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Solving Multi-Objective Linear Fractional Programming Problems by Novel Methods

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حل مسائل البرمجة الجزئية الخطية متعددة الأغراض بطرق جديدة

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ABSTRACT

In this paper, we suggested two new techniques for transforming multi-objective linear fractional programming problems into single-objective linear fractional programming problems (SOLFPP) and then solving them by methods of SOLFPP. Illustrative numerical examples are presented for demonstration purposes. Also, the obtained results are compared with another method studied previously.

Keyword: Galton skewness, Pearson 2 skewness coefficient, Single Objective Linear Fractional programming, Multi-objective linear programming.

الخلاصة

في هذا البحث، استخدمنا طريقتين جديدتين لتحويل مسائل البرمجة الجزئية الخطية متعددة الأهداف إلى المسائل البرمجة الخطية الجزئية ذات هدف واحد (SOLFPP) ثم حلها بطرقSOLFPP . يتم تقديم أمثلة عددية توضيحية لهذا الغرض. أيضا، يتم مقارنة النتائج تم الحصول عليها مع طريقة أخرى تمت دراستها سابقاً.

الكلمات المفتاحية:

انحراف جالتون، معامل الانحراف بيرسون ٢، البرمجة الجزئية الخطية ذات الهدف واحد، مشاكل البرمجة الجزئية الخطية متعددة الأهداف

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INTRODUCTION

Linear fractional Programming (LFP) could be a generalization of straight programming (LP), but a linear program has a linear goal function. Any linear fraction program's objective function is the proportion of two linear functions. [1]. The fractional programming problems are particularly useful in the solution of economic problems in which various activities use certain resources in various proportions [2].

In the beginning, multi-objective linear fractional programming problems (MOLFPP) posed some difficulties, so, they were converted into single objectives and were solved by the methods of Charnes and Cooper and Bitran and Novaes. In 1983, Charnes and Cooper [3]] studied studied conversion from linear fractional programming to equivalent linear programming. In 1973, Bitran, and Novaes [4] studied linear programming with a fractional objective function. In 1981, Kornbluth and Steuer [5] introduced goal programming techniques to solve multi-objective linear fractional programming problems based on the variable change method. In 1983, Chandra Sen [6] suggested an approach to transform multi-objective optimization. In 2008, Duran [7] studied fuzzy multi objective linear fractional programming. In 2010, Sulaiman and Salih [8] solved multi-objective linear fractional programming using mean and median values. In 2013, Sulaiman, N.A., Abdulrahim, B. K. [9] studied multi-objective linear fractional programming problems by transforming method. In 2007, Sulaiman, and Othman, [10] for solving multi-objective linear programming problems suggested an Optimal Transformation technique.

In this paper, we are using two techniques (Galton skewness and Pearson 2 skewness coefficient) to transform and solve multi-objective linear fractional programming problems to the Single objective linear fractional programming problem, then compare the results by different techniques.

Materials and Methods

- **Form of MOLPP in Mathematics**

The following is the MOLPP mathematical form:

$$\left.\begin{aligned} Max.Z_1 &= c_1^t x + \alpha_1 \\ Max.Z_2 &= c_2^t x + \alpha_2 \\ &\vdots \\ Max.Z_r &= c_r^t x + \alpha_r \\ Min.Z_{r+1} &= c_{r+1}^t x + \alpha_{r+1} \\ &\vdots \\ Min.Z_n &= c_n^t x + \alpha_n \end{aligned}\right\} \quad \dots (1)$$

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$$\text{Subject to: } Ax\begin{Bmatrix}\le\\ \ge\\ =\end{Bmatrix} b \qquad \ldots (2)$$

$$x \ge 0 \qquad \ldots (3)$$

Where n is the total number of objective functions that need to be maximized and minimized, r is denoted of Goal functions that need to be maximized, the number of objective functions that need to be minimized is denoted by $n-r$ and vector of decision variables indicated by x is a with n-dimensions, c is indicated for a vector of constants with n-dimensions, b is m −dimensional vector of constants, A defines a coefficients matrix as having $(m \times n)$ items. Assumed that every vector are column vectors unless transported, the scaler constant α_i is used for each $i = 1, 2, \ldots, n$.

- **Mathematical form of MOLFPP**

The Multi-Objective Linear Fractional Programming Problem is generally expressed as follows:

$$\left.\begin{array}{l} Max.Z_1 = \frac{c_1^t x+\alpha_1}{d_1^t x+\beta_1} \\ Max.Z_2 = \frac{c_2^t x+\alpha_2}{d_2^t x+\beta_2} \\ \quad\vdots \\ Max.Z_r = \frac{c_r^t x+\alpha_r}{d_r^t x+\beta_r} \\ Min.Z_{r+1} = \frac{c_{r+1}^t x+\alpha_{r+1}}{d_{r+1}^t x+\beta_{r+1}} \\ \quad\vdots \\ Min.Z_n = \frac{c_n^t x+\alpha_n}{d_n^t x+\beta_n} \end{array}\right\} \qquad \ldots (4)$$

$$\text{Subject to: } Ax\begin{Bmatrix}\le\\ \ge\\ =\end{Bmatrix} b \qquad \ldots (5)$$

$$x \ge 0 \qquad \ldots (6)$$

Where r denotes the quantity of objective functions that must be maximized, $n-r$ is the number of objective functions that need to be minimized and n is the total number of objective functions that need to be maximized and minimized, x is a vector of decision variables with n-dimensions, c, d are a vector of constants with n −dimensions, b is m −dimensional vector of

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constants, A is a matrix of coefficients with $(m \times n)$ elements. All vectors are presumed to be column vectors unless they are transferred, and α_i and β_i are scaler constants, for each $i = 1, 2, \ldots, n$.

- **Utilizing the Chandra Sen. Technique to solve (MOLFPP)**

The constraint objective function for the (MOLFPP) is formulated using the same method as in Sen. (1983) [6]. Let's say we are able to determine a single value for each of the (MOLFPP's) objective functions of equation (4). According to the constraints (5) and (6), they are being optimized separately as follows:

$$\left.\begin{array}{l} Max.Z_1 = \psi_1 \\ Max.Z_2 = \psi_2 \\ \quad . \\ \quad . \\ \quad . \\ Max.Z_r = \psi_r \\ Min.Z_{r+1} = \psi_{r+1} \\ \quad . \\ \quad . \\ \quad . \\ Min.Z_n = \psi_n \end{array}\right\} \quad \ldots (7)$$

Where ψ_i, $i = 1, 2, \ldots, n$ are values of the goal functions, the level of the decision variable may not always be the same for all optimal solutions in presence of conflicts among objectives. But the common sets of decision variable between objective functions are necessary in order to select the best compromise solution. Applied Chandra Sen's technique to solve MOLFPP, which is of the form:

$$Max.Z = \sum_{i=1}^{r} \frac{Z_i}{|\psi_i|} - \sum_{i=r+1}^{n} \frac{Z_i}{|\psi_i|} \quad \ldots (8)$$

And solve (4.2) by [3].

- **Using New Techniques to solve MOLFPP**

To determine the common set of decision variables, we formulate the combined objective function (8) as follows. Using (Galton skewness and Pearson 2 skewness coefficient) procedures to solve MOLFPP examine the following:

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- **Using Galton skewness Technique**

We suggested a thought by using Galton skewness method to solve multi-objective linear fractional programming problems, to combined objective function (4) depending on the idea that generating the combined objective function like the formula (8) to solve (MOLFPP).

$$Max.Z = \frac{S_1 - S_2}{GS} \qquad \dots (9)$$

Subject to constraints (5) and (6).

Where GS is a Galton skewness

$$GS = \left|\frac{(Q_1 + Q_3 - 2Q_2)}{Q_3 - Q_1}\right|,$$

where ($Q_1$is the lower quartile, Q_3 is the upper quartile, and Q_2 is the median) for the value of the all objective functions.

$S_1 = \sum_{i=1}^{r} Max.Z_i$, for all $i = 1, 2, \dots, r.$, $S_2 = \sum_{i=r+1}^{n} Min.Z_i$, for all $i = r+1, r+2, \dots, n.$

- **Algorithm for Galton skewness technique**

The following is a brief description of the procedure for finding the best answer to the (MOLFPP) that described in equation (4):

Step1: Each of the individual objective functions that must be maximized or minimized are given arbitrary values.

Step2: For linear fractional programming according to the restrictions, solve the first objective function by the [3].

Step3: Before moving on to step 4, make sure the solution found in step 2 is feasible.

Step4: Give the objective function Z_i's optimal value a name, say ψ_i for $i = 1,2,3,\dots,n$ as before.

Step5: Find lower quartile (Q_1), upper quartile(Q_3) , and the median (Q_2) to all absolute value of ψ_i, for all $i = 1, 2, \dots, n$

Step6: Calculate $GS = \left|\frac{(Q_1 + Q_3 - 2Q_2)}{Q_3 - Q_1}\right|$.

Step7: Create a combined objective function with the given formula (9).

Step8: Under the same constraints (5) and (6), Optimize the combined objective function.

- **Using Pearson 2 skewness coefficient Technique**

The combined objective function (4) is expressed as follows to solve MOLFPP by Pearson 2 skewness coefficient technique consider as follows:

$$Max. Z = \frac{S_1 - S_2}{S_{k2}} \qquad \dots (10)$$

Subject to constraints (5) and (6).

Where S_{k2} is a Pearson 2 skewness coefficient,

$$S_{k2} = 3 \left| \frac{mean(\psi_i) - median(\psi_i)}{S} \right|,$$

where S is a standard deviation for the value of the all objective functions.

$S_1 = \sum_{i=1}^{r} Max. Z_i$, for all $i = 1, 2, \dots, r.$, $S_2 = \sum_{i=r+1}^{n} Min. Z_i$, for all $i = r+1, r+2, \dots, n$.

- **Algorithm for Pearson 2 skewness coefficient Technique**

Following is a summary of an approach for finding the best solution for the MOLFPP described in equation (4):

Step1: Solve each of the objective functions individually, that required to be maximized or minimized.

Step2: For linear fractional programming according to the restrictions, solve the first objective function by the [3].

Step3: Step 2's solution should be tested for feasibility before moving on to step 4.

Step4: Set a name to the optimal value of the objective function Z_i, as before, let ψ_i for $i = 1,2,3, \dots, n$

Step5: Calculate $S_{k2} = 3 \left| \frac{mean(\psi_i) - median(\psi_i)}{S} \right|$, where S is a standard deviation for the value of the all objective functions, for all $i = 1, 2, \dots, n$

Step6: Create the formula-based composite objective function (10).

Step7: Under the same constraints (5) and (6), Optimize the combined objective function.

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- **Numerical Examples**

Example 1) Consider the MOLFP Problem

$$Max. Z_1 = (x_1 + 2x_2 + 1)/(x_1 + x_2 + 1)$$

$$Max. Z_2 = (x_1 + 2x_2 + 4)/(x_1 + x_2 + 1)$$

$$Max. Z_3 = (x_1 + x_2 + 7)/(3x_1 + 3x_2 + 3)$$

$$Min. Z_4 = (-2x_1 - 3x_2 + 2)/(3x_1 + 3x_2 + 3)$$

$$Min. Z_5 = (-x_1 - x_2 - 4)/(x_1 + x_2 + 1)$$

Subject to: $x_1 + 2x_2 \leq 8$

$$x_1 + x_2 \geq 2$$

$$x_1, x_2 \geq 0$$

Solution 1)

Solve each of the objective function under the giving constraints by [3], that present in the Table 1.

Table 1. The result of solution of all objective functions of Example (1)

No.	(x_1,x_2)	$Z_i = \|\psi_i\|$	Q_1	Q_2	Q_3	Mean ψ_i	Median ψ_i	Standard Deviation
1	(0, 4)	9/5	0.8335	1.8	2.3335	1.6268	1.8	0.8
2	(0,2)	8/3						
3	(1, 1)	1						
4	(0,4)	2/3						
5	(1,1)	2						

$S_1 = \sum_{i=1}^{r} Max. Z_i = \frac{7x_1+13x_2+22}{3(x_1+x_2+1)}$, $S_2 = \sum_{i=r+1}^{n} Min. Z_i = \frac{-5x_1-6x_2-10}{3(x_1+x_2+1)}$

By our methods:

1) **Galton skewness**

$$GS = \left|\frac{(Q_1+Q_3-2Q_2)}{Q_3-Q_1}\right| = 0.9$$

By formula (9) we get, $Max.Z = \frac{12x_1+19x_2+32}{0.9(x_1+x_2+1)}$,

Using the same constraints for solving this problem, we receive, $Max.Z = 25.93$, at (0, 2).

2) Pearson 2 skewness coefficient

$$S_{k2} = 3\left|\frac{mean(\psi_i)-median(\psi_i)}{S}\right| = 0.64$$

By formula (10) we get

$$Max.Z = \frac{12x_1+19x_2+32}{1.92(x_1+x_2+1)}$$

With the same restrictions we solve this problem we obtain, $Max.Z = 12.15$, at (0, 2).

Solve example (1) by other methods such as:

a) **By Cahndra Sen.** [6], the result $Max.Z = 4.59$, at (0,2).
b) **By Mean** [8], we get, $Max.Z = 2.8$, at (0,2).
c) **By Median** [8], the result $Max.Z = 4.8$, at (0,2).
d) **By average Mean** [9], the result $Max.Z = 2.27$, at (0,2).
e) **By average Median** [9], the result $Max.Z = 4.96$, at (0,2).

Example 2) Consider the MOLFP Problem

$$Max.Z_1 = (x_1 - x_2 + 7.5)/(2x_1 + 2x_2 + 2)$$

$$Max.Z_2 = (x_1 + x_2 + 4)/(x_1 + x_2 + 1)$$

$$Max.Z_3 = (x_1 - x_2 + 4.5)/(3x_1 + 3x_2 + 3)$$

$$Max.Z_4 = (x_1 + x_2)/(x_1 + x_2 + 1)$$

$$Min.Z_5 = (-2x_1 - 2x_2)/(3x_1 + 3x_2 + 3)$$

$$Min.Z_6 = (-5x_1 - x_2)/(x_1 + x_2 + 1)$$

$$Min.Z_7 = (-4x_1 + x_2)/(2x_1 + 2x_2 + 2)$$

Subject to: $-x_1 + x_2 \le 2$

$$2x_1 + 2x_2 \ge 1$$

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$$9x_1 + 3x_2 \le 3$$

$$x_2 \le 1$$

$$x_1, x_2 \ge 0.$$

Solution 2)

Solve each of the objective function with the same constraints by [3], showed in the Table 2.

Table 2. The result of solution of all objective functions of Example (2)

No.	(x_1, x_2)	$Z_i = \|\psi_i\|$	Q_1	Q_2	Q_3	Mean ψ_i	Median ψ_i	Standard Deviation
1	(1/4,1/4)	5/2	0.33	1	2.5	1.23	1	1.09
2	(0,1/2)	3						
3	(1/4,1/4)	1						
4	(0, 1)	1/2						
5	(0, 1)	1/3						
6	(1/4,1/4)	1						
7	(1/4,1/4)	1/4						

$$S_1 = \sum_{i=1}^{r} Max.Z_i = \frac{17x_1 + 7x_2 + 55.5}{6(x_1 + x_2 + 1)}, \qquad S_2 = \sum_{i=r+1}^{n} Min.Z_i = \frac{-46x_1 - 7x_2}{6(x_1 + x_2 + 1)}$$

1) **Galton skewness**

$$GS = \left|\frac{(Q_1 + Q_3 - 2Q_2)}{Q_3 - Q_1}\right| = 0.4$$

By formula (9) we get, $Max.Z = \frac{63x_1 + 14x_2 + 55.5}{2.4(x_1 + x_2 + 1)}$, solve this problem with the same constraints we get, $Max.Z = 20.76$, at (1/4, 1/4).

2) **Pearson 2 skewness coefficient**

$$S_{k2} = 3\left|\frac{mean(\psi_i) - median(\psi_i)}{S}\right| = 0.6$$

By formula (10) we get, $Max.Z = \frac{63x_1 + 14x_2 + 55.5}{3.6(x_1 + x_2 + 1)}$

solve the above problem with the same constraints we get, $Max.Z = 13.84$, at (1/4, 1/4).

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Solve example **(2)** by other methods such as:

a) The result is $Max.Z = 6.333333$, at (1/4, 1/4), **By Cahndra Sen [6].**
b) We get, $Max.Z = 6.69$, at (1/4, 1/4)), **By Mean [8].**
c) The result is $Max.Z = 8.32$, at (1/4, 1/4), **By Median [8].**
d) The solution is $Max.Z = 7.29$, at (1/4, 1/4), **By average Mean [9].**
e) the result is $Max.Z = 7.97$, at (1/4, 1/4), **By average Median [9].**

- **Comparison of the Proposed Method with another Techniques**

  Based on the numerical outcomes, we are comparing the methods as below in table 3:

Table 3: Comparing results of Numerical Methods

Techniques		Example 1	Example 2
Chandra Sen. Technique		4.59	6.333333
Mean Technique		2.8	6.69
Median Technique		4.8	8.32
Average Techniques	Mean Technique	2.27	7.29
	Median Technique	4.96	7.97
Pearson 2 skewness coefficient		12.15	13.84
Galton skewness		25.93	20.76

It is evident from Table 3 that the outcomes from the examples 1, 2 when using two new techniques called Pearson 2 skewness coefficient and Galton skewness the results are better than other techniques such as Chandra Sen. Technique, mean & Median, and Average Technique (Mean & Median).

Results and Discussion

In this paper, we introduced two methods for solving multi-objective linear fractional programming problem (MOLFPP), we use Pearson 2 skewness coefficient and Galton skewness techniques to change MOLFPP into LFPP, and discussed a number of techniques which are studied to get the optimal solution of the MOLFPP. We illustrate some examples to compare these techniques which are based on the value of the objective functions. The two novel strategies optimize the problem better than that of other methods.

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Conflict of interests.

There are non-conflicts of interest.

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