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Fast Uncertainty Propagation of Ultrasonic Testing Simulations for MAPOD and Sensitivity Analysis

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***Abstract*—Model-assisted probability of detection (MAPOD) and sensitivity analysis (SA) are widely used for measuring the reliability of nondestructive testing (NDT) systems, such as ultrasonic testing (UT), and understanding the effects of uncertainty parameters. In this work, a stochastic expansion-based metamodel is used in lieu of the physics-based NDT simulation model for efficient uncertainty propagation while keeping satisfactory accuracy. The proposed stochastic metamodeling approach is demonstrated for MAPOD and SA on a benchmark case for UT simulations on a fused quartz block with a spherically-void defect. The proposed approach is compared with direct Monte Carlo sampling (MCS), and MCS with Kriging metamodels. The results indicate that around one order of magnitude reduction in the number of model evaluations required for MAPOD analysis can be obtained. Moreover, the results indicate around two orders of magnitude reduction of the number of model evaluations for the convergence of the statistical moments and obtaining the problem sensitivities.**

Keywords—MAPOD; sensitivity analysis; nondestructive testing; NDT; stochastic metamodeling; MCS; Kriging metamodels

I. INTRODUCTION

The concept of probability of detection (POD) [1] was initially proposed for experimentally measuring the reliability of nondestructive testing (NDT) systems, such as ultrasonic testing (UT) and eddy current testing. Sensitivity analysis (SA) [2, 3] on the other hand is mainly used for quantifying the effects of each uncertainty parameter has on the model response. With the development of accurate physics-based NDT models, such as the full wave ultrasonic model, model-assisted POD (MAPOD) [4, 5] and SA can be utilized for reducing the amount of experimental information required in traditional POD analysis.

Current state-of-the-art methods for both MAPOD and SA rely heavily on the Monte Carlo sampling (MCS) method on the true physics-based model [4, 5]. The MCS-based uncertainty propagation (UP) is computationally costly because it requires large numbers of model evaluations, and it can become impractical when each model evaluation is time-consuming. In order to alleviate the computational burden, metamodeling approaches, such as Kriging interpolation metamodeling, and polynomial chaos expansion (PCE) methods [6, 7], have recently been introduced to the NDT area.

In this work, we apply the PCE-based metamodel with the least-angle regression sparse (LARS) technique [7] for efficient MAPOD and SA of NDT systems. The approach is demonstrated on a benchmark case for ultrasonic testing of a fused quartz block with a spherically-void defect. The results, including POD analysis, convergence on statistical moments and sensitivity analysis, are compared against state of the art approaches in the NDT area, i.e., MCS of the true physics-based model, MCS with Kriging interpolation metamodels, and PCE-based metamodel with the ordinary least squares (OLS) method [6].

This paper is organized as follows. Section II describes NDT, MAPOD, and sensitivity analysis using Sobol' indices. Section III gives the details of Kriging metamodeling, and the PCE metamodels, including the proposed PCE-based LARS metamodel. Section IV gives the results of the numerical results of demonstration case. The paper ends with conclusion.

II. MAPOD AND SOBOL' INDICES

A. Nondestructive Testing

Nondestructive testing (NDT) refers to the process of inspecting, testing, or evaluating materials, components or assemblies for discontinuities, or differences in characteristics without destroying the serviceability of the part or system. Modern NDT systems are widely used for manufacturing, fabrication, and in-service inspections to ensure product integrity and reliability, in order to control the manufacturing process, reduce production costs, and maintain uniform quality level.

Varieties of NDT techniques have been developed, such as ultrasonic, electromagnetic, and radiographic. In this work, the focus is ultrasonic testing, which has the same principle with naval SONAR. Ultra-high frequency sound is introduced into the part or system being inspected, and if the sound hits a material with a different acoustic impedance (density or acoustic velocity), some of the sound will reflect back to the sending unit and can be presented on a visual display. Based on the strength of the signal, the defect size or degree of damage can be characterized.

B. Model-Assisted Probability of Detection

Model-assisted probability of detection (MAPOD) is developed for reducing experimental budgets for measurement of the reliability of NDT systems. Commonly used terminologies in probability of detection (POD) analysis are "90% POD" and "90% POD with 95% confidence interval", which are written as a_{90} and $a_{90/95}$, respectively [3].

In this work, POD is calculated from the correlation of "$\hat{a}$ vs. a" data, where $\hat{a}$ is the model response and a is the defect size. Based on experimental observations a log-log scale relationship between $\hat{a}$ and a is modeled, or

$$\ln \hat{a} = \beta_0 + \beta_1 \ln a + \delta, \tag{1}$$

where the coefficients β_0 and β_1 can be determined by the maximum likelihood method, and the δ has a Gaussian distribution with zero mean and standard deviation σ_δ, Gaussian (0, σ_δ). This standard deviation can be determined by the residuals of the observed data.

The POD can be obtained as the probability that the obtained ultrasonic signal lies above arbitrary user-defined threshold $\hat{a}_{threshold}$, or

$$POD(a) = 1 - \Phi\left[\frac{\ln \hat{a}_{threshold} - (\beta_0 + \beta_1 \ln a)}{\sigma_\delta}\right], \tag{2}$$

where Φ is the standard normal distribution function.

From the equation above, it is straightforward to obtain

$$POD(a) = \Phi\left[\frac{\ln a - \dfrac{\ln \hat{a}_{threshold} - \beta_0}{\beta_1}}{\dfrac{\sigma_\delta}{\beta_1}}\right], \tag{3}$$

which is the cumulative density function of a log-normal distribution with mean μ and standard deviation σ given by

$$\mu = \frac{\ln \hat{a}_{threshold} - \beta_0}{\beta_1}, \tag{4}$$

$$\sigma = \frac{\sigma_\delta}{\beta_1}, \tag{5}$$

where the parameters β_0, β_1, and σ_δ can be obtained by maximum likelihood method.

C. Sobol' Indices

Sobol' indices (also known as variance-based sensitivity analysis) work as a global sensitivity analysis approach which is popularly used and proved to be robust, interpretable and efficient in various areas including NDT field [2]. It decomposes model response as

$$Y = f_0 + \sum_{i=1}^{d} f_i(X_i) + \sum_{i<j}^{d} f_{i,j}(X_i, X_j) + \cdots + f_{1,2,\ldots,d}(X_1, X_2, \ldots, X_d), \tag{6}$$

where X is design variable, f_0 is a constant, and f_i is orthogonal function in terms of conditional expected value of X_i.

Further simplification can be made, to make the first-order and total-order Sobol' indices as

$$S_i^{1st} = \frac{Var_{X_i}(E_{\mathbf{X}\sim i}(Y \mid X_i))}{Var(Y)}, \tag{7}$$

$$S_{T_i} = \frac{E_{\mathbf{X}_{\sim i}}(Var_{X_i}(Y \mid \mathbf{X}_{\sim i}))}{Var(Y)} = 1 - \frac{Var_{\mathbf{X}_{\sim i}}(E_{X_i}(Y \mid \mathbf{X}_{\sim i}))}{Var(Y)}, \tag{8}$$

where $\mathbf{X}_{\sim i}$ indicates the set of all variables except X_i.

III. Metamodeling Methods

Metamodeling methods, such as Kriging and polynomial chaos expansions (PCE), have been successfully introduced to NDT area for efficient uncertainty propagation [4-6]. This section describes the generalized Kriging metamodel and PCE integrated with state-of-the-art regression techniques.

A. Kriging interpolation

Kriging interpolation is a type of deterministic metamodel widely used, because it can model highly-nonlinear functions with multiple extremes well. The basic function of Kriging interpolation can be written as the sum of a global trend function, $\mathbf{f}^T(\mathbf{x})\boldsymbol{\beta}$, and a Gaussian random function, $Z(\mathbf{x})$, as follows [8]

$$y(\mathbf{x}) = \mathbf{f}^T(\mathbf{x})\boldsymbol{\beta} + Z(\mathbf{x}), \tag{9}$$

where $\mathbf{x} \in \mathbb{R}$ and $\mathbf{f}^T(\mathbf{x})\boldsymbol{\beta}$ is taken as either constant or low-order polynomials, and $Z(\mathbf{x})$ denotes a stationary random process with zero mean, variance σ2 and nonzero covariance.

From the derivation by Sacks [8], the Kriging predictor for untried $\mathbf{x}$ can be written as

$$\hat{y}(\mathbf{x}) = \beta_0 + \mathbf{r}^T(\mathbf{x})\mathbf{R}^{-1}(y_s - \beta_0 \mathbf{1}), \tag{10}$$

where the generalized least squares estimation of β_0 is

$$\beta_0 = (\mathbf{1}^T \mathbf{R}^{-1} \mathbf{1})^{-1} \mathbf{1}^T \mathbf{R}^{-1} y_s, \tag{11}$$

where $\mathbf{1}$ is a vector filled with ones, and $\mathbf{R}$, $\mathbf{r}$ are the correlation matrix and the correlation vector, respectively.

B. Polynomial Chaos Expansions

Polynomial chaos expansions (PCEs) are stochastic metamodels consisting of orthogonal bases corresponding with the statistical distributions of random inputs. The PCE metamodel has a generalized format of

$$Y = M(\mathbf{X}) = \sum_{i=1}^{\infty} \alpha_i \mathbf{\Psi}_i(\mathbf{X}) \approx \sum_{i=1}^{P} \alpha_i \mathbf{\Psi}_i(\mathbf{X}), \quad (12)$$

where, $\mathbf{X} \in \mathbb{R}^M$ is a vector with M random independent components, described by a probability density function f_X, $Y \equiv M(\mathbf{X})$ is a map of $\mathbf{X}$, i is the index of ith polynomial term, $\mathbf{\Psi}$ is multivariate polynomial basis, α is corresponding coefficient of basis function, and P is the truncated number of polynomial terms calculated as

$$P = \frac{(p+n)!}{p!n!}, \quad (13)$$

where, p is the required order of PCE, and n is the total number of random variables.

Solving for the coefficients $\boldsymbol{\alpha}$ is the core of constructing a PCE model, and is typically formulated as a least-squares minimization problem

$$\hat{\boldsymbol{\alpha}} = \arg\min E[\boldsymbol{\alpha}^T \Psi(\mathbf{X}) - M(\mathbf{X})]. \quad (14)$$

This work refers to this approach as ordinary least-squares (OLS) with the coefficients obtained by solving

$$\hat{\boldsymbol{\alpha}} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{Y}, \quad (15)$$

where $\mathbf{Y}$ is vector of model response, $A_{ji} = \Psi_i(x^j)$, $j = 1, \ldots, n$, $i = 1, \ldots, P$.

The proposed method of this work is the least-angle regression (LARS) method which modifies (14) by adding a regularization term to favor low-rank solution

$$\hat{\boldsymbol{\alpha}} = \arg\min E[\boldsymbol{\alpha}^T \boldsymbol{\psi}(\mathbf{x}) - M(\mathbf{x})] + \lambda \|\boldsymbol{\alpha}\|_1, \quad (16)$$

where λ is a penalty factor, $\|\boldsymbol{\alpha}\|_1$ is L1 norm of $\boldsymbol{\alpha}$.

Comparing (6) and (12), allows the PCE-based models to give the Sobol' indices without additional computational efforts as

$$S_k^{1st} = \sum_{\boldsymbol{\alpha}} \alpha_{1st}^2 / D, \qquad S_k^{Total} = \sum_{\boldsymbol{\alpha}} \alpha_{total}^2 / D, \quad (17)$$

where D is the total variance of model response, α^{1st} is coefficients of terms containing only k^{th} parameter, α^{total} is coefficients of any terms containing k^{th} parameter.

IV. Test Case

The proposed approach is demonstrated on the spherically-void-defect benchmark problem, as shown in Fig. 1. In this work, the physics-based ultrasonic testing model consists of Thompson-Gray model for transmission and reception process, multi-Gaussian beam model, and separation of variable model for the scattering process. The validation of the physics model is shown in Fig. 2, showing satisfactory match with the experimental results.

Three uncertainty parameters are considered: the probe angle (θ), the probe F number (F), and the x location of the transducer (x). Here, the following distributions are assumed: $\theta \sim N(0, 1)$ deg., $F \sim U(13, 15)$, and $x \sim U(0, 1)$ mm.

Comparison of the metamodels, based on the root mean squared error (RMSE) and the normalized RMSE (NRMSE), is shown in Fig. 3. It can be seen that the proposed LARS-based PCE outperforms the traditionally used Kriging interpolation metamodel and the OLS-based PCE method. In particular, to reach 1%σ of the testing points, LARS-based PCE reduces the number of sample points by around one order of magnitude, compared with the state-of-the-art MCS and Kriging.

After constructing the PCE metamodels, the POD curves are calculated as shown in Fig. 4. The Sobol' indices for this case are given in Figs. 5 and 6. The results indicate that the probe angle θ has a significantly larger impact on the model response than the probe F number and probe x location. In fact, the probe F number has an insignificant impact on the model response in this case. Furthermore, the results indicate that there is not much interaction effects of the random input on the model response since the first order and total Sobol' indices are of comparable absolute values for each variable.

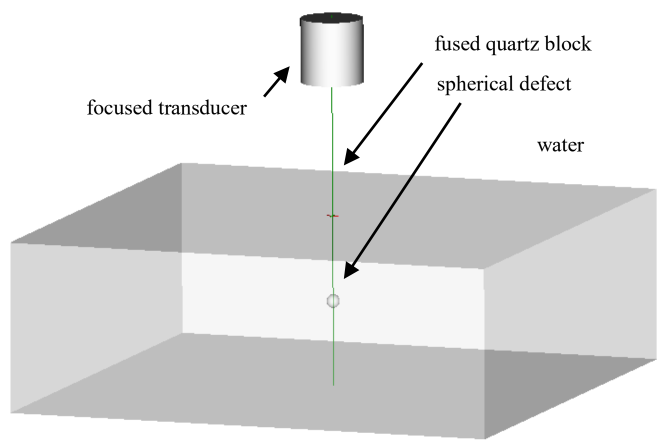

Fig. 1. Spherically void defect under focused transducer.

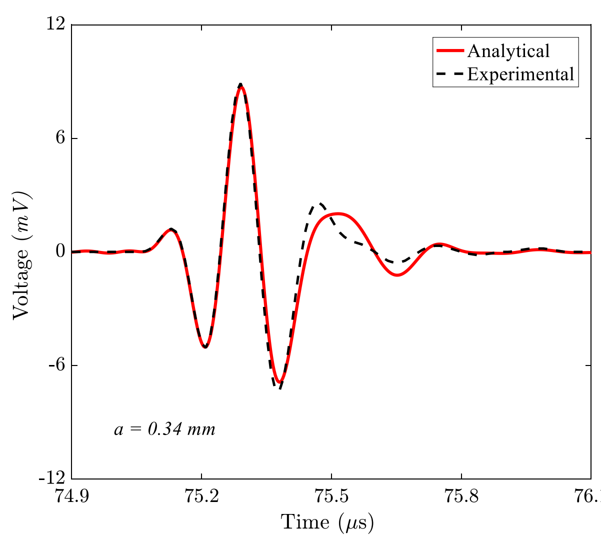

Fig. 2. Validation on physics model (analytical solution).

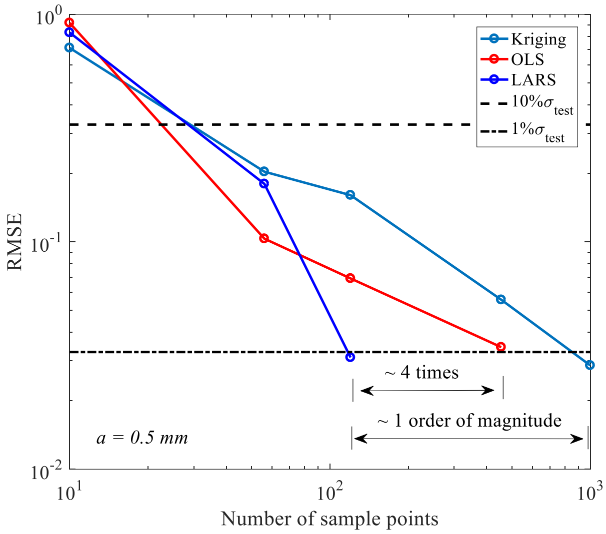

Fig. 3. Validation and comparison of metamodels on RMSE, a = 0.5mm.

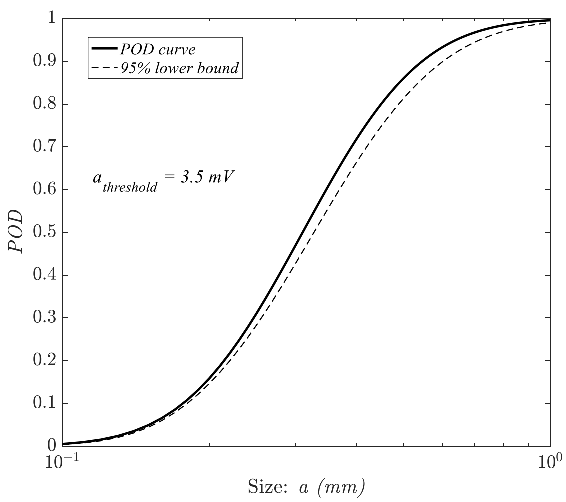

Fig. 4. POD curves generated by LARS PCE.

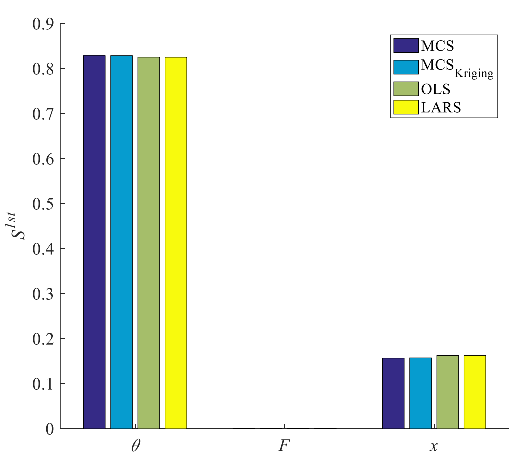

Fig. 5. 1st order Sobol' indices.

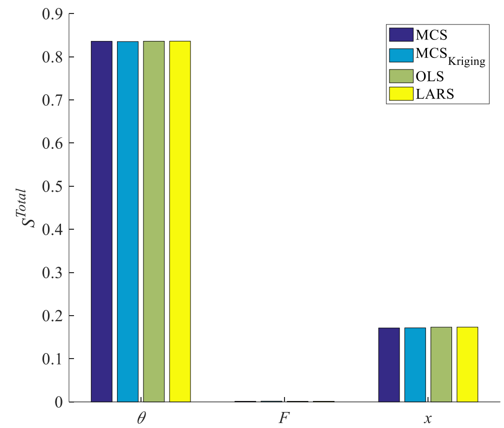

Fig. 6. Total order Sobol' indices.

CONCLUSION

In this paper, the proposed LARS PCE metamodel is demonstrated on a model-assisted probability of detection analysis using the spherically-void-defect benchmark problem under ultrasonic testing. The proposed approach outperforms the current state-of-the-art methods in terms of computational efficiency in terms of uncertainty propagation and sensitivity analysis. Future work will consider industry-relevant problems of higher complexities.

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