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Model-Assisted Probability of Detection of Flaws in Aluminum Blocks using Polynomial Chaos Expansions

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Abstract. Probability of detection (POD) is widely used for measuring reliability of nondestructive testing (NDT) systems. Typically, POD is determined experimentally, while it can be enhanced by utilizing physics-based computational models in combination with model-assisted POD (MAPOD) methods. With the development of advanced physics-based methods, such as ultrasonic NDT testing, the empirical information, needed for POD methods, can be reduced. However, performing accurate numerical simulations can be prohibitively time-consuming, especially as part of stochastic analysis. In this work, stochastic surrogate models for computational physics-based measurement simulations are developed for cost savings of MAPOD methods while simultaneously ensuring sufficient accuracy. The stochastic surrogate is used to propagate the random input variables through the physics-based simulation model to obtain the joint probability distribution of the output. The POD curves are then generated based on those results. Here, the stochastic surrogates are constructed using non-intrusive polynomial chaos (NIPC) expansions. In particular, the NIPC methods used are the quadrature, ordinary least-squares (OLS), and least-angle regression sparse (LARS) techniques. The proposed approach is demonstrated on the ultrasonic testing simulation of a flat bottom hole flaw in an aluminum block. The results show that the stochastic surrogates have at least two orders of magnitude faster convergence on the statistics than direct Monte Carlo sampling (MCS). Moreover, the evaluation of the stochastic surrogate models is over three orders of magnitude faster than the underlying simulation model for this case, which is the UTSim2 model.

NOMENCLATURE

a_{90}	=	90% probability of detection
$a_{90/95}$	=	90% probability of detection, corresponding with 95% confidence bounds
$\mathbf{A}$	=	matrix containing polynomial basis functions
C	=	beam diffraction correction
E	=	expectation value
fx	=	joint probability density function of random vector **X**
i	=	basis function index of stochastic expansions
j	=	index of random design variables
M	=	map between independent variables and dependent outputs
M^{PC}	=	approximate map by polynomial chaos expansions
n	=	total number of random variables
N	=	total number of sample points
NI	=	current number of iterations
NI_{max}	=	maximum number of iterations
p	=	required order of polynomial chaos expansions
P	=	total number of sample points needed for truncated polynomial chaos expansions
P_a	=	beam propagation and attenuation
T	=	fluid-solid transmission coefficient
R	=	joint-distributed output
x	=	independent component

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$\mathbf{X}$	=	random vector containing independent components
Y	=	response from actual model
Y^{PC}	=	prediction from polynomial chaos expansions
$\hat{\boldsymbol{\alpha}}$	=	approximated coefficient vector of stochastic expansions
α	=	coefficient of stochastic expansions
β	=	system efficiency factor of non-destructive evaluation
ε_{LOO}	=	leave-one-out error
ε_T	=	error threshold
ω	=	frequency in radians per second
μ^{PC}	=	mean value obtained from polynomial chaos expansions
σ^{PC}	=	standard deviation from polynomial chaos expansions
$\mathbf{\Psi}$	=	multivariate polynomial basis
d$\mathbf{\Gamma}$	=	spectrum of the incident field
Abbreviations		
CFD	=	Computational Fluid Dynamics
FBH	=	Flab Bottom Hole
LARS	=	Least Angle Regression Sparse
LOO	=	Leave-One-Out error
MAPOD	=	Model-Assisted Probability of Detection
MCS	=	Monte Carlo Sampling
NDT	=	Non-Destructive Testing
OLS	=	Ordinary Least Squares
PCE	=	Polynomial Chaos Expansions
POD	=	Probability of Detection
RMSE	=	Root Mean Squared Error
UQ	=	Uncertain Quantification

INTRODUCTION

Uncertainty quantification (UQ) [1,2] is the science of quantitative characterization and reduction of uncertainties in applications. It tries to determine how likely certain outcomes are if some aspects of the system are not exactly known, bridging a more reliable and robust application from theoretical and experimental models to the practical world. However, the computational cost of the widely used Monte Carlo Sampling (MCS) method [3], is often unaffordable, when calculating necessary statistics, even though MCS has the best accuracy level among all methods. Polynomial Chaos expansion (PCE) [4,5], a type of non-intrusive stochastic expansion method, is a very efficient and effective alternative computational model for uncertain quantification.

This work introduces PCE into the area of nondestructive testing (NDT) [6] for the probability of detection (POD) analysis. POD is widely used for measuring reliability of NDT systems. Typically, POD is determined experimentally, although it can also be enhanced by utilizing physics-based computational models, which is called model-assisted POD (MAPOD) methods. In this paper, a state-of-the-art PCE method, containing adaptive-degree stochastic basis [7] and Least-Angle Regression Sparse (LARS) method [8,9] is utilized in lieu of MCS in the MAPOD analysis. The approach is demonstrated on the MAPOD analysis of flat-bottom-hole flaws in an aluminum block using an ultrasonic testing measurement model. The results show that the PCE requires significantly fewer samples than the MCS approach.

The paper is organized as follows. Section II gives a description of the MAPOD method. Section III describes the methods for statistical coefficients calculation and state-of-the-art PCE techniques used in this work. Section IV gives the results on the test case. Finally, the paper ends with conclusions.

MODEL-ASSISTED PROBABILITY OF DETECTION

POD [10] is essentially the quantification of inspection capability starting from the distributions of variability, and describes its accuracy with confidence bounds, also known as uncertain bounds. In many cases, the final product of a POD curve is the flaw size, a, for which there is a 90% probability of detection. This flaw size is denoted a_{90}. The 95% upper confidence bound on a_{90} is denoted as $a_{90/95}$.

POD is important for quantifying the efficacy of inspection in components designed and used in accordance with damage tolerant concepts. The POD is typically determined through experiments that are both time-consuming and costly. This motivated the MAPOD methods with the aim for reducing the number of experimental sample points by introducing insights from controlled experiments using information from physics-based simulations [11,12]. However, when it comes to a large amount of simulations, especially when containing statistical uncertainty in random inputs and exploring the joint distributed statistical moments (Fig. 1), performing MAPOD analyses with physics-based simulations can be impractical in a reasonable timeframe. This work addresses the issue of the computational expense by using stochastic surrogate models in lieu of MCS within the MAPOD process (Fig. 1).

POLYNOMIAL CHAOS EXPANSIONS

In this section, the general formulation of PCEs is presented along with the basis-adaptive technique, methods for calculating coefficients, and calculation of statistical moments.

General Formulation

PCE is one type of a stochastic expansion, initially developed by Wiener [13], and applied to uncertainty quantification by Ghanem and Spanos [14, 15]. It has the general formulation of

$$Y = M(\mathbf{X}) = \sum_{i=1}^{\infty} \alpha_i \mathbf{\Psi}_i(\mathbf{X}), \tag{1}$$

where, $\mathbf{X} \in \mathbb{R}^M$ is a vector with random independent components, described by a probability density function $f_{\mathbf{X}}$, $Y \equiv M(\mathbf{X})$ is a map of $\mathbf{X}$, i is the index of ith polynomial term, $\mathbf{\Psi}$ is multivariate polynomial basis, and α is corresponding coefficient of basis function. In practice, the total number of sample points needed does not have to be infinite; instead, a truncated form of the PCE is used

$$M(\mathbf{X}) \approx M^{PC}(\mathbf{X}) = \sum_{i=1}^{P} \alpha_i \mathbf{\Psi}_i(\mathbf{X}), \tag{2}$$

where, $M^{PC}(\mathbf{X})$ is the approximate truncated PCE model, P is the total number of sample points needed, which can be calculated as

$$P = \frac{(p+n)!}{p!n!}, \tag{3}$$

where, p is the required order of PCE, and n is the total number of random variables.

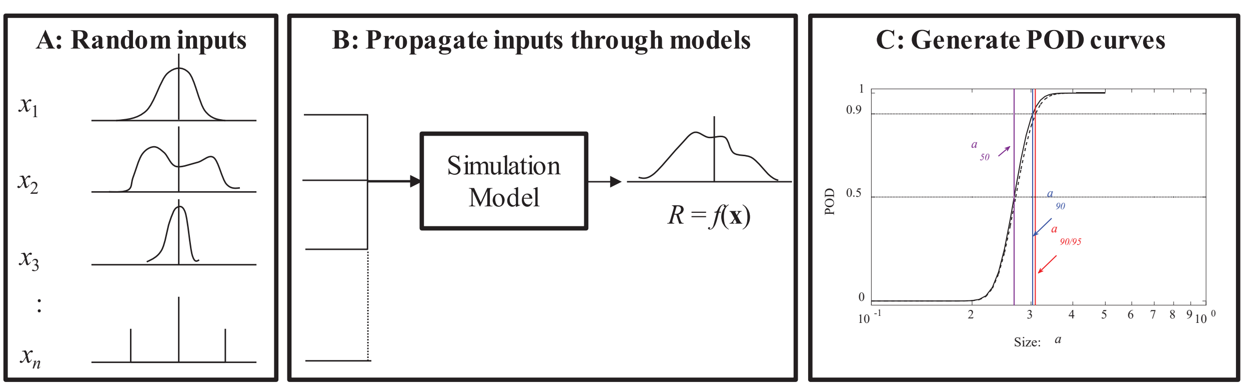


FIGURE 1. Flowchart showing the main elements of model-assisted probability of detection.

Basis-Adaptive PCE

The basis-adaptive technique [7, 8] is used for determining the optimal degree of the PCE from available data. It starts from low-order PCE, which needs less sample points, gradually increases the degree of the polynomial and calculates corresponding residual. Once the residual is small enough, the iteration is stopped. A commonly used iteration-based basis-adaptive method has the framework shown in Fig. 2.

In the algorithm of Fig. 2, ε_{LOO} is the leave-one-out (LOO) error, which is defined as the norm of difference between model response and prediction from PCE, or

$$\varepsilon_{LOO} = \frac{1}{N}\sum_{i=1}^{N}(Y_i - Y_i^{PC})^2, \tag{4}$$

where, Y is the model response, Y^{PC} is the prediction from PCE.

Solving for Coefficients

Since a polynomial basis has the characteristics of orthonormality, the equation can be solved by taking the expectation of (1) multiplied by Ψ_j, or

$$\alpha_i = E[\Psi_i(\mathbf{X}) \cdot M(\mathbf{X})]. \tag{5}$$

However, in this work the standard method, called the Gaussian quadrature method [16], is applied with the coefficients calculated as

$$\alpha_i = \int M(x)\Psi_i(x) f_{\mathbf{X}}(x)dx \approx \sum_{k=1}^{N}\omega^k M(x^k)\Psi_i(x^k), \tag{6}$$

where, the weights ω^k and quadrature points x^k are derived from Lagrange polynomial interpolation. This method guarantees exactness of the evaluation of integrals, while the total number of sample points needed increase rapidly for high-dimensional design variables. It has the relationship with PCE order and number of input variables as $N = (p+1)^n$, which is called curse of dimensionality.

Another method is to treat the model response as a summation of PCE prediction and corresponding residual

$$M(\mathbf{X}) = M^{PC}(\mathbf{X}) + \varepsilon_P = \sum_{i=1}^{P}\alpha_i \boldsymbol{\Psi}_i(\mathbf{X}) + \varepsilon_P \equiv \boldsymbol{\alpha}^T \boldsymbol{\Psi}(\mathbf{X}) + \varepsilon_P, \tag{7}$$

where, ε_p is the residual between $M(\mathbf{X})$ and $M^{PC}(\mathbf{X})$, which is to be minimized in least-squares methods.

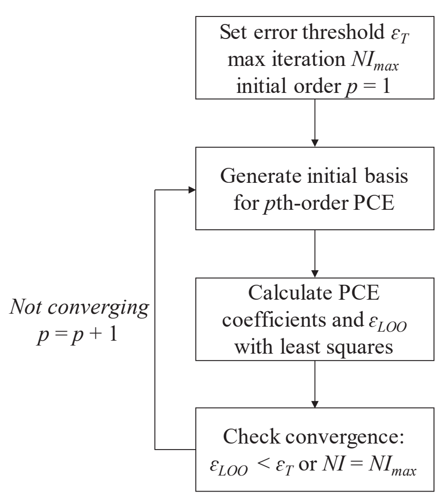


FIGURE 2. Framework of the basis-adaptive technique (adapted from [7, 8]).

Then the initial problem can be converted to a least-squares minimization problem [17]

$$\hat{\boldsymbol{\alpha}} = \arg\min E[\boldsymbol{\alpha}^T \Psi(\mathbf{X}) - M(\mathbf{X})]. \tag{8}$$

The first method, used for solving this problem above and applied in this work, is called ordinary least-squares (OLS) [18], with the coefficients obtained by solving

$$\hat{\boldsymbol{\alpha}} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{Y}, \tag{9}$$

where **Y** is vector of model response, $A_{ji} = \Psi_i(x^j)$, $j = 1, \ldots, n$, $i = 1, \ldots, P$.

The second method used for solving (8) is the least-angle regression sparse (LARS) [8, 9], a type of sparse method, adding one more regularization term to favor low-rank solution.

$$\hat{\boldsymbol{\alpha}} = \arg\min E[\boldsymbol{\alpha}^T \boldsymbol{\psi}(\mathbf{x}) - M(\mathbf{x})] + \lambda \| \boldsymbol{\alpha} \|_1, \tag{10}$$

where λ is a penalty factor, $\|\boldsymbol{\alpha}\|_1$ is L1 norm of the coefficients of PCE

Calculation of Statistical Moments

The PCE basis has the orthonormal characteristics, so the first two statistical moments can be obtained from coefficients. The mean value of PCE is

$$\mu^{PC} = E[M^{PC}(\mathbf{X})] = \alpha_1, \tag{11}$$

where α_1 is the coefficient of the constant basis term $\Psi_1 = 1$. The standard deviation of PCE is

$$\sigma^{PC} = E[(M^{PC}(\mathbf{X}) - \mu^{PC})^2] = \sum_{i=2}^{P} \alpha_i^2, \tag{12}$$

where it is the summation on coefficients of non-constant basis terms only.

NUMERICAL CASE STUDY

In this section, a typical model, the Black Beauty (Fig. 3) is tested. The ultrasonic simulation utilizes a Gaussian-Hermite beam model [19], modeling ultrasonic beam with Gaussian-weighted Hermite polynomials coupled with a paraxial approximation, working as part of Thompson-Gray measurement model [20]. There are three uncertain variables in the test case, namely, the probe angle as $N(0, 0.5)$ deg., the probe F number as $U(7, 9)$, and the flaw size as $U(0.5, 8)$. MAPOD analysis is performed using MCS, and the NIPC methods, quadrature, OLS, and LARS. The methods are compared based on the convergence on statistics and the cost of the prediction.

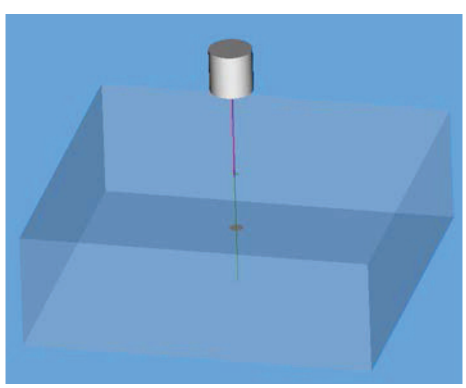


FIGURE 3. The Black Beauty ultrasonic model.

The root mean squared error (RMSE) is used to validate generated surrogate model. The RMSE is defined as

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(Y_i^{PC} - Y_i)^2}{n}}, \tag{13}$$

where n is the total number of testing points.

In this case, RMSE goes close to zero with very few sample points (Fig. 4). In this two-uncertain-variable case, it takes 5,000 samples for MC to get convergence on mean value and standard deviation, while it takes 10 samples for Quadrature, 20 samples for OLS/LARS PCE method (Fig. 5). Reasonable POD curves are obtained (Fig. 6).

In addition to the case shown above, the variation of the computation cost with the number of uncertain variables is investigated. For this purpose, two additional cases are considered: (1) one uncertain variable: the probe angle as N(0, 0.5) deg., (2) three uncertain variables: the probe angle as N(0, 0.5) deg., the probe F number as U(7, 9) and the flaw size as U(0.5, 8) mm. The sampling cost for convergence is plotted in Fig. 7. It is clear that the MCS method always needs much more sample points, about 2 to 3 orders of magnitude more than the PCE methods, to reach converged statistical moments. When it comes higher dimension, sample points needed by MCS increase significantly, while the PCE still only needs about 100 samples, which is very efficient for the use of exploring statistics. When it comes to the utilization of predicting response, PCE methods have very good accuracy, based on RMSE results. UTSim2 needs around 0.22 seconds per simulation, while the PCE model (almost the same efficiency between coarse and fine PCE model) needs around 0.02 seconds per 100 evaluations. The simulation time ratio in this case is around 1,100.

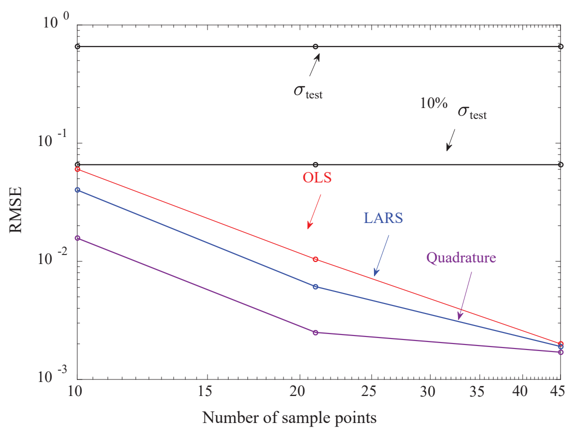
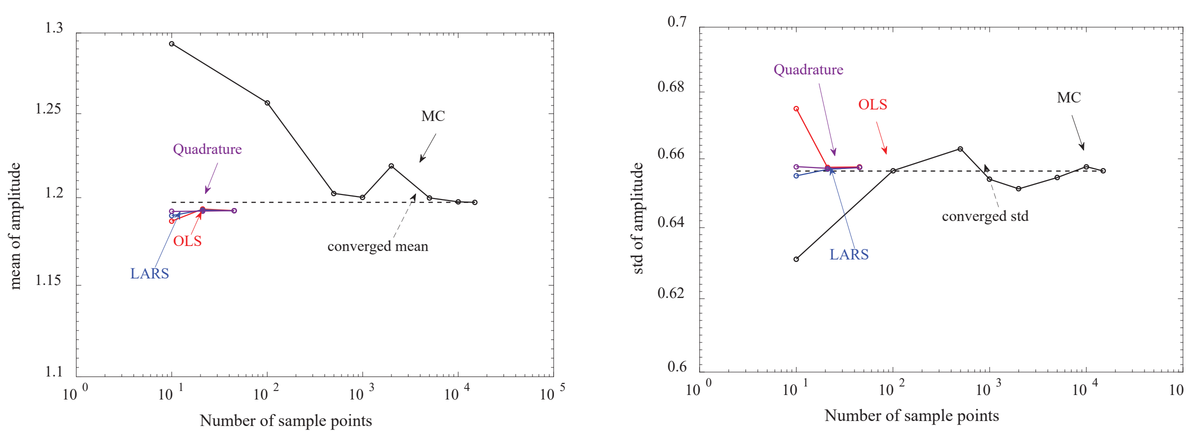


FIGURE 4. RMSE tests on the PCE methods.



(a) (b)

FIGURE 5. Convergence comparison on statistics with MCs and PCE methods: (a) mean; (b) standard deviation.

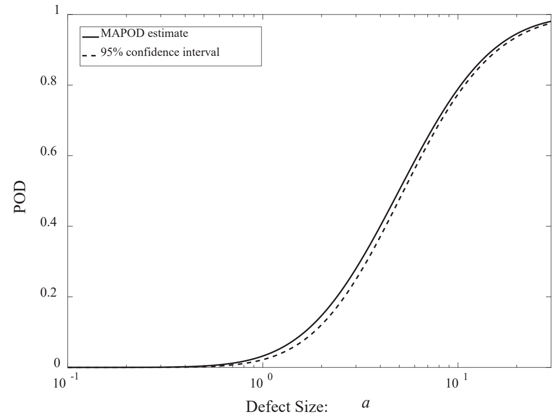


FIGURE 6. POD curves for test case.

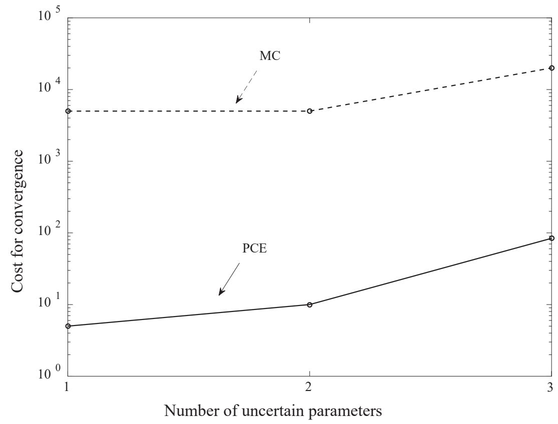


FIGURE 7. Sample cost versus number of uncertain variables.

CONCLUSION

In this work, MAPOD analysis of three ultrasonic testing simulations cases, containing one, two, and three uncertain variables, respectively, are tested on the Black Beauty flaw in an aluminum block, using the direct MCS method and the proposed PCE methods. In all the cases tested, the PCE models outperform the MCS method on both the convergence of the statistics and on the prediction efficiency. Among the PCE methods, the quadrature method has the best performance for the case with one uncertain variable, while the LARS method outperforms all the methods for the cases with two and three variables. Finally, reasonable POD curves are obtained with all the PCE surrogate models. Future work will consider problems of higher complexity in terms of the physics-based simulations and higher input space dimensionality, as well as other modalities.

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