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Abstract: With the tremendous increase in spending on public projects, contractors need to employ efficient and effective bidding strategies to cope with the competitive bidding environment. Usually, general contractors carry a portion of the work and subcontract other parts to eventually submit a holistic joint bid. This bidding setting is referred to as multistage bidding where subcontractors submit their quotations/bids to the general contractor, after which the general contractor submits a final joint bid for the whole project. In a multistage bidding environment, general contractors may be faced with an increase in the probability of negative or below normal profits. Despite previous research efforts for developing bidding models, there is a need for the extension of existing literature to tackle the multistage bidding environment, referred to hereinafter as multistage game (MSG). As such, the goal of this paper is to develop a bidding model for the MSG. The authors followed a multistep research methodology comprised of: (1) defining MSG in terms of game theory; (2) deriving a game-theoretic bid function for general contractors to determine the final joint bid to submit in MSG; and (3) developing a simulation model for MSG, using a data from 2,235 US public infrastructure projects. Results demonstrate that the new bid function gives general contractors a competitive advantage by avoiding the occurrence of negative profits in their part of the project. Also, results show a reduction in the occurrence and magnitude of the negative profits in relation to the final joint bids. This research significantly contributes to the body of knowledge by providing an innovative bid function for MSG. In addition, it offers substantial practical benefits for general contractors by providing a tool that facilitates dealing with the inherent complexity and uncertainties related to actual cost estimation within the MSG decision-making process. **DOI: 10.1061/(ASCE)ME.1943-5479.0000997.** © *2021 American Society of Civil Engineers.*

Introduction

The construction industry is considered a mainstay of the US economy and an indicator of the effectiveness and efficiency of its economy (Ahmed 2015). According to the Fails Management Institute (FMI) (Fails Management Institute 2019), the construction industry contributes around 7% of the total US Gross Domestic Product (GDP). Public infrastructure projects are considered a key portion of the construction industry and the global and national spending on public infrastructure projects continues to increase tremendously. More specifically, the global spending on infrastructure projects is anticipated to reach more than \$53 trillion between 2010 and 2030 (US Department of Commerce 2020). With this massive increase in infrastructure projects, it is essential to understand the construction industry-related processes for contractors to maintain long-term competitiveness for effective and efficient functioning of the economy.

The construction industry is a complex sector (Assaad et al. 2020), where construction and infrastructure bidding is considered one of the most complex and highly competitive inherent processes. During the bidding process, submitted bids are evaluated from the technical perspective, and then the technically approved bids are evaluated from the financial perspective (Ahmed et al. 2016). Moreover, there are various project delivery methods including design/bid/build (DBB), design/build, construction manager at risk, integrated project delivery, among others. DBB is the most commonly used project delivery method in the US construction industry, especially in public projects (Antoine et al. 2019; Ling et al. 2004). In addition, there are also various project award methods including competitive bid, best value selection, negotiated selection, qualification-based selection, among others (Messner 2019; Chinowsky and Kingsley 2008). In the public sector, DBB accompanied by a competitive bidding process is usually implemented for contractors' selection and procurement (Messner 2019). That said, it is worth highlighting that the focus of this paper is on competitive bidding as the project award method, in which determining the bid amount to submit is one of the greatest challenges and critical decisions that face contractors (Assaad et al. 2021). This decision depends on many factors including the bid financial evaluation method. There are many bid financial evaluation methods such as the low-bid method, the second-lowest bid method, the average bid method, and the below-average bid method (Ioannou and Awwad 2010). In the US construction industry, the low-bid method is the

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most commonly used method for the financial evaluation of bids (Ioannou and Awwad 2010). Accordingly, it is imperative to highlight that the focus of this paper is on the low-bid method as the used method for the financial evaluation of bids. In the low-bid method, the contract is allocated following a competitive bidding process and awarded to the lowest qualified bidder (Seydel 2003).

For projects where the low bid method is implemented as the method for the financial evaluation of bids, contractors strive to be the lowest qualified bidder and be awarded the project contract. Such a strategy may lead the contractor to face what is called in game theory the "winner's curse." According to Ahmed et al. (2016), the winner's curse is the situation in which the bidder who has most underestimated the true cost wins the project with a bid that is less than the true cost of the project, and consequently, will be expected to earn negative or below normal profits. One of the main reasons for the existence of the winner's curse in the construction and infrastructure bidding is that at the time of submission of bids, contractors do not know with certainty all the costs related to project execution (Awwad et al. 2015; Ahmed et al. 2015). Hence, it is difficult for contractors to fully avoid the winner's curse, but its occurrence can be decreased by means of game-theoretic bidding models (Dyer et al. 1989; Assaad et al. 2021).

In general, construction and infrastructure bidding can be either: (1) single-stage bidding, in which general contractors compete in between for the project and the winning general contractor execute the whole project on his\her own; or (2) multistage bidding, in which the final bid is a compilation of multiple bids where subcontractors submit their quotations\bids to the general contractor, after which general contractor submits a final joint bid for the whole project. If the subcontractor wins, the general contractor signs a contract with the winning subcontractor (i.e., the subcontractor with the lowest price following the low bid method) and pays the amount of the subcontractor's quotation upon full execution of their part of the project. In other words, the received quotation from the subcontractor constitutes a bid in an informal bidding setting.

In fact, subcontracting is a common practice in the construction industry (Lew et al. 2020). The use of subcontracting is more frequent for larger and more complex projects such as infrastructure megaprojects (Lew et al. 2018; Kardes et al. 2013; Tam et al. 2011). Generally, in the selection of subcontractors, general contractors experience multistage bidding. Such a multistage bidding setting could lead to an increased probability and magnitude of the winner's curse. More specifically, Ahmed et al. (2016) highlighted that the winner's curse may happen at each stage, one on part of the winning general contractor and the other on part of its winning subcontractor(s). There is a plethora of research efforts on developing bidding models; however, there is still a lack of research that tackles the multistage construction and infrastructure bidding, referred to hereinafter as multistage game (MSG). Being the case, this paper addresses this important research need by incorporating game theory and its related concepts to investigate and analyze MSG.

Goal and Objectives

The goal of this paper is to develop a bidding model to be utilized by general contractors in MSG. The associated objectives include: (1) presenting a solution for MSG in the form of a bid function, (2) simulating the real-world MSG environment following the rules of the competitive construction and infrastructure bidding and low bid method, and (3) testing and validating the performance of the derived bid function and model. To this end, this research should aid contractors in dealing with the uncertainties related to actual cost estimation within MSG decision-making processes.

Background Information

Auction Theory and the Construction and Infrastructure Bidding

Auction theory is a subdiscipline of game theory; thus, game theory and auction theory concepts are interconnected. Game theory is one of the most important established mathematical tools to illustrate and model the human decision-making process. Over hundreds of years, Auctions have been utilized for selling and distributing goods and services. More recently, auctions are of substantial importance in both public and private sector transactions. From the informational perspective, there are two major types of auctions: (1) private value auctions, in which each bidder knows, with certainty, its valuation or cost of the item being auctioned, but it does not know other bidder's valuations or costs; and (2) common value auctions, in which all bidders have the same valuation or true cost of the item being auctioned, but no bidder knows it with certainty before submitting their bid. According to Kagel and Levin (2002), in common value auctions, each bidder develops an estimate about the true value of the auctioned item at the time of bidding; and the winning bidder is the only one to observe the true value or cost of the auctioned item.

Generally, construction and infrastructure bidding are considered a common value auction (Dyer and Kagel 1996). In the construction and infrastructure bidding, the project cost constitutes the information variable for bidders. In fact, bidders develop independent estimates about the true cost of the project, and this true cost will not be known until the completion of the project by the winning bidder. Estimates vary as each bidder has different information and/or beliefs about the factors that affect the final project cost. Generally, in the construction industry, bidders have two sources of incomplete information: (1) the actual cost of the project; and (2) the estimates of their competitors of the actual cost of the project. Construction and infrastructure bidding can also be referred to as a reverse first-price (low bid) sealed-bid auction (Ahmed et al. 2016). Unlike auctions in which bidders aim to purchase goods and services, in the construction and infrastructure bidding, bidders aim to sell their services to project owners. In such a setting, the winner is determined as the bidder with the lowest submitted bid. Due to incomplete information, bidders are subject to the adverse selection problem, which is the situation when the winner is the one who has most underestimated the true project cost and won the project with a bid less than the true project cost. This results in negative expected earnings. Not accounting for the adverse selection problem results in the winner's curse. The winner's curse was firstly introduced by Capen et al. (1971) who analyzed its existence in outer continental shelf (OCS) oil lease auctions. Thereafter, many researchers have investigated common value auctions and either account for or identify the occurrence of the winner's curse in various domains including construction bidding (Ahmed 2015).

Auctions with Private and Common Values

Despite that the widely accepted theoretical categorization of auctions to either private or common value auctions, most real-world auctions are a combination of both types of auctions. In that regard, Laffont (1997) conducted an extensive survey of empirical studies on auctions and concluded that "most empirical studies clearly involve some private value element as well as some common value element". Moreover, Goeree and Offerman (2002) stated that "most real-world auctions exhibit both private and common value elements" (p. 627). Regarding auctions with private and common values, Goeree and Offerman (2002) presented optimal bid functions for the first-price (high bid) sealed-bid auction with two-dimensional (2D) value information comprised of private and common values. Further, Goeree and Offerman (2002) conducted a series of laboratory experiments, in which bidders simultaneously receive a private value and an independent signal for the common value portion. The authors found that bidders suffered from the winner's curse and its degree is increasing with the increase of uncertainty about the common value. As per Kagel and Levin (2014), in their survey of experimental research related to auctions, the aforementioned experiments were the only known laboratory experiments of auctions with both common and private value elements. Thereafter, various researchers utilized and extended the bid functions presented by Goeree and Offerman to investigate and model different aspects related to competitive bidding and auctions (De Silva et al. 2003; Fatima et al. 2005; Ye 2007; Levin et al. 2007; Heumann 2019).

Previous Research on Construction and Infrastructure Bidding

Over the last 60 to 70 years, various models have been developed by many researchers to be applied in the construction and infrastructure bidding. More specifically, various studies have investigated the bidding-related decisions in competitive bidding since the primary research by Friedman (1956) and Rastegar et al. (2021). Mainly, these models aim primarily in providing the contractors with a criterion to determine the optimal bid value that maximizes the probability of winning and the earned profit. In general, existing bidding models' approaches can be classified into the following: (1) statistics; (2) utility theory; (3) artificial intelligence; (4) operations research; and\or (5) game theory and auction theory (Rastegar et al. 2021; Abotaleb and El-adaway 2017). Statistical models were utilized for estimation of the bid price based on a statistical analysis of historical behavior of competitors (Friedman 1956; Gates 1967; Abotaleb and El-adaway 2017) and detection of bid price irregularity in bidding situations (Erfani et al. 2021). Artificial intelligence–based models were utilized to estimate the bid price through the application of artificial neural networks, for example (Li 1996; Liu and Ling 2005). Utility theory–based models were utilized to estimate various bidding decisions considering a variety of bidding-related factors (Leśniak and Plebankiewicz 2015; Marzouk and Moselhi 2003; Jarkas et al. 2013; Chou et al. 2013). Operations research–based models were utilized for the estimation of bid price through applying various optimization techniques (Davatgaran et al. 2018; Rastegar et al. 2021).

Concerning game theory and auction theory approaches, various previous research efforts have applied auction theory and its related concepts to study and model the construction bidding decision-making process. For instance, Dyer et al. (1989) presented the symmetric risk neutral Nash equilibrium (SRNNE) bidding function for the reverse first-price sealed-bid common value auctions. The authors then conducted laboratory experiments to investigate the performance of experienced construction executives versus inexperienced students. They found that, like the students, experienced executives fell prey to adverse selection and the resulting winner's curse. Further, Ahmed et al. (2016) utilized the SRNNE function to analyze the occurrence of the winner's curse, mainly, in the single-stage construction bidding, and compare it with MSG. The authors found that contractors suffer from the winner's curse in approximately 83% and 92% of all the projects being bid in the single-stage construction bidding (SSG) and MSG, respectively. Moreover, Drew and Skitmore (2006) conducted a laboratory experiment to investigate the feasibility of Vickery's revenue equivalence theory, an auction theory concept, in construction bidding. Dong-hong and Xi-yan (2009) developed an auction theory–based bidding model that can benefit contractors in identifying the basis of bidding price decisions and guidelines on how to improve their competitiveness within the construction market. AbouRizk et al. (2009) developed a bidding game training tool, using the high-level architecture approach, to be used by students and practitioners to enhance their bidding decision-making skills. Tan and Suranga (2008) analyzed the occurrence of the winner's curse in the construction industry in Sri Lanka and concluded the existence of a significant effect of the winner's curse in the studied sector. Ho and Hsu (2014) utilized auction theory concepts to analyze the interfaces between heterogeneous bidders and concluded that, under specific conditions, bid compensation can motivate bidders to exert more efforts in the early stage. Moreover, Awwad et al. (2015) developed an agent-based model for construction bidding to investigate different bidding strategies of contractors and analyze bidding patterns and market behavior. De Clerck and Demeulemeester (2016) developed an auction theory–based bidding model that represents the public-private partnership (PPP) procurement setting and analyzes the effect of various mechanisms for governmental policies on the behavior of bidders. Nichols (2018) analyzed the reverse auction aiming to create an artificially intelligent player who maximizes its returns from the game. More recently, Assaad et al. (2021) utilized algorithmic game theory and auction theory concepts and developed a simulation model to study and analyze the effect of learning in the construction bidding decision-making process.

Knowledge Gap

Based on the aforementioned information, various valuable previous research efforts have developed bidding models following different approaches to aid contractors in various aspects related to the construction and infrastructure bidding decision-making process. Despite that, there is still a lack of research that tackles and models MSG, where subcontractors bid first and general contractors bid second. In that regard, Dyer and Kagel (1996) highlighted that existing game-theoretic research has yet to consider MSG, where the lowest bidder wins the contract. Awwad et al. (2015) highlighted the need for bidding models that represent the actual dynamics within the construction and infrastructure bidding since existing research studies have restricted applicability due to their methodological constraints. In addition, Ahmed et al. (2016) emphasized the need for a more realistic bidding model that accurately represents the real-world MSG by considering the general contractors' and subcontractors' bids interrelated rather than independent. Being the case, this paper tackles this critical research gap by providing a game-theoretic–based bid function and model for MSG in which the general contractors' and subcontractors' bids are considered interrelated.

It is worth noting that bidding models that are based on game theory and auction theory approaches have a distinct feature compared to other bidding models. Game-theoretic bidding models are basically mathematical models that study interactions between rational players, where the decision of each player will impact the others' payoff. These models do not require an extensive amount of data about competitors as game theory enables players to heuristically and mathematically formulate reasonable expectations about other players' behavior (Kadane and Larkey 1982). However, other bidding models that are following other approaches–such as statistics, utility theory, artificial intelligence, and\or operations research–requires extensive data about historical bid prices of competitors to be able to reach sound statistical inferences and provide reliable recommendation for bid price (Abotaleb and El-adaway 2017). Such data is even harder–if not impossible–to acquire in MSG as the general contractor needs to acquire data not only about its competitors but also about their subcontractors in order to be

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able to model them and predict their behavior. All the aforementioned details highlight the main reason for considering game theory and its related concepts to investigate and analyze MSG.

Methodology

To achieve the research goal and objectives, the authors followed a multistep research methodology, as depicted in Fig. 1.

In the followed methodology, the authors: (1) defined MSG in terms of game theory; (2) derived an innovative bid function for MSG to be utilized by the general contractors in determining their final joint bid to submit; and (3) developed MSG simulation model to investigate the impact of the derived bid function on the general contractor's expected earnings and the winner's curse from bidding on multiple projects. Subsequent paragraphs provide details pertaining to each followed step.

Step 1: Defining MSG in terms of game theory

In the first methodology step, the authors defined MSG in terms of game theory. The main purpose of this step was to identify game-theoretic approaches upon which a solution for MSG can be developed. In MSG, the general contractor has two sources of information at the time of bidding: (1) common value element that is represented in its estimates about the cost for their part of the project; and (2) the private value element that is represented in its winning subcontractor's bid. Therefore, the authors found it more appropriate to consider MSG as an auction with both private and common values from the general contractor's perspective. Upon critical review of existing models for auctions with both private and common values, it is important to highlight that the existing Goeree and Offerman's (2002) bid function is derived for the case of the first-price auction with a 2D valuation representing private and common values where the highest bidder is the winner. Despite many other extensions to the Goeree and Offerman's (2002) bid function, existing research has yet to consider deriving a bid function for reverse first-price auctions with private and common values where the lowest bidder is the winner. This paper and research take the lead on this important contribution.

Step2: Derivation of a Bid Function for MSG

In the second methodology step, the authors derived an innovative bid function for MSG to be utilized by the general contractors in determining their final joint bid to submit. This derivation is an extension to the derived optimal bid function by the Goeree and Offerman (2002) to consider the reverse first-price auctions with both private and common values. That being said, the subsequent paragraphs present an illustration of the mathematical derivation of the optimal bid function for the reverse first-price auctions with both private and common values in the context of MSG.

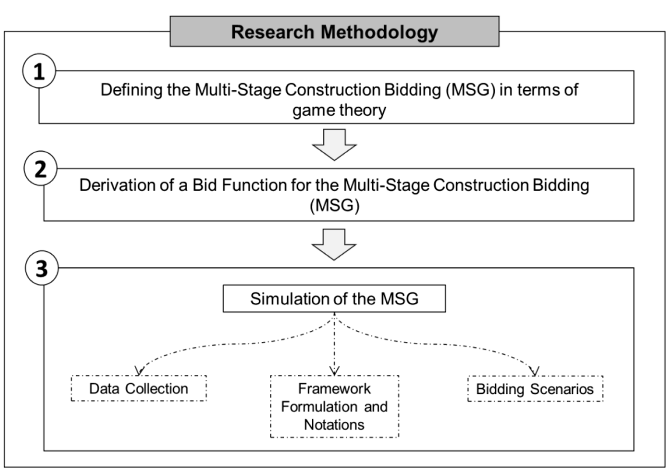

Fig. 1. Research methodology.

Let I denote the index of the general contractor. Let k denote the index of the project that general contractors bid on. Let LB_{iksc} represent the lowest subcontractor bid among the bids received by ith general contractor from its subcontractors for project k. In MSG, for each project k, each general contractor i receives two elements of information: (1) private value element that is represented in the bid of its winning subcontractor (LB_{iksc}); and (2) common value element that is represented in its estimate about the cost for their part of the project (S_{ik}). In simple words, the ith general contractor's valuation for the actual cost of the project is the sum of LB_{iksc} and S_{ik}. It is important to highlight that in this derivation, the true realization of the common value, which represents the actual cost of the general contractor's part of the project k (C_{kgc}), is represented as the average of general contractors' common value signals as shown in Eq. (1)

$$C_{kgc} = \frac{1}{n_{gc}} \sum_{i=1}^{n_{gc}} S_{ik} \tag{1}$$

where n_{gc} = total number of general contractors in the bidding competition. This straightforward representation of the common value has been used in previous theoretical and experimental research work. The traditional approach, however, is more complicated requiring a known distribution for the true common values and a known distribution for the signals centered around any particular true common value (Wilson 1977; Goeree and Offerman 2002). In this average representation of common value, the two different pieces of information can be combined into a single summary statistic of the true common value (Milgrom and Weber 1982). However, the traditional approach does not lend itself easily to the construction of a single summary statistic in the presence of both private and common values. To this end, let s_i denotes the summary statistics term that combines both private and common value elements into one single piece of information, referred to hereafter as the surplus variable. Accordingly, the surplus variable is depicted in Eq. (2)

$$s_i = \frac{S_{ik}}{n_{gc}} + LB_{iksc} \tag{2}$$

In Eq. (2), the surplus variable s_i is the sum of the averaging of the ith general contractor's common value signal S_{ik} plus the private value LB_{iksc}. Without loss of generality, focus on arbitrary general contractor denoted as General Contractor 1, for example. Following a derived extension of Goeree and Offerman's (2002) general solution of the optimal bid function, Eq. (3) presents an equilibrium bid function for the reverse first-price auction in presence of private and common values in the context of MSG

$$\begin{aligned} B(x) = {} & \mathrm{E}(C_{kgc} + LB_{1ksc} | s_1 = x, Y_1 = x) \\ & + \mathrm{E}(y_1 - Y_1 | s_1 = x, Y_1 = x) \end{aligned} \tag{3}$$

where $B(x)$ = bid value; x = random variable that represents the surplus; s_1 = surplus of General Contractor 1; and y_1 = lowest surplus among $(n_{gc} - 1)$ contractors, which equals $\min((S_{lk}/n_{gc}) + LB_{lksc})$ for $l = 2, 3, \ldots, n_{gc}$, and Y_1 is the lowest surplus among n_{gc} contractors which equals $\min\left(\left(\frac{S_{mk}}{n_{gc}}\right) + LB_{mksc}\right)$ for $m = 1, 2, \ldots, n_{gc}$. In general, the first term on the right side of Eq. (3)

represents the general contractor's valuation of the actual cost of the project assuming that its surplus value is the lowest. While the second term on the right side of Eq. (3) represents how much shall the general contractor shades up its joint bid to account for adverse selection and the winner's curse.

Recall that in Eq. (2), the term S_{ik} refers to the ith general contractor's estimate of the cost for its part of the project (common value signal), and LB_{iksc} refers to the winning subcontractor bid cost corresponding to the ith general contractor (private value). To simplify mathematical derivation, S_{ik} and LB_{iksc} are assumed to be uniformly distributed for all general contractors, following assumptions of Goeree and Offerman (2002), as follows: $S_{ik} \sim U[a,b]$ and $LB_{iksc} \sim U[c,d]$, where $b > a > d > c > 0$ assuming that the general contractor will construct most of the project on its own; in other words, the percentage of work constructed by the general contractor will be greater than the percentage of work subcontracted. Such assumption was considered in this paper because it is more common in practice that the percentage of work of the general contractor is more than the percentage of the work subcontracted, especially in infrastructure projects. For instance, various departments of transportations including the Arizona Department of Transportation, Colorado Department of Transportation, Utah Department of Transportation as well as the US Army Corps of Engineers impose conditions on their contractors to restrict the percentage of work being subcontracted considering the detrimental impacts associated with excessive subcontracting (Ng and Luu 2008).

It is important to note that the values of a, b, c, and d are deemed to be common knowledge for all general contractors considering that they are experienced in the construction market sector and the type of projects; thus, they approximately know the possible range for cost estimates for their part of the project and the cost of subcontracting portion of the project based on project type, location, and other various attributes. As such, the surplus variable s (i subscripts dropped for simplicity) can be the sum of two uniformly distributed variables. The support of s can be decomposed into the following three regions: $R_1 = [c + (a/n_{gc}), c + (b/n_{gc})] \cup R_2 = [c + (b/n_{gc}), d + (a/n_{gc})] \cup R_3 = [d + (a/n_{gc}), d + (b/n_{gc})]$. In addition, the density function of s is as follows: $f_{1s} = (n_{gc}(s-c)-a)/((b-a)(d-c))$, $f_{2s} = (1/(d-c))$, $f_{3s} = ((b-n_{gc}(s-d)/((b-a)(d-c))$ corresponding to R_1, R_2, R_3, respectively. It is worth noting that the authors plotted the density of the s and found that it has a trapezoidal shape. Given the previous specificity, the general solution of Eq. (3) is as follows:

$$B(x) = \frac{n_{gc}-1}{n_{gc}} \mathrm{E}(C_{kgc}|s \geq x) + \mathrm{E}(y_1|y_1 \geq x) \tag{4}$$

where

$$\mathrm{E}(C_{kg}|s \geq x) = \int_x^{d+\frac{b}{n_{gc}}} \mathrm{E}(C_{kgc}|s = y)\frac{f_s(y)}{1 - F_s(x)} dy \tag{5}$$

$$\mathrm{E}(y_1|y_1 \geq x) = \int_x^{d+\frac{b}{n_{gc}}} y \frac{(n_{gc}-1)f_s(y)(1-F_s(y))^{n_{gc}-2}}{(1-F_s(x))^{n_{gc}-1}} dy \tag{6}$$

where F_s = cumulative distribution corresponding to f_s. The corresponding regional conditional expectations of $\mathrm{E}(C_{kgc}|s = y)$ are as follows: $\mathrm{E}_1(C_{kgc}|s = y) = (1/2)(a + n_{gc}(y-c))$, $\mathrm{E}_2(C_{kgc}|s = y) = (a+b)/2$, $\mathrm{E}_3(C_{\mathrm{kgc}}|s = y) = (1/2)(b + n_{gc}(y-d))$. Therefore, each regional bidding function can be computed following Eqs. (4)–(6).

Consider the situation when $n_{gc} = 3$; in other words, three general contractors are competing for a project. The explicit formulas for the bidding function for each of the three regions are provided in Eqs. (7)–(9)

$$\begin{aligned} B_1(x) = {} & \frac{1}{45(a^2 + 6b(c-d) + 9(c-x)^2 + 6a(d-x))^2} \\ & \times [22a^5 + b^5 + 15b^4(c-d) + 810b^3(c-d)^2 \\ & + 5a^4(b + 51d - 42x) - 243(7c - 22x)(c-x)^4 \\ & + 4860b(c-d)(c-x)^2 x \\ & + 270b^2(c-d)(7c^2 - 2d^2 + 3x^2 - 2c(d+3x)) \\ & - 10a^3(b^2 + 12b(d-c) \\ & - 9(2c^2 + 9d^2 - 4x(c+3d) + 5x^2)) \\ & - 5a(b^4 + 12b^3(c-d) + 54b^2(c-d)(c-3d+2x) \\ & + 108b(c-d)(c^2 - 2cd - 2d^2 + 3x^2) \\ & + 81(c-x)^2(c^2 - 2x(c+6d) + 13x^2)) \\ & + 10a^2(b^3 + 18b^2(c-d) + 27b(c-d)(c+3d-2x) \\ & + 27(3c^2 d + 2d^3 - 6cdx - 3dx^2 + 4x^3))] \end{aligned} \tag{7}$$

$$\begin{aligned} B_2(x) = {} & \frac{1}{90}\left[35(a+b) + 30\mathrm{d} - \frac{(a-b)^3}{(a+b+6d-6x)^2} \right. \\ & \left. + \frac{10(a-b)^2}{a+b+6d-6x} + 60x \right] \end{aligned} \tag{8}$$

$$B_3(x) = \frac{1}{45}[23\mathrm{b} - 21\mathrm{d} + 66\mathrm{x}] \tag{9}$$

It is important to note that while the functions seem complex, especially for region 1 (R_1), they can be easily programmed when values for a, b, c, and d are specified.

Step 3: Simulation of MSG

In the third methodology step, the authors developed an MSG simulation model to investigate the impact of the derived bid function on the general contractor's expected earnings and the winner's curse from bidding on multiple projects. The following subsections provide the details related to the simulation model developed in this paper in terms of data collection, simulation framework, used notations, as well as simulation assumptions.

Data Collection

The simulation model developed in this paper is implemented using a dataset provided by the Ohio Department of Transportation (ODOT). The data consists of 2,235 public infrastructure projects managed and funded by ODOT. The data is for multimillion projects with values ranging from \$1 million to \$10 million. For each project, the data includes the name and description of the project, the value of the winning bid for each project, and the actual cost realized after the completion of the project by the winning contractor based on the initial scope of the project at the time of bidding. It is worth highlighting that some projects experienced change orders. However, the values of the change orders, which represent additional compensable costs, were not included in the actual cost of the project based on its initial scope as change orders cannot be foreseen at the time of bidding and its impact on cost shall be compensable.

The types of projects were limited to infrastructure projects conducted by ODOT such as freeway and interchange construction, construction and rehabilitation of bridges, roadway construction, among others. Table 1 shows the types of projects and their numbers on the dataset. In addition, it is imperative to highlight that all

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Table 1. Types and numbers of projects on the dataset

Project type	Number of projects	Percentage (%)
Freeway and interchange construction	734	32.84
Roadway construction	576	25.77
Utilities works	473	21.16
Construction of new bridges	223	9.98
Rehabilitation of bridges	217	9.71
Railway construction and rehabilitation	12	0.54

the considered projects are delivered following the DBB delivery method. Moreover, all the projects were awarded through competitive bidding to the lowest qualified bidder as per the low bid method. In fact, this was anticipated because most of the infrastructure projects are publicly funded and they are subject to the conditions of the Federal Acquisition Regulation (FAR), where the low bid method is considered a legal requirement, and contracts are often awarded to the lowest qualified bidder (US General Services Administration 2016). Accordingly, Fig. 2 and Table 2 show the distribution of the winning bid values and the actual costs of the projects of the collected dataset and provides brief descriptive statistics. This actual infrastructure dataset of projects was used in simulating the real-world MSG process.

Framework Formulation, Notations, and Assumptions

In the framework formulation for the simulation model, the authors implemented notations and assumptions. That being said, the following notations have been used in this paper:

- Recall that n_{gc} denotes the number of general contractors bidding for the project, such that $n_{gc} = 1, 2, \ldots, i$;
- Let n_{isc} be the number of subcontractors submitting their bids for the ith general contractor, such that $n_{isc} = 1, 2, \ldots, j$;
- Let n be the number of sequential projects, such that $n = 1, 2, \ldots, k$; and
- Let n_{gc} and n_{isc} bid on each project independently without knowledge of future projects.
- *In the first stage of bidding*: Considering subcontractors do not know the actual cost for their part of the project during bid submission, it can be represented as a random variable in the simulation model. To this end, let C_{ksc} denotes the actual cost of the subcontractors' part of the project k. Moreover, at the time of bidding, each subcontractor j develops its estimate of the actual cost of its part of the construction project C_{ksc}. In general, these estimates vary between projects depending on the scope, type, and complexity of the work. In addition, these estimates vary from one subcontractor to the other depending on their experience level, and the competency of their estimation team (Assaad et al. 2021). That said, let S_{jk} denotes the estimate of subcontractor j for the actual cost of its portion of project k at the time of bidding. Furthermore, subcontractors encounter an error in their estimation for the actual cost of their part of the project C_{ksc}. Let ε_{sc} denote the maximum error percentage around the actual cost of the subcontractors' part the project C_{ksc}. As highlighted by Assaad et al. (2021), the value of ε_{sc} can be modeled as a fixed percentage. Therefore, each subcontractor submits its bid considering both its developed estimate of the project's actual cost, as well as the expected error in its developed estimate. Thus, let B_{jki} represents the bid submitted by subcontractor j for its part of project k to the general contractor i; where $B_{jki} = f(S_{jk}, \varepsilon_{sc})$.

  That being said, the winning subcontractor is the one who has submitted the lowest bid to the general contractor. Recall that LB_{iksc} denotes the lowest subcontractor bid among the bids received by the ith general contractor from its subcontractors for project k; thus, $LB_{iksc} \in [B_{jki} \quad \forall \text{ i and j}]$
- *In the second stage of bidding*: Similar to subcontractors, general contractors do not know the actual cost for their portion of the project at the time of bidding, it can also be represented as a random variable. To this end, recall that C_{kgc} denotes the actual cost of the general contractors' part of the project k. In addition, let C_k denotes the actual cost of the whole project; thus, $C_k = C_{ksc} + C_{kgc}$. Moreover, at time of bidding, each general contractor i develops its estimate of the actual cost of its part of the construction project C_{kgc}. Similar to subcontractors, these estimates differ from one project to the other, as well as from one general contractor to the other. As such, let S_{ik} denotes the estimate of general contractor i for the actual cost of its part of project k at the time of bidding. Furthermore, similar to subcontractors, general contractors encounter an error in their estimation for the actual cost of their part of the project C_{kgc}. Let ε_{gc} denotes the maximum error percentage around the actual cost of the general contractors' part the project C_{kgc}. Therefore, each general contractor prepares its bid for its part of the project

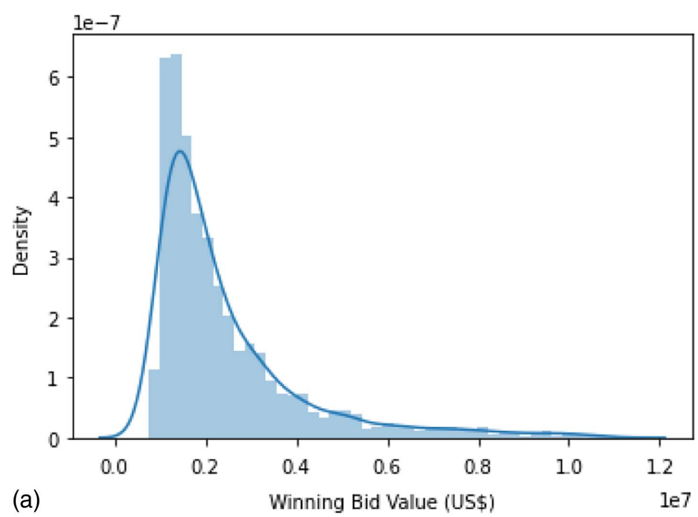

Fig. 2. Distribution of (a) winning bids; and (b) actual costs of the projects of the dataset.

Table 2. Descriptive Statistics of the winning bids and the actual cost of the projects o the dataset

Data	Mean (million \$)	Std (million \$)	Minimum (million \$)	Maximum (million \$)
Winning bid values	2.449	1.696	0.765	11.024
Actual costs	2.461	1.716	1.000	10.951

Note: Std = standard deviation.

considering both its developed estimate of the actual cost of its part of the project, as well as the expected error in its developed estimate. Thus, let B_{ik} represents the bid prepared by general contractor i for its part of project k; where $B_{ik} = f(S_{ik}, \varepsilon_{gc})$.

This being the case, in MSG, each general contractor submits a joint bid for the whole project that includes both its portions of the project and those of the subcontractor(s). Let JB_{ik} denotes the joint bid submitted by the ith general contractor for project k. Finally, the contract is awarded to the general contractor with the lowest joint bid, and consequently, its winning subcontractor wins the project contract. Being the case, the joint bid submitted by a general contractor is depending on the lowest bid among its subcontractors' bids as well as its prepared bid for its part of the project, thus $JB_{ik} = f(B_{ik}, LB_{iksc})$.

In addition, for the simulation model, some assumptions have been made to facilitate mimicking the real-world MSG, while simplifying the inherent uncertainties and complexities. That said, the following assumptions have been made:

- For simulation purposes and to maintain symmetry among bidders in both stages, the number of general contractors is assumed to be 3 ($n_{gc} = 3$) and the number of subcontractors is assumed to be 9, with 3 subcontractor bids for each general contractor ($n_{isc} = 3$), as shown in Fig. 3. However, the derived bid function and the developed simulation model can be implemented for any number of general contractors and subcontractors;
- For all the projects, it is assumed that the general contractors subcontract 30% of the project work based on the low bid method; thus, for each project, $C_{ksc} = 0.3C_k$ and $C_{kgc} = 0.7C_k$. This assumption is made to maintain symmetry among the general contractors and subcontractors. However, any set of percentages can be considered;
- As previously highlighted, the actual cost of the project is unknown to contractors, either general contractors or subcontractors, at the time of bid submission. Therefore, each contractor generates an estimate for the actual cost of its portion of the project. Moreover, contractors have a maximum error percentage around the actual cost of their part of the project. For simulation purposes, the maximum error percentage is assumed to be 2%; in other words, $\varepsilon_{sc} = \varepsilon_{gc} = 2\%$. This percentage is assumed based on a review of the literature (Ahmed et al. 2016; Assaad et al. 2021) and consultation with experienced individuals in the construction industry. That being said, for each project, each general contractor and subcontractor is randomly given an independent private signal which represents its estimate for the actual cost of its part of the project; in other words, $S_{jk} \in [C_{ksc}(1-\varepsilon_{sc}), C_{ksc}(1+\varepsilon_{sc})]$ and $S_{ik} \in [C_{kgc}(1-\varepsilon_{gc}), C_{kgc}(1+\varepsilon_{gc})]$;
- All subcontractors are assumed to bid exactly their estimates for the actual cost for their part of the project; thus, $B_{jki} = S_{jk}$;
- As previously highlighted, the winning subcontractor is the one with the lowest submitted bid among its competitors. Its corresponding general contractor will treat the winning subcontractor's bid as a private cost and is obligated to pay the winning subcontractor the total amount of its bid at end of the project;
- For their part of the project, general contractors are assumed to bid exactly their estimates for the actual cost; thus, $B_{ik} = S_{ik}$. Thereafter, general contractors submit a joint bid to the owner (ODOT in the developed simulation model). For the purpose of this research, the authors considered three bidding scenarios. For simulation purposes, it is assumed that under Scenario 1, all general contractors submit the summation of their estimates and the lowest bid of their corresponding subcontractors; in other words, $JB_{ik} = B_{ik} + LB_{iksc}$. Under Scenario 2, only General Contractor 1 will change to bid based on the derived bid function. Under Scenario 3, all general contractors will bid based on the derived bid function. Further details are provided under the subsection on bidding scenarios; and
- Eventually, in the utilization of the derived bid functions Eqs. (7)–(9) and for simulation purposes, the values of a and b for are modeled as the range within ±2% around the cost of the general contractor's part of the project. In addition, each general contractor will consider the values of c and d are within ±2% around its winning subcontractor's bid value. The range percentage (2%) is assumed based on the contractor's expectation of the cost estimation error. Such assumptions are made based on the principle that in the real world, general contractors can determine these values based on their experience from past similar projects without knowing the actual true cost with certainty.

It is worth highlighting that the aforementioned assumptions have been made for simulation purposes only, and contractors can adjust them as needed when utilizing the developed model in this paper.

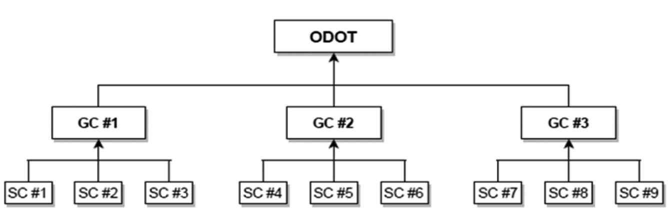

Fig. 3. MSG in the simulation model whereGC = general contractor, and SC = subcontractor.

Bidding Scenarios

In the developed simulation model, the authors considered three bidding scenarios. The first bidding scenario (Bidding Scenario 1) considered in this paper is the scenario in which all general contractors are preparing their joint bid for the project according to their cost estimates and the lowest bid of their corresponding subcontractors, as shown in Eq. (10)

$$JB_{ik} = B_{ik} + LB_{iksc} \tag{10}$$

The second bidding scenario (Bidding Scenario 2) considered in this paper is the scenario in which General Contractor 1 is considered to be the only general contractor using the derived bid function [Eqs. (7)–(9)] in preparation of its joint bid for the project, while other general contractors are following Eq. (10) in preparation for their joint bids.

The third bidding scenario (Bidding Scenario 3) considered in this paper is the scenario in which all the general contractors bid using the derived bid function [Eqs. (7)–(9)]. These three scenarios were considered to facilitate the determination of the impact of deviation to the derived bid function on winning projects and reducing the winner's curse.

It is worth noting the developed simulation model reports the percentage of earnings for each general contractor for its part of the project following Eq. (11), and the expected profit following Eq. (12)

$$\%\text{Earnings}_{ik} = \begin{cases} \dfrac{B_{ik} - C_{kgc}}{C_{kgc}} \times 100 & \text{if winning the project } k \\ 0 & \text{if loosing the project } k \end{cases} \tag{11}$$

$$\text{Expected profit}_i = \text{Probability of winning}_i \times \text{Average profit conditional on winning}_i \tag{12}$$

In Eq. (11), the general contractor observes zero as earnings percentage value if it did not win the project. The general contractor observes a positive value as earnings percentage if it wins the project with a joint bid value higher than the actual cost of the whole project; thus, earning positive profits. On the other hand, the general contractor receives a negative value as earnings percentage if it wins the project with a joint bid value less than the actual cost of the project, thus encountering negative profits.

In addition, to take into account both the probability of winning and the earnings of contractors while comparing the simulated bidding scenarios, the authors determined the expected profit of each general contractor for each followed bidding strategy following Eq. (12). In Eq. (12), the probability of winning for the ith general contractor equals the number of projects it won divided by the total number of projects, while the average profit conditional on winning has two values. In the case of determining the expected profit on the part of the general contractor, the average profit conditional on winning for the ith general contractor equals the sum of total profits or losses it earned based on its submitted bid for its parts of the projects divided by the number of projects it won. In the case of determining the expected profit on the part of the whole joint bid, the average profit conditional on winning for the ith general contractor equals the sum of total profits or losses observed on the part of the submitted joint bid divided by the number of projects it won. In fact, from a game-theoretic perspective, expected profit/payoff is considered the appropriate way for comparison between various bidding strategies/scenarios (Milgrom 1989). In other words, the main purpose of the expected profit is to compare between followed bidding strategies in repeated games like bidding for projects in the simulation model. That being said, to determine the value of the exact earnings from following a specific strategy in bidding for multiple projects, one must multiply the expected profit by the total number of projects they bid on.

As a general rule, the contractors aim to maximize their percentage of earnings and expected profit value for various reasons such as increasing expected earned profits and minimizing losses especially at recession periods (Ahmed et al. 2016).

Simulation Model Development and Aiding Software Packages

For the simulation model developed in this paper, the programming language Python was used. Python is an object-oriented, high level, and easy to interpret programming language (Oliphant 2007; Millman and Aivazis 2011). More specifically, it is important to highlight that the simulation was executed using Project Jupyter as an environment for the development of Python code. Project Jupyter is a nonprofit organization providing open-source software that supports multiple programming languages including Python. Moreover, some open-source packages have been used for the development of the simulation framework. For instance, NumPy package was used, which facilitates mathematical computing performed on homogenous multidimensional matrices and arrays (Oliphant 2006; Van Der Walt et al. 2011). Furthermore, the SymPy package was used, which is an open-source package that facilitates symbolic computing in Python (Meurer et al. 2017). In addition, the Pandas package was used, which is an open-source package that facilitates data manipulation and analysis (McKinney 2010). For visualization, Matplotlib package was used to provide insightful illustrations through visualization of the results of the simulation (Hunter 2007).

Results and Analysis

This section presents the results and analysis for the three bidding scenarios considering in the simulation model developed in this paper. It is imperative to highlight that in MSG, each party (general contractor and\or subcontractor) is liable to the submitted bid for its part of the project. As such, the party who experiences losses in its part of the project is considered liable to them, while the other party may earn profits on its part (Ahmed et al. 2016). In addition, the derived bid functions, depicted in Eqs. (7)–(9), are mainly for the general contractor to determine the final joint bid to submit. That being said, the conducted analysis primarily focuses on analyzing the status of each general contractor in terms of negative earnings and expected profit based on its submitted bids for its part of the project.

Bidding Scenario 1: Bidding According to the Cost Estimate

This subsection presents the obtained results of Bidding Scenario 1 from the developed simulation model where all general contractors submit their joint bid based on their cost estimates and their corresponding winning subcontractor's bid. Table 3 shows the results of the simulation model under Bidding Scenario 1.

As shown in Table 3, General contractor 1 won 712 out of 2,235, where General Contractor 2 won 761 projects and General Contractor 3 won 762 projects. The winning percentages for the General Contractors 1, 2, and 3 are approximately 31.9%, 34.0%, and

Table 3. Results of the simulation model under Bidding Scenario 1

			Negative earnings					
	Probability of winning		Joint bid		GC's part		Expected profit ($)	
Contractor	No. of projects	Percentage (%)	No. of projects	Percentage (%)	No. of projects	Percentage (%)	Joint bid	GC's part
GC 1	712	31.9	673	94.5	600	84.3	−8,131.3	−5,117.4
GC 2	761	34.0	712	93.6	651	85.5	−8,452.6	−5,503.9
GC 3	762	34.1	720	94.5	656	86.1	−8,276.1	−5,510.3

Note: GC = general contractor.

34.1%, respectively. This result is anticipated as the winning percentage shall be more or less equally divided between the general contractors since all of them are utilizing the same bidding strategy.

In addition, in terms of negative earnings, the results indicate that the three general contractors, approximately, experienced equally negative earnings in their part of the project. As shown in Table 3, in general contractor's part of the project, General Contractor 1 experienced negative earnings in 84.3% of the project it won. Similarly, General Contractors 2 and 3 experienced negative earnings in 85.5% and 86.1% of the projects they won, respectively. Furthermore, the developed simulation model reported the expected profit, based on Eq. (12), of the three general contractors from the followed bidding strategy. From a game-theoretic perspective, it is worthy to note that the appropriate comparison of bidding strategies shall be based on the expected profit from playing the game given a particular strategy. As shown in Table 3, in general contractor's part of the project, the three general contractors experienced negative expected profit: -\$5,117.4 for General Contractor 1, -\$5,503.9 for General Contractor 2, and -\$5,510.3 for General Contractor 3. These negative expected profits are a consequence of the winner's curse which is faced by contractors.

Because the simulated projects have a wide range of actual costs, from \$1 million to \$10 million, the authors determined the earnings, based on Eq. (11), for each of the three general contractors to analyze the magnitude of negative earnings. Fig. 4(a) shows the earnings for each of the three general contractors on their part of each project. In addition, Fig. 4(b) shows the cumulative earnings for each of the three general contractors under Scenario 1. Approximately, each of the three general contractors experienced cumulative earnings of −700%. This result indicates that the three general contractors experienced, approximately, the same magnitude of the winner's curse at the end of the 2,235 projects.

In summary, results of the developed simulation model under Bidding Scenario 1 reflect that experienced general contractors are expected to have approximate similar winning percentages; in other words, experienced general contractors are expected to share the market equally in case they are following the same bidding strategies. Moreover, the results reflect that general contractors experience similar magnitude and probability of the winner's curse in MSG in their part of the project compared to a percentage around 83% in the single-stage construction bidding (SSG) (Ahmed et al. 2016). However, in terms of the whole joint bid for both parts of general contractor and subcontractors, the results indicate the magnitude and probability of the winner's curse are high in MSG than in SSG (94% versus 83%, approximately). This result was anticipated as the winner's curse is expected to increase and be combined in the joint bids in MSG as it can happen twice; one on part of the general contractor, and the other on part of the subcontractor (Ahmed et al. 2016).

Bidding Scenario 2: Bidding According to the Derived Bid Function by Only One General Contractor

This subsection presents the obtained results of Bidding Scenario 2 from the developed simulation model where General Contractor 1 utilizes the derived bid function [Eqs. (7)–(9)] in preparation of its joint bid for the project, while General Contractors 2 and 3 submit their joint bid based on their cost estimates and their corresponding winning subcontractor's bid. In this bidding scenario, General Contractor 1 is completely surrounded by irrational bidders. Table 4 shows the results of the simulation model under Bidding Scenario 2.

As shown in Table 4, General contractor 1 won 268 out of 2,235, where General Contractor 2 won 970 projects and General contractor 3 won 997 projects. The winning percentage for General Contractor 1 decreased to 12.0% under Building Scenario 2 compared to 31.9% under Bidding Scenario 1. While the winning percentages increased for General Contractors 2 and 3 are to 43.4% and 44.6% under Bidding Scenario 2 compared to 34.0%, and 34.1% under Bidding Scenario 1, respectively. However, the results indicate that General Contractor 1 was in a significant advantage position in terms of the negative earnings. General Contractor 1 did not experience negative earnings in its part in any of the projects it won under Bidding Scenario 2 compared to a percentage of 84.3% under Bidding Scenario 1. More specifically, General Contractor 1 earned 268 projects with positive profits under Bidding Scenario 2, while it earned positive profits in only 112 projects under Bidding Scenario 1. It is worth noting that in 166 of the 268 projects won by General Contractor 1, the submitted joint bid was less than the

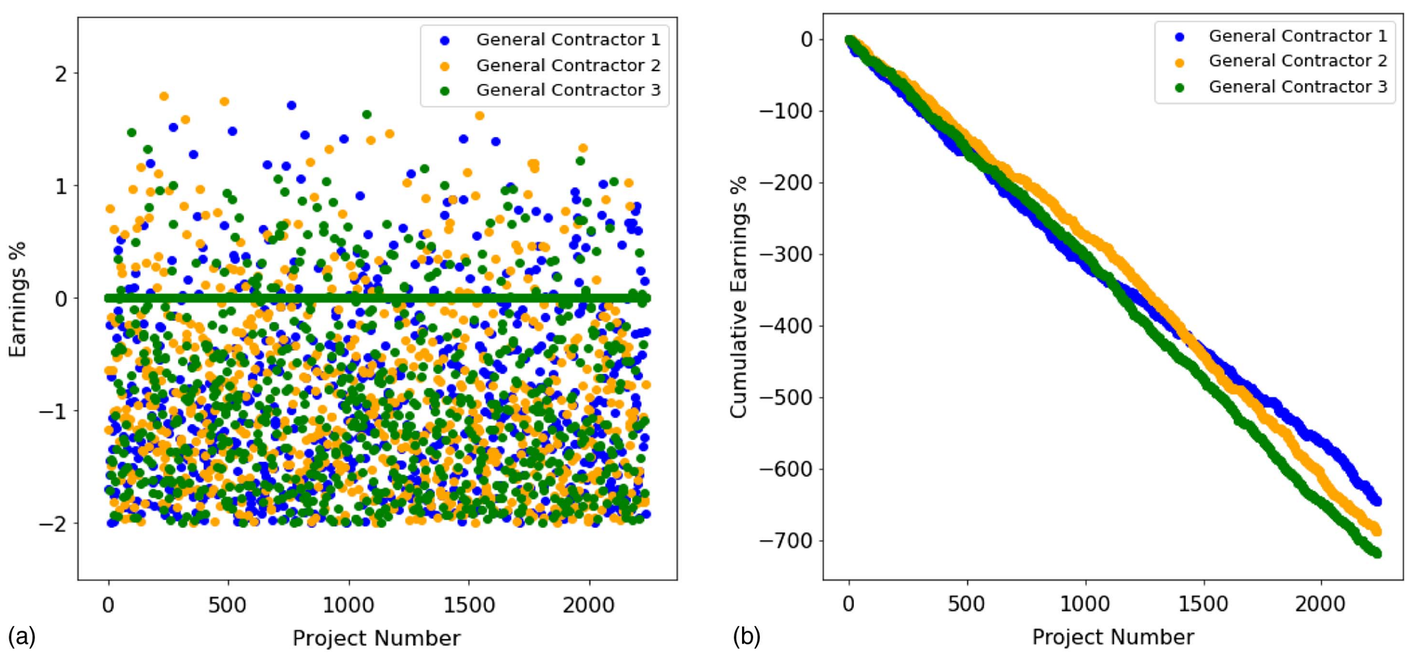

Fig. 4. (a) Earnings; and (b) cumulative earnings for the general contractors under Bidding Scenario 1.

Table 4. Results of the simulation model under Bidding Scenario 2

Contractor	Probability of winning		Negative earnings				Expected profit ($)	
			Joint bid		GC's part			
	No. of projects	Percentage (%)	No. of projects	Percentage (%)	No. of projects	Percentage (5)	Joint bid	GC's part
GC 1	268	12.0	166	61.9	0	0.0	−218.2	1,045.6
GC 2	970	43.4	906	93.4	812	83.7	−9,970.3	−6,324.5
GC 3	997	44.6	937	94.0	826	82.8	−10, 056.1	−6,293.2

Note: GC = general contractor.

actual true cost of the whole project due to losses on the part of its winning subcontractor. On the other hand, General Contractors 2 and 3 continued to experience almost the same percentages of suffering from negative earnings and the resulting winner's curse, i.e., 83.7% and 82.8% under Bidding Scenario 2, respectively.

Furthermore, as shown in Table 4, the three general contractors experienced expected profits as follows: $1,045.6 for General Contractor 1, -$6,324.5 for General Contractor 2, and -$6,293.2 for General Contractor 3. Accordingly, the use of the derived bid function has substantially assisted General Contractor 1 in overcoming the occurrence of negative earnings and increasing its expected profits. More specifically, General Contractor 1's expected profit increased from -$5,117.4 to $1,045.6, which represents approximately 120.4% as a percentage increase in expected profit compared to Bidding Scenario 1. This is a substantial result because, despite the situation that General Contractor 1 was surrounded by irrational bidders (general contractors and subcontractors), it still makes a positive profit. This sheds the light on the game-theoretic concept that there is an incentive for general contractors to deviate from the norm and utilize the derived bid function. In relation to General Contractors 2 and 3, these negative expected profits are a consequence of the winner's curse which is experienced by contractors, similar to Bidding Scenario 1.

In terms of the joint bids, General Contractor 1 could not fully avoid the negative earnings and the resulting winner's curse due to losses on the part of the winning subcontractor. These results were expected as for General Contractor 1 to avoid\reduce the winner's curse, it had to rise the value of its submitted joint bids; hence, its winning percentage is expected to decrease under the low bid method for contract allocation. These results are in line with what Assaad et al. (2021) highlighted that general contractor should make a trade-off between (1) winning more projects by lowering joint bids, against (2) reducing the winner's curse and expected losses conditional on winning by increasing value of joint bids and hence, decreasing the probability of winning. This means that utilizing the derived bid function aided General Contractor 1 in winning more projects with positive profits in terms of the whole joint bid. It is worth highlighting that a percentage of 61.9% for the negative earnings is considered a substantial improvement in terms of the joint bids in MSG compared to previous research that tackled the negative earnings issue in SSG (Assaad et al. 2021).

In relation to the magnitude of negative earnings, Fig. 5(a) shows the earnings for each of the three general contractors on their part of each project. The results indicate that General Contractor 1 achieved higher earnings, ranging from 0.0% to +1.5%, in case of winning. In addition, Fig. 5(b) shows the cumulative earnings for each of the three general contractors under Bidding Scenario 2. General Contractor 1 obtained a cumulative earnings value of +170%, approximately. That being said, General Contractor 1's cumulative earnings value increased from −700% to +170%, achieving approximately 124.3% as an improvement percentage in terms of the cumulative earnings. On the other hand, General Contractors 2 and 3 obtained

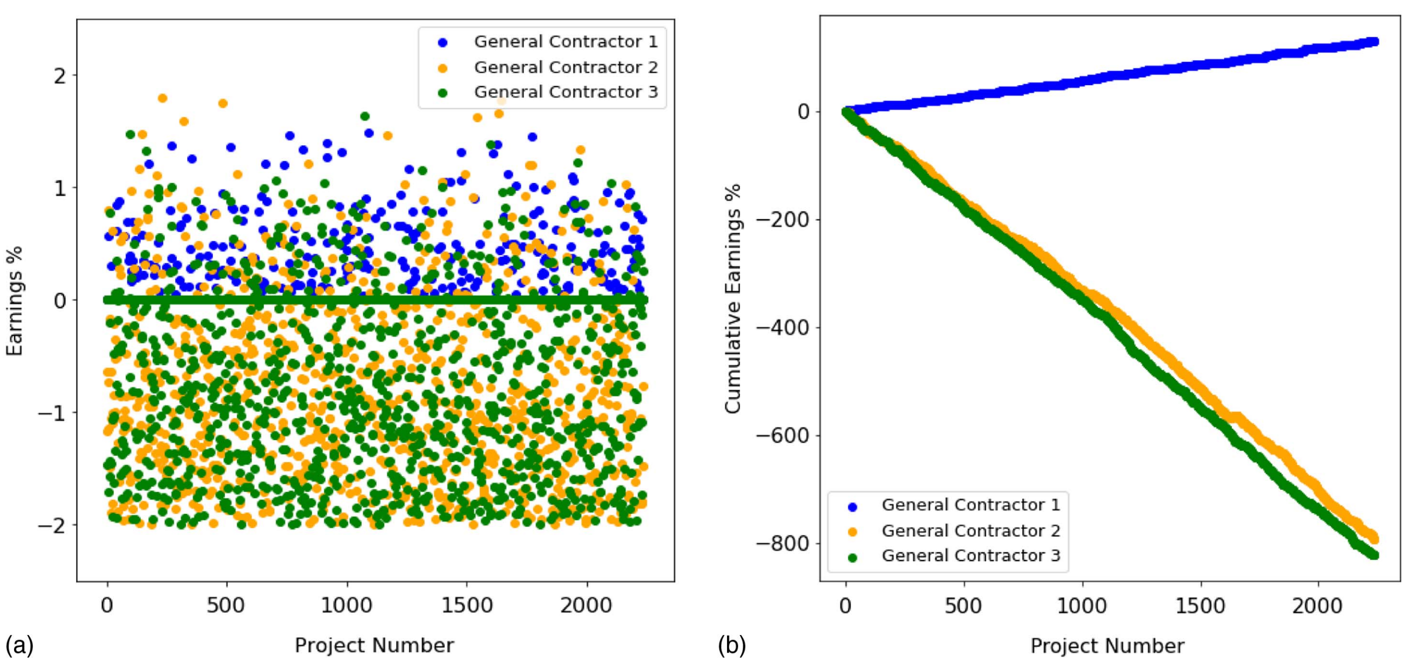

Fig. 5. (a) Earnings; and (b) cumulative earnings for the general contractors under Bidding Scenario 2.

a cumulative earnings value of −790% and −810%, respectively. It is worth noting that Fig. 5(b) shows that the curve for General Contractor 1 is increasing over time compared to the curves of General Contractors 2 and 3, which implies that General Contractor 1 succeeded in overcoming the occurrence of negative earnings by utilizing the derived bid function.

Bidding Scenario 3: Bidding According to the Derived Bid Function by All General Contractors

This subsection presents the obtained results of Bidding Scenario 3 from the developed simulation model where the three general contractors utilize the derived bid function [Eqs. (7)–(9)] in preparation of their joint bids for the project. Table 5 shows the results of the simulation model under Bidding Scenario 3.

As shown in Table 5, the three general contractors approximately observed similar results in terms of winning status, negative earnings percentage, and expected profit. This is expected as the three general contractors are following similar bidding strategies in preparation for their joint bids. In addition, three general contractors were in an advantageous position in terms of the negative earnings compared to Scenario 1. More specifically, utilizing the derived bid function assisted general contractors in full avoidance of negative earnings in their part of the project. Furthermore, as shown in Table 5, the three general contractors experienced positive expected profits as follows: $2,564.9 for General Contractor 1, $2,251.3 for General Contractor 2, and $2,290.5 for General Contractor 3. As such, each of the three general contractors experienced approximately 140% as an improvement percentage in the expected profits. Furthermore, an interesting result is that General Contractor 1 experienced a higher expected profit value than in Building Scenario 2 ($2,564.9 versus $1,045.6). This emphasizes that utilization of the derived bid function by all general contractors serves for their mutual benefit. It aids all general contractors to achieve higher expected profits and thus, maintain long-term competitiveness within the construction market as well as promote effective and efficient functioning of the economy. In addition, it is imperative to highlight that the negative expected profit in terms of the submitted joint bid is due to losses on the parts of the winning subcontractors similar to Bidding Scenario 2. However, the three general contractors experienced a reduction in the magnitude and occurrence of negative earnings in terms of the final joint bids.

In relation to the magnitude of negative earnings, Fig. 6(a) shows the earnings for each of the three general contractors on their part of each project under Bidding Scenario 3, which is ranging from 0.0% to +1.5%. In addition, Fig. 6(b) shows the cumulative earnings for each of the three general contractors under Bidding Scenario 3. Approximately, each of the three general contractors experienced cumulative earnings of +300% under Bidding Scenario 3 compared to −700% under Bidding Scenario 1. This represents around 143% as an improvement percentage in the value of cumulative earnings.

Table 5. Results of the simulation model under Bidding Scenario 3

	Probability of winning		Negative earnings				Expected profit ($)	
			Joint bid		GC's part			
Contractor	No. of projects	Percentage (%)	No. of projects	Percentage (%)	No. of projects	Percentage (%)	Joint bid	GC's part
GC 1	722	32.3	500	69.3	0	0.0	−887.9	2,564.9
GC 2	758	33.9	534	70.4	0	0.0	−1,098.5	2,251.3
GC 3	755	33.8	525	69.5	0	0.0	−995.7	2,290.5

Note: GC = general contractor.

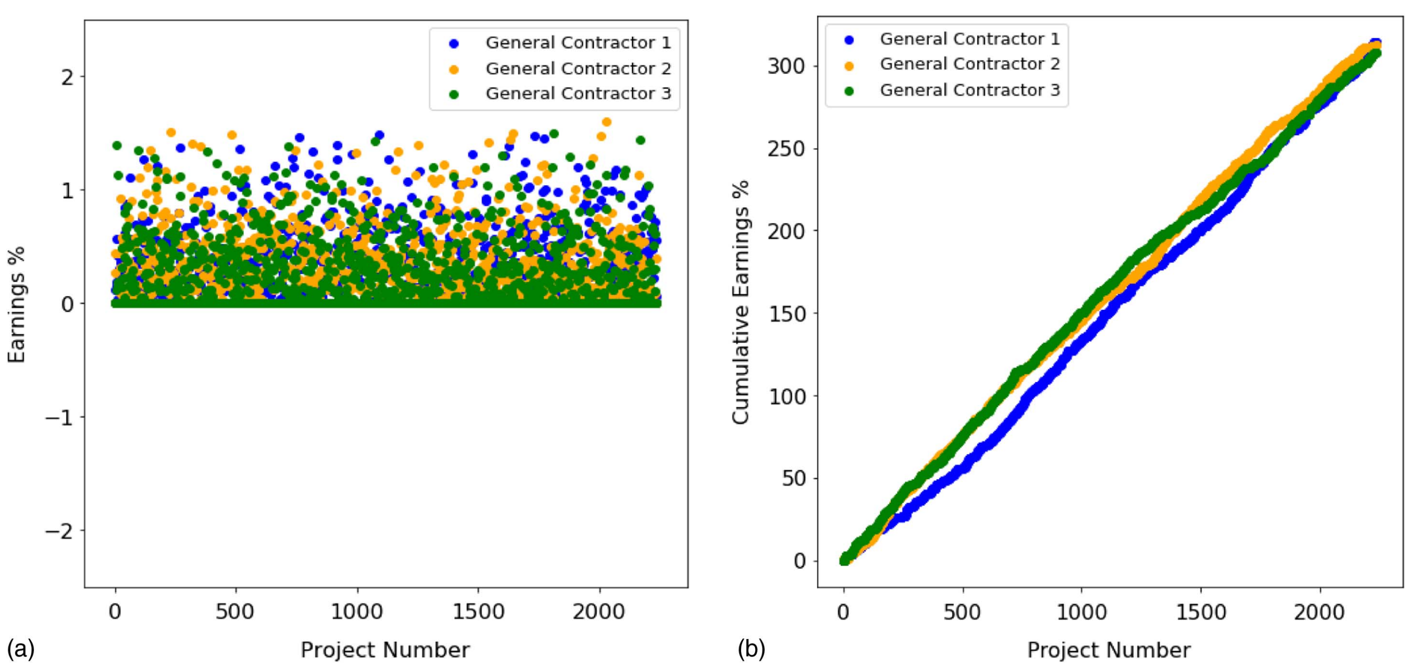

Fig. 6. (a) Earnings; and (b) cumulative earnings for the general contractors under Bidding Scenario 3.

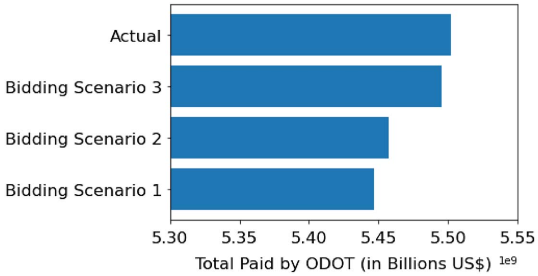

Fig. 7. Total amount paid by ODOT versus amount paid under each simulation scenario for executing the 2,235 projects.

Discussion

Benefits of the Developed MSG Bidding Function and Model

In light of the aforementioned simulation results, it is worth recalling that the differences between the results of the three Bidding Scenarios 1, 2, and 3 are due to the different combinations of bidding strategies followed under each bidding scenario. In other words, in Bidding Scenario 1, all general contractors are preparing their joint bid for the project according to their cost estimates. In Bidding Scenario 2, General Contractor 1 is considered to be the only general contractor using the derived bid function [Eqs. (7)–(9)] in preparation of its joint bid for the project, while other general contractors are preparing their joint bid for the project according to their cost estimates. In Bidding Scenario 3, all the general contractors bid using the derived bid function [Eqs. (7)–(9)]. As such, it can be concluded that utilizing the derived bid function shall aid general contractors in avoiding the occurrence of negative earnings in their part of the project and thus, reducing the impact of the winner's curse in MSG while maintaining a reasonable probability of winning with a desirable result of more projects with positive profits. In addition, Fig. 7 shows that clients are anticipated to slightly pay a lower total monetary amount to execute their projects in the long run when general contractors are utilizing the derived bid function.

Validity of the Developed MSG Bidding Function and Model

Concerning validation of the results of the simulation model, it is worth highlighting that the derived bid function has been mathematically proven and validated to represent the equilibrium bid function for the reverse first-price auction in presence of private and common values, as shown under Step 2 of the methodology section. Furthermore, it is worth noting that actual datasets are often used to conduct simulation-based research (Assaad et al. 2021). Using actual datasets promotes the generation of realistic results under reasonable assumptions. Moreover, in simulation-based research, validation of the simulation results can be accomplished through: (1) comparison with results obtained from theoretical models when such models exist (which is not the case in this research); and (2) comparison with results obtained from real historical or actual operation data (Rekapalli and Martinez 2011; AbouRizk and Halpin 1990). Accordingly, the authors investigated and compared the simulation results against the actual dataset of 2,235 ODOT infrastructure projects described under the data collection subsection of the methodology section. Table 6 shows the percentage of times the winning general contractor experienced negative earnings in their part of the project under the three scenarios of the simulation model as well as based on the actual dataset of ODOT.

Table 6. Percentage of winning general contractors who experienced negative earnings under the simulation model's scenarios versus the actual dataset of ODOT

Scenario 1	Scenario 2	Scenario 3	Actual
85.3%	73.3%	0.0%	50.11%

Note: Actual refers to the actual dataset of ODOT.

To this effect, the results demonstrated the improvement in terms of avoidance of negative earnings from one scenario to another within the simulation model. In addition, comparing Bidding Scenario 3, in which all general contractors are homogenous in utilizing the derived bid function, with the actual dataset clarified the advantage of using the derived bid function by general contractors in determining the final joint bid to submit. This exercise provides validation of the results of the simulation model regarding its usefulness in dealing with the inherent uncertainties about the cost estimation and complexity within MSG.

Another method that can be examined for validation of agent-based simulation models is sensitivity analysis (SA) (Asgari 2020). In this study, SA was applied to investigate the impact of uncertainty about maximum error percentage around the actual costs of the general contractors' part and the subcontractors' part on the general contractors' status in terms of the expected profit, probability of winning, and percentage of occurrence of negative earnings. In doing so, the authors examined five possible percentages for the maximum error (1%, 25, 3%, 4%, and 5%). Further, the authors investigated five scenarios for SA, as shown in Table 7. It is worth noting that for each SA scenario, the three bidding scenarios (Bidding Scenario 1, Bidding Scenario 2, and Bidding Scenario 3) were examined for each maximum error percentage. Moreover, as previously highlighted, the conducted SA focuses on analyzing the status of the general contractors (especially General Contractor 1) as the main focus of the paper.

Figs. 8–10 show the results of the conducted SA in terms of expected profit, probability of winning, and percentage of occurrence of negative earnings on the part of the general contractors, respectively. In relation to the expected profit, the results showed that the expected profit of General Contractor 1 decreased with the increase in uncertainty level about the cost estimation for its part of the project (see subparts under SA Scenario 2 of Fig. 8). However, when the level of uncertainty is the same among all general

Table 7. Sensitivity analysis scenarios

SA scenario	Description
SA Scenario 1	In this SA scenario, the maximum error percentage was changed for all GCs simultaneously.
SA Scenario 2	In this SA scenario, the maximum error percentage was changed for only GC 1.
SA Scenario 3	In this SA scenario, the maximum error percentage was changed for all SCs simultaneously.
SA Scenario 4	In this SA scenario, the maximum error percentage was changed for only SC 1, SC 2, and SC 3, simultaneously. In other words, the subcontractors under GC 1.
SA Scenario 5	In this SA scenario, the maximum error percentage was changed for only SC 1 under GC 1.

Note: SA = sensitivity analysis; GC = general contractor; and SC = subcontractor.

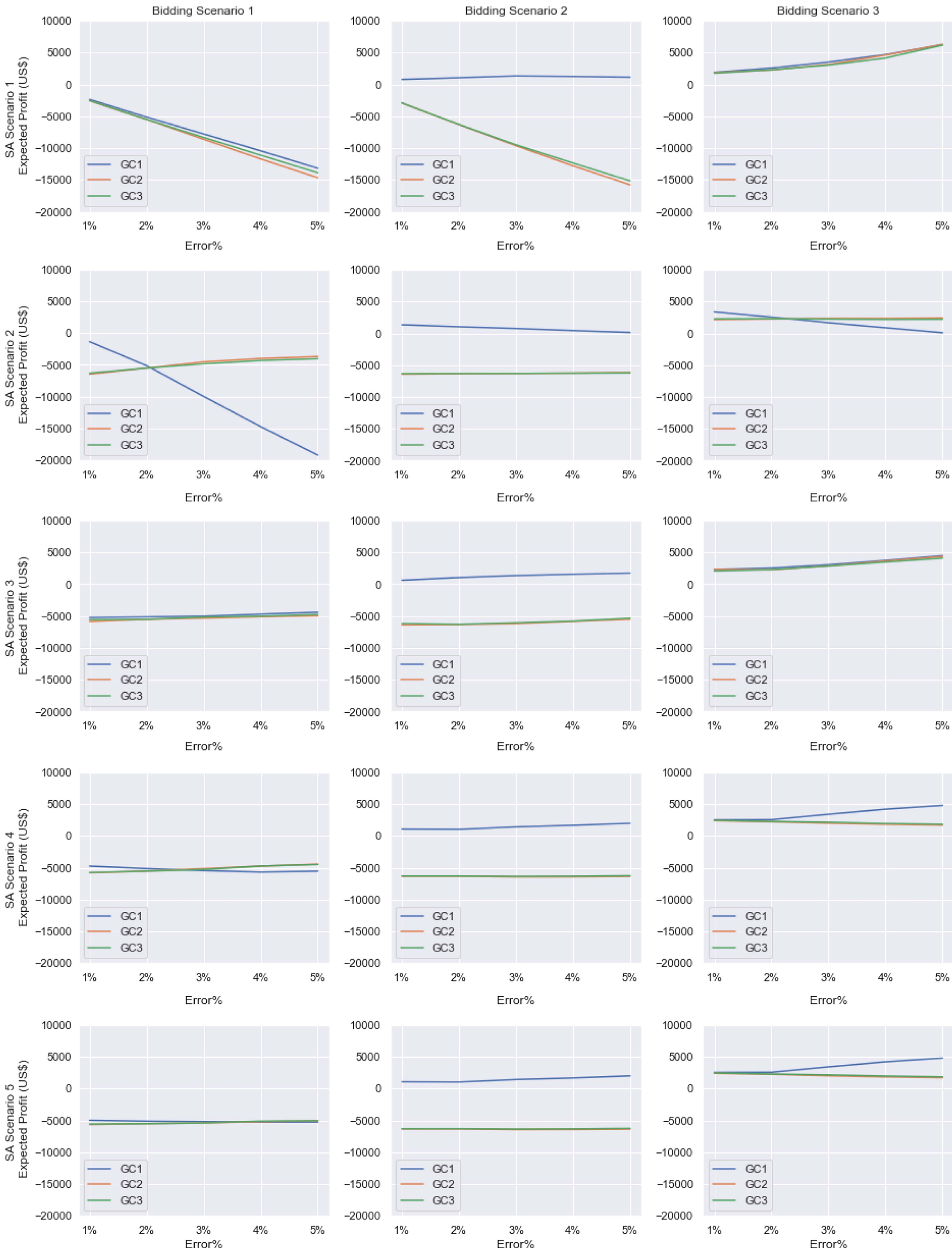

Fig. 8. Sensitivity analysis results in terms of expected profits on the part of the general contractors.

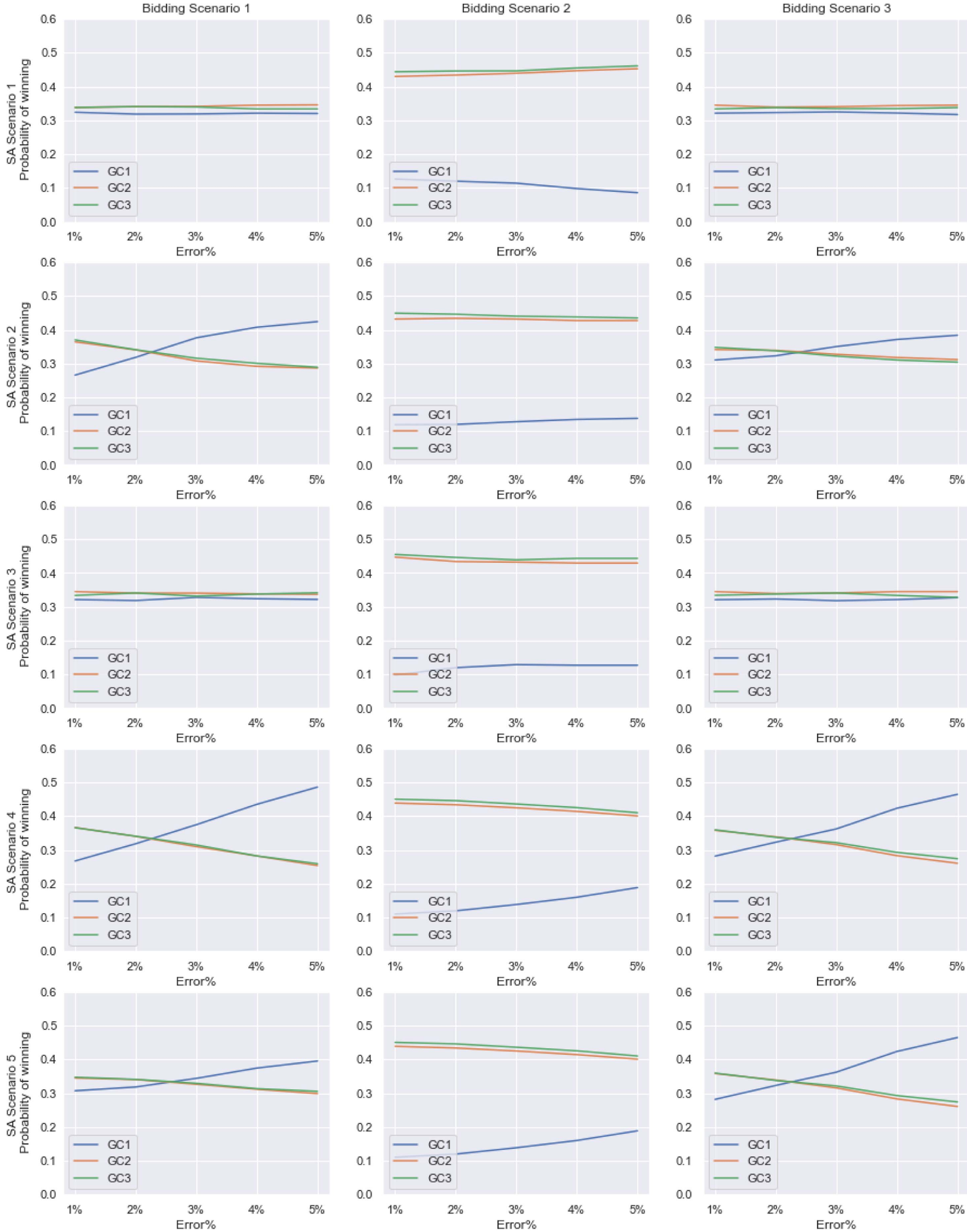

Fig. 9. Sensitivity analysis results in terms of probability of winning on the part of the general contractors.

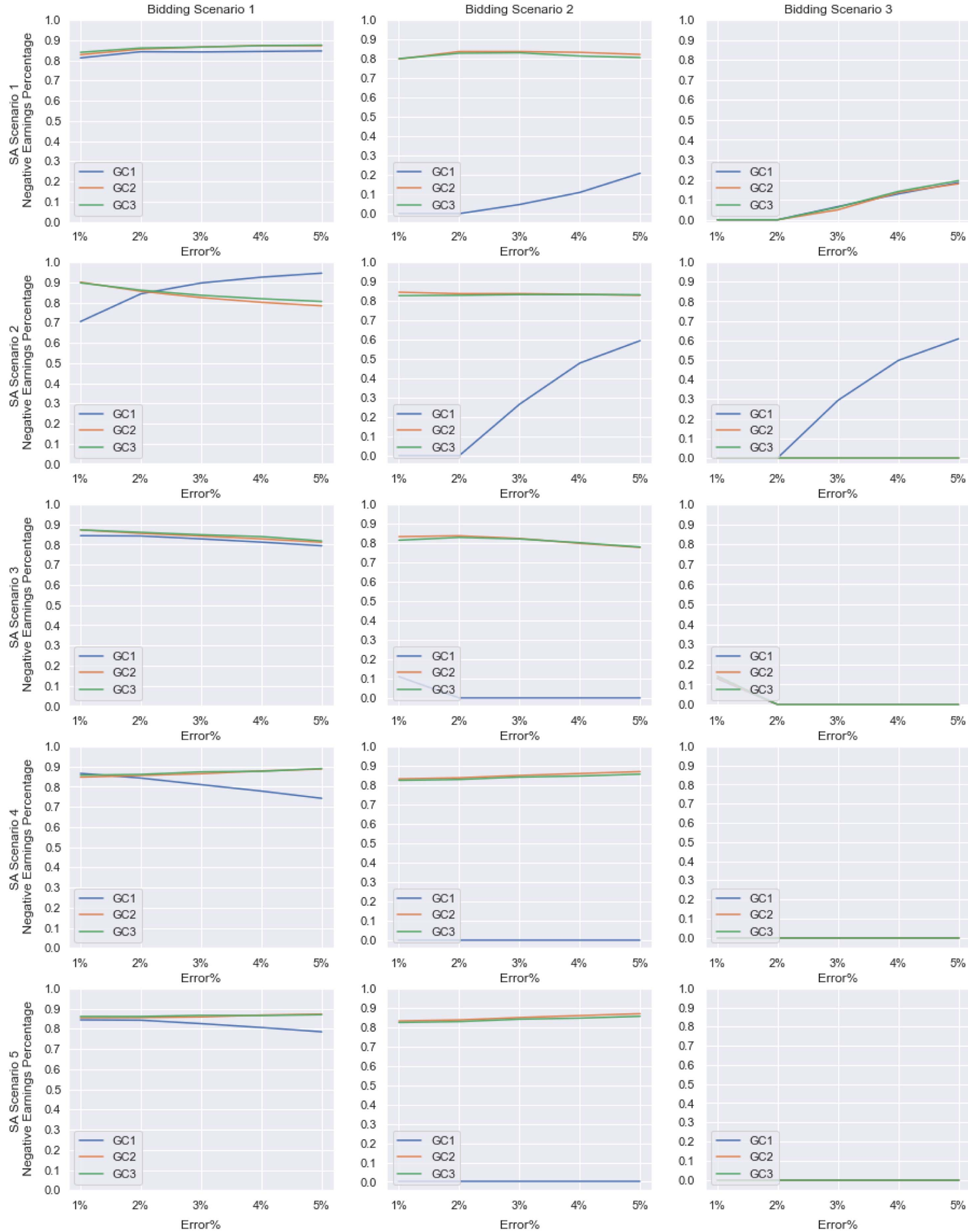

Fig. 10. Sensitivity analysis results in terms of percentage of negative earnings on the part of the general contractors.

contractors, General Contractor 1 experienced a decrease in its expected profit with increased uncertainty level about the cost estimation for its part of the project when using the derived bid function (see subparts under SA Scenario 1 and Bidding Scenario 2 and 3 of Fig. 8). In addition, the results showed that General Contractor 1 slightly benefited from the increase in uncertainty level about the cost estimation on the part of its subcontractors by obtaining higher expected profits (see subparts under SA scenarios 3, 4, and 5 of Fig. 8).

In relation to the probability of winning, the results showed that the probability of winning of General Contractor 1 increased with the increase in uncertainty level about the cost estimation for its part of the project or the part of its subcontractors (see subparts under SA Scenarios 2, 3, 4, and 5 of Fig. 9). This result was anticipated as increased uncertainty about the cost estimation increases the probability of having accidentally or deliberately the lowest bid among competitors, thus winning more projects. However, when the level of uncertainty is the same among all general contractors, General Contractor 1 experienced a decrease in its probability of winning with increased uncertainty level about the cost estimation for its part of the project when using the derived bid function (see subparts under SA Scenario 1 and Bidding Scenario 2 of Fig. 9). This implies that the derived bid function aids General Contractor 1 in mitigating the effect of the high level of uncertainty in cost estimation by submitting higher bids compared to its competitors to avoid falling prey to the winner's curse; thus, winning fewer projects.

In relation to the percentage of occurrence of negative earnings, the results showed that the general contractors experienced an increase in the percentage of occurrence of negative earnings with increased uncertainty in the cost estimation of its part of the project (see subparts under SA Scenarios 1, and 2 in Fig. 10). However, as reasonably anticipated, the general contractors experienced almost no change in the percentage of occurrence of negative earnings in their part of the project with increased uncertainty in the cost estimation on the part of their subcontractors. This can be attributed to the aforementioned fact that in MSG, each party (general contractor and\or subcontractor) is liable to the submitted bid for its part of the project. As such, the party who experiences losses in its part of the project is considered liable to them, while the other party may earn profits on its part (Ahmed et al. 2016).

Overall, the results of the conducted SA demonstrated that General Contractor 1 benefited from utilizing the derived bid function in mitigating/avoiding the occurrence of negative earnings and the associated winner's curse in its part of the project. In other words, utilization of the derived bid function enabled General Contractor 1 in dealing with the uncertainties related to the cost estimation in MSG and mitigating its associated impacts.

Guidelines for Utilization of the Developed MSG Bidding Function and Model

Contractors may utilize the derived bid function for bidding for a project in real-world as follows. First, general contractors must acquire information about the possible range of project's cost estimates of their part of the project, which is the range within which the actual cost of their part of the project is expected to be (a and b in the derived bid function), and the possible range of the cost of subcontracting portion of the project (c and d in the derived bid function). It is anticipated that general contractors, considering that they are experienced in the construction market sector and the type of projects, will be able to approximately determine the possible range for cost estimates for their part of the project and the cost of subcontracting portion of the project based on project type, location, and other various attributes. Second, a general contractor can determine the final joint bid amount to submit based on its surplus value through the implementation of Eqs. (7)–(9), if the number of general contractors bid for the project is 3. Otherwise, if the number of general contractors bidding for the project is not 3, the general contractor can refer to Eqs. (4)–(6) to obtain explicit formulas [similar to Eqs. (7)–(9)] to determine the final joint bid amount to submit based on surplus value. Third, it is worth noting that the derived bid function and model account only for the general contractor's direct cost estimate plus the cost of subcontracting portion of the project, which implies the direct cost, indirect cost, and markup of the subcontractor; however, the derived bid function and model do not account for other factors such as indirect cost and markup of the general contractor, which can be addressed in an extension of this research as highlighted under the section on limitations and future work recommendations.

In general, indirect cost consists of site overheads and head office overheads, while markup consists of profit, financial charges, and risk allowance. Contractors usually have different estimates for indirect cost and markup based on the size of the firm, the anticipated rate of return, among other factors. Such difference may affect their probability of winning the project by being the lowest. However, it is imperative to note that contractors expect to obtain at least the certain estimated monetary values of their indirect cost and markup. As such, dealing with uncertainty in their direct cost estimation to reduce the impact of the winner's curse shall not be confused with the expected markup or indirect cost to be fully attained. As such, the general contractor shall consider adding additional value, to account for its indirect cost and markup, to the determined final joint bid amount using the derived bid function in this paper.

Moreover, in the developed simulation model as well as the utilized dataset, the authors did not consider change orders that may have been altered and/or added to the initial scope of the project. The main reason is following a similar logic to the aforementioned one that in case of change orders, rational contractors shall price change orders reasonably and fairly and be compensated accordingly. Ideally, this shall have no impact on the experience of the winner's curse in relation to actual cost estimation based on the initial scope of the project at the time of bidding as well as the expected profit from following a specific bidding strategy. However, according to Dyer and Kagel (1996), general contractors sometimes tend to utilize change orders as a strategy to avoid falling prey to the winner's curse. Usually, the price of a change order is established through negotiations between associated stakeholders. Through tough negotiations, contractors, who underbid a project, can recover at least some losses, or even make some profit. However, such a strategy is considered ineffective as it is most likely to result in adversarial relationships, claims, and disputes between contracting parties as well as increase legal costs. Therefore, to avoid the winner's curse, contractors must carefully consider utilizing efficient bidding strategies to deal with the inherent uncertainties and complexities within the MSG, including actual cost estimation.

Research Contributions

The research conducted in this paper has substantial contributions to body of knowledge, including: (1) deriving a bid function for the reverse first-price auction with both private and common values in which the lowest bidder is the winner, which is a significant addition to both game theory and construction management domains, especially given that existing game-theoretic research has not considered MSG setting previously (Dyer and Kagel 1996); (2) tackling a research need highlighted by Ahmed et al. (2016) and Awwad et al. (2015) through providing a more realistic bidding model that considers the actual dynamics of MSG in which the general

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contractors' and subcontractors' bids interrelated rather than independent; (3) being the first research work to apply the approach of reverse first-price auction with private and common values for MSG and provide a solution for MSG that aids general contractors to reduce\overcome the impact of the winner's curse by winning more projects with positive profits; and (4) acting as a foundation for future research work that tackles the winner's curse problem and other related issues in MSG.

In addition, this research has practical implications to practitioners in the construction industry by: (1) providing general contractors with a tool, in the form of a bidding function, that shall enable them to account for the inherent uncertainties as related to actual cost estimation within MSG while deciding the joint bid value; (2) enabling general contractors to avoid the occurrence of negative profits, and associated winner's curse in relation to the actual cost of their part of the project, as well as reduce in the occurrence and magnitude of the negative profits in relation to the final joint bids; and (3) facilitating a healthier contracting environment between project parties and minimize the risk of disputes arising from underestimated bids (Ahmed et al. 2016) because the developed MSG bidding function and model enables more accurate bid price determination in relation to the actual cost of the project.

Limitations and Future Work Recommendations

Any conducted research is frequently escorted with limitations and areas for promising improvements in future research work.

First, the developed simulation model considered three scenarios in studying and analyzing the derived bid function. Future research work is recommended to consider other scenarios such as general contractors are utilizing other bidding strategies such as fixed markup versus the derived bid function for MSG.

Second, in this paper, the derived bid function [mainly the explicit formulas in Eqs. (5)–(9)] were based on the assumption that $b > a > d > c > 0$, which implies that general contractor will construct most of the project on its own; in other words, the percentage of work constructed by the general contractor will be greater than the percentage of work subcontracted. Such assumption was made considering such situation as the most common in practice, especially in infrastructure projects. However, in case the percentage of work of the general contractor is less than the percentage of work of subcontractor, a new set of equations can be derived based on the basic Eq. (3), and following the steps illustrated under section of the paper regarding Step 2: Derivation of a Bid Function for MSG, considering a new setup that $d > c > b > a > 0$. It is imperative to note that such a situation shall have no impact on the experience of the winner's curse on part of the general contractor because as previously highlighted in MSG, each party (general contractor and\or subcontractor) is liable to the submitted bid for its part of the project (Ahmed et al. 2016). However, the impact of the winner's curse on the part of the whole joint bid is expected to increase in case of losses on the part of the subcontractor due to the increase in its percentage of the project.

Third, in this paper, the subcontractors are bidding according to their estimates of their part of the project; thus, they are not accounting for adverse selection and the resulting winner's curse, nor are they raising their bid as if it is a first-price common value auction. As such, future research can examine applying other bidding strategies for the subcontractor part such as the application of learning algorithms along with the derived bid function in this paper to facilitate learning from previous decisions and examine the impact of that from the winner's curse and probability of winning perspective.

Fourth, the validation of the benefits of the derived bid function in reducing the impact of the winner's curse in MSG is based on the results of the developed simulation model, which is based on real-world data of 2,235 infrastructure projects managed by ODOT. In general, results of simulation-based research are often validated through comparison with the actual dataset as well as SA. However, it could be argued that in-depth validation would be beneficial. As a result, providing in-depth validation of the results would require applying the derived bid function in the real-world MSG by general contractors. However, such a process may take years to obtain results that enable further analysis, and thus it is way beyond the reasonable control of the authors. As such, the authors believe that the validation based on the real-world dataset, which is conducted in this paper, is sufficient for the purpose of this research. Nevertheless, the proposed framework could benefit from further validation efforts. Therefore, future research work is recommended to consider further study and investigation of the derived bid function through applying it using different datasets for various types of projects, and\or executing laboratory experiments with construction and infrastructure bidding executives as a trial to mimic the real-world MSG as much as possible.

Finally, the derived bid function shall aid the general contractors in deciding their joint bid amount in relation to their cost estimate plus the cost of subcontracting portion of the project. However, other factors may affect the bidding decision of the general contractor in reality such as indirect costs, risks, and markup. As such, the authors recommend future research work to consider the extension of the derived bid function as well as the developed model and integrate with other concepts such as Bayesian statistics, learning algorithms, and decision theory to facilitate inclusion of other factors associated with the bidding decision in MSG.

Conclusion

Acquiring the service of subcontractors is a common practice by general contractors in the construction industry to handle public construction and infrastructure projects. At the time of bidding, general contractors are challenged to deal with the increased complexity in MSG while maintaining a competitive position and avoiding the winner's curse as much as possible. In this paper, the authors derived a game-theoretic bid function for the reverse first-price auction with private and common values, where the lowest bidder is the winner. In addition, a simulation model was developed to study and analyze the impact of utilization of the derived bid function in MSG on winning projects and reducing the winner's curse. The simulation model was developed based on an actual dataset of 2,235 infrastructure projects managed by ODOT. The results of the developed simulation model demonstrated that utilizing the derived bid function shall aid general contractors in avoiding the occurrence of negative earnings in their part of the project and as such, reducing the impact of the winner's curse in terms of the final joint bid in MSG while maintaining a reasonable probability of winning with a desirable result of more projects with positive profits. Ultimately, it is anticipated that this research would provide general contractors with guidelines to mitigate\avoid the winner's curse in the MSG, and consequently have positive impacts on the associated contracting parties, projects, and overall construction industry.

Data Availability Statement

All data and models generated or analyzed during the study are included in the published paper.

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