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Analytical Method for Joint Optimization of Ffe and Dfe Equalizations for Multi-Level Signals

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PAPERS

Analytical method for joint optimization of FFE and DFE equalizations for multi-level signals

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Channel equalization is the efficient method for recovering distorted signal and correspondingly reducing bit error rate (BER). Different type of equalizations, like feed forward equalization (FFE) and decision feedback equalization (DFE) are canceling channel effect and recovering channel response. Separate optimization of tap coefficients for FFE and DFE does not give optimal result. In this case FFE and DFE tap coefficients are found separately and they are not collaborating. Therefore, the final equalization result is not global optimal. In the present paper new analytical method for finding best tap coefficients for FFE and DFE joint equalization is introduced. The proposed method can be used for both NRZ and PAM4 signals. The idea of the methodology is to combine FFE and DFE tap coefficients into one optimization problem and allow them to collaborate and lead to the global optimal solution. The proposed joint optimization method is fast, easy to implement and efficient. The method has been tested for several measured channels and the analysis of the results are discussed.

Keywords: bit error rate, channel equalization, channel response, DFE, FFE, joint optimization, PAM4, tap coefficients

1 Introduction

Constant increase in data rates introduce new challenges and concepts. Signal in high-speed channels is attenuated by channel loss, inter-symbol interference (ISI), jitter, noise and crosstalk, see [1-5]. Channel equalization is an efficient method for recovering distorted signal and correspondingly reducing bit error rate in high-speed channels. Higher data rates require more precise tap coefficients for usually used equalization algorithms such as FFE and DFE. This is more crucial for 4-level pulse amplitude modulation (PAM4) signal that brings in new challenges due to inherent jitter and three slicers, [6-9]. Multi-level signal represents innovative alternative approach to increase data rates and is expected to be frequently used in future in the devices. While multi-level signals significantly increase data rate without improving component material, nonlinear effects and other factors of signal distortion pose more challenges for several levels, therefore tap coefficients of equalizers have more effect on signal restoration, see [10-14].

Widely used optimization algorithms for FFE and DFE equalizations can only efficiently optimize each equalization separately. FFE Least Mean Square optimization algorithm uses signal energy maximally to remove ISI noise and therefore significantly decreases the signal amplitude level. DFE optimization removes the pulse response tail that remains after FFE equalization [13]. Therefore, these two equalizers work independently from each other, which does not yield the best possible results. Combined optimization of these equalization mostly uses sweeping through coarsely pre-chosen subsets of tap coefficients for each equalizer [1],[5] and adaptively choosing the best combination, that is time-consuming and inefficient.

In the work [15] we proposed a fast and efficient algorithm that calculates the best tap coefficients for combined FFE and DFE equalizers. This methodology removes part of the noise from the signal using FFE while preserving the maximal cursor value to keep the eye opening, so that DFE can be applied. The next, optimal DFE is used to remove remaining ISI without reducing the signal level. Proposed FFE and DFE joint optimization algorithm forms one goal function, where both FFE and DFE tap coefficients are included as optimization parameters. The goal function minimizes signal to noise ratio based on collaboration of FFE and DFE tap coefficients. The proposed optimization problem is linear regarding FFE and DFE tap coefficients and therefore can be solved analytically. The algorithm does not use any iteration or parameter sweeping procedure, therefore is very fast and easy to implement. In the present work this optimization algorithm is generalized for any multi-level signal, including PAM4. The method is tested for several measured channels and analysis of the results is presented.

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In this paper we describe FFE and DFE equalizations for multi-level signals and benefits of the joint equalization algorithm. Further we give formulation of separate optimizations for FFE and DFE equalizations for multilevel signals and construct FFE and DFE joint optimization algorithm for multi–level signals and proposes analytical solution of the problem; Proposed joint optimization algorithm is compared to separate FFE and DFE equalizations and the results are summarized.

2 Optimization of FFE equalization for PAM4 signals

The goal of the paper is to evaluate the algorithm that can analytically solve optimization problem of finding the best tap coefficients of FFE and DFE joint equalization for PAM4 signals. The optimal tap coefficients allow to minimize difference between equalized signal and training signal at the center location of the unit intervals in terms of least mean square. To understand proposed algorithm, let us formulate FFE and DFE optimization problems separately.

FFE equalized signal v_k^{FFE} can be calculated as a convolution of tap coefficients c_i, $i = -m, \dots, n$, and the the channel output v_k, $k = 1, 2, \dots, N$

$$v_k^{\mathrm{FFE}} = \sum_{i=-m}^{n} c_i v_{k-i}, \tag{1}$$

where, N – is the number of training bits. Schematic of FFE equalization is given in Fig. 1.

The goal of FFE optimization is to find such tap coefficients c_i that minimize the difference between training "signal" u_k and equalized signal calculated by (1). Since the number of training signal samples (N) may be greater than number of unknown tap coefficients ($m + n + 1$) the task can be solved using minimum mean square error (MMSE) algorithm. That is we will minimize the cost function at the central location of the unit interval

$$\min_{c_i, i=-m,\dots,n} ||u - v^{\mathrm{FFE}}||^2 =$$

$$\min_{c_i, i=-m,\dots,n} \sum_{k=1}^{N} \Big(u_k - \underbrace{\sum_{i=-m}^{n} c_i v_{k-i}}_{v_k^{\mathrm{FFE}}} \Big)^2 . \tag{2}$$

Let us note that for FFE equalization and corresponding optimization algorithm (2), there is no difference between NRZ and PAM4 signals, as in both cases tap coefficients are convolved similarly with a channel output.

3 Optimization of DFE equalization for PAM4 signals

DFE is applied after FFE. The goal of DFE is to directly remove ISI effect from the signal. At every moment ISI tail from the previously transferred bits are added to the signal. The goal of DFE optimization is to estimate ISI tail values and subtract from the signal. These estimated ISI tail values are tap coefficients. The i-th DFE tap coefficient corresponds to the i-th unit interval ISI value. In case of NRZ, for each bit we have two different values: positive and negative, but with the same amplitude. Depending on the sign of the previous bit value, the ISI tail should be either added or removed from the signal. If DFE tap coefficients are known, DFE equalized signal for NRZ is calculated using the following formula

$$v_k^{\mathrm{DFE}} = v_k - \sum_{i=1}^{p} d_i \operatorname{sgn}(v_{k-i}), \tag{3}$$

where $d_i, i = 1, \dots, p$, are DFE tap coefficients. The termn $\operatorname{sgn}(x)$ in (3) is needed to identify previous bit values and correspondingly subtract or add tap coefficient to cancel ISI. In case of PAM4 signals, the signum function is not enough to accurately remove ISI. For each bit, PAM4 has 4 different values - two positive and two negatives. Both, positive and negative values include high voltage amplitude and low voltage amplitude values. It means that for each bit, ISI values not just added and subtracted but also scaled based on previous bit values.

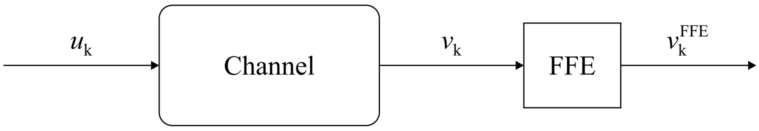

Fig. 1. FFE equalization schematics

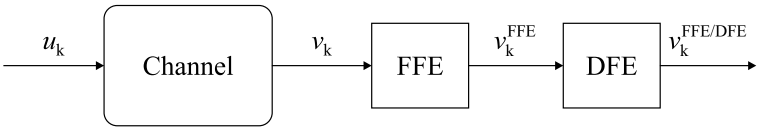

Fig. 2. FFE and DFE equalization schematics

To remove ISI properly from PAM4 signal simple signum function is not enough and should be replaced by the following function

$$F(v_k) = \begin{cases} 1 & \text{if } v_k \text{ is a positive high amplitue} \\ 1/3 & \text{if } v_k \text{ is a positive low amplitude} \\ -1/3 & \text{if } v_k \text{ is a negative low amplitude} \\ -1 & \text{if } v_k \text{ is a positive high amplitude} \end{cases}. \tag{4}$$

Here we are assuming that ratio between high amplitude and low amplitude bits is equal to 3. In general case ratio between $F(v_k)$ values should coincide to the ratio of high amplitude and low amplitude bits. Using $F(v_k)$ function DFE for PAM4 signals should be calculated using the following formula

$$\begin{aligned} v_k^{\text{DFE}} &= v_k - S_{\text{v}} \\ \text{where} \quad S_{\text{v}} &= \sum_{i=1}^{p} d_i F\big(v_{k-i}\big) \end{aligned}. \tag{5}$$

Formulas (3) and (5) can be combined if we will assume that in case of PAM4 $F(v_k)$ is calculated according to the formula (5) and in case of NRZ it just equals to $\operatorname{sign}(v_k)$. Schematic of FFE and DFE equalizations are given in Fig. 2.

Optimal DFE tap coefficients will create opposite of Inter Symbol Interference of pulse response and remove channel effect from the channel response. The goal of DFE optimization is to find such tap coefficients that can cancel channel effect and minimize difference between input and equalized signals (see [5], [6]). It means that we face again an over determined set of equations

$$u_k = v_k^{\text{DFE}} - S_{\text{v}}. \tag{6}$$

Moreover, (6) represent set of non-linear equations as function $F\big(v_{k-i}^{\text{DFE}}\big)$ has already DFE equalized signal value at the previous bits as an argument which itself depends on unknown DFE tap coefficients. This nonlinearity is pretty strong, and it makes hard to solve the set of equations. Fortunately, function $F\big(v_{k-i}^{\text{DFE}}\big)$ values are discrete and just needs information which bit was transmitted. Optimal tap coefficients are found using training signal, which means that transmitted bit values are known and the argument of this function can be replaced by training signal. After this replacement we will obtain the following linear set of equations

$$\begin{aligned} u_k &= v_k^{\text{DFE}} - S_{\text{u}}, \\ \text{where} \quad S_{\text{u}} &= \sum_{i=1}^{p} d_i F\big(u_{k-i}\big) \end{aligned}. \tag{7}$$

As in case of FFE the system (7) can be solved using MMSE algorithm, that will minimize the mean square error between equalized and training signals at the central location of the unit interval

$$\begin{aligned} &\min_{d_i, i=-m,\dots,p} ||u - v^{\text{DFE}}||^2 = \\ &\min_{d_i, i=-m,\dots,p} \sum_{k=1}^{N} \Big(u_k - S_{\text{v}}\Big)^2. \end{aligned} \tag{8}$$

Equations (7) and (8) represent algorithm to solve DFE optimization problem using MMSE for both NRZ and PAM4.

4 Joint optimization of FFE and DFE equalization for PAM4 signals

Solving FFE optimization problem and DFE optimization problem separately will not give the best tap-coefficients. The most efficient way is to solve FFE-DFE joint optimization problem for finding the best tap-coefficients. Let us merge two optimization problem into one. First, we need to apply FFE on a channel response and then DFE. Applying both equalization we will get the following formula

$$v_k^{\text{FFE/DFE}} = v_k^{\text{FFE}} - S_{\text{u}}.$$

Here c_i and d_i are unknown tap coefficients. The goal of the optimization algorithm is to find such c_i and d_i tap coefficients that will minimize difference between equalized signal and training signal. It means to meet the conditions

$$u_k = v_k^{\text{FFE}} - S_{\text{u}}.$$

We can write this set into matrix form

$$\begin{aligned} &\mathbf{u} = \mathbf{A}\mathbf{x}, \\ &\mathbf{x} = \big(c_{-m}, \dots, c_n, d_1, \dots, d_p\big)^{\top}, \\ &\mathbf{u} = \big(u_0, \dots, u_N\big)^{\top}. \end{aligned} \tag{9}$$

$\mathbf{u}$ being a vector of given training signal and $\mathbf{A}$ is known matrix containing values of the channel response and training signal

$$\mathbf{A} = \begin{bmatrix} v_m & \dots & v_{-n}, & -F(u_{-1}) & \dots - F(u_{-p}) \\ v_{1+m} & \dots & v_{1-n}, & -F(u_0) & \dots - F(u_{1-p}) \\ v_{2+m} & \dots & v_{2-n}, & -F(u_1) & \dots - F(u_{2-p}) \\ \dots & \dots & \dots & \dots & \dots \quad \dots \\ v_{N+m} & \dots & v_{N-n}, & -F(u_{N-1}) & \dots - F(u_{1-p}) \end{bmatrix}.$$

Here $v_k = 0$ and $u_k = 0$ for $k < 0$ and $k > N$. Let us note again that (10) is an over determined system of equations corresponding to the number of training bits and number of unknowns thus *ie*total number of FFE and

DFE tap coefficients. These equations should be solved using MMSE, to minimize the difference

$$\min_{\mathbf{x}} ||\mathbf{Ax}-\mathbf{u}||^2 = \min_{\substack{c_i,\, i=-m,..,n\\ d_j,\, j=1,\dots,p}} \Big\{ g(x) \Big\},$$

where the cost function is (10)

$$g(x) = \sum_{k=1}^{N} \Big(u_k - \underbrace{\sum_{i=-m}^{n} c_i v_{k-i}}_{S_\mathrm{o}} + \underbrace{\sum_{i=1}^{p} d_i F(u_{k-i})}_{S_\mathrm{u}} \Big)^2 .$$

To solve optimization problem we need to derivate $g(x)$ by tap coefficients and make it equal to zero

$$\frac{\partial g(x)}{\partial c_j} = 2\sum_{k=1}^{N} v_{k-j}\Big(u_k - S_\mathrm{o} + S_\mathrm{u}\Big),$$
$$\frac{\partial g(x)}{\partial d_j} = 2\sum_{k=1}^{N} F(u_{k-j})\Big(u_k - S_\mathrm{o} + S_\mathrm{u}\Big).V \qquad (11)$$

After simplification we will get the following linear set of equations

$$\sum_{i=-m}^{n} B_{i,j}^{(1)} c_i - \sum_{i=1}^{p} C_{i,j}^{(1)} d_i = \sum_{k=1}^{N} v_{k-j}\, u_k,$$
$$j = -m,\dots,n,$$
$$\sum_{i=-m}^{n} B_{i,j}^{(2)} c_i - \sum_{i=1}^{p} C_{i,j}^{(2)} d_i = \sum_{k=1}^{N} F(v_{k-j}) u_k, \qquad (12)$$
$$j = 1,\dots,p,$$

where

$$B_{i,j}^{(1)} = \sum_{k=1}^{N} v_{k-j} v_{k-i},$$
$$C_{i,j}^{(1)} = \sum_{k=1}^{N} v_{k-j} F(u_{k-i}),$$
$$B_{i,j}^{(2)} = \sum_{k=1}^{N} F(u_{k-j}) v_{k-i}, \qquad (13)$$
$$C_{i,j}^{(2)} = \sum_{k=1}^{N} F(u_{k-j}) F(u_{k-i}).$$

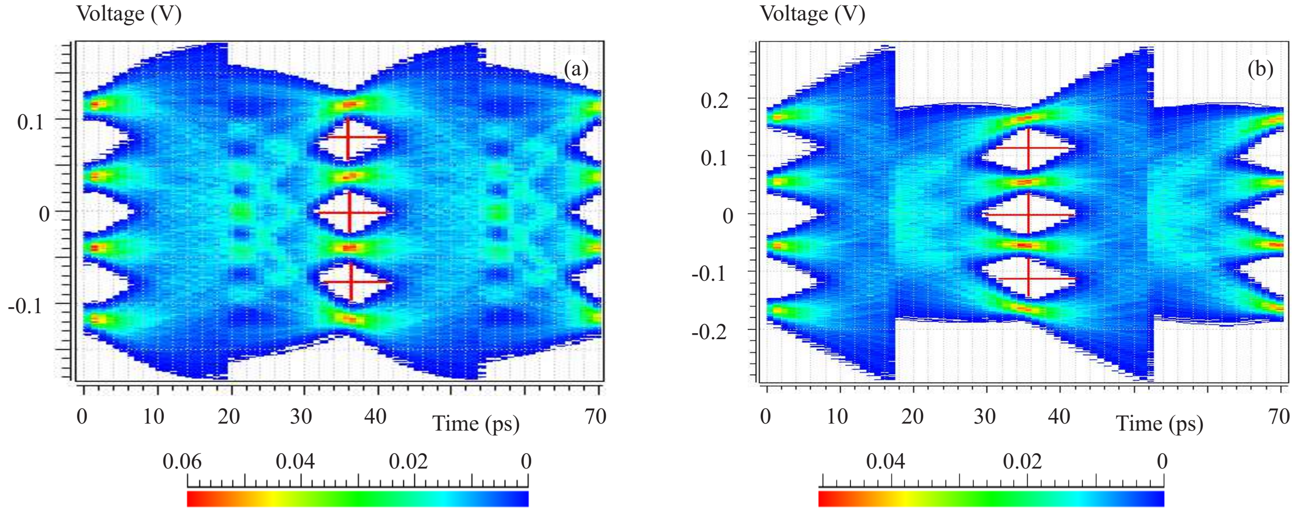

Fig. 3. Eye diagram (3-tap FFE, 11-tap DFE) for channel 1: (a) – separate equalization, (b) – joint equalization

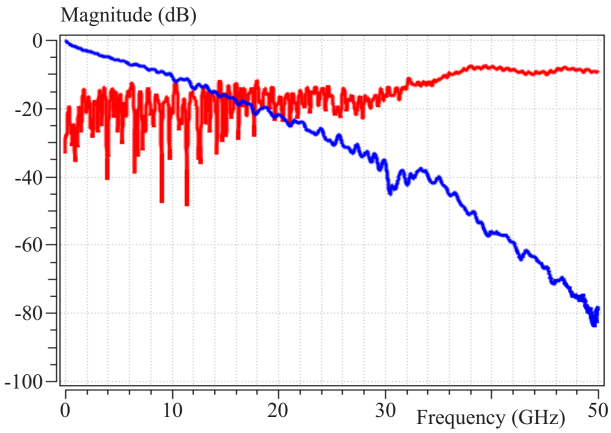

Fig. 4. Insertion loss and reflection loss of test channel 1

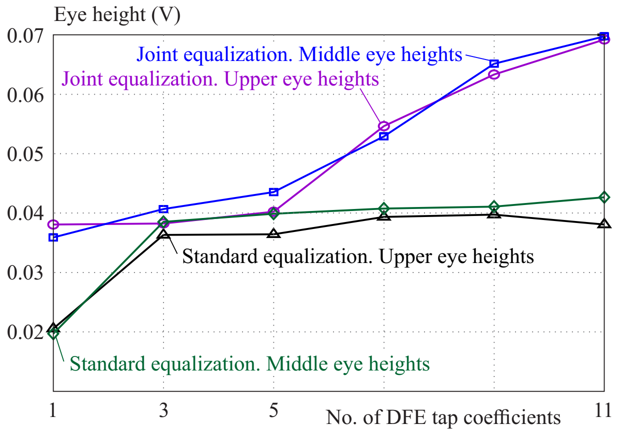

Fig. 5. Eye opening comparison of joint and separate equalization algorithms for channel 1 in case of PAM4 signal

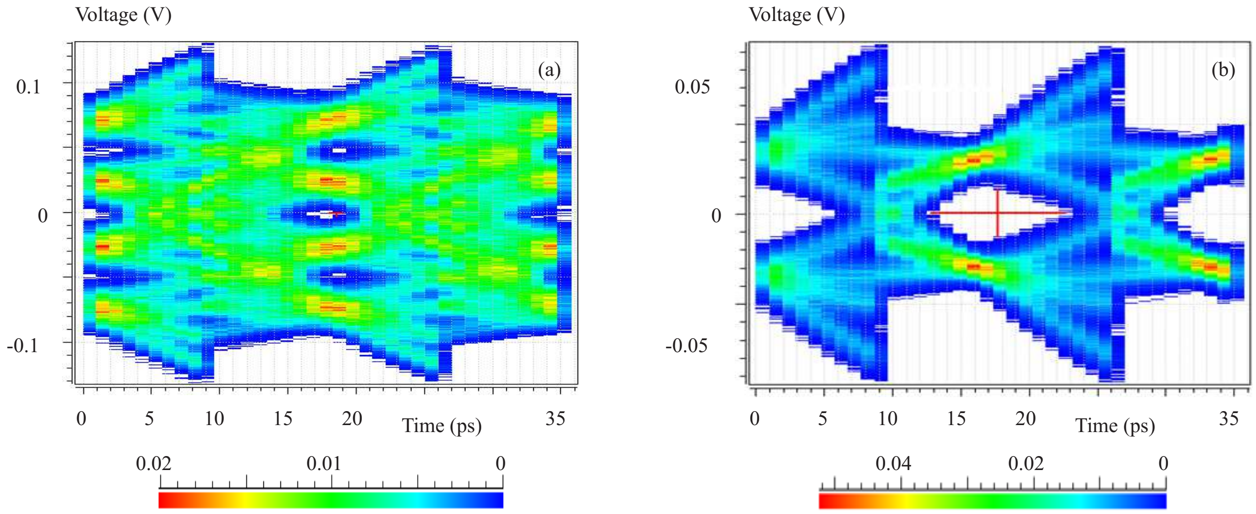

Fig. 6. Eye diagram for NRZ signal(3-tap FFE,m 11-tap DFE) for channel 1: (a0 – separate equalization, (b) – joint equalization

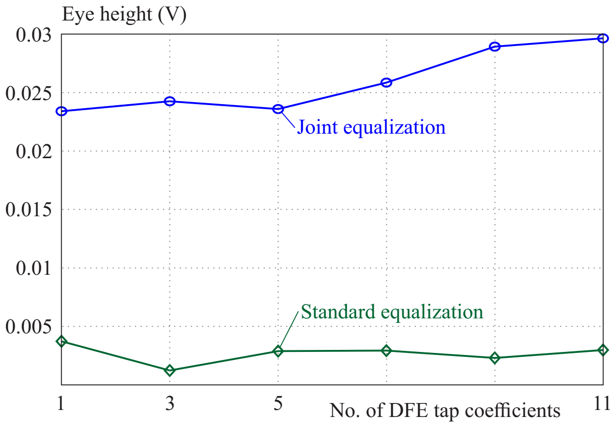

Fig. 7. Eye opening comparison between joint equalization and separate equalization algorithms for channel 1 in case of NRZ signal

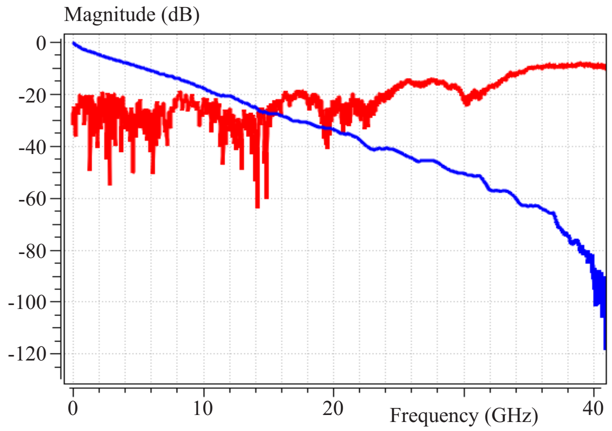

Fig. 8. Insertion loss and reflection loss of test channel 2

In this st of linear equations the number of unknowns is the same as the number of equations, hence it can be solved directly. The result will be a vector of optimal FFE and DFE tap coefficients, minimizing mean square error between equalized and training signals.

5 Test results

The proposed method was tested for two different measured channels.

5.1 Test results for channel 1

Channel 1 comprises of 4.3” transmission line on a board with IS415 material (mid-low loss material) connecting to another board with 4.3 of transmission line on IS415. There are total of 4 vias in the path with each via having less than 10 mils of via stub. The connector used for connecting the boards is a high frequency connector designed for 50 Gbps speeds.

For channel 1 proposed joint equalization algorithm was compared to the standard separate FFE and DFE optimizations. The three tap FFE (one post-cursor one cursor and one pre-cursor) and from 1 to 11 tap DFE equalization were used.

Figure 3 shows eye diagrams for both equalization methods with 3-tap FFE and 11-tap DFE in case of channel 1. Fig. 3a corresponds to separate optimization and Fig. 3(b) corresponds to the joint optimization. The minimum eye heights for separate equalization are about 38 mV, see Fig. 3(a) and for joint equalization equal to 68m V approximately, see Fig. 3(b). Joint equalization improvement is about 79%. Figure 4 shows the insertion and reflection loss of test channel 1.

Summarized comparison results for the channel 1 is given in Fig. 5. Number of FFE tap coefficients are fixed and equals to 3. Number of DFE tap coefficients varies from 1 to 11. As we see from Fig. 3 for higher number of DFE the tab coefficients eye height difference between the joint and separate optimization methods is increasing sig-

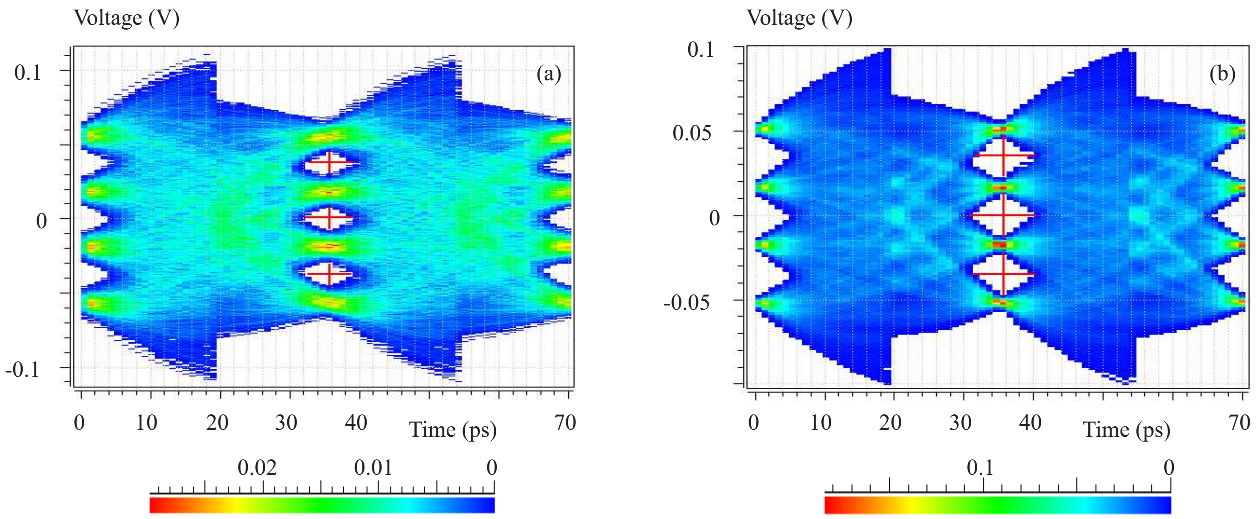

Fig. 9. Eye diagram for channel 2: separate equalization; 3-tap FFE; 7-tap DFE,Eye diagram for channel 2: joint equalization; 3-tap FFE; 7-tap DFE.

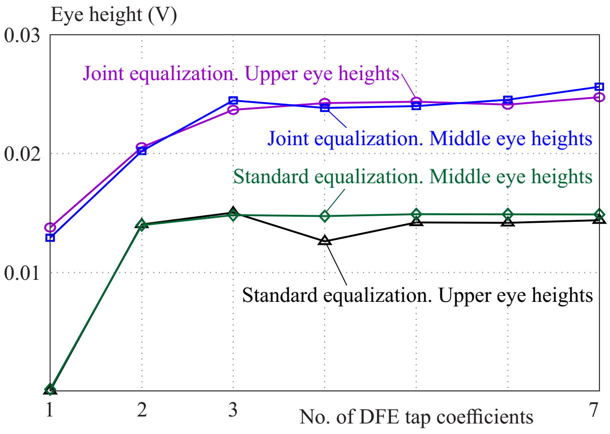

Fig. 10. Eye opening comparison between joint equalization and separate equalization algorithms for channel 2 in case of PAM4 signal.

nificantly. Starting from DFE 4-tap coefficients separate equalization cannot improve eye opening significantly, but joint equalization still is efficient and joint equalization improves eye opening from 30-69%. Figure 3 depicts eye heights for upper and middle eyes. Eye heights for bottom eyes are not plotted here due to almost symmetry.

For channel 1 we also did the comparison for NRZ signal. Figure 6 shows eye diagrams for both equalization methods with 3-tap FFE and 7-tap DFE in case of channel 1 for NRZ signal. Figure 6(a) corresponds to separate optimization and Fig. 6(b) corresponds to the joint optimization.

As we see separate optimization almost could not open eye-eye height is about 3 mV, see Fig. 6(a), while for joint equalization the eye height is about 29 mV, see Fig. 6(b). The joint equalization improves eye opening almost ten times.

Summarized comparison results for the channel 1 in case of NRZ signal is given in Fig. 7. Number of FFE tap coefficients are fixed and equals to 3. Number of DFE tap coefficients varies from 1 to 11. As we see from Fig. 7, for higher number of DFE tab coefficients, separate equalization cannot improve eye opening, whereas joint equalization still is efficient and joint equalization improves eye opening approximately from 5 to 10 times.

5.2 Test results for channel 2

Channel 2 comprises of 5” transmission line on a board with Megtron-6 material (ultra-low loss material) connecting to another board with 13 of transmission line on Megtron-6. There are total of 4 vias in the path with each via having less than 10 mils of via stub. The connector used for connecting the boards is a high frequency connector designed for 50 Gbps speeds [7].

Figure 8 shows insertion loss and return loss for the considered channel. For channel 2 proposed joint equalization algorithm was compared to the standard separate FFE and DFE optimizations. The three tap FFE (one post-cursor one cursor and one pre-cursor) and from 1 to 7 tap DFE equalization was used.

Figure 9 shows eye diagrams for both equalization methods with 3-tap FFE and 7-tap DFE in case of channel 2. The minimum of eye heights for separate equalization is about 14.3 mV, see Fig. 9(a) and for joint equalization equals to $\approx$ 24.9 mV, see Fig. 9(b). Joint equalization improves eye opening by 74%.

Summarized comparison results for the channel 2 is given in Fig. 10. Number of FFE tap coefficients are fixed and equals to 3. Number of DFE tap coefficients varies from 1 to 7. As we see from it, for any number of DFE tab coefficients joint equalization improves eye opening more than 65%. Likewise to Fig. 5, due to symmetricity, in Fig. 10 we plotted only upper and middle eyes.

This eye opening comparison between joint equalization and separate equalization algorithms for channel 2 in case of PAM4 signal.

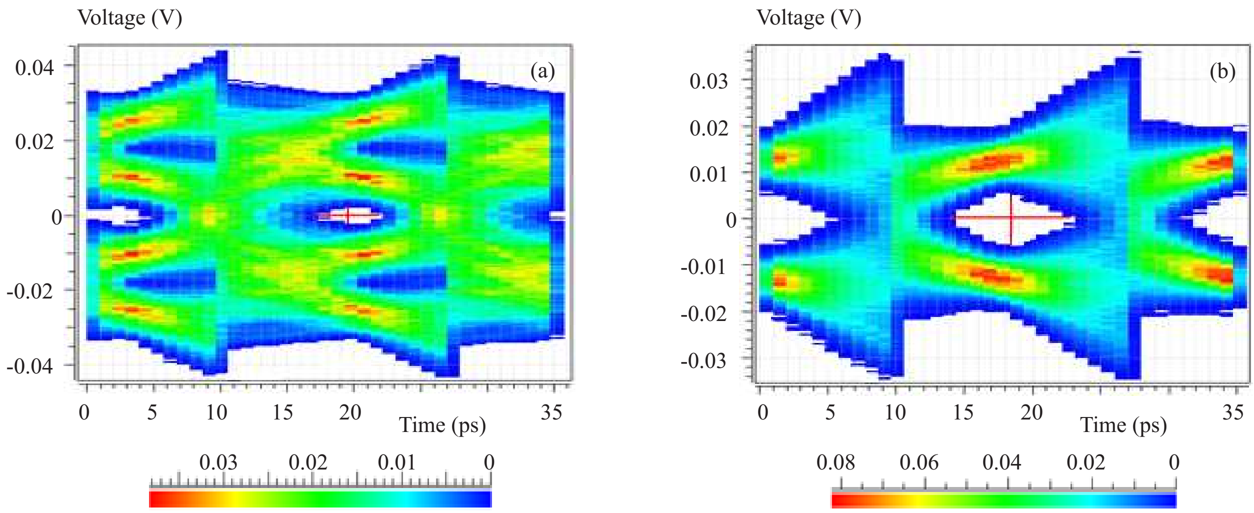

Fig. 11. Eye diagram for channel 2 (NRZ signal): separate equalization; 3-tap FFE; 7-tap DFE, Eye diagram for channel 2 (NRZ signal): joint equalization; 3-tap FFE; 7-tap DFE

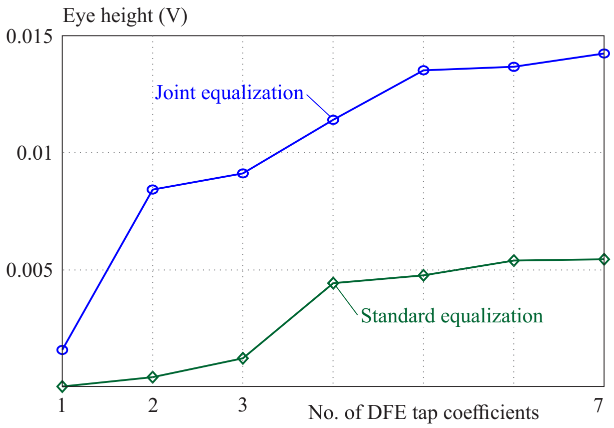

Fig. 12. Eye opening comparison between joint equalization and separate equalization algorithms for channel 2 in case of NRZ signal

For channel 2 we also did the comparison for NRZ signal. Fig. 11 shows eye diagrams for both equalization methods with 3-tap FFE and 7-tap DFE in case of channel 2 for NRZ signal. Figure 11(a) corresponds to separate optimization and Fig. 11(b) corresponds to the joint optimization. As we see separate optimization barley opens eye–eye height equals to 5mV, see Fig. 11(a). For joint equalization the eye height is almost 15mV, see Fig. 6(b). Joint equalization improves eye opening nearly three times.

Summarized comparison results for the channel 2 in case of NRZ signal is given in Fig. 12. Number of FFE tap coefficients are fixed and equals to 3. Number of DFE tap coefficients varies from 1 to 7.

As we see from Fig. 12 for higher number of DFE tab coefficients eye height difference between joint and separate optimization methods is increasing significantly. Starting from DFE 4-tap coefficients separate equalization cannot improve eye opening significantly, but joint equalization still is efficient and joint equalization improves eye opening approximately three times.

6 Conclusions

In the paper we proposed new analytical method for finding the best tap coefficients for FFE and DFE joint optimization. The proposed method combines FFE and DFE tap coefficients into one optimization problem. The goal function of the optimization problem is to minimize mean square error between equalized and training signals and therefore decrease bit error rate. Combining DFE and FFE tap coefficients into one optimization problem allows them to collaborate and lead to the global optimal solution. The paper gives analytical solution of the proposed optimization problem. The proposed method is robust and can be applied for both NRZ and multi-level signals. The method was tested for several measured channels. Analysis of the testing shows that the efficiency of the proposed method is increasing for the larger number of DFE tap coefficients and gives more than 70% better eye opening than separate optimization. The proposed method is efficient, and its practical implementation is easy.

Acknowledgements

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