Wolfgang Reuter

Original or Pupil?

Possible applications of Artificial Intelligence in attribution issues using the example of the Rembrandt Research Project

Artificial Intelligence in the arts

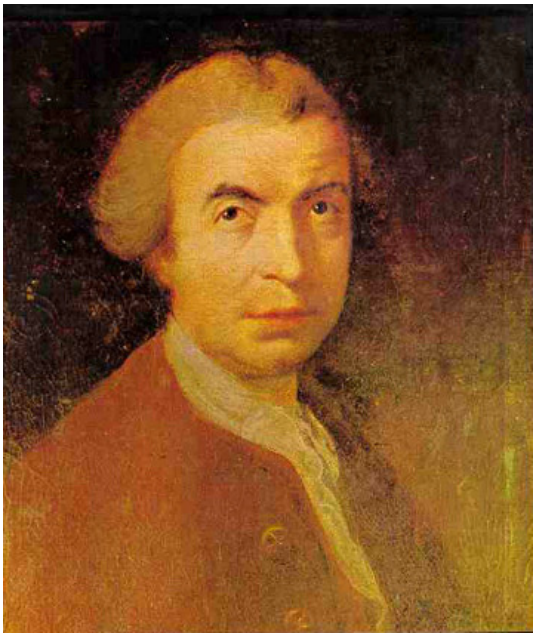
Artificial intelligence and Machine Learning, a subfield of so-called Artificial Intelligence (AI), have long been accepted as a technology in many industries and are an important part of their value chains. Most art historians, however, find it difficult to accept the methods and procedures, let alone apply them. The reasons for this may be manifold, but we would like to at least mention two of them here and take a closer look at them.

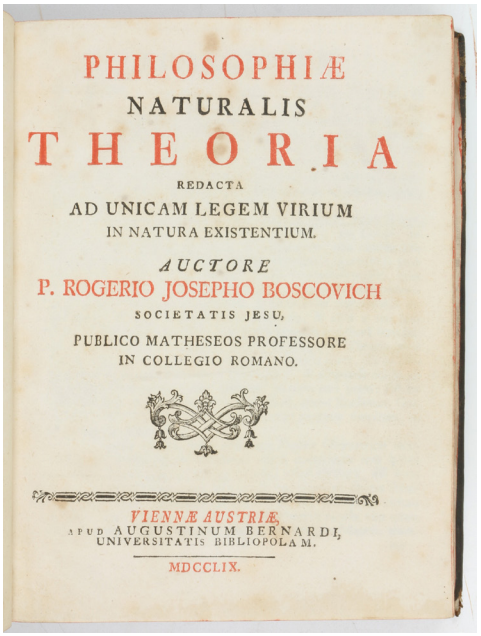
Above all, it is the term 'Artificial Intelligence' that art historians as well as experts from other sectors and/or industries find threatening. 'Will we all soon be superfluous?' – AI experts and practitioners are confronted with such and similar questions on a daily basis. Yet what is commonly referred to as Artificial Intelligence has nothing whatsoever to do with actual intelligence.

All algorithms within this field are based on minimizing a so-called cost or error function. A method that was intuitively applied for the first time around 1750 by the polymath Roger Joseph Boscovich, shown in Figure 1. Together with the English Jesuit priest and astronomer Christopher Maire, he calculated the ellipticity of the earth based on the measured distances between five different latitudes[1]. A good 30 years later, the method was then formalized in mathematically precise terms by Carl Friedrich Gauss and Adrien-Marie Legendre and has been significantly improved since the 1940s. However, the basic principle has not changed until today: Machine Learning algorithms are based on minimizing the distances between calculated predictions and actual measured values[2].

Let us imagine that the technology didn't have the word 'intelligence' in its name, but would, for example, be called, 'Optimized Error Function Minimization' and considered a subfield of applied mathematics, which is a much more likely description and actually quite accurate to its core. Then there would be no ethical discussion about the technology, nor would people be afraid of it. We, the most evolved and intelligent species on this planet (at least we think we are), can easily accept that a calculator multiplies two nine-digit numbers faster than our brain. But the vague idea that a machine could be intelligent, and thus perhaps at some point more intelligent than we are, frightens us because it calls into question our position within the ecosystem Earth.

However, the AI community and the faculties and organizations that study and research this technology are partly to blame for people's concerns and fears about AI. The reason is a completely meaningless contest that always measures AI models against the performance of the best experts. The latter often feel they are competing against an opponent they can neither grasp nor understand, which reinforces their dismissive attitude. Especially when the perceived challenge, for example in the medical field, results in headlines such as "Artificial Intelligence beats dermatologists in cancer diagnosis."[3]




PHILOSOPHIÆ
NATURALIS
THEORIA
REDACTA
AD UNICAM LEGEM VIRIUM
IN NATURA EXISTENTIUM.
AUCTORE
P. ROGERIO JOSEPHO BOSCOVICH
SOCIETATIS JESU,
PUBLICO MATHESEOS PROFESSORE
IN COLLEGIO ROMANO.
VIENNÆ AUSTRIÆ,
APUD AUGUSTINUM BERNARDI,
UNIVERSITATIS BIBLIOPOLAM.
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1a/b Roger Joseph Boscovich, painted by R. Edge Pine, 1760, and the title page of his publication *Theoria philosophiae naturalis redacta ad unicam legem virium in natura existentium*, published in Vienna in 1758, in which he presented a first method for minimizing an error function

Yet the public perception of Artificial Intelligence is at the very least distorted and in some cases completely wrong. For example, these models quiet often work well only with certain data but fall far behind physicians with others[4]. Most importantly, several experiments have shown that the best results occur when subject matter experts, whether physicians, physicists, or chemists, use the technology, incorporate the results into their reasoning and considerations – but do not slavishly adhere to them. In other words, when Artificial Intelligence assists human intelligence[5].

In this context, it makes sense to know the accuracy of human experts in certain tasks, such as here in the recognition of painting styles. After all, Artificial Intelligence needs one thing above all: labeled data. The algorithms learn from examples, in this case paintings *and* the associated painters – and these assignments to specific artists come from art historians. For this reason, we will, in the following, also evaluate the accuracy of the experts in attributing paintings from Rembrandt's school and studio.

However, we will not engage in the nonsensical competition that compares human intelligence with Artificial Intelligence. Instead, we will demonstrate on the one hand, that so-called Artificial Intelligence[6] is indeed very effective in recognizing patterns that, in this case, constitute the painting styles of artists. On the other hand, and most importantly, we will demonstrate how this technology can help art historians attribute controversial paintings to specific painters. In addition, we will point out, wherever necessary, the sensitivities and shortcomings of this method.

The difficulty of attribution in paintings by workshop painters

Dead men don't paint, that much is certain. And yet the catalogues raisonnés by popular and expensive painters regularly swell over the years and decades. Heinrich Campendonk's oeuvre, for example, counted only 115 works in 1960[7], in 1989 there were 1215[8], including some works that have since been proven to be forgeries. And a good 600 watercolors by Alexej von Jawlensky were rejected in 1998. They had only emerged in the early 1990s and were recognized as genuine by the Jawlensky Archive in Locarno[9]. Since then, a heated debate has been going on about the updating of the catalogue raisonné, in which numerous forged paintings are also suspected. The question 'Original or forgery?' arises with almost all painters who fetch at least five-figure sums on the art market – and art historians often disagree about the authenticity of certain works.

2 School of Rembrandt, *Old man contemplating in a study*, Oslo, National Museum

It is even more difficult to assign works of art to the workshop painters of the 16th and 17th centuries. The studios of Leonardo da Vinci, Raphael, Rembrandt, Rubens, and others were basically medium-sized businesses with dozens of employees in some cases. Their only goal was to produce paintings in the style of the master. To exaggerate, they were copy and forgery manufactories. In addition, the respective master often painted individual parts of the picture himself or at least retouched or completed them. Consequently, the assignment of works by these artists is particularly difficult. Some of the paintings have been passed around between different pupils for decades like traveling trophies. The painting *Old man contemplating in a study* (fig. 2), is currently considered to be a work of the Rembrandt school, but in the past has also been attributed, according to the Netherlands Institute for Art History (RKD), to Adriaen Verdoel, Abraham van den Hecken, Karel van der Pluym, Gerbrand van den Eeckhout, Willem Drost and of course the master himself, that is Rembrandt[10]. Other paintings have undergone similar attribution and rejection odysseys.

Overall, the authorship of almost three quarters of the works by Rembrandt and his students is disputed. In other words, for three quarters of these works there is on average at least one rejected attribution. So how certain are art historians when it comes to judging works of art by Rembrandt or his circle? Or how uncertain? What is the probability that a painting is correctly attributed?

To our knowledge, there are no robust studies on this topic. We have taken a systematic approach to measuring the accuracy of attributions to Rembrandt and his pupils, fully realizing that it is not possible to answer the above questions exactly. However, the upper and lower limits can be calculated precisely – and we will transparently disclose the approach in the following chapter.

Human Level Performance in the Attribution of Works of Art from Rembrandt's School and Circle

The data basis for the study are the online available findings of the Netherlands Institute for Art History, RKD. The RKD undoubtedly has the world's largest collection of research on the paintings of Rembrandt and his pupils, as well as the most comprehensive, though not in all cases complete, list of literature sources on these works. For each individual painting, all essential data are compactly summarized and presented on the corresponding web page.

Based on this information, we have developed two Confidence Scores for calculating the accuracy of art historians on works by Rembrandt and his pupils. The Base Score can be considered as a lower limit for the accuracy of the assignment of Rembrandt's works as well as paintings of his pupils, the Literature Score as an upper limit. The main parameters for the calculation are:

1. "Current attribution", i.e., the painter (or in rare cases the painters) to whom a work is currently attributed.
2. "Rejected attribution", a list of all artists to whom a work was attributed in the past or is still attributed by individual art historians, although this attribution is rejected by most experts.
3. "Literature", a list of sources, usually catalogue raisonnés, that include an attribution.

Both scores are discussed in detail below. The Base Score is calculated as follows:

Base Score = 1 / (n_current + n_rejected).

3 Rembrandt painting *Bust of a youth*, c. 1660. The work was considered an original by the master from 1836 to 1969, from 1969 to 1992 it was rejected, in 1992 it was classified by Slatkes as partly by Rembrandt – and in 1997 by Blankert as entirely by Rembrandt

Where n_current is the number of current attributions (usually there is only one) and n_rejected is the number of previous or rejected attributions.

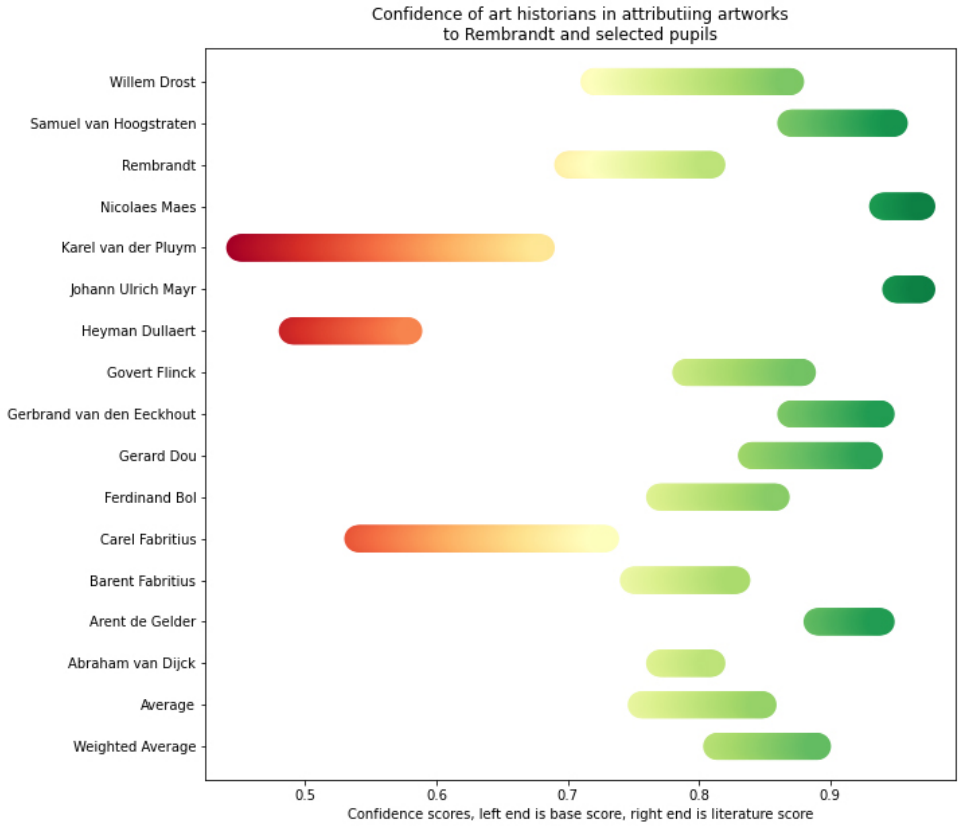
The logic behind the base score is that each attribution is weighted equally, regardless of how many art historians support the attribution or how significant the art historians are. Therefore, the corresponding value is definitely too small.

Rembrandt's work *Bust of a youth* (fig. 3), formerly titled *Portrait of Titus, the Artists Son*, is attributed to Rembrandt. According to RKD, it has one rejected attribution to "Follower of Rembrandt". This painting would enter the calculation with a Base Score of 0.5.

The weighting is done in the literature score in such a way that this score represents an upper limit for the accuracy of the experts. The formula is:

Literature Score = 1 / n_current - n_rejected / (n_literature + n_rejected),

where n_literature is the number of literature sources for the respective work.



4 Bands of accuracy of art historians in attributing paintings of the Rembrandt school

The logic of this calculation leads on the one hand to a weighting. Rembrandt's work *Bust of a youth* has 24 sources, so n_literature is 24. The literature score is 0.96 (1 - 1/25), a very high certainty of this attribution in the context of the turbulent attribution and rejection history (see caption to fig. 3)[11].

This is intentional, as the literature score is aimed to lead to an upper bound in calculating the accuracy or certainty of the art experts in the attribution of artworks. It is therefore implicitly assumed in the formula that all literature sources support the current attribution, which in most instances is not the case. Furthermore, it is implicitly assumed that each rejected attribution is based on the judgment of only one art historian, which is also not the case for most artworks. Each additional attribution to a rejected artist of a work would reduce the literature score. Both assumptions therefore lead to an overly high attribution certainty score.

If no literature source is given, n_literature is replaced by n_literature_avg, the average number of literature sources for the respective painter. However, this is rarely the case; on average, each work in the dataset has more than seven literature sources.

While the method does not exactly determine the accuracy of art experts in attributing paintings by Rembrandt and his students, the actual value is certainly somewhere between the base score and the literature score.

Painter Name	Base Score	Lit Score	Nr. Images	Average Nr. References
Abraham van Dijck	0.770	0.810	27	0.889
Arent de Gelder	0.890	0.940	123	4.504
Barent Fabritius	0.750	0.830	64	3.062
Carel Fabritius	0.540	0.730	31	7.903
Ferdinand Bol	0.770	0.860	206	4.053
Gerard Dou	0.840	0.930	200	4.355
Gerbrand van den Eeckhout	0.870	0.940	130	1.854
Govert Flinck	0.790	0.880	182	3.747
Heyman Dullaert	0.490	0.580	8	0.875
Johann Ulrich Mayr	0.950	0.970	41	2.463
Karel van der Pluym	0.450	0.680	12	15.917
Nicolaes Maes	0.940	0.970	594	1.086
Rembrandt	0.700	0.810	625	26.078
Samuel van Hoogstraten	0.870	0.950	97	3.351
Willem Drost	0.720	0.870	37	12.270
Average	0.756	0.850	158.467	9.117
Weighted Average	0.813	0.891	158.467	9.117

Tab. 1 Base and Literature Score for paintings by Rembrandt and 14 selected pupils as well as number of paintings and literature sources

The results for Rembrandt and 14 of his students are visualized in Figure 4 and explicitly listed in Table 1. It is striking that especially works by Karel van der Pluym, Heyman Dullaert but also by Carel Fabritius are very poorly assigned. One possible reason for this is the number of paintings – there are few works by these three painters, as can be seen in Table 1. However, this also applies to Johann Ulrich Mayr, for example, whose works art historians have little doubt about.

It should also be noted that the accuracy for the attributions to Nicolaes Maes is markedly high. Surely, this is consistent with the number of works (594 in all) for this artist. However, Maes worked only briefly in Rembrandt's studio and later turned to Flemish painters, such as Anthonis van Dyck. Compared to other Rembrandt pupils, the distinction of most of Maes's works is relatively easy – nevertheless, not only the early works, which were created in Rembrandt's style, but all his paintings were used for the calculation of the scores.

Our intention is not to further investigate the accuracy of art historians here, but to show an approach how it can be measured, being open to criticism and suggestions. The main goal of this study is to investigate whether Artificial Intelligence can be helpful in attributing paintings to either Rembrandt or particular pupils of this baroque workshop painter. In the following, we will explain, present

and evaluate the approach. Further below, we will use Artificial Intelligence to individually analyze some works by Rembrandt and his students that are controversial regarding authorship.

The Accuracy of Artificial Intelligence in Classifying Artworks from Rembrandt's Circle and School

The accuracy of Machine Learning algorithms depends less on the architecture of the corresponding neural networks than on the quality of the data on which they are trained. And, of course, on the labels, in this case the painters, to whom the corresponding works are attributed. Here, as shown above, a high degree of uncertainty can be assumed among the respective experts.

However, this problem of so-called false labels occurs not only in the recognition of painting styles, but also in other disciplines, for example the medical field, in the recognition of diabetic retinopathy (blindness) based on images of the eyeball, where the assessing experts notoriously disagree about the (residual) vision of the respective patient. Nevertheless, Machine Learning algorithms achieve very good results here. Various authors have also shown that neural networks trained with a manageable number of false labels can be very robust. As an example, a recent study by Alexander Thamm GmbH and LMU-Munich with synthetically generated data is referred to[12].

For the purpose of this study, we downloaded the relevant works by Rembrandt and the afore mentioned students from the corresponding websites of the RKD. This dataset is largely complete in terms of the current attributions of the paintings or drawings – only about two dozen works are missing, which are not available online from the RKD. However, the larger side of these images is limited to 650 pixels. The resolution is not particularly good, but sufficient for the analysis.

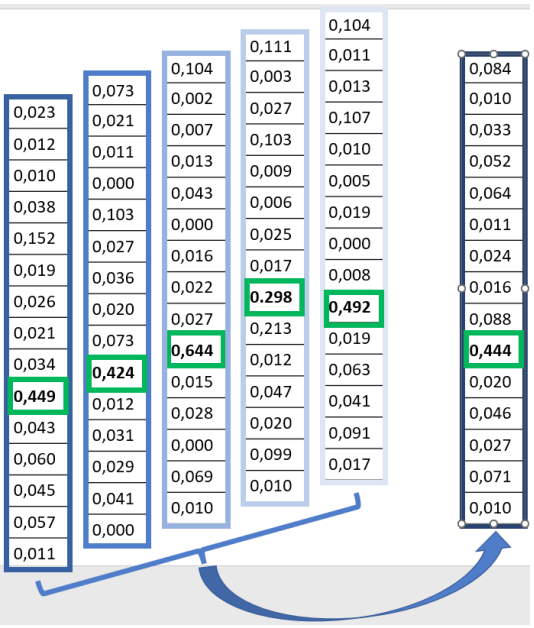
Moreover, we have only taken works that are clearly attributed by the RKD. Interestingly, this is not the case with the vast majority of the Rembrandt Research Project's judgements - which allows for an unprejudiced test of these paintings. A bias regarding the view of this research team is thus also largely excluded.

The dataset was split 85:15 into a training and test set. Then, five neural networks each were trained using a cross-validated approach. This means that of the training data, i.e., of 85 percent of the images, we used 80 percent each for the actual training of each neural network, while the remaining 20 percent were used for validation and optimization of the respective neural network. The distribution of the

5 Schematic representation of the cross-validated approach in training the individual neural networks, training data in blue, validation data in green

Van Dijck	0,010	0,040	0,020	0,023
De Gelder	0,003	0,010	0,025	0,012
B. Fabritius	0,020	0,003	0,007	0,010
C. Fabritius	0,002	0,105	0,008	0,038
Bol	0,102	0,000	**0,350**	0,152
Dou	0,000	0,000	0,056	0,019
Eeckhout	0,015	0,021	0,043	0,026
Flinck	0,007	0,009	0,047	0,021
Dullaert	0,000	0,102	0,001	0,034
Mayr	**0,567**	**0,444**	0,335	**0,449**
v. d. Pluym	0,040	0,000	0,090	0,043
Maes	0,008	0,170	0,003	0,060
Rembrandt	0,120	0,001	0,013	0,045
Hoogstraten	0,106	0,063	0,002	0,057
Drost	0,000	0,032	0,000	0,011

SoftMax-Output für Ausschnitte eines Werkes, gemittelte Werte rechts (blau)



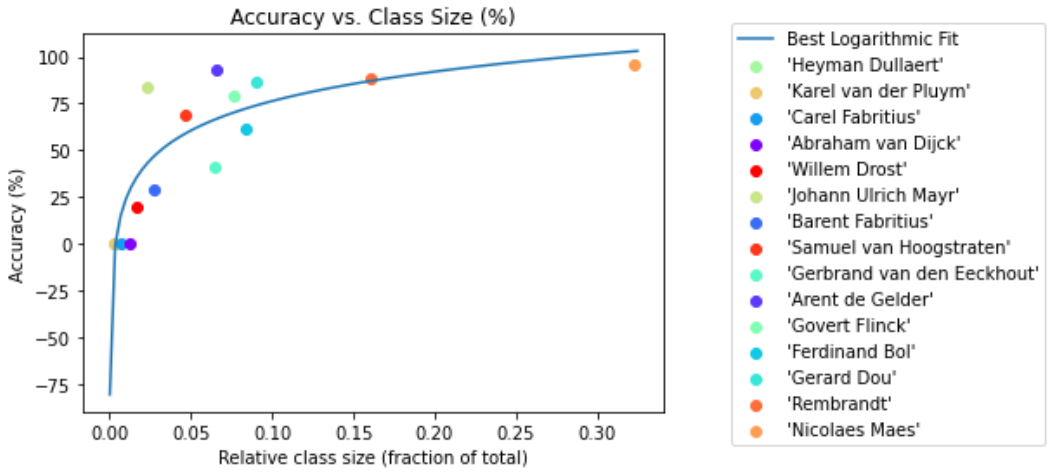
6a/b Evaluation of the results of the ensemble of five CNNs, schematically for one work: First, the SoftMax output is averaged over the individual image sections (6a). The values outlined in red each represent the maximum SoftMax output for a painter within an image section. The green bordered value on the right is the maximum value of the average values of all entries for the different painters. In this case, this is the average of 0.567, 0.444 and 0.335. This is done separately in each of the neural networks. Then, the averaged results of each CNN are averaged again for each painter (6b). The vector generated in this way is used as the final result. The painter's prediction is based on the highest SoftMax output, here 0.444 – the average of 0.449, 0.424, 0.644, 0.213 and 0.492

data within the training set was done in such a way that each painting occurs at least once in the validation set. Thus, all the five networks have been trained on slightly different data (fig. 5).

The individual Convolutional Neural Networks (CNNs) were evaluated on the test set. It should be noted that the works in the test set did not encounter any of the networks during training. To understand the final accuracy of the ensemble of CNNs, the so-called SoftMax output of a neural network must be considered. For each individual image, this is a vector with entries for each of the painters in the dataset. These entries sum to 1 for each individual artwork and are interpreted by many AI experts as probabilities that a painting is by a particular painter. However, this view is mathematically incorrect and also problematic, as we will show later in this chapter.

In fact, the networks have not been trained with the entire images – but with several randomly selected image sections or crops from each of the individual works. The SoftMax output was first averaged over all the sections of a work – and then over the different neural networks. Schematically, this procedure is shown in Figure 6a/b.

Understanding the output of the individual networks is very important, especially for this use case. This is because looking at the individual SoftMax output values provides clues as to how confident the ensemble is in attributing to a particular artist. For this purpose, so-called confidence scores can be cal-



7 Accuracy of the ensemble of five CNNs in recognizing Rembrandt and 14 of his students as a function of the percentage of paintings by each artist in the dataset. The neural networks were trained on the RKD dataset (of parsed images)

culated, which can help art historians to assess how seriously a vote of the neural network ensemble is to be taken. We will come back to this further below.

First, however, we want to present, evaluate, and discuss the results of the AI analysis for all images. On the RKD dataset, the ensemble of five CNNs achieves 79.3 percent accuracy, using only uniquely attributed works. But the recognition for individual painters varies widely. There is a (logarithmic) dependency with respect to the number of paintings in the dataset (see fig. 7).

Altogether four Rembrandt students are not recognized at all; there are too few paintings with clear attribution for them. These are Abraham van Dijck (21 works), Carel Fabritius (12 works), Heyman Dullaert (4 works) and Karel van der Pluym (4 works). The number of works by the painters in question refers in each case to the entire data set, i.e., training and test data. Of the four clearly attributed paintings by Heyman Dullaert and Karel van der Pluym, three each are in the training data set and only one in the test data set. Other artists, however, are very well recognized by the ensemble. Of Arendt de Gelder, Gerard Dou, Johann Ulrich Mary, Nocholaes Maes and Rembrandt himself, well over 80 percent of the works are correctly classified in each case. A detailed list can be found in Table 2.

From a mathematical point of view, the accuracy of the ensemble's predictions cannot be equated with the calculated scores of art historians. For a true 'human level performance' one would need the predictions of art historians, i.e., a test of how well the experts assign certain paintings to the currently assumed artist. Intuitively, however, the values are quite comparable.

A closer look shows that art historians are (often clearly) more certain in their attribution to particular students, but with Rembrandt himself it is the other way around. Here, the accuracy of the ensemble is distinctly above the range of scores of human experts.

Against the background of the high degree of uncertainty in the attribution of numerous works, a clumsy comparison along the lines of 'Who is better at assigning painting styles – human intelligence or Artificial Intelligence?' makes no sense at all in our view. The question is rather: Can Artificial In-

Painter	Precision	Recall	F1-Score	Train Size (in %)
Abraham van Dijck	0	0	0	1.244
Arent de Gelder	0.929	0.929	0.929	6.584
Barent Fabritius	0.667	0.286	0.400	2.707
Carel Fabritius	0	0	0	0.732
Ferdinand Bol	0.684	0.619	0.650	8.339
Gerard Dou	0.909	0.870	0.889	8.998
Gerbrand van den Eeckhout	0.636	0.412	0.500	6.437
Govert Flinck	0.600	0.789	0.682	7.608
Heyman Dullaert	0	0	0	0.219
Johann Ulrich Mayr	0.455	0.833	0.588	2.341
Karel van der Pluym	0	0	0	0.219
Nicolaes Maes	0.854	0.962	0.905	32.260
Rembrandt	0.760	0.884	0.817	16.094
Samuel van Hoogstraten	1	0.692	0.818	4.609
Willem Drost	1	0.200	0.333	1.609
Overall (weighted) wrt Train Set	0.775	0.793	0.770	100
Overall (unweighted) wrt Train Set	0.566	0.498	0.501	100

Tab. 2 Accuracy (Recall) and Precision of the ensemble of five CNNs trained on the RKD dataset for individual painters, and their percentage of paintings within the training dataset

telligence be helpful as an additional indication in the attribution of works to artists of the Rembrandt School, besides scientific and art historical expertise? And if so, how? To answer these questions, we need to look more closely at the output of the model.

Critical discussion of the model results

To this end, let us consider two – admittedly extreme – examples from the test dataset. They are the paintings *Isaac lying in bed blesses Jacob who, disguised in Esau's clothes, brings food to his father* (fig. 8a), by Gerbrand van den Eeckhout, and the portrait *Johannes de Doper* (fig. 8b), by Gerard Flinck, of which only a black-and-white image is available. The painting by Gerbrand van den Eeckhout is not recognized correctly by the model, but the portrait by Govert Flinck is. So far, so good. But, if we look at the SoftMax output for the two works (fig. 9a/b), questions inevitably arise.

For the work of Gerbrand van den Eeckhout, the maximum SoftMax output is 0.1989. It refers to Govert Flinck, which is why the image is misclassified. The SoftMax output values for eight other artists, namely Arent de Gelder, Barent and Carel Fabritius, Ferdinand Bol, Gerard Dou, Gerbrand van den Eeckhout, Nicolaes Maes and Rembrandt are between 0.05 and 0.15, comparatively close to the maximum value.

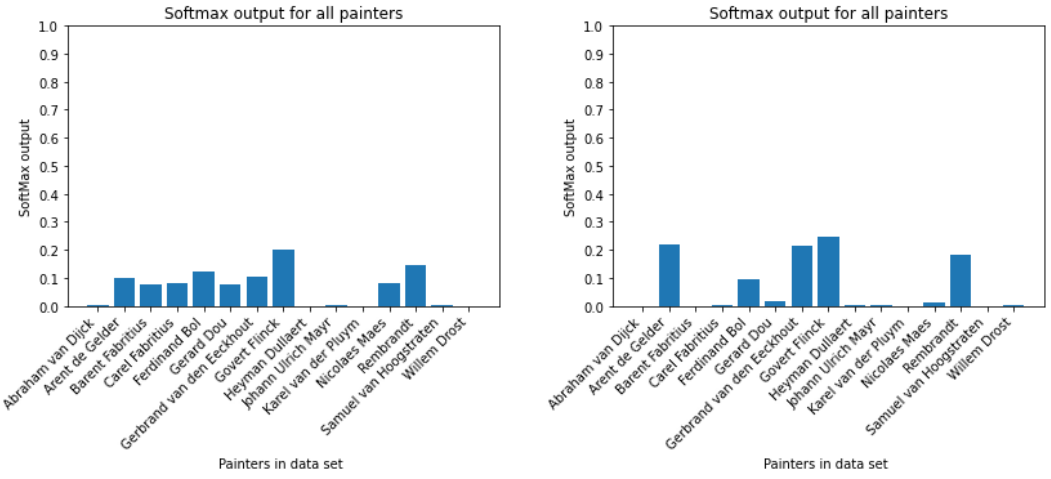
The situation is quite similar for the portrait by Govert Flinck. The SoftMax output for this artist is maximum with 0.2473, which is why this work is predicted correctly. However, the difference to Arent de Gelder (0.2191), Gerbrand van den Eeckhout (0.2147) and Rembrandt (0.1836) is extremely small.

If we assume for a moment the – mathematically incorrect – interpretation of the SoftMax output as probability, the model says about the portrait of *Johannes de Doper*: With a probability of 24 per-

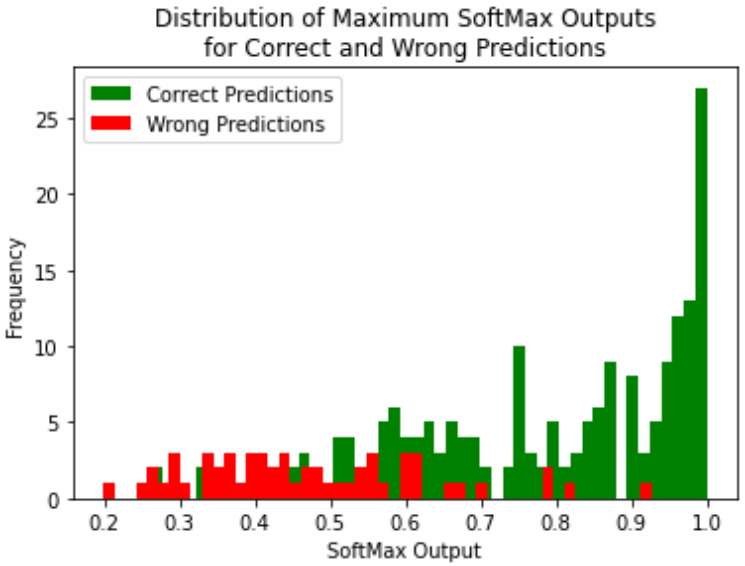
8a/b Gerbrand van den Eeckhout, *Isaac lying in bed blesses Jacob who, disguised in Esau's clothes, brings food to his father*, New York, Metropolitan Museum of Art (8a) and Govaert Flinck, Johannes de Doper, Los Angeles, Los Angeles County Museum of Art (8b)

cent, it originates from Govert Flinck, with a probability of just under 22 percent from Arent de Gelder, also from Gerbrand van den Eeckhout and with a probability of 18.36 percent from Rembrandt. What are we to make of this statement?

The honest answer is: Such a statement is completely worthless for an attribution or rejection. The ensemble of CNNs does not know who painted this picture, it is completely undecided – as art historians are with numerous other works. Consequently, an attribution to the artist with the highest Soft-



9a/b SoftMax output for the works by Gerbrand van den Eeckhout and Govert Flinck shown in Figure 8a/b



10 Maximum SoftMax output for all images in the test set, red for incorrect, green for correct predictions

Max output would be simply irresponsible due to the high uncertainty of the model. In other words, attributing a painting to the painter with the highest SoftMax output is legitimate and correct for an initial evaluation of the model as a whole. For the evaluation of individual works, however, this approach is only conditionally applicable.

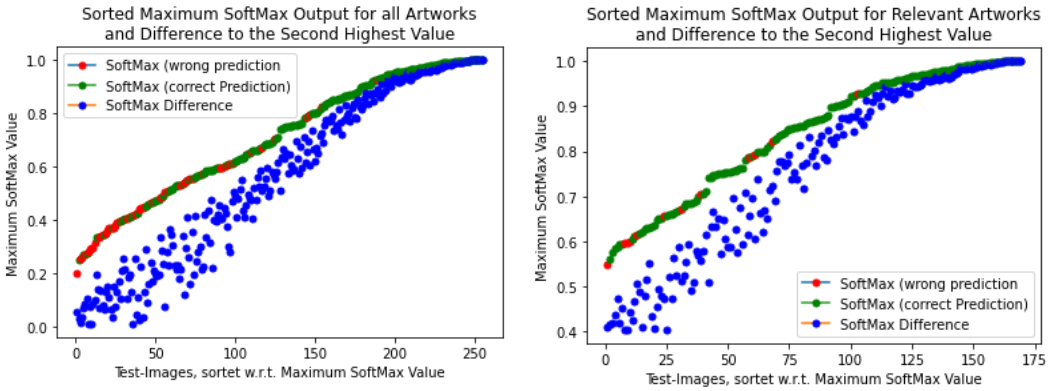
The good news, however, is that we can intuitively see how certain the model is in assigning a picture to a painter – and subsequently formulate this mathematically. This, in turn, is extremely helpful. The model tells us when we should take its predictions seriously and when not.

The situation is possibly comparable to the statement of an art historian who is asked, "What do you think, is this painting by Rembrandt or by Gerard Dou?" And his answer is: "The color palette speaks for Gerard Dou, but the brushstroke fits more to Rembrandt. For that reason, I must pass here." We do not know whether art historians (occasionally) express themselves in this way, and at this point we want to emphasize once again that this study does not pursue the question of whether Artificial Intelligence can judge art better than human intelligence and expert knowledge. We want to discuss the question whether Artificial Intelligence can be helpful in attributing controversial paintings – and if so, how?

A first insight is that there are images, including the two extreme examples (see fig. 8a/b), where classification using an ensemble of CNNs does not help, and the model tells us so. But what about the other cases?

In order to judge not only intuitively when we should take a prediction of the model seriously, we consider the distribution of the SoftMax output separately – once for the correctly predicted, but also for the incorrectly classified images. The corresponding histograms are summarized in Figure 10.

It becomes clear that the SoftMax output is generally *clearer* for the correct predictions than for the incorrect predictions. However, there are exceptions, for both categories. Some correct predictions have a very low SoftMax output, such as the *Johannes de Doper* portrait by Govert Flinck mentioned



11a/b Maximum SoftMax output for all images in the test set, red for wrong, green for correct predictions as well as the difference to the second highest SoftMax value (11a). Same plot for all images with a difference between the highest and second highest SoftMax value of more than 0.4 (11b). For clarity, the images are sorted by their highest SoftMax value

earlier. In some rare cases, the model is very confident even with incorrect predictions. But what is a clear prediction?

To trust a prediction, we want the SoftMax output on the one hand to be as high as possible – and on the other hand, we want it to be distinctly different from the other artists in the dataset for the predicted painter. The SoftMax output for the predicted painter must therefore be significantly higher than the values for the other painters. This is always the case when it is significantly higher than the SoftMax output of the painter with the second highest value – because the values of all other artists are lower. The distance to the painter with the second highest SoftMax output is therefore a minimum value.

We have calculated the difference between the highest and the second highest SoftMax output for each example in the test set. These differences, along with the highest SoftMax output in each case, can be seen in Figure 11a. In addition, we have set a threshold of 0.4 and used it to define a SoftMax difference criterion. Thus, if the maximum SoftMax output of a prediction is also at least 0.4 higher than the second highest value, we take the prediction seriously and believe it, otherwise we reject it.

Mathematically, this threshold means that the lowest possible SoftMax output for a serious attribution is 0.44. However, since the SoftMax output across all painters always adds up to 1, all other values would have to be exactly 0.04, which is highly unlikely. In fact, the lowest SoftMax output in the test set that meets this criterion is 0.549.

The threshold also means that any SoftMax output value higher than 0.7 will result in a serious attribution. The math is quite simple. If the SoftMax output for one painter is 0.7, it can be a maximum of 0.3 for one other artist, assuming it is 0 for all other painters. But this satisfies the difference criterion.

Why so complicated? We could simply fix a prediction to be accepted at the absolute SoftMax output value, for example, accept every prediction as correct if the threshold value is above 0.5. This must be a safe attribution, after all, no other value can be above 0.5 in this case.

In fact, many AI experts calibrate their models with an absolute threshold. However, in our case this would have the consequence that we would have to attribute a painting with a SoftMax output

Painter	Precision	Recall	F1-Score	Train Size (in %)
Abraham van Dijck	0	0	0	1.244
Arent de Gelder	1	1	1	6.584
Barent Fabritius	1	1	1	2.707
Ferdinand Bol	1	0.600	0.750	8.339
Gerard Dou	0.947	1	0.973	8.998
Gerbrand van den Eeckhout	1	0.833	0.909	6.437
Govert Flinck	0.818	1	0.900	7.608
Heyman Dullaert	0	0	0	0.219
Johann Ulrich Mayr	0.600	1	0.750	2.341
Karel van der Pluym	0	0	0	0.219
Nicolaes Maes	0.947	0.986	0.966	32.260
Rembrandt	0.912	0.939	0.925	16.094
Samuel van Hoogstraten	1	0.800	0.889	4.609
Overall (weighted) wrt Train Set	0.900	0.892	0.889	97.659
Overall (unweighted) wrt Train Set	0.710	0.705	0.697	97.659

Tab. 3 Precision of the ensemble of five CNNs trained on the RKD dataset for individual painters, as well as their percentage of paintings in the training dataset. The recall values refer - unlike the precision values – only to cases in which the model makes a prediction

for Rembrandt of 0.51, for Carel Fabritius or 0.49, and for all other artists of 0, to Rembrandt, which we are reluctant to do. Our interpretation of this hypothetical example would be that the model is not sure whether the work is by Rembrandt or by Fabritius, which is why we reject an attribution to Rembrandt.

For the sake of completeness, we need to re-evaluate the model with the SoftMax difference rule. The accuracy drops to 62.4 percent. That means: Of all works, no longer 79.3 percent are correctly recognized, but only 62.4 percent – which is obvious, because we have classified some works, such as Govert Flinck's portrait *Johannes de Doper* as wrong, because the result of the model result seemed too uncertain to us. However, accuracy is not the metric we are interested in. In mathematical terms, it represents the probability that the model makes a correct statement, i.e., that it for example predicts Govert Flinck, under the condition that the painting in question is actually by Govert Flinck.

When we test paintings of which we do not know who painted them, we need the inverse probabilities. We need to know: What is the probability that an artwork is by Govert Flinck – given that the model predicts Govert Flinck. The corresponding metric is called precision, and it increases from 77.5 to 90.0 percent with the difference rule.

In other words, if the model predicts a painter with this stricter criterion, the painting is actually by that artist with an average 90 percent probability. The values do fluctuate, as shown in Table 3. Again, Abraham van Dijck, Heyman Dullaert and Karel van der Pluym are not recognized at all. For all other artists in the data set, however, the certainty of an attribution increases significantly if they are predicted by the model. For example, if the model predicts Rembrandt as the creator for a particular work under the strict difference rule, we can assume with a probability of 91.2 percent that this painting is indeed by Rembrandt.

In summary, we have recalibrated the model with a stricter prediction rule. As a result, this model does not provide any information for 33 percent of the images in the test data set, but we accept this

12a/b Rembrandt, *Simeon's Song of Praise*, high-resolution image from Wikimedia Commons (12a), photograph from the National Museum in Stockholm (12b)

for this use case. For the remaining two thirds of the cases, we can trust the model prediction much more.

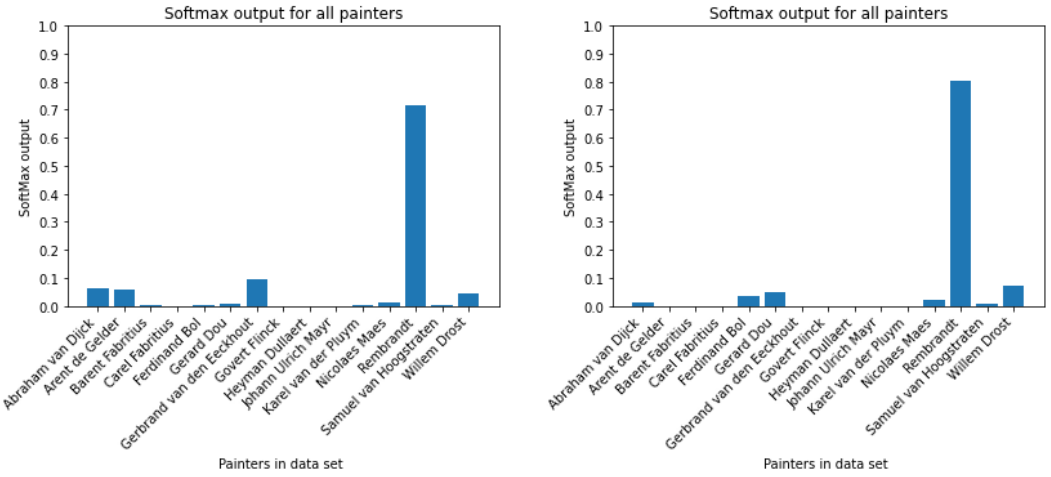
In the following detailed analysis, we will examine 15 disputed paintings, all of which are not present in either the training or the test dataset. The model has therefore not 'learned' these images - and can consequently judge them without bias.

The focus is mainly on whether these works are Rembrandt originals. In the *Further Analysis* section, we will show how Artificial Intelligence can assist art historians with a variety of other methods. In particular, when this first model does not provide any information about the authorship of a work due to the stricter prediction criteria.

Detailed analysis of controversial works

Undoubtedly partly by Rembrandt is the painting *Simeon's Song of Praise* (c. 1669). The two painters Allaert and Cornelis van Everdingen testified that they saw Rembrandt working on the painting in the months before his death in 1669. However, the artist did not complete this work. The woman in the background, often identified as Hanna or Mary, was, according to Bredius[13] and Bauch[14], probably added later.

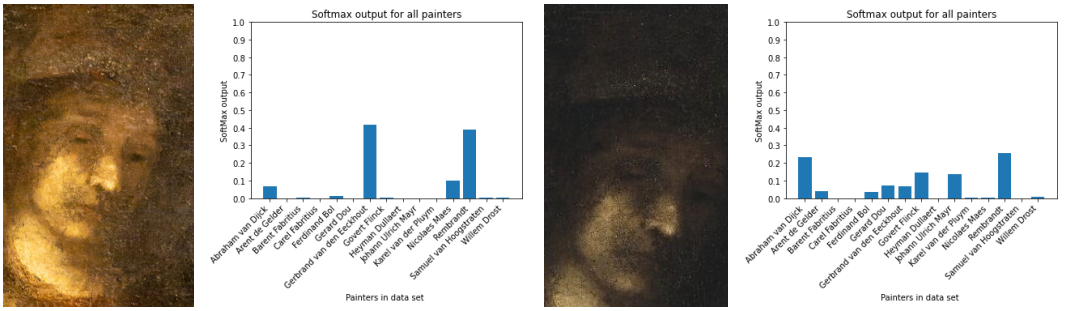
We first tested the entire painting – in two different publicly available variants (fig. 12a/b). Both variants are clearly identified as Rembrandt. The distribution of the SoftMax output is shown in Figure 13a/b. The question remains, however, whether Rembrandt painted the woman in the background as well, or whether the work was completed by a pupil. And if so, by whom?



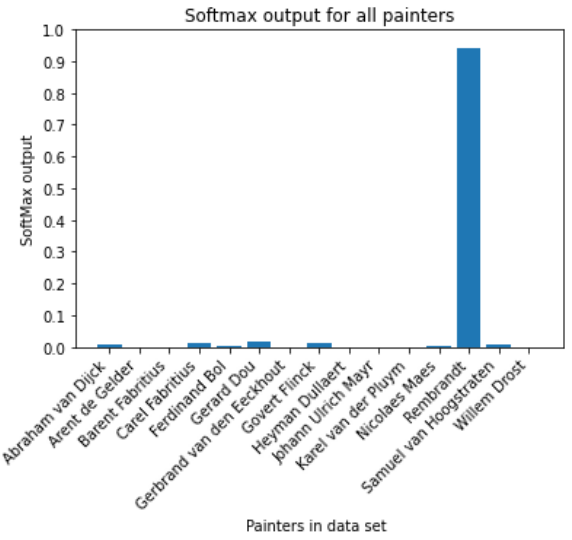
13a/b SoftMax output for the two variants of the Rembrandt painting *Simeon's Song of Praise* shown in Figure 12a/b

For this purpose, the figure was cut out of the two paintings and tested separately in each case. The results are shown in Figure 14a–d. Neither of the two cutouts is recognized as an original by Rembrandt. The ensemble suspects Gerbrand van den Eeckhout as the author for the left cutout, but for the right cutout the SoftMax output is relatively evenly distributed across eight different artists. This is a typical distribution when the actual painter is not present in the dataset. The two overall images were tested again, with the image sections from Figure 13 blacked out. This resulted in both cases in an even higher SoftMax output for Rembrandt than shown in Figure 13.

As a conclusion, the assumption of Bredius and Bauch (7, 8) can be considered very plausible. The female figure in the background was almost certainly not painted by Rembrandt. However, the ensemble from CNNs does not provide any valid clues as to the artist who completed the painting.



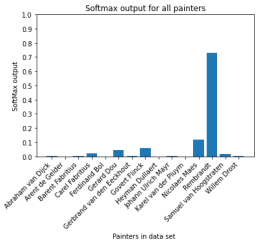
14a–d Extracted female figure (14a/c) from the variants of the Rembrandt painting *Simeon's Song of Praise* shown in Figure 12a/b, along with the associated SoftMax output (14b/d)

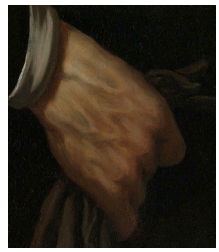


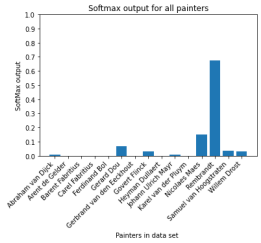
15a/b Rembrandt painting *Portrait of Johannes Wttenbogaert*, including the corresponding SoftMax output

A similar situation is found in the painting *Portrait of Johannes Wttenbogaert* (1633), The Rembrandt Research Project (RRP) initially had doubts about the signature of the painting but dropped them after a restoration in 1992. Van de Wetering[15] suspected that the portrayed's hands were executed by one of the master's assistants. In order to be able to cut out and test the hands in passable resolution, an image from Wikimedia Commons (4818 × 5992 pixels) was also used for the test here. The overall image is once again clearly and undoubtedly recognized as an original by Rembrandt, as, by the way, is the low-resolution variant of the image shown at the RKD. The SoftMax output is over 0.94 in each case (fig. 15a/b).

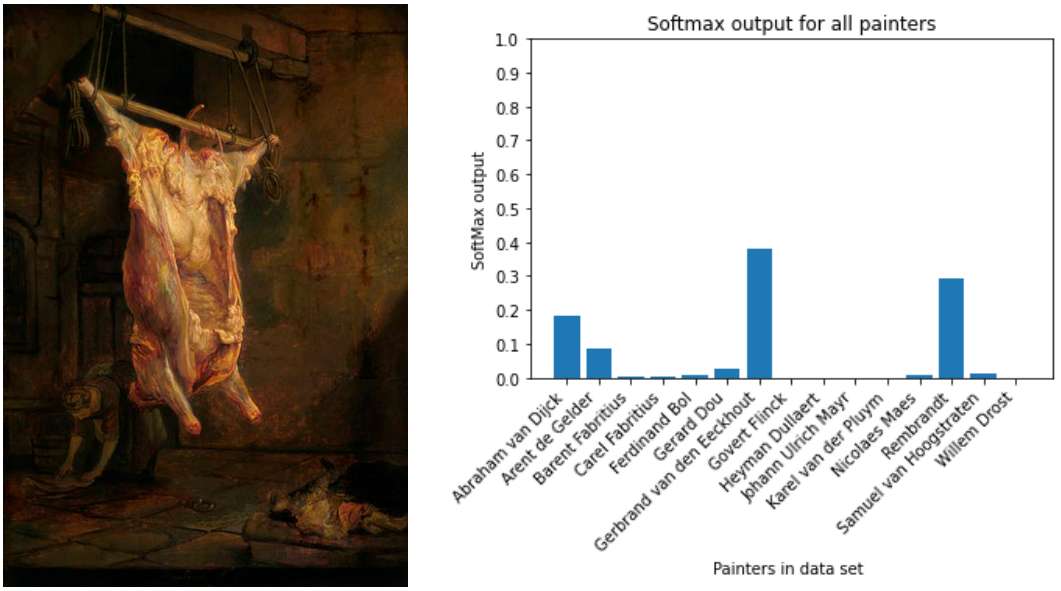
The cutouts of the hands do drop in SoftMax output from the overall image (fig. 16a/b), but they are still classified as original by Rembrandt, with values of 0.739 and 0.645, even under the stricter SoftMax difference rule. The distance to the next highest SoftMax value is greater than 0.4 in both cases, and while the drop is striking, it is rather unlikely that the hands were painted entirely by another artist.







16a–d Extracted hands from the Rembrandt painting *Portrait of Johannes Wttenbogaert* (16a/c), including the corresponding SoftMax output (16b/c)



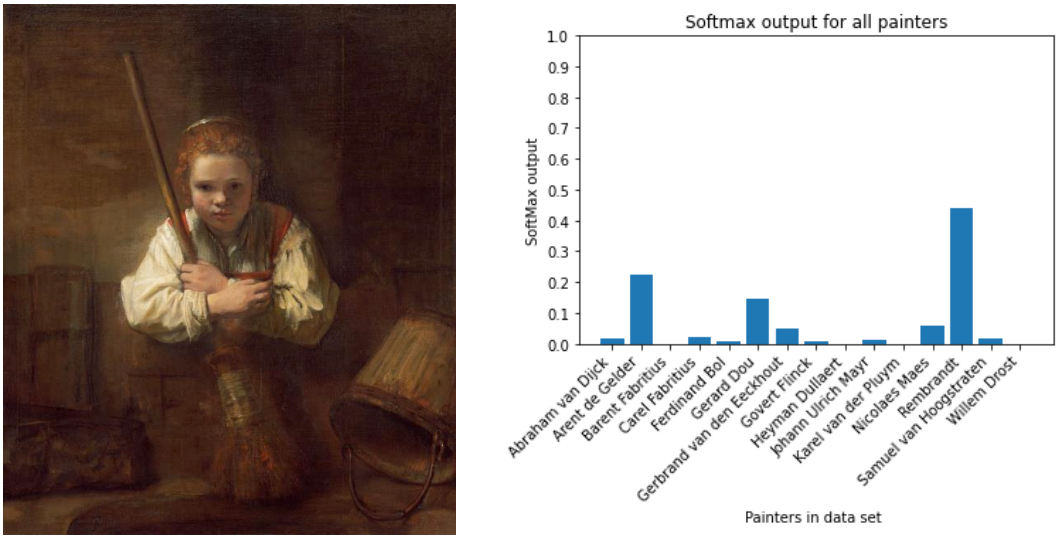
17a/b *The Slaughtered Ox*, formerly a Rembrandt original, now a workshop image, along with the associated SoftMax output

One of the less surprising rejections of the Rembrandt Research Project was the painting *The Slaughtered Ox* (c. 1640). According to RKD, numerous art historians had doubts about the authenticity of the work, yet it was commonly considered an original by the master and was listed as such in various catalogue raisonnés[16] – until the RRP copied the work in 1989 and declared it a workshop painting ("Studio of Rembrandt"), explicitly mentioning Carel Fabritius as a possible author. The work is shown in Figure 17a/b, along with the SoftMax output of the ensemble of CNNs. Although Rembrandt's style is made out by the ensemble in the artwork (which is not surprising for a workshop painting), it is neither seen as an original Rembrandt by the model as a whole, nor by any of the individual CNNs (tab. 4).

Instead, the style of both Abraham van Dijck and Gerbrand van den Eeckhout is dominant. However, this result should not be overinterpreted, firstly because the distribution of the values does not provide a clear picture as the SoftMax difference rule is violated, and secondly because the actual pain-

Modell	SoftMax Rembrandt	Predicted	SoftMax Predicted
CNN1	0.31733	Abraham van Dijck	0.34047
CNN2	0.39493	Abraham van Dijck	0.41836
CNN3	0.21116	G. van den Eeckhout	0.49480
CNN4	0.26749	G. van den Eeckhout	0.65851
CNN5	0.26196	G. van den Eeckhout	0.39202
Ensemble	0.29057	G. van den Eeckhout	0.37824

Tab. 4 SoftMax results and predicted painters of the individual CNNs and the ensemble as a whole for the Rembrandt workshop image *The Slaughtered Ox*



18a/b Painting *Girl with a Broom*, formerly attributed to Rembrandt and today, according to RKD, "possibly Carel Fabritius", together with the corresponding SoftMax output

ter may not even be present in the data set. Also, despite the very low SoftMax value for this probably most talented student of Rembrandt, the possible attribution to Carel Fabritius cannot be rejected based on the available data. Too few paintings have survived of Fabritius, who died at the age of 32 in the explosion of the Delft Powder Tower, where he lived and had his studio. Therefore, the neural networks cannot learn the style of this painter.

As a conclusion, however, it remains that the rejection of the painting by the RRP from an original to a workshop painting was certainly justified – and in view of the long prevailing doubt among art historians probably also overdue.

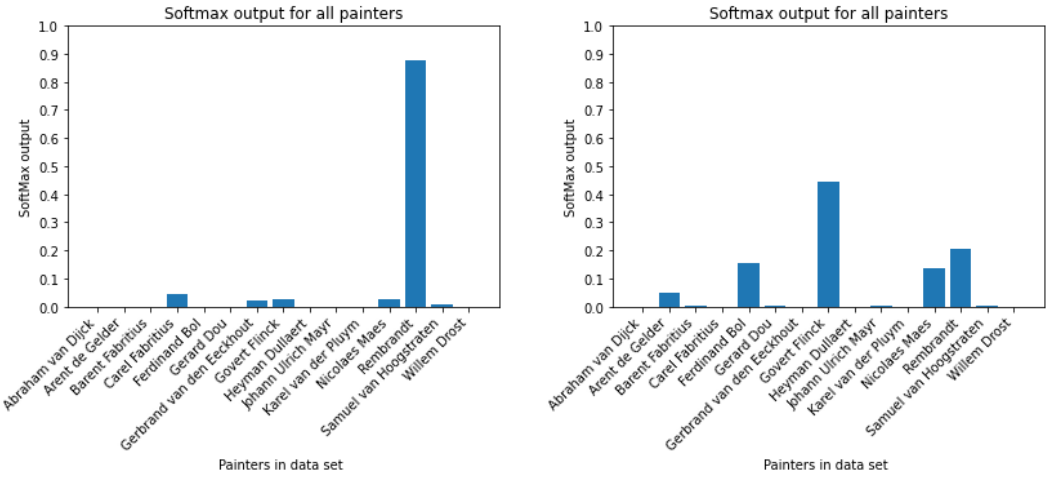
Also controversial is the painting shown in Figure 18a *Girl with a Broom* (between 1646 and 1651). In 1969, Horst Gerson raised doubts about the authenticity of the work as a Rembrandt original (which it had been seen as until then)[17], the proposals of art historians concerning the authorship accumulated. Werner Sumowski suggested Samuel van Hoogstraten as the creator[18], Görel Cavalli-Björkmann considered Carel Fabritius and Rembrandt's workshop as the originators[19], Arthur K. Wheelock agreed with Sumowski that the painting was painted by a pupil of Rembrandt, however, he assumed that it was most likely by Carel Fabritius[20].

The ensemble of CNNs does recognize Rembrandt's style in the work (fig. 18a), but the SoftMax output of 0.438 is low on the one hand – and above all, the SoftMax difference rule is violated. The gap to Arent de Gelder (0.224) is 0.214, that to Gerard Dou (0.148) still 0.29. The difference rule is also violated regarding the SoftMax output of Nicolaes Maes (0.058) and Gerbrand van den Eeckhout (0.051), albeit just barely. For this reason, the rejection seems justified, even if the situation is not quite as clear as with the painting *The Slaughtered Ox*. However, the SoftMax output across all artists is too diffuse to make a statement regarding the student who painted it.

19a/b Undoubted Rembrandt original *Portrait of a Man, Seated*, as well as its counterpart *Portrait of a Woman, Seated*, doubted by some authors, both Kunsthistorisches Museum, Vienna

Also interesting, are the paintings (19a/b), which are designed as complementary counterparts. Both works are painted on walnut, a rather unusual substrate. The postures of the two figures are mirror images of each other. In the case of the portrait of the man, art historians have no doubts about its authenticity as a Rembrandt original, and the portrait of the woman was also considered a genuine Rembrandt for centuries. In 1986, however, Tümpel doubted Rembrandt's authorship of the Portrait of the Woman[21], an opinion shared by the authors of the catalogue raisonné *A Corpus of Rembrandt Paintings*[22]. There it says: "Although it was already regarded as a work of Rembrandt in the 18th century together with its counterpart and has always been accepted as such in modern literature, this portrait of a woman cannot be regarded as his work due to the different design and execution. It was certainly intended as a companion piece to 'Portrait of a Man, Seated', as is evident from the use of the same, unusual type of wood – walnut – and the sitter's complementary poses. Judging by the handling of the paint, the work was probably executed in Rembrandt's workshop by an assistant already trained elsewhere."

The ensemble of CNNs can only underline the assessment of Tümpel and the authors of *A Corpus of Rembrandt Paintings*. The SoftMax output for the portrait of a woman is clear (fig. 20b). The painting is not even recognized as an original by Rembrandt, instead Govert Flinck shows the highest SoftMax output. In contrast, the portrait of the man is clearly recognized as a Rembrandt original. However, the SoftMax output is not representative here; the work, unlike the portrait of the woman, is in the training dataset of the model due to the unequivocal attribution to Rembrandt.



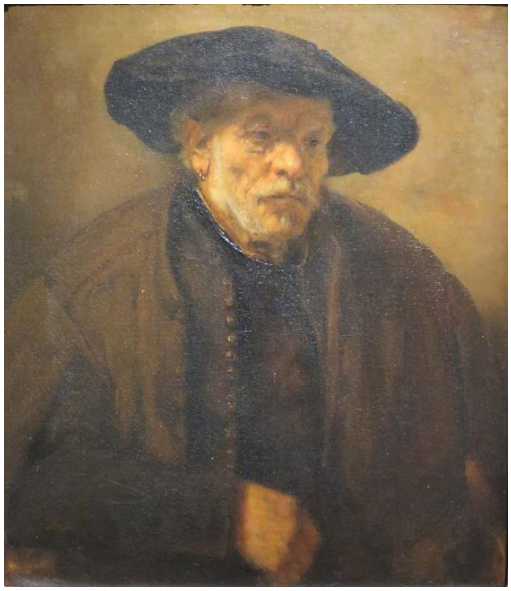
20a/b SoftMax output for Rembrandt original *Portrait of a Man, Seated* (20a) and for its doubted counterpart *Portrait of a Woman, Seated* (20b)

However, the presence of its counterpart in the training dataset makes the rejection of the portrait of the woman even more plausible. If a painting very similar to a test image is present in the training dataset, the test image should actually be recognized better. If the similarity is too great, this is called 'data leakage', which makes the algorithm appear better than it really is. However, this effect does not occur here. Although the neural networks have learned the style of the man's portrait directly, the counterpart is not recognized as a Rembrandt original. The SoftMax output for Govert Flinck is 0.445, that for Rembrandt only 0.205. Thus, the difference rule is violated, an attribution to Flinck cannot be justified based on the model results, but also not rejected. The model makes no statement in this regard.

The difference, however, is relatively large, especially because the SoftMax output is with respect to a student who are generally recognized worse by the model. The model result could therefore be understood by art historians as an incentive to check the portrait of the woman for the style of Govert Flinck, especially because the historical key data fit. Flinck worked as a pupil in Rembrandt's workshop from 1632 and had also previously been trained (since 1629) by Lambert Jacobsz in Leeuwarden.

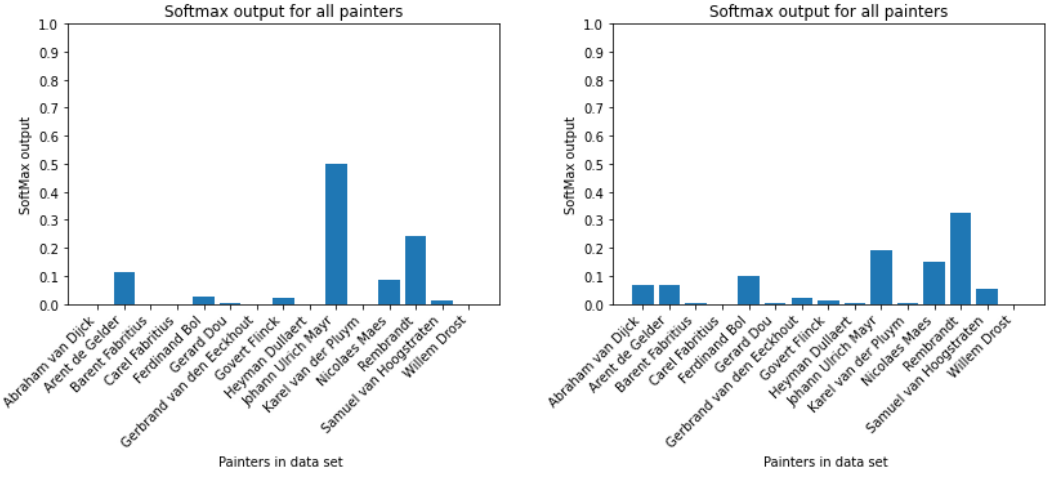
Also, the two paintings *Half Length Figure of an Old Man with Beret* (1654) and *Half Length Figure of an Old Woman* (1654) are seen by most art historians as complementary portraits[23]. Leonhard J. Slatkes, however, rejected this view; he did not consider the works to be counterparts[24]. Moreover, both portraits were thought of as undisputed originals by Rembrandt until 1986. After Tümpel doubted the authenticity[25], other art historians followed. The two works (fig. 21a/b) are attributed by the Rembrandt Research Project to Rembrandt's circle. They were not included in the latest edition of the *Corpus of Rembrandt Paintings*[26] as originals by the master. The rejections are widely accepted. The Pushkin Museum, however, still considers the works to be originals by Rembrandt.

Whether the two portraits are counterparts cannot be determined with the architecture of the ensemble of CNNs used here. Their authenticity as Rembrandt originals, however, is rightly questioned, as can be seen in Figure 22a/b.

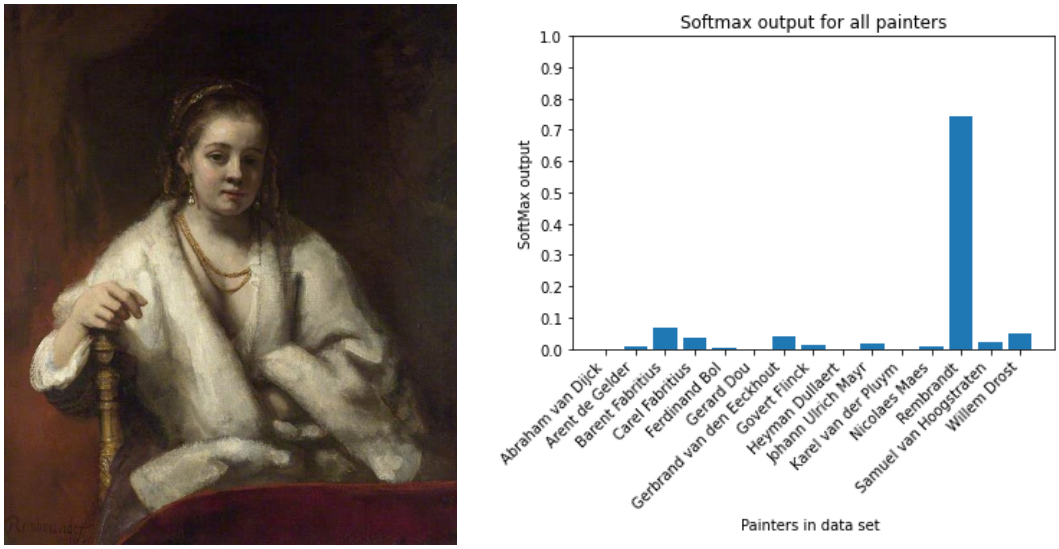


21a/b Possible counterparts *Half Figure of an Old Man with Beret* and *Half Figure of an Old Woman*, both in the Pushkin Museum in Moscow. The works were considered originals of the artist until the 1980s, today they are attributed to the school or circle of the master

The portrait of the man (fig. 21a) possibly representing Rembrandt's brother Adriaen is not recognized by the model as Rembrandt, but as a work by Johann Ulrich Mayr, a Rembrandt pupil from Augsburg. The difference of the SoftMax output for Mayr (0.501) to Rembrandt (0.240) is 0.261 – according to the admittedly strict difference rule, no attribution to Mayr can be made based on this



22a/b SoftMax output for *Half Figure of an Old Man with Beret* (22a) and *Half Figure of an Old Woman* (22b)



23a/b Rembrandt original *Half Figure of a Woman with a White Wrap*, consistently identified as depicting Hendrickje Stoffels, along with the associated SoftMax output

result. The nevertheless high difference could, however, be used by art historians as an indication of a possible authorship by Mayr and justify further stylistic investigations.

The portrait of the woman (fig. 21b), on the other hand, is recognized as Rembrandt, but the SoftMax output of 0.326 is very low, and the difference to several painters is extremely small, in particular to Johann Ulrich Mayr, who is in second place with a value of 0.192. An attribution to Rembrandt based on this diffuse result picture would be irresponsible from our point of view. However, the result can very well be seen as a confirmation that the two works were rightly rejected as originals, by the RRP.

The opinions of art historians and the museum that owns it also diverge on another portrait of Rembrandt. This is the painting shown in Figure 23a, *Half Figure of a Woman with a White Wrap* (1654–1656). Rembrandt did not name the figure in the painting, but she is consistently identified as Hendrickje Stoffels, Rembrandt's maid and later mistress, with whom he fathered a daughter in 1654. The description from the National Gallery in London states that the sitter "has an almost regal poise. She looks down on us from a slight height, her right hand resting on what must be part of the arm of a chair, but which has the air of a sceptre. She wears expensive pearl earrings and jewellery and what seems to be a fur mantle. All these might hint at the trappings of royalty, yet this is also an intimate – even erotically charged – portrait. Her dress seems to be unfastened and the mantle falls slightly open, revealing much more of her breast than would have been acceptable in a formal portrait of the time."[27]

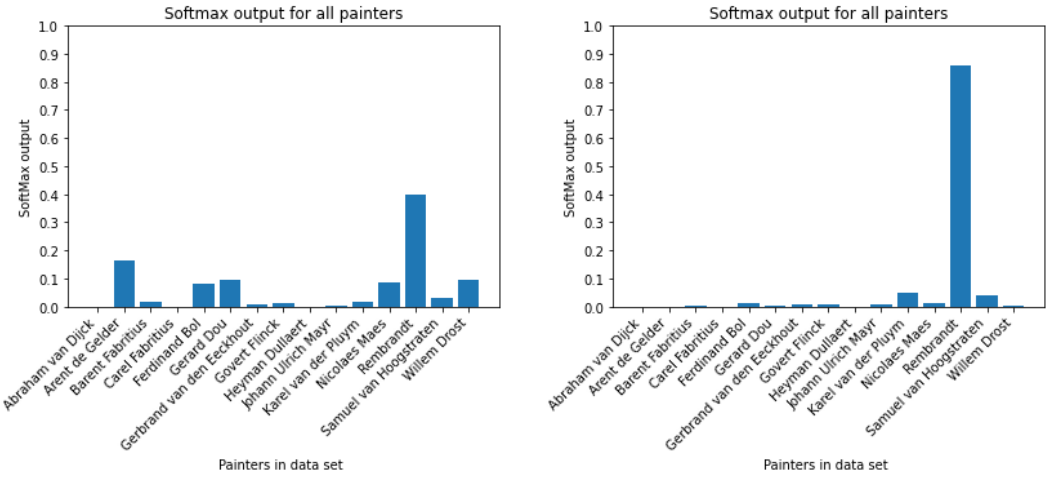
Human intelligence or intuition would perhaps question that a painter who was already famous at the time would have his mistress painted like this by a student. In any case, the painting was always considered an original by Rembrandt until Tümpel attributed it to an unknown painter from the master's circle in 1096[28]. Most art historians, however, did not follow this assessment and insisted on an attri-

24a/b Rejected Rembrandt painting *Pallas Athena*, Lisbon, Museo Calouste Gulbenkian (24a) and according to newspaper reports recently recognized as Rembrandt original work *Hendrickje Stoffels as Pallas Athena* from a European private collection (24b)

bution to Rembrandt, or at least to his workshop[29]. A test with the ensemble of CNNs confirms the attribution to Rembrandt (fig. 23b). The work is clearly identified as Rembrandt with a SoftMax output of 0.742; the values for all other artists are less than 0.1. As a conclusion, it can be said that the National Gallery is quite right to classify the painting as Rembrandt's original on its website to date[30].

Another portrait of Hendrickje Stoffels (fig. 24b) has recently caused a furor. It is the depiction of Rembrandt's later companion as Pallas Athena, which was since 2019 shown in various exhibitions, including in Aalen, Augsburg as well as in the Museo Nacional De Arte Munal in Mexico City[31]. According to the weekly newspaper *Kontext*, the work appeared in 1678 in the estate of Herman Becker, a Rembrandt admirer[32]. The art historian Werner Sumowski discovered the painting in poor condition in the art market in the late 1950s and had it restored. In 1962, he published an article in Pantheon about the work, suggesting it was a depiction of Pallas Athena painted by Rembrandt[33].

However, first, the restoration had failed; according to Dietrich Heißenbüttel[34], the face was painted over so that the work looked like a 19th-century painting; second, Sumowski's article included only a small black-and-white photograph of the mythological representation. Sumowski's attribution was not accepted by other art historians. During a new restoration, the overpaintings from the 1950s were removed. Afterwards, the Institut für Technologie der Malerei at the Stuttgart Art Academy examined the painting, analyzing the priming and pigment layers. Prof. Dr. Christoph Krekel concluded: "All pigments and fillers found on the painting were available at the time indicated. The structure of the primer can be regarded as typical for Rembrandt"[35]. In various newspaper articles as well as mu-



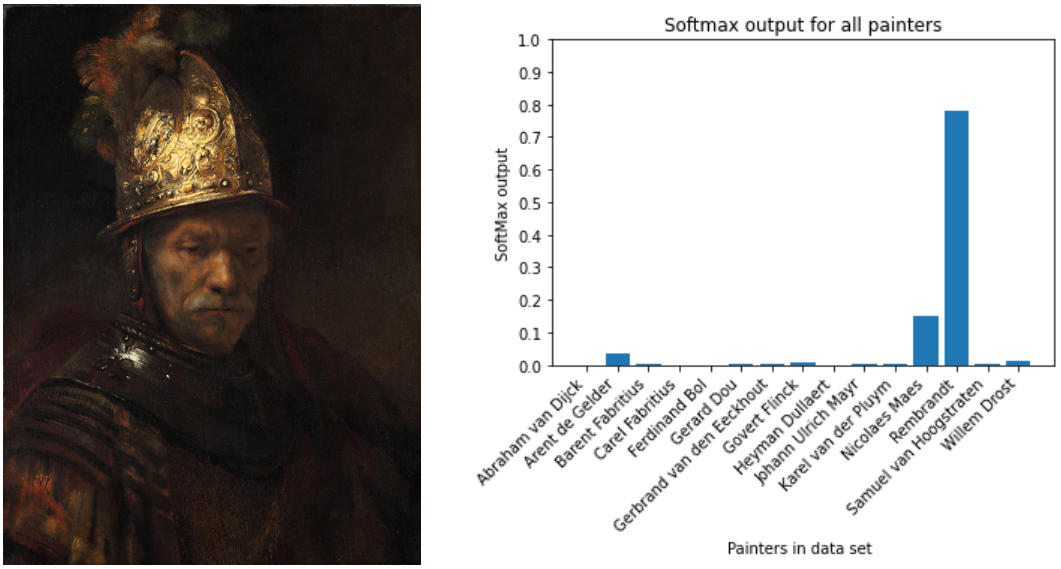
25a/b SoftMax output for *Pallas Athena*, Lisbon, Museo Calouste Gulbenkian (25a) and *Hendrickje Stoffels als Pallas Athena*, European private collection (25b)

seum publications, the work is therefore described as "recently attributed to Rembrandt" or "recently recognized as an original"[36], which however, represents an overinterpretation of the expert opinion. Krekel emphasized upon request that he did not conduct a stylistic examination of the painting[37].

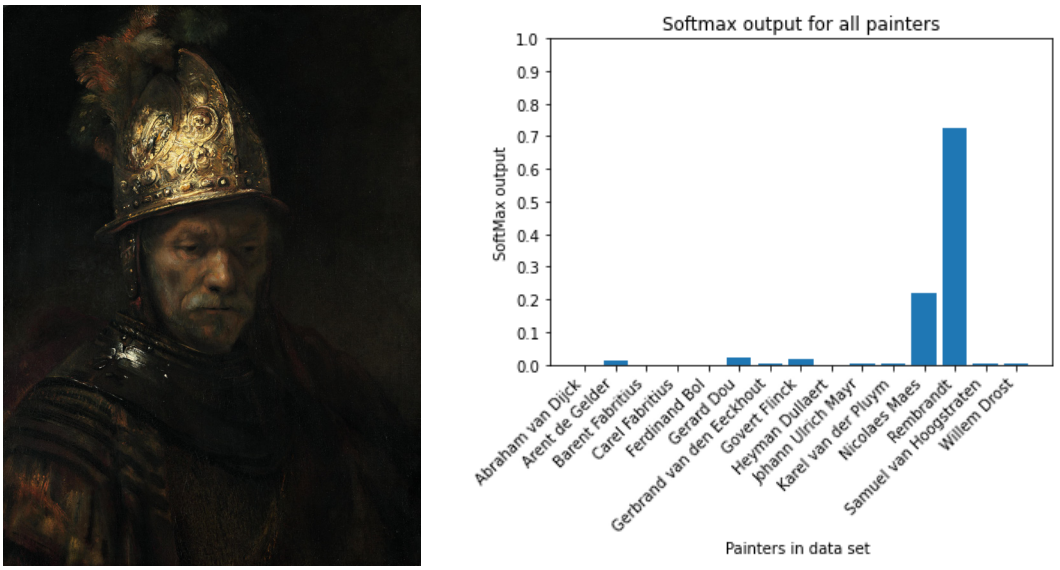
This painting is not available in the online archive of the RKD, but another depiction of Pallas Athena, which was rejected by the Rembrandt Research Project (fig. 24a) is. The painting in Lisbon was always considered a Rembrandt original, although the figure has also been interpreted as Minerva, Mars, Alexander the Great, or simply as a warrior. Christopher Brown attributed it to an anonymous pupil from Rembrandt's workshop in 1991[38]. Jeroen Giltaij followed in 1999 with doubts about its authenticity[39] and van de Wetering classified the work in 2006 as painted by Rembrandt with the help of an employee of his workshop[40]. This view is consistent with the Rembrandt Research Project's assessment and prevails today[41]. It is also covered by the ensemble from CNNs. The SoftMax output of the work in Lisbon (fig. 25a) is 0.400 for Rembrandt – making it the highest among all artists in the dataset, but just not unequivocal, since the difference rule is violated. In particular, the style of Arent de Gelder is also prominently represented with a SoftMax output of 0.164.

The situation is quite different with the depiction of *Hendrickje Stoffel as Pallas Athena* (fig. 25b). This painting is clearly classified as an original by Rembrandt with a SoftMax output of 0.859. The corresponding values for the other artists are all below 0.05.

As a conclusion it can be said: The SoftMax output for the painting from Lisbon does not allow a clear statement due to the difference rule, but it agrees with the estimation of van de Wetering and the RRP that Rembrandt painted the painting together with a pupil. The rejection as an original by the master is justified, as is even more clearly, the attribution of the depiction of *Hendrickje Stoffel as Pallas Athena* as an authentic work by Rembrandt.

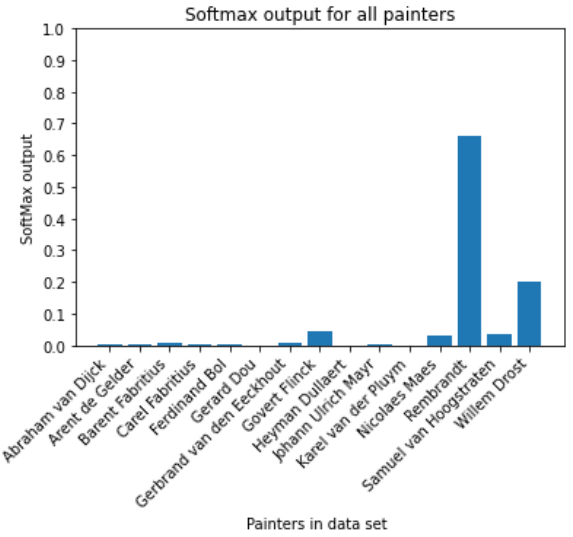


26a/b Rejected Rembrandt painting *The Man with the Golden Helmet*, Gemäldegalerie Berlin, including the corresponding SoftMax output



27a/b Rejected Rembrandt painting The Man with the Golden Helmet, here the high-resolution version from the website of the Gemäldegalerie Berlin, including the corresponding SoftMax output

The Rembrandt Research Project also rejected a painting very similar in color and motif to the Lisbon work, which was always considered an original by the artist (fig. 26a). It is *The Man with the Golden Helmet* (c. 1650–1660). The first to doubt its authenticity was Benjamin A. Rifkin in 1969[42].



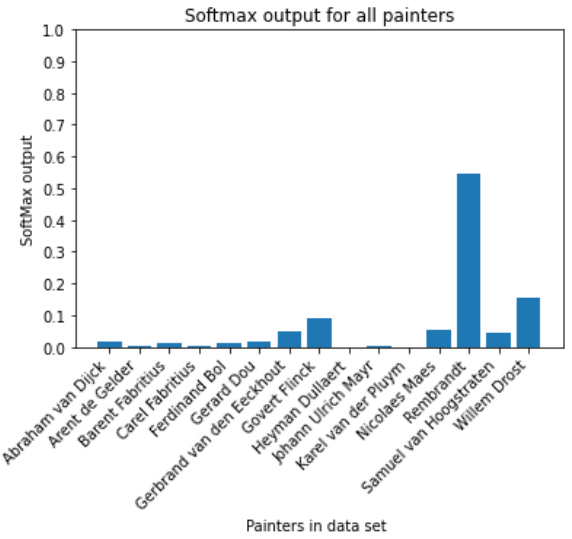
28a/b Rejected Rembrandt painting *Old Woman Cutting her Nails*, New York, Metropolitan Museum, along with associated SoftMax output

Keith Roberts[43], Christopher Brown[44], and Claus Grimm[45] followed until the Staatliche Gemäldegalerie accepted the rejection[46]. Hennig Bock, director of the Gemäldegalerie at the time, wrote in summary: "The result of all the efforts is probably a final confirmation that the 'Man with the Gold Helmet' cannot be painted by Rembrandt himself," but the artistic value of the work remains undiminished. The RRP eventually attributed the painting to Rembrandt's circle[47]. In this case, however, the rejection does not seem to be justified. The model (fig. 26b) recognizes the style of Rembrandt with a SoftMax output of 0.780, the painter with the second highest value is Nicolaes Maes (0.149). The result meets the difference criterion by far.

To be sure, the image was tested again, this time in a high-resolution version (2982 × 4000 pixels) downloaded from the Gemäldegalerie Berlin website (fig. 27a). The result was confirmed. The SoftMax output is now only 0.723, but the difference criterion is still met (fig. 27b). The second highest SoftMax value, again with Nicolaes Maes, is 0.217.

As a conclusion, it can be said that this work is very likely an original by Rembrandt. Further stylistic and technical studies by art historians and scientists are definitely indicated in this case.

A similar, though not quite as clear, situation arises with the painting *Old Woman Cutting her Nails* (1650s). The painting has had one of the most tumultuous attribution and rejection odysseys, reflected in the 89 literature references on the Metropolitan Museum website[48]. The work was long considered a Rembrandt before it was called into question. According to RKD, provenance can be traced back to 1764. The Metropolitan Museum states that, Abraham van Dijck and Nicolaes Maes were or are primarily considered candidates for authorship, while Sumowski attributed it to Karel van der Pluym[49]. At the RKD it is today listed as "Follower of Rembrandt", the Metropolitan Museum limits itself to the designation "In the style of Rembrandt", which can be interpreted as an elegant paraphrase for the



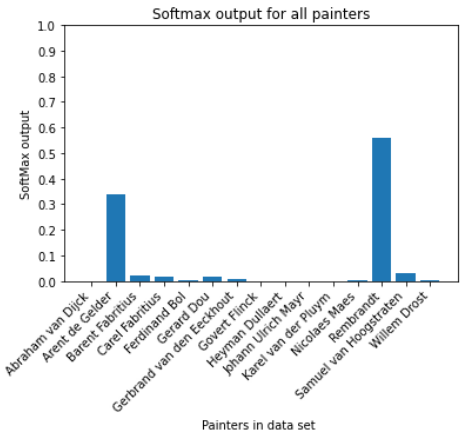
29a/b Rejected Rembrandt painting *Old Woman Cutting her Nails*, New York, Metropolitan Museum, shown here in the higher-resolution version along with the associated SoftMax output

fact that nobody really knows who the creator is. The museum's website goes on to say, "The monumental work is certainly inspired by Rembrandt." Or is it painted by Rembrandt after all?

The ensemble from CNNs at least provides veritable clues for that. The painting, along with its associated SoftMax output, is shown in Figure 28a/b. The SoftMax output for Rembrandt is not only the highest at 0.660, but the difference criterion is also met. The second highest SoftMax value, for Willem Drost, is 0.199 and the difference is thus above the value of 0.4. However, to be sure, this image was also downloaded and tested in the higher resolution version of the Metropolitan Museum's website (974 × 1200 pixels; fig. 29a). In this version, the SoftMax output (fig. 29b) for Rembrandt is 0.547, and that for Willem Drost is 0.155. The difference criterion is violated here, albeit only just.

Although the model, due to the strict difference criterion, does not make a determination for the higher resolution version of the painting, the two results should be considered together. Further stylistic study of the painting, along with scientific analyses, is certainly indicated here.

The difficulty of classifying paintings as originals by Rembrandt, works by students, or even as early copies of a masterpiece is illustrated by four examples where the Rembrandt Research Project has corrected itself over time. The first of the works considered below is the mythological representation *Jupiter and Mercury visiting Philemon and Baucis* (c. 1658). According to Arthur K. Wheelock, the work was transferred to a ribbed, horizontally grained piece of wood, and the original support medium was probably also wood. It has no primer; it was presumably removed when the painting was transferred. Instead, the wood and the painting layers are now separated with gauze[50]. This information is relevant because the priming structure of about 150 Rembrandt works has been studied and documented as part of the Rembrandt Research Project[51]. The priming can therefore in many cases provide important



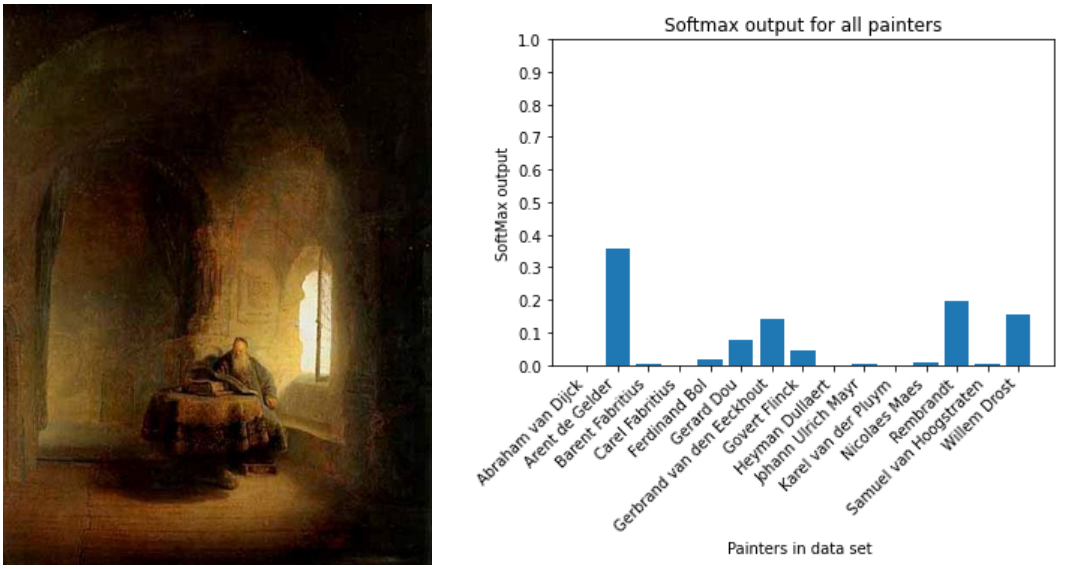
30a/b Controversial Rembrandt painting *Jupiter and Mercury visiting Philemon and Baucis*, Washington, National Gallery, along with associated SoftMax output

clues to the authorship of a work, especially if it is atypical for Rembrandt and his workshop. But not in this case.

The painting was always considered to be Rembrandt's original until it was challenged by Tümpel in 1986[52]. Van de Wetering considered the signature questionable[53]. The Rembrandt Research Project initially classified the work as "created by Rembrandt or one of his pupils,"[54] but later corrected this view and has since considered it an "original by Rembrandt."[55] The image is shown in Figure 30, along with the ensemble's SoftMax output from CNNs. As can be seen there, the work is identified as Rembrandt with a SoftMax output of 0.560, but the distance to Arent de Gelder (0.339) with 0.161 is too small to fulfill the difference criterion. According to the strict rules for attribution applied here, the model does not make a decision.

Nevertheless, the strong focus in recognizing the painting styles of Rembrandt and his last pupil is striking. This is not necessarily significant but could warrant further stylistic investigation. If de Gelder executed some passages of the painting, this would imply that the dating of the painting to "c. 1658" is not entirely correct, since de Gelder did not work for Rembrandt until 1661.

Another painting about which the Rembrandt Research Project was undecided and corrected itself, is *An old Scholar in a Vaulted Room* (1631). The first doubts about its authenticity as a Rembrandt original date back to 1882, when Alfred von Wurzbach described the work as an early forgery that "must have been painted roughly between 1631 and 1650." He argues, on the one hand, stylistically, finding the painting "held in a light, characterless and dull tone" that contrasts strongly with "the deep, mysterious chiaroscuro in which the 'Representation in the Temple,' in the Hague [meaning the Royal Picture Cabinet in The Hague] and the two [in Wurzbach's original text] aforementioned philosophers in the Louvre are painted." His main argument, however, is technical. Rembrandt would have had to mirror a figure from the third state of an etching, in which Wurzbach identifies a person as identical to the scholar depicted in the painting, and at the same time copy this figure exactly. He would have thus, according to Wurzbach, "mindlessly and slavishly copied himself."[56]



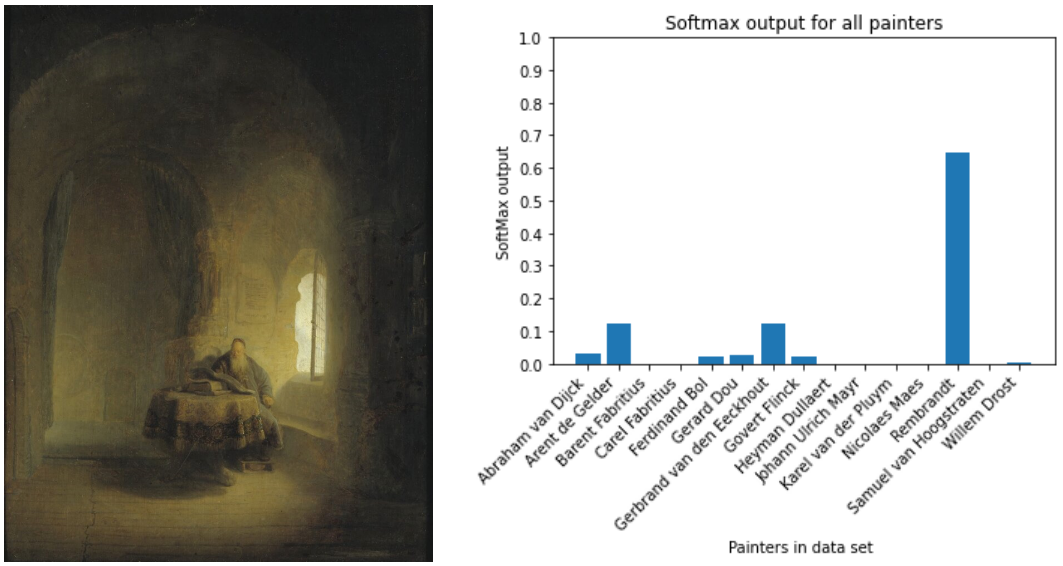
31a/b Controversial Rembrandt painting *An old Scholar in a Vaulted Room*, Stockholm, Nationalmuseum, along with associated SoftMax output

The Rembrandt Research Project classified the work 100 years later as an "old copy" of a lost work by Rembrandt[57], without naming an author, but attributed it again in 2015[58]; since then it has been considered a Rembrandt original. An important indication of the rejection by the RRP in 1982 was the signature. In the letter or plaque to the left of the window, the painting is signed "Rembrant fc. 1631". But Rembrandt did not sign with his first name until 1632, which Wurzbach also mentions[59]. In 1631, he still used the monogram "RHL" However, there is one exception, namely the painting *Self Portrait in Oriental Costume with Poodle* in the Musée du Petit Palais, Paris, which was painted between 1631 and 1633. The RRP concluded that Rembrandt predated the painting – and it (later) adopted this view for the work under consideration here, which is shown with the associated SoftMax output in Figure 31a/b.

Neither is the painting recognized as Rembrandt, nor can it be assigned to a student from the data set according to the difference criterion. Any speculations regarding the comparatively high SoftMax value for Arent de Gelder are forbidden from our point of view, at least if the dating of the work to the year 1631 is correct. De Gelder was not born until 1645. The wood on which the work is painted dates from 1624 according to a dendrochronological study by the National Museum in Stockholm but was probably painted after 1630[60].

The value must therefore be interpreted as an artifact because the actual creator of the work is very likely not present in the data set. In 2005, Görel Cavalli-Björkman, then head of the research department at the National Museum in Stockholm, classified the work as from a "successor of Rembrandt."[61] Today, the museum lists this work as an original[62]. And possibly rightly so.

We have tested the image, like any other with a result different from the RRP's verdict, in a higher resolution. For this purpose, we have usually, and also in this case, downloaded the variant of the respective museum. However, in this case the color scheme of the online available image from the Na-

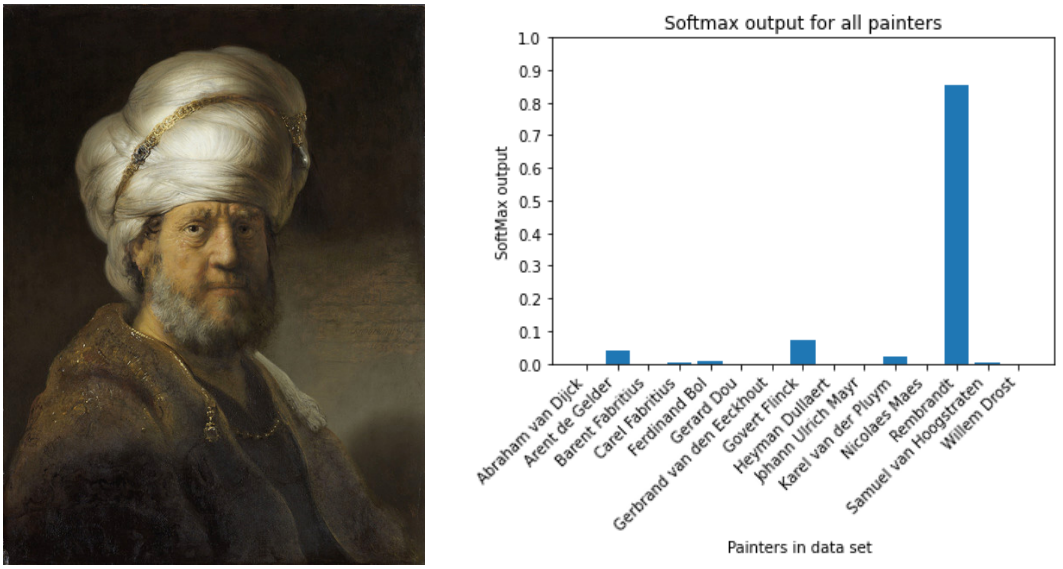


32a/b Controversial Rembrandt painting *An old Scholar in a Vaulted Room*, Stockholm, Nationalmuseum, shown here in the higher-resolution version, along with the associated SoftMax output

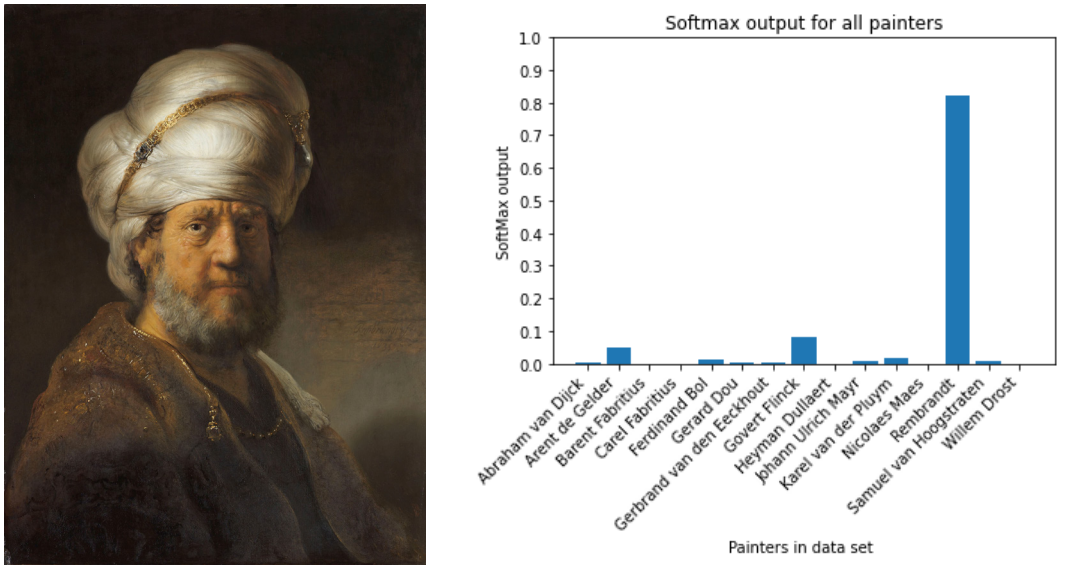
tional Museum Stockholm (700 × 1000 pixels) is extremely different from the variant deposited at the RKD. The image is shown in Figure 32a/b together with the corresponding SoftMax output. In this rather greenish image with weaker contrasts, the painting is recognized as Rembrandt with a SoftMax output of 0.648. Moreover, the difference criterion is fulfilled, the distance to Gerbrand van den Eeckhout, the painter with the second highest value, is 0.524. We show this example explicitly to point out the uncertainties, indeed possibly shortcomings, of an analysis with CNNs. In the section *Further Analyses* we will show ways to deal with such uncertainties.

As a conclusion, it remains that the model contradicts itself here, as, however, do the members of the Rembrandt Research Project and other art historians. The result cannot be used without further analysis, especially because we do not know which of the two variants available online corresponds more closely to the original. However, this is the only case where the model results for two variants of the same work differ dramatically.

The Rembrandt Research Project has also corrected itself for the next two paintings we discuss here. First, it concerns the work *Bust of a Man in Oriental Dress* (1635). Until 1989, the work (fig. 33a) was considered an original by Rembrandt. In the third edition of the *Corpus of Rembrandt Paintings* the RRP classified this painting as a workshop product[63], while in the latest edition it is again listed as Rembrandt's original[64]. In between, some members of the RRP and experts, especially Norbert Middelkoop, considered it either an original or the work of a pupil[65]. Werner Sumowski attributed it to Govert Flinck[66] but could not prevail with this view. The painting is clearly recognized as Rembrandt with a SoftMax output (fig. 33b) of 0.855, the values for all other painters in the data set are below 0.05.

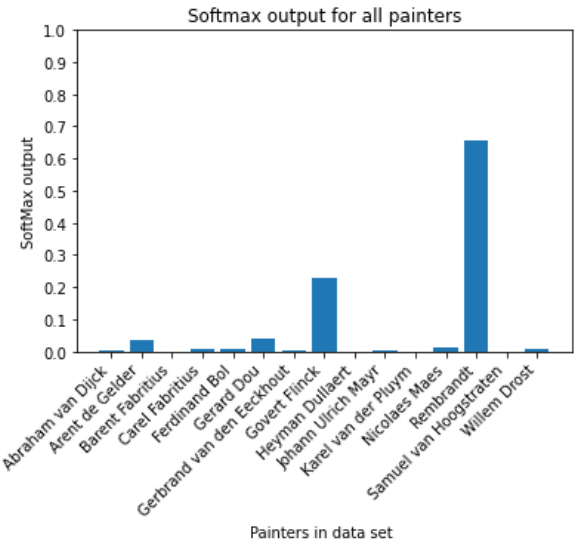


33a/b Considered to be an original by Rembrandt again today. The painting *Bust of a Man in Oriental Dress*, Amsterdam, Rijksmuseum, together with the accompanying SoftMax output



34a/b The painting *Bust of a Man in Oriental Dress*, Amsterdam, Rijksmuseum, shown here in the higher-resolution version, along with the corresponding SoftMax output

The high-resolution image on the Rijksmuseum website (4282 × 5690 pixels) was also tested (fig. 34a/b). In this case, the SoftMax output is 0.820 and, except for Govert Flinck (0.080), the values for all painters are below 0.05. The conclusion in this case is that the model refutes the original rejection



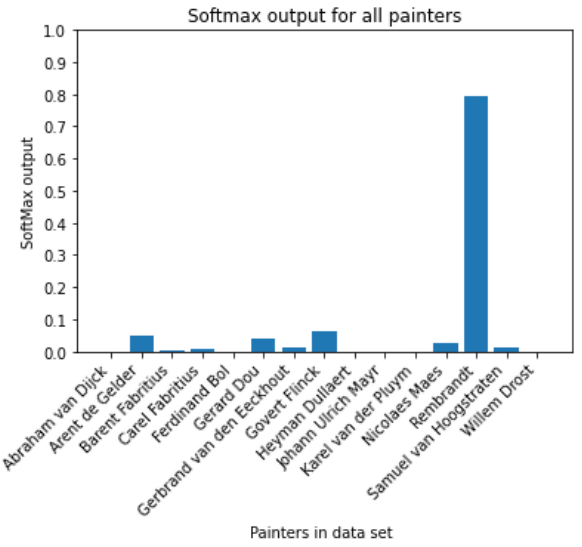
35a/b The painting *Self-Portrait in a Black Cap*, London, Wallace Collection, together with the corresponding SoftMax output

of the painting by the Rembrandt Research Project as an original but strongly confirms the later renewed attribution.

A dendrochronological report played an essential role in the renewed attribution, proving that the wood on which the *Bust of a Man in Oriental Dress* is painted comes from the same tree as the wood that served as the ground for three other paintings by the master[67] (43, 9). Two of these works, namely *Landscape with a Thunderstorm* (1638–1640), Herzog Anton Ulrich-Museum, Braunschweig, and *Self Portrait with a Velvet Baret* (1634), Staatliche Gemäldegalerie Berlin, have never been challenged as originals. They are in the model's training dataset.

The third painting, *Self Portrait in a Black Cap* (c. 1637), although always considered an original, was initially doubted by Gerson in 1968[68]. Tümpel categorized it as a workshop painting in 1986[69], an assessment that the Rembrandt Research Project initially adopted[70] but recanted in 2005[71] and has since considered it an original. The RKD still lists this work as "attributed to Rembrandt or Studio of Rembrandt," and because of this ambiguous attribution, this painting is not in the dataset and could be tested. The work, along with the associated SoftMax output, is shown in Figure 35a/b. It is clearly recognized as an original by Rembrandt with a SoftMax output of 0.658. The second highest SoftMax value, for Govert Flinck, is 0.230, so the difference criterion is met.

This is also true for the higher resolution variant from the Wallace Collection website (1491 × 1960 pixels; fig. 36a). The SoftMax output (fig. 36b) for Rembrandt is even higher here at 0.7936, while that of Govert Flinck, still the painter with the second highest value, is lower (0.062). Again, the model can confirm the final attribution by the Rembrandt Research Project.



36a/b The painting *Self-Portrait in a Black Cap*, London, Wallace Collection, here in the high-resolution version, together with the corresponding SoftMax output

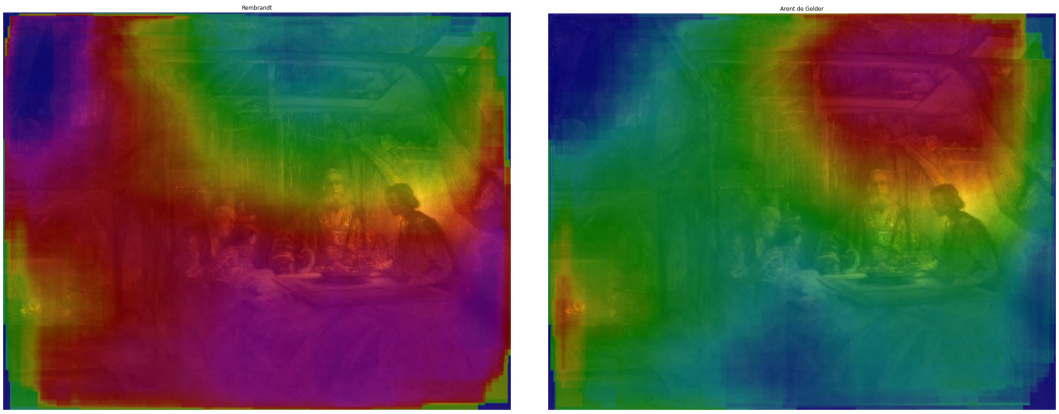
Further Analysis

Class Activation Maps (CAMs) are a useful method for analyzing paintings. They show in which parts of the work a neural network accounts for the style of a certain painter.

An example of such a visualization can be seen in Figure 37 *for the painting Jupiter and Mercury visiting Philemon and Baucis*, at the National Gallery in Washington. The analysis of the painting (fig. 30b) with the ensemble of CNNs did not yield a result because the SoftMax difference criterion was violated. However, without the strict difference rule, the work is recognized as Rembrandt, with the style of Arent de Gelder also prominent. The visualization with CNNs now shows that Rembrandt's style is mainly represented in the central parts of the image, while de Gelder's is recognized especially in the upper right. This is a typical pattern. Artists from all periods have always spent more time and effort on central parts of the picture, and the edges of their works are often only rudimentarily executed. The same is true for workshop painters, who often left backgrounds or marginal sections to students. We will not revise the judgment of the model, which in this case did not lead to any result, based on this visualization. We merely want to show that an analysis with Class Activation Maps can give art historians further clues that may be helpful in assessing and attributing a work to a particular artist. In any case, an attribution to Rembrandt now seems more plausible than without this result.

Regarding the involvement of Arent de Gelder, the CAMs do not help much either, especially because not all of Rembrandt's students are represented in the dataset. However, the CAM analysis may also provide clues for art historians as to which parts of the picture they should possibly examine stylistically for a certain artist (or even several possible artists). From our point of view, CAMs can be another valuable analysis method to support art experts in their work.

In presenting and evaluating the ensemble from CNNs, we also saw that while this model architecture is very good at recognizing Rembrandt (and also Nicolaes Maes) – it fails completely or at least



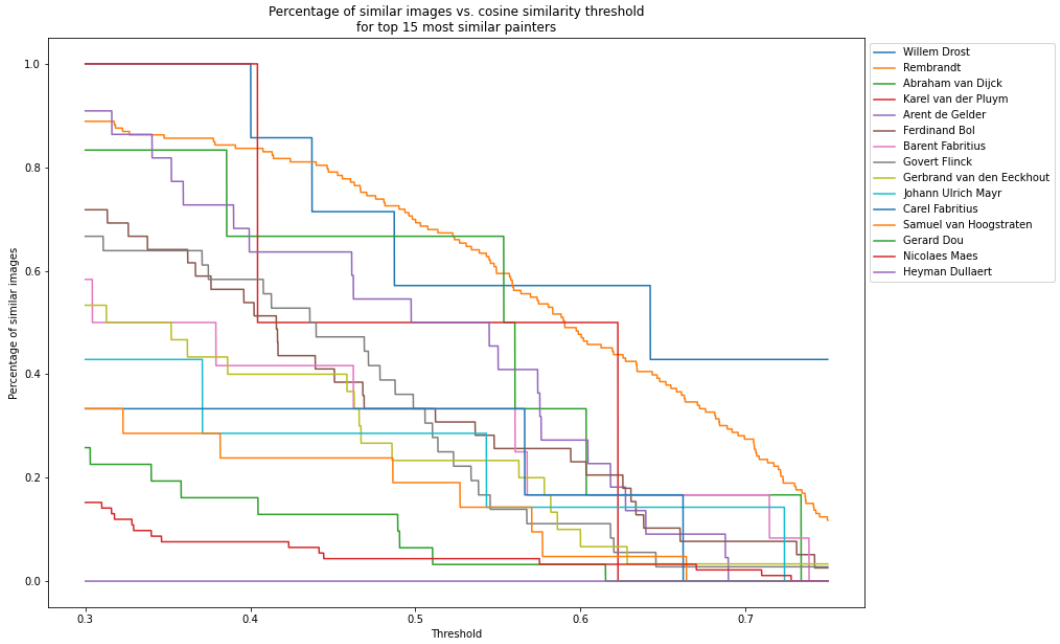
37a/b The painting *Jupiter and Mercury visiting Philemon and Baucis*, overlaid with a Class Activation Map for the painting style of Rembrandt (37a) and Arent de Gelder (37b). The more the colors go into the reddish, the more the painter in question is recognized

largely, with some other students. This has primarily to do with the very small number of secured works by these artists. Siamese networks, on the other hand, are largely robust to so-called class differences, i.e., a widely varying number of data points per class, in this case of paintings per painter. They are often used in face recognition, but also for signature verification. The main difference to classical CNNs is that these networks do not learn the style of each painter in the dataset – but only whether two paintings are similar or not. We trained a Siamese network using the dataset of undisputed paintings by Rembrandt and the 14 selected students. The weighted average of accuracy across all painters is 59 percent. The difficulty in handling Siamese networks is the evaluation of similarity. To do this, a threshold is usually calibrated. If the measure of similarity is above the threshold, the images are assigned to the relevant class, i.e., in the case of the similarity of two faces, to the person in question, and in the present case, to the painter in question. The problem with this is that the choice of threshold can be arbitrary and need by no means be unambiguous.

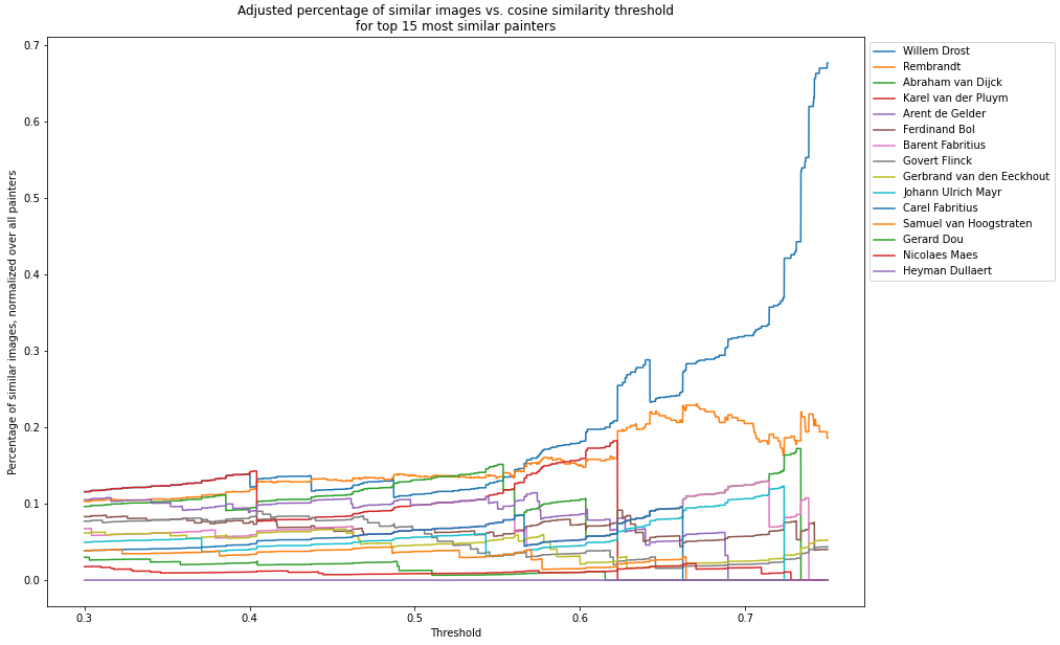
We have therefore decided on a slightly modified procedure. Here, an entire series of thresholds is considered, each spaced 0.0001 apart. For each threshold, we then determine the percentage of images that are similar to the test image, i.e., *Girl with a Broom* (fig. 18a) at that threshold. The result is shown in Figure 38.

It is clearly evident that the number of similar images decreases the higher the threshold value is. But the ambiguity of this metric also becomes clear. For example, for almost all thresholds, the most similar images in percentage terms are detected by Willem Drost – but for thresholds in the (very small) range of 0.4003 to 0.4043 by Karel van der Pluym, in the ranges of 0.4374 to 0.5226 and 0.5534 and 0.5588 by Rembrandt, and between 0.5227 and 0.5533 by Abraham van Dijck.

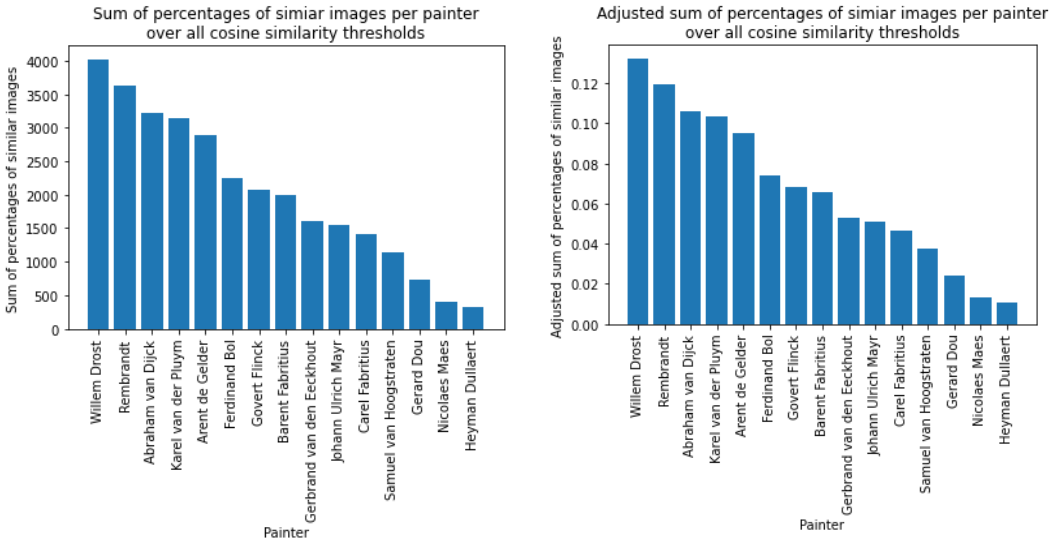
The similarity of the test image to Willem Drost becomes even more obvious when the percentages are normalized so that they always sum to 1 (fig. 39). Here we can clearly see that the percentage of paintings by Willem Drost that are similar to the test image increases more strongly relative to other painters the higher the threshold value is. To our knowledge, there is no attribution to Willem Drost for this work. It is not mentioned in Jonathan Bikker's standard catalog raisonné for this Rembrandt



38 Percentage of works similar to the test image *Girl with a Broom* for different painters and thresholds



39 Percentage of works similar to the test image *Girl with a Broom* for different painters and thresholds. In this graph, the percentage values are normalized across the different painters



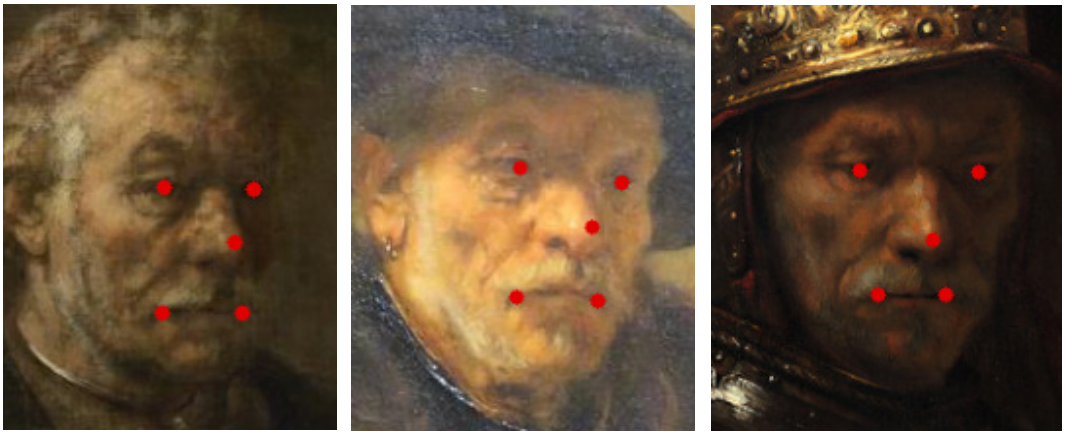
40a/b Sum of the percentages of works similar to the image under test for different painters across all thresholds, in absolute values (40a), adjusted or normalized to the value 1 (40b)

pupil, not even as a rejected attribution[72]. However, the frame data would fit, the assumed date of creation of the painting is between 1646 and 1651 according to RKD, Drost worked as a student in Rembrandt's workshop from 1648 to 1654.

To avoid the often arbitrary calibration of a threshold, we averaged the percent similarities for all painters across all thresholds. The corresponding value for each painter corresponds to the integral under the line of the painter in question (fig. 40). In the right part of the graph, the values are normalized to 1 – and thus, although calculated differently, intuitively comparable to the SoftMax output from the previous two approaches. It now allows – without calibrating a specific threshold – the assignment to a painter by selecting the maximum value.

For the present painting *Girl with a Broom*, Willem Drost is the most similar painter, closely followed by Rembrandt. We did not define and evaluate a difference criterion for the Siamese network, however, the distance of the metric used here between Willem Drost and Rembrandt, as well as the two next most similar artists, i.e., Abraham van Dijck and Karel van der Pluym is intuitively too small for us to suggest an attribution to Drost based on this result. In any case, this result could be taken as a hint for further stylistic investigations concerning an authorship or at least participation of the mentioned painters and especially of Willem Drost.

Another often helpful clue in the attribution of paintings is the identity of the person portrayed in each case. For example, Wilhelm Bode identified the person in the painting *The Man with the Golden Helmet* (fig. 26a) as Rembrandt's brother Adriaen[73], based on the painting *Half Figure of a Man with Gray Curly Hair* (1650). The person depicted is identified by most experts as Adriaen, i.e. Rembrandt's brother, but some also considered him to be Rembrandt's father. Adriaen Harmensz. van Rijn is also thought to be in the painting *Half Length Figure of an Old Man with Beret* (fig. 21) in the Pushkin Museum, with the earlier title *Portrait of an Old Man, Rembrandt's Brother Adriaen*. However, Bode's



41a–c Extracted faces and identified biometric points for *Figure of a Man with Gray Curly Hair* (41a) from the Mauritshuis in The Hague, *Half Length Figure of an Old Man with Beret* (41b) from the Pushkin Museum, and *The Man with the Golden Helmet* (41c) from the Staatliche Gemäldegalerie Berlin. According to many experts, the left and the middle picture represent Rembrandt's brother Adriaen, Wilhelm Bode also suspected him as a model for the right picture

identification was rejected a few years later. *The Man with the Golden Helmet*, like the painting in the Pushkin Museum, was created in 1654, but Rembrandt's brother died in 1652[74].

We applied the so-called Mtcnn face recognition algorithm to the paintings in question (fig. 41a–c) – as well as to 369 other extracted faces from Rembrandt paintings. This model recognizes five biometric points in the faces: the two pupils, the tip of the nose, and the corners of the mouth. However, this algorithm is trained with photographs of people. Since the expressions and the positions of faces in paintings are often more extreme and vary more than in photographs, individual points were corrected manually.

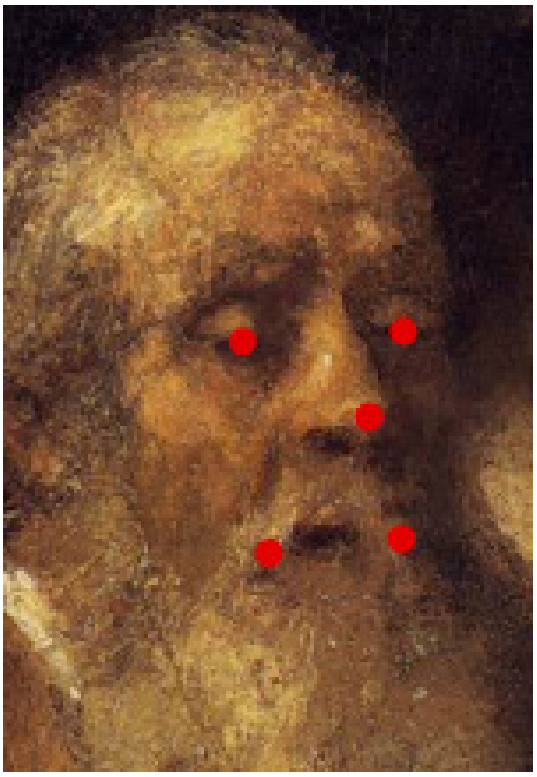
The actual similarity determination, on the other hand, does not use any Machine Learning methods; it is measured based on the so-called Procrustes distance, a metric by which shapes are compared adjusted for their angle of rotation and size. The smaller the Procrustes distance, the more similar the shapes are. The Procrustes distance for the facial points of *Half Figure of a Man with Gray Curly Hair* from the Mauritshuis and *Half Length Figure of an Old Man with Beret* from the Pushkin Museum is 0.00422964. Among the 369 faces extracted, there is only one which has a lower Procrustean distance to the two faces (fig. 42). It is the face of the man in Figure 12.

On the other hand, the Procrustean distance between the face on the portrait *The Man with a Golden Helmet* and that of *Half Figure of a Man with Gray Curly Hair* is 0.03384419. This means that 194 faces in the dataset are more similar to the face from the Gemäldegalerie Berlin than the (presumed) portrait of Rembrandt's brother Adriaen from the Mauritshuis. To the face from the Pushkin Museum, the Procrustes distance is 0.02707823, making 160 faces from the dataset more similar to the face from the Gemäldegalerie Berlin.

While working on this study, we came across another portrait of a person that at first glance resembles the face from the Gemäldegalerie Berlin (fig. 43a/b). The Procrustes distances of this face to the faces in the paintings from the Gemäldegalerie Berlin, the Pushkin Museum and the Mauritshuis are 0.03527768, 0.06847077 and 0.07925262. Thus 105 images from the dataset are more similar to

the face from the Gemäldegalerie Berlin than the face from Bol's painting. For the face from Pushkin Museum, it is 295 and for the face from Mauritshuis 312.

We will not get into the exact evaluation metrics for similarity comparisons at this point – however, we have shown in various tests that even more modern artists, who alienate persons and faces more than the painters of the baroque period, intuitively reproduce biometric features correctly. For example, the algorithm recognizes the two portraits of Adele Bloch-Bauer by Gustav Klimt as identical persons. This also applies to the person in Raphael's sketch of the Mona Lisa, which he made during the weeks he worked in Leonardo da Vinci's workshop, and da Vinci's painting of the portrayed person in the Louvre itself – as well as Rembrandt's depiction of the surgeon Dr. Tulp, who was also painted by Nicolaes Eliaszoon Pickenoy. Even in the case of Picasso, probably the most insensitive painter as far as the faithful reproduction of facial features is concerned, three of the depictions of his last muse, Sylvette David, are still classified as identical persons.[75]



42 Only image more similar to the faces from the portraits *Half Figure of a Man with Gray Curly Hair*, Mauritshuis, The Hague, and *Half Length Figure of an Old Man with Beret*, Pushkin Museum, Moscow, from the dataset of 369 extracted faces

To summarize the analysis of the similarities of the faces considered here, it can be said that the two paintings from the Pushkin Museum and the Mauritshuis most probably depict the same person. Whether it is Rembrandt's brother Adriaen, we are not able to judge due to the not completely sure identification of the person from the Mauritshuis and the already mentioned inconsistency concerning the year of origin of the work from the Pushkin Museum and Adriaen's year of death.

Wilhelm Bode's assumption, on the other hand, that *The Man with the Golden Helmet* represents the same person as the painting from the Mauritshuis must be clearly rejected on the basis of these results. Also, the person on Ferdinand Bol's depiction of a man as Mars is with probability bordering on certainty neither identical with the model from the Gemäldegalerie Berlin nor with the most likely identical person on the paintings in the Pushkin Museum and the Mauritshuis.

Further analysis options

We have given some examples of more advanced or additional analyses here, but the possibilities of Machine Learning and statistics are far from exhausted. For example, so-called multilayer perceptrons can be used for the attribution or rejection of paintings.

43a/b Ferdinand Bols' *Portrait of a Man as Mars*, Warsaw, Museum Narodowe w Warszawie, (43a), and extracted face with biometric points from this work (43b)

Multilayer perceptrons are neural networks, but unlike the Convolutional Neural Networks (CNNs) used here, they do not work with the raw data from paintings, i.e. the individual pixels – but with extracted features from the paintings. These include, for example, statistical parameters of color distribution, contrasts, luminance (brightness), and so on. We have shown that, for example, the distribution of the angles of the color gradients are very characteristic of the styles of individual painters.[76]

In addition, Fourier analyses can be used to study the color frequencies and spectra of artworks. Fourier-transformed images of paintings can also be used as input images for various Machine Learning algorithms.

With the help of clustering algorithms, images can be reduced to a few color tones, which in turn allows a quantitative analysis of the flatness or granularity of a painting style with the help of graph or network algorithms. For example, paintings by Claude Monet are significantly more fine-granular than works by Picasso.

These are only a few examples from a multitude of other possibilities. It is important to note, however, that the choice of analysis method or procedure used must be based on the particular question posed by the art historian. Artificial Intelligence and Machine Learning methods are very effective. But, it will always require human intelligence to select the right approaches and procedures for a given problem and to correctly interpret the results.

Future Work

In this study, we have shown exemplarily how Artificial Intelligence and Machine Learning methods can be used in art history. In our view, the results are promising and a complete review of the pain-

tings of the Rembrandt School using these methods and approaches and the expertise of art historians would make great sense.

The data basis of the RKD is certainly suitable for this, also or especially because it makes it very easy to remove controversial paintings from the data set or not to include them in the first place. This avoids a distortion of the data base with regards to the view of individual art historians or a certain group of experts.

However, if possible, all of Rembrandt's pupils known by name should be present in the data set; in addition, it would be useful and desirable to work with higher-resolution images. If there are strongly differing variants of different paintings, as in the example of the painting *An old Scholar in a Vaulted Room* from the Nationalmuseum Stockholm, all of these variants should be included in the dataset – whereby strict care should be taken that all variants are always used in either the training or the test dataset. Otherwise, due to the so-called data leakage, it would be very likely that the models seem to provide better results than is actually the case.

Summary

Based on the online available dataset of the RKD, we trained an ensemble of Convolutional Neural Networks with undisputed works by Rembrandt and 14 selected pupils of the Dutch artist. This model does not recognize some of the pupils at all, which is due to the small number of undisputed surviving works by these artists. Originals by Rembrandt, on the other hand, are recognized very well.

In addition, we have introduced a strict SoftMax difference criterion for the evaluation. It results in works being classified as originals by Rembrandt only if the result is highly unambiguous. For this, the SoftMax output for the predicted painter must be 0.4 or more above that of the artist with the second highest value. With this additional criterion, the model achieves a precision of 90.0 percent on average and across all painters; for Rembrandt, it is 91.2 percent. In other words: If a painting is classified as Rembrandt with this additional criterion, then there is a 91.2 percent probability that it is indeed an original by the master.

Using this model, we tested 15 works for which the authorship is disputed among art historians and, in some cases, among members of the Rembrandt Research Project. In the vast majority of cases, we were able to confirm the final judgment of the Rembrandt Research Project, be it attribution or rejection.

Only in the case of two works does the model provide very clear indications that the respective rejections as Rembrandt originals are incorrect. This concerns in particular The Man with the Golden Helmet (fig. 26a), Staatliche Gemäldegalerie Berlin, which is with very high probability an original by Rembrandt. The same applies, somewhat less clearly, to Old Woman Cutting her Nails (fig. 28a), Metropolitan Museum, New York. Both works, as well as numerous other paintings, were tested in different image variations and resolutions.

Regarding the work *Hendrickje Stoffels as Pallas Athena* from a European private collection, we were able to prove very clear indications of authenticity. They are, besides a dendrochronological expert opinion, to be considered as a further, clear indication and also certainly justify additional stylistic expertises and analyses by art historians, especially since this work, to our knowledge, has not been examined by the Rembrandt Research Project and is not mentioned in the online archive of the RKD.

In other cases, unlike the works from the Gemäldegalerie Berlin, the Metropolitan Museum in New York, and the *Hendrickje Stoffels as Pallas Athena* (fig. 24b) mentioned above, the model results, while not unambiguous, are clear enough that, in our view, further stylistic and/or other procedures are indicated to verify the results. As an example, we would like to mention Govert Flinck as the possible author of the painting Portrait of a Woman, Seated (fig. 19b), in the Kunsthistorisches Museum, Vienna.

The presented model, together with the restriction regarding the unambiguousness of an attribution, is very helpful and meaningful for a first analysis of a painting. However, we accept the fact that this ensemble of CNNs does not make a statement in almost one third of the cases, which we interpret as a rejection of the predicted painter or the respective hypothesis.

However, analysis results that do not allow clear conclusions are nothing unusual. They also occur frequently in scientific procedures. If, for example, a chemical analysis of a painting shows that the pigments were commercially available and in common use at the time the work was assumed to have been created, this result does not allow any conclusions to be drawn about the actual artist or the authenticity of the painting. While this makes it more likely that the work under study dates from the assumed period, even a modern forgery cannot be ruled out. It is possible that a skilled forger has been meticulous in using only materials that also date from the assumed period. To rule this out, a further analysis would have to investigate whether the aging process of the work was artificially accelerated.

It is quite similar with the present procedure. We have shown that in many cases where the model does not come to a clear decision, it nevertheless provides clues, including hints regarding the involvement of certain students. They could, and in some cases should, be grounds for further stylistic or scientific investigation.

In addition, we have shown some further analysis possibilities by way of example. Again, these methods do not provide unambiguous results in all cases, but nevertheless provide further clues that can be considered or rejected by art historians. As an example, Willem Drost may be mentioned as a possible (co-)creator of the painting *Girl with a Broom* (fig. 18a).

Overall, we have shown that Artificial Intelligence and Machine Learning methods are very powerful methods that can support art historians – as long as they are used with a sense of proportion as well as a healthy dose of skepticism and are interpreted correctly. They are very valuable as an additional method in the attribution and/or rejection of paintings as well as in other art-historically relevant questions, such as the identification of persons in paintings.

Endnotes

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2 Strictly speaking, this statement only applies to algorithms and methods of so-called supervised learning, in which an actual value, the technical term for this is "label", is available for each data point. In this case, each painting has an associated artist.
But in fact, the methods and procedures of the so-called Unsupervised Learning are also based on the minimization of a cost function. In Unsupervised Learning, the data are not labeled. In the present application, this would be a collection of paintings without the assigned artists in each case. Clustering algorithms, for example, always first assign each data point to the nearest cluster center in two iterative steps, then the cluster center is shifted so that the distances of all points within a cluster to the respective center are minimized.
In concrete terms, this could look like this in the present case: From the collection of images, certain features are extracted for each painting, for example regarding color distribution and contrasts. Then, each painting is assigned a vector containing these features. Now the number of clusters is determined, i.e., the number of groups into which the paintings are to be divided. Then, the cluster centers are initialized randomly. These are vectors, whose dimension corresponds to the painting vectors. For each painting vector, the distance to all cluster centers is now measured. Afterwards the painting is assigned to the cluster center with the smallest distance. This is the minimization of a distance. When all painting vectors are correctly assigned, the second step is to move the center within each cluster so that the distance to the paintings within the cluster is minimized, which again is the minimization of a distance or error function. After dozens or hundreds of iterations, similar images are found in the respective clusters, whether in terms of painting styles, genres, techniques – or other characteristics.

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Fig. 1a: R. Edge Pine, Portrait Rudjer Josip Bošković, 1760: Public domain, https://commons.wikimedia.org/wiki/File:Rudjer_Boskovic.jpg (03.02.2023)

Fig. 1b: Title page of Rudjer Josip Bošković, *Theoria philosophiae naturalis redacta ad unicam legem virium in natura existentium*, Vienna 1759: Antiquariat Inlibris, Gilhofer Nfg. GmbH, Vienna

Fig. 2: School of Rembrandt, *Old man contemplating in a study*, 1640s, oil on canvas, 96.5 × 82 cm, Oslo, National Museum: Public domain, https://commons.wikimedia.org/wiki/File:Rembrandt_-_A_Scholar_in_his_Study_-_NG.M.01365.jpg (03.02.2023)

Fig. 3: Rembrandt, *Bust of a youth/Portrait of Titus, the artists' son*, c. 1660, oil on canvas, 81.5 × 78.5 cm, Baltimore Museum of Art: Public domain, https://de.wikipedia.org/wiki/Datei:Rembrandt_-_Titus,_the_Artist%27s_Son,_1660_%28Baltimore_Museum_of_Art%29.jpg (03.02.2023)

Abb. 4–7, 9a–11b, 13a/b, 14b, 14d, 15b, 16b, 16d, 17b, 18b, 20a/b, 22a/b, 23b, 25a/b, 26b, 27b, 28b, 29b, 30b, 31b, 32b, 33b, 34b, 35b, 36b–40b: Art Intelligence GmbH

Fig. 8a: Gerbrand van den Eeckhout, *Isaac lying in bed blesses Jacob who, disguised in Esau's clothes, brings food to his father*, 1642, oil on canvas, 100.6 × 128.3 cm, New York, Metropolitan Museum of Art: Public domain, https://commons.wikimedia.org/wiki/File:Isaac_Blessing_Jacob_MET_DP145918.jpg (03.02.2023)

Fig. 8b: Govaert Flinck, *Johannes de Doper*, 1632, oil on panel, 64.8 × 48.3 cm, Los Angeles, Los Angeles County Museum of Art: RKD – Netherlands Institute for Art History, RKDimages (50356), https://rkd.nl/nl/explore/images/50356 (03.02.2023)

Fig. 12a: Rembrandt, *Simeon's Song of Praise*, c. 1669, oil on canvas, 98.5 × 79.5 cm, Stockholm, Nationalmuseum: Public domain, https://commons.wikimedia.org/wiki/File:Rembrandt_Simeon%E2%80%99s_Song_of_Praise.jpg (03.02.2023)

Fig. 12b: Rembrandt, *Simeon in the Temple*, c. 1669, oil on canvas, 98.5 × 79.5 cm, Stockholm, Nationalmuseum: Photo: Erik Cornelius / Nationalmuseum Stockholm, CC BY-SA 4.0

Fig. 14a: Rembrandt, *Simeon's Song of Praise*, c. 1669, oil on canvas, 98.5 × 79.5 cm, Stockholm, Nationalmuseum: Public domain, https://commons.wikimedia.org/wiki/File:Rembrandt_Simeon%E2%80%99s_Song_of_Praise.jpg (detail; 03.02.2023)

Fig. 14c: Rembrandt, *Simeon in the Temple*, c. 1669, oil on canvas, 98.5 × 79.5 cm, Stockholm, Nationalmuseum: Photo: Erik Cornelius / Nationalmuseum Stockholm (detail), CC BY-SA 4.0

Fig. 15a: Rembrandt, *Portrait of Johannes Wttenbogaert*, 1633, oil on canvas, 103.5 × 104.5 cm, Amsterdam, Rijksmuseum: Public domain, https://commons.wikimedia.org/wiki/Category:Portrait_of_Johannes_Wtenbogaert_(Rembrandt) (03.02.2023)

Fig. 16a/c: Rembrandt, *Portrait of Johannes Wttenbogaert*, 1633, oil on canvas, 103.5 × 104.5 cm, Amsterdam, Rijksmuseum: Public domain, https://commons.wikimedia.org/wiki/Category:Portrait_of_Johannes_Wtenbogaert_(Rembrandt) (detail; 03.02.2023)

Fig. 17a: *The Slaughtered Ox*, 1655, oil on canvas, 95,5 × 68,8 cm, Glasgow, Kelvingrove Art Gallery and Museum: Public domain, https://commons.wikimedia.org/wiki/File:El_cad%C3%A1ver_de_un_buey,_atribuido_a_Rembrandt.jpg (03.02.2023)

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Fig. 21a: *Half Figure of an Old Man with Beret*, 1654, oil on canvas, 74 × 63 cm, Moscow, Pushkin Museum: Public domain, RKD – Netherlands Institute for Art History, RKDimages (39465), https://rkd.nl/en/explore/images/39465 (03.02.2023)

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Fig. 23a: Rembrandt, *Half Figure of a Woman with a White Wrap*, 1654–1656, oil on canvas, 101.9 × 83.7 cm, London, National Gallery: Public domain, RKD – Netherlands Institute for Art History, RKDimages (41279), https://rkd.nl/en/explore/images/41279 (03.02.2023)

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Fig. 26a: *The Man with the Golden Helmet*, c. 1650–1660, oil on canvas, 67.5 × 50.7 cm, Berlin, Gemäldegalerie: Staatliche Museen zu Berlin, Gemäldegalerie (Property Kaiser Friedrich Museumsverein) / Christoph Schmidt

Fig. 27a: *The Man with the Golden Helmet*, c. 1650–1660, oil on canvas, 67.5 × 50.7 cm, Berlin, Gemäldegalerie: Staatliche Museen zu Berlin, Gemäldegalerie (Eigentum Kaiser Friedrich Museumsverein) / Christoph Schmidt

Fig. 28a: *Old Woman Cutting her Nails*, 1650er, oil on canvas, 126.1 × 101.9 cm, New York, Metropolitan Museum of Art: Public domain, Metropolitan Museum of Art, https://www.metmuseum.org/art/collection/search/437412 (03.02.2023)

Fig. 29a: *Old Woman Cutting her Nails*, 1650er, oil on canvas, 126.1 × 101.9 cm, New York, Metropolitan Museum of Art: Public domain, Metropolitan Museum of Art, https://www.metmuseum.org/art/collection/search/437412 (03.02.2023)

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Fig. 31a: *An old Scholar in a Vaulted Room*, 1631, oil on panel, 60 × 48 cm, Stockholm, Nationalmuseum: RKD – Netherlands Institute for Art History, RKDimages (206145), https://rkd.nl/en/explore/images/206145 (03.02.2023)

Fig. 32a: *An old Scholar in a Vaulted Room*, 1631, oil on panel, 60 × 48 cm, Stockholm, Nationalmuseum: Public domain, Nationalmuseum Stockholm, https://collection.nationalmuseum.se:443/eMP/eMuseumPlus?service=ExternalInterface&module=collection&objectId=17582&viewType=detailView (03.02.2023)

Fig. 33a: *Bust of a Man in Oriental Dress*, 1635, oil on canvas, 72 × 54.5 cm, Amsterdam, Rijksmuseum: Public domain, Rijksmuseum Amsterdam, https://www.rijksmuseum.nl/en/collection/SK-A-3340 (03.02.2023)

Fig. 34a: *Bust of a Man in Oriental Dress*, 1635, oil on canvas, 72 × 54.5 cm, Amsterdam, Rijksmuseum: Public domain, Rijksmuseum Amsterdam, https://www.rijksmuseum.nl/en/collection/SK-A-3340 (03.02.2023)

Fig. 35a: *Self Portrait in a Black Cap*, c. 1637, oil on panel, 63 × 50.7 cm, London, Wallace Collection: Public domain, RKD – Netherlands Institute for Art History, RKDimages (30284), https://rkd.nl/en/explore/images/30284 (03.02.2023)

Fig. 36a: *Self Portrait in a Black Cap*, c. 1637, oil on panel, 63 × 50.7 cm, London, Wallace Collection: Wallace Collection, CC-BY-NC-ND 4.0, https://wallacelive.wallacecollection.org/eMP/eMuseumPlus?service=ExternalInterface&module=collection&objectId=63454&viewType=detailView (03.02.2023)

Fig. 41a: *Half Figure of a Man with Gray Curly Hair*, 1654, oil on canvas, 80.5 × 66.5 cm, The Hague, Mauritshuis: Public domain, Mauritshuis, The Hague und Art Intelligence GmbH

Fig. 41b: *Half Figure of an Old Man with Beret*, 1654, oil on canvas, 74 × 63 cm, Moscow, Pushkin Museum: Public domain, RKD – Netherlands Institute for Art History, RKDimages (39465), https://rkd.nl/en/explore/images/39465 (detail; 03.02.2023) und Art Intelligence GmbH

Fig. 41c: *The Man with the Golden Helmet*, c. 1650–1660, oil on canvas, 67.5 × 50.7 cm, Berlin, Gemäldegalerie: Staatliche Museen zu Berlin, Gemäldegalerie (property Kaiser Friedrich Museumsverein) / Christoph Schmidt und Art Intelligence GmbH

Fig. 42: Rembrandt, *Simeon's Song of Praise*, c. 1669, oil on canvas, 98.5 × 79.5 cm, Stockholm, Nationalmuseum: RKD – Netherlands Institute for Art History, RKDimages (51422), https://rkd.nl/en/explore/images/51422 (detail; 03.02.2023) und Art Intelligence GmbH

Fig. 43a: Ferdinand Bol, *Portrait of a Man as Mars*, c. 1657, oil on canvas, 72.5 × 62.5 cm, Warsaw, Museum Narodowe w Warszawie: RKD – Netherlands Institute for Art History, RKDimages (59613), https://rkd.nl/en/explore/images/59613 (03.02.2023)

Fig. 43b: Ferdinand Bol, *Portrait of a Man as Mars*, c. 1657, oil on canvas, 72.5 × 62.5 cm, Warsaw, Museum Narodowe w Warszawie: RKD – Netherlands Institute for Art History, RKDimages (59613), https://rkd.nl/en/explore/images/59613 (detail; 03.02.2023) und Art Intelligence GmbH

Tab. 1–4: Art Intelligence GmbH

The author is the founder of Art Intelligence GmbH, a start-up that deals with the analysis of works of art and is recognized as a research project by the German Federal Ministry of Research and Development.

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