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Integration of STEM Education in Differential and Integral Calculus classes: Aspects Evidenced in a Mathematical Modelling Activity

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ABSTRACT

Background: The integration of STEM areas (science, technology, engineering and mathematics) in engineering courses has been the focus of research in the educational context and mathematical modelling has the potential for its success. **Objectives**: The research question that guides the discussion is: What aspects of STEM education can be evidenced when engineering students develop mathematical modelling activities in a virtual environment within the scope of a Differential and Integral Calculus course? **Design**: Qualitative research carried out in the second half of 2020, based on research design guidelines, in the Differential and Integral Calculus subject during remote teaching. **Setting and Participants**: Virtual environment shared by two classes with 72 students from different courses for the development of a mathematical modelling activity. **Data collection and analysis**: Data from 18 groups were collected virtually from notes on the wiki, audio or video recordings of group meetings or orientation meetings, activity communication video and individual response to a questionnaire. The analysis was based on an initial coding of data from all groups, carried out with the support of AtlasTi, in which three groups were selected for a detailed analysis related to aspects of STEM education. **Results**: Some groups naturally integrate the four STEM areas and others carry out this integration in a partial way. **Conclusions**: The virtual environment made it possible for students to interact in their groups and collaborate with each other to solve a problem in a way that knowledge of basic sciences and mathematics were articulated.

Keywords: STEM education; Mathematical modelling activity; Remote teaching; Differential and Integral Calculus; Fixed radar.

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Integração da Educação STEM em aulas de Cálculo Diferencial e Integral: aspectos evidenciados em uma atividade de Modelagem Matemática

RESUMO

Contexto: A integração das áreas STEM (*Science*, *Technology*, *Engineering and Mathematics*) nos cursos de Engenharia tem sido foco de pesquisas no contexto educacional e a Modelagem Matemática se apresenta com potencial para seu êxito. **Objetivo:** A questão de pesquisa que direciona a discussão é: Que aspectos da educação STEM podem ser evidenciados quando alunos de Engenharia desenvolvem atividades de Modelagem Matemática em um ambiente virtual no âmbito de uma disciplina de Cálculo Diferencial e Integral? **Design:** Pesquisa qualitativa realizada no segundo semestre de 2020, baseada nas orientações da Research Design, na disciplina de Cálculo Diferencial e Integral durante o ensino remoto. **Ambiente e participantes**: Ambiente virtual compartilhado por duas turmas com 72 alunos de diferentes cursos para o desenvolvimento de uma atividade de Modelagem Matemática. **Coleta e análise de dados:** Dados de 18 grupos coletados de forma virtual a partir de anotações na *wiki*, gravações em áudio ou vídeo dos encontros dos grupos ou dos encontros de orientação, vídeo de comunicação da atividade e resposta individual a questionário. A análise se deu a partir de uma codificação inicial dos dados de todos os grupos, realizada com suporte do AtlasTI, em que três grupos foram selecionados para uma análise pormenorizada relativa a aspectos da educação STEM. **Resultados**: Alguns grupos integraram naturalmente as quatro áreas STEM e outros realizam essa integração de forma parcial. **Conclusões**: O ambiente virtual possibilitou aos alunos interagirem em seus grupos e colaborarem entre si para a resolução de um problema de forma que conhecimentos de Ciências Básicas e Matemática se articularam.

Palavras-chave: educação STEM; atividade de modelagem matemática; ensino remoto; Cálculo Diferencial e Integral; radar fixo.

INTRODUCTION

The National Curriculum Guidelines for the Engineering course (DCN, in the Portuguese acronym, Resolution N. 2, of April 24, 2019) brings recommendations for the development of competencies during the qualification of future engineers. Such recommendations include, for example, encouraging the development of activities linking theory, practice, and the application context; the development of individual or group work, under effective teachers' guidance; the implementation, from the beginning of the course, of activities that promote integration and interdisciplinarity coherently with the curriculum development axis to integrate the technical, scientific, economic, social, environmental, and ethical dimensions. Finally, as for the methodological options, they suggest activities that provide active and more student-centred learning (Brasil, 2019). These and other guidelines align with the STEM education movement (*science*, *technology*, *engineering, and mathematics*), an education format focused on the topics (Pugliese, 2020).

In Brazil, this is a recent movement. However, it has already impacted educational policies, such as the high school reform according to the National Common Curriculum Base (BNCC, in the Portuguese acronym), wich was influenced by educational policies from other countries, including the United States, where the STEM movement became popular (Pugliese, 2020). According to the author, "Given the STEM scenario and the effects that its reform has on the educational system, it is valid to seek ways to deeply understand its real meaning and impact in Brazil" (Pugliese, 2020, p. 2019). He adds that educational research from a science-teaching point of view is needed.

We also emphasise that, among the sub-competencies provided for in the engineering DCNs, are: being able to model phenomena, physical and chemical systems, using mathematical, statistical, computational, and simulation tools, among others; predicting the results of the systems through the models; designing experiments that generate real results for the behaviour of the phenomena and systems under study; verifying and validating the models through appropriate techniques (Brasil, 2019).

Considering the studies and even debates related to the implementation of STEM education, English (2017) listed five central issues that should be considered in curricula: (a) perspectives on STEM education, (b) approaches to STEM integration, (c) discipline of STEM representation, (d) equality in access to STEM education, and (e) STEM extension to STEAM (*science*, *technology*, *engineering, art*, *and mathematics*). In her investigation, the author highlights the importance of pedagogical resources that enable the integration of STEM education in the classroom, including mathematical modelling.

Mathematical modelling, a pedagogical alternative where a mathematical approach is carried out from a reality-based problem situation in which it is possible to present a solution to a problem (Almeida et al., 2012), is part of the classroom practice of the authors of this article. As educators in an institution with an essentially technological tradition, especially engineering, we seek, through mathematics education, to contribute to forming human beings within the undergraduate scope.

Hallström e Schönborn, (2019) consider it a great challenge for teachers to design classroom activities that integrate two or more STEM areas in a relevant way for students' teaching and learning. The authors cite several studies that discuss practices with model exploration and modelling to increase relevance and authenticity in STEM subjects. However, “[...] it is imperative that model-based pedagogical practices for STEM education classrooms be further researched and tested in real educational settings to contribute to an integrated STEM literacy” (Hallström & Schönborn, 2019, p. 9).

The subject of differential and integral calculus of a real variable (Calculus 1) is offered in the first semester (of a semi-annual academic regime) of the pedagogical political project of the engineering courses –mechanics, materials, environmental, chemical and production engineering– at the institution where the research was developed. In this sense, it can be considered a subject with the potential to promote, from the beginning of the educational process, the integration between the curricular units of basic sciences and mathematics and establish relationships with specific engineering units. Research that deals with mathematical modelling has presented results that allow us to infer this integration (Hallström & Schönborn, 2019, English, 2017, English & Mousoulides, 2015).

The development of mathematical modelling activities in subjects offered in person is part of the authors’ pedagogical practice. However, the years 2020 and 2021 challenged the educational context, demanding different actions in the conduct of classes at different levels of education. In higher education, in the first semester of 2020, the classes were entirely remote and with the development of synchronous and asynchronous activities due to the Covid-19 pandemic. Therefore, we implemented the first *design* for the Calculus 1 subject, offered to students who had already failed the subject at least once (dependents).

Considering the pandemic context and our intention to promote an educational environment conducive to the integration between the basic sciences and mathematics curricular units, we outlined the research question: What aspects of STEM education can be evidenced when engineering students develop mathematical modelling activities in a virtual environment within the scope of a differential and integral calculus course?

To support the discussions and reflections on the research question, the following sections bring the theoretical framework of STEM education and mathematical modelling articulated to STEM education. The next section resumes the research context to present the methodological option based on qualitative research. Then, we describe and discuss the mathematical modelling activity developed. Finally, we make some remarks and present future implications.

STEM EDUCATION

STEM education, STEM literacy, STEM integration, and STEM careers are some expressions used to refer to the approximation of the areas of science, technology, engineering, and mathematics. While there are different approaches to STEM, Baptista e Martins (2019) argue that STEM education can be understood as an approach that aims to meet the current needs of society for the literacy of its citizens and to encourage students to pursue STEM careers. According to the authors, the literature indicates that, besides the need to qualify human resources to work in STEM areas, other careers benefit from or even require STEM knowledge and competencies. This justifies the growing interest in initiatives to promote STEM education.

Jonhson (2013) argues that the initiatives to promote STEM education are diverse, from the mere expansion of the workload of specific subjects, by offering independent courses without much connection with the curriculum but with an interdisciplinary appeal, to an approach called integrated STEM, which aims to "create a meaningful integration of STEM disciplines in the context of real-world challenges and problems" (Johnson, 2013, p. 367). In part, this is due to the lack of consensus on what STEM education means and how it should be carried out in practice, as: "In a broad sense, it can refer to the sum of the individual disciplines involved in STEM or to an interdisciplinary approach to STEM education that emphasises connections between disciplines" (Gao et al., 2020, p. 1).

Promoting the integration of knowledge from different areas can be crucial for STEM education because, as Zawojewski (2016) warns, although the concepts covered in conventional mathematics curricula are fundamental to the work of professionals in STEM areas such as engineering and computing, the mathematics they use to solve problems in their field of activity often does not refer to formal mathematics learned at school.

In this sense, in recent decades, scholars have searched for a concept for STEM education and how it can be integrated, considering that, according to Baptista and Martins (2019, p. 100), "[...] it cannot be perceived as a set of disconnected and independent contents of the four STEM fields".

According to Johnson (2013, p. 367),

> [...] integrated STEM education is an instructional approach that integrates the teaching of science and mathematics subjects through the infusion of scientific research practices,

> technological and engineering *design*, mathematical analysis, and 21st-century interdisciplinary themes and skills.

It seems evident that engaging students in the learning process through integrated STEM education requires choosing real-world problems that are relevant to them (Baptista & Martins, 2019). Gao et al. (2020, p. 3) attribute to this approach the improvement of students in the affective domain, which includes: “students’ interest, engagement, attitude, and motivation towards STEM content and practices and career aspiration for STEM professions”.

Gao et al. (2020) present a literature review covering the last two decades, focused on the evaluation of interdisciplinary STEM education, analysing articles focused on research with students at secondary and higher levels. The authors indicate that research in STEM education is still on the rise. Among the analysed articles, most include both science and engineering, but few of them show, in fact, the integration of the four areas. Another aspect concerns the implementation context: most studies were carried out in non-formal educational environments. Moreover, few studies encompassed higher education. From the literature review, the authors provide a two-dimensional framework for assessing student learning in STEM education and conclude that few assessments have paid attention to interdisciplinary learning and practices. Among the suggestions that aim to contribute to the advancement of research in the area, Gao et al. (2020) indicate that connections between STEM disciplines need to be operationalised to provide feedback to students, and the curriculum must spell out the expected connections between disciplines.

Hallström and Schönborn (2019, p. 8-9) point out implications for STEM education, among which we highlight the view that “models and modelling should be used as means to promote STEM literacy and the transfer of knowledge and skills between contexts inside and outside STEM disciplines” and the understanding that STEM education should focus on de-focusing the vocational vision of STEM “[...] as a way of increasing economic competitiveness in favour of promoting STEM as an interdisciplinary way of authentically learning science, technology, engineering and mathematics”.

This study focuses on Calculus 1 classes, which means that the understandings that supported the approach are allocated in mathematical modelling within the scope of mathematics education.

MATHEMATICAL MODELLING AND STEM EDUCATION

Within the scope of mathematics education, there are different characterisations for mathematical modelling, depending on the modellers' objectives. Stillman et al. (2015) consider that mathematical modelling can be seen as the definition of a problem from a real situation that can be interpreted in mathematical terms.

The mathematical interpretation that emerges in the search for a solution to a problem is supported by a set of procedures in which those involved look for information, simplify, identify variables, define hypotheses, deduce a mathematical model, validate that model, and communicate their results (Almeida et al., 2012). Those actions comprise what we mean by a mathematical modelling activity.

Generally speaking, "modelling problems are open to various approaches and solutions; thus, they encourage creative, critical and flexible thinking" (English & Mousoulides, 2015, p. 532). In this sense, defining a problem that is amenable to a mathematical approach is an action that requires a glimpse of a mathematical apparatus that allows the real situation to be mathematised through its translation in the mathematical domain into a mathematical model that needs to be interpreted and validated to be solved. Thus, "the problems require students to take ownership of the situation, to mathematise it in the most meaningful and reasonable way for them" (Baioa & Carreira, 2019, p. 11).

In general, the deduction of a mathematical model is subsidised in formulating hypotheses that guide the investigation. Thus, Bassanezi (2002) indicates that formulating hypotheses comes from observing data or information about the real situation, and comparing resolutions for similar problems, besides the modeller's experience. In research carried out in undergraduate and graduate courses, Almeida et al. (2021, p. 89) show that "the formulation of hypotheses by the students signals a way of seeing the students; it is anchored in their experiences and provides elements for subsequent actions". Those actions culminate in the deduction of a mathematical model.

The development of a modelling activity has as its main objective the search for a solution to the problem subsidised in a mathematical model. In this search, "a complex structural relationship is created between two entities of different epistemological nature: the situation to be modelled and the mathematical system" (Almeida & Silva, 2015, p. 45). The situation to be modelled allows “an interdisciplinary look, in which, in addition to teaching mathematics, physics, chemistry or biology, it aims to teach the student to think, investigate, and solve problems”(Setti & Vertuan, 2021, p. 5). According to Malheiros (2011, p. 82), “when concerned with looking for solutions to a given problem, it is often necessary to use concepts that are not always directly related to the question studied”.

Ärlebäk and Doerr (2015) make a characterisation between mathematical modelling activities of model elicitation and model application activities. According to the authors, the model elicitation activity aims to make the student explore the mathematical structure of the deduced model, i.e., with this type of activity, the focus of mathematical modelling is the mathematical content covered. On the other hand, a model application activity should enable students to reflect, based on the model they deduced, on different situations, applying them in other contexts or even relating them to other models.

Problems that emerge in model elicitation activities can “promote the design of processes that apply in STEM” (English & Mousoulides, 2015, p. 534). The model elicitation process allows students to critically analyse the relevance of their solutions while documenting their thought processes along the way.

Given the results of research on the integration of STEM education in classrooms, specifically regarding the presence of mathematics as calculations or representations of science data, Baker e Galanti (2017) promoted a teacher education course considering model elicitation activities. During four days, the teachers in training engaged in activities to elicit models as learners, contrasted such activities with problem-based learning and highlighted the potential of these activities in modifying curriculum tasks to broaden the approach to mathematics in the scope of the STEM integration.

From a STEM educational perspective, including experimentation and development of a prototype, Carreira and Baioa (2018) showed that students attributed credibility to the mathematical modelling activity through the simulation of paint manufacturing. Although they were based on a fictitious reality brought to the class, the students “used everyday artefacts, school materials and knowledge of the situation (even though they were not experts in such knowledge) to reach results considered reasonable” (Carreira & Baio, 2018, p. 213). When developing a modelling activity with the biometrics theme of the hand with basic education students, subsidised in STEM education, Baioa and Carreira (2019, p. 11) clarified that this referral

> allows and promotes the use of materials and equipment, encourages hands-on work, cooperative learning, discussion and research, questioning and conjecture, production of justifications, writing of reports, problem-solving activity, including the use of technologies.

For STEM integration within mathematics classes, English and Mousoulides (2015) are based on engineering-based model elicitation activities. Faced with "multifaceted and idea-generating" problems (English & Mousoulides, 2015, p. 534), the researchers highlighted students' frustrations. However, as they progressed in the activity, students became more actively involved and enjoyed the referral, working in mixed-skill groups where they could express their ideas.

The studies mentioned above guided our investigation in considering a STEM approach in the development of mathematical modelling activities with engineering students in a virtual environment within the scope of a Calculus 1 course.

METHODOLOGICAL ASPECTS

The implementation of the first design, in the pandemic context of the first semester of 2020, involved two classes of Calculus 1 for repeaters, with 72 students from the different engineering courses (environmental, materials, mechanics, production, and chemistry) of the institution. We, the authors of this article, were the responsible professors of the classes. However, we collaboratively planned the subject and implemented synchronous and asynchronous activities. Thus, the classes shared the same virtual teaching and learning environment (AVEA, in the Portuguese acronym) organised in Moodle, which allowed us to organise the groups more flexibly, with participants from both classes.

The theme developed in the first cycle was *fixed radar*, according to our suggestion by proposing: *How do fixed radars in urban area and highways operate?* This suggestion followed our understanding that it can potentially integrate the STEM areas, to be developed as a mathematical modelling activity in groups, according to the guidelines of the engineering DCNs.

The students formed 18 groups (A, B, C, D,..., and R), with four members each, who could interact online remotely, synchronously, or asynchronously from August 31, 2020, to September 25, 2020. They could also schedule guidance meetings with the professors remotely and synchronously.

Each group had a Wiki in Moodle where the development of the activity was recorded and a web conferencing space integrated into Moodle, the BigBlueButton, with the option to record the group meetings. The groups were instructed to follow a schedule so that they could receive feedback at each stage: interaction with the problem and definition of the group's study problem; mathematisation and problem solving; interpretation and validation of results; video with a summary of the development of the activity (Table 1).

Table 1

Schedule with steps in the development of the activity

Dates	Activity Stages	Evaluation of the stage
Week 08/31 to 09/04	Interaction with the problem and definition of the study problem of the group.	7 - Sept
Week 08 to 11/09	Mathematisation and resolution of the group problem.	14 - Sept
Week 14 to 09/18	Interpretation and validation of the results of the group.	21-Sept
Week 21 to 09/25	Video with the summary of the activity development.	25-Sep
MM grade result	Publication of the assessment by the professors.	29 - Sep

Although the research theme was the same, the groups' approaches to the problems of the mathematical modelling activities were not the same "since each one has its methods and goals, considering their interests, objectives and experiences" (Malheiros, 2011, p. 81). The approaches, based on the theme *fixed radar*, were: *the simulation of a new system* (A group), *speed calculation accuracy* (Group B and K), the *relationship between speed and fines* (Group C, G, H, and L), the *relationship between camera angle and image capture* (Group D, E, F, I, J, and O), the *relationship between camera height and camera reach* (Group M), *problems that interfere in the assessment of infringing vehicles* (Group N and Q), and *speed calculation* (Group P). Only Group R did not finish the activity, and its members did not conclude the course.

To present reflections on which aspects of STEM education can be evidenced when engineering students develop mathematical modelling activities in a virtual environment within the scope of a differential and integral calculus course, we rely on qualitative research methodology based on research design. Research design "involves new ways of thinking about the nature of students' developing mathematical knowledge and skills, and new ways of thinking about the nature of the teaching, learning, and efficient problem solving"(Lesh, 2002, p. 29).

In our investigation, the "new ways of thinking" were structured according to STEM integration and engineering DCNs. For this, we analysed the students' registers produced while developing activities, whether notes in the wiki of each group, *audio* or video recordings of group meetings or guidance meetings, activity communication videos, and individual answers to the activity completion questionnaire. The research is part of a project approved by the Research Ethics Committee (CEP) through opinion number 3.318.427, May 10, 2019.

The research data were organised with the support of Atlas.TI software, in a project where all material was analysed. Initially, 18 text files with the conversion to pdf of each group's wiki and eight video files from the groups' presentations and some guidance meetings were coded. The coding of the texts was based on reading and highlighting excerpts. The coding of the videos was based on visualising/listening and highlighting significant excerpts to identify the groups' approach to the proposed problem and how each group's mathematical modelling activity was developed.

The encoding process of the 26 documents generated 34 codes, which we list in Table 2 together with the number of times they were used:

Table 2

Codes generated and number of times they were used

Code	Freq.	Code	Freq.
anticipation	6	parameters	1
Arduino	1	problem	14
search for generalisation	1	textbook type problem	12
communication	5	proposal	2
fictitious data	3	question system	1
real data	1	additional resource	9
misunderstanding	5	graphic representation	5
experimentation	1	system representation	10
the lack of validation	1	resolution	1
illustrative image	1	result	5
STEM information	3	optical sensors	1
conflicting information	2	simplifications	1
intent of generalisation	1	simulation	4
peer interaction	6	professors' suggestion	7
model	4	Tracker	1
classic model	2	voltage valleys	1
new system	3	validation	2

This process allowed a first perception of the presence of aspects of STEM education throughout the development of the modelling activities.

Considering the interaction of the groups with the professors and the connection between the different STEM areas, we chose to analyse the activities developed by Groups A, K, and M. Group A was formed by mechanical engineering students, designated as AM1, AM2, AM3, and AM4; Group K was formed by three environmental engineering students (KA1, KA2, and KA3) and one materials engineering student (KMT4); Group M was formed by three environmental engineering students (MA1, MA2, and MA3) and one mechanical engineering student (MM4). Once the groups were defined, we began to analyse the material seeking to highlight science, technology, engineering, and mathematics. In the next topic, we present a description and discussion of the activities developed by those three groups.

DESCRIPTION AND DISCUSSION OF MODELLING ACTIVITIES

Initially, each student was asked *how fixed radars in urban areas and highways operate.* So, gathered in groups, they continued *interacting with the problem and defining the group's study problem*. In the way it was presented, the problem elucidates the theme to be investigated; however, "the core of ideas is not presented in advance, but is inserted in the problem and must be elicited and operated to produce a solution model" (English & Mousoulides, 2015, p. 534).

At this stage, in the first individual interaction, the internet was the only information source the students used. The groups' discussions guided the choices, and according to each group's registers in the Moodle wiki, we saw that the majority chose to study a system called fixed radar of inductive loops. Such motor vehicle speed meters are based on measuring the vehicle's passage time between two or three sensors of fixed and known distance installed on the lane pavement. The vehicle's passage time over the sensors is measured from an electromagnetic field, and the speed is calculated by the ratio between the space and time variation. If the vehicle exceeds the speed allowed on the road, a camera fixed to a pole is activated to register the offender's license plate, as shown in Figure 1, inserted in some groups' registers.

The system represented in Figure 1 makes it possible to cover contents related to the discipline of physics, such as mechanics, which studies phenomena related to the movement of bodies that are generally studied since basic education.

Groups A and K were initially interested in studying how to improve the way of considering the speed of vehicles by the radar system, which is based on average speed. Group M directed its interest to the physical structure of the system, considering the positioning of the cameras and the relationship with the image captured of the offending vehicles.

Aiming at the phase of *mathematisation and problem solving*, the professors asked the three groups, via the Moodle platform, whether they had in mind how to proceed with the study and what mathematics they thought was necessary. With this, the professors tried to evidence the students' anticipation regarding the mathematical contents that could emerge from the problems, as well as the knowledge related to the problem addressed. Anticipation aims to predict what "will be mathematically useful subsequently in the transitions between the phases of the modelling process" (Stillman et al., 2015, p. 95), according to the DCNs (Brasil, 2019). Predicting the results of the systems through the models are sub-competencies to be developed from the first education terms of the future engineer.

Figure 1

Fixed radar of inductive loops

During the guidance meeting, Group A presented new information and the intention to understand a new system, built with optical instead of electromagnetic sensors, as expressed by AM3:

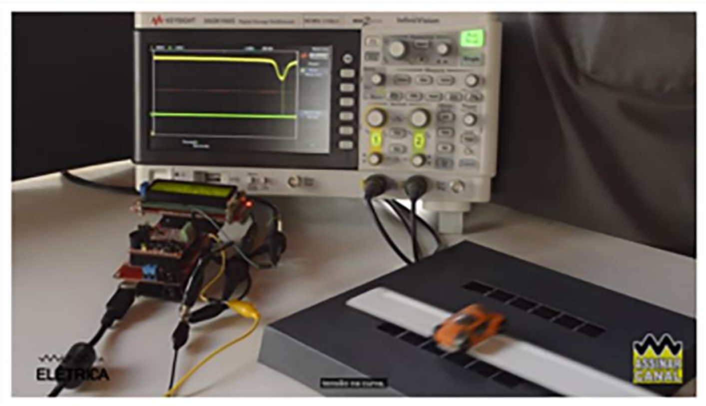
> *At first, we had a little difficulty finding the problem to be solved, but after talking to you and each other, we decided to rely on two main sources, those that are in Moodle. On this one* [opens a link and scrolls through the website] *a little of the test is explained by the Arduino, but as we do not have the equipment, nor experience with the Arduino, we chose to use*

this one, based more on this [opening another link], *which was more..., more practical, because we could film the cart and then use Tracker to do the tests and be able to analyse the data. After explaining the speed calculation and when the radar limit is surpassed, there are some valleys on the graph… which is how we explained it in the video above. Because the valleys have this relationship with speed.*

Interested in continuing to study the radar system, Group A looked for ways to collect data on the speed of vehicles that pass over optical sensors and found a simulation video using Arduino. Figure 2 shows a video frame in which one of the research sources displays a simulation using Arduino.

Figure 2

Speed simulation with optical sensor using Arduino



The use of Arduino for data collection was a reason for students to advance in the development of the activity. Understanding this system made the members of Group A integrate their knowledge with what they were investigating to make viable the search for a solution to the problem.

As the members of Group A explained, the Tracker[1] software had already been presented in a physics subject of the mechanical engineering

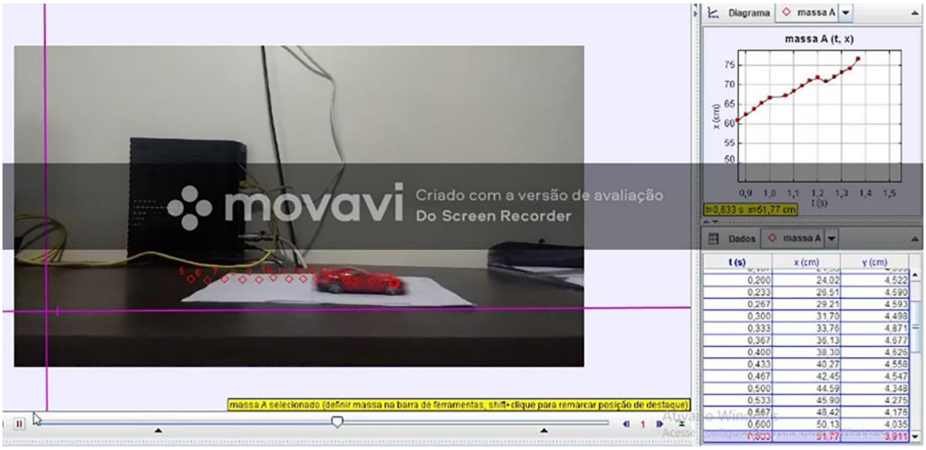
[1] Free video analysis software developed by Open Source Physics (OSP) and available at https://physlets.org/tracker/

course and the students perceived in the mathematical modelling activity a possibility to implement this knowledge to solve the problem. With this, we recognise “the importance of the ability to mobilise mathematical and extra-mathematical knowledge to solve problems” (Baioa & Carreira, 2019, p. 11) that emerge from the investigated problem situation, which allows connection with different areas.

Figure 3 represents an image of the group’s explanatory video showing a simulation that aimed to illustrate the effect of a vehicle passing through two sensors and the perception of disturbance in the tension curve graph, which was supposedly being simulated.

Figure 3

Student simulation for speed reduction when passing the radar and its effect on the graph



The mathematical models obtained by Group A consist of graphical and tabular representations (as in Figure 3) that describe the space covered by a vehicle as a function of time when passing through a radar. We can see that the models are not realistic, in the sense of describing the optical system (Figure 2) to obtain the speed of vehicles when passing through the radar, as they represent a fictitious reality in which students “used everyday artefacts, school materials, and knowledge of the situation (even not being experts in such knowledge) to reach results considered reasonable” (Carreira & Baio, 2018, p.

213). When answering the questionnaire, student AM1 stated that by developing the activity and studying the theme, "*I learned a lot about radars, even though I didn't know much about it*".

However, the development of the activity mobilised the students to reflect on how to integrate this knowledge into a speed recognition system performed by a machine and this, in a way, allowed: the understanding of the concepts of the electrical engineering area associated with those of physics when comparing the functioning of two systems; the development of simulations using free and accessible software, which made it possible to carry out an experiment with resources available at the AM2 student's home; establish relationships between the operation of the speed measurement system and mathematical concepts, which can be seen in the graphic analysis and the analogy of AM2 in the presentation of the results:

> [...] *I'm going to talk a little about the valleys, which are found on speed cameras... and in this case, this first experiment here, that I did using the Tracker software. I did the car first, as it would normally be going without any radar, not having speed deceleration, just in a straight line.* [the student shows with the mouse cursor an excerpt of the video that illustrates the experiment in Tracker] *I calculated here, here the results of time and distance came out. First, I measured the distance here from the table, as a guide for the distance problem that the car covers, right? Then, it calculates the time until it arrives at the end, which is where I marked... er... in the measurement. Here it shows the graph it makes. There was a slight deviation here because it is not in the ideal shooting conditions and angle, because I did not have a tripod and the lighting in my room is not very good, but I got this result here in this graph... you can see that it is continuous and there is no change in it. And here are the data that the program makes available to us. Here is the second test I did... I put this sheet here, I improvised and put this sheet with relief for the car to slow down, and for us to have a change here and create these valleys here that appear here in the graph.* [...] *You can see that it's been coming and it's behaving like in the other graph, but here it's already doing more... it already makes the valleys, not so big, because, you know, here it's reduced in size and in a worse condition, but it's possible to show you the files from the Mundo da Elétrica website, where you can see that the valleys are very different,*

> *that trucks make bigger valleys, motorcycles make smaller ones. Optical radars behave differently, but here we can see how they work.* [...]

The data seem to indicate that the integration between the knowledge of the STEM areas needs improvement since the simulation, illustrated in Figure 3, reproduced the new system that the group proposed to study only to a certain extent. They understood the simulation and mathematisation of the vehicle movement well but did not actually represent models with information from the optical system. In this regard, Hallström and Schönborn (2019) claim that modelling can establish significant links between STEM areas. However, it is crucial to distinguish between models for educational purposes and models as part of authentic professional practices.

Simulation, very present in engineering education and, according to the DCNs in the area, should be stimulated since the first terms of the course, was an alternative explored in approaching the problem by Group A in the mathematical modelling activity. Carreira and Baioa (2018, p. 203) state that simulation helps "reflect on what is happening in the real world, besides being an adaptation of reality under controlled conditions, a search for similarity with reality and a way to verify how the practice works in reality".

The mathematisation allowed the students of Group A to carry out prototyping, experimentation, and simulation in a system that involved the construction of a model based on mathematics, supported by computational software in which relationships between the real model, the physical model, and the mathematical model were strengthened and consolidated. Therefore, it is an activity that provided "multifaceted, and idea-generating" problems (English & Mousoulides, 2015, p. 534) so that the students liked the results obtained: "*It is an interesting activity to further stimulate the development of mathematisation and also leaves that area of sameness of tests and lists*" (AM3's answer to the questionnaire).

The simulation was also an alternative chosen by the K Group for the study of an integrated system of radars located in the city of São Paulo. Group K was based on information and an image of news on an integrated system of radars (Figure 4) that fines a vehicle if the average of its speeds registered by the radars of the stretch exceeds the speed allowed on the road.

Figure 4

Section of the highway that inspired the group

Figura 2. Radares instalados na cidade de São Paulo-SP. Fonte: Site G1.

Thus, based on the real situation, K Group sought to:

> *[...] examine, through the variation rate, whether the calculations performed by the software are really accurate and also, whether there is a way system could be circumvented by drivers, since the calculation is made from the average, i.e., not considering whether the driver performed any braking or acceleration during the journey* (K Group Report, 2020).

At first, Group K members seemed to be carrying out “mathematical simulations, generating representations of problematic real-world situations, engaging in a two-way process of translating between a real-world situation and mathematics” (English, 2017, p. 12). However, considering the pandemic context, which made it impossible for them to collect empirical data in a part of the city where the university is located, the approach was based on simulated data, as justified by the group’s report.

> *Due to the current moment, it was not possible to go to the field to collect real data from a road with fixed radars. Because of this impasse, we formulated two simulations, distributing points with a determined interval for each simulation along the*

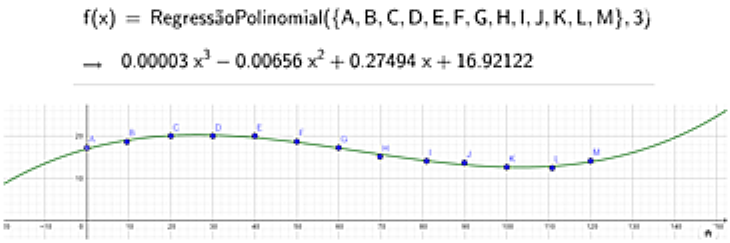
x-axis in the GeoGebra software, 10 seconds for one and 15 for the other (K Group Report, 2020).

From what they called experimental procedure, which used the information from Figure 4 and manipulations in the GeoGebra software[2], they explained how they obtained the model in Figure 5, where the speed of a vehicle would be recorded every 10 seconds on a 2 km route, obtaining the third-degree polynomial model: $f(x) = 0{,}00003x^3 - 0{,}00656x^2 + 0{,}27494x + 16{,}92122$

> *We perform the data distribution procedures, firstly in the example where we work with an interval of 10 seconds. [...] Creating scores (A to M) in the software, we could create a 3rd- degree polynomial regression, automatically obtained by the software* (K Group Report, 2020).

Figure 5

Mathematical model of a simulation



In the work systematisation, Group K was concerned with linking the mathematisation of the problem to the concepts studied in the subject, which was shown in the guidance meetings and the group's report. The concepts of average and instantaneous (derived) variation rates and the concept of integral appeared systematised and discussed in terms of the situation studied, supported by the academic literature of physics and calculus and websites with technical information about radar operation. In fact, we can show that the activity allowed students to "appreciate how their school learning in mathematics and science applies to the problems of the outside world" (English

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[2] Free, online, dynamic math software available at https://www.geogebra.org/

& Mousoulides, 2015, p. 532). According to KA2's answer to the questionnaire: "*we used the tools we studied during the content to apply to radar operation*."

In the stage of *interpretation and validation of results*, the group warns of the need to assess the feasibility of the results obtained. This is an important aspect in the engineer's education, who will have to provide real and viable solutions in their professional practice.

> *After interpreting the data, we realised that some drivers can indeed be fined for this proposed model, which may cause inconvenience to other drivers, since its speed is not constant. The model should help prevent the driver from rushing other cars and overtaking motorcycles, among other problems that could be avoided with this more precise analysis of the speed. Moreover, the cost for the implementation of this system must be considered. For example, in the first simulation, eight more radars would have to be added to have this precision and, 13 in the second simulation, so, it is necessary a more detailed analysis of the economic factors involved. But, as this is not the main objective of this work, and for the short time given, the question remains open, whether the fines would generate sufficient revenue for the implementation of more radars.*

The considerations above reveal that the students of Group K considered technical, scientific, economic, and social dimensions regarding the implementation of this integrated radar system. These subcompetencies are necessary for engineering education, as stated in the DCNs (Brasil, 2019).

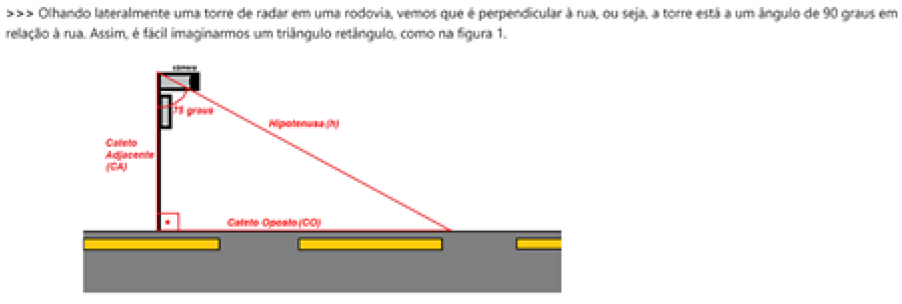
Group M started based on technical information about the system configured for the radar equipment so that infringing vehicle license plates could be identified:

> *The new CONTRAN resolution of 09/02/2020 determines that all radar equipment must have OCR (optical character recognition) software to recognise vehicle license plates and consult them in databases, which enables the identification of stolen cars, for example.* (Group M Report, 2020).

Before starting the mathematisation, Group M made some simplifications regarding the physical structure of a radar, approaching it to a right triangle, as shown in Figure 6.

Figure 6

Physical structure of a radar



This approach required students to use mathematical procedures generally studied in basic education and necessary for students to develop the activity "in the most meaningful and reasonable way for them" (Baioa & Carreira, 2019, p. 11). Although the mathematical content seemed simple for the level of education, students MA1 and MA3 stated, respectively, in response to the questionnaire: "*I had to understand the laws of sines. I'm not very good at trigonometry, but with the help of the other members, it was easy* ", " *I needed a lot of help, including basic math* ".

Considering some information that the group found in previous research, she established hypotheses to guide the mathematisation:

> *Suppose that: the software operation requires that the radar camera position reaches a shooting range of at least 18 meters, with a 75 degree inclination in relation to the pole; and that the standard height of the pole should be between 2.80m and 6m. Thus, a model for the height of the pole as a function of the camera's reach is:"*[presenting referrals - Figure 7] (Group M Report, 2020).

The formulation of the hypotheses by Group M reflects the students' way of seeing the problem, is "anchored in their experiences and provides elements for subsequent actions"(Almeida et al., 2021, p.89), culminating in the mathematical model.

Figure 7

Obtaining the mathematical model

PARTE 4: Relações Trigonométricas

>>> Agora que todos os ângulos internos são conhecidos (90°, 75° e 15°), podemos usá-los para determinar os lados usando fórmulas de Relações Trigonométricas:

$h = \frac{a}{cos15°}$ em que h=hipotenusa, a=cateto oposto (alcance da câmera)

$H = h.cos75°$ em que H=cateto adjacente (altura do poste), h=hipotenusa

>>> Desta forma, é necessário descobrir primeiro a hipotenusa **h** para depois achar a altura **H**, da seguinte forma:

$$h = \frac{a}{cos15°} = \frac{18}{cos15°} = 18{,}63$$

$$H = h.cos75° = 18{,}63.cos75° = 4{,}82\ metros$$

PARTE 5: O modelo matemático

>>> A partir das Relações Trigonométricas demonstradas, podemos usá-las como modelo matemático para a altura do poste em função do alcance da câmera:

$$H(a) = \frac{a}{cos15°}\ .cos75°$$

PARTE 6: A derivada

>>> O cálculo da derivada da função se dá da seguinte forma:

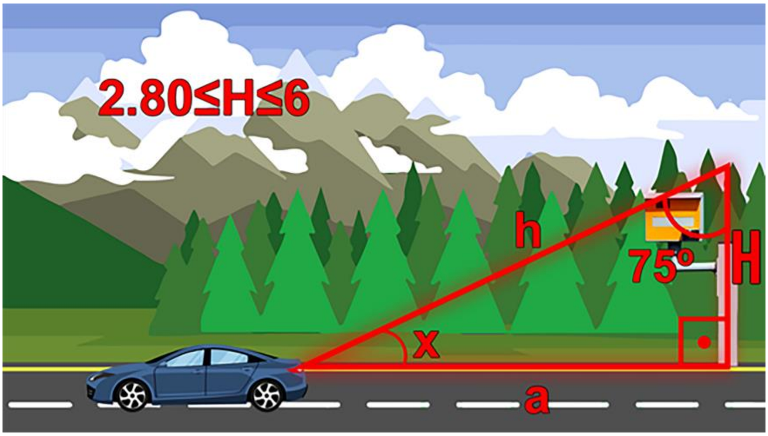
In this case, obtaining the mathematical model was supported by basic mathematics concepts addressed at the beginning of the course and was not supported by the use of digital technologies, although, to illustrate the situation, in communicating the results, the group created an animation that highlights the height of the pole, the tilt angle of the camera and the distance from the car to the pole (Figure 8).

Even if the approach is simplified and unrealistic, in the sense that the mathematical model $H(a) = \frac{a}{cos(75°)} cos(15°)$ describes the variation in the height of the pole in relation to the reach of the camera, the problem was answered and allowed the group to understand how the radar operates and to mathematise part of the system. This can be understood as an initial approach that has the potential to be expanded, for example, based on the proposition of simulations with a dynamic geometry software.

Group M created a digital animation with background and audio effects, the results of which may not have been relevant for modelling, that provided opportunities for learning that contribute to professional education. This aligns with a discussion of STEM or STEAM education, where the "A" stands for Arts in a broad sense. According to Pugliese (2020), its integration into STEM can cover aspects of sociology, history, and visual arts, among others, and can have a sensitising, educating, creative, and critical function. According to Jhonson (2013, p. 367), this can expand the meaningful context of the real world of STEM and value the integrated nature of learning, “while supporting the work of building student access to interdisciplinary themes and skills of the 21st century”.

Figure 8

Animation in the video presentation



The mathematical modelling activity implemented enabled “a problem structure in which mathematics and engineering form the primary content areas, with supporting science content” (Lesh, 2017, p. 5). For the MA2 student of Group M:

> *With the topics used by the professors, she made a guide on which I will base myself when I have a problem simulating the work environment, in which people I don’t know discuss the related problem and argue about the possible answers to the problem. With this work, I got a better view of the steps for the formulation of a problem situation.*

Table 3 brings together aspects evidenced based on data analysis throughout the groups’ development of the mathematical modelling activity.

Table 3

Aspects highlighted in each area of STEM

	Group A	Group K	Group M
S	Study of a system that replaces electromagnetic sensors with optical ones; valleys in the voltage curve; mention of physics concepts about kinematics; search for knowledge in scientific articles and websites.	Technical information on the operation of different types of systems; use of academic references in physics.	It was not evident.
T	Mention to Arduino; the use of the Tracker as an alternative for simulating part of the radar system.	Use of GeoGebra to obtain mathematical models.	Use of GeoGebra to graphically represent the model obtained and discuss its validity; realisation of animations illustrating part of the fixed radar system.
E	Attempt to simulate the new system; use of available material resources that provide reasonable results, recognising that they are not ideal.	Analyse a new traffic monitoring system to improve the efficiency of the conventional system; draft a proposal and recognise the need to analyse its economic viability.	Search for a model that meets a recently approved CONTRAN resolution (09/2020). Recognition of a framework applicable to professional career situations.
M	Graphical comparison from the models obtained by	They sought to systematise mathematical	Use of metric and trigonometric relationships in the

the Tracker and analysis seeking comparison with the disturbance in the voltage curve of the optical system.	knowledge about average and instantaneous rates of change, as well as associate the concept of integral; obtained models by polynomial regression to simulate the speed as a function of time in a fixed section.	triangle; obtain a composite function to describe the relationship between pole height and camera reach; interpret the model's rate of change through the derivative.

The professors guided the groups of students' development of the mathematical modelling activity under the theme of fixed radars. However, the guidelines were mediated by questioning so that the groups were free to accept them or not. The professors did not impose. Thus, the aspects summarised in Table 2 indicate that some groups naturally integrate the four STEM areas, while in others, this integration occurs partially. This is in line with Hallström and Schönborn (2019), who consider that, through modelling in STEM education, students may feel the need to move between different areas of knowledge when engaging in scientific, mathematical, and technological activities, establishing a synergy relationship between the disciplines.

CONCLUDING REMARKS

The proposed problem can potentially study several aspects that can link the STEM areas, providing the integration between the curricular units of basic sciences and mathematics. With this, we aim to bring to the debate results of our research that elucidated a pedagogical practice based "on models for STEM education classrooms [...] in real educational environments" (Hallström & Schönborn, 2019, p. 9). In this way, we understand that we have contributed, to a certain extent, to a STEM literacy integrated into a Calculus 1 subject, which, although intended for repeaters, reflects a formal educational environment during remote teaching.

We understand that mathematical modelling can provide the teacher with opportunities to encourage students to establish connections between

STEM areas to improve or build their knowledge from a problem situation, as in this research. In this sense, Baioa and Carreira (2019, p. 10) suggest mathematical modelling as "a strong possibility to consider whether we want to promote an integrated education and a less fragmented teaching, more focused on the student and on the ability to solve problems".

From a partnership between educators and researchers in the area, we highlight aspects of STEM education, when engineering students develop mathematical modelling activities in a virtual environment within the scope of a differential and integral calculus course. Even though we have proposed a ready-formulated theme by asking a question for students to research, the groups had the initiative and autonomy to encompass problems to which they intended to present a solution.

In this sense, the need for understandings related to science, technology, engineering, and mathematics permeated the routing of the activities, either by seeking information about the situation through a system that simulated what was intended to be investigated, using computational software that could allow adjusting curves and then enabling an albeit simplified interpretation of reality (Biembengut, 2016) or by drawing attention to a situation through animation. We consider that there was a practical, shared, cooperative work, of questioning, and of elaboration of conjectures and justifications (Baioa & Carreira, 2019).

Modelling elicitation activities based on a problem situation stated by the teachers of two Calculus 1 classes helped show how students worked collaboratively to solve problems in a way that allowed them to elucidate how they "apply their mathematical knowledge, explore possible strategies, evaluate their thinking, compare solutions, and produce a prototype" (Baker & Galanti, 2017, p. 4). In the search for information about the problem situation, students integrate knowledge from different areas - technical, scientific, economic, social, environmental and ethical - so that they perceive the existence of mathematics as an ally to present a solution to the problem, as the DCNs suggest (Brasil, 2019).

Each group discussed here chose to investigate partial aspects of the fixed radar system, which did not allow for an understanding of the system as a whole. Thus, the communication of the results of each group and the guarantee of space for discussions, which was not made available in the first cycle of the design, would certainly contribute significantly to the understanding of the problem and to STEM education. We did not share the *videos with a summary of the development of the activity* with the groups.

Simulations like the ones undertaken by Groups A and K led to the exploration of different technologies, whether digital or not. With this, new learning opportunities were presented. The students did not know the functionalities to which they resorted when using the Tracker and GeoGebra and had to learn them to perform the mathematical modelling activity. Although Group M did not use any software, they developed skills with digital technologies to produce, edit, and bring forward the video presentation.

However, even though the analysed groups made efforts to conduct and complete the activity, one did not complete it. Others mentioned that the time –approximately 30 days– was not enough for a more detailed approach. Because of this remark, in a second design, we consider extending the activity development time to a school term and discussing the pertinence of the theme investigated by the groups.

AUTHORSHIP CONTRIBUTIONS STATEMENTS

K. A. P. S. e A. H. B. were the professors of the differential and integral calculus classes of a real variable in which the mathematical modelling activities were developed. In partnership, the three authors (K. A. P. S., A. H. B. e E. C. F.) appropriated the theoretical foundations of the article and carried out data collection and analysis. The text was also written jointly and collaboratively and a preliminary, more concise version was presented at a scientific event of mathematics education in 2021.

DATA AVAILABILITY STATEMENT

The data that support the results of this study will be made available by the corresponding author, A. H. B., upon duly justified request.

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