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On Sw*- Regular Spaces

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Sw*- حول الفضاءات المنتظمة من النمط

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ABSTRACT

The purpose of this paper is to present and investigate a new class of topological spaces known as S_w*-regular spaces, by utilizing the concept of Sw-regular sets and some of its properties. which is introduced in 2009 by L. S. Abddullah and A. B. Khalaf [1], the new class is properly contained in S^*- regular space [2], [3], means that S_w*- regular spaces is a stronger form to the space S^*- regular. Several characterizations, properties and relationships of S_w*- regular space with other spaces such as, Sw-compact, extremally disconnected, regular, semi-regular, Sw-T_2 and Urysohn spaces has been studied. Furthermore, several properties of S_w*- regular spaces with some functions such as, continuous, strongly continuous, open, clopen and Somewhat open functions are also explored. In addition we investigate that $S_w{}^*$- regular space has a topological property, while it has not a hereditary property, only by adding certain conditions such as, a subspace is open or, if the subspace of an $S_w{}^*$-regular submaximal space is preopen, then it becomes an S_w*- regular.

Key words:

Sw-regular set, Sw-open set, S^*-regular space, Semi-regular space and Somewhat open function.

الخلاصة

الهدف من هذا البحث هو تقديم ودراسة فئة جديدة من الفضاءات التبولوجية والتي اسميناها بالفضاءات المنتظمة من النمط S_w*- باستخدام مفهوم المجموعة المنتظمة - Sw وبعض خصائصها والتي قدمت من قبل L. S. Abddullah, A. B. Khalaf [1] في عام 2009 ، حيث ان هذا الفضاء هو فضاء جزئي من الفضاء المنتظم -S* [2], [3]. اي ان الفضاء المنتظم - S_w* تكون اقوى من الفضاء المنتظم -S*. تمت دراسة العديد من خصائص هذا الفضاء و علاقة الفضاء المنتظم - S_w* مع الفضاءات الاخرى كالفضاءات المتراصة - Sw ،غير متصل للغاية ، المنتظمة ، شبه المنتظمة ، Sw-T_2 و يوريسون. علاوة على ذلك تم دراسة العديد من الصفات للفضاء المنتظم -S_w* مع بعض الدوال كالدوال المستمرة ، المستمرة بقوة ، الدوال المفتوحة، الدوال المفتوحة المغلقة والدوال المفتوحة إلى حد ما. بالإضافة إلى ذلك ، قمنا بالتحقق من ان الفضاء المنتظم - S_w* لها خاصية تبولوجية ، في حين أنها لا تمتلك الخاصية الوراثية ، الا باضافة شروط معينة كأن يكون الفضاء الجزئي مفتوح أو إذا كان الفضاء الجزئي من الفضاء المنتظم -* S_w دون الحد الأقصى مفتوح قبلا و عندها يصبح الفضاء الجزئي فضاء منتظم من النمط -S_w* .

الكلمات المفتاحية:المجموعة المنتظمة – Sw، المجموعة المفتوحة – Sw ، الفضاء المنتظم –S^* ، الفضاء شبه المنتظم والدالة المفتوحة الى حد ما..

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INTRODUCTION

Topological space, unless otherwise stated, no separation axioms are assumed. Assume that H ⊆ X, "the closure and interior of H, denoted by Cl(H) and Int(H)", while, "S_w-Cl(H) and S_w-Int(H) denotes the Sw-closure and Sw-interior of H respectively". In 2009 L. S. Abddullah and A. B. Khalaf defined Sw- regular space by using the notion of Sw-open sets, while in this paper S_w^*-regular space is defined by using the concept of Sw-regular set.

A subset H of a space (X, τ) is called "semi-open [4] (resp., nearly open[5], or preopen[6], and α-open [7]) set if H⊆Cl(Int(H)) (resp., H⊆ Int(Cl((H)) and H=Int(Cl(Int(H))). The complement of a semi-open (resp., preopen and α-open) set is called semi-closed [8] (resp., preclosed [5] and α-closed [8])". We established several characterizations and some properties of S_w^*-regular space. Also we investi2. Preliminaries

Recall some basic definitions and results which will be used in the next section.

Definition 2.1[9]: A subset H of a space X is said to be semi-regular if it is both semi-open and semi-closed.

Definition 2.2 [1]: Let (X, τ) be a topological space, and let H ⊆X, then H together with the empty set is called an Sw-open set if Int(H) ≠ φ . An Sw-closed set is the complement of a Sw-open set.

Definition 2.3 [1]: If a subset H of a space X is both an Sw-open and an Sw-closed set, then it is called Sw-regular.

Theorem 2.4 [1]: Let (X, τ) be any topological space; then the family of all Sw-open sets in (X, τ) is identical to the family f all Sw-open sets in (X, τα). That is, SwO(X, τ) SwO(X, τα).

Proposition 2.5 [1]: If Y is a subspace of a space X, and if H is a subset in Y and H is an Sw-open set in X, then H is Sw-open in Y. If Y is open in X, the converse is also true.

Lemma 2.6 [1]: If H Y X, then H is a Sw-closed set in X if H is a proper Sw-closed set of a subspace Y.

Lemma 2.7 [1]: Every super set of an Sw-open set is Sw-open.

Definition 2.8 [1]: If there are two disjoint Sw-open sets U and V of X such that (briefly s.t.) x∈U and y∈V, a space (X, τ) is called a Sw-T2 space.

Theorem 2.9 [1]: A topological space X is Sw-T2, iff there is an Sw-regular set U containing one of the points but not the other for each pair of distinct points x, y in X.

Definition 2.10 [1]: The space X is Sw-compact if every Sw-open cover of X has a finite subcover.

Definition 2.11: A space X is called S*-regular [3] (resp., semi-regular [10]) if for each a in X and any semi-regular (resp., semi-closed) set A in X such that a∉ A, there exist disjoint open (resp., semi-open) sets L and K in X such that a∈K and A⊂ K.

Theorem 2.12 [11]: A space X, is semi-regular if and only if there exists a semi-open set B such that x∈B⊂sCl(B)⊂A for each point x∈X and each semi-open E containing x.

Definition 2.13 [12]: If there exists an open set F such that x∈F⊂Cl(F)⊂E for each x∈X and each open set E containing x, then the space X is called regular.

Definition 2.14 [13]: If the closure of each open set in X is open, or if the interior of each closed set in X is closed, a space X is said to be an extremally disconnected.

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Definition 2.15 [14]: "A function f from a space X into a space Y is said to be somewhat open function provided that for E ∈ τ and E ≠ φ , there exists a set F which is open in Y such that F ≠ φ and F ⊂ f (E)".

Theorem 2.16 [15]: A function f : X →Y is somewhat open if and only if for each G⊂X and Int(G)≠φ, then Int(f (G)) ≠ φ.

Theorem 2.17[16]: A function f from a space X into a space Y is closed iff ; ∀ subset F of X , Cl(f(F))⊂f(Cl(F)).

Definition 2.18 [17]: Let f be a function from a space X into a space Y, if f -1(U) is clopen in X for each subset U in Y, f is said to be strongly continuous.

Theorem 2.19 [1]: f: (X, τ) □ (Y, σ) is Sw-irresolute, if it is a surjective continuous function.

Theorem 2.20 [1]: The following statements are equivalent for a function f : X Y,

f is Sw-irresolute.

There is an inverse Sw-open set in X; for every Sw-open set in Y.

There is an inverse Sw-closed set in X for every Sw-closed set in Y.

Sw*- Regular Spaces

Definition 3.1: A topological space (X, τ) is called Sw*- regular, if for every Sw-regular set E and each a ∉ E in X, there exist open sets L and K in X that are disjoint; such that; a∈L and F⊂ K.

It is important to note that all discrete and indiscrete spaces are Sw*- regular spaces. It is also clear from the preceding definition that the class of Sw*- regular spaces is contained in the class of S*-regular spaces., mean that every Sw*- regular is S*- regular , however, as shown in the following example. In general, the reverse is not true.

Ex. 3.2: Let X = {a, b, c, d} with a topology τx ={φ, X, {b}, {c}, {d}, {a, d}, {b, c}, {c, d}, {b, d}, {b, c, d}, {a, c, d}, {a, b, d}} on X. Then (X, τx) is an S*- regular space but not Sw*- regular.

The following theorem is a characterization of an Sw*- regular spaces:

Theorem 3.3: The following propositions are equivalent for a topological

space (X, τ):

1. (X, τ) is an Sw*- regular space.

2. For each a ∈ X and each Sw-regular set V containing a, ∃ an open set U, s.t.

a∈U⊂ Cl(U)⊂V.

3. The intersection of all closed neighborhoods of a Sw-regular set E is E itself.

4. For each non-empty set E; and each Sw-regular set F of X; s.t. E∩F≠φ, ∃ an open subset U of X s.t. E∩U ≠ φ and Cl(U) ⊂F.

5. For any non-empty subset E and Sw –regular set F of X, s.t. E∩F =φ, there are open sets C and D of X; that are disjoint s.t. E∩C≠φ and F⊂D.

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Proof: (1) ⇒(2)

Let V be an Sw-regular set in X containing a, therefore a ∉ X\V and X\V

is also an Sw-regular set, then by Sw*- regularity of X, ∃ two disjoint open sets U1 and U2 of X s.t. a∈U1 and X\V⊆U2 . Since U1 and U2 are disjoint, then U1⊆X\U2 and since X/ U2 is closed, then Cl(U1)⊆ X\U2⊆V. Thus a∈ U1⊆ Cl(U1)⊆V. This gives (2).

(3) ⇒ (2)

Let E be Sw-regular set in X, then X\E be also Sw-regular. By the hypothesis, for all a∈X\E, there is a set which is open Ua in X such that a∈ Ua⊂ Cl(Ua)⊂X\E. Then $\bigcup_{a\in(X\setminus E)} \{a\}\subset \bigcup_{a\in(X\setminus E)} U_a\subset X\setminus E$, and so $X\setminus E\subset\bigcup_{a\in(X\setminus E)} U_a\subset X\setminus E$. That is,

$X\setminus E=\bigcup_{a\in(X\setminus E)} U_a$. Therefore; $E=\bigcap_{a\in E} (X\setminus U_a)$), where $X\setminus U_a$ is a closed neighborhood of E. This completes the proof.

(4) ⇒ (3)

Let E and F be two non-empty disjoint subsets of X; such that F is an Sw-regular set. Then, there exists a∈E∩F. Thus a∉X\F and X\F is an Sw-regular set, so by the hypothesis there exists a closed neighborhood H such that a∉H and X\F⊂ H, then X\F⊂G⊂H, G is open. Let X\H=U, then a∈U where U is open. Hense E∩U≠φ, and since G is open, then X\G is closed. This implies that X\H⊂X\G⊂F, and so U⊂Cl(U)⊂Cl(X\G)= X\G⊂F. That is U⊂Cl(U)⊂F, thus Cl(U)⊂F.This proves (4).

(5) ⇒ (4)

Let E be non-empty subset of X; and F be an Sw-regular set of X; s.t. E∩F = φ, therefore; X\F is also an Sw-regular set in X and E∩X\F ≠ φ. Using (4) ∃ an open subset C of X; s.t. E∩C ≠φ and Cl(C)⊂X\F and then F⊂X\Cl(C). Put D = X\Cl(C) ⊂X\C. Thus D is an open set, s.t. F⊂D. As a result C, D are open sets with E∩C ≠φ, F⊂D and D∩C =φ.

(1) ⇒ (5)

Let a∉ E, where E is Sw-regular set of X and let K={a} ≠ φ. Then by using (5) there exist two open sets C and D of X, such that K∩C ≠φ, C∩D = φ and E⊂D. Therefore a∈C, E⊂D and C∩D = φ. That is, X is an Sw*- regular space.

Theorem 3.4: Disjoint open sets Sw*- regular space X can separate each disjoint pair consisting of a compact set E and an Sw-regular set F.

Proof: Since X is an Sw*- regular space with E∩F=φ in X, then for every x∈ E, x ∉ F, where F is an Sw-regular set. Therefore; ∃ disjoint open sets Lx and Kx of X; s.t. x∈Lx and F⊆Kx. Obviously, the compact set E is covered by {Lx: x∈E}. Thus ∃ a finite subfamily {Lxi: i= 1, 2,…, m} which covers E. As a result, E⊂⋃{Lxi: i= 1, 2,…, m} and F⊂⋂{Kxi: i = 1, 2,…, m}. Put L= ⋃{Lxi: i= 1, 2,…, m} and K=⋂{Kxi: i= 1, 2,…, m}. Since L∩K=φ, then L⊂X\K and so Cl(L)⊂Cl(X\K)=X\K. That is Cl(L)⋂K=φ, by the same way L⋂Cl(K)=φ, so L and K are separated. Then the needed disjoint open sets are L and K.

Corollary 3.5: Let (X, τ) be an Sw*- regular space. If E is a compact subset of X, and F is a Sw-regular set that contains E, then ∃ an Sw-regular set K, s.t. E⊂K⊂Sw-Cl(K) ⊂F.

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Proof: Since F is an Sw-regular set, so X\F is also Sw-regular; and E⋂X\F = φ in X, where E is compact, then by Theorem 3.4; ∃ disjoint open sets L1 and L2 of X; s.t. E⊂L1 and X\F ⊆ L2. But L1⋂L2=φ, so L1⊂X\L2 and since L1 is open, so it Sw-open and then by Lemma 2.7 X\L2 is also an Sw-open set, furthermore, X\L2 is a closed set and consequently it is an Sw-closed set, and so X\L2=Sw-Cl(X\L2). That X\L2 is Sw-regular set. Put K=X\L2, then E⊂L1⊂ X\L2⊂F. Means that E⊂K⊂Sw-Cl(K)=Sw-Cl(X\L2)=X\L2⊂F. Thus E⊂K⊂Sw-Cl(K)⊂F. This is the end of the proof.

Corollary 3.6: Let (X, τ) be an Sw*- regular space and let E , F be two disjoint subsets of X, with E being compact and F being a Sw-regular set. Then there exists Sw-regular sets L and K s.t. E⊂ L, F ⊂ K and L⋂K=φ.

Proof: By Theorem 3.4; ∃ disjoint open sets L and K of X s.t. E⊂L and F⊆K. But L⋂K=φ, so L⊂X\K and since L is open, so it is Sw-open. Furthermore; X\K is closed, then it is Sw-closed. That is, Cl(X\K)≠X and since L⊂X\K, then Cl(L)≠X. That is, L also is an Sw-closed set. Thus L is an Sw-regular set. By the same way K is also Sw-regular. Thus L and K are the required Sw-regular sets.

In the following theorem we show that Sw*- regular space has not a hereditary property.

Theorem 3.7: If a space X is an Sw*- regular and E be an open subspace of X, then E is Sw Sw*- regular space.

Proof : Suppose that E is an open subspace of the Sw*- regular space X. To demonstrate E's Sw*- regularity, suppose that G is an Sw-regular set in E and let a∉ G; s.t. a∈E. Since G⊂E, G is Sw-open in E and E is open in X, then by Proposition 2.5, G is an Sw-open set in X, also since G⊂E⊂X and G is Sw-closed, then from Lemma 2.6 G is Sw-closed set in X, such that a∉ G, so by Sw*- regularity of X, ∃ two disjoint open sets La and Ka of X s.t. a∈ La and G⊂Ka. Let L=La⋂E and K=Ka⋂E, clearly a∈ L and G⊂ K; where L and K are open sets that are disjoint in E. Therefore E is an Sw*- regular space.

The following example shows that the condition of openness on A in Theorem 3.7 is necessarily.

Ex. 3.8 : Let X = {a, b, c, d}; with τ = {φ, X, {a}, {a, b}, {a, c}, {a, b, c}}. So the space (X, τ) is Sw*-regular, while A = {b, c, d} is not Sw*- regular though A is closed. It follows that Sw*- regularity is not a hereditary property.

Recall that a topological space (X, τ) is said to be submaximal [5], if every preopen is open.

Corollary 3.9: Let X be an Sw*- regular submaximal space, then every preopen subspace of X is Sw*-regular.

Proof : Suppose that A is a preopen subspace of X, since X is Submaximal, then A is open and by Theorem 3.7 A is Sw*- regular.

Theorem 3.10: Every Hausdorff Sw-compact space is Sw*- regular.

Proof: Let E be any Sw-regular set containing a point say b in X, so X\E is also Sw-regular set s.t. b∉ X\E. But X is a T2 space, therefore; for every a∈ X\E, ∃ open sets La and Ka s.t. x∈ La , b∈ Ka and La⋂Ka=φ. Obviously {La: a∈ X\E} is a cover of X\E by Sw-open sets of X and since E is Sw-regular, then N={La: a∈ X\E}⋃E is an Sw-open cover of X, but X is Sw-compact space, then ∃ a finite subfamily of N covers X. That is, X= ⋃{Lai: i=1, 2,…,m}⋃E. Therefore X\E⊂⋃{Lai: i=1, 2,…,m}. Let L=⋃{Lai: i=1, 2,…,m} and K=⋂{Kai: i=1, 2,…,m}. Then b∈K and X\E⊂L, such that L and K are open sets in X. As a result, the space X is Sw*- regular.

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Theorem 3.11: A topological space (X, τ) is regular if it is semi-regular and Sw*- regular.

Proof: Let L be any open set of X and a∈L. But X is semi-regular, so by

Theorem 2.12; ∃ a semi-open set M in X s.t. a∈M⊂sCl(M)⊂L. But Sw-Cl(M) ⊂sCl(M) ⊂Cl(M) for any M⊂ X. So a∈ M ⊂ Sw-Cl(M) ⊂ sCl(M) ⊂ L, and since M is semi-open, then it is Sw-open. That is, Sw-Cl(M) is Sw-regular set and since X is Sw*- regular space, thus by Theorem 3.3, ∃ an open set E s.t. a∈E⊂Cl(E)⊂Sw-Cl(M)⊂L. Thus, a∈E⊂Cl(E)⊂L. As a result, X is a regular space.

Proposition 3.12: If a topological space (X, τ) is Sw*- regular, then it is extremally disconnected.

Proof: Let K be any non-empty open subset of X, so Cl(K) is an Sw-regular set and since X is an Sw*- regular space, then by Theorem 3.3(2) for each a∈Cl(K), there exists an open set La such that a∈La⊂Cl(La)⊂Cl(K). Thus Cl(K)=⋃{La: a∈Cl(K)} which it is open. Therefore X is extremally disconnected.

The following theorem give another characterization of an Sw*- regular space.

Theorem 3.13: Let (X, τ) be any topological space. Then X is Sw*- regular iff for all Sw-regular set M and a point p∈X such that p∉M, there exist open sets R and S of X such that p∈R, M⊂S and Cl(R)⋂Cl(S)=φ.

Proof : Suppose that (X, τ) is an Sw*- regular space; and p∉M, s.t. M is an Sw-regular set in X, so ∃ two disjoint open sets Ro and S such that p∈Ro and M⊂S, further Ro⋂Cl(S)=φ, if not suppose that Ro⋂Cl(S)≠ φ, then there exists a∈ Ro⋂Cl(S), so a∈ Ro and a∈Cl(S), then for all open set K of X and a∈K, K⋂S≠ φ and since Ro is an open set which containing a, then Ro⋂S≠ φ which is contradiction, thus Ro⋂Cl(S)= φ and Cl(S) is an Sw-regular set and since p∈Ro , then p∉Cl(S), again by Sw*- regular of X, there exist open sets A and B of X such that p∈A, Cl(S)⊂B and A⋂B=φ, hence Cl(A)⋂B=φ. Put R=Ro⋂A, then R is open s.t. p∈R, M⊂ S and Cl(R)⋂Cl(S) = Cl(Ro⋂A) ⋂ Cl(S) ⊂ Cl(Ro)⋂ Cl(A)⋂ Cl(S) ⊂ Cl(Ro)⋂Cl(A)⋂B = φ. Thus Cl(R)⋂Cl(S)=φ.

Conversely; suppose that p∉M, with M is an Sw-regular set in X, so ∃ two open sets R and S such that p∈R, M⊂S and Cl(R)⋂Cl(S)=φ, means that R⋂S=φ. Therefore X is an Sw*- regular space.

Recalling that a topological space (X, τ) is called a Urysohn[18] if there are neighbours K of a and L of b with Cl(K)⋂Cl(L)=φ, whenever a ≠ b in X.

Proposition 3.14: A topological space (X, τ) is Urysohn, if it is Sw*- regular and Sw-T2 space.

Proof: Assume that a, b are any two different points in X, but X is Sw-T2 space, then by Theorem 2.9 there exists Sw-regular set A of X contains a but not b, or contains b but not a , say a∈A and b∉A, so by Theorem 3.13, there exists two open set K and L of X s.t. b∈K, A⊂L and Cl(K)⋂Cl(L)=φ. That is a∈A⊂L, means a∈L, b∈K and Cl(K)⋂Cl(L)=φ. Therefore X is a Urysohn Space.

Proposition 3.15: If (X,τ) is an Sw*- regular space, then so is (X, τα).

Proof: This follows from Theorem 2.4 and the fact that every open set is α-open.

Theorem 3.16: Let Y be an Sw*- regular space, if f : X→Y is a bijective, continuous, closed and Sw-open function, then X is also an Sw*- regular space.

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Proof : Assume that F is an Sw-regular set in X, a∈ X with a∉ F, and so there exists b∈ Y s.t. f(a)= b. But f is an Sw-open function, so f(F) is Sw-open in Y, that is Int(f(F))≠φ. Also F is Sw-closed, then Cl(F)≠X, but f is closed, then by Theorem 2.17

Cl(f(F))⊂f(Cl(F)). Now Cl(F)≠X, this implies that f(Cl(F))≠f(X)=Y and so Cl(f(F))⊂f(Cl(F))≠Y. That is Cl(f(F))≠Y, so f(F) is Sw-closed in Y. So f(F) is Sw-regular set in Y with b∉ f(F), then by Sw*- regularity of Y; ∃ two disjoint open sets L and M of Y s.t. b∈ L and f(F)⊂M. Then by the continuity of f [19] f -1(L) and f -1(M) are open in X, such that a = f -1(b)∈f -1(L), F⊂f -1(M) and f -1(L)⋂f -1(M)=φ, consequently; X is an Sw*- regular space.

Recall that a function f : (X, τ) → (Y, σ) is said to be clopen [2] if it is both open and closed.

Corollary 3.17: If f is a bijective continuous and clopen function from a space X into an Sw*- regular space Y, then X is also Sw*- regular.

Proof : This is a direct result of Theorem 3.16 and the fact that each open function is Sw-open

Theorem 3.18: If X Sw*- regular and f is a strongly continuous and open function from a space X onto a space Y. Then Y is also Sw*- regular.

Proof: Let K be an Sw-regular set of Y, and b∈Y s.t. b∉K, but f is a surjective function, so ∃ a∈X s.t. f (a) = b. In addition to f is strongly continuous function and K is a subset of Y, so f -1(K) is clopen set in X, that is f -1(K) is Sw-regular set in X, where a∉ f -1(K), then by Sw*- regularity of X, ∃ two disjoint open sets A and B in X whereas a∈A and f -1(K)⊂B, but f is open, so f(A) and f(B) are open sets in Y, s.t. b∈ f(A), F⊂ f(B) and f(A)⋂f(B)=φ. As a result, Y is an Sw*- regular space.

Theorem 3.19: If X is an Sw*- regular space and f is a bijective Sw-irresolute and open function from a space X into space Y, then Y is also Sw*- regular space

Proof : Suppose that F is an Sw-regular set in Y and b∈Y s.t. b∉F, then ∃ a∈X s.t. f(a) = b, but f is Sw-irresolute, then f -1(F) is an Sw-regular set in X, where

a∉ f -1 (F), then by Sw*- regularity of X, ∃ two disjoint open sets L and K of X s.t. a∈L and f -1(F)⊂K, this implies that f(a)= b∈ f(L), F= f(f -1(F))⊂f(K), where f(L) and f(K) are open sets in Y with f(L)⋂f(K)=φ. As a result, Y is an Sw*- regular space.

Corollary 3.20: Let X be an Sw*- regular space. If f: X→Y is an open, bijective and continuous function, then Y is also Sw*-regular.

Proof: Follows directly from Theorem 3.19 and Theorem 2.19.

Corollary 3.21: Sw*- regularity is a topological property.

Proof: Follows directly from the concepts of a homeomorphism [12 Theorem 2.4] and Corollary 3.20.2.
Preliminaries

Recall some basic definitions and results which will be used in the next section.

Definition 2.1[9]: A subset H of a space X is said to be semi-regular if it is both semi-open and semi-closed.

Definition 2.2 [1]: Let (X, τ) be a topological space, and let H ⊆X, then H together with the empty set is called an Sw-open set if Int(H) ≠ φ . An Sw-closed set is the complement of a Sw-open set.

Definition 2.3 [1]: If a subset H of a space X is both an Sw-open and an Sw-closed set, then it is called Sw-regular.

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Theorem 2.4 [1]: Let (X, τ) be any topological space; then the family of all Sw-open sets in (X, τ) is identical to the family f all Sw-open sets in (X, τα). That is, SwO(X, τ) SwO(X, τα).

Proposition 2.5 [1]: If Y is a subspace of a space X, and if H is a subset in Y and H is an Sw-open set in X, then H is Sw-open in Y. If Y is open in X, the converse is also true.

Lemma 2.6 [1]: If H Y X, then H is a Sw-closed set in X if H is a proper Sw-closed set of a subspace Y.

Lemma 2.7 [1]: Every super set of an Sw-open set is Sw-open.

Definition 2.8 [1]: If there are two disjoint Sw-open sets U and V of X such that (briefly s.t.) x∈U and y∈V, a space (X, τ) is called a Sw-T2 space.

Theorem 2.9 [1]: A topological space X is Sw-T2, iff there is an Sw-regular set U containing one of the points but not the other for each pair of distinct points x, y in X.

Definition 2.10 [1]: The space X is Sw-compact if every Sw-open cover of X has a finite subcover.

Definition 2.11: A space X is called S*-regular [3] (resp., semi-regular [10]) if for each a in X and any semi-regular (resp., semi-closed) set A in X such that a∉ A, there exist disjoint open (resp., semi-open) sets L and K in X such that a∈K and A⊂ K.

Theorem 2.12 [11]: A space X, is semi-regular if and only if there exists a semi-open set B such that x∈B⊂sCl(B)⊂A for each point x∈X and each semi-open E containing x.

Definition 2.13 [12]: If there exists an open set F such that x∈F⊂Cl(F)⊂E for each x∈X and each open set E containing x, then the space X is called regular.

Definition 2.14 [13]: If the closure of each open set in X is open, or if the interior of each closed set in X is closed, a space X is said to be an extremally disconnected.

Definition 2.15 [14]: "A function f from a space X into a space Y is said to be somewhat open function provided that for E ∈ τ and E ≠ φ , there exists a set F which is open in Y such that F ≠ φ and F ⊂ f (E)".

Theorem 2.16 [15]: A function f : X →Y is somewhat open if and only if for each G⊂X and Int(G)≠φ, then Int(f (G)) ≠ φ.

Theorem 2.17[16]: A function f from a space X into a space Y is closed iff ; ∀ subset F of X , Cl(f(F))⊂f(Cl(F)).

Definition 2.18 [17]: Let f be a function from a space X into a space Y, if f -1(U) is clopen in X for each subset U in Y, f is said to be strongly continuous.

Theorem 2.19 [1]: f: (X, τ) □ (Y, σ) is Sw-irresolute, if it is a surjective continuous function.

Theorem 2.20 [1]: The following statements are equivalent for a function f : X Y,

f is Sw-irresolute.

There is an inverse Sw-open set in X; for every Sw-open set in Y.

There is an inverse Sw-closed set in X for every Sw-closed set in Y.

Sw*- Regular Spaces

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Definition 3.1: A topological space (X, τ) is called Sw*- regular, if for every Sw-regular set E and each a ∉ E in X, there exist open sets L and K in X that are disjoint; such that; a∈L and F⊂ K.

It is important to note that all discrete and indiscrete spaces are Sw*- regular spaces. It is also clear from the preceding definition that the class of Sw*- regular spaces is contained in the class of S*-regular spaces., mean that every Sw*- regular is S*- regular , however, as shown in the following example. In general, the reverse is not true.

Ex. 3.2: Let X = {a, b, c, d} with a topology τx ={φ, X, {b}, {c}, {d}, {a, d}, {b, c}, {c, d}, {b, d}, {b, c, d}, {a, c, d}, {a, b, d}} on X. Then (X, τx) is an S*- regular space but not Sw*- regular.

The following theorem is a characterization of an Sw*- regular spaces:

Theorem 3.3: The following propositions are equivalent for a topological

space (X, τ):

1. (X, τ) is an Sw*- regular space.

2. For each a ∈ X and each Sw-regular set V containing a, ∃ an open set U, s.t.

a∈U⊂ Cl(U)⊂V.

3. The intersection of all closed neighborhoods of a Sw-regular set E is E itself.

4. For each non-empty set E; and each Sw-regular set F of X; s.t. E∩F≠φ, ∃ an open subset U of X s.t. E∩U ≠ φ and Cl(U) ⊂F.

5. For any non-empty subset E and Sw –regular set F of X, s.t. E∩F =φ, there

are open sets C and D of X; that are disjoint s.t. E∩C≠φ and F⊂D.

Proof: (1) ⇒(2)

Let V be an Sw-regular set in X containing a, therefore a ∉ X\V and X\V

is also an Sw-regular set, then by Sw*- regularity of X, ∃ two disjoint open sets U1 and U2 of X s.t. a∈U1 and X\V⊆U2 . Since U1 and U2 are disjoint, then U1⊆X\U2 and since X/ U2 is closed, then Cl(U1)⊆ X\ U2⊆V. Thus a∈ U1⊆ Cl(U1)⊆V. This gives (2).

(3) ⇒ (2)

Let E be Sw-regular set in X, then X\E be also Sw-regular. By the hypothesis, for all a∈X\E, there is a set which is open Ua in X such that a∈ Ua⊂ Cl(Ua)⊂X\E. Then ⋃_(a∈(X\E))▒〖 {a}⊂〗 ⋃_(a∈(X\E))▒〖U_a⊂X\E〗 , and so X\E⊂⋃_(a∈(X\E))▒〖U_a⊂X\E〗 . That is,

X\E=⋃_(a∈(X\E))▒U_a . Therefore; E=⋂_(a∈E)▒〖(X\U_a 〗), where X\U_a is a closed neighborhood of E. This completes the proof.

(4) ⇒ (3)

Let E and F be two non-empty disjoint subsets of X; such that F is an Sw-regular set. Then, there exists a∈E∩F. Thus a∉X\F and X\F is an Sw-regular set, so by the hypothesis there exists a closed neighborhood

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H such that a∉H and X\F⊂ H, then X\F⊂G⊂H, G is open. Let X\H=U, then a∈U where U is open. Hense E∩U≠φ, and since G is open, then X\G is closed. This implies that X\H⊂X\G⊂F, and so U⊂Cl(U)⊂Cl(X\G)= X\G⊂F. That is U⊂Cl(U)⊂F, thus Cl(U)⊂F.This proves (4).

(5) ⇒ (4)

Let E be non-empty subset of X; and F be an Sw-regular set of X; s.t. E∩F = φ, therefore; X\F is also an Sw-regular set in X and E∩X\F ≠ φ. Using (4) ∃ an open subset C of X; s.t. E∩C ≠φ and Cl(C)⊂X\F and then F⊂X\Cl(C). Put D = X\Cl(C) ⊂X\C. Thus D is an open set, s.t. F⊂D. As a result C, D are open sets with E∩C ≠φ, F⊂D and D∩C =φ.

(1) ⇒ (5)

Let a∉ E, where E is Sw-regular set of X and let K={a} ≠ φ. Then by using (5) there exist two open sets C and D of X, such that K∩C ≠φ, C∩D = φ and E⊂D. Therefore a∈C, E⊂D and C∩D = φ. That is, X is an Sw*- regular space.

Theorem 3.4: Disjoint open sets Sw*- regular space X can separate each disjoint pair consisting of a compact set E and an Sw-regular set F.

Proof: Since X is an Sw*- regular space with E∩F=φ in X, then for every x∈ E, x ∉ F, where F is an Sw-regular set. Therefore; ∃ disjoint open sets Lx and Kx of X; s.t. x∈Lx and F⊆Kx. Obviously, the compact set E is covered by {Lx: x∈E}. Thus ∃ a finite subfamily {Lxi: i= 1, 2,…, m} which covers E. As a result, E⊂⋃{Lxi: i= 1, 2,…, m} and F⊂⋂{Kxi: i = 1, 2,…, m}. Put L= ⋃{Lxi: i= 1, 2,…, m} and K=⋂{Kxi: i= 1, 2,…, m}. Since L∩K=φ, then L⊂X\K and so Cl(L)⊂Cl(X\K)=X\K. That is Cl(L)⋂K=φ, by the same way L⋂Cl(K)=φ, so L and K are separated. Then the needed disjoint open sets are L and K.

Corollary 3.5: Let (X, τ) be an Sw*- regular space. If E is a compact subset of X, and F is a Sw-regular set that contains E, then ∃ an Sw-regular set K, s.t. E⊂K⊂Sw-Cl(K) ⊂F.

Proof: Since F is an Sw-regular set, so X\F is also Sw-regular; and E⋂X\F = φ in X, where E is compact, then by Theorem 3.4; ∃ disjoint open sets L1 and L2 of X; s.t. E⊂L1 and X\F ⊆ L2. But L1⋂L2=φ, so L1⊂X\L2 and since L1 is open, so it Sw-open and then by Lemma 2.7 X\L2 is also an Sw-open set, furthermore, X\L2 is a closed set and consequently it is an Sw-closed set, and so X\L2=Sw-Cl(X\L2). That X\L2 is Sw-regular set. Put K=X\L2, then E⊂L1⊂ X\L2⊂F. Means that E⊂K⊂Sw-Cl(K)=Sw-Cl(X\L2)=X\L2⊂F. Thus E⊂K⊂Sw-Cl(K)⊂F. This is the end of the proof.

Corollary 3.6: Let (X, τ) be an Sw*- regular space and let E , F be two disjoint subsets of X, with E being compact and F being a Sw-regular set. Then there exists Sw-regular sets L and K s.t. E⊂ L, F ⊂ K and L⋂K=φ.

Proof: By Theorem 3.4; ∃ disjoint open sets L and K of X s.t. E⊂L and F⊆K. But L⋂K=φ, so L⊂X\K and since L is open, so it is Sw-open. Furthermore; X\K is closed, then it is Sw-closed. That is, Cl(X\K)≠X and since L⊂X\K, then Cl(L)≠X. That is, L also is an Sw-closed set. Thus L is an Sw-regular set. By the same way K is also Sw-regular. Thus L and K are the required Sw-regular sets.

In the following theorem we show that Sw*- regular space has not a hereditary property.

Theorem 3.7: If a space X is an Sw*- regular and E be an open subspace of X, then E is Sw Sw*- regular space.

Proof : Suppose that E is an open subspace of the Sw*- regular space X. To demonstrate E's Sw*- regularity, suppose that G is an Sw-regular set in E and let a∉ G; s.t. a∈E. Since G⊂E, G is Sw-open in E and E is

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open in X, then by Proposition 2.5, G is an Sw-open set in X, also since G⊂E⊂X and G is Sw-closed, then from Lemma 2.6 G is Sw-closed set in X, such that a∉ G, so by Sw*- regularity of X, ∃ two disjoint open sets La and Ka of X s.t. a∈ La and G⊂Ka. Let L=La⋂E and K=Ka⋂E, clearly a∈ L and G⊂ K; where L and K are open sets that are disjoint in E. Therefore E is an Sw*- regular space.

The following example shows that the condition of openness on A in Theorem 3.7 is necessarily.

Ex. 3.8 : Let X = {a, b, c, d}; with τ = {φ, X, {a}, {a, b}, {a, c}, {a, b, c}}. So the space (X, τ) is Sw*-regular, while A = {b, c, d} is not Sw*- regular though A is closed. It follows that Sw*- regularity is not a hereditary property.

Recall that a topological space (X, τ) is said to be submaximal [5], if every preopen is open.

Corollary 3.9: Let X be an Sw*- regular submaximal space, then every preopen subspace of X is Sw*-regular.

Proof : Suppose that A is a preopen subspace of X, since X is Submaximal, then A is open and by Theorem 3.7 A is Sw*- regular.

Theorem 3.10: Every Hausdorff Sw-compact space is Sw*- regular.

Proof: Let E be any Sw-regular set containing a point say b in X, so X\E is also Sw-regular set s.t. b∉ X\E. But X is a T2 space, therefore; for every a∈ X\E, ∃ open sets La and Ka s.t. x∈ La , b∈ Ka and La⋂Ka=φ. Obviously {La: a∈ X\E} is a cover of X\E by Sw-open sets of X and since E is Sw-regular, then N={La: a∈ X\E}⋃E is an Sw-open cover of X, but X is Sw-compact space, then ∃ a finite subfamily of N covers X. That is, X= ⋃{Lai: i=1, 2,…,m}⋃E. Therefore X\E⊂⋃{Lai: i=1, 2,…,m}. Let L=⋃{Lai: i=1, 2,…,m} and K=⋂{Kai: i=1, 2,…,m}. Then b∈K and X\E⊂L, such that L and K are open sets in X. As a result, the space X is Sw*- regular.

Theorem 3.11: A topological space (X, τ) is regular if it is semi-regular and Sw*- regular.

Proof: Let L be any open set of X and a∈L. But X is semi-regular, so by

Theorem 2.12; ∃ a semi-open set M in X s.t. a∈M⊂sCl(M)⊂L. But Sw-Cl(M) ⊂sCl(M) ⊂Cl(M) for any M⊂ X. So a∈ M ⊂ Sw-Cl(M) ⊂ sCl(M) ⊂ L, and since M is semi-open, then it is Sw-open. That is, Sw-Cl(M) is Sw-regular set and since X is Sw*- regular space, thus by Theorem 3.3, ∃ an open set E s.t. a∈E⊂Cl(E)⊂Sw-Cl(M)⊂L. Thus, a∈E⊂Cl(E)⊂L. As a result, X is a regular space.

Proposition 3.12: If a topological space (X, τ) is Sw*- regular, then it is extremally disconnected.

Proof: Let K be any non-empty open subset of X, so Cl(K) is an Sw-regular set and since X is an Sw*-regular space, then by Theorem 3.3(2) for each a∈Cl(K), there exists an open set La such that a∈La⊂Cl(La)⊂Cl(K). Thus Cl(K)=⋃{La: a∈Cl(K)} which it is open. Therefore X is extremally disconnected.

The following theorem give another characterization of an Sw*- regular space.

Theorem 3.13: Let (X, τ) be any topological space. Then X is Sw*- regular iff for all Sw-regular set M and a point p∈X such that p∉M, there exist open sets R and S of X such that p∈R, M⊂S and Cl(R)⋂Cl(S)=φ.

Proof : Suppose that (X, τ) is an Sw*- regular space; and p∉M, s.t. M is an Sw-regular set in X, so ∃ two disjoint open sets Ro and S such that p∈Ro and M⊂S, further Ro⋂Cl(S)=φ, if not suppose that Ro⋂Cl(S)≠ φ, then there exists a∈ Ro⋂Cl(S), so a∈ Ro and a∈Cl(S), then for all open set K of X and a∈K, K⋂S≠ φ

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and since Ro is an open set which containing a, then Ro⋂S≠ φ which is contradiction, thus Ro⋂Cl(S)= φ and Cl(S) is an Sw-regular set and since p∈Ro , then p∉Cl(S), again by Sw*- regular of X, there exist open sets A and B of X such that p∈A, Cl(S)⊂B and A⋂B=φ, hence Cl(A)⋂B=φ. Put R=Ro⋂A, then R is open s.t. p∈R, M⊂ S and Cl(R)⋂Cl(S) = Cl(Ro⋂A) ⋂ Cl(S) ⊂ Cl(Ro)⋂ Cl(A)⋂ Cl(S) ⊂ Cl(Ro)⋂Cl(A)⋂B = φ. Thus Cl(R)⋂Cl(S)=φ.

Conversely; suppose that p∉M, with M is an Sw-regular set in X, so ∃ two open sets R and S such that p∈R, M⊂S and Cl(R)⋂Cl(S)=φ, means that R⋂S=φ. Therefore X is an Sw*- regular space.

Recalling that a topological space (X, τ) is called a Urysohn[18] if there are neighbours K of a and L of b with Cl(K)⋂Cl(L)=φ, whenever a ≠ b in X.

Proposition 3.14: A topological space (X, τ) is Urysohn, if it is Sw*- regular and Sw-T2 space.

Proof: Assume that a, b are any two different points in X, but X is Sw-T2 space, then by Theorem 2.9 there exists Sw-regular set A of X contains a but not b, or contains b but not a , say a∈A and b∉A, so by Theorem 3.13, there exists two open set K and L of X s.t. b∈K, A⊂L and Cl(K)⋂Cl(L)=φ. That is a∈A⊂L, means a∈L, b∈K and Cl(K)⋂Cl(L)=φ. Therefore X is a Urysohn Space.

Proposition 3.15: If (X,τ) is an Sw*- regular space, then so is (X, τα).

Proof: This follows from Theorem 2.4 and the fact that every open set is α-open.

Theorem 3.16: Let Y be an Sw*- regular space, if f : X→Y is a bijective, continuous, closed and Sw-open function, then X is also an Sw*- regular space.

Proof : Assume that F is an Sw-regular set in X, a∈ X with a∉ F, and so there exists b∈ Y s.t. f(a)= b. But f is an Sw-open function, so f(F) is Sw-open in Y, that is Int(f(F))≠φ. Also F is Sw-closed, then Cl(F)≠X, but f is closed, then by Theorem 2.17

Cl(f(F))⊂f(Cl(F)). Now Cl(F)≠X, this implies that f(Cl(F))≠f(X)=Y and so Cl(f(F))⊂f(Cl(F))≠Y. That is Cl(f(F))≠Y, so f(F) is Sw-closed in Y. So f(F) is Sw-regular set in Y with b∉ f(F), then by Sw*- regularity of Y; ∃ two disjoint open sets L and M of Y s.t. b∈ L and f(F)⊂M. Then by the continuity of f [19] f -1(L) and f -1(M) are open in X, such that a = f -1(b)∈f -1(L), F⊂f -1(M) and f -1(L)⋂f -1(M)=φ, consequently; X is an Sw*- regular space.

Recall that a function f : (X, τ) → (Y, σ) is said to be clopen [2] if it is both open and closed.

Corollary 3.17: If f is a bijective continuous and clopen function from a space X into an Sw*- regular space Y, then X is also Sw*- regular.

Proof : This is a direct result of Theorem 3.16 and the fact that each open function is Sw-open

Theorem 3.18: If X Sw*- regular and f is a strongly continuous and open function from a space X onto a space Y. Then Y is also Sw*- regular.

Proof: Let K be an Sw-regular set of Y, and b∈Y s.t. b∉K, but f is a surjective function, so ∃ a∈X s.t. f (a) = b. In addition to f is strongly continuous function and K is a subset of Y, so f -1(K) is clopen set in X, that is f -1(K) is Sw-regular set in X, where a∉ f -1(K), then by Sw*- regularity of X, ∃ two disjoint open sets A and B in X whereas a∈A and f -1(K)⊂B, but f is open, so f(A) and f(B) are open sets in Y, s.t. b∈ f(A), F⊂ f(B) and f(A)⋂f(B)=φ. As a result, Y is an Sw*- regular space.

Theorem 3.19: If X is an Sw*- regular space and f is a bijective Sw-irresolute and open function from a space X into space Y, then Y is also Sw*- regular space

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Proof : Suppose that F is an Sw-regular set in Y and b∈Y s.t. b∉F, then ∃ a∈X s.t. f(a) = b, but f is Sw-irresolute, then f -1(F) is an Sw-regular set in X, where

a∉ f -1 (F), then by Sw*- regularity of X, ∃ two disjoint open sets L and K of X s.t. a∈L and f -1(F)⊂K, this implies that f(a)= b∈ f(L), F= f(f -1(F))⊂f(K), where f(L) and f(K) are open sets in Y with f(L)⋂f(K)=φ. As a result, Y is an Sw*- regular space.

Corollary 3.20: Let X be an Sw*- regular space. If f: X→Y is an open, bijective and continuous function, then Y is also Sw*-regular.

Proof: Follows directly from Theorem 3.19 and Theorem 2.19.

Corollary 3.21: Sw*- regularity is a topological property.

Proof: Follows directly from the concepts of a homeomorphism [12 Theorem 2.4] and Corollary 3.20.

Conflict of interests.

There are non-conflicts of interest.

References

1. L. S. Abddullah and A. B. Khalaf, “An Extension of Semi-Open Sets with its Applications on Spaces and Functions”, Ph.D. dissertation, University of Sulaimaniya, Sulaimani, Iraq, 2009.
2. M. Khan , “Weak forms of continuity, compactness and connectedness”, Ph. D. dissertation, Bahauddin Zakariya University Multan, Pakistan, 1997.
3. S. Hussain and B. Ahmad, "On γ-s*-regular spaces and almost γ-s-continuous functions", Lobachevskii Journal of Mathematics, vol. 30, no. 4, pp. 263-268, 2009.
4. N. Levine, “Semi-open sets and semi-continuity in topological spaces”, Amer. Math. Monthly, vol. 70, pp. 36-41, 1963.
5. J. Dontchev , “Survey on preopen sets”, ArXiv: Math., vol. 1, pp. 1-18, 1998.
6. J. H. Park, "Almost p-normal, mildly p-normal spaces and some functions", Chaos, Solitons and Fractals, vol.18, Issue 2, pp. 267-274, October 2003.
7. O. Njastad , “On some classes of nearly open sets”, Pacific J. Math., vol. 15, no. 3, pp. 961-970, 1965.
8. H. Z. Ibrahim , "Bc-Open Sets in Topological Spaces", Advances in Pure Mathematics, vol. 3, pp. 34-40, January 2013.
9. A. Sabaha , M. Din Khanb and L. Kocinac, “Covering properties defined by semi-open sets”, J. Nonlinear Sci. Appl. , pp. 1-10, Feb. 2016.
10. J. Dontchev and M. Ganster , “On covering spaces with semi- regular sets*”, Ricerche di Matematica, vol. 45, no. 1, pp. 229-245, 1996.
11. G. D. Maio and T. Noiri , "On s-closed spaces", India J. Pure Appl. Math., vol. 18, pp. 226-233, 1987.
12. J. N. Sharma, Topology. Krishna Prakashna Mandir, 1977.
13. M. Mirmiran, “A survey on extremally disconnected spaces”, Math. Dept., Isfahan Univ., Iran, pp. 1-3, 2000.

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14. K.Rekha and T.Indira, “Somewhat *b-continuous and Somewhat *b-open Functions in Topological spaces “,Intern. J. Fuzzy Mathematical, vol. 2, pp. 17-25, July 2013.
15. C. W. Baker , “Weak forms of openness based upon denseness”, Tr. J. of Mathematics, no. 20, pp. 389-394, 1996.
16. N. S. Noorie and R. Bala, “Some Characterizations of Open, Closed, and Continuous Mappings”, International Journal of Mathematics and Mathematical Sciences, vol. 2008, pp. 1-5, January 2008.
17. R. S. Wali, B. B. Mathad and N. Laxmi, “ Some Semi-Regular Weakly Continuous Functions in Topological Spaces”, International Journal of Advanced Research (IJAR), vol. 5, no. 7 , pp. 2261-2270, July 2017.
18. V. Popa , "Characterizations of H-almost continuous functions", Glas. Mat. Ser., vol. 22, no. 1, pp. 157-161, 1987.
19. Sidney A. Morris, Topology Without Tears, version of June 28, 2020.:pp.87–89.

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