCOVA should be used to examine the effectiveness of MEP in controlling the pre-test of motivation to learn mathematics. Therefore to answer this question, as this study was applied to separate schools for males and females, the researcher sought to study separately the effect of the MEP on male and female students' motivation to learn mathematics with pre-test control, and then compare the difference in the effect on them using ANCOVA.

4.2.1 First Sub-question

RQ2a: What is the impact of the Mathematics Enrichment Program on tenth grade female students’ motivation to learn mathematics?

H0: There is no statistically significant difference between the experimental group and the control group on tenth grade female students’ motivation to learn mathematics.

Following the same procedure, before conducting the one-way ANCOVA analysis, the researcher examined six additional assumptions for the use of an ANCOVA analysis (Laerd Statistics, 2020b). For this research question, the dependent variable was the motivation to learn as measured by posttest and the independent variable was the availability of the MEP. There were no outliers in the data, as assessed by no cases with standardized residuals greater than ±3 standard deviations as revealed by Figure 22 that represents the whisker plot graph for both experimental and control female student groups below.

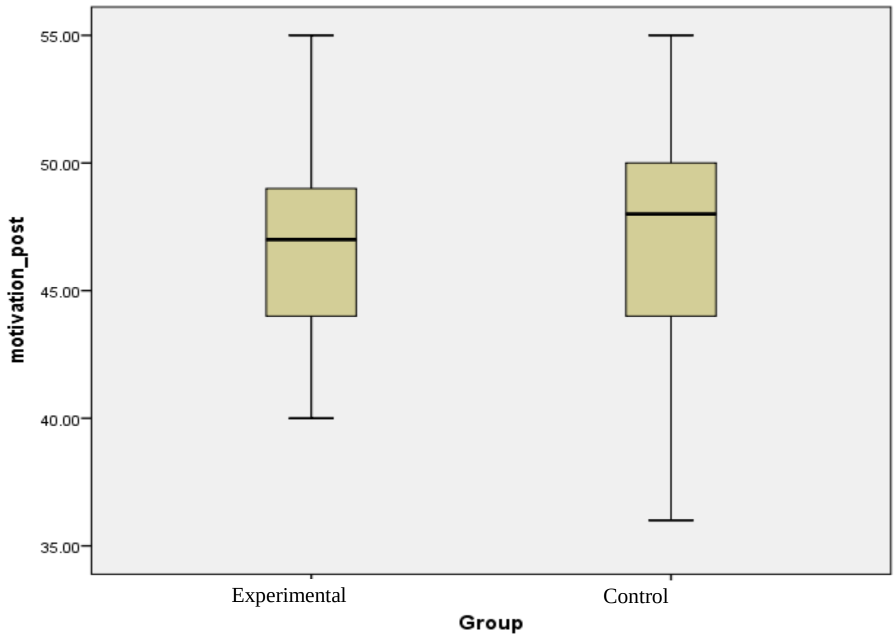

Figure 22: The whisker plot graph of female group’s motivation

There was a linear relationship between pretest and posttest motivation to learn mathematics for experimental and control groups, as assessed by visual inspection of a scatter plot as represented in Figure 23 below.

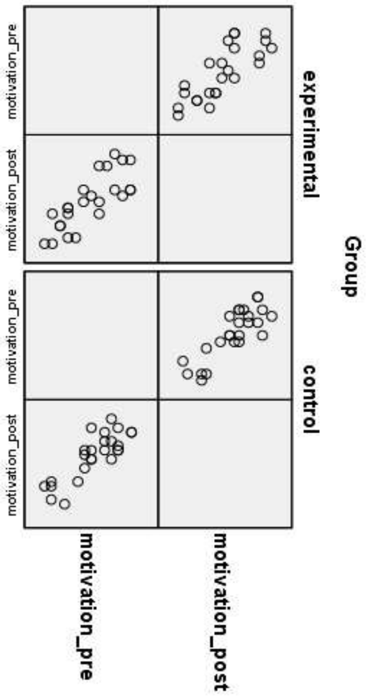

Figure 23: Scatter plot of female student’s motivation pretest and posttest

There was homogeneity of regression slopes as the interaction term was not statistically significant, F (1,45) = 0.392, p = 0.534. The data of experimental (p = 0.256) and control group (p = 0.170) was normally distributed as assessed by Shapiro-Wilk's test (p > 0.05) as revealed in Table 23 below.

Table 23: Tests of Normality of the motivation of the female groups

		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Group	Statistic	df	Sig.	Statistic	Df	Sig.
posttest	experimental	0.127	24	0.200*	0.949	24	0.256
	Control	0.158	25	0.106	0.943	25	0.170

The homogeneity of variance was assumed as shown by Levene's test of homogeneity of variance (p = 0.794). Moreover, the homoscedasticity was assumed as assessed by visual inspection of a scatter plot as represented by Figure 24 below.

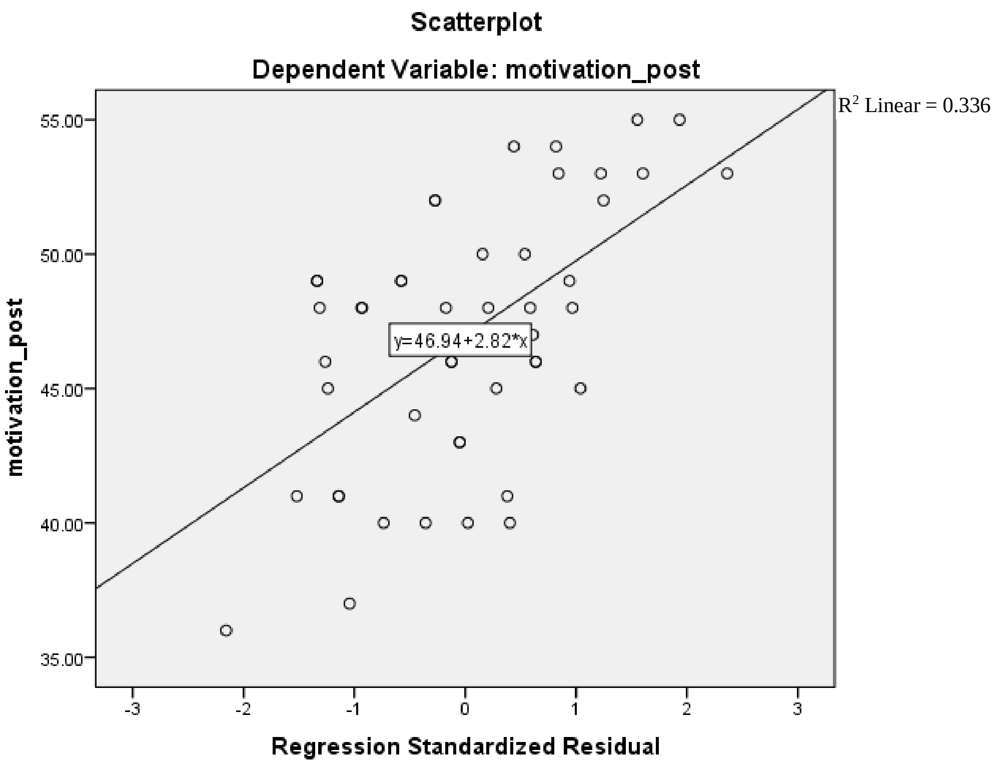

Figure 24: Scatter plot of the female students' motivation

The effect of MEP on the adjusted mathematical literacy posttest of female students was examined using ANCOVA. The results of one-way ANCOVA showed that there was no statistically significant difference at the $p < 0.05$ level in the posttest of motivation to learn mathematics results between the control group and the experimental group of the female students when adjusted for motivation to learn mathematics pretest results as shown in Table 24.

Table 24: ANCOVA results for female 10th grade students' motivation.

Source	df	Mean Square	F	P	η^2
Pretest	1	735.499	90.867	0.000	0.664
Group	1	0.219	0.027	0.870	0.001
Error	46	8.094			
Total	49				

Table 24 illustrates that the ANCOVA test was not significant $F(1, 46) = 0.027$, $p = 0.870$, partial $\eta^2 = 0.001$ indicating that there was no statistically significant difference in the posttest of motivation to learn mathematics results between the control group and the experimental group when adjusted for pretest results. Therefore, there was no need to perform post hoc analyses. Table 25 shows the adjusted mean scores for motivation to learn mathematics of the 10^{th} grade female students in the experimental and control groups using pre-test as a covariate as follows.

Table 25: Adjusted and unadjusted means for motivation of female students

		Unadjusted		Adjusted	
Female Groups	N	M	SD	M	SE
Experimental Group	24	47.08	4.51	46.87	0.581
Control group	25	46.80	5.16	47.00	0.569

Based on the results, one-way ANCOVA failed to reject the null hypothesis to demonstrate that there is no statistically significant difference between the means of female students in the results of the experimental and control group regarding their motivation to learn mathematics results after the implementation of the MEP.

4.2.2 Second Sub-question

RQ2b: What is the impact of the Mathematics Enrichment Program on tenth grade male students' motivation to learn mathematics?

H0: There is no statistically significant difference between the experimental group and the control group on tenth grade female students' motivation to learn mathematics.

As in the previous research questions, before conducting the one-way ANCOVA analysis, the researcher examined six additional assumptions for the use of an ANCOVA analysis (Laerd Statistics, 2020b). For this research question, the dependent variable was the mathematical literacy as measured by posttest and the independent variable was the availability of the MEP. There were no outliers in the data, as assessed by no cases with standardized residuals greater than ±3 standard deviations as revealed by Figure 25 that represents the whisker plot graph for both experimental and control female student groups below.

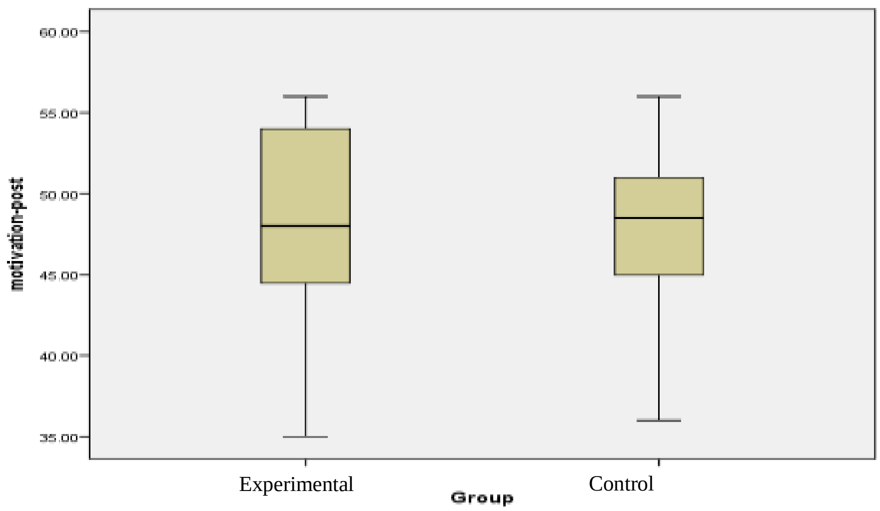

Figure 25: The whisker plot graph of male group's motivation

There was a linear relationship (to some extent) between pretest and posttest mathematical literacy for experimental and control groups, as assessed by visual inspection of a scatter plot as represented in Figure 26 below.

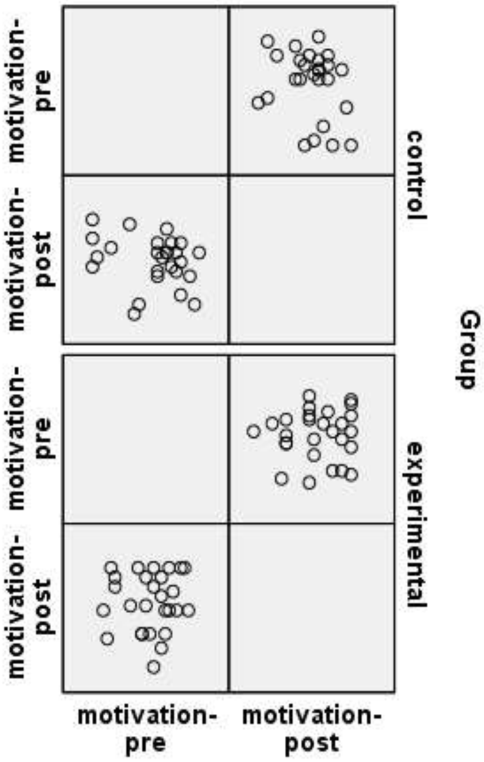

Figure 26: Scatter plot of male student’s motivation pretest and posttest

There was homogeneity of regression slopes as the interaction term was not statistically significant, F (2, 50) = 0.718, p = 0.493. The data of the control group was normally distributed as assessed by Shapiro-Wilk's test (p > 0.05) as revealed in Table 26 below but for the experimental group, p = 0.032.

Table 26: Tests of Normality for male students' motivation

		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Group	Statistic	Df	Sig.	Statistic	Df	Sig.
posttest	experimental	0.137	27	0.200*	0.916	27	0.032
	Control	0.131	26	0.200*	0.950	26	0.227

Nevertheless, despite looking at a histogram distribution that somewhat reflects a normal distribution as shown in Figure 27 below, ANCOVA can still be operated because it is robust to the violation of normality (Laerd Statistics, 2020b).

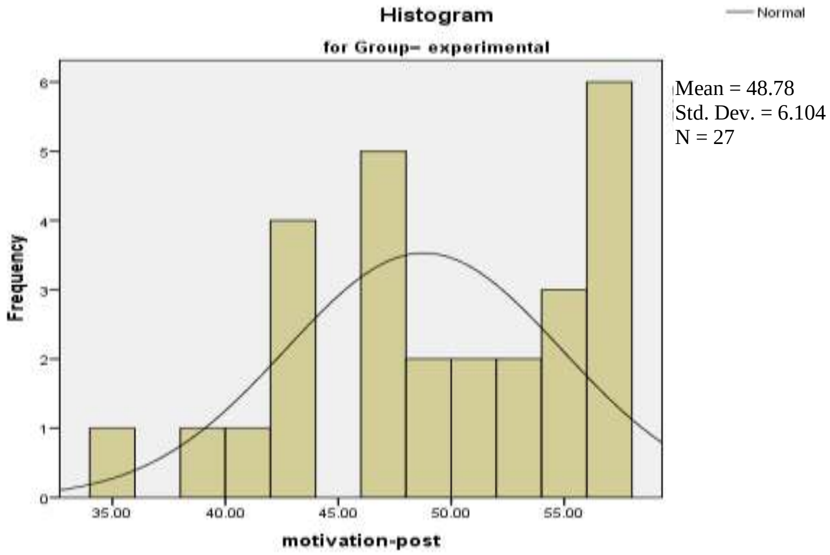

Figure 27: Histogram of the male experimental group’s motivation

The homogeneity of variance was assumed as shown by Levene's test of homogeneity of variance (p = 0.245). Moreover, the homoscedasticity was assumed as assessed by visual inspection of a scatter plot as represented by Figure 28 below.

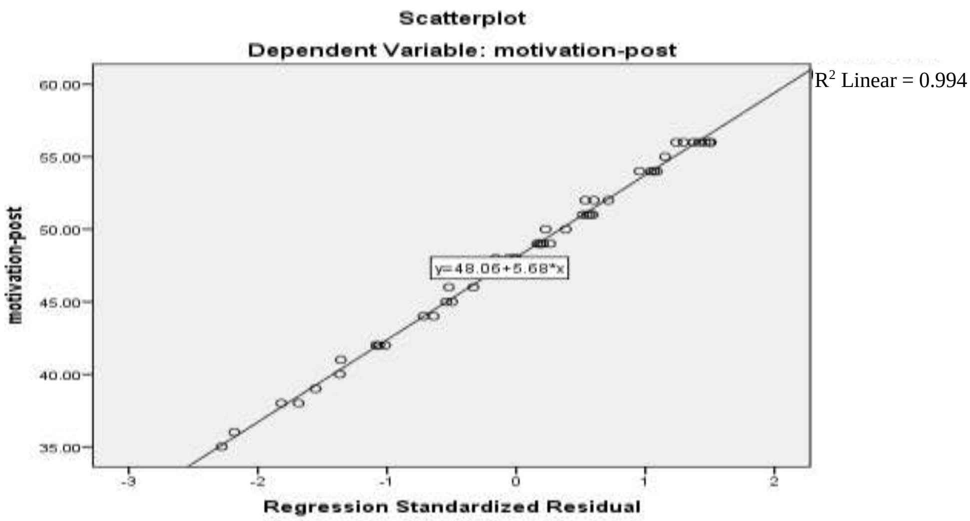

Figure 28: Scatter plot of male's motivation posttest

The effect of the MEP on the adjusted mathematical literacy posttest of male students was examined using ANCOVA. The results of one-way ANCOVA showed that there was no statistically significant difference at the $p < 0.05$ level in the posttest of motivation to learn mathematics results between the control group and the experimental group of the male students when adjusted for motivation to learn mathematics pretest results as shown in Table 27 below.

Table 27: ANCOVA results for the male 10th grade students' motivation

Source	df	Mean Square	F	P	η^2
Pretest	1	11.561	0.341	0.562	0.007
Group	1	33.573	0.989	0.325	0.019
Error	50	33.933			
Total	53				

Similar to the results of the female students, Table 27 shows that the ANCOVA test was not significant $F(1, 50) = 0.989$, $p = 0.325$, partial $\eta^2 = 0.019$ indicating that there was no statistically significant difference in the posttest of motivation to learn mathematics results between the control group and the experimental group when adjusted for pretest results. Therefore, there was no need to perform post hoc analyses. Table 28 shows the adjusted mean scores for motivation to learn mathematics for the 10^{th} grade female students in the experimental and control groups using pre-test as a covariate.

Table 28: Adjusted and unadjusted means for motivation of male students.

		Unadjusted		Adjusted	
Male Groups	N	M	SD	M	SE
Experimental Group	27	48.87	6.10	48.85	1.127
Control group	26	47.31	5.44	47.24	1.149

Based on the results, one-way ANCOVA failed to reject the null hypothesis to demonstrate that there is no statistically significant difference between the means of male students in the results of the experimental and control group regarding their motivation to learn mathematics results after the implementation of the MEP.

Considering the results of RQ2a and RQ2b, there is no statistically significant difference in motivation to learn mathematics in experimental and control groups for both male and female 10^{th} grade students. To get a more comprehensive understanding, the researcher sought to find if there is a significant difference between males' and females' motivation to learn mathematics after implementing the MEP by answering the following question.

4.2.3 Third Sub-question

RQ2c: Are there any gender-based significant differences in motivation to learn mathematics in response to the Mathematics Enrichment Program?

H0: There is no statistically significant influence of gender on students' motivation to learn mathematics in response to the Mathematics Enrichment Program.

For this research question, all six additional assumptions for the use of an ANCOVA analysis were examined previously in RQ2a and RQ2b.

The effect of the MEP on the adjusted motivation to learn mathematics posttest of male students was examined using ANCOVA. The results of one-way ANCOVA showed that there was no statistically significant difference at the $p < 0.05$ level in the posttest of motivation to learn mathematics results between the control group and the experimental group when adjusted for motivation to learn mathematics pretest results as shown in Table 29 below.

Table 29: ANCOVA results for the male 10th grade students' motivation

Source	Df	Mean Square	F	P	η^2
Pretest	1	92.679	3.207	0.080	0.063
Group	1	51.807	1.793	0.187	0.036
Error	48	28.901			
Total	51				

Table 29 confirms that the ANCOVA test was not significant $F(1, 48) = 1.793$, $p = 0.187$, partial $\eta^2 = 0.036$ indicating that there was no statistically significant difference in the posttest of mathematical literacy results between the female and male groups when adjusted for pretest results. As such, there was no need to perform post hoc analyses. Table 30 shows the adjusted mean scores for motivation to learn mathematics for the 10th grade female and male students in the experimental and control groups using pre-test as a covariate.

Table 30: Adjusted and unadjusted means for motivation for all students

		Unadjusted		Adjusted	
Groups	N	M	SD	M	SE
Female students	24	47.08	4.51	47.81	1.169
Male students	27	48.78	6.10	45.54	1.095

Based on the results, one-way ANCOVA, the null hypothesis was accepted to prove that there is no statistically significant difference between the means of the female and male groups regarding their motivation to learn mathematics results after the implementation of the MEP.

As an answer to the second question, taking into account its three sub-questions, no effect was found to the MEP regarding their motivation to learn mathematics in both genders. Moreover, no difference was found between male and female students.

4.3 Third Research Question

RQ3: What are the students' perceptions towards the Mathematics Enrichment Program after its implementation?

To answer this question, it was sought to collect quantitative and qualitative data to learn about how the students perceive their experience of participating in the MEP then analyze it to answer this question. The perceptions survey was analyzed after it was distributed to each of the two experimental groups, male and female. The perceptions survey consists of two parts; the quantitative part and the qualitative part. Thus, descriptive analyses were used to analyze the quantitative portion in the survey analysis as well as thematic analysis for analyzing the qualitative portion of this data. Moreover, the qualitative part also provides valuable suggestions on how to improve the MEP.

4.3.1 The Quantitative Part of the Perceptions Survey

The quantitative part of the perceptions survey consisted of 8 items of 5 point Likert scale statements. Descriptive statistical analyses were employed; mean scores rather than total scores were analyzed, following Gagné’s (1991) interpretations. The mean scores were categorized as follows: mean scores of 4–5 points were classified as high positive (HP), between 3.24–3.99 as positive (P), 2.75–3.25 as ambivalent (A), and 2–2.74 as negative (N). Scores under 2 were considered high negative (HN). The means and standard deviations of the results for individual statements in the survey were found. In addition, the mean results per individual statement rating were estimated using Gagné’s (1991) interpretation and demonstrated in Table 31 as follows:

Table 31: Perceptions Survey Mean Scores

Program Evaluation Survey Items	M	SD	Rating
1- I loved the mathematical contextual problem solving presented in this program.	3.80	1.15	P
2- This program made me feel more confident about my mathematics ability.	3.92	0.94	P
3- This program helped me to do better in my regular Mathematics class.	4.06	0.86	HP
4- This program made me see and appreciate the importance of mathematics in life.	4.18	0.87	HP
5- This program made me more motivated and engaged in my mathematics study.	3.86	0.98	P
6- This program made me more prepared to take the PISA test in mathematics.	4.20	0.66	HP
7- It is important to spend time studying contextual problem-solving in mathematics classes.	4.10	0.94	HP
8- Deducting time from math classes to implement this program did not present a challenge to complete the required curriculum on time.	2.90	1.15	A
Survey Average	3.88	0.80	P

Table 31 reveals that students who enrolled in MEP showed a general positive feeling (M=3.88) about the program. They were very positive about 4 aspects of the program: it made them more prepared for the PISA test (M = 4.20), it also made them see and appreciate the importance of mathematics in life (M = 4.18), they see that spending part of mathematics classes time to study this type of problems is important (4.10) and that the program helped them to do better in their regular mathematics class. The students were positive about 3 aspects of the program; they loved the mathematical contextual problems (M = 3.80) that made them more confident in their mathematical ability (3.92) and motivated them to learn mathematics (M = 3.86).

Noteworthy, the students were only ambivalent (M = 2.90) about deducting time from mathematics classes to implement the program.

Furthermore, the students were asked "Do you recommend applying this program to students to improve their mathematical literacy?" if they recommend this program for improving mathematics literacy. Frequencies and percentages of students who recommend the program from both genders presented in Table 32:

Table 32: Percentages of students who recommend the program

Students who recommend the program		F	%
All Students	Yes	44	86.3
	No	7	13.7
	Total	51	100.0
female students	Yes	22	91.7
	No	2	8.3
	Total	24	100.0
Male students	Yes	22	81.5
	No	5	18.5
	Total	27	100.0

Table 28 shows that most students would recommend this program to improve students' mathematical literacy and literacy based on their experience. The number of students who recommended this program was 44 (86.3%), while those who did not recommend it were 7 (13.7%). In more detail, most of the students who don't recommend this program were males, 5 males versus 2 females.

The majority of female and male students recommended this program. However, for further understanding, another sub-question has been added to understand if there are differences between female and male perceptions as follows:

RQ3a: Are there statistically significant differences between female and male perceptions of tenth grade students about MEP?

H0: There are no statistically significant differences between female and male perceptions of tenth grade students.

An independent-samples t-test was conducted to compare the perceptions towards the program for female and male students to answer this question. As usual, test assumptions are examined before any statistical test is performed. There are six assumptions for the use of the independent-samples t-test (Laerd Statistics, 2020a). The first three assumptions are related to the choice of research design and do not need statistical tests to check them. The research design of this study, a quasi-experimental, non-equivalent control group design supports these assumptions; there is one dependent variable (students evaluation to MEP) that is measured on a continuous scale; there is one independent variable of two categorical independent groups (Gender), and independence of observations as there is no relationship between the participants in either of the groups.

The other three assumptions were normally distributed residuals, homogeneity of variance, and no outliers (Field, 2013; Laerd Statistics, 2020a). There were no outliers in the students' responses to the program evaluation survey, as assessed by no cases with standardized residuals greater than ±3 standard deviations. The data of male and female groups were not normally distributed as assessed by Shapiro-Wilk's test ($p = 0.046$ for females; $p = 0.024$ for males) but their histograms showed approximate normality. However, despite looking at a histogram distribution that to some extent reflected a normal distribution, the t-test was described as a robust test regarding the assumption of normality. This means that some deviation from normal does not have a significant effect on Type 1 error rates (Laerd Statistics, 2020a). For homogeneity of variance, based on Levene's test of equality of variances (F = 0.093, P = 0.762) equal variances were assumed.

After testing all the assumptions, the independent t-test was performed, and its results are shown in the following Table 29.

Table 33: Comparison of perceptions about the MEP

Variable	Female (N = 24)		Male (N =27)		T	df	P
	M	SD	M	SD			
MEP perceptions	4.03	.74	3.74	0.84	1.306	49	.198

Table 33 above, shows that there was no statistically significant difference between female (M= 4.03, SD = .74) and male students (M= 3.74, SD = .84) on program evaluation (df = 49, t = 1.306, p > .05). The researcher failed to reject the null hypothesis. This indicates that female and male students are roughly holding the same perceptions about MEP.

4.3.2 The Qualitative Part of the Perceptions Survey

Students were asked two open-ended questions to explain how they felt about their experience and offer their recommendations. The two questions were as follows:

1- "Do you feel that Mathematical Enrichment Program helped you or not? In what Aspects?"

2- “What can be done to improve this program?”

As for the first question, the majority of students expressed positive or negative feelings with an explanation, while a small number of students answered yes or no; the percentages of student opinions are reported in Table 34 below.

Table 34: Positive Vs. Negative responses by gender

Positive Vs. Negative Responses		N	%
All students	Positive	42	82.4
	Negative	9	17.6
	Total	51	100.0
Female students	Positive	21	87.5
	Negative	3	12.5
	Total	24	100.0
Male students	Positive	21	77.8
	Negative	6	22.2
	Total	27	100.0

Table 34 shows that students generally viewed the program as positive and that the biggest source of this positivity was the female students. The students reported 42 (82.4%) positive comments towards the program versus 9 (17.6%) negative comments.

Patterns formed naturally while reviewing responses to the first open-ended question explaining students' feelings about the program. All student comments had been reviewed and then classified. Tables 35 and 36 represent the general positive and negative themes respectively, with some examples of comments representing most of the groups.

Table 35: Positive trends of students' comments

Theme	N	Examples of positive comments
1-Understanding	14	"Yes, it helped me to understand school topics." "I understood the mathematics concepts better and differently, now I got it." "The time taken from class to study this program was good, I used the program information in the mathematics periods." "It broadens my understanding of mathematics."
2-Mathematics in life	6	"This program makes me see the importance of mathematics in life and not to study only equations." "It showed me applications of mathematics in life." "It is very useful and could answer many questions in life."

Table 35: Positive trends of students' comments (Continued)

Theme	N	Examples of positive comments
3-Preparing for PISA	6	"Yes, I think I can now take the PISA test well because I am the first time to know about it through the exercises of this program." "It helped me to know about PISA, I liked this kind of problems, it makes me feel like this is the real math not only the formulas that we learn and use." "Yes, it helped me a lot, for example, increased my self-confidence in solving math problems, and also helped me to know the way the questions of the PISA exams."
4-Excitement	6	"It makes me excited about the tricks, I never thought of mathematics in this way." "It's fun and makes me love math." "It was exciting, even when I don't understand the meaning for every word in the question, I tried to understand the question and tried to solve it and I succeeded many times."
5-Revision	3	"Yes, the revision part was very important because it is the basics that help in solving the story problems." "The program covered all mathematics we learned. " "I liked it to make me revise previous math concepts."
6-Discussion	2	"Some questions challenged me, I liked spending time discussing these questions with teacher and colleagues." "I am excited in these classes; I like the discussion with my teacher and friends."
7-With no comments	5	"Yes." "Yes, it helped me. "
Total	42 positive comments	

Table 35 displays trends in positive student comments and presents these comments in seven groups resulting in six themes. Students generally held positive impressions about the impact of the program. The most positive theme was "understanding". Students reported 14 comments about the program indicating the impact of this program in increasing their understanding of the curriculum. Three themes of the positive responses were on “Mathematics in Life”, “Preparing for PISA” and “Excitement” with 6 positive comments each. The last two topics on "review" and "discussion" were in 3 and 2 comments respectively, while 5 comments expressed students' positive feelings about the program without explanation.

Table 36: Negative trends of students’ comments

Theme	N	Examples of negative comments
1-Time	3	“No, because time is not enough.” “It is not part of the curriculum, why should we waste time.” “It needs more effort and time because sometimes we don’t study the same things we study from the book.”
2-Language of problem	3	“Not too much. I don’t understand the question, the words are very hard and make me feel nervous.” “No, I don’t understand this kind of problems there are many difficult words.” “No, I can’t understand these questions.”
3-Problem solving skills	3	“I don’t like these questions; I can’t understand how to solve them.” “I don’t like it because I don’t like this kind of math because I don’t know how to solve it.” “I don’t like it because I don’t like story problems. it needs more efforts to understand what method to use to solve it.”
Total	9	

Through the students ’response to the first open question about their feelings about the applied enrichment program, they indicated 9 negative comments that are mentioned in Table 36. These negatives responses were divided into three main themes: “Time”, “Language” and “Problem Solving”. The “Time” theme indicates that students didn't like deducting part from mathematics classes for the program implementation. The "language" theme mainly reflected the students feeling of the difficulty of words that results in a misunderstanding of the question. Some students were struggling in deciding how to solve the problems as presented in the "Problem Solving" theme.

The second open-ended question asked the students to explain "What can be done to improve this program?" Students' suggestions are shown in Table 37 below.

Table 37: Improvement suggestions to MEP

Theme	N	Examples of improvement suggestions to MEP
1-Time	16	"I liked the way the teacher discusses the program lessons so providing more time is good to finish all the questions in the lesson with the teacher."
		"Make one period weekly for these problems."
		"Not to use the period time, or make it an official part of the curriculum."
		"Add specific lessons for PISA problems in the school timetable."
2-Official curriculum	9	"A formal addition of the PESA exam questions method for mathematics, as it is taught within the mathematics curriculum in schools."
		"Make it a regular part of the curriculum."
		"I think this program should be used with all classes not only grade 10 because it is useful in understanding what mathematics is used for."
3-Language	6	"I liked the way the teacher discusses the program lessons so providing more time is good to finish all the questions in the lesson with the teacher."
		"Make one period weekly for these problems."
		"Not to use the period time, or make it an official part of the curriculum."
		"Add specific lessons for PISA problems in the school timetable."

Table 37: Improvement suggestions to MEP (Continued)

Theme	N	Examples of improvement suggestions to MEP
4-Assessment	5	"Make more time for the program and make extra marks for the solution of problems." "Despite it motivates us to learn better, I think you need to make exams because we study if we have an exam." "No need to have this program there are no marks for it so why should we study it?"
5-Flipped classroom	4	"Let the students prepare and come later to discuss." "PISA sometimes is applied online. So, if this program is made online and the students can use it independently then discuss with the teacher." "If the teacher gave us the problems to prepare at home this gives us the chance to prepare and be more engaged in the discussion."
6-Revision	3	"Increase the revision part because it was very important and helped me." "Increase the revision part." "We can add the number of revision lessons to take this program."
7-No suggestions	8	
Total	51	

There was nothing better than seeking assistance from students who had undergone the implementation of the enrichment program to present their suggestions for developing the program. The students' suggestions for improvements were in six themes. The largest share of suggestions went for "Time" with 16 suggestions. Some students were not satisfied with taking periods of math to implement the program and suggested making a special time for the program. In addition, they suggested more time to discuss all the problem solving provided in the program. Other students 'suggestions weren't in Table 37, however, the students' ideas were all about adding extra time above math classes. As for the second theme of the students 'suggestions, it revolved around nine suggestions that focused on making a formal addition to the enrichment program subject to mathematics curricula. Moreover, students requested that this program be applied to all grades. The students' six suggestions about language were centered on using easier language. Then students focused on the assessment noting the importance of testing students' knowledge and making it accountable because students would make no effort if the program was not counted in their homework. There were four other suggestions regarding providing students with enrichment materials in advance and giving students time to prepare by using their own time as an indicator of the flipped classroom. Finally, three students' suggestions were to increase the review portion of MEP.

4.4 Summary

This chapter provided data analyses to address the study's three research questions. The data provided insights into the MEP. Analysis of pretest and post-test showed that a student who enrolled in the MEP improved their scores on mathematics literacy, while it failed to improve the students to learn mathematics. An analysis of the perceptions survey showed that students felt better about their mathematical literacy and thought the MEP was helpful. The students also expressed a positive feeling about the program, and there were no differences between male and female students' perceptions. Students provided some important suggestions for improvement too. The fifth chapter of this dissertation discussed the research findings, recommendations, and implications for future research.

Chapter 5: Discussion

5.1 Introduction

The purpose of this chapter is to discuss the findings and implications of this study. This study aimed to investigate the effect of the MEP on the mathematical literacy of tenth graders and their motivation to learn mathematics. The study used the mixed methods design, where the quantitative and qualitative results were drawn. The tools used in this research were: pre and post-test, the motivation to learn scale, and the scale of the perception, which consisted of quantitative and qualitative parts. Descriptive statistics, an analysis of covariance, and a t-test for independent means were conducted to examine the significance of differences in the scores. In addition, thematic analysis was used to analyze qualitative data. This chapter provides a summary of the findings and an interpretation of the results as they relate to the research questions. This chapter also presents the study implications for instructional designers and recommendations for future research.

5.2 Discussion of Research Questions

After building the mathematical enrichment program based on the defined principles of preparing the proposed MEP, the main purpose of this research was to study the effect of this program to determine if its application had a significant impact on the tenth-grade student's mathematical literacy and motivation toward mathematics. This study was applied to two experimental classes and two control classes over a period of ten weeks. All students received a pretest of mathematical literacy and a motivation to study mathematics in the first week of the study. For the following eight weeks, the MEP was implemented for the experimental male and female groups at a weekly lesson rate, at two periods per week. During mathematics lessons, students received PBL instruction that depended on discussion and active learning to allow the students to build their own knowledge, while in the same period, the two male and female control groups received traditional instruction. Then, in the last week of the study, all students received a mathematical literacy posttest and a motivation to learn mathematics survey. Then they were asked about their perceptions towards the implemented MEP.

To achieve the purpose of this study, three research questions were assigned to this research. Two of the research questions were related to the two dependent variables-- mathematical literacy and motivation. The first research question examined the effect of MEP on students' mathematical literacy, while the second question dealt with its effect on student motivation. The third question sought to find students' perceptions of MEP.

5.2.1 First Research Question

RQ1: What is the impact of the Mathematics Enrichment Program on the mathematical literacy of tenth grade students?

The main finding related to this question was that there was an observable general increase in students' mathematical literacy of the experimental group more than the control group. This increase was explained by the results of the post-test participants compared to the pretest with the pretest as a covariate to control students' previous levels. The mean scores for both genders on the posttest of mathematical literacy were increased reflecting the good impact of the MEP on the students' mathematical literacy. This finding supported the arguments discussed by (e.g., Nuurjannah & Sayoga, 2019; Laurens, Batlolona, Batlolona & Leasa, 2018) who indicated that using contextual problems improved students' achievement in mathematics. The results showed that increased mathematics literacy was more evident among female students than male students.

All potential confounding variables such as time difference, teacher influence, and topics to be covered were controlled. Thus, it is clear to conclude the effect of the treatment. The study showed that students who were participants in MEP were more likely to improve their mathematical literacy. This study demonstrated the positive impact of MEP that is designed to focus on solving contextual problems and reasoning as main components of mathematical literacy. Under the pressure to improve mathematical literacy to meet the UAE Vision 2021 to be among the top 20 countries in PISA, this study was a major first step in a positive direction in establishing data-based decision-making protocols and processes for analyzing instructional programs.

This improvement in mathematical literacy for both male and female students is also consistent with the positive effect of PBL and is in line with previous research findings such as Wardono et al. (2016) and Firdaus, Wahyudin, and Herman (2017). Wardono et al. (2016) investigated the PBL using Realistic Mathematics education with 7^{th} graders to find that the average mathematical literacy ability for two experimental classes using PBL was better than the control group. Similarly, another study by Firdaus, Wahyudin, and Herman (2017) also applied a quasi-experimental study to investigate the effect of PBL in improving the students' mathematical literacy. The results of their research indicated that the PBL was more effective in improving students' mathematical literacy model than direct instruction.

Most students perceive mathematics as a difficult subject due to the lack of real-life connection and unattractive teaching methods. However, the proposed MEP was successful and increased the students' mathematical literacy. The MEP consisted mainly of contextual mathematical problems that are taught in a PBL environment as one of the CTL approaches. CTL is a method that helps the teachers to relate subject content to real-world applications and motivate students to make connections (Berns & Ericson, 2001; Hudson & Whistler, 2007). Thus, the increase in students' achievement in mathematical literacy is likely due to meaningful learning that allows information to be stored more quickly and remembered more easily for retrieval (Taylor & Parson, 2011).

More recently, a lot of previous research has given great importance to producing valid and practical PISA-like questions because the PISA test is considered one of the most powerful tests measuring students' literacy in mathematics. These questions were designed and examined by various researchers such as Dewantara, Zulkardi and Darmawijoyo (2015), Efriani, Putri, and Hapizah (2019), Nizar, Putri and Zulkardi (2018), and Putri and Zulkardi (2020). These researchers developed PISA-like problems in different content areas of mathematics and reached the same conclusion of the positive impact of these problems on the students' mathematical literacy.

The students' results in MLT didn't reflect the existence of gifted students in the tested classes. Moreover, although students' mathematical literacy increased in this study, a closer look at the results showed a disturbing result because students barely passed the test (half of the full test mark) which is unsatisfactory. The researcher observed, consistent with the research of Depaepe, De Corte and Verschaffel (2010) that some students tend to only select numbers from the text and perform operations without understanding, and continue to think of word problems as an exercise without looking at the real limitations. This answer-getting mindset that is called “suspension of sense-making” is seen as a serious challenge in mathematics education (Schoenfeld, 1991 & 1992; Verschaffel et al., 2000).

Moreover, this low level of student performance may prompt us to pay attention to the quality and method of evaluating students, as it is assumed that students in the academic stream are distinguished students, how can they perform high in the grade-level achievement test, while their results show a low level in the mathematical literacy test?. It is possible that the grade-level achievement test is very easy and does not have enough items of appropriate difficulty for the student, and therefore the result may not indicate the true level of his understanding (Rotigel & Fello, 2004). Thus, a higher-level test containing more difficult items may be necessary to be administered in grade-level achievement tests.

Moreover, student scores on the MLT test may reflect the prevailing testing culture among students because they only make effort to study upon testing and do not care if this test is informal. In addition, some schools focus more on student performance on the test only. Hence, teachers teach for the tests and students focus on memorizing for the test without understanding the concept of real mathematics. Changing these malpractices may take more time and effort.

Even when students want to study an informal subject such as that offered by the MEP program, another important factor that plays an important role in students' achievement in mathematics is self-regulated learning because it is a factor that makes the learning process more effective (Fauzi & Widjajanti, 2018). The purpose of Fauzi and Widjajanti’s (2018) research was to analyze previous research findings on the effects of self-regulated learning on student achievement in mathematics. They concluded, based on 11 related research articles reviewed, that students with a high degree of self-regulated learning tend to be highly motivated with high achievement, and vice versa. Therefore, in the current study, the reason for students' low achievement may be poor self-regulatory learning.

The results of the current study revealed that there is no significant difference between male and female students like many research (Ghasemi, Burley & Safadel, 2019; Reis & Park, 2001). However, females in the current study showed slightly better mathematical literacy than males. Similar results were revealed by the study of Ajai and Imoko (2015), which followed the same design of the current study, a quasi-experimental pretest, and applied PBL as well. Moreover, the results of the present study were consistent with PISA 2018 results of the UAE where girls outperformed boys in mathematics by nine score points and were in contrast to the trend in OECD countries where males scored five points higher than females (OECD, 2019b).

Student performance showed overall improvement in all six levels of mathematical literacy. The experimental group in both male and female students showed better improvement at all levels compared to the control groups. Female students recorded greater improvement than male students at all levels. Remarkably, the improvement of students at higher levels is almost negligible, especially at the sixth level problems. However, females' performance in levels 5 and 6 was better than male students that is consistent with Innabi and Dodeen's (2018) results. Additionally, similar results were found by Edo, Hartono, and Putri (2013) who investigated secondary school students' difficulties in solving PISA problems of levels 5 and 6. Their research revealed the students' difficulty in formulating situations mathematically and evaluating the reasonableness of a mathematical solution in the context of a real-world problem.

Thus, having a deep look at the processes that students use is also important to understand the strengths and weaknesses of students thinking such as the research of Dewantara, Zulkardi, and Darmawijoyo (2015) which developed and examined PISA-like problems. The research focused on the three mathematical processes that were used in the modeling cycle; formulate, employ and interpret, the results showed that the highest percentage of students' achievement in interpreting tasks was more than employ and formulate. The results of the present study were consistent with research by Dewantara, Zulkardi and Darmawijoyo (2015) and Edo, Hartono and Putri (2013) in that the higher percentage of students' achievement was interpreting tasks was more than employ and formulate. However, more attention should be given also to the reasoning that has been added to the PISA 2021 mathematical literacy framework (OECD, 2018a, 2018b). Consequently, in the current research, reasoning has been embedded in the MEP content and MLT. In the UAE, girls got nine points higher than boys in mathematics in the PISA 2018 results, while In OECD countries, boys outperformed girls by five points (OECD, 2019b). The results of current research, and in line with the results of PISA 2018 for the UAE, showed that females outperformed males in each of the modeling processes (formulate, employ, and interpret). In contrast, males outperformed female students in solving complex problem solving as found in some studies (e.g., Hyde, Fennema & Lamon, 1990; Fennema, 2000). However, performance on problems that require reasoning was against the trend of results in the UAE, where males outperformed females. Similar results were found by Stage, Kreinberg, Eccles, and Becker (1985) in their review of studies related to gender and achievement found that males perform better on reasoning tasks, while females perform better on computational tasks.

The students were examined in four main content areas; Change and relationship, Space and shapes, Quantity, and Uncertainty. Results showed perfect performance in problems that covered the “uncertainty” content area, while the worst performance was in “change and relationship” problems. This result was also experienced by Indonesia (Edo, Hartono & Putri, 2013).

The four contexts (personal, occupational, societal, and scientific) reflected the wide range of situations in which individuals may meet mathematical opportunities. The students were most successful in dealing with personal problems in the present study.

5.2.2 Second Research Question

RQ2: What is the impact of the Mathematics Enrichment Program on tenth grade students’ motivation to learn mathematics?

The main conclusion regarding this question is that there was no statistically significant difference between students in the control and experimental group in their motivation. Moreover, the results indicated that there was no improvement in student motivation due to MEP which mainly involved contextual problems and was managed using PBL. The results of the current study were inconsistent with the claim that contextual tasks can enhance student participation (Hernández, Levy, Felton-Koestler & Zbiek, 2016; NCTM, 2000, 2014) and also disagree with Hmelo-Silver (2004) who found that PBL leads students to become intrinsically passionate about learning. However, similar results were found for both male and female students in this study. Wernet (2015) supports these findings and has argued that contextual problems can play different roles in terms of improving students' understanding of mathematical concepts, while finding that contextual tasks do not necessarily have the same ability to motivate students to learn.

Although previous research indicates that educators tend to look at contextual problems primarily in terms of their ability to motivate their students (Lee, 2012; Pierce & Stacey, 2006), the results of this study have not supported this possibility. This indicated that some other factors determine students' motivation to learn. This might be due to the idea in the school culture that students fear problems with the story (Wilson, 2003). Thomas and Gerofsky (1997) referred to them as “the hated word problems” (p. 21). The reason for this fear has been revealed by another study of 526 high school students that found that student participation was significantly affected by the balance between the challenge posed by the task and the possession of the necessary skills, perceived control overactivity, and relevance of the task (Shernoff, Csikszentmihalyi, Schneider & Shernoff, 2003). This indicates the need to take a closer look at the relationship between students' motivations and contextual tasks.

The literature revealed that some researchers found that using contextual tasks was problematic and called for a reconsideration of their role in the curriculum (Gerofsky, 2004; Verschaffel et al., 2000). Wernet (2015) suggested two main arguments that challenge the motivational potential of contextual tasks. The first that the notion of the real world itself in contextual tasks is problematic because there is no agreement on its definitive meaning (Gerofsky, 2004). He argued that contextual tasks reflected the nature of word problems rather than reflecting the students' “real life”. Therefore, Verschaffel et al. (2000) supported the argument that students engage with contextual tasks with established beliefs and expectations about how to solve them as rote applications of algorithms. This indicated that contextual tasks are unlikely to support meaningful connections between school and everyday mathematics or to communicate the value of mathematics (Wernet, 2015). The second argument is about the students’ diverse experiences and interests. It suggests that it is unrealistic to find contextual tasks that can relate to all students’ experiences and goals. This may isolate or exclude some students, resulting in students' disengagement in mathematics (Middleton & Jansen, 2011; Sullivan et al., 2003). Consequently, this underscores the importance of contexts that are personally relevant or fit with the broader identities of students and highlights the difficulty of developing contexts that will stimulate diverse groups of students at scale.

Wernet (2017) stated that it was assumed that contextual tasks supported student participation and sense-making. However, he argued that conflicting ideas about the role of these tasks were found in the lessons, and more research was needed to explore how classroom interactions might help achieve the intended purposes. Thus, he investigated how teachers and students interacted about problem contexts in written assignments. After analyzing the videos of lessons in three eighth grade classrooms using a problem-based curriculum, the findings showed that how teachers discussed contexts fell into five general categories: referral, positioning, elaboration, illustration, and meta-level commentary. These findings provided a framework for interpreting contextually relevant classroom interactions. This pointed to the importance of studying the role contexts might play in mathematics lessons.

However, it was noticed that both experimental and control groups experienced a slight increase in their mean levels of motivation. This was not a surprising finding because these students already possessed high levels of motivation. Students in both experimental and control groups belonged to the advanced stream in which most students were high achieving students and were self-motivated to their study. This might be considered another reason for this contradictory result.

Despite that research has confirmed the gender differences in mathematics motivation, the results of this question revealed that there are no gender differences regarding students’ motivation. Some research indicates that males generally have better motivational profiles in mathematics than females (Kurtz-Costes et al., 2008; Rodríguez, Regueiro, Piñeiro, Estévez & Valle, 2020), while Frost, Hyde, and Fennems (1994) found that attitudes and affect of females were more negative with respect to mathematics. Generally, gender differences in math performance and attitudes and affect appeared to be small to moderate only.

In summary, the results of the current study showed no significant differences between males and females students’ motivation. As a result, it could be included that the MEP failed to improve the students’ motivation.

5.2.3 Third Research Question

RQ3: What are the students' perceptions towards the Mathematics Enrichment Program after its implementation?

The proposed MEP in the present study, was evident in improving students’ mathematical literacy based on students’ performance, while there was no improvement in students’ motivation. However, reaching this conclusion about student motivation was based on a self-report survey of some Likert survey questions, and it might not be sufficient to judge the program's inability to motivate students to study mathematics. Therefore, qualitative data was very important to know how students perceive their participation in this program, how it had affected their learning process, what were their comments about the program, and what could be done to improve it. Therefore, in this question, students’ opinions were surveyed through a specially designed survey to measure the impact of this experience from the students’ point of view as well as a follow-up qualitative investigation using open-ended questions that sought their comments and suggestions for improving this program.

Student responses to a self-reported survey designed specifically to measure the impact of an enriching math program showed that students were generally very positive about the program. This finding was supported by a qualitative analysis of open-ended questions about how students felt about the program if it was helpful and about their suggestions for improving the program. The percentage of 86.3 reported that they recommend this program for other students. There was no significant difference between male students and females. This indicates that female and male students are roughly holding the same perceptions about MEP.

After reviewing the general response trends regarding students' sentiment towards the program, it was concluded that students perceived the program generally positively, with 42 students giving positive comments compared to only 9 students offering negative reviews. A thematic analysis was applied to positive and negative student comments. The majority of students' comments about how they felt about the program fall into seven positive themes and this corroborated the results of the self-reported survey. Themes of positive comments were about: “Understanding”, “Mathematics in Life”, “Preparing for PISA”, “Excitement”, “Review”, “Discussion” and the last category was “Yes” without comments, while the rest of the comments fell into three negative themes. Themes of negative comments were: "Time", "Problem language" and "Problem-solving".

Student responses to the self-reported survey generally yielded four very positive responses and three positive responses, while responses to only one statement reflected an ambivalent feeling about the program. The highest positive response in the survey was that the MEP made the students more prepared to take the mathematics PISA test (M = 4.20). This statement was supported by 6 positive comments that fell in the theme "Preparing for PISA" as one of the representative comments was "Yes, I think I can now take the PISA test well because I am the first time to know about it through the exercises of this program." The second very positive response in the survey was about appreciating the importance of mathematics in life (M = 4.18). There were 6 positive comments in the "Mathematics in Life" theme that support this view. One of the comments was "This program makes me see the importance of mathematics in life and not to study only equations." These comments reflected the important role of mathematics in life, which was more than just studying abstract and separate topics from life.

The students' responses to the importance of spending time studying contextual problem-solving in mathematics were the third most positive views (M = 4.10). This might be also reflected by the 6 comments in the "Mathematics in life" theme and another two responses in the "Discussion" theme such as "I am excited in these classes; I like the discussion with my teacher and friends."

The responses were also very positive to the survey item "This program helped me to do better in my regular Mathematics class" (M = 4.6). In addition to the positive responses to the survey item "This program made me feel more confident about my mathematics ability" (M = 3.92). Both survey items were reflected hugely by comments on two positive themes: 14 comments in the "Understanding" theme such as the comment “I understood the mathematics concepts better and differently, now I got it” and three more comments in the “Revision” theme, for example, “The program covered all mathematics we learned.” Students also demonstrated positive feelings about another survey item “This program made me more motivated and engaged in my mathematics study” (M = 3.86). This statement was supported by 6 positive comments in the “Excitement” theme such as “It makes me excited about the tricks, I never thought of mathematics in this way.” Any of the previous themes and positive comments could explain the students’ positive rating to the statement “I loved the mathematical contextual problem solving presented in this program” (M = 3.80). So, overall, the students loved this MEP.

However, the students held an ambivalent feeling towards how time was used to implement this program as their responses indicate to this survey statement “Deducting time from math classes to implement this program did not present a challenge to complete the required curriculum on time” (M = 2.90). This was reflected by three negative comments in the “Time” theme where these comments about the students benefit from the program were as follows: “No because time is not enough”, “It is not part of the curriculum, why should we waste time” and “It needs more efforts and time because sometimes we don’t study the same things we study from the book.” These comments reflected some students’ rejection of the program because they perceived it as “a waste of time” and repetition of their previous study or because they didn’t have enough time, while two more themes explain the students’ rejection of this program where 3 negative comments fell in the “Language of problems” theme such as “Not too much. I don’t understand the question, the words are very hard and make me feel nervous.” This indicated the difficulty faced by students regarding understanding the word problems generally. In addition to another three negative comments that fell in the "Problem-solving process" theme where the students indicate their inability to decide how to solve the word problem such as "I don't like these questions; I can't understand how to solve them."

Finally, the students suggested some solutions to improve implementing this program for more student's benefit and relieve negativity. Thematic analysis of 42 presented comments resulted in 6 themes, while 8 students do not provide any comments. There are 16 suggestions about the "Time" theme. These suggestions reflect two main ideas that the students need for more time to discuss and solve problems such as "I liked the way the teacher discusses the program lessons so providing more time is good to finish all the questions in the lesson with the teacher". This suggestion for more time was explained by Van Dooren, Verschaffel, Greer, and De Bock (2006) in which they emphasized the importance of not only posing problems to students but also listening to their methods and explanations for solving problems. Despite the importance of these practices to promote students' understanding of mathematical processes, most teachers prefer safety by showing the students how to do mathematics without understanding (Burns & Lash, 1988). The second main idea about the " Time" suggestion was about the need to specify an official time for this program away from mathematics classes as reflected by "Not to use the period time or make it an official part of the curriculum." This idea regarding time might support the claim that students need to improve their self-regulated learning to improve their mathematical literacy because the time allocated for solving these problems was supported by experts' opinions and the implementation of PISA. In addition, 4 more suggestions fell in the "Flipped classroom" theme that offered a solution to save the limited students' time at school. For example, one of the students suggested to "Let the students prepare and come later to discuss", while another supported his suggestion of the possibility of applying PISA online as stated by the statement “PISA sometimes is applied online. So, if this program made online and the students can use it independently then discuss with the teacher”

The second major 9 suggestions fall in the “Official curriculum” theme where the students suggested to “Make it a regular part of the curriculum” not all classes as one of the suggestions stated that “I think this program should be used with all classes not only grade 10 because it is useful in understanding what mathematics is used for”. This might explain the generally low level of the students’ performance in the MLT because the students do not care about MEP as it was informal. Additionally, another 5 suggestions were classified as the “Assessment” theme that also supports the claim that students would not make efforts if there is no official assessment. This was reflected by statements like “Despite it motivates us to learn better, I think you need to make exams because we study if we have an exam.” Moreover, a student raised a question about the necessity of applying to the program because it is not counted in the students ’grades by saying that “No need to have this program there are no marks for it so why should we study it?” Some students stated additional suggestions, 6 centered on the “Language” theme such as “Make the language clear” and 3 on the “Revision” theme, as represented by the statement “Increase the revision part because it was very important and helped me.” The difficulty of language may explain why this program failed to improve the students’ motivation to learn mathematics.

The results of the first research question showed that students possessed a low level of mathematical literacy despite improvement in mathematical literacy after implementing the MEP, while the students did not show any improvement in their motivation as a result of the second research question. Many positive themes emerged from students’ answers to the first qualitative question about how they felt whether the MEP has helped them and in what aspects. The improvement in mathematical literacy could be the result of applying a CTL approach that reflects the “ mathematics in life” theme where teachers could relate subject content to real-world applications and motivate students to make connections (Berns & Ericson, 2001; Hudson & Whistler, 2007). This connection between content and real-life problems might lead to meaningful learning allowing the student to retrieve stored information more quickly (Taylor & Parson, 2011). This meaningful learning was reflected by the largest positive theme about “understanding” where the students reported that their understanding of their school mathematics topics was increased which is also supported by “revision” and “discussion” themes. Furthermore, students’ suggestions to improve the MEP also supported the role of MEP in improving the student's mathematical literacy as revealed by the positive themes. Students' suggestions were about adding the MEP as an “official curriculum” and including it in the “assessment” as well as increasing the “review” portion.

Moreover, negative themes that emerged from students’ responses to the first qualitative question of how they felt whether the MEP helped them and in what aspects, might explain the low level of the students’ mathematical literacy and the lack of improvement in motivation level. Researchers cited two reasons for students' inability to solve problems: the comprehension stage and the solving stage (Koedinger & Nathan, 2004). They note that students have difficulty understanding linguistics when it comes to understanding a problem stage, while the problem-solving stage focuses on students' problem-solving strategies, the equations they use, and how they progress (Koedinger & Nathan, 2004). The students' “problem-solving skills” was one of the students' concerns about MEP. They reported that they had difficulty in solving contextual problems that refer to the modeling and reasoning processes. Mathematical modeling begins with a problem in the context where the students attempt to “formulate” the problem by transforming this problem from a real-life context into a mathematical problem, then “employ” mathematical concepts, procedures, facts, and tools to find mathematical results and then “interpret, apply, and evaluate” this mathematical solution in the real world context (OECD, 2013).

The results of this study showed that students' performance in “interpreting” the problem was better than “formulating” and “employing” stages of the modeling process. This was also supported by the literature (Efriani, Putri & Hapizah, 2019; Nizar, Putri & Zulkardi, 2018; Putri & Zulkardi, 2020). Students’ inability to solve the contextual problems might also be a reason why students' motivation has not improved as well. This may be due to the notion in the school culture that students fear story problems (Wilson, 2003) which may be because of students' established beliefs and expectations about how to solve contextual problems as algorithmic implementations as supported by Verschaffel et al. (2000). This indicates that contextual tasks are unlikely to support meaningful connections between school and everyday mathematics or to communicate the value of mathematics (Wernet, 2015). Moreover, the contexts may not be personally related to or resonate with students' identities. This led to students' detachment from mathematics (Middleton & Jansen, 2011; Sullivan et al., 2003).

Another negative theme was the “language of the problem” where students reported that they do not understand the language of the problem in the first place and therefore they cannot solve this kind of problem. The student’s inability to solve word problems may be due to a lack of understanding of the wording rather than the process

(Clement, 1982). Although this issue was very old, it still persists, and students are struggling with it. This indicates the student's need to simplify the language of problems. The idea of simplifying problem wording was tested by Abedi and Lord (2001) where the researchers wanted to see if modified wording of standardized test word problems would increase success in problem-solving. Most of the students said that the modified questions were easier to understand which made them prefer the modified questions because they were better able to understand what they are supposed to do to solve the problem (Abedi & Lord, 2001).

Students also mentioned their concern about "time". Moreover, most of the students' suggestions for improving MEP revolved around the theme of "time". Students suggested that a specific time should be allocated to study this program separately from the mathematics classes, so as not to deduct part of the time allotted for finishing the mathematics curriculum. Students also mentioned that they need additional time to discuss how to solve the problems that were included in this program. The students' need for more time for discussion might explain the findings of the first question where it showed that students' low level of mathematical literacy despite the improved mathematical literacy. This supports the importance of discussion with the teacher and colleagues because according to the emergent perspective of constructivism, learning occurs in the social context of the classroom. Classroom discourse is one of the seven NCTM Standards for teaching and learning mathematics (NCTM, 2007). From an emerging perspective, mathematics is a social activity as well as an individual activity. Constructivists depend on teaching practices that are rich in a conversation where learners create their own knowledge based on interaction with other people (Draper, 2002).

5.4 Implications

5.4.1 Future Research

Based on the results of this study, the proposed MEP was successful in improving the students’ mathematical literacy. This improvement was significant for the experimental group compared to the control group for female students and males. Despite the stable motivation to learn mathematics after implementing the MEP, analysis of qualitative data reflected the students’ positive feelings towards the program. For further research, students could be interviewed to gain more insights to enhance the program in addition to measuring students’ actual engagement in the classroom through observations or teacher reports. Moreover, triangulation of data could be achieved by studying deeply the instructional strategies used by teachers who implemented this program and compared it with other teachers’ strategies. Furthermore, research should study the students’ self-regulated learning skills as it could be another important factor that might explain the results of this study. This view was also supported by Fauzi and Widjajanti (2018).

Additionally, other studies can be conducted to draw a comparison between the UAE context and other highly performed countries' contexts to adopt their best practices in curriculum and teaching methods. Moreover, another opportunity for further research may be by extending the implementation of this program to the different grade levels to verify its effectiveness depending on the grade level. This information may provide decision-makers with valuable information if the scope of program implementation is expanded.

5.4.2 Curriculum Design

The implications of the study results for curriculum design to include modeling problems in the curriculum regularly. Mathematical modeling such as in contextual problems is recommended for all curricula and grades as it is considered a standard for mathematical practice in the Common Core State Standards. Thus, curriculum writers are encouraged to consider the potential of modeling in promoting mathematical proficiency and engagement when problems are meaningful to students.

NCTM (2000) indicated that assessment and education should be complementary so that the assessment provides information for the teacher to use in making educational decisions. Thus, it may be necessary to apply higher-level tests that have more difficult elements of achievement tests at grade level. Thus, if the test does not contain sufficient elements of difficulty appropriate for the student, the result may not indicate the true level of his understanding (Rotigel & Fello, 2004).

5.4.3 Instruction

The implications for mathematics teachers came from one of the main findings of this study, the percentage of students who solved level 5 and 6 problems in the mathematical literacy posttest was very low. Thus, this encourages teachers to use constructivist approaches in teaching such as PBL to focus on deep understanding, in addition, to investigate what reasons prevent them from solving these problems. In addition, the findings suggest giving more attention to improving the students' self-regulated learning strategies because many students in their comments suggested more time for problem-solving and discussion of these problems with their teachers. This leads us to another finding of this study that the MEP program failed to improve students' motivation to learn mathematics. This suggests that presenting the contextual problems that fit the purpose of improving students' mathematical literacy is not sufficient to motivate students to learn. Thus, teachers are encouraged to adopt more engaging methods, providing the students with appropriate support, especially when presenting problems that require high ordered thinking skills as this might be the demotivating factor for their inability to solve these problems.

5.5 Recommendations

The implementation of the MEP and similar programs, if there is any, should be continued. However, more efforts should be considered in the status quo to increase the UAE chances to reach the National Agenda goal to be among the best 20 countries all over the world in PISA international tests. Most importantly, realizing the constant need for change is an integral part of developing the educational process. In light of the global outbreak of the COVID 19 pandemic, the MEP program should be adapted to be implemented in the distance learning situation. This need is consistent with the OECD recent trend. For example, the items for PISA were implemented using web-based technologies in many countries since 2015. For PISA 2021, a digital-based assessment will be used for assessing mathematical literacy as this makes it possible to measure skills and processes such as simulations that cannot be measured using the traditional methods of assessment (OECD, 2018a).

Although word problems such as mathematical contextual problems recently exist in all mathematics curricula, teachers continue to ignore them for many reasons such as lack of time to search for appropriate contextual problems to use to improve students' mathematical literacy and time to apply them due to the intensive curriculum. Thus, to help educators overcome these hurdles, it is recommended that policymakers devote formal time to implementing similar programs in schools that pose contextual problems such as mathematical literacy curricula. Moreover, to ensure that students make the appropriate effort to benefit from the program, it should not only be a formal part of the mathematics curriculum, but it should be taken into account in the formal assessment as well.

Teachers must use methods that evoke the students' interest to be active in their learning, otherwise, teachers will not succeed in teaching their students. The CTL is one method that can help teachers relate math content to real-world situations and motivate students to make a connection between them. Moreover, teachers should be involved in professional development activities programs that train them to use constructivist theory-based strategies in teaching, such as CTL and PBL.

5.6 Conclusion

The success of this MEP is a major first step towards achieving the goal of the National Agenda 2021 for the UAE to be among the top 20 countries in the international PISA test. Mathematical literacy in this study was based on using the contextual mathematical problems that reflected the student's ability to use what is learned in school in life problems. This is a major goal that is the essence of the students learning and it is unlikely to change. Leaders in mathematics education, such as NCTM (2000) and CCSSI (2010), call for more focus on mathematics problems based on real-life situations that promote mathematical modeling and quantitative reasoning. Thus, researchers, teachers, and curriculum designers need to continue their efforts to support the use of contextual problems and its roles in students learning.

This study, to the researcher's knowledge, is the first study that proposed a program to improve mathematical literacy in the UAE. The results of this study presented proof of the ability of the proposed mathematical program to improve the students’ mathematical literacy. Yet, this is not enough. More research is needed to investigate more issues related to this study such as the students’ self-regulated learning and what should be considered to improve the students’ motivation. In short, this study was one step up, and in the long term, much remains to be investigated and learned about mathematical literacy.

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List of Publications

Jarrah, A. M., & Almarashdi, H. S. (2019). Mathematics teachers' perceptions of teaching gifted and talented learners in general education classrooms in the UAE. *Journal for the Education of Gifted Young Scientists, 7*(4), 835-847. Doi: http://dx.doi.org/10.17478/jegys.628395

Appendices

Appendix A

UAEU College of Education

جامعة الإمارات العربية المتحدة
United Arab Emirates University

Mathematics Enrichment Program

Table of contents

PhD student: Hanan Almarashdeh

Advisor: Dr. Adeeb Jarrah

Source: OECD
The Problems are in the form of units

UAEU College of Education

جامعة الإمارات العربية المتحدة United Arab Emirates University

Change and Relationships

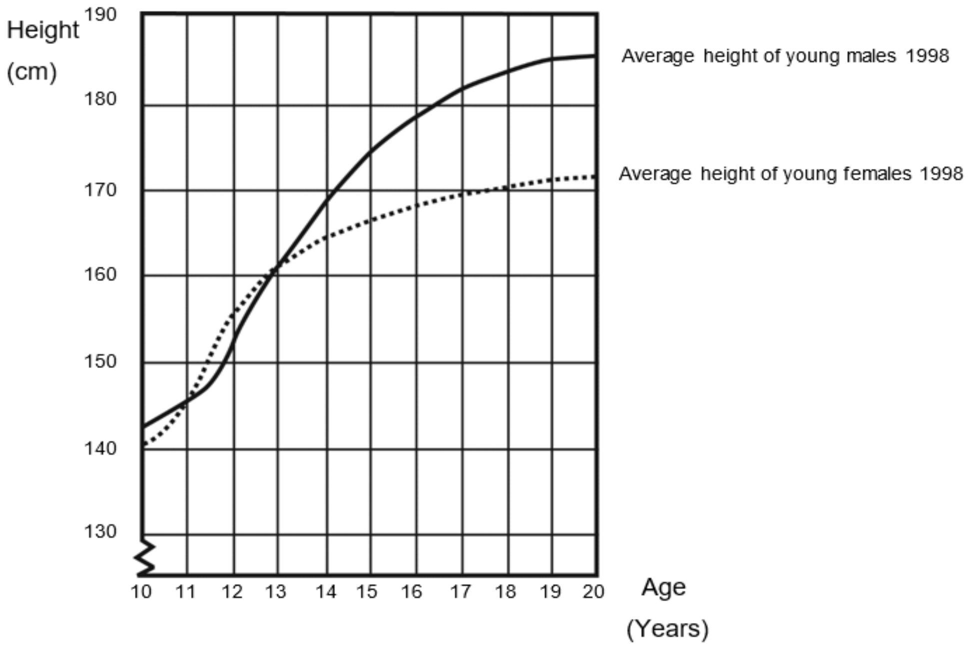
LESSON 1: Functions and variations

<u>**Learning outcome:** ***Identify relevant information for a simple mathematical model to calculate a number***</u>

Growing up

YOUTH GROWS TALLER

In 1998 the average height of both young males and young females in the Netherlands is represented in this graph.



Question 1:

Since 1980 the average height of 20-year-old females has increased by 2.3 cm, to 170.6 cm. What was the average height of a 20-year-old female in 1980?

……………………………………………………………..cm

Question 2:

Explain how the graph shows that on average the growth rate for girls slows down after 12 years of age.

Question 3:

According to this graph, on average, during which period in their life are females taller than males of the same age?

DVD RENTAL

Jenn works at a store that rents DVDs and computer games. At this store the annual membership fee costs 10 zeds. The DVD rental fee for members is lower than the fee for non-members, as shown in the following table:

Non-member rental fee for one DVD	Member rental fee for one DVD
3.20 zeds	2.50 zeds

Question 1:

Troy was a member of the DVD rental store last year.

Last year he spent 52.50 zeds in total, which included his membership fee.

How much would Troy have spent if he had not been a member but had rented the same number of DVDs?

Number of zeds: ....

Question 2:

What is the minimum number of DVDs a member needs to rent so as to cover the cost of the membership fee? Show your work.

....

....

....

Number of DVDs: ....

SELLING NEWSPAPERS

In Zedland there are two newspapers that try to recruit sellers. The posters below show how they pay their sellers.

> **ZEDLAND STAR**
>
> **NEED EXTRA MONEY?**
>
> **SELL OUR NEWSPAPER**
>
> You will be paid:
> 0.20 zeds per newspaper for the first 240 papers you sell in a week, plus 0.40 zeds for each additional newspaper you sell.

> **ZEDLAND DAILY**
>
> **WELL PAID JOB THAT TAKES LITTLE TIME!**
>
> Sell the *Zedland Daily* and make 60 zeds a week, plus an additional 0.05 zeds per newspaper you sell.

Question 1:

On average, Frederic sells 350 copies of the *Zedland Star* every week.

How much does he earn each week, on average?

Amount in zeds: ....

Question 2:

Christine sells the *Zedland Daily*. One week she earns 74 zeds.

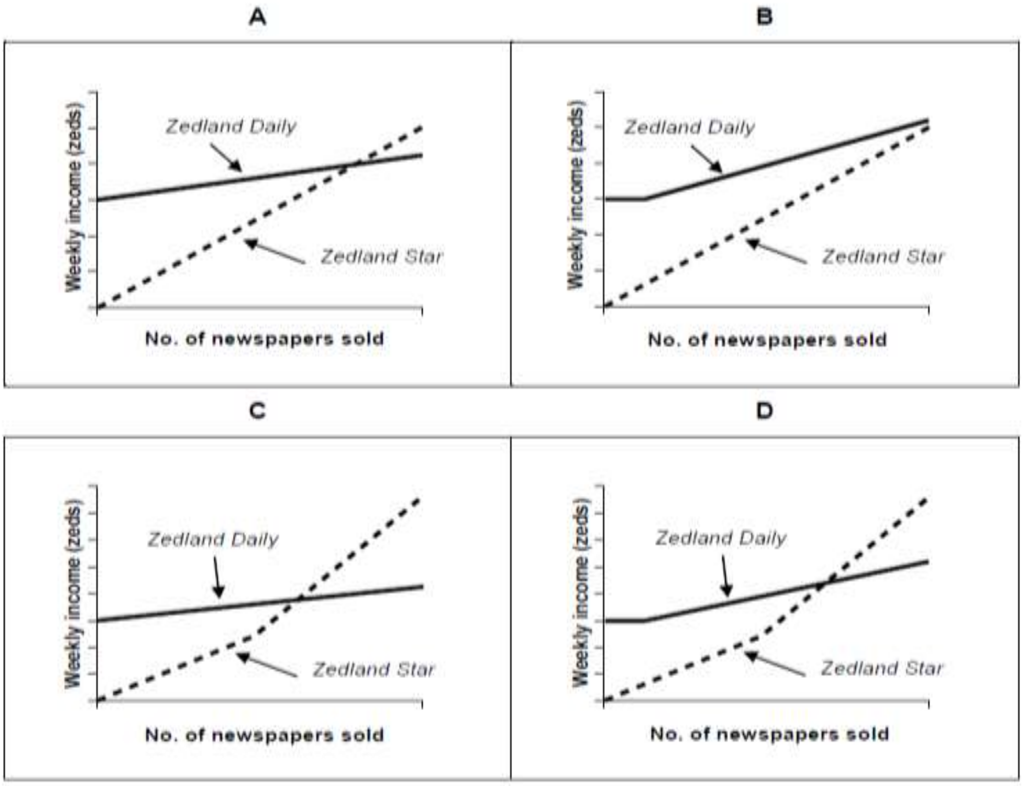
How many newspapers did she sell that week?

Number of newspapers sold: ....

Question 3:

John decides to apply for a newspaper seller position. He needs to choose the *Zedland Star* or the *Zedland Daily*.

Which one of the following graphs is a correct representation of how the two newspapers pay their sellers? Circle A, B, C or D.



Revision

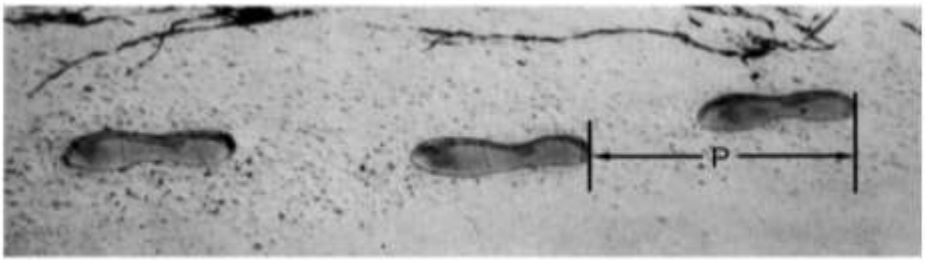
Considering the formula $Y = \frac{2x}{z}$, if x increases then Y increases and if z increases then Y decreases , Answer the following questions :

a- Describe how Y changes if x is doubled but z does not change ?

b- Describe how Y changes if z is doubled but x does not change ?

c- If Y = 12 and z=3 what is the value of x?

Walking



The picture shows the footprints of a man walking. The pace length P is the distance between the rears of two consecutive footprints.

For men, the formula, $\frac{n}{P} = 140$, gives an approximate relationship between n and P where,

n = number of steps per minute, and

P = pace length in meters

Question 1

If the formula applies to Heiko's walking and Heiko takes 70 steps per minute, what is Heiko's pace length? Show your work.

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Question 2:

Bernard knows his pace length is 0.80 meters. The formula applies to Bernard's walking.

Calculate Bernard's walking speed in meters per minute and in kilometers per hour.

Show your work

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Change and Relationships

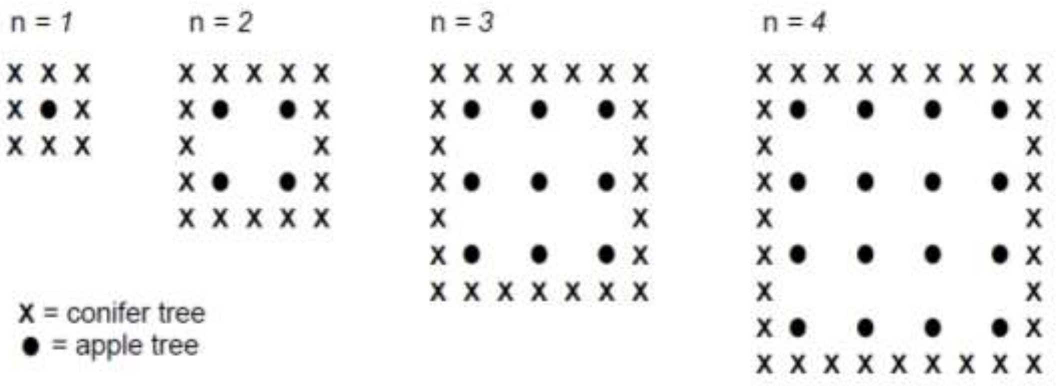
LESSON 2: Numerical trends and patterns

Learning outcome: ***to solve real life problems within patterns and numerical trends***

Apples

A farmer plants apple trees in a square pattern. In order to protect the apple trees against the wind he plants conifer trees all around the orchard.

Here you see a diagram of this situation where you can see the pattern of apple trees and conifer trees for any number (n) of rows of apple trees:



Question 1

Complete the table:

n	Number of Apple trees	Number of conifers
1	**1**	**8**
2	**4**	
3		
4		
5		

Question 2:

There are two formulas you can use to calculate the number of apple trees and the number of conifers for the pattern described above:

Number of apple trees = n^2

Number of conifers = $8n$

where n is the number of rows of apple trees.

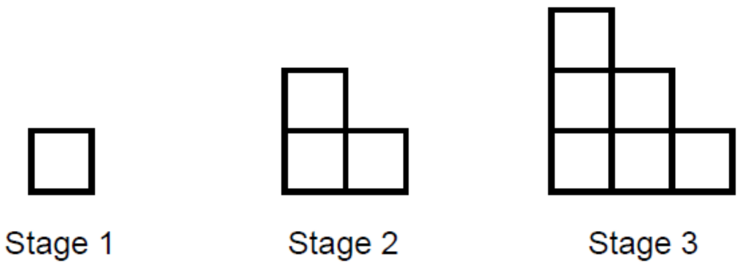
There is a value of n for which the number of apple trees equals the number of conifers. Find the value of n and show your method of calculating this.

Question 3:

Suppose the farmer wants to make a much larger orchard with many rows of trees. As the farmer makes the orchard bigger, which will increase more quickly: the number of apple trees or the number of conifers? Explain how you found your answer.

Step Pattern

Question 1: Robert builds a step pattern using squares. Here are the stages he follows.



for Stage 3.

How many squares should he use for the fourth stage?

Answer: ....squares.

The Best Car

A car magazine uses a rating system to evaluate new cars, and gives the award of "The Car of the Year" to the car with the highest total score. Five new cars are being evaluated, and their ratings are shown in the table.

Car	Safety Features (S)	Fuel Efficiency (F)	External Appearance (E)	Internal Fittings (T)
Ca	3	1	2	3
M2	2	2	2	2
Sp	3	1	3	2
N1	1	3	3	3
KK	3	2	3	2

The ratings are interpreted as follows:
3 points = Excellent
2 points = Good
1 point = Fair

Question 1:

To calculate the total score for a car, the car magazine uses the following rule, which is a weighted sum of the individual score points:

Total Score = (3 x S) + F + E + T

Calculate the total score for Car "Ca". Write your answer in the space below.

Total score for "Ca": ....

Question 2:

The manufacturer of car "Ca" thought the rule for the total score was unfair.

Write down a rule for calculating the total score so that Car "Ca" will be the winner.

Your rule should include all four of the variables, and you should write down your rule by filling in positive numbers in the four spaces in the equation below.

Total score = ………× S + ………× F + ………× E + ………× T.

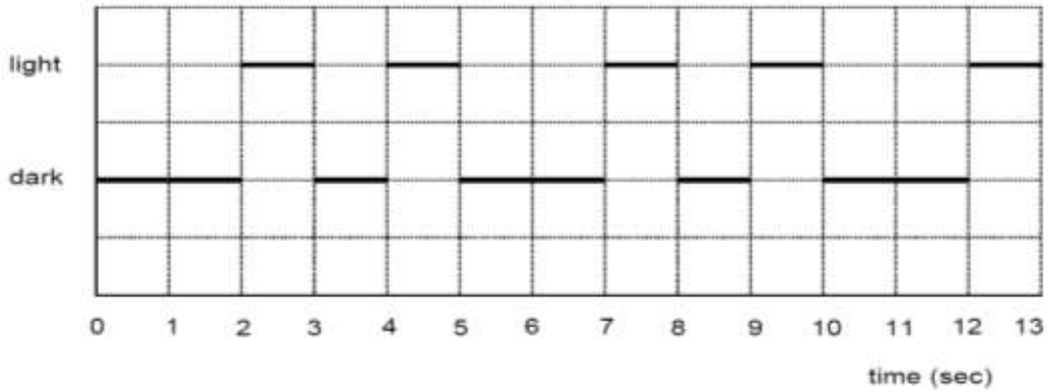
Lighthouse

Lighthouses are towers with a light beacon on top. Lighthouses assist sea ships in finding their way at night when they are sailing close to the shore.

A lighthouse beacon sends out light flashes with a regular fixed pattern. Every lighthouse has its own pattern.

In the diagram below you see the pattern of a certain lighthouse.

The light flashes alternate with dark periods.



It is a regular pattern. After some time the pattern repeats itself. The time taken by one complete cycle of a pattern, before it starts to repeat, is called the *period*. When you find the period of a pattern, it is easy to extend the diagram for the next seconds or minutes or even hours.

Question 1:

Which of the following could be the period of the pattern of this lighthouse?

A 2 seconds.

B 3 seconds.

C 5 seconds.

D 12 seconds.

Question 2:

For how many seconds does the lighthouse send out light flashes in 1 minute?

A 4

B 12

C 20

D 24

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Quantity

LESSON 3: Percentages

Learning outcome: use percentages within a real context.

Revision: Basic skills ……………Note: not all is needed, it depends on the student level

1. Ahmed has travelled 120km of the 180 km to the airport from his home. What percentage of the journey has he covered?

2. A jar contains 16 cubes. 12 ½ % are white, 37 ½ % are red, and 50% are orange. How many cubes of each colour are in the jar?

3. 25 out 40 students turned up for a practice for the school concert. What % of students came to the practice?

4. In trials for the local team 42 players attended.
   The coach said later that he got 84% of the attendance he expected.
   How many players was the coach expecting?

5. A house which costs 550,000 last year has decreased in price by 20%. How much will you save on last year's price by buying now?

6. The school office has ordered a new table for 152.99 Dhs. Vat at 21 % must be added to this price. The secretary needs a quick estimate of the final price. What will she do?

Drug Concentrations

Question 1:

A woman in hospital receives an injection of penicillin. Her body gradually breaks the penicillin down so that one hour after the injection only 60% of the penicillin will remain active.

This pattern continues: at the end of each hour only 60% of the penicillin that was present at the end of the previous hour remains active.

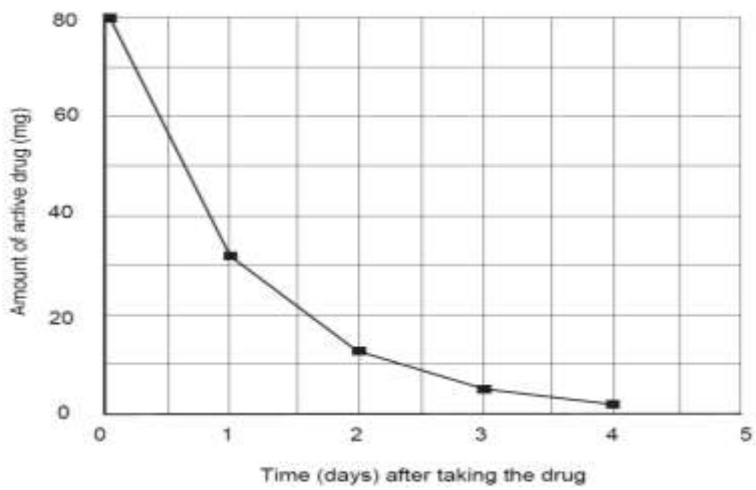
Suppose the woman is given a dose of 300 milligrams of penicillin at 8 o'clock in the morning.

Complete this table showing the amount of penicillin that will remain active in the woman's blood at intervals of one hour from 0800 until 1100 hours.

Time	0800	0900	1000	1100
Penicillin (mg)	300			

Question 2:

Peter has to take 80 mg of a drug to control his blood pressure. The following graph shows the initial amount of the drug, and the amount that remains active in Peter's blood after one, two, three and four days.



How much of the drug remains active at the end of the first day?

A 6 mg.

B 12 mg.

C 26 mg.

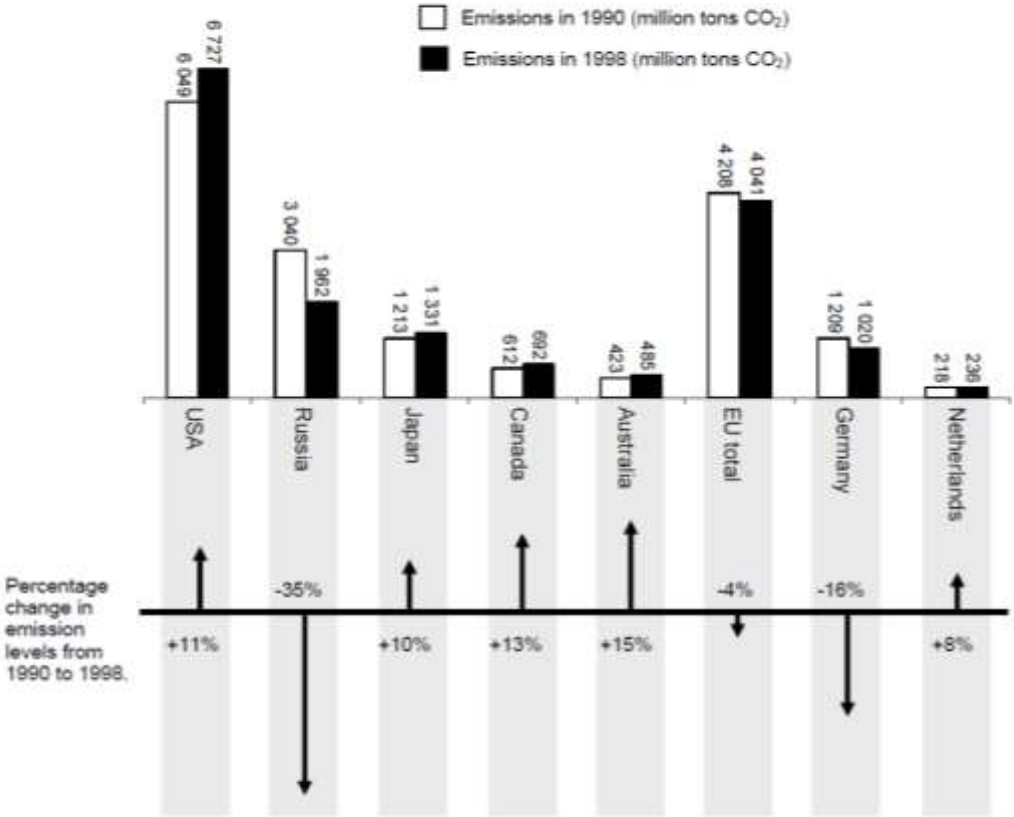
D 32 mg.

Decreasing CO2 Levels

Many scientists fear that the increasing level of CO2 gas in our atmosphere is causing climate change.

The diagram below shows the CO2 emission levels in 1990 (the light bars) for several countries (or regions), the emission levels in 1998 (the dark bars), and the

Percentage change in emission levels between 1990 and 1998 (the arrows with percentages).



Question 1:

In the diagram you can read that in the USA, the increase in CO2 emission level from 1990 to 1998 was 11%.

Show the calculation to demonstrate how the 11% is obtained.

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Question 2:

Mandy analysed the diagram and claimed she discovered a mistake in the percentage change in emission levels: “The percentage decrease in Germany (16%) is bigger than the percentage decrease in the whole European Union (EU total, 4%). This is not possible, since Germany is part of the EU.”

Do you agree with Mandy when she says this is not possible? Give an explanation to support your answer.

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Question 3:

Mandy and Niels discussed which country (or region) had the largest **increase** of CO2 emissions.

Each came up with a different conclusion based on the diagram.

Give two possible ‘correct’ answers to this question, and explain how you can obtain each of these answers.

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Space Flight

Space station Mir remained in orbit for 15 years and circled Earth some 86 500 times during its time in space.

The longest stay of one cosmonaut in the Mir was around 680 days.

Question 1:

Approximately how many times did this cosmonaut fly around Earth?

A 110

B 1 100

C 11 000

D 110 000

Payments by Area

People living in an apartment building decide to buy the building. They will put their money together in such a way that each will pay an amount that is proportional to the size of their apartment.

For example, a man living in an apartment that occupies one fifth of the floor area of all apartments will pay one fifth of the total price of the building.

Question 1:

Circle Correct or Incorrect for each of the following statements.

Statement	Correct / Incorrect
A person living in the largest apartment will pay more money for each square metre of his apartment than the person living in the smallest apartment.	Correct / Incorrect
If we know the areas of two apartments and the price of one of them we can calculate the price of the second.	Correct / Incorrect
If we know the price of the building and how much each owner will pay, then the total area of all apartments can be calculated.	Correct / Incorrect
If the total price of the building were reduced by 10%, each of the owners would pay 10% less.	Correct / Incorrect

Question 2: PAYMENTS BY AREA

There are three apartments in the building. The largest, apartment 1, has a total area of $95m^2$. Apartments 2 and 3 have areas of 85m2 and $70m^2$ respectively. The selling price for the building is 300 000 zeds.

How much should the owner of apartment 2 pay? Show your work.

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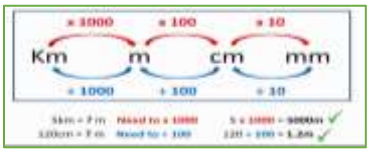
Quantity

LESSON 4: Quantification

Learning outcome: understanding measurements, counts, magnitudes, units, indicators, relative size, and numerical trends and patterns.

1 kilolitre = 1000 L

1 kilogram = 1000 g



Remember : Converting time units

Times decimal of hour by 60 to get minutes: 1.15hrs means 1hr 9 minutes, not 1 hr 15 minutes
Divide minutes by 60 to get decimal of hour: 3hrs 45 minutes = 3.75 hours, not 3.45 hours

Distance travelled
A man walks for 3 hours at a speed of 3 miles per hour. How far has he travelled?

…………………………………………………………………………………………

A man walks for 45 minutes at a speed of 4 miles per hour. How far has he travelled?

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Average Speed maintained
A man covers a distance of 21 miles in 7 hours. Calculate his average speed.

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A man covers a distance of 232 miles in 7 hours and 15 minutes. What was his average speed?

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Time taken
How long does it take an aircraft travelling at 580 km per hour to travel a distance of 232 kilometres?

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A man drove a distance of 250 miles at an average speed of 70 mph. How long did the journey take ? Give your answer in hours, minutes and seconds.

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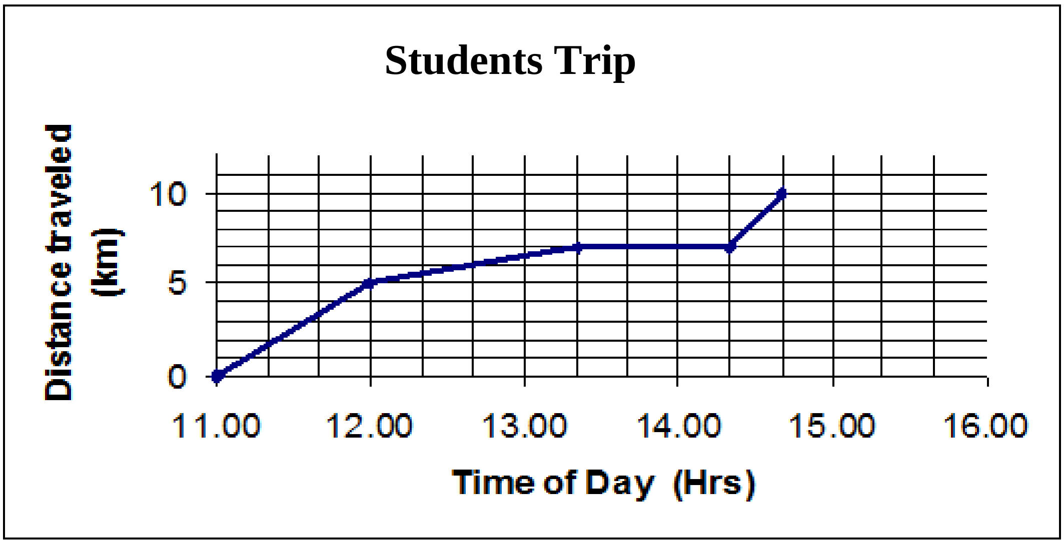
Revision: Distance – Time Graphs

The distance travelled is plotted on the y - axis, the time period is plotted on the x – axis.



a- When did the students start their trip?

....

b- How many kilometers did the students travel by midday?

....

c- How long did they stop?

....

d- What was the travelled distance before their stop?

....

e- When did they restart after rest? For how long? How many kilometers ?

....

f- What was the total distance travelled?

....

g- What was the average speed for their last part of their trip?

....

h- When was their slowest pace?

....

i- What was the average speed from the start until the rest?

....

Exchange Rate

Mei-Ling from Singapore was preparing to go to South Africa for 3 months as an exchange student. She needed to change some Singapore dollars (SGD) into South African rand (ZAR).

Question 1:

Mei-Ling found out that the exchange rate between Singapore dollars and South African rand was:

1 SGD = 4.2 ZAR

Mei-Ling changed 3000 Singapore dollars into South African rand at this exchange rate.

How much money in South African rand did Mei-Ling get?

Answer: ....

Question 2:

On returning to Singapore after 3 months, Mei-Ling had 3 900 ZAR left. She changed this back to Singapore dollars, noting that the exchange rate had changed to:

1 SGD = 4.0 ZAR

How much money in Singapore dollars did Mei-Ling get?

Answer: ....

Question 3:

During these 3 months the exchange rate had changed from 4.2 to 4.0 ZAR per SGD.

Was it in Mei-Ling's favour that the exchange rate now was 4.0 ZAR instead of 4.2 ZAR, when she changed her South African rand back to Singapore dollars? Give an explanation to support your answer.

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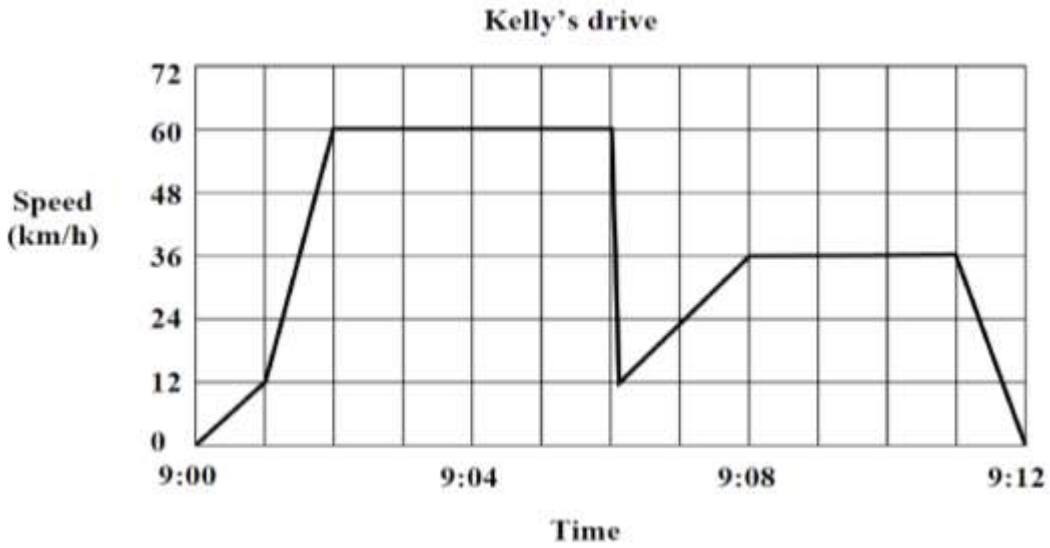
CAR DRIVE

Kelly went for a drive in her car. During the drive, a cat ran in front of the car. Kelly slammed on the brakes and missed the cat.

Slightly shaken, Kelly decided to return home.

The graph below is a simplified record of the car's speed during the drive.



Question 1:

What was the maximum speed of the car during the drive?

Maximum speed: .... km/h.

Question 2:

What time was it when Kelly slammed on the brakes to avoid the cat?

Answer: ....

Question 3:

Was the route Kelly took to return home shorter than the distance she had travelled from home to the place where the incident with the cat occurred? Give an explanation to support your answer, using information given in the graph.

....
....

Skateboard

Eric is a great skateboard fan. He visits a shop named SKATERS to check some prices.

At this shop you can buy a complete board. Or you can buy a deck, a set of 4 wheels, a set of 2 trucks and a set of hardware, and assemble your own board.

The prices for the shop's products are:

Product	Price in zeds	
Complete skateboard	82 or 84	
Deck	40, 60 or 65	
One set of 4 Wheels	14 or 36	
One set of 2 Trucks	16	
One set of hardware (bearings, rubber pads, bolts and nuts)	10 or 20	

Question 1:

Eric wants to assemble his own skateboard. What is the minimum price and themaximum price in this shop for self-assembled skateboards?

(a) Minimum price: ....zeds.

(b) Maximum price: ....zeds.

Question 2:

The shop offers three different decks, two different sets of wheels and two different sets of hardware. There is only one choice for a set of trucks.

How many different skateboards can Eric construct?

A 6
B 8
C 10
D 12

Question 3:

Eric has 120 zeds to spend and wants to buy the most expensive skateboard he can afford.

How much money can Eric afford to spend on each of the 4 parts? Put your answer in the table below.

Part	Amount (zeds)
Deck	
Wheels	
Trucks	
Hardware	

Choices

Question 1: CHOICES

In a pizza restaurant, you can get a basic pizza with two toppings: cheese and tomato. You can also make up your own pizza with **extra** toppings. You can choose from four different extra toppings: olives, ham, mushrooms and salami. Ross wants to order a pizza with two different **extra** toppings.

How many different combinations can Ross choose from?

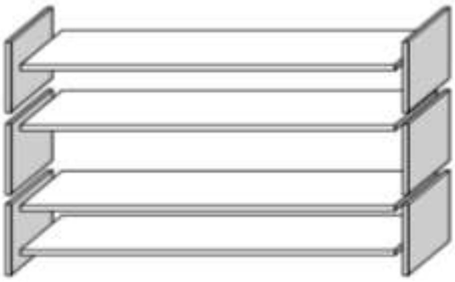
Answer: ....combinations.

Bookshelves

Question 1: BOOKSHELVES

To complete one set of bookshelves a carpenter needs the following components:

4 long wooden panels,
6 short wooden panels,
12 small clips,
2 large clips and
14 screws.



The carpenter has in stock 26 long wooden panels, 33 short wooden panels, 200 small clips, 20 large clips and 510 screws.

How many sets of bookshelves can the carpenter make?

Answer: ....

Reaction Time……HW

In a Sprinting event, the 'reaction time' is the time interval between the starter's gun firing and the athlete leaving the starting block. The 'final time' includes both this reaction time, and the running time.

The following table gives the reaction time and the final time of 8 runners in a 100 metre sprint race.

Lane	Reaction time (sec)	Final time (sec)
1	0.147	10.09
2	0.136	9.99
3	0.197	9.87
4	0.180	Did not finish the race
5	0.210	10.17
6	0.216	10.04
7	0.174	10.08
8	0.193	10.13

with the medallists' lane number, reaction time and final time.

Medal	Lane	Reaction time (secs)	Final time (secs)
GOLD			
SILVER			
BRONZE			

Question 2:

To date, no humans have been able to react to a starter's gun in less than 0.110 second.

If the recorded reaction time for a runner is less than 0.110 second, then a false start is considered to have occurred because the runner must have left before hearing the gun.

If the Bronze medallist had a faster reaction time, would he have had a chance to win the Silver medal? Give an explanation to support your answer.

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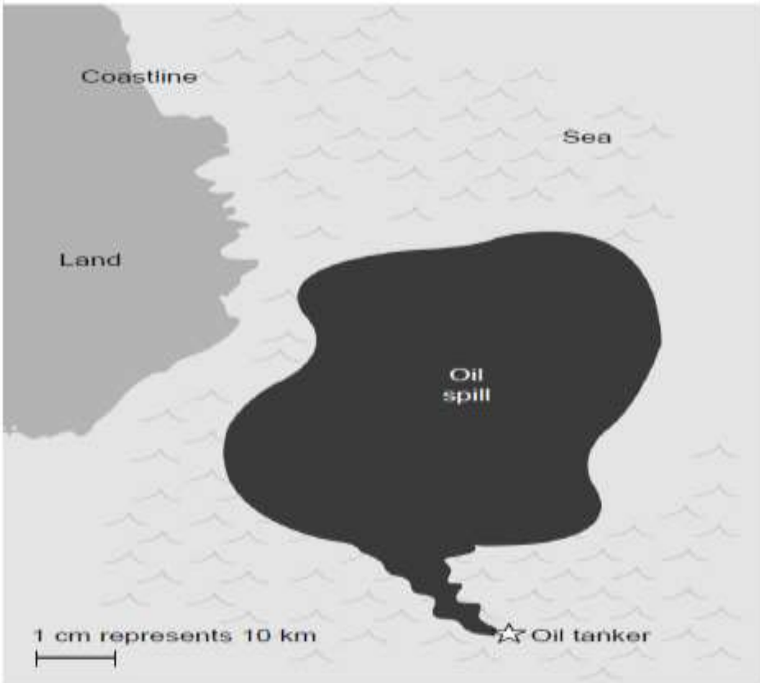
Space and shape

LESSON 5: Geometric approximation

Learning outcome: ***interpret, understand, classify, appreciate and describe the world through two-dimensional shapes and three-dimensional objects, their location, movement and relationships.***

OIL SPILL

An oil tanker at sea struck a rock, making a hole in the oil storage tanks. The tanker was about 65 km from land. After a number of days the oil had spread, as shown on the map below.



Question 1:

Using the map scale, estimate the area of the oil spill in square kilometres (km^2).

Answer: .... km^2

Question 2:

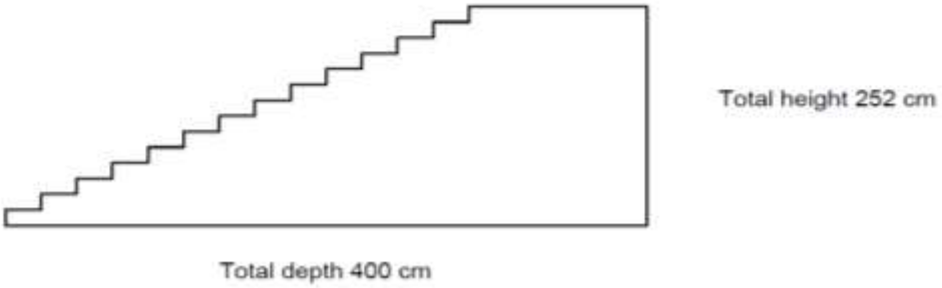
Explain how could you estimate the perimeter of the oil spill?

……………………………………………………………………………………………

Staircase

Question 1:

The diagram below illustrates a staircase with 14 steps and a total height of 252 cm:



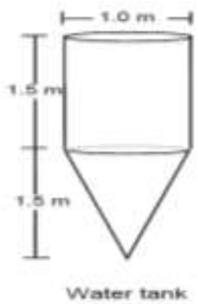
What is the height of each of the 14 steps?

Height: ....cm.

Water Tank

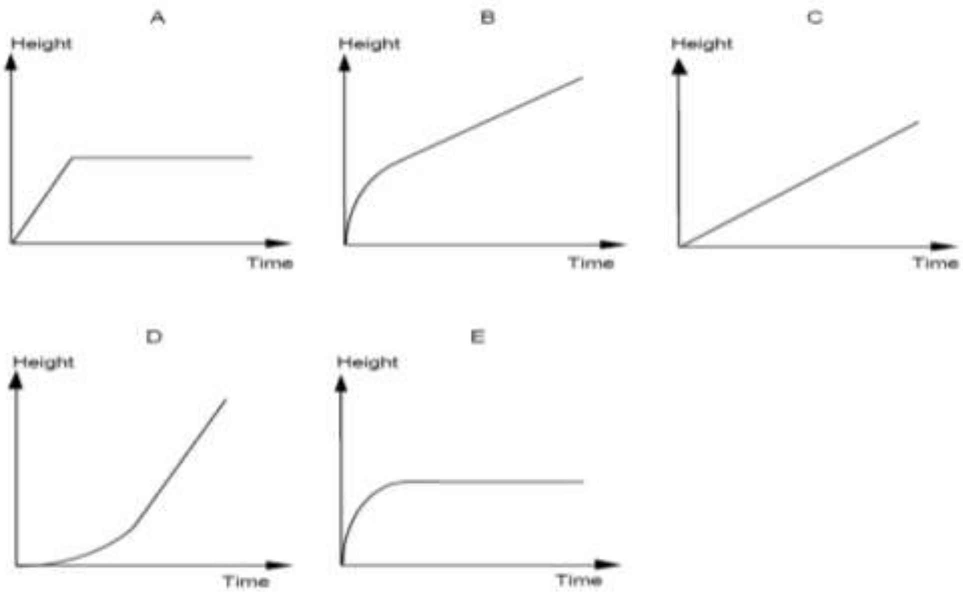
Question 1:

A water tank has shape and dimensions as shown in the diagram.



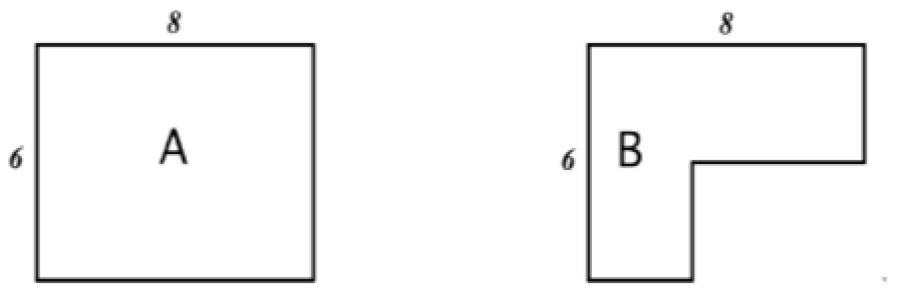
At the beginning the tank is empty. Then it is filled with water at the rate of one litre per second.

Which of the following graphs shows how the height of the water surface changes over time?



Perimeter and Area

Q1: Which perimeter is the greatest? Explain your reasoning



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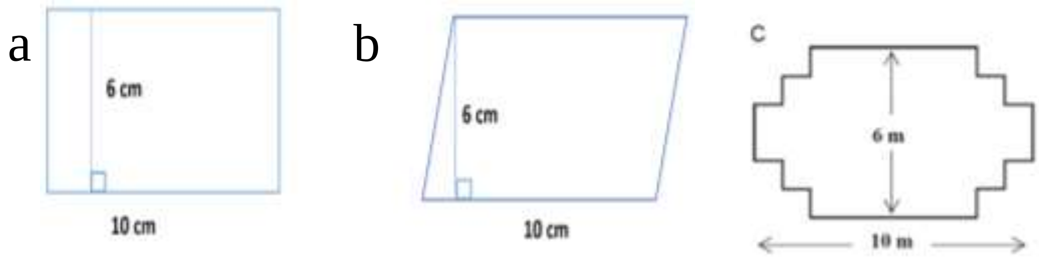
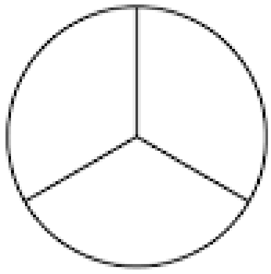
Q2 : what the minimum information that is needed to find the area of shape B? is there only one solution ?

Q3: which shape has the greatest perimeter? Explain



Q4 : what is the size of the angle of each part of the circle? If the arc length of each part is 4л, what is the radius of the circle?



ROCK CONCERT

For a rock concert a rectangular field of size 100 m by 50 m was reserved for the audience. The concert was completely sold out and the field was full with all the fans standing.

Which one of the following is likely to be the best estimate of the total number of people attending the concert?

A 2 000

B 5 000

C 20 000

D 50 000

E 100 00

Space and shape

LESSON 6: visual and physical world

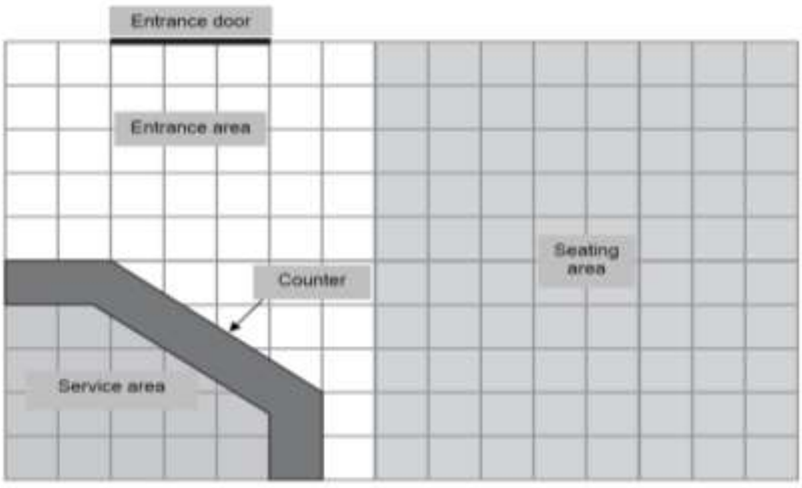
Learning outcome: ***Use Pythagorean Theorem within a real geometric context***

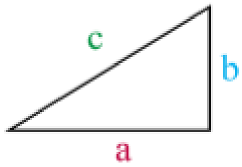
ICE-CREAM SHOP

This is the floor plan for Mari's Ice-cream Shop. She is renovating the shop.

The service area is surrounded by the serving counter.

Review Pythagorean theorem





$a^2 + b^2 = c^2$

Note: Each square on the grid represents 0.5 metres × 0.5 metres.

Question 1:

Mari wants to put new edging along the outer edge of the counter. What is the total length of edging she needs? Show your work.

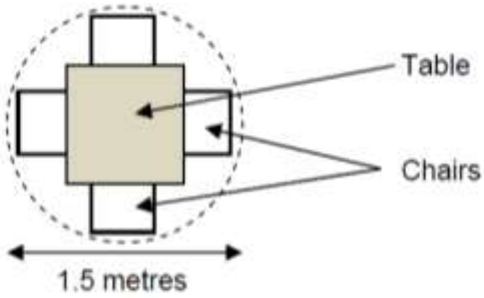
....
....
....

Question 2:

Mari is also going to put new flooring in the shop. What is the total floor space area of the shop, excluding the service area and counter? Show your work.

....
....
....

Question 3:



Mari wants to have sets of tables and four chairs like the one shown above in her shop. The circle represents the floor space area needed for each set.

For customers to have enough room when they are seated, each set (as represented by the circle) should be placed according to the following constraints:

Each set should be placed at least at 0.5 metres away from walls.

Each set should be placed at least at 0.5 metres from other sets.

What is the maximum number of sets that Mari can fit into the shaded seating area in her shop?

Number of sets: ....

A CONSTRUCTION WITH DICE (1 ITEM)

In the picture below a construction has been made using seven identical dice with their faces numbered from 1 to 6.

When the construction is viewed from the top, only 5 dice can be seen.

Question 1:

How many dots in total can be seen when this construction is viewed from the top?

Number of dots seen: ....

Number Cubes

Question 1:

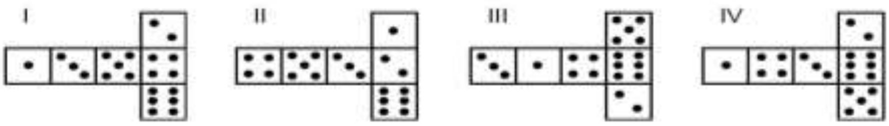
On the right, there is a picture of two dice.

Dice are special number cubes for which the following rule applies:

The total number of dots on two opposite faces is always seven.

You can make a simple number cube by cutting, folding and gluing cardboard. This can be done in many ways. In the figure below you can see four cuttings that can be used to make cubes, with dots on the sides.

Which of the following shapes can be folded together to form a cube that obeys the rule that the sum of opposite faces is 7? For each shape, circle either "Yes" or "No" in the table below.



Shape	Obeys the rule that the sum of opposite faces is 7?
I	Yes / No
II	Yes / No
III	Yes / No
IV	Yes / No

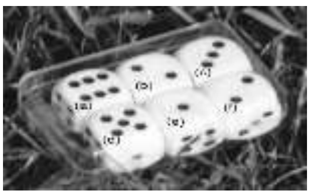
Cubes

Question 1: CUBES *M145Q01*

In this photograph you see six dice, labelled (a) to (f). For all dice there is a rule:

The total number of dots on two opposite faces of each die is always seven

Write in each box the number of dots on the **bottom** face of the dice corresponding to the photograph.

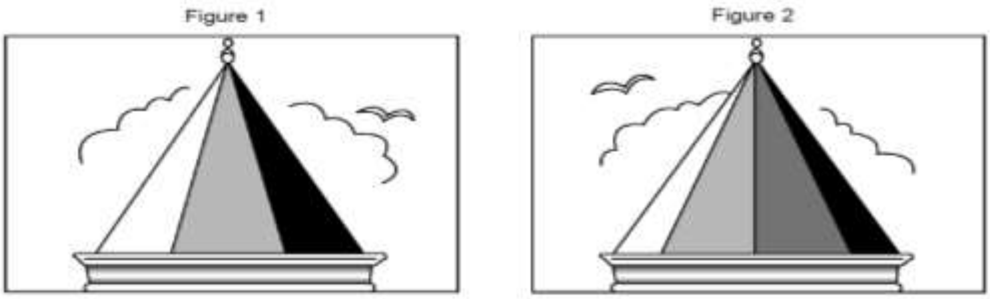


(a)	(b)	(c)
(d)	(e)	(f)

SEEING THE TOWER

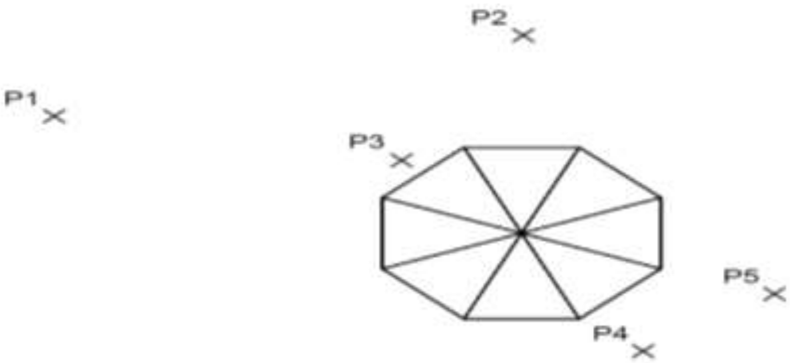
Question 1:

In Figures 1 and 2 below, you see two drawings of the **same** tower. In Figure 1 you see **three** faces of the roof of the tower. In Figure 2 you see **four** faces.



In the following diagram, the view of the roof of the tower, from above, is shown. Five positions are shown on the diagram. Each is marked with a cross (×) and they are labelled P1 – P5.

From each of these positions, a person viewing the tower would be able to see a number of faces of the roof of the tower.



In the table below, circle the number of faces that could be seen from each of these positions.

Position	Number of faces that could be seen from that position (circle the correct number)				
P1	1	2	3	4	more than 4
P2	1	2	3	4	more than 4
P3	1	2	3	4	more than 4
P4	1	2	3	4	more than 4
P5	1	2	3	4	more than 4

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Uncertainty and

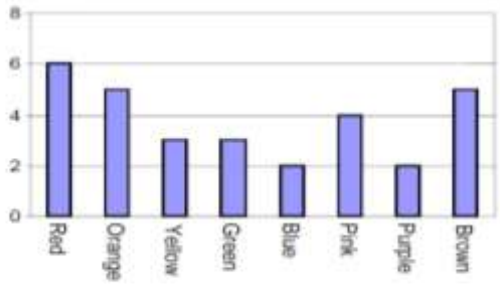
LESSON 7: Probability

Learning outcome: Solve applications involving probabilities.

What is the probability? the probability of something means the chance of its occurrence or the chances that we will observe an event at a certain time

Coloured Candies

Question 1: Robert's mother lets him pick one candy from a bag. He can't see the candies. The number of candies of each colour in the bag is shown in the following g



What is the probability that Robert will pick a red candy?

A 10%
B 20%
C 25%
D 50%

FORECAST OF RAINFALL

Question 1: On a particular day, the weather forecast predicts that from 12 noon to 6 pm the chance of rainfall is 30%.

Which of the following statements is the best interpretation of this forecast?

A 30% of the land in the forecast area will get rain.
B 30% of the 6 hours (a total of 108 minutes) will have rain.
C For the people in that area, 30 out of every 100 people will experience rain.
D If the same prediction was given for 100 days, then about 30 days out of the 100 days will have rain.
E The amount of rain will be 30% of a heavy rainfall (as measured by rainfall per unit time).

FAULTY PLAYERS

The *Electrix Company* makes two types of electronic equipment: video and audio players. At the end of the daily production, the players are tested and those with faults are removed and sent for repair.

The following table shows the average number of players of each type that are made per day, and the average percentage of faulty players per day.

Player type	Average number of players made per day	Average percentage of faulty players per day
Video players	2000	5%
Audio players	6000	3%

Question 1: Below are three statements about the daily production at *Electrix Company*. Are the statements correct?

Circle "Yes" or "No" for each statement.

Statement	Is the statement correct?
One third of the players produced daily are video players.	Yes / No
In each batch of 100 video players made, exactly 5 will be faulty.	Yes / No
If an audio player is chosen at random from the daily production for testing, the probability that it will need to be repaired is 0.03.	Yes / No

Question 2: One of the testers makes the following claim:

"On average, there are more video players sent for repair per day compared to the number of audio players sent for repair per day."

Decide whether or not the tester's claim is correct. Give a mathematical argument to support your answer.

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Question 3: The *Tronics Company* also makes video and audio players. At the end of the daily production runs, the *Tronics Company's* players are tested and those with faults are removed and sent for repair.

The tables below compare the average number of players of each type that are made per day, and the average percentage of faulty players per day, for the two companies.

Company	Average number of video players made per day	Average percentage of faulty players per day
Electrix Company	2000	5%
Tronics Company	7000	4%

Company	Average number of audio players made per day	Average percentage of faulty players per day
Electrix Company	6000	3%
Tronics Company	1000	2%

Which of the two companies, *Electrix Company* or *Tronics Company*, has the lower overall percentage of faulty players? Show your calculations using the data in the tables above.

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Earthquake

Question 1: A documentary was broadcast about earthquakes and how often earthquakes occur. It included a discussion about the predictability of earthquakes.

A geologist stated: "In the next twenty years, the chance that an earthquake will occur in Zed City is two out of three".

Which of the following best reflects the meaning *of the geologist's statement*?

A $\frac{2}{3} \times 20 = 13.3$, so between 13 and 14 years from now there will be an earthquake in Zed City.

B $\frac{2}{3}$ *is more than* $\frac{1}{2}$, so you can be sure there will be an earthquake in Zed City at some time during the next 20 years.

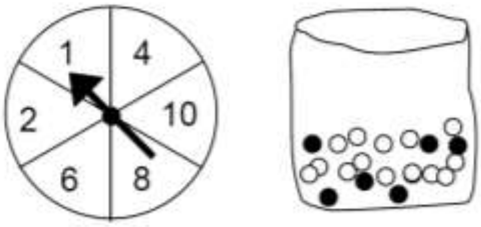
C The likelihood that there will be an earthquake in Zed City at some time during the next 20 years is higher than the likelihood of no earthquake.

D You cannot tell what will happen, because nobody can be sure when an earthquake will occur.

Spring Fair

Question 1:

A game in a booth at a spring fair involves using a spinner first. Then, **if** the spinner stops on an even number, the player is allowed to pick a marble from a bag. The spinner and the marbles in the bag are represented in the diagram below.



Prizes are given when a black marble is picked. Sue plays the game once.

How likely is it that Sue will win a prize?

A Impossible.
B Not very likely.
C About 50% likely.
D Very likely.
E Certain.

Table Tennis Tournament

Question 1:

Teun, Riek, Bep and Dirk have formed a practice group in a table tennis club. Each player wishes to play against each other player once. They have reserved two practice tables for these matches.

Complete the following match schedule; by writing the names of the players playing in each match.

	Practice Table 1	Practice Table 2
Round 1	Teun - Riek	Bep - Dirk
Round 2	.... -	.... - ....
Round 3	.... -	.... - ....

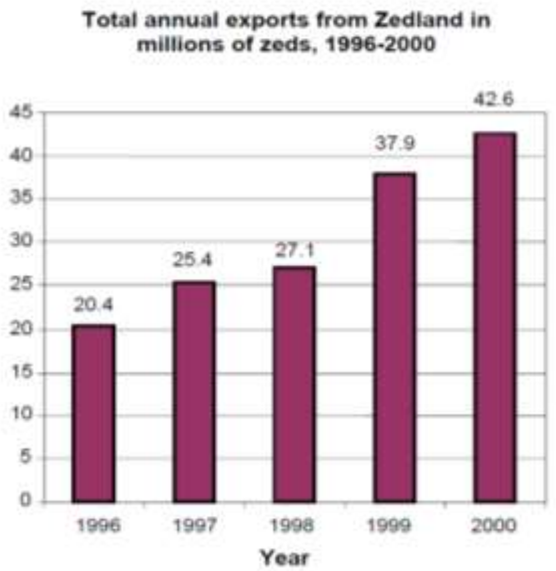
Uncertainty and

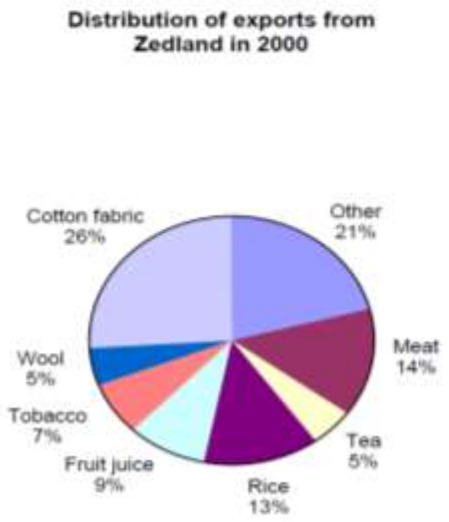
LESSON 8: Statistics

Learning outcome1: Read a bar chart, pie chart and compare

Exports

The graphics below show information about exports from Zedland, a country that uses zeds as its currency.





Question 1: What was the total value (in millions of zeds) of exports from Zedland in 1998?

Answer: ....

Question 2: What was the value of fruit juice exported from Zedland in 2000?

A 1.8 million zeds.

B 2.3 million zeds.

C 2.4 million zeds.

D 3.4 million zeds.

E 3.8 million zeds.

Litter

Question 1: For a homework assignment on the environment, students collected information on the decomposition time of several types of litter that people throw away:

Type of Litter	Decomposition time
Banana peel	1–3 years
Orange peel	1–3 years
Cardboard boxes	0.5 year
Chewing gum	20–25 years
Newspapers	A few days
Polystyrene cups	Over 100 years

A student thinks of displaying the results in a bar graph.

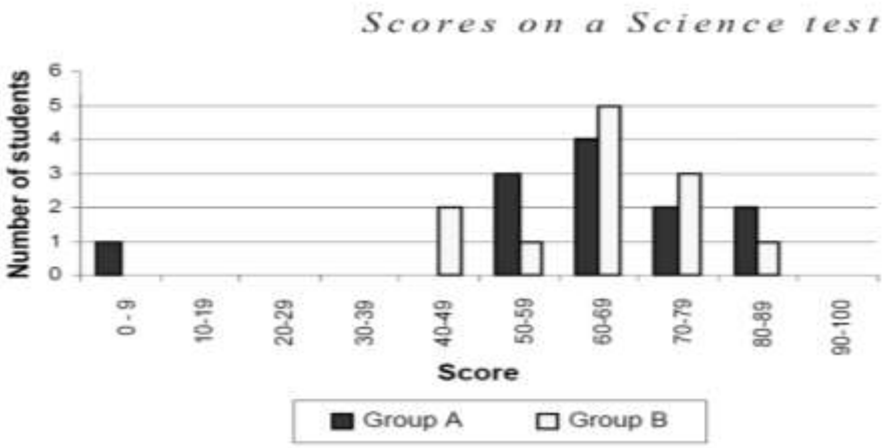
Give **one** reason why a bar graph is unsuitable for displaying these data.

……………………………………………………………………………………………

TEST SCORES

The diagram below shows the results on a Science test for two groups, labeled as Group A and Group B.

The mean score for Group A is 62.0 and the mean for Group B is 64.5. Students pass this test when their score is 50 or above.



Looking at the diagram, the teacher claims that Group B did better than Group A in this test.

The students in Group A don't agree with their teacher. They try to convince the teacher that Group B may not necessarily have done better.

Question 1: Give one mathematical argument, using the graph that the students in Group A could use.

……………………………………………………………………………………………

Learning outcome2: find average, reason and complete data sets when given mean averages

Science Tests

Question 1:

In Mei Lin's school, her science teacher gives tests that are marked out of 100. Mei Lin has an average of 60 marks on her first four Science tests. On the fifth test she got 80 marks.

What is the average of Mei Lin's marks in Science after all five tests?

Average: ....

HEIGHT

There are 25 girls in a class. The average height of the girls is 130 cm.

Question 1: Explain how the average height is calculated.

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....
....

Question 2: Circle either "True" or "False" for each of the following statements.

Statement	True or False
If there is a girl of height 132 cm in the class, there must be a girl of height 128 cm.	True / False
The majority of the girls must have height 130 cm.	True / False
If you rank all of the girls from the shortest to the tallest, then the middle one must have a height equal to 130 cm.	True / False
Half of the girls in the class must be below 130 cm, and half of the girls must be above 130 cm.	True / False

Question 3: An error was found in one student's height. It should have been 120 cm instead of 145 cm. What is the corrected average height of the girls in the class?

A 126 cm
B 127 cm
C 128 cm
D 129 cm
E 144 cm

CABLE TELEVISION

The table below shows data about household ownership of televisions (TVs) for five countries.

It also shows the percentage of those households that own TVs and also subscribe to cable TV.

Country	Number of households that **own TVs**	Percentage of households that **own TVs** compared to **all households**	Percentage of households that **subscribe to cable television** compared to households that **own TVs**
Japan	48.0 million	99.8%	51.4%
France	24.5 million	97.0%	15.4%
Belgium	4.4 million	99.0%	91.7%
Switzerland	2.8 million	85.8%	98.0%
Norway	2.0 million	97.2%	42.7%

Source: ITU, World Telecommunication Indicators 2004/2005
ITU, World Telecommunication/ICT Development Report 2006

Question 1: The table shows that in Switzerland 85.8% of all households own TVs.

Based on the information in the table, what is the closest estimate of the total number of households in Switzerland?

A 2.4 million

B 2.9 million

C 3.3 million

D 3.8 million

Question 2: Kevin looks at the information in the table for France and Norway.

Kevin says: "Because the percentage of all households that own TVs is almost the same for both countries, Norway has more households that subscribe to cable TV."

Explain why this statement is incorrect. Give a reason for your answer.

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SUPPORT FOR THE PRESIDENT

In Zedland, opinion polls were conducted to find out the level of support for the President in the forthcoming election. Four newspaper publishers did separate nationwide polls. The results for the four newspaper polls are shown below:

Newspaper 1: 36.5% (poll conducted on January 6, with a sample of 500 randomly selected citizens with voting rights)

Newspaper 2: 41.0% (poll conducted on January 20, with a sample of 500 randomly selected citizens with voting rights)

Newspaper 3: 39.0% (poll conducted on January 20, with a sample of 1000 randomly selected citizens with voting rights)

Newspaper 4: 44.5% (poll conducted on January 20, with 1000 readers phoning in to vote).

Question:

Which newspaper's result is likely to be the best for predicting the level of support for the President if the election is held on January 25? Give two reasons to support your answer.

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THE END

Appendix B

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Mathematical Literacy Test

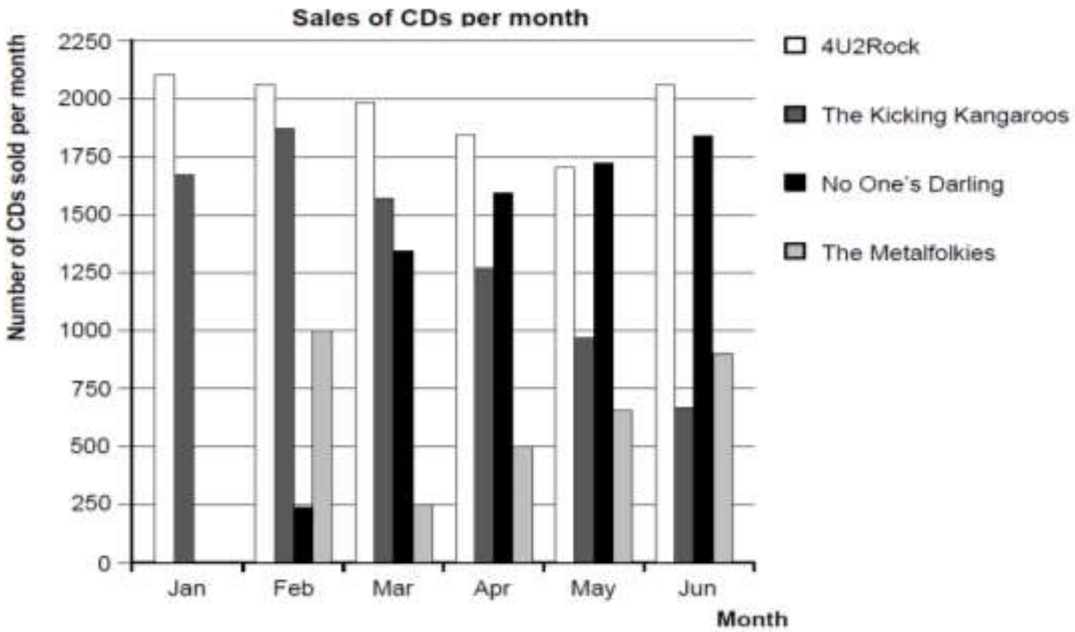
Source: OECD
The Problems are in the form of units

NOTE: PISA questions often refer to situations that take place in the fictional country of Zedland, where the Zed is the unit of currency.

Numbers in green boxes aims to order the whole test questions

CHARTS

In January, the new CDs of the bands *4U2Rock* and *The Kicking Kangaroos* were released. In February, the CDs of the bands *No One's Darling* and *The Metalfolkies* followed. The following graph shows the sales of the bands' CDs from January to June.



1 Question 1: CHARTS

How many CDs did the band *The Metalfolkies* sell in April?

A) 250

B) 500

C) 1000

D) 1270

2 Question 2: CHARTS

In which month did the band *No One's Darling* sell more CDs than the band *The Kicking Kangaroos* for the first time?

A) No month

B) March

C) April

D) May

3

Question 5: CHARTS

The manager of *The Kicking Kangaroos* is worried because the number of their CDs that sold decreased from February to June.

What is the estimate of their sales volume for July if the same negative trend continues?

A) 70 CDs

B) 370 CDs

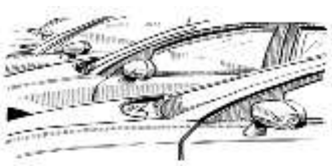
C) 670 CDs

D) 1340 CDs

WHICH CAR?

Chris has just received her car driving licence and wants to buy her first car.

This table below shows the details of four cars she finds at a local car dealer.



Model:	Alpha	Bolte	Castel	Dezal
Year	2003	2000	2001	1999
Advertised price (zeds)	4800	4450	4250	3990
Distance travelled (kilometres)	105 000	115 000	128 000	109 000
Engine capacity (litres)	1.79	1.796	1.82	1.783

4

Question 1: WHICH CAR?

Chris wants a car that meets **all** of these conditions:
The distance travelled is **not** higher than 120 000 kilometres.
It was made in the year 2000 or a later year.
The advertised price is **not** higher than 4500 zeds.

Which car meets Chris's conditions?

A) Alpha

B) Bolte

C) Castel

D) Dezal

5

Question 2: WHICH CAR?

Which car's engine capacity is the smallest?

A) Alpha
B) Bolte
C) Castel
D) Dezal

6

Question 3: WHICH CAR?

Chris will have to pay an extra 2.5% of the advertised cost of the car as taxes.

How much are the extra taxes for the Alpha?

Extra taxes in zeds: ....

GARAGE

A garage manufacturer's "basic" range includes models with just one window and one door.

George chooses the following model from the "basic" range. The position of the window and the door are shown here.

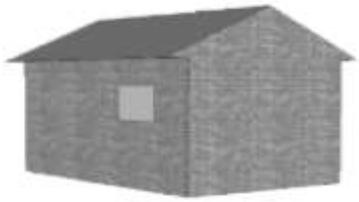
7

Question 1: GARAGE

The illustrations below show different "basic" models as viewed from the back. Only one of these illustrations matches the model above chosen by George.

Which model did George choose? Circle A, B, C or D.

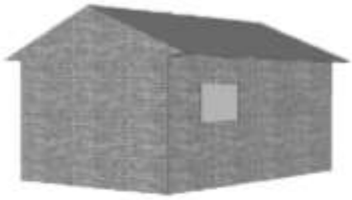
A



B

C

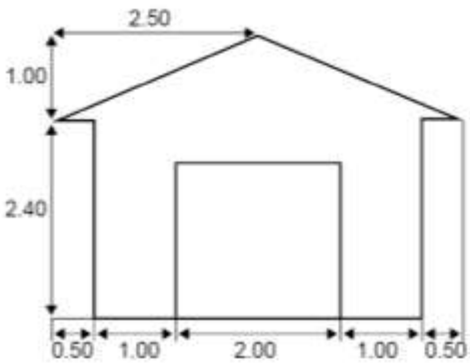
D



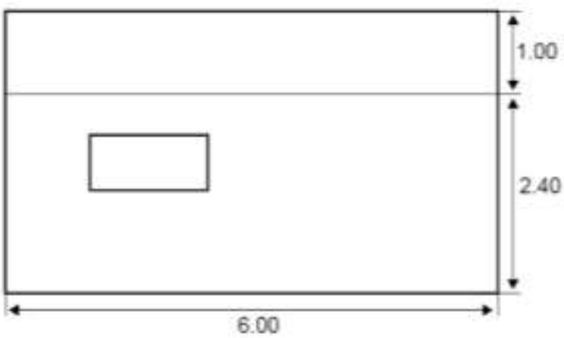
8

Question 2: GARAGE

The two plans below show the dimensions, in metres, of the garage George chose.



Front view



Side view

Note: Drawing not to scale.

The roof is made up of two identical rectangular sections.

Calculate the **total** area of the roof. Show your work.

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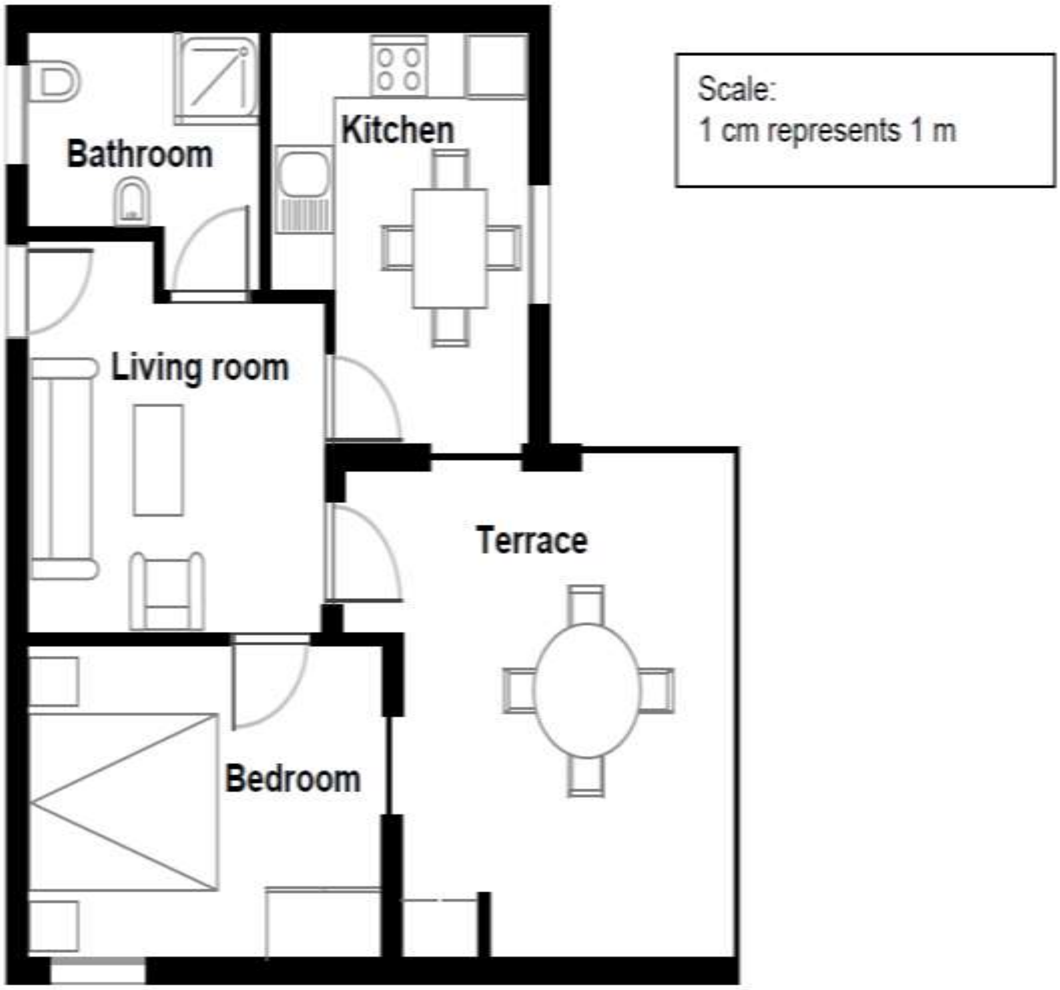
APARTMENT PURCHASE

This is the plan of the apartment that George's parents want to purchase from a real estate agency.



9 **Question 1: APARTMENT PURCHASE**

To estimate the total floor area of the apartment (including the terrace and the walls), you can measure the size of each room, calculate the area of each one and add all the areas together.

However, there is a more efficient method to estimate the total floor area where you only need to measure 4 lengths. Mark on the plan above the **four** lengths that are needed to estimate the total floor area of the apartment.

DRIP RATE

Infusions (or intravenous drips) are used to deliver fluids and drugs to patients.

Nurses need to calculat the drip rate, *D*, in drops per minute for infusions.

They use the formula $D = \frac{dv}{60n}$ where

d is the drop factor measured in drops per millilitre (mL)

v is the volume in mL of the infusion

n is the number of hours the infusion is required to run.

10 **Question 1: DRIP RATE**

A nurse wants to double the time an infusion runs for.

Describe precisely how *D* changes if *n* is **doubled** but *d* and *v* do not change.

....

....

....

11 **Question 3: DRIP RATE**

Nurses also need to calculate the volume of the infusion, *v*, from the drip rate, *D*.

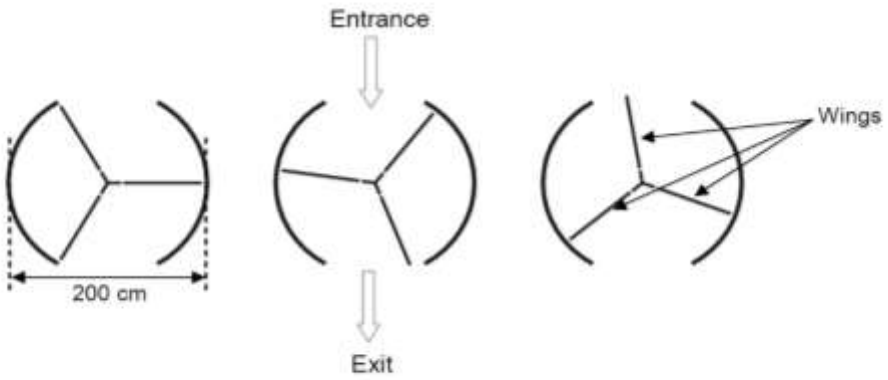
An infusion with a drip rate of 50 drops per minute has to be given to a patient for 3 hours. For this infusion the drop factor is 25 drops per millilitre.

What is the volume in mL of the infusion?

Volume of the infusion: .... mL

REVOLVING DOOR

A revolving door includes three wings which rotate within a circular-shaped space. The inside diameter of this space is 2 metres (200 centimetres). The three door wings divide the space into three equal sectors. The plan below shows the door wings in three different positions viewed from the top.



12

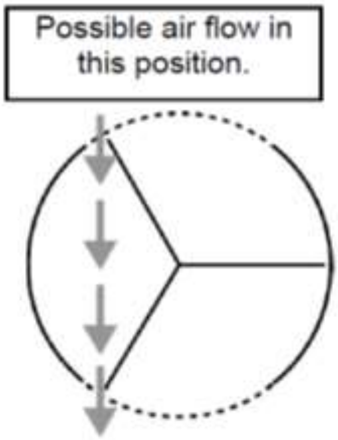
Question 1: REVOLVING DOOR

What is the size in degrees of the angle formed by two door wings?

Size of the angle: .... º

13

Question 2: REVOLVING DOOR



The two door **openings** (the dotted arcs in the diagram) are the same size. If these openings are too wide the revolving wings cannot provide a sealed space and air could then flow freely between the entrance and the exit, causing unwanted heat loss or gain. This is shown in the diagram opposite.

What is the maximum arc length in centimetres (cm) that each door opening can have, so that air never flows freely between the entrance and the exit?

Maximum arc length: .... cm

14 **Question 3: REVOLVING DOOR**

The door makes 4 complete rotations in a minute. There is room for a maximum of two people in each of the three door sectors.

What is the maximum number of people that can enter the building through the door in 30 minutes?

A) 60

B) 180

C) 240

D) 720

SAUCE

15 **Question 2: SAUCE**

You are making your own dressing for a salad.

Here is a recipe for 100 millilitres (mL) of dressing.

Salad oil:	60 mL
Vinegar:	30 mL
Soy sauce:	10 mL

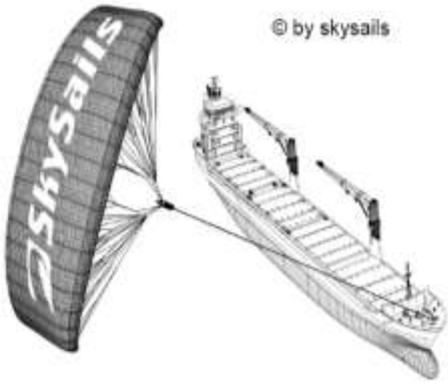
How many millilitres (mL) of salad oil do you need to make 150 mL of this dressing?

Answer: ……………….. mL

SAILING SHIPS

Ninety-five percent of world trade is moved by sea, by roughly 50 000 tankers, bulk carriers and container ships. Most of these ships use diesel fuel.

Engineers are planning to develop wind power support for ships. Their proposal is to attach kite sails to ships and use the wind's power to help reduce diesel consumption and the fuel's impact on the environment.



16

Question 1: SAILING SHIPS

One advantage of using a kite sail is that it flies at a height of 150 m. There, the wind speed is approximately 25% higher than down on the deck of the ship.

At what approximate speed does the wind blow into a kite sail when a wind speed of 24 km/h is measured on the deck of the ship?

A) 6 km/h

B) 18 km/h

C) 25 km/h

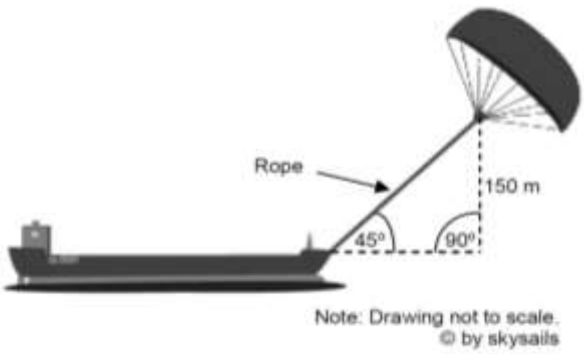
D) 30 km/h

E) 49 km/h

17

Question 3: SAILING SHIPS

Approximately what is the length of the rope for the kite sail, in order to pull the ship at an angle of 45° and be at a vertical height of 150 m, as shown in the diagram opposite?



A) 173 m

B) 212 m

C) 285 m

D) 300 m

18

Question 4: SAILING SHIPS

Due to high diesel fuel costs of 0.42 zeds per litre, the owners of the ship *NewWave* are thinking about equipping their ship with a kite sail.

It is estimated that a kite sail like this has the potential to reduce the diesel consumption by about 20% overall.

Name: *NewWave*

Type: freighter

Length: 117 metres

Breadth: 18 metres

Load capacity: 12 000 tons

Maximum speed: 19 knots

Diesel consumption per year without a kite sail: approximately 3 500 000 litres

The cost of equipping the *NewWave* with a kite sail is 2 500 000 zeds.

After about how many years would the diesel fuel savings cover the cost of the kite sail? Give calculations to support your answer.

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....

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....

....

Number of years: ....

CLIMBING MOUNT FUJI

Mount Fuji is a famous dormant volcano in Japan.

19

Question 1: CLIMBING MOUNT FUJI

Mount Fuji is only open to the public for climbing from 1 July to 27 August each year. About 200 000 people climb Mount Fuji during this time.

On average, about how many people climb Mount Fuji each day?

A) 340

B) 710

C) 3400

D) 7100

E) 7400

20 **Question 2: CLIMBING MOUNT FUJI**

The Gotemba walking trail up Mount Fuji is about 9 kilometres (km) long.

Walkers need to return from the 18 km walk by 8 pm.

Toshi estimates that he can walk up the mountain at 1.5 kilometres per hour on average, and down at twice that speed. These speeds take into account meal breaks and rest times.

Using Toshi's estimated speeds, what is the latest time he can begin his walk so that he can return by 8 pm?

………………………………………………………………………………………

21 **Question 3: CLIMBING MOUNT FUJI**

Toshi wore a pedometer to count his steps on his walk along the Gotemba trail.

His pedometer showed that he walked 22 500 steps on the way up.

Estimate Toshi's average step length for his walk up the 9 km Gotemba trail. Give your answer in centimetres (cm).

Answer: .... cm

HELEN THE CYCLIST

Helen has just got a new bike. It has a speedometer which sits on the handlebar.

The speedometer can tell Helen the distance she travels and her average speed for a trip.

22

Question 1: HELEN THE CYCLIST

On one trip, Helen rode 4 km in the first 10 minutes and then 2 km in the next 5 minutes.

Which one of the following statements is correct?

A) Helen’s average speed was greater in the first 10 minutes than in the next 5 minutes.

B) Helen’s average speed was the same in the first 10 minutes and in the next 5 minutes.

C) Helen’s average speed was less in the first 10 minutes than in the next 5 minutes.

D) It is not possible to tell anything about Helen’s average speed from the information given.

23 Question 2: HELEN THE CYCLIST

Helen rode 6 km to her aunt’s house. Her speedometer showed that she had averaged 18 km/h for the whole trip.

Which one of the following statements is correct?

A) It took Helen 20 minutes to get to her aunt’s house.

B) It took Helen 30 minutes to get to her aunt’s house.

C) It took Helen 3 hours to get to her aunt’s house.

D) It is not possible to tell how long it took Helen to get to her aunt’s house.

24 Question 3: HELEN THE CYCLIST

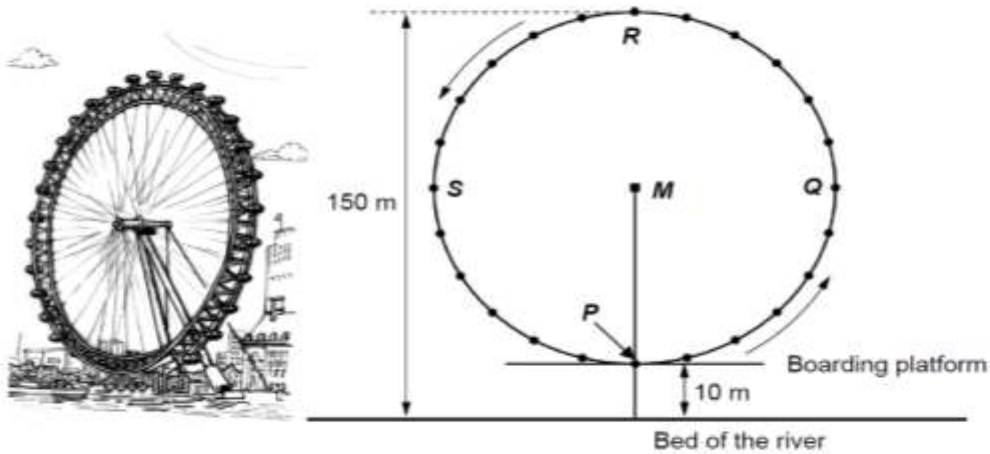
Helen rode her bike from home to the river, which is 4 km away. It took her 9 minutes. She rode home using a shorter route of 3 km. This only took her 6 minutes.

What was Helen’s average speed, in km/h, for the trip to the river and back?

Average speed for the trip: .... km/h

FERRIS WHEEL

A giant Ferris wheel is on the bank of a river. See the picture and diagram below.



The Ferris wheel has an external diameter of 140 metres and its highest point is 150 metres above the bed of the river. It rotates in the direction shown by the arrows.

25

Question 1: FERRIS WHEEL

The letter *M* in the diagram indicates the centre of the wheel.

How many metres (m) above the bed of the river is point *M?*

Answer: .... m

26

Question 2: FERRIS WHEEL

The Ferris wheel rotates at a constant speed. The wheel makes one full rotation in exactly 40 minutes.

John starts his ride on the Ferris wheel at the boarding point, *P*.

Where will John be after half an hour?

A) At *R*
B) Between *R* and *S*
C) At *S*
D) Between *S* and *P*

Continent Area

Below is a map of Antarctica.

27

Question 2:

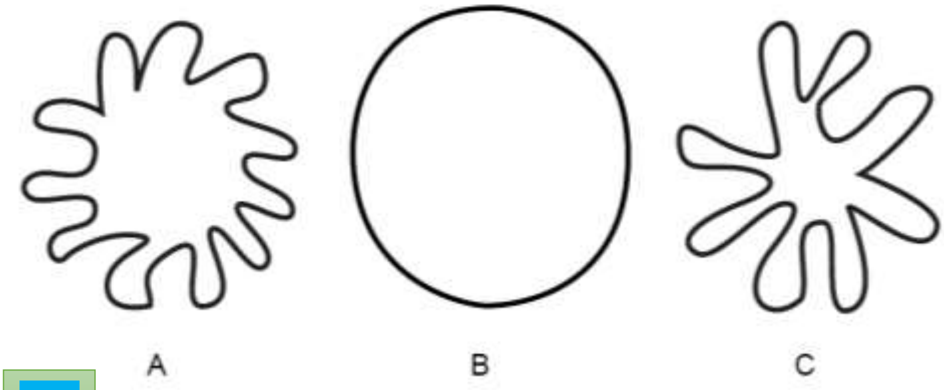
Estimate the area of Antarctica using the map scale.

Show your working out and explain how you made your estimate. (You can draw over the map if it helps you with your estimation)

………………………………………………………………………………………
………………………………………………………………………

………………………………………………………………………………………
………………………………………………………………………

Shapes



28

Question 1:

Which of the figures has the largest area? Explain your reasoning.

29

Question 2:

Describe a method for estimating the area of figure C.

30

Question 3:

Describe a method for estimating the perimeter of figure C.

Pizzas

A pizzeria serves two round pizzas of the same thickness in different sizes. The smaller one has a diameter of 30 cm and costs 30 zeds. The larger one has a diameter of 40 cm and costs 40 zeds.

31 Question 1:

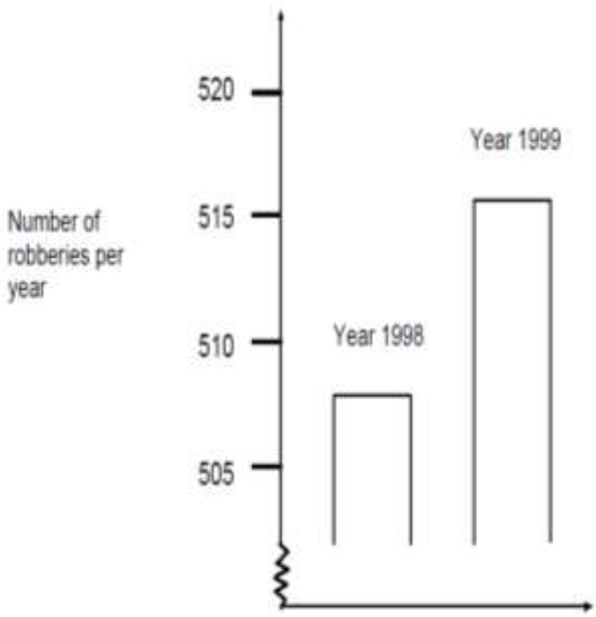
Which pizza is better value for money? Show your reasoning.

Robberies

32 Question 1:

A TV reporter showed this graph and said:

“The graph shows that there is a huge increase in the number of robberies from 1998 to 1999.”



Do you consider the reporter’s statement to be a reasonable interpretation of the graph? Give an explanation to support your answer.

………………………………………………………………………………………

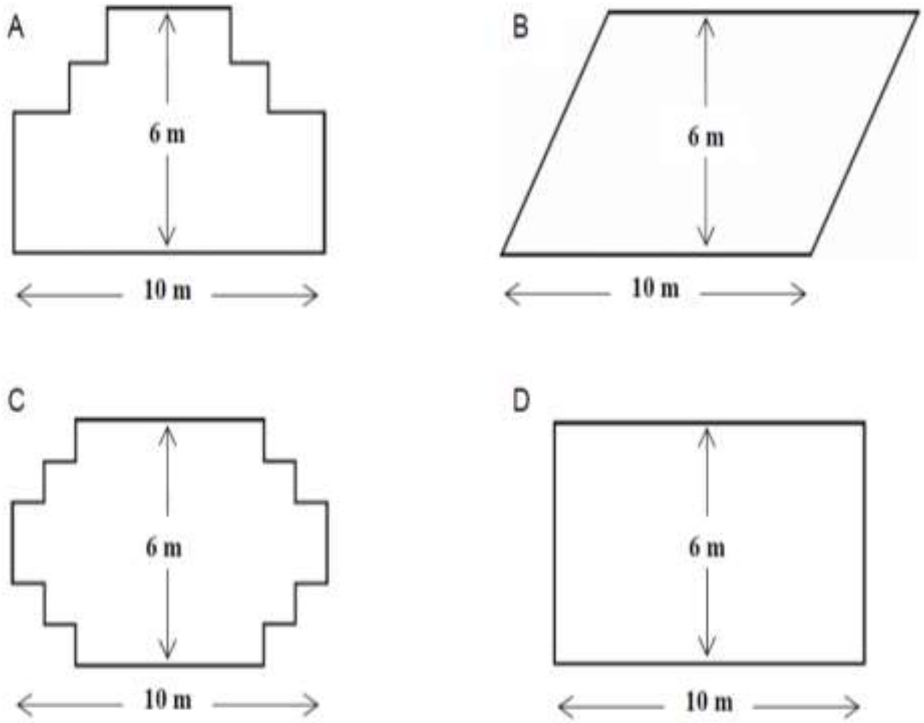
………………………………………………………………………………………

………………………………………………………………………………………

CARPENTER

33 Question 1:

A carpenter has 32 metres of timber and wants to make a border around a garden bed. He is considering the following designs for the garden bed.



Circle either "Yes" or "No" for each design to indicate whether the garden bed can be made with 32 metres of timber.

Garden bed design	Using this design, can the garden bed be made with 32 metres of timber?
Design A	Yes / No
Design B	Yes / No
Design C	Yes / No
Design D	Yes / No

Swing

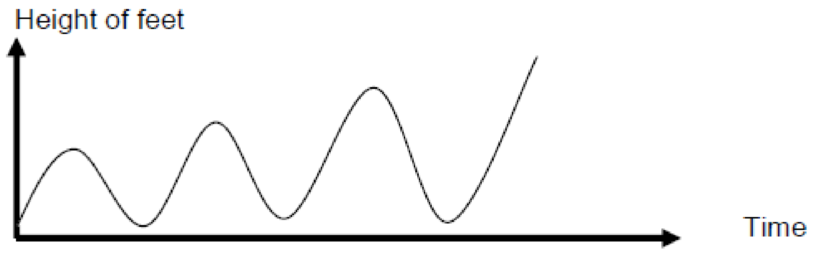
34

Question 1:

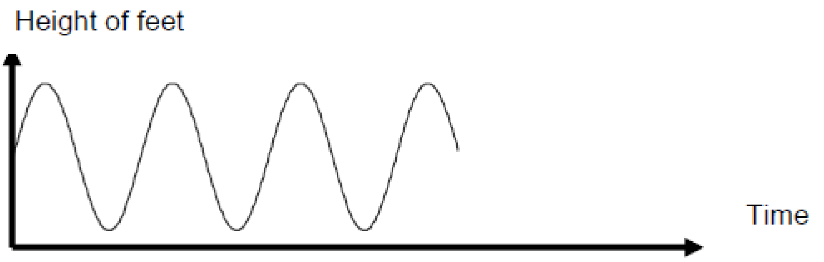
Mohammed is sitting on a swing. He starts to swing. He is trying to go as high as possible.

Which diagram best represents the height of his feet above the ground as he swings?

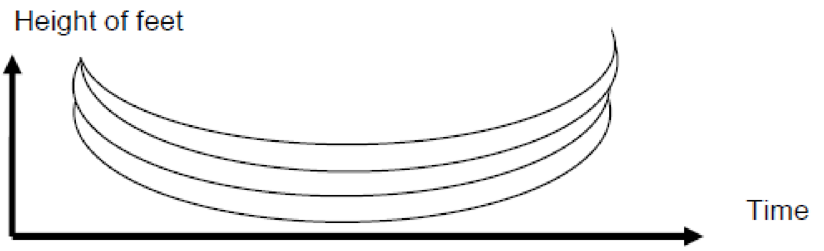
A



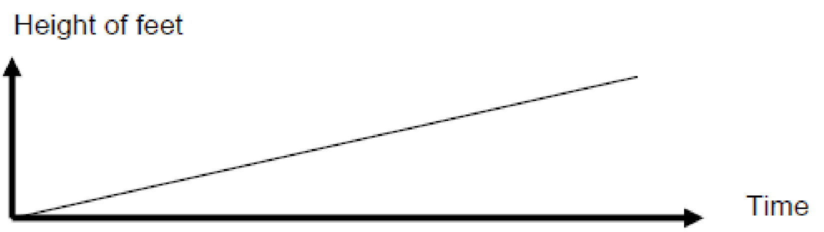
B



C



D



The End

Appendix C

UAEU College of Education

The Motivation Survey

جامعة الإمارات العربية المتحدة
United Arab Emirates University

This survey aims to learn about your motivations for learning mathematics in mathematic classes. Your participation in this survey is completely voluntary. The researcher highly appreciates your cooperation in taking the time and effort to answer this survey.

A: Demographic data

1- **Your Gender:** **Male** **Female**
2- **Your Age:**
3- **Your class section:**

B- How much do you disagree or agree with the following statements about your motivation to learn mathematics?

Very true to me.................. Not at all true to me
7 6 5 4 3 2 1

		Strongly Agree	Agree	Somewhat Agree	Neutral	Somewhat Disagree	Disagree	Strongly disagree
1	I prefer course material that really challenges me so I can learn new things	7	6	5	4	3	2	1
2	I prefer the material that arouses my curiosity, even if it is difficult to learn.	7	6	5	4	3	2	1
3	The most satisfying thing for me in math subject is trying to understand the content as thoroughly as possible.	7	6	5	4	3	2	1
4	When I have the opportunity in math class, I prefer assignments that I can learn from even if they don't guarantee a good grade.	7	6	5	4	3	2	1
5	Getting a good grade in math is the most satisfying thing for me right now.	7	6	5	4	3	2	1
6	The most important thing for me right now is improving my overall grade, so my main concern in math is getting a good grade.	7	6	5	4	3	2	1
7	If I can, I want to get better grades in this class than most of the other students.	7	6	5	4	3	2	1
8	I want to do well in the math class because it is important to show my ability to my family, friends, employer, or others.	7	6	5	4	3	2	1

Thank you

Appendix D

جامعة الإمارات العربية المتحدة
United Arab Emirates University

UAEU College of Education

Perceptions Survey

This survey is intended to investigate student' perceptions regarding their perceptions and evaluation of the Mathematical Enrichment Program. Your participation in this questionnaire is completely voluntary. The researcher highly appreciates your cooperation for taking the time and effort to answer this questionnaire.

A: Demographic data

1- Your Gender: Male Female
2- Your Age:
3- Your class section:

B- How much do you disagree or agree with the following statements about the Mathematical Enrichment program?

		Strongly Agree	Agree	Neutral	Disagree	Strongly disagree
1	I liked the Mathematical Contextual Problem solving provided in this program.	5	4	3	2	1
2	This program made me feel more confident about my ability to do Mathematics.	5	4	3	2	1
3	This program helped me to do better in my regular Mathematics class.	5	4	3	2	1
4	This program made me see and appreciate the importance of Mathematics in life.	5	4	3	2	1
5	This program made me more motivated and engaged in my Mathematics study.	5	4	3	2	1
6	This program made me more prepared to take the PISA test in Mathematics.	5	4	3	2	1
7	It is important to spend time in Mathematics classes to study contextual problem solving.	5	4	3	2	1
8	Deducting time from math classes to implement this program did not present a challenge to complete the required curriculum on time.	5	4	3	2	1

 # Do you recommend applying this program to students to improve their mathematical literacy?

1- Yes **2- No**

C- Feeling and perceptions about the Mathematical Enrichment Program

1- Do you feel that Mathematical Enrichment Program helped you or not? In what Aspects? Explain

………

………

………

………

………

………

2-What can be done to improve this program? Explain.

………

………

………

………

Thank you

Appendix E

UAEU College of Education

جامعة الإمارات العربية المتحدة
United Arab Emirates University

التاريخ: 2019/9/12

السادة / وزارة التربية و التعليم المحترمين

لمن يهمه الأمر

نود إفادتكم علماً بأن الطالبة: حنان شاهر المراشدة -ID: 201080035.
التخصص : مناهج وطرق التدريس – مسار الرياضيات

مسجله في برنامج الدكتوراة في كلية التربية، وتقوم بإعداد بحث بعنوان:

Investigation of the impact of a proposed enrichment Program based on problem solving on the mathematical literacy of UAE's grade 10 students

نرجو التكرم بالموافقة على تسهيل مهمه الباحثه حيث تحتاج لتطبيق بحثها على: مدرسة الجاهلي للبنات و مدرسة الدهماء للبنين.

يرجى التفضل بتوفير المساعدة للطالبة
هذا وتفضلوا بقبول فائق التحية والتقدير.

Ali Ibrahim

د. علي إبراهيم
مساعد العميد لشؤون البحث العلمي والدراسات العليا

College of Education
Assistant Dean for Research and Graduate Studies
PO BOX 15551, Al Ain, UAE
T +971 3 713 6221 T +971 3 713 6249
/graduateprogram/www.cedu.uaeu.ac.ae

كلية التربية
مساعد العميد لشؤون البحث العلمي والدراسات العليا
ص.ب 15551، العين، الإمارات العربية المتحدة
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Appendix F

11/14/2020 FW: تسهيل مهمة باحثة من جامعة الامارات - Hanan Shaher Saadeh Almarashdeh

FW: تسهيل مهمة باحثة من جامعة الامارات

Muna Mohammad Janahi

Mon 11/4/2019 7:18 AM

To: Hanan Shaher Saadeh Almarashdeh <hanan.almarashdeh@moe.gov.ae>;

Importance: High

FYI

UNITED ARAB EMIRATES
MINISTRY OF EDUCATION

الإمارات العربية المتحدة
وزارة التربية والتعليم

منى محمد نور جناحي
اداري رئيسي - مستشارو الوزير
Muna Mohammad Janahi
Administrative - Advisors to Minister

TEL : 042176201 : مباشر
FAX : 04-2176803 : فاكس

P.O.Box : 3962 : صندوق بريد
دبي,الإمارات العربية المتحدة
Dubai, United Arab Emirates

www.moe.gov.ae

عـام التـسـامـح
YEAR OF TOLERANCE

From: Operation Center Abu Dhabi <OperationCenter.AD@moe.gov.ae>
Sent: Sunday, November 3, 2019 4:09 PM
To: Al Jahili <1087@moe.gov.ae>; AL DAHMA'A MODEL <354@moe.gov.ae>
Cc: Lubna Alshamsi <lubna.alshamsi@moe.gov.ae>; Humaid Abdulla :ADEK- HQ <humaid@moe.gov.ae>; Khaled Al Abri <Khaled.AlAbri@moe.gov.ae>; Khaled Al Ansari <khaled.alansari@moe.gov.ae>; Muna Mohammad Janahi <Muna.Janahi@moe.gov.ae>; Hessa Al Wahabi <hessa.alwahhabi@moe.gov.ae>; Rahma Al Rubaei <rahma-km.alrubaei@moe.gov.ae>; Khaled Alahbabi <khaled.alahbabi@moe.gov.ae>

https://mail.moe.gov.ae/owa/#viewmodel=ReadMessageItem&ItemID=AAMkADYxMjYxNjBjLTI0NmEtNDE4Yi1iZDE3LTQyZDc1MWNmNWJiMAB... 1/3

Subject: تسهيل مهمة باحثة من جامعة الامارات
Importance: High

Subject: Facilitating researcher's study from UAE University.

Dear Al Jahli school Director,
Dear Al Dahma'a school Director,

The researcher Hanan Al Marashdeh is pursuing Higher Education to receive the Doctorate degree from the UAE University, that is conducting a study entitled: "*Investigation of the impact of a proposed enrichment Program based on problem solving on the mathematical literacy of UAE's grade 10 students in UAE.*".

This study aims to improve mathematical literacy skills of 10th grade students in the Emirate of Abu Dhabi (Al Ain) to assess and develop their abilities in problem-solving to receive high results in the International Student Assessment (PISA), that is also part of achieving the vision of the UAE National Agenda.

The researcher will be applying PISA trail assessment (Problem Solving Test) on 10th grade students in Al Jahli and Al Dahma'a schools in the Emirate of Abu Dhabi as well as taking part in completing survey about students' motivation in learning mathematics on the following link: https://e.moe.gov.ae/ords/f?p=112:Q::F0JY::::

Guidelines for students participating in the survey and PISA trail assessment:
- Participation is voluntary and the test is distributed randomly to students participating in the class.
- Students' Grades and information is confidential and not to be shared with the researcher.

Your corporation in facilitating the researcher's study is highly appreciated.

Kind Regards.

For further inquiries kindly email the Researcher:
hanan.almarashdeh@moe.gov.ae

الموضوع: تسهيل مهمة باحثة من جامعة الإمارات.

السيد / مدير مدرسة الجاهلي للتعليم الثانوي للبنات المحترم
السيد / مدير مدرسة الدهماء للتعليم الثانوي للبنين المحترم

تقوم الباحثة حنان المراشدة باستكمال دراستها العليا للحصول على شهادة الدكتوراه من جامعة الإمارات حيث تقوم بدراسة تحت عنوان: (تأثير برنامج إثرائي قائم على حل المشكلات والاستدلال لتحسين مهارات الرياضيات لدى طلاب الصف العاشر بدولة الامارات في ضوء برنامج تقييم الطلاب الدولي PISA).

حيث تهدف الدراسة إلى تحسين مستوى مهارات الطلبة في الصف العاشر لمادة الرياضيات وتطوير قدراتهم لرفع نتائج تقييم الطلاب في الاختبار الدولي PISA لتحقيق رؤية الاجندة الوطنية.

وعليه ستقوم الباحثة بزيارة مدرستي الجاهلي للتعليم الثانوي للبنات والدهماء للبنين بمدينة العين بإمارة أبو ظبي لتطبيق اختبار PISA التقييمي باللغة الإنجليزية وسيتم استكمال طلبة الصف العاشر استبيان عن دافعية تعلم مادة الرياضيات باللغة العربية على الرباط التالي:
https://e.moe.gov.ae/ords/f?p=112:Q::F0JY::::

الإرشادات الخاصة للطلبة المشاركين في الاستبانة والاختبار التجريبي PISA:
- المشاركة في استكمال الاختبار والاستبانة تطوعية ويتم توزيع الاختبار بشكل عشوائي على الطلبة المشاركين في الفصل.
- بيانات الدرجات والمعلومات الخاصة بالطالب سرية ولا يتم المشاركة بها مع الباحث.

وعليه نرجو الإيعاز لما يلزم بتسهيل مهمة الباحثة المذكورة،

وتفضلوا بقبول وافر التقدير والإحترام.

للتواصل مع الباحثة على البريد الالكتروني:
hanan.almarashdeh@moe.gov.ae

UNITED ARAB EMIRATES
MINISTRY OF EDUCATION

الإمارات العربية المتحدة
وزارة التربية والتعليم

قطاع العمليات المدرسية
School Operations Sector

Appendix G

Investigation of the impact of a proposed enrichment Program based on problem solving on the mathematical literacy of UAE's students

Consent Form

Dear student's parents,

I am a PhD student in Education college in the United Arab Emirates University. I am writing to you to request your permission to allow your daughter/son to participate in the Mathematics Enrichment Program. In this program, the students will participate in weekly session that is expected to improve their mathematical literacy for about 10 weeks during the first semester 2019-2020. Mathematical literacy is about the students' use mathematics to reflect on their lives, plan their futures and mind and solve meaningful problems related to a range of important issues in their lives. Students will also participate in a pretest and a post-test along with a perceptions survey to present their opinions on the implemented program. The collected data will be used for research purposes only and will not, under any circumstances, be shared with anyone. Moreover, participation in this study is voluntary, and the student has the right to withdraw at any time during the intervention without any consequences. It would be appreciated if you could inform me of that withdrawal.

Should you have any questions about the research please call the researcher Hanan Almarashdi on 0507339896 or contact on 201080035@uaeu.ac.ae

Please check the box that represents your opinion.

I agree to let my daughter/son …………………………..take part in the above study.

Yes ☐ No ☐

Participant's Parent	Date	Signature
……………………..	…………………	……………………..