Journal of Materials and Engineering Structures

Research Paper

Approximation solution for steel concrete beam accounting high-order shear deformation using trigonometric-series

Ta Duy Hien [a,*], *Nguyen Tan Khoa* [b,c], *Dao Ngoc Tien* [d], *Do Thi Hang* [a], *Nguyen Xuan Tung* [a], *Nguyễn Đăng Điềm* [e]

[a] *Faculty of Civil Engineering, University of Transport and Communications, Hanoi, Vietnam*

[b] *Institute of Research and Development, Duy Tan University, 550000, Da Nang, Vietnam*

[c] *Faculty of Civil Engineering, Duy Tan University, 550000, Da Nang, Vietnam, Vietnam*

[d] *Hanoi Architectural University, Hanoi, Vietnam*

[e] *Campus in Ho Chi Minh City, University of Transport and Communications, Ho Chi Minh City, Vietnam*

ARTICLE INFO

Article history :

Received : 15 November 2022

Revised : 16 December 2022

Accepted : 18 December 2022

Keywords:

Steel concrete beam

Spproximate solution

Higher-order beam

Trigonometric series.

ABSTRACT

Steel concrete beams have a reasonable structure in terms of using material and high load carrying capacity. This paper deals with an approximate solution based on a trigonometric series for the static of steel concrete beams. The displacement field is based on the higher-order theory using Reddy's hypothesis. The governing equations are derived from variation principles. An approximate solution based on the representation of displacement fields by trigonometric series is developed to solve the static problem of steel concrete beams. In order to verify the accuracy of the present approximate solution, numerical results are compared with those of exact solutions using classical beam theory. The displacements and nominal stress distributions in the depth direction are obtained with various high of beams. The present approximate approach can accurately predict the displacements and stresses of steel concrete beams.

F. ASMA & H. HAMMOUM (Eds.) special issue, 4th International Conference on Sustainability in Civil Engineering ICSCE 2022, Hanoi, Vietnam, J. Mater. Eng. Struct. 9(4) (2022)

1 Introduction

The steel-concrete beam is commonly used in civil engineering, which is composed of steel beams cast in situ with reinforced concrete slabs. This configuration utilizes sufficiently high capacities in the compression of concrete and steel

* *Corresponding author*
E-mail address: tdhien@utc.edu.vn

e-ISSN: 2170-127X, (cc) BY-SA

RESEARCH REVIEW of Sciences and Technologies

tension. Thus, the steel-concrete beam has improved the stiffness and load capacity of the structure compared to those of the individual ones.

The literature reviews showed several works concerned with experimental and computational research on steel-concrete composite structures in civil and bridge engineering. P.V Phe and N.X. Huy [1] analysed the interfacial shear and peeling stresses of FRP-strengthened beams subjected to load and thermal effects by the finite difference method. Wang [2] studied the deflection of steel-concrete beams when there is a partial adhesion between steel and concrete. Chen Tao et al. [3] focused on the behaviour of steel-concrete beams with web openings under a negative moment. Jorge Luis Palomino et al. [4] studied the short-term performance of steel-concrete beams by developing a finite element model using plate elements. Pham Van Phe et al. [5] presented the computing of steel beams strengthened with composite panels by the finite element method based on the principle of stationary complementary energy. Sliseris and Korjakins [6] conducted an experimental study on high-performance concrete containing steel fibre beams by numerical method. Sugar Bükülmez Pınar and Celik Oguz [7] presented an experimental work on the fire resistance capacity of steel-concrete beams with a large openings ratio. Mahmoud Ashraf Mohamed [8] used finite element software to simulate steel-concrete beams with a nonlinear material.

There are many different computation beam theories depending on the members' dimensions, materials, and behavior that can be used when computing beams or plates. Classical beam theory can be used as presented on the issues of the strength of materials or structural mechanics[9], Timoshenko beam theory [10], and beam theory considering higher-order shear deformation [11]. L.T Ha and N.T.K Khue calculated the free oscillation of porous nanobeams using classical beam theory [12]. N.T Nhung et al. [13] investigated the stochastic free vibration of non-uniform beams using stochastic finite element method. Gianluca Ranzia and Alessandro Zona [14] presented a study on computing steel-concrete beams using both classical beam theory and Timoshenko beam theory. Due to the neglect of shear strain in classical beam theory, many researchers have used higher-order beam theory. In which, the higher-order shear deformation term has been considered. Thus, increasing the accuracy of the observations. Atilla Ozutoka and Emrah Madenci [15] calculated multilayer composite beams using the finite element method by considering the higher-order shear deformation. T.D Hien et al. [16] developed the stochastic finite element method for higher-order beams involving the random field of elastic modulus. P.B Thang and L.L.Anh [17] analyzed the steel-concrete beam by using classical beam theory and Timoshenko beam theory considering the interaction between connectors and reinforced concrete slab. Fabrizio Gara et al. [18] proposed the finite element model steel–concrete beams considering the partial shear interaction between concrete and steel.

Although numerous types of research on steel-concrete beams have been presented in the literature, there are still a few works that consider the higher-order shear deformation of beams. Thus, it is necessary to research the computing method for steel-concrete beams, in which, the higher-order shear deformation is taken into account. Finally, the observations on internal forces and displacements are predicted close to the actual behaviour of the beams.

2 Governing equation of steel-concrete beam using Reddy's beam theory

Considering a schematic of the steel-concrete beam as shown in Fig. 1, a popular structure used in bridge construction.

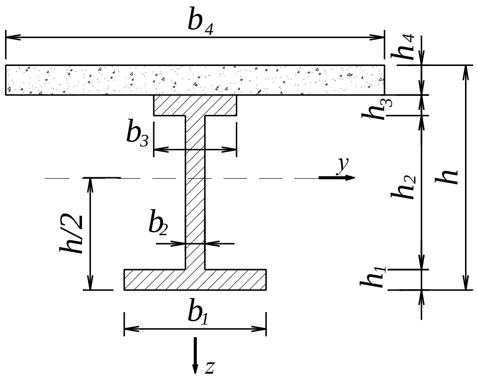

Fig. 1 – Cross-section of steel-concrete beam

Following the classical beam theory, the reference coordinate is usually chosen at the centroid of the section. Then, the axial displacement will be neglected. In this study, when the theory of higher-order beams is applied, determining the neutral axis is complex following the above. Hence, the reference axis is determined at the middle of the cross-section's height. Therefore, axial displacement exists. The displacement field of the steel-concrete beam under Reddy's theory [11] is determined with the origin located in the middle of the beam's height as follows:

$$u(x,z)=u_0(x)+z\psi(x)-\frac{4z^3}{3h^2}\left(\psi(x)+\frac{\partial w_0}{\partial x}\right);\quad w(x,z)=w_0(x) \tag{1}$$

where u_0, w_0, ψ are the displacement components at the mid-axis in the x, z directions, and the displacement parameter, respectively.

From the displacement field in Eq.(1), deformation can be calculated as follows:

$$\varepsilon_x=\frac{\partial u_0}{\partial x}+\frac{\partial \psi}{\partial x}z-\frac{4z^3}{3h^2}\left(\frac{\partial \psi}{\partial x}+\frac{\partial^2 w_0}{\partial x^2}\right);\quad \gamma_{xz}=\psi\left(1-\frac{4z^2}{h^2}\right)+\frac{\partial w_0}{\partial x}\left(1-\frac{4z^2}{h^2}\right) \tag{2}$$

The deformation potential of the beam:

$$U=\frac{1}{2}\int_\Omega E(z)\left[\frac{\partial u_0}{\partial x}+\frac{\partial \psi}{\partial x}z-\frac{4}{3h^2}z^3\left(\frac{\partial \psi}{\partial x}+\frac{\partial^2 w_0}{\partial x^2}\right)\right]d\Omega+\frac{1}{2}\int_\Omega G(z)\left[\left(1-\frac{4z^2}{h^2}\right)\left(\psi+\frac{\partial w_0}{\partial x}\right)\right]dx \tag{3}$$

The beam's equilibrium position is determined according to the minimum potential energy principle:

$$\delta(U+\Pi)=0 \tag{4}$$

The Eq. (4) can write as variations formulation:

$$\begin{aligned}&\int_\Omega E(z)\left[\frac{\partial u_0}{\partial x}+\frac{\partial \psi}{\partial x}z-\frac{4}{3h^2}z^3\left(\frac{\partial \psi}{\partial x}+\frac{\partial^2 w_0}{\partial x^2}\right)\right]\times\delta\left[\frac{\partial u_0}{\partial x}+\frac{\partial \psi}{\partial x}z-\frac{4}{3h^2}z^3\left(\frac{\partial \psi}{\partial x}+\frac{\partial^2 w_0}{\partial x^2}\right)\right]d\Omega\\&+\int_\Omega G(z)\left[\left(1-\frac{4z^2}{h^2}\right)\left(\psi+\frac{\partial w_0}{\partial x}\right)\right]\delta\left[\left(1-\frac{4z^2}{h^2}\right)\left(\psi+\frac{\partial w_0}{\partial x}\right)\right]dx-\int_\Omega q(x)\delta w_0(x)d\Omega=0\end{aligned} \tag{5}$$

Transforming and isolating the differential variables in Eq. (5) we have a system of differential equations of the beam:

$$\begin{aligned}&\delta u_0:\ \frac{\partial N_x}{\partial x}=0\quad \delta w_0:\ \frac{\partial Q_x}{\partial x}-\frac{4}{h^2}\frac{\partial R_x}{\partial x}+\frac{4}{3h^2}\frac{\partial^2 P_x}{\partial x^2}+q=0\\&\delta\psi:\ \frac{\partial M_x}{\partial x}-\frac{4}{3h^2}\frac{\partial P_x}{\partial x}-Q_x+\frac{4}{h^2}R_x=0\end{aligned} \tag{6}$$

In which the internal force components are defined as follows:

$$\begin{Bmatrix}N_x\\M_x\\P_x\end{Bmatrix}=\int_A E(z)\varepsilon_x\begin{Bmatrix}1\\z\\z^3\end{Bmatrix}dA;\quad \begin{Bmatrix}Q_x\\R_x\end{Bmatrix}=\int_A G(z)\gamma_{xz}\begin{Bmatrix}1\\z^2\end{Bmatrix}dA \tag{7}$$

Substituting the expression of displacement in Eq. (2) into Eq. (8), we have:

$$\begin{Bmatrix}N_x\\M_x\\P_x\end{Bmatrix}=\int_A E(z)\begin{bmatrix}\frac{\partial u_0}{\partial x}+\frac{\partial \psi}{\partial x}\left(z-\frac{4z^3}{3h^2}\right)\\-\frac{4z^3}{3h^2}\frac{\partial^2 w_0}{\partial x^2}\end{bmatrix}\begin{Bmatrix}1\\z\\z^3\end{Bmatrix}dA;\quad \begin{Bmatrix}Q_x\\R_x\end{Bmatrix}=\int_A G(z)\begin{bmatrix}\psi\left(1-\frac{4z^2}{h^2}\right)\\+\frac{\partial w_0}{\partial x}\left(1-\frac{4z^2}{h^2}\right)\end{bmatrix}\begin{Bmatrix}1\\z^2\end{Bmatrix}dA \tag{8}$$

Computing the integrals in Eq. (9) we obtain the internal force expressions under the displacement parameters as follows:

$$\begin{Bmatrix} N_x \\ M_x \\ P_x \end{Bmatrix} = \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ D_1 & D_2 & D_3 \end{bmatrix} \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial \psi}{\partial x} \\ \dfrac{\partial^2 w_0}{\partial x^2} \end{Bmatrix} ; \quad \begin{Bmatrix} Q_x \\ R_x \end{Bmatrix} = \begin{bmatrix} F_1 & F_2 \\ H_1 & H_2 \end{bmatrix} \begin{Bmatrix} \psi \\ \dfrac{\partial w_0}{\partial x} \end{Bmatrix} \tag{9}$$

In which the constants of the properties of a plane area of the cross-section are defined as follows:

$$\begin{Bmatrix} A_1 \\ A_2 \\ A_3 \end{Bmatrix} = \int_A E(z) \begin{Bmatrix} 1 \\ z - \dfrac{4z^3}{3h^2} \\ -\dfrac{4z^3}{3h^2} \end{Bmatrix} dA; \quad \begin{Bmatrix} B_1 \\ B_2 \\ B_3 \end{Bmatrix} = \int_A zE(z) \begin{Bmatrix} 1 \\ z - \dfrac{4z^3}{3h^2} \\ -\dfrac{4z^3}{3h^2} \end{Bmatrix} dA; \quad \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \int_A z^3 E(z) \begin{Bmatrix} 1 \\ z - \dfrac{4z^3}{3h^2} \\ -\dfrac{4z^3}{3h^2} \end{Bmatrix} dA;$$
$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \int_A G(z) \left(1 - \frac{4z^2}{h^2}\right) \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} dA; \quad \begin{Bmatrix} H_1 \\ H_2 \end{Bmatrix} = \int_A z^2 G(z) \left(1 - \frac{4z^2}{h^2}\right) \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} dA \tag{10}$$

3 Analytical solution

Considering a beam with simple hinges at both ends, from the boundary conditions, we use the approximate solution based on a trigonometric series of Navier [11]:

$$u_0 = \sum_{i=1}^{\infty} U_i \cos(\alpha x); \quad w_0 = \sum_{i=1}^{\infty} W_i \sin(\alpha x); \quad \psi_0 = \sum_{i=1}^{\infty} \Psi_i cos(\alpha x) \tag{11}$$

where $\alpha = \dfrac{i\pi}{L}$

By substituting into Eq. (9), we observe:

$$\begin{Bmatrix} N_x \\ M_x \\ P_x \end{Bmatrix} = \sum_{i=1}^{\infty} \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ D_1 & D_2 & D_3 \end{bmatrix} \begin{Bmatrix} -\alpha U_i \\ -\alpha \Psi_i \\ -\alpha^2 W_i \end{Bmatrix} \sin(\alpha x); \quad \begin{Bmatrix} Q_x \\ R_x \end{Bmatrix} = \sum_{i=1}^{\infty} \begin{bmatrix} F_1 & F_2 \\ H_1 & H_2 \end{bmatrix} \begin{Bmatrix} \alpha \Psi_i \\ \alpha W_i \end{Bmatrix} cos(\alpha x) \tag{12}$$

Substituting the internal force expressions from Eq. (12) into Eq. (6) and using the orthogonal property of the sine and cosine functions, Eq. (6) is altered as follows:

$$\begin{bmatrix} A_1 & A_2 & \alpha A_3 \end{bmatrix} \begin{Bmatrix} U_i \\ \Psi_i \\ W_i \end{Bmatrix} = 0$$
$$\begin{bmatrix} \alpha^3 \dfrac{4}{3h^2} D_1 & \alpha^3 \dfrac{4}{3h^2} D_2 - \alpha^2 F_1 + \alpha^2 \dfrac{4}{h^2} H_1 & \alpha^4 \dfrac{4}{3h^2} D_3 - \alpha^2 F_2 + \alpha^2 \dfrac{4}{h^2} H_2 \end{bmatrix} \begin{Bmatrix} U_i \\ \Psi_i \\ W_i \end{Bmatrix} = 0 \tag{13}$$
$$\begin{bmatrix} -\alpha^2 B_1 - \alpha^2 \dfrac{4}{3h^2} D_1 & \begin{matrix} -\alpha^2 B_2 - \alpha^2 \dfrac{4}{3h^2} D_2 \\ -\alpha F_1 + \alpha \dfrac{4}{h^2} H_1 \end{matrix} & \begin{matrix} -\alpha^3 B_3 - \alpha^3 \dfrac{4}{3h^2} D_3 \\ -\alpha F_2 + \alpha \dfrac{4}{h^2} H_2 \end{matrix} \end{bmatrix} \begin{Bmatrix} U_i \\ \Psi_i \\ W_i \end{Bmatrix} = 0$$

or abbreviation form:

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{Bmatrix} U_i \\ \Psi_i \\ W_i \end{Bmatrix} + \begin{Bmatrix} 0 \\ q_i \\ 0 \end{Bmatrix} = 0 \tag{14}$$

where load parameter:

$$q_i = \frac{2}{L}\int_0^L q(x)\sin(\alpha x)dx \tag{15}$$

and S_{ij} is the parameter of stiffness

$$\begin{aligned}
&S_{11} = A_1;\ \ S_{12} = A_2;\ \ S_{13} = \alpha A_3 \\
&S_{21} = \alpha^3 \frac{4}{3h^2} D_1;\ \ S_{22} = \alpha^3 \frac{4}{3h^2} D_2 - \alpha^2 F_1 + \alpha^2 \frac{4}{h^2} H_1; \\
&S_{23} = \alpha^4 \frac{4}{3h^2} D_3 - \alpha^2 F_2 + \alpha^2 \frac{4}{h^2} H_2 \\
&S_{31} = -\alpha^2 B_1 - \alpha^2 \frac{4}{3h^2} D_1 \\
&S_{32} = -\alpha^2 B_2 - \alpha^2 \frac{4}{3h^2} D_2 - \alpha F_1 + \alpha \frac{4}{h^2} H_1 \\
&S_{33} = -\alpha^3 B_3 - \alpha^3 \frac{4}{3h^2} D_3 - \alpha F_2 + \alpha \frac{4}{h^2} H_2
\end{aligned} \tag{16}$$

By solving the system of equations (14) to obtain the displacement constants U_i, Ψ_i, W_i . So, the displacement is determined following Eq. (11). Based on those results, the deformation and stress of the beam can be calculated.

4 Numerical examples

To estimate the accuracy of the approximate solution based on a trigonometric series, we consider a simple span of steel-concrete beam, length *L*=15m, and cross-sectional dimensions are defined in Fig. 1.

Dimensions of the cross-section:

$h = 1,2m;\ h_1 = 0,02m;\ \ h_3 = 0,02m;\ \ h_4 = 0,2m;$

$b_1 = 0,3m;\ \ b_2 = 0,014m;\ b_3 = 0,3m;\ b_4 = 2,5m;$

Applied uniform load: *q*=10kN/m.

We use the classical beam theory to estimate the simulation results as presented in the same material. Then, the beam deflection is observed of 3.025 mm (*w*=3,025(mm)). In addition to using the theoretical formula, the displacements can be checked by the finite element method. Specifically, in this present work, Sap2000 software is utilized, a beam element simulate the steel beam, and the concrete slab is simulated by a plate element. The simulation result shows the displacement in the middle of the beam is *w*=3.205(mm).

Using the trigonometric series in Eq. (11), the displacement observation in the middle of the beam is w=3.131(mm) when computing with the first 15 terms of the series. Thus, it shows that the displacement result computed according to the theory of higher-order beams is 3.3 % higher than that computed according to classical beam theory but lower than that calculated by finite element simulation by 2.36%. When computing the beam using classical theory, because of the neglection of the shear deformation, the observed displacement is lower. However, when simulating finite elements using a plate model for concrete slab, the result will be close to the actual behavior of the beam, the displacement results are larger than the two theoretical ones using the classical beam theory and higher-order beam theory.

For analyzing the influence of shear deformation on the internal force and displacement of the beam, we investigate the internal force and displacement of the beam, which the height varies from 0.8m to 1.5m, and the observations are compared with the observations according to the classical beam theory without considering the shear deformation. Table 1 presents the observations of the maximum deflection and normal stress at the mid-span and lower boundary fiber. The observations clearly show that when the height increases, the influence of the shear deformation increases, leading to an increase significantly of the deflection computed according to the higher-order shear deformation theory compared that computed according to the classical beam theory. As shown in Table 1, when the beam height reaches a ratio of 1/10 of the span length, the deflection computed according to the higher-order beam theory is higher than that computed according to the classical beam theory by 6.1%. In the case of the normal stress at the lower boundary fiber of the beam, it remained unchanged when computing the normal stress according to the classical beam theory and the higher-order shear theory, although the difference increases as the height increases.

Table 1 – Comparison the displacement and normal stress at of mid-span.

h(m)	Displacement (mm)			Normal stress (MPa)		
	Higher-order beam theory	Classical beam theory	Error (%)	Higher-order beam theory	Classical beam theory	Error (%)
0,8	8,46	8,379	0,96%	43,822	43,813	0,02%
0,9	6,341	6,252	1,40%	37,171	37,157	0,04%
1,0	4,895	4,799	1,96%	31,93	31,911	0,06%
1,1	3,874	3,772	2,63%	27,739	27,716	0,08%
1,2	3,131	3,025	3,39%	24,313	24,342	0,12%
1,3	2,576	2,467	4,23%	21,548	21,514	0,16%
1,4	2,152	2,042	5,11%	19,225	19,186	0,20%
1,5	1,822	1,711	6,09%	17,271	17,227	0,25%

5 Conclusions

The work presents an approach to compute the internal forces and displacements of steel-concrete beams by using the approximate solution based on a trigonometric series, in which, the higher-order shear deformation is considered. The comparison of the results based on the present approach and the results based on classical beam theory and finite element simulation shows the reliability of the approximate solution based on a trigonometric series. The observations from a steel-concrete beam with a height to span length ratio of 1/20 - 1/10 show higher displacement and stress based on the higher-order beam theory compared to those based on classical beam theory. As the beam height increases, the displacement increases significantly while the normal stress increases slightly. As a result, the effect of the shear deformation on computing the beam, which is height relatively higher than the span length, is clarified.

Acknowledgments

This research is funded by the University of Transport and Communications (UTC) under grant number T2021-CT-007TĐ

REFERENCES

[1]- V.P. Phe, X.N. Huy, An analytical solution for FRP-strengthened beams under load and thermal effects. Transp. Com. Sci. J. 71(2) (2020) 80-90. doi:10.25073/tcsj.71.2.3.

[2]- Y.C. Wang, Deflection of Steel-Concrete Composite Beams with Partial Shear Interaction. J. Struct. Eng., 124(10) (1998) 1159-1165. doi:10.1061/(ASCE)0733-9445(1998)124:10(1159).

[3]- T. Chen, X. Gu, H. Li, Behavior of steel-concrete composite cantilever beams with web openings under negative moment. Int. J. Steel Str., 11(1) (2011) 39-49. doi:10.1007/S13296-011-1004-8.

[4]- J.L.P. Tamayo, I.B. Morsch, A.M. Awruch, Short-time numerical analysis of steel–concrete composite beams. Journal of the Brazilian Society of Mechanical Sciences and Engineering, 37(4) (2015) 1097-1109. doi:10.1007/s40430-014-0237-9.

[5]- P.V. Pham, M. Mohareb, A. Fam, Shear deformable super-convergent finite element for steel beams strengthened with glass-fiber reinforced polymer (GFRP) plate. Can J Civ Eng, 46(4) (2019) 338-351. doi:10.1139/cjce-2018-0259.

[6]- J. Sliseris, A. Korjakins, Numerical Modeling of the Casting Process and Impact Loading of a Steel-Fiber-Reinforced High-Performance Self-Compacting Concrete. Mech. Compos. Mater., 55(1) (2019) 29-40. doi:10.1007/s11029-019-09789-x.

[7]- P. Sunar Bükülmez, O.C. Celik, Experimental Study on Fire Behavior of Steel–Concrete Composite Cellular Beams with Large Opening Ratio. Int. J. Steel Str., 20(1) (2020) 207-231. doi:10.1007/s13296-019-00281-9.

[8]- A.M. Mahmoud, Finite element modeling of steel concrete beam considering double composite action. Ain Shams Eng. J., 7(1) (2016) 73-88. doi:10.1016/j.asej.2015.03.012.
[9]- S.P. Timoshenko, Strength of materials. New Delhi: CBS Publishers & Distributors, 2004.
[10]- I. Elishakoff, Who developed the so-called Timoshenko beam theory? Mathematics and Mechanics of Solids, 25(1) (2020) 97-116. doi:10.1177/1081286519856931.
[11]- J.N. Reddy, Mechanics of laminated composite plates and shells : theory and analysis. 2nd ed. Boca Raton: CRC Press. xxiii, 831 p., 2004.
[12]- L.T. Ha, N.T.K. Khue, Free vibration of functionally graded porous nano beams. Transp. Commun. Sci. J., 70(2) (2019) 95-103. doi:10.25073/tcsj.70.2.32.
[13]- N.T. Nhung, T.D. Hien, N.V. Thuan, D.N. Tien, Stochastic finite element analysis of the free vibration of non-uniform beams with uncertain material. J. Mat. Eng. Str. «JMES», 9(1) (2022) 29-37.
[14]- G. Ranzi, A. Zona, A steel–concrete composite beam model with partial interaction including the shear deformability of the steel component. Eng. Struct., 29(11) (2007) 3026-3041. doi:10.1016/j.engstruct.2007.02.007.
[15]- A. Özütok, E. Madenci, Static analysis of laminated composite beams based on higher-order shear deformation theory by using mixed-type finite element method. Int. J. Mech. Sci., 130 (2017) 234-243. doi:10.1016/j.ijmecsci.2017.06.013.
[16]- T.D. Hien, B.T. Thanh, N.N. Long, N. Van Thuan, D.T. Hang. Investigation into the response variability of a higher-order beam resting on a foundation using a stochastic finite element method. in CIGOS 2019, Innovation for Sustainable Infrastructure. Ha Noi: Springer. (2020), 117-122.
[17]- P.B. Thang, L.V. Anh, Structural analysis of steel-concrete composite beam bridges utilizing the shear connection model. Transp. Commun. Sci. J., 72(7) (2021) 811-823. doi:10.47869/tcsj.72.7.4.
[18]- F. Gara, S. Carbonari, G. Leoni, L. Dezi, A higher order steel–concrete composite beam model. Eng. Struct., 80 (2014) 260-273. doi:10.1016/j.engstruct.2014.09.002.