01 Jan 2017

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Recommended Citation

X. Du et al., "Airfoil Design under Uncertainty using Non-Intrusive Polynomial Chaos Theory and Utility Functions," *Procedia Computer Science*, vol. 108, pp. 1493 - 1499, Elsevier, Jan 2017.
The definitive version is available at https://doi.org/10.1016/j.procs.2017.05.079

Available online at www.sciencedirect.com

ScienceDirect

Procedia Computer Science 108C (2017) 1493–1499

Procedia Computer Science

International Conference on Computational Science, ICCS 2017, 12-14 June 2017, Zurich, Switzerland

Airfoil Design Under Uncertainty Using Non-Intrusive Polynomial Chaos Theory and Utility Functions

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Abstract

Fast and accurate airfoil design under uncertainty using non-intrusive polynomial chaos (NIPC) expansions and utility functions is proposed. The NIPC expansions provide a means to efficiently and accurately compute statistical information for a given set of input variables with associated probability distribution. Utility functions provide a way to rigorously formulate the design problem. In this work, these two methods are integrated for the design of airfoil shapes under uncertainty. The proposed approach is illustrated on a numerical example of lift-constrained airfoil drag minimization in transonic viscous flow using the Mach number as an uncertain variable. The results show that compared with the standard problem formulation the proposed approach yields more robust designs. In other words, the designs obtained by the proposed approach are less sensitive to variations in the uncertain variables than those obtained with the standard problem formulation.

Keywords: Design under uncertainty, stochastic surrogates, utility theory, transonic airfoil design.

1 Introduction

Aerodynamic design plays an important role in the design of various engineering systems where the focus is largely on deterministic approaches (see, e.g., Jameson, 1988; and Ouellet et al., 2004). For example, single-point and multi-point deterministic optimization are widely used. Designs obtained by these approaches often suffer from poor off-design behavior, i.e., their performance degrades in conditions other than the design points. Design under uncertainty (or robust design) aims at designing the system to be insensitive to changes in the design parameters (see, e.g., Zhang et al., 2012; Yao et al., 2011). The formulation of such design problems is, however, not well defined.

This work investigates the use of utility theory (Neumann and Morgenstern, 1947) to rigorously formulate the design under uncertainty problem. Utility theory is a rigorous decision making method

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10.1016/j.procs.2017.05.079

based on the designer's risk preferences and has not yet been introduced in aerodynamic design. In this paper, utility theory is integrated with efficient methods to compute statistical information to create an efficient and effective aerodynamic design under uncertainty approach. A numerical example of transonic airfoil design is used to illustrate the approach.

2 Problem Formulation

This section describes the standard and the proposed approach to formulating the design problem.

2.1 Standard Formulation

The aim is to design transonic airfoil shapes insensitive to uncertainties in the operational parameters. More specifically, the airfoil performance, such as the drag coefficient, should vary as little as possible for a given range of operational condition. This is often called robust design, or design under uncertainty (see, e.g., Shah et al., 2015). The most widely adopted approach for robust airfoil design is to find the deterministic shape parameters to minimize the sum of the mean drag coefficient and the standard deviation of the drag coefficient subject to constraints on the mean lift coefficient and the airfoil thickness for a range of uncertain operational Mach numbers.

In this work, the problem is formulated using the conventional approach as follows. For the uncertain Mach number M_∞, find the deterministic airfoil shape parameters $\mathbf{x}$ to minimize (Shah et al., 2015)

$$f(\mathbf{x}) = \mu_{Cd}(\mathbf{x}) + \sigma_{Cd}(\mathbf{x}), \tag{1}$$

subject to

$$h(\mathbf{x}) = \mu_{Cl}^{\ *} - \mu_{Cl}(\mathbf{x}) = 0, \tag{2}$$

and

$$g_j(\mathbf{x}) = t_j^* - t_j(\mathbf{x}) \le 0, \tag{3}$$

where μ_{Cd} is the mean drag coefficient, σ_{Cd} is the standard deviation of the drag coefficient, μ_{Cl} is the mean lift coefficient, μ_{Cl}^* the required mean lift coefficient, t_j is the airfoil thickness at location j, with $j = 1$ to m, and t_j^* is the minimum airfoil thickness at location j. The airfoil shape parameters have the upper and lower bounds $\mathbf{u}$ and $\mathbf{l}$, respectively, i.e., $\mathbf{x} \in [\mathbf{l},\mathbf{u}]$. It is assumed that the uncertain Mach number is distributed uniformly within the range $M_\infty \in [M_{\infty,l}, M_{\infty,u}]$, where $M_{\infty,u}$, and $M_{\infty,l}$ are the upper and lower bounds on the Mach number, respectively.

2.2 Formulation by Utility Functions

An alternative formulation to the conventional approach of (1)-(3) is investigated. In particular, airfoil design under uncertainty is formulated based on utility theory (Neumann and Morgenstern, 1947). Utility theory translates a range of targeted responses, due to uncertainty in operational parameters, at a specific design point, using a specific function, called the utility function, to metrics used for design comparison. Within the utility function, one can change a parameter to obtain a given risk preference of the designer-called risk aversion (or risk avoiding), risk neutral, and risk loving (risk taking) of the targeted function. These three types of risk preferences are shown graphically in Fig. 1.

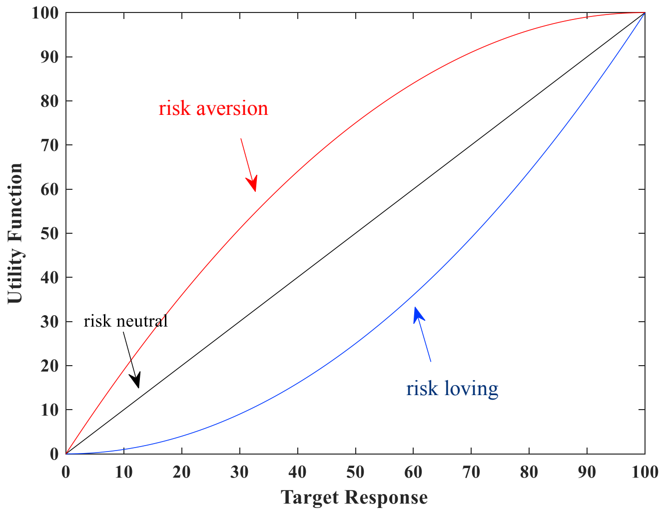


Figure 1: Utility functions for a given range of target responses with different risk preferences of the designer.

A commonly used utility function U is written as

$$U(V) = -\frac{1}{\alpha} e^{-\alpha V}, \tag{4}$$

where, V denotes a scalar, or a vector, with the objective function responses, and α is the constant controlling risk reference. Positive α represents risk aversion, while negative α represents risk loving. Given the probability $P(V_i)$ and utility function response $U(V_i)$ of V_i, the ith objective function response, the expected utility of objective function responses at this specific design point, is defined as

$$E_U = \sum_i U(V_i) \cdot P(V_i) \cdot \tag{5}$$

Another term, called the certainty equivalent, is defined as

$$C = U^{-1}(E_U) \cdot \tag{6}$$

In terms of the airfoil design problem, the problem formulation with utility functions is defined as follows. For the uncertain Mach number M_∞, find the deterministic airfoil shape parameters $\mathbf{x}$ to minimize

$$f(\mathbf{x}) = C(\mathbf{x}), \tag{7}$$

where $C(\mathbf{x})$ is given by (6) with $V = -C_d(\mathbf{x})$, and subject to the same constraints (2) and (3).

3 Stochastic Surrogates by Polynomial Chaos Theory

In this work, the stochastic expansions are generated using non-intrusive polynomial chaos theory. For this purpose, the open-source UQLab (Marelli and Sudret, 2014) is utilized, which is a MATLAB-based uncertainty quantification framework, and contains state-of-the-art, highly optimized algorithms, making it easy to use and deploy. Stochastic surrogates are constructed in UQLab using Polynomial Chaos expansions (PCE) (Xiong et al., 2011). In particular, this work utilizes the least angle regression sparse (LARS) algorithm (Xiong et al., 2010) in combination with the non-intrusive polynomial chaos (NIPC) theory (Shah et al., 2015).

4 Numerical Example

The proposed approach is illustrated on a numerical example involving the robust design of transonic airfoil shapes. This section gives the details of the problem statement, modeling, and optimization results.

4.1 Problem Statement

The goal is to obtain an optimal airfoil shape which could be insensitive with respect to the uncertain characteristics of Mach number. Mach number is of uniform distribution [0.70, 0.75], with target lift coefficient set as 0.5, and Reynolds number of $6.5 \cdot 10^6$, and subjected to thickness constraints at 20% chord and 80% chord locations. The design variable bounds are set as 1±25% of the baseline airfoil RAE 2822. More specifically, the problem is formulated as

$$
\begin{aligned}
&\min_{\mathbf{l} \le \mathbf{x} \le \mathbf{u}} f(\mathbf{x}) \\
&s.t.\ \ C_l^* = 0.5 \\
&\qquad t/c_{x/c=0.2} \ge t/c_{RAE2822,x/c=0.2} \\
&\qquad t/c_{x/c=0.8} \ge t/c_{RAE2822,x/c=0.8}
\end{aligned}
$$

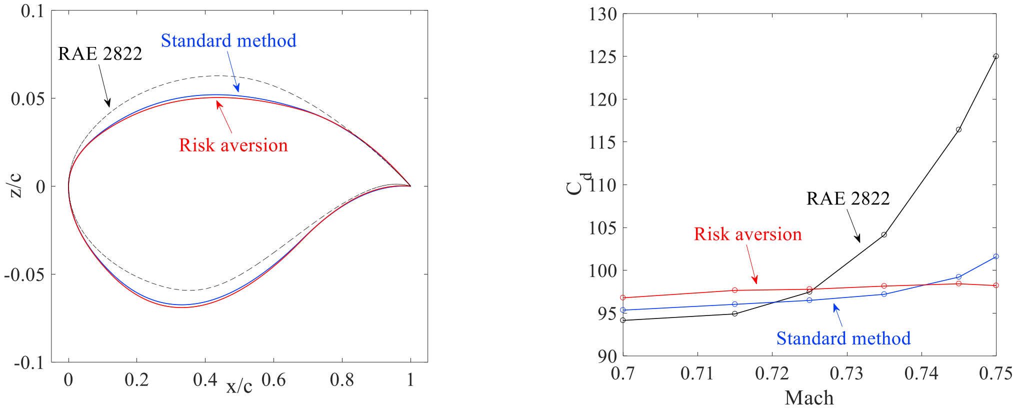
where $f(\mathbf{x})$ is the objective function (described in Sect. 2), $\mathbf{x}$ is the vector of design variables (in this case we use six B-spline control points), C_l is the lift coefficient, and t/c is the thickness to chord ratio.

4.2 Computational Modeling

In this work, the Stanford University Unstructured (SU2) is used as the flow solver (Palacios et al., 2013). SU2 is an open-source integrated computational environment for multi-physics simulation and design. The Reynolds-Averaged Navier-Stokes (RANS) equations are solved using the Spalart-Allmaras (SA) turbulence model. The computational grid is generated using Pointwise.

4.3 Optimization Results

The baseline and optimized designs are given in Fig. 3. It can be seen that the design based on the utility function with risk aversion is less sensitive in the objective function to the uncertain Mach number than the design obtained by the standard method. Figure 4 shows Mach contour plots of the baseline design and the design optimized by the utility function. There we can see that the optimized design is nearly shock-free throughout the range of Mach numbers, which is the reason for the constant drag coefficient with Mach number. Figures 5, 6, and 7 show the pressure coefficient distributions at several free-stream Mach numbers.



(a) (b)

Figure 3: Baseline and optimized designs: (a) shapes, and (b) drag divergence plot.

5 Conclusion

Utility theory has been integrated with stochastic expansions to form an efficient and rigorously formulated approach for aerodynamic design under uncertainty. The results of an application to the design of airfoils in two-dimensional viscous transonic flow with the Mach number as an uncertain variable show that designs with drag coefficients insensitive to the Mach number can be obtained at a low cost.

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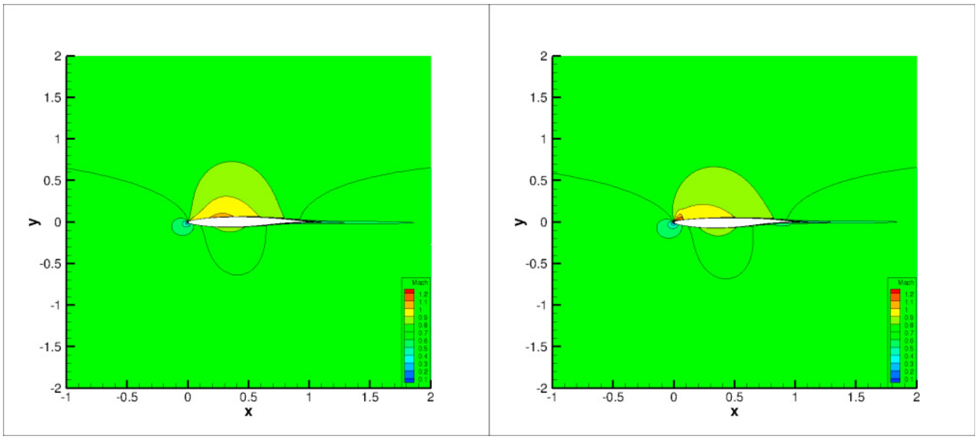
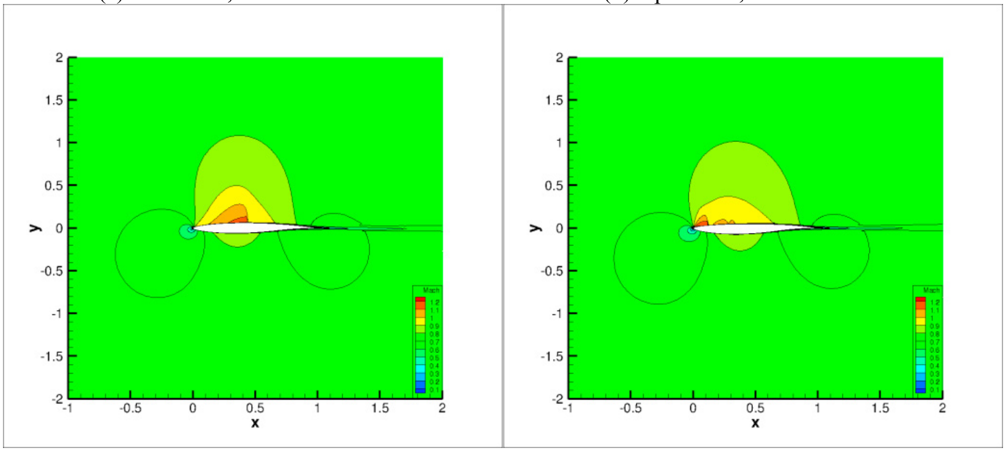
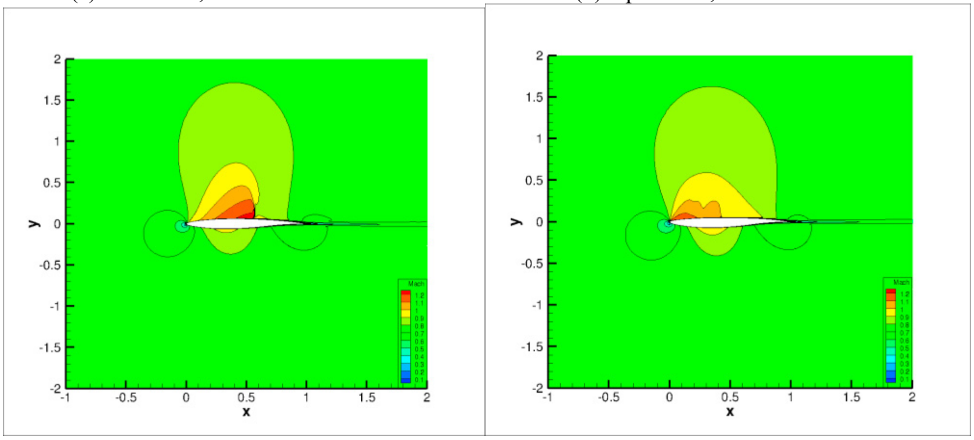


(a) RAE2822, $M_\infty = 0.70$ (b) Optimized, $M_\infty = 0.70$



(c) RAE2822, $M_\infty = 0.725$ (d) Optimized, $M_\infty = 0.725$



(e) RAE2822, $M_\infty = 0.75$ (f) Optimized, $M_\infty = 0.75$

Figure 4: Mach contour plots for the benchmark RAE 2822 airfoil and optimized airfoil (risk aversion) at several free-stream Mach numbers.

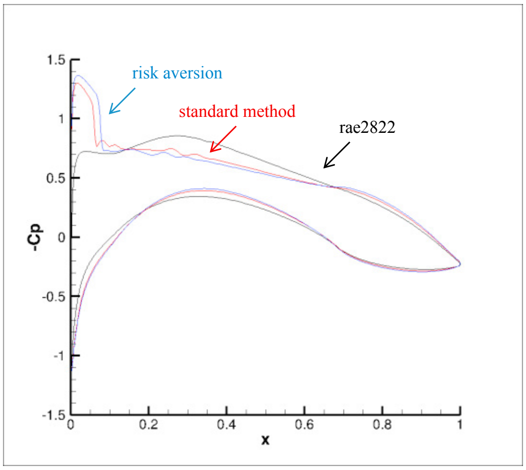


Figure 5: Pressure distributions at M = 0.70 for the RAE 2822 airfoil and airfoils optimized with the standard method, and risk aversion.

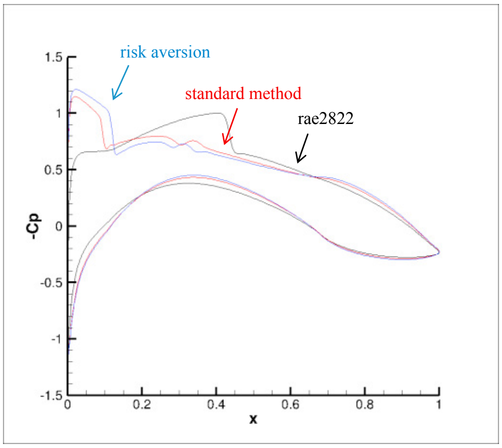


Figure 6: Pressure distributions at M = 0.725 for RAE 2822 airfoil and airfoils optimized with the standard method, and risk aversion.

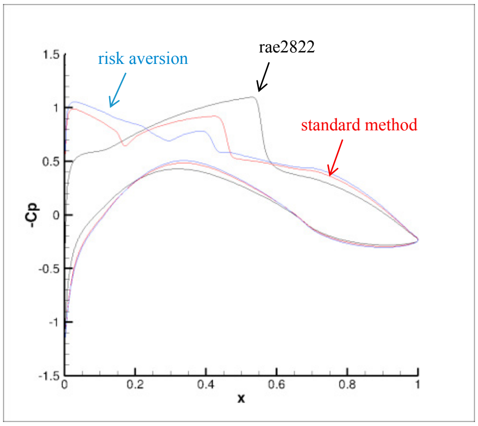


Figure 7: Pressure distributions at M = 0.75 for RAE 2822 airfoil and airfoils optimized with the standard method, and risk aversion.